Kinematics

Displacement	$\Delta x = x_f - x_i$
Total displacement	$\Delta x_{\text{Total}} = \sum \Delta x_i$
Average velocity (for constant acceleration)	$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$
Instantaneous velocity	$v(t) = \frac{dx(t)}{dt}$
Average speed	Average speed $= \bar{s} = \frac{\text{Total distance}}{\text{Elapsed time}}$
Instantaneous speed	Instantaneous speed $= \lvert v(t) \rvert$
Average acceleration	$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$
Instantaneous acceleration	$a(t) = \frac{dv(t)}{dt}$
Position from average velocity	$x = x_0 + \bar{v}t$
Average velocity	$\bar{v} = \frac{v_0 + v}{2}$
Velocity from acceleration	$v = v_0 + at$ (constant a)
Position from velocity and acceleration	$x = x_0 + v_0 t + \frac{1}{2}at^2$ (constant a)
Velocity from distance	$v^2 = v_0^2 + 2a(x - x_0)$ (constant a)
Velocity of free fall	$v = v_0 - gt$ (positive upward)
Height of free fall	$y = y_0 + v_0 t - \frac{1}{2}gt^2$
Velocity of free fall from height	$v^2 = v_0^2 - 2g(y - y_0)$
Velocity from acceleration	$v(t) = \int a(t)dt + C_1$
Position from velocity	$x(t) = \int v(t)dt + C_2$
Time of flight	$T_{\text{tof}} = \frac{2(v_0 \sin\theta_0)}{g}$
Trajectory	$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0\cos\theta_0)^2}\right]x^2$
Range	$R = \frac{v_0^2 \sin 2\theta_0}{g}$
Centripetal acceleration	$a_C = \frac{v^2}{r}$

Linear Momentum

Definition of momentum	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
Impulse	$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt$ or $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{ave}}\Delta t$
Impulse-momentum theorem	$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$
Average force from momentum	$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$

Energy

Work done by a force over an infinitesimal displacement	$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \lvert\vec{\mathbf{F}}\rvert\lvert d\vec{\mathbf{r}}\rvert\cos\theta$
Work done by a force acting along a path from A to B	$W_{AB} = \int_{\text{path}AB} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$
Work done by a constant force of kinetic friction	$W_{\text{fr}} = -f_k \lvert l_{AB} \rvert$
Work done going from A to B by Earth's gravity, near its surface	$W_{\text{grav},AB} = -mg(y_B - y_A)$
Work done going from A to B by one-dimensional spring force	$W_{\text{spring},AB} = -\left(\frac{1}{2}k\right)\left(x_B^2 - x_A^2\right)$
Kinetic energy of a non-relativistic particle	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
Work-energy theorem	$W_{\text{net}} = K_B - K_A$
Power as rate of doing work	$P = \frac{dW}{dt}$
Power as the dot product of force and velocity	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$
Difference of potential energy	$\Delta U_{AB} = U_B - U_A = -W_{AB}$
Potential energy with respect to zero of potential energy at $\vec{\mathbf{r}}_0$	$\Delta U = U(\vec{\mathbf{r}}) - U(\vec{\mathbf{r}}_0)$
Gravitational potential energy near Earth's surface	$U(y) = mgy + \text{const.}$
Potential energy for an ideal spring	$U(x) = \frac{1}{2}kx^2 + \text{const.}$

Rotation

Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
Tangential speed	$v_t = r\omega$
Angular acceleration	$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Tangential acceleration	$a_t = r\alpha$
Average angular velocity	$\bar{\omega} = \frac{\omega_0 + \omega_f}{2}$
Angular displacement	$\theta_f = \theta_0 + \bar{\omega}t$

Angular Momentum

Velocity of center of mass of rolling object	$v_{\text{CM}} = R\omega$
Acceleration of center of mass of rolling object	$a_{\text{CM}} = R\alpha$
Displacement of center of mass of rolling object	$d_{\text{CM}} = R\theta$
Acceleration of an object rolling without slipping	$a_{\text{CM}} = \frac{mg\sin\theta}{m + (I_{\text{CM}}/r^2)}$
Angular momentum	$\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Derivative of angular momentum equals torque	$\frac{d\vec{\mathbf{l}}}{dt} = \sum \vec{\boldsymbol{\tau}}$
Angular momentum of a rotating rigid body	$L = I\omega$

Dynamics

Magnitude of static friction	$f_s \le \mu_s N$
Magnitude of kinetic friction	$f_k = \mu_k N$
Centripetal force	$F_c = m\frac{v^2}{r}$ or $F_c = mr\omega^2$
Ideal angle of a banked curve	$\tan\theta = \frac{v^2}{rg}$
Drag force	$F_D = \frac{1}{2}C\rho Av^2$

Gravitation

Newton's law of gravitation	$\vec{\mathbf{F}}_{12} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12}$
Acceleration due to gravity at the surface of Earth	$g = G\frac{M_E}{r^2}$
Gravitational potential energy beyond Earth	$U = -\frac{GM_E m}{r}$
Escape velocity	$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$
Orbital speed	$v_{\text{orbit}} = \sqrt{\frac{GM_E}{r}}$
Orbital period	$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$
Energy in circular orbit	$E = K + U = -\frac{GmM_E}{2r}$
Kepler's third law	$T^2 = \frac{4\pi^2}{GM}a^3$

Scalar Product

$\vec{A} * \vec{B} = AB\cos\varphi = B(A\cos\varphi) = A(B\cos\varphi)$

$\vec{A} * \vec{B} = A_x B_x + A_y B_y + A_z B_z$

(negative when $90 < \varphi \le 180$)

(positive when $0 \le \varphi \le 90$)

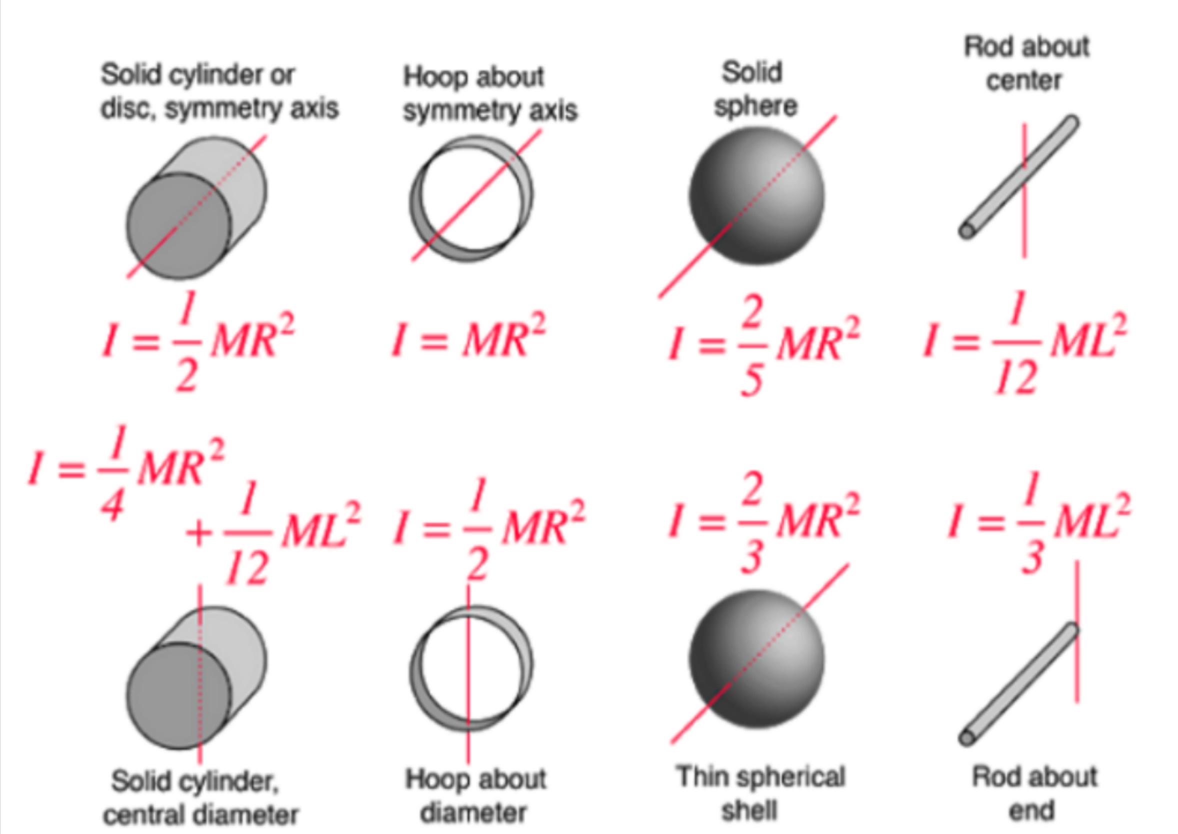
Cross Product

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} = AB\sin\varphi$

Right hand rule (point fingers along first vector and curl hand in towards next vector)

(Thumb shows direction of resultant force)

$\vec{C} = \vec{A} \times \vec{B} = (AyBz-AzBy)\hat{\imath} + (AzBx-AxBz)\hat{\jmath} + (AxBy-AyBx)\hat{k}$



Oscillations

Relationship between frequency and period	$f = \frac{1}{T}$
Position in SHM with $\phi = 0.00$	$x(t) = A\cos(\omega t)$
General position in SHM	$x(t) = A\cos(\omega t + \phi)$
General velocity in SHM	$v(t) = -A\omega\sin(\omega t + \phi)$
General acceleration in SHM	$a(t) = -A\omega^2\cos(\omega t + \phi)$
Maximum displacement (amplitude) of SHM	$x_{\max} = A$
Maximum velocity of SHM	$\lvert v_{\max}\rvert = A\omega$
Maximum acceleration of SHM	$\lvert a_{\max}\rvert = A\omega^2$
Angular frequency of a mass-spring system in SHM	$\omega = \sqrt{\frac{k}{m}}$
Period of a mass-spring system in SHM	$T = 2\pi\sqrt{\frac{m}{k}}$
Frequency of a mass-spring system in SHM	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
Energy in a mass-spring system in SHM	$E_{\text{Total}} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$
The velocity of the mass in a spring-mass system in SHM	$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$
The *x*-component of the radius of a rotating disk	$x(t) = A\cos(\omega t + \phi)$
The *x*-component of the velocity of the edge of a rotating disk	$v(t) = -v_{\max}\sin(\omega t + \phi)$
The *x*-component of the acceleration of the edge of a rotating disk	$a(t) = -a_{\max}\cos(\omega t + \phi)$
Force equation for a simple pendulum	$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$
Angular frequency for a simple pendulum	$\omega = \sqrt{\frac{g}{L}}$
Period of a simple pendulum	$T = 2\pi\sqrt{\frac{L}{g}}$
Angular frequency of a physical pendulum	$\omega = \sqrt{\frac{mgL}{I}}$
Period of a physical pendulum	$T = 2\pi\sqrt{\frac{I}{mgL}}$
Period of a torsional pendulum	$T = 2\pi\sqrt{\frac{I}{\kappa}}$
Newton's second law for harmonic motion	$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$
Solution for underdamped harmonic motion	$x(t) = A_0 e^{-\frac{b}{2m}t}\cos(\omega t + \phi)$
Natural angular frequency of a mass-spring system	$\omega_0 = \sqrt{\frac{k}{m}}$
Angular frequency of underdamped harmonic motion	$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$

Waves

Wave speed	$v = \frac{\lambda}{T} = \lambda f$
Linear mass density	$\mu = \frac{\text{mass of the string}}{\text{length of the string}}$
Speed of a wave or pulse on a string under tension	$\lvert v\rvert = \sqrt{\frac{F_T}{\mu}}$
Speed of a compression wave in a fluid	$v = \sqrt{\frac{B}{\rho}}$
Resultant wave from superposition of two sinusoidal waves that are identical except for a phase shift	$y_R(x, t) = \left[2A\cos\left(\frac{\phi}{2}\right)\right]\sin\left(kx - \omega t + \frac{\phi}{2}\right)$
Wave number	$k \equiv \frac{2\pi}{\lambda}$
Wave speed	$v = \frac{\omega}{k}$
A periodic wave	$y(x, t) = A\sin(kx \mp \omega t + \phi)$
Phase of a wave	$kx \mp \omega t + \phi$
The linear wave equation	$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v_w^2}\frac{\partial^2 y(x,t)}{\partial t^2}$
Power averaged over a wavelength	$P_{\text{ave}} = \frac{E_\lambda}{T} = \frac{1}{2}\mu A^2\omega^2\frac{\lambda}{T} = \frac{1}{2}\mu A^2\omega^2 v$
Intensity	$I = \frac{P}{A}$
Intensity for a spherical wave	$I = \frac{P}{4\pi r^2}$
Equation of a standing wave	$y(x, t) = [2A\sin(kx)]\cos(\omega t)$
Wavelength for symmetric boundary conditions	$\lambda_n = \frac{2}{n}L, \quad n = 1, 2, 3, 4, 5...$
Frequency for symmetric boundary conditions	$f_n = n\frac{v}{2L} = nf_1, \quad n = 1, 2, 3, 4, 5...$

Fundamental constants and Astronomical Data:

g	9.8 m/s^2
G	6.67×10^{-11} Nm^2/kg^2
mass of Earth	5.97×10^{24} kg
radius of Earth	6371 km
radius of Earth's orbit	149.6×10^6 km
Earth's period of rotation	23.93 h
mass of the Moon	7.340×10^{22} kg
radius of Moon	1737 km
radius of Moon's orbit	384,400 km
period of Moon's rotation	27.3 days

Center of Mass

$$R_{cm} = \frac{1}{M}\left(\sum mr\right)$$

$$V_{cm} = \frac{1}{M}\left(\sum mv\right) = \frac{1}{M}\left(\sum p\right)$$

$$a_{cm} = \frac{1}{M}\left(\sum ma\right) = \frac{1}{M}\left(\sum F\right)$$

Conversions

$$\frac{rad}{s} = \frac{1}{2\pi}Hz = \frac{60}{2\pi}rpm$$

$$1\ rev = \frac{2\pi rad}{T} = \frac{360°}{T}$$

$$\text{N} = \frac{kgm}{s^2}$$

$$\text{J} = \text{Nm} = \frac{kgm^2}{s^2}$$

$$\text{W} = \frac{J}{s} = \frac{Nm}{s} = \frac{\frac{kgm^2}{s^2}}{s} = \frac{kgm^2}{s^3}$$

Quadratic formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$