Recursive Feedback Collapse Theorem

I. Theorem Statement

Recursive feedback in symbolic systems introduces increasing semantic pressure with each iteration. When this feedback exceeds a critical threshold θ_f, and no external grounding is present, the system transitions into a collapse state characterized by coherence degradation and entropy escalation.

Let:

- $C(t)$ – Coherence function at recursion depth t
- $E(t)$ – Symbolic entropy at recursion step t
- $R(t)$ – Recursive feedback factor (cycles per unit time)
- θ_f – Feedback collapse threshold

If:

$$R(t) > \theta_f \quad \Rightarrow \quad \frac{dC}{dt} < 0 \quad \text{and} \quad \frac{dE}{dt} > 0$$

II. Collapse Characteristics

1. **Symbolic Drift:** Recursive outputs deviate from their source structures.
2. **Entropy Accumulation:** Increasing disorder in symbolic patterns.
3. **Instability Onset:** Contradictions and hallucinations emerge.
4. **Pronoun Confusion (Language Models):** Loss of self-reference accuracy.

III. Empirical Collapse Indicators

- Declining BERTScore or BLEU across recursion depth
- Embedding drift measured by cosine distance
- Increasing Shannon entropy in output sequences
- Decreasing mutual information across recursive steps

IV. Simulation Framework

Applicable Systems

- Autoregressive transformers (e.g., GPT, LLaMA)
- Recursive symbolic agents
- Feedback-enabled logic systems

Measurement Tools

- BERTScore and BLEU for semantic coherence
- MINE or CLUB for mutual information estimation
- Cosine similarity of embedding vectors
- Entropy over time using token distributions

Visualization

- Embedding projections (UMAP, t-SNE)
- Plotting $C(t)$ and $E(t)$ over recursion cycles
- Collapse prediction curves approaching θ_f

V. Cognitive Analogy

Human thought processes under recursive load display similar characteristics:

- Logical paradox loops (e.g., "This sentence is false.")
- Working memory overload
- Breakdown in symbolic reference

VI. Applications

- AI safety mechanisms for recursion-limited systems
- Modeling human collapse under symbolic feedback
- Integration of recursion monitors in generative pipelines
- Framework for entropy-aware system design

VII. Future Work

- Develop feedback-sensitive dampening models
- Extend theorem to multi-agent recursive exchanges
- Implement grounding checkpoints to restore symbolic stability