Recursive Drift Collapse Theorem

I. Theorem Statement

Recursive information processing systems exhibit progressive degradation in output quality as recursion depth or density increases. There exists a critical threshold θ_c beyond which semantic coherence decreases, and the information gained per recursion step approaches zero.

Let:

- $C(t)$ – Coherence function at recursion step t
- $\Delta I(t)$ – Information gain between recursive iterations
- $D_a(t)$ – Active recursion density
- θ_c – Critical recursion threshold

If:

$$D_a(t) > \theta_c \quad \Rightarrow \quad \frac{d^2C}{dt^2} > 0 \quad \text{and} \quad \Delta I(t) \to 0$$

This defines the onset of recursive drift collapse.

II. Collapse Mechanism

1. **Cumulative Drift:** Small output variations accumulate across iterations, reducing semantic integrity.
2. **Reduced Information Gain:** $\Delta I(t)$ declines toward zero as outputs become less novel.
3. **Accelerated Degradation:** Coherence loss accelerates near θ_c, often exhibiting short-term oscillations before collapse.

III. Collapse Conditions

- Recursion depth or density exceeds θ_c
- No external semantic grounding is applied
- Input structure induces increased entropy over iterations

IV. Simulation Design

Suggested Models

- Autoregressive transformers (e.g., GPT variants)
- Recursive neural networks (RNNs)
- Symbolic reasoning systems with feedback loops

Metrics

- Coherence scoring (e.g., BERTScore)
- Mutual Information estimation (MINE, CLUB)
- Cosine similarity of latent embeddings
- $\Delta I(t)$ trends over recursion steps

Visualization Techniques

- Embedding projections using UMAP or t-SNE
- Acceleration of coherence loss $\left(\frac{d^2C}{dt^2}\right)$
- Threshold indicators for $\Delta I(t) \to 0$

V. Human Cognition Analogy

Human reasoning exhibits similar degradation under recursive load:

- Breakdown in symbolic consistency during paradox tasks
- EEG entropy increases during recursive reasoning
- Working memory failures under semantic recursion pressure

VI. Applications

- AI failure detection in recursive text generation
- Modeling cognitive collapse under symbolic stress
- Recursion-safe design in language and symbolic systems
- Theoretical bounding of symbolic regressions

VII. Future Work

- Formal models of entropy buildup in recursion
- Design of stabilizing feedback mechanisms
- Exploration of drift-correction strategies in symbolic AI