$$T_M(n) = \frac{2}{n} \sum_{m=0}^{n-1} T_M(m) + b \cdot n$$

Per Induzione

$\text{I})$ $\quad T_M(n) \leq c \cdot n \log n$

BASE

$n=1 \qquad \cancel{a \leq c \cdot 1 \underbrace{\log 1}_{0}}$

$n=2 \qquad T_M(2) = \cancel{\frac{2}{2}}\left(\overset{a}{\cancel{T_M(0)}} + \overset{a}{\cancel{T_M(1)}}\right) + b \cdot 2$

$$T_M(2) = 2a + 2b \underset{??}{\leq} c \cdot 2 \overset{1}{\cancel{\log 2}}$$

$$\boxed{c \geq a + b}$$

PASSO

$$T_M(n) = \frac{2}{n} \sum_{m=0}^{n-1} T_M(m) + bn$$

$$\underset{\text{Hp Ind}}{\leq} \frac{2c}{n} \underbrace{\sum_{m=0}^{n-1} m \log m}_{\underset{*}{\leq} \frac{1}{2} n^2 \log n - \frac{1}{8} n^2} \overset{\leq\, n^2 \log n}{} + bn \quad \underset{??}{\leq} c\, n \log n$$

$$\leq \frac{2c}{\cancel{n}} \left(\frac{1}{2} n^2 \log n - \frac{1}{8} n^2 \right) + bn$$

$$\leq c\, n \log n \; - \frac{c}{4} n + b n \leq c\, n \log n$$
??

?? ≤ 0

$$b\cancel{x} \leq \frac{c}{4}\cancel{x} \qquad \boxed{c \geq 4b}$$

$$* \quad \sum_{m=\cancel{0}1}^{n-1} m \log m = \sum_{m=0}^{\lfloor n/2 \rfloor} m \log m + \sum_{m=\lfloor \frac{n}{2} \rfloor + 1}^{n-1} m \log m$$

$$\leq \log \frac{n}{2} \sum_{m=0}^{\lfloor n/2 \rfloor} m + \log n \sum_{m=\lfloor \frac{n}{2} \rfloor + 1}^{n-1} m$$

$$= \underbrace{\log n \sum_{m=0}^{\lfloor n/2 \rfloor} m} - \sum_{m=0}^{\lfloor n/2 \rfloor} m + \underbrace{\log n \sum_{m=\lfloor \frac{n}{2} \rfloor + 1}^{n-1} m}$$

$$= \log n \sum_{m=0}^{n-1} m - \sum_{m=0}^{\lfloor n/2 \rfloor} m$$

$$= \log n \cdot \frac{n(n-1)}{2} - \frac{\left(\lfloor \frac{n}{2} \rfloor + 1\right) \cdot \lfloor \frac{n}{2} \rfloor}{2}$$

$$= \frac{1}{2} n^2 \log n - \frac{1}{2} n \log n - \frac{n^2}{8} - \frac{n}{4}$$

$$\leq \frac{1}{2} n^2 \log n - \frac{n^2}{8}$$

$\blacksquare$

$$T_M(n) = \frac{2}{n} \sum_{m=0}^{n-1} T_M(m) + bn$$

$$n T_M(n) = 2 \sum_{m=0}^{n-1} T_M(m) + bn^2$$

$$= 2 T_M(n-1) + 2 \sum_{m=0}^{n-2} T_M(m) + bn^2$$

$$(n-1) T_M(n-1) = 2 \sum_{m=0}^{n-2} T_M(m) + b(n-1)^2$$

$$n T_M(n) - (n-1) T_M(n-1) = 2 T_M(n-1) + bn^2 - b(n-1)^2$$

$$n T_M(n) = (n+1) T_M(n-1) + \cancel{bn^2} - b(n-1)^2$$

$$\cancel{-bn^2} - b + 2bn$$

Divido $n(n+1)$

$$\frac{T_M(n)}{n+1} = \frac{T_M(n-1)}{n} + \cancel{\frac{2bn}{n+1}} + \frac{2b}{n+1} - \frac{b}{n(n+1)}$$

$$\frac{T_M(n)}{n+1} = S(n) = O(\log n) \Longrightarrow T_M(n) = O(\log n) \cdot (n+1) = O(n \log n)$$

$$S(n) \leq S(n-1) + \frac{2b}{n+1}$$

$\frac{2b}{n+1}$

$\frac{2b}{n}$

$\frac{2b}{1}$

$$S(n) = 2b \underbrace{\sum_{i=1}^{n+1} \frac{1}{i}}_{\text{Serie armonica}} = O(\log n)$$