

SKETCH OF THE “ALTERNATING SQP” METHOD FOR FITTING POISSON TOPIC MODELS

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1. Some derivations. Given an $n \times p$ matrix of counts X with individual entries $x_{ij} \geq 0$, our aim is to fit a Poisson model of the counts,

$$(1.1) \quad \begin{aligned} p(x) &= \prod_{i=1}^n \prod_{j=1}^p p(x_{ij}) \\ &= \prod_{i=1}^n \prod_{j=1}^p \text{Poisson}(x_{ij}; \lambda_{ij}), \end{aligned}$$

in which the Poisson rates are given by the mixture $\lambda_{ij} = \sum_{k=1}^K l_{ik} f_{jk}$. Therefore, the Poisson model is specified by a $p \times K$ matrix F with entries $f_{ik} \geq 0$ (the “factors”), and an $n \times K$ matrix L with entries $l_{ik} \geq 0$ (the “loadings”). Fitting F and L is equivalent to non-negative matrix factorization with the “beta-divergence” cost function [2]. It can also be used to recover a maximum-likelihood estimate for the latent Dirichlet allocation (LDA) model [1]. So fitting this model is useful for a wide range of applications.

The log-likelihood for the Poisson model is

$$(1.2) \quad \log p(x | F, L) \propto \sum_{i=1}^n \sum_{j=1}^p x_{ij} \log(\sum_{k=1}^K l_{ik} f_{jk}) - \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^K l_{ik} f_{jk},$$

where the constant of proportionality is obtained from factorial terms in the Poisson densities. We propose a slight modification to this log-likelihood to allow for missing counts, which may be useful for some applications:

$$(1.3) \quad \log p(x | F, L) \propto \sum_{i=1}^n \sum_{j=1}^p x_{ij} \log(\sum_{k=1}^K l_{ik} f_{jk}) - \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^K a_{ij} l_{ik} f_{jk},$$

in which we introduce an $n \times p$ matrix A such that $a_{ij} = 0$ and $x_{ij} = 0$ whenever the count is missing. Our specific aim is to find a F and L that maximizes the log-likelihood (1.3). In the remainder, we derive an efficient approach to doing this.

REFERENCES

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- [2] D. D. LEE AND H. S. SEUNG, *Algorithms for non-negative matrix factorization*, in Advances in Neural Information Processing Systems 13, 2001, pp. 556–562.

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