Predator-Prey Dynamics and Dynamic Carrying Capacity: Non-linear Approaches to Ecological Stability

Overview

This theorem extends the traditional Lotka-Volterra predator-prey model by introducing a **dynamically fluctuating carrying capacity**. It proposes that environmental variability alters equilibrium population sizes, stabilizing predator and prey populations below the fixed carrying capacity assumed in classical models.

Theorem 1 (Predator-Prey Dynamics and Dynamic Carrying Capacity)**.** *Fluctuations in carrying capacity $K(t)$ due to environmental and resource changes drive non-linear dynamics in predator-prey systems, leading to dynamic equilibrium points. Predator and prey populations stabilize below the traditionally fixed carrying capacity in response to variability in available resources.*

Mathematical Formulation

1. Dynamic Carrying Capacity

$$K(t) = K_0 \left(1 + \lambda \cdot \sin(\omega t) + \varphi \cdot \cos(\omega t)\right)$$

Where:

- K_0: Baseline carrying capacity
- λ, φ: Amplitude coefficients
- ω: Frequency of environmental oscillations

2. Modified Lotka-Volterra Equations

Prey Dynamics:

$$\frac{dP}{dt} = \alpha P \left(1 - \frac{P}{K(t)}\right) - \beta PQ$$

Predator Dynamics:

$$\frac{dQ}{dt} = \delta PQ - \gamma Q$$

Where:

- P: Prey population
- Q: Predator population
- α: Prey growth rate
- β: Predator-prey interaction coefficient
- δ: Predator reproduction rate per prey
- γ: Predator death rate

3. Equilibrium Populations

Time-dependent equilibrium values:

$$P^*(t) = \frac{\alpha K(t)}{\beta + \gamma}, \quad Q^*(t) = \frac{\delta \alpha K(t)}{\beta(\gamma + \delta)}$$

These equilibria vary over time and remain below the fixed K_0, reflecting ecological response to environmental variation.

Testable Predictions

1. In ecosystems with fluctuating resources (e.g., seasonal or climate-driven), predator and prey populations will stabilize below the fixed carrying capacity, following $K(t)$.
2. A dynamic carrying capacity alters the amplitude and frequency of predator-prey oscillations compared to classical models.
3. Empirical observations from marine or terrestrial ecosystems with resource variability will support the predictions of this model over static-capacity models.

Peer Review Considerations

Logical Consistency:

- Is the use of fluctuating $K(t)$ justified biologically?
- Could other models explain the same behavior?

Mathematical Soundness:

- Are the modified equations solvable and stable?
- How sensitive is the model to the parameters λ, φ, ω?

Empirical Feasibility:

- Can this model be validated using ecological field data?
- Are there documented cases of fluctuating equilibrium populations?

Simulation Viability:

- Could platforms like MESA or NetLogo be used to simulate this model?
- What simulation parameters would best reflect real-world variability?

Conclusion

This theorem introduces a more ecologically realistic view of predator-prey systems by embedding time-dependent environmental changes directly into the carrying capacity term. The implications for ecological modeling, conservation, and resource management are substantial, particularly in the face of climate change and habitat variability.