Predator-Prey Resilience Theorem

I. Theorem Statement

Ecosystems with predator-prey interactions exhibit variable resilience depending on the presence of:

- Time-delayed predator response
- Nonlinear predator sensitivity to external disturbance
- Species migration and spatial dispersal
- Multi-trophic redundancy and substitution

These structural components influence whether systems recover, absorb, or collapse in response to external shocks.

II. Model Components

1. Time Delay in Predator Response

We introduce a delay-differential system:

$$\frac{dP}{dt} = \alpha P \left(1 - \frac{P}{K}\right) - \beta P(t-\tau)Q(t)$$

$$\frac{dQ}{dt} = \delta P(t-\tau)Q(t) - \gamma Q$$

Where τ is the delay between prey abundance and predator behavioral response.

2. Nonlinear Resilience Function

$$R(t) = \psi \cdot \exp(-\varphi \cdot D(t))$$

Where:

- $R(t)$: Resilience of predator response
- $D(t)$: Disturbance input (e.g., climate, disease)
- ψ: Maximum adaptation amplitude
- φ: Sensitivity to disturbance

3. Spatial Migration (Diffusion)

$$\frac{\partial P}{\partial t} = \cdots + D_p \nabla^2 P(x,t) \qquad \frac{\partial Q}{\partial t} = \cdots + D_q \nabla^2 Q(x,t)$$

Where D_p, D_q are diffusion coefficients and ∇^2 is the Laplacian.

III. Model Predictions

1. Systems with delayed predator response exhibit prey overshoot or undercompensation.
2. High φ (sensitivity) causes rapid collapse after shocks.
3. Migration buffers local collapse but amplifies system-wide volatility.
4. Predator redundancy increases resilience in multi-trophic networks.

IV. Simulation Design

- Use delay-differential solvers with spatial dynamics.
- Model $D(t)$ as stochastic or event-driven disturbance.
- Explore ranges of ψ, φ, and τ under various ecological scenarios.
- Recommended tools: MATLAB, Julia, SciPy, R (‘deSolve‘), NetLogo.

V. Empirical Foundation

- **Isle Royale**: Collapse and partial rebound in predator-prey cycles.
- **Atlantic Cod**: Delayed predation and trophic restructuring.
- **Ecological Resilience Studies**: Threshold behavior and regime shifts.

VI. Applications

- Conservation planning and recovery window identification.
- Ecosystem sensitivity testing under stochastic climate inputs.
- Cross-scale resilience mapping in complex adaptive networks.