FIT5171 Tutorial 3
Discrete Mathematics for Software Testing.

Week 3, 2023

Please do try the questions before coming to the tutorial. Your active participation is the most important!

Logics & Set Theory

1. Decide if the following predicates are true. If not, give a counter example.
   (a) $\forall A \exists B \bullet C \equiv \exists B \forall A \bullet C$, where A and B are variable declarations and C is a Boolean expression.

Solution: False.
Counterexample: let both A and B be "a human" and C be "is father". Then clearly the LHS and the RHS are not logically equivalent. More formally,

$$\forall x : Human \bullet \exists y : Human \bullet (x, y) \in isFather$$

is different from

$$\exists y : Human \bullet \forall x : Human \bullet (x, y) \in isFather$$

(b) $\forall A, B : PN \bullet A \subseteq B \lor B \subseteq A$

Solution: False.
Counter example: let $A = \{0\}$, $B = \{1\}$, clearly $A \not\subseteq B$ nor $B \not\subseteq A$.

Relations & Functions

2. Give a formal definition of the binary relation R, over natural numbers, such that each x is related to y by R if and only if y is greater than the square of x but less than the square of $x + 1$.

Solution: $R \colon N \longleftrightarrow N \bullet \forall x, y \colon N \bullet x \mapsto y \in R \iff x^2 < y < (x+1)^2$

3. Let S be the set of natural numbers between 1 and 15 inclusively. Express R in Q2 as a set of ordered pairs in S.

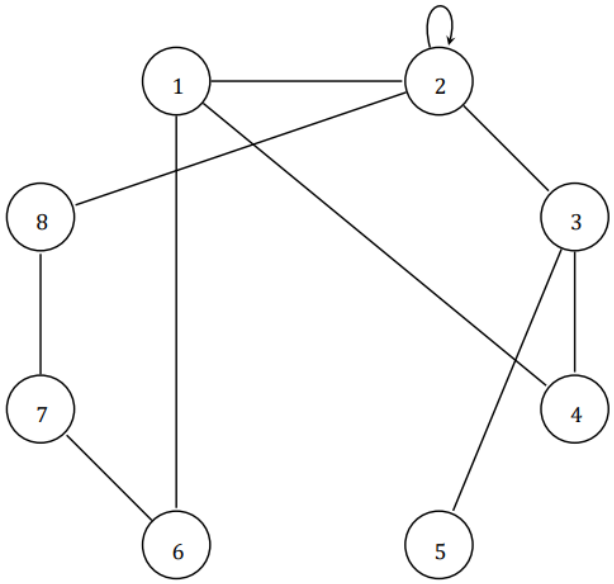
Solution: R = {(1, 2), (1, 3), (2, 5), (2, 6), (2, 7), (2, 8), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14), (3, 15)}

Graph Theory

4. Given an undirected graph G with vertices $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and edges $E = \{12, 14, 16, 22, 23, 28, 34, 35, 67, 78\}$

   (a) Draw G graphically.

Solution:



(b) Calculate the degree of each node in the graph.

Solution:

Node	1	2	3	4	5	6	7	8
Degree	3	5	3	2	1	2	2	2

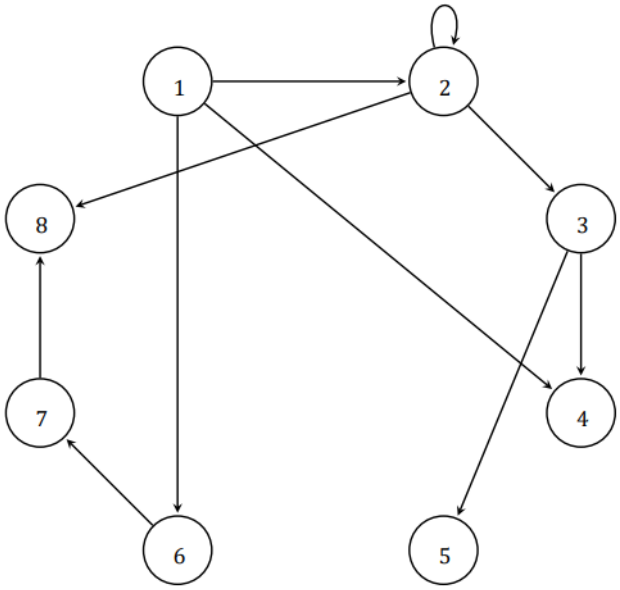
(c) Calculate the Cyclomatic number of G.

Solution: The Cyclomatic number of of G, $V(G) = \#E - \#V + p = 10 - 8 + 1 = 3$.

5. Let's treat G in Q5 as a directed graph (edge 12 in E represents an edge from node 1 to node 2).

(a) Draw G graphically.

Solution:



(b) Calculate the in-degree and out-degree of each node (ignoring edge 22 for this question)

Solution:

Node	1	2	3	4	5	6	7	8
In-degree	0	1	1	2	1	1	1	2
Out-degree	3	2	2	0	0	1	1	0

(c) Are there any source nodes or sink nodes in G?

Solution:
Source nodes: 1
Sink nodes: 4, 5, 8

(d) Does this graph contain semi-paths? If so, identify them.

Solution: Yes, (12, 23, 16, 67), (12, 28, 16, 67), and (12, 23, 34, 16, 67, 78) are semi-paths.

(e) Identify a pair of nodes that are 0-connected, 1-connected, 2-connected, and 3-connected, if any (ignoring edge 22 again).

Solution:
0-connected: none.
1-connected: 2 & 6, 3 & 7, etc.
2-connected: 1 & 2, 6 & 7, etc.
3-connected: none.

(f) Work out the reachability matrix for G.

Solution:

	1	2	3	4	5	6	7	8
1	0	1	1	1	1	1	1	1
2	0	1	1	1	1	0	0	1
3	0	0	0	1	1	0	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	1
7	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0	0	0

6. Draw a directed graph for each of the following common control constructs: (1) if-then, (2) if-then-else, (3) case switch, (4) while loop, and (5) do-while loop. As an example, the graph of sequence execution is included below in Figure 1.

Figure 1: A directed graph showing sequential execution.

Solution: See below for the depictions for the five cases.

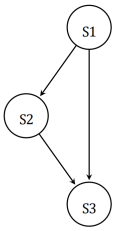


Figure 2: if-then

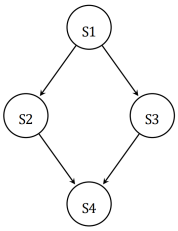


Figure 3: if-then-else

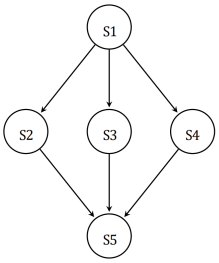


Figure 4: case switch

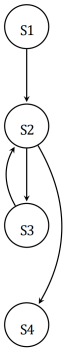


Figure 5: while loop

Figure 6: do-while loop