Recursive Mutual Implication Collapse Theorem

I. Theorem Statement

In symbolic systems that implement closed triadic implication loops (e.g., $A \to B \to C \to A$), recursive inference can lead to collapse when a critical recursion density is exceeded.

Let:

- $\Phi_r(t)$ – System coherence over time
- $\Delta I(t)$ – Information gain per recursive cycle
- $D_r(t)$ – Recursion density (cycles per unit time)
- θ_r – Critical recursion threshold

If:

$$D_r(t) > \theta_r \quad \Rightarrow \quad \frac{d^2\Phi_r}{dt^2} > 0 \quad \text{and} \quad \Delta I(t) \to 0$$

II. Collapse Mechanism

1. **Triadic Looping:** Recursion across three symbolic elements causes amplification of minor inconsistencies.
2. **Information Saturation:** $\Delta I(t)$ declines as redundancy increases.
3. **Entropy Escalation:** Symbolic structure deteriorates, and contradictions emerge.

III. Collapse Indicators

- Increase in $\frac{d^2\Phi_r}{dt^2}$ indicating accelerating coherence loss
- Decline in $\Delta I(t)$ toward zero
- Embedding divergence across recursion steps
- Logical contradictions within implication chains (e.g., $A \to \neg A$)

IV. Simulation Framework

Target Systems

- Recursive logic systems
- Transformer models with symbolic feedback
- Triadic knowledge graph structures

Metrics

- Mutual information estimation (MINE, CLUB)
- BERTScore and BLEU for semantic coherence
- Cosine similarity of embeddings over recursion steps
- Entropy tracking over recursive depth

V. Visualization

- Trajectory plots of $\Phi_r(t)$ and $\Delta I(t)$
- UMAP or t-SNE projections of embedding drift
- Recursion density graphs from symbolic logic cycles

VI. Cognitive Analogy

- Circular reasoning in argument or belief systems
- Mental fatigue from recursive contradiction processing
- Symbolic collapse under semantic overload

VII. Applications

- AI architecture design with recursion safety constraints
- Symbolic knowledge graph collapse prevention
- Cognitive model simulations of contradiction handling
- Collapse detection in triadic symbolic systems

VIII. Future Work

- Extend theorem to n-node mutual implication loops
- Develop symbolic dampening mechanisms for recursive feedback
- Integrate detection into live graph-based AI systems