1 Kinematics

Scalar Product

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Cross Product

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A} = AB\sin\theta$$
$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}$$

Use right hand rule (point fingers along the first vector, curl hand in towards next vector).

1D/2D Kinematics

$$v_i = v_o + at$$
$$\Delta x = v_o t + \tfrac{1}{2}at^2$$
$$v_f^2 = v_o^2 + 2a\Delta x$$
$$\Delta x = \tfrac{1}{2}t\,(v_o + v_i)$$

Projectile Motion

$$t = \frac{2v_o\sin\theta}{-g}$$
$$\Delta x = \frac{v_o^2\sin(2\theta)}{-g} = \frac{2v_o^2\sin\theta\cos\theta}{-g}$$

Relative Motion

$$v_{pw} = v_{pg} + v_{gw}$$

DRAW VECTOR DIAGRAMS

2 Newton's Laws of Motion

$$\vec{F} = m\vec{a}$$
$$F_g = mg = weight$$
$$F_g = \frac{GMm}{r^2}$$
$$g = \frac{GM}{r^2} = \frac{F_g}{m}$$
$$F_N = mg \quad \text{(horizontal surface)}$$
$$F_N = mg\cos\theta \quad \text{(angled surface)}$$
$$F_{f_s} = \mu_s F_N$$
$$F_{f_k} = \mu_k F_N$$
$$\mu_k < \mu_s \quad \text{(always)}$$
$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$
$$F_{drag} = \tfrac{1}{2}C\rho A v^2$$
$$\tan\theta = \frac{v^2}{rg} \quad \text{(banked curve)}$$
$$F_c = mg\tan\theta = F_{Nx} \quad \text{(banked curve)}$$

FREE-BODY DIAGRAMS ONLY INCLUDE EXTERNAL FORCES

3 Work Power Energy

Energy

$$E_k = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$$
$$E_{pg} = mgh = \frac{-GMm}{r}$$
$$E_{ps} = \tfrac{1}{2}k\,(\Delta x)^2$$

Work

$$\Delta\sum E = \Delta E_k + \Delta E_p = W$$
$$= Fd\cos\theta \quad \text{(Force parallel to direction of motion)}$$

Power

$$P = \frac{W}{t} = Fv$$
$$h_{min} = \frac{5r}{2} \quad \text{(rollercoaster loop)}$$

4 Linear Momentum/Collisions

Momentum

$$p = mv$$

Impulse

$$J = \Delta p = Ft$$

Conservation of Momentum

$$p_i = p_f$$
$$p_{i_x} = p_{f_x}$$
$$p_{i_y} = p_{f_y}$$

Centre of Mass

$$R_{cm} = \frac{1}{M}\left(\sum r\right)$$
$$V_{cm} = \frac{1}{M}\left(\sum mv\right) = \frac{1}{M}\left(\sum p\right)$$
$$a_{cm} = \frac{1}{M}\left(\sum ma\right) = \frac{1}{M}\left(\sum F\right)$$

LINEAR MOMENTUM MUST BE CONSERVED IN AT LEAST ONE DIRECTION

5 Rotational Motion

Rotational Kinematics

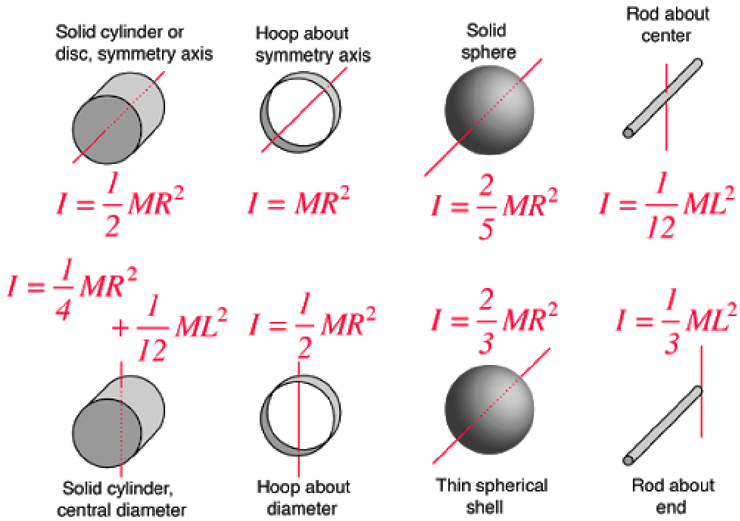
$$
\begin{aligned}
r\theta &= s_t \\
r\omega &= v_t \\
r\alpha &= a_t \\
\theta_f &= \theta_i + \omega t \\
\omega_f &= \omega_i + \alpha t \\
\Delta\theta &= \omega_i t + \tfrac{1}{2}\alpha t^2 \\
\omega_f^2 &= \omega_i^2 + 2\alpha\Delta\theta
\end{aligned}
$$

Rotational Work Power Energy

$$
\begin{aligned}
E_k &= \tfrac{1}{2} I\omega^2 \\
\tau &= I\alpha \\
\tau &= Fd_| = rF\sin\theta \\
W_\tau &= \Delta E_k = \tau\theta \\
P &= \tau\omega
\end{aligned}
$$

Inertia

I changes depending on the system.



6 Angular Momentum

Centre of Mass

$$
\begin{aligned}
V_{cm} &= R\omega \\
a_{cm} &= R\alpha \\
d_{cm} &= R\theta \\
L &= I\omega \\
L &= mvr \quad \text{(for point object)} \\
\Delta L &= \tau\Delta t \\
T &= \frac{2\pi}{\omega} = \frac{1}{f}
\end{aligned}
$$

Terms/Definitions

Free fall $a = g$

Average $\frac{\Delta y}{\Delta x}$

Instantaneous $\frac{dx}{dt}$

Newton's 1st Law "A body at rest tends to stay at rest unless acted upon by a large enough force"

Newton's 2nd Law $F = ma$

Newton's 3rd Law "Every action has an equal and opposite reaction"

Effective weight F_N

Mechanical Energy Sum of all kinetic, potential, and rotational kinetic energy

Inelastic Energy is not conserved

Elastic Energy is conserved

Perfectly Inelastic All kinetic energy is lost

Constants

$$
\begin{aligned}
G &= Universal Gravitational Constant \\
&= 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}} = 9.8\,\mathrm{m\,s^{-2}} \quad \text{(on Earth)} \\
C &= drag coefficient \\
\rho &= density of fluid \\
A &= surface area \\
k &= spring constant
\end{aligned}
$$

Conversions

$$
\begin{aligned}
\mathrm{rad\,s^{-1}} &= \frac{1}{2\pi}\mathrm{Hz} = \frac{60}{2\pi}\mathrm{rpm} \\
\mathrm{N} &= \mathrm{kg\,m\,s^{-2}} \\
\mathrm{J} &= \mathrm{N\,m} = \mathrm{kg\,m^2\,s^{-2}} \\
\mathrm{W} &= \mathrm{J\,s^{-1}} = \mathrm{kg\,m^2\,s^{-3}}
\end{aligned}
$$