Exact linearization and noninteracting control of a 4 rotors helicopter via dynamic feedback

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This paper presents a nonlinear dynamic model for a four rotors helicopter in a form suited for control design. We show that the input-output decoupling problem is not solvable for this model by means of a static state feedback control law. Then, a dynamic feedback controller is developed which render the closed-loop system linear, controllable and noninteractive after a change of coordinates in the state-space. Finally, the stability and the robustness of the proposed control law in the presence of wind, turbulences and parametric uncertainties is analyzed through a simulated case study.

1 Introduction

Autonomous UAVs[1] are increasingly popular platforms, due to their potential use in search and rescue, surveillance, law enforcement, inspection, mapping, aerial cinematography [1][2]. For these applications, the ability of helicopters to take off and land vertically, to perform hover flight, as well as their agility, controllability, make them ideal vehicles.

In order to accomplish high level human-planned missions, flight control systems able to track accurate reference trajectories in the presence of wind or turbulences are required. Most of the realized flight control systems have been designed by applying classical synthesis techniques (such as single-loop PD systems, root locus, Bode plots etc.) to an approximate linear model of the vehicle dynamics . But the trend of escalating performance, the increasing maneuverability, the unpredictable changes in the environment, the stronger dynamic coupling and nonlinearities necessitate more sophisticated control systems [3] [4].

[1]Unmanned Aerial Vehicle

In this context, our efforts have been directed to the development of global feedback controllers for a miniature four rotors helicopter on the basis of a comprehensive nonlinear dynamic model. Similar efforts have already been accomplished for standard model helicopters (See e.g. [5][6][7][8][9]).

The paper is divided as follows: in Section II, a dynamic model for a miniature four rotors helicopter is developed. Based on this nonlinear model, we design in Section III a dynamic feedback control law which renders the system linear, controllable and noninteractive. In section IV, some simulations are carried out, allowing the analysis of the stability - robustness of the proposed controller in the presence of wind, turbulences and parametric uncertainties. Finally, in section V, our conclusions and directions for future work are presented.

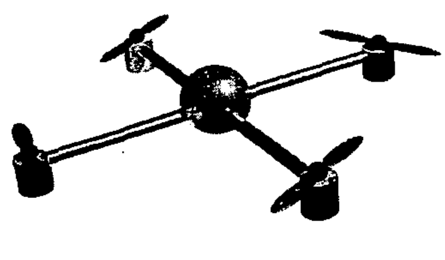

Figure 1: The miniature four rotors helicopter

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2 Dynamic modeling of a 4 rotors helicopter

In this section we develop the dynamic model describing the UAV position and attitude. The considered UAV is a miniature four rotors helicopter (Figure 1). Each rotor consists of an electric DC motor, a drive gear and a rotor blade. Forward motion is accomplished by increasing the speed of the rear rotor while simultaneously reducing the forward rotor by the same amount. Aft, left and right motion work in the same way. Yaw command is accomplished by accelerating the two clockwise turning rotors while decelerating the counter-clockwise turning rotors.

The equations describing the attitude and position of an UAV are basically those of a rotating rigid body with six degrees of freedom [3][10]. They may be separated into *kinematic* equations and *dynamic* equations [11].

The kinematic equations may be represented as follows. The *absolute position* of the UAV is described by the three coordinates (x_0, y_0, z_0) of its center of mass with respect to an earth fixed inertial reference frame and its *attitude* by the three Euler's angles (ψ,θ,ϕ). These three angles are respectively called yaw angle $(-\pi \leq \psi < \pi)$, pitch angle $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$ and roll angle $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$.

The derivatives with respect to time of the angles (ψ,θ,ϕ) can be expressed in the form

$$col(\dot{\psi}, \dot{\theta}, \dot{\phi}) = M(\psi, \theta, \phi)\omega \tag{1}$$

in which $\omega = col(p, q, r)$ is the angular velocity expressed with respect to a body reference frame and $M(\psi, \theta, \phi)$ is the 3x3 matrix given by

$$M(\psi,\theta,\phi) = \begin{bmatrix} 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \\ 0 & \cos\phi & -\sin\phi \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \end{bmatrix}$$

This matrix, as shown, depends only on (ψ, θ, ϕ) and is invertible if the above conditions on (ψ, θ, ϕ) hold.

Similarly, the derivative with respect to time of the position (x_0, y_0, z_0) is given by

$$col(\dot{x}_0, \dot{y}_0, \dot{z}_0) = V_0 \tag{2}$$

where $V_0 = col(u_0, v_0, w_0)$ is the absolute velocity of the UAV expressed with respect to an earth fixed inertial reference frame. Let $V = col(u, v, w)$ be the absolute velocity of the UAV expressed in a body fixed reference frame. V and V_0 are related by

$$V_0 = R(\psi, \theta, \phi)V$$

where $R(\psi, \theta, \phi)$ is the rotation matrix given by[2]

$$R = \begin{bmatrix} C\theta C\psi & C\psi S\theta S\phi - C\phi S\psi & C\phi C\psi S\theta + S\phi S\psi \\ C\theta S\psi & S\theta S\phi S\psi + C\phi C\psi & C\phi S\theta S\psi - C\psi S\phi \\ -S\theta & C\theta S\phi & C\theta C\phi \end{bmatrix}$$

(1) and (2) are the kinematic equations. The dynamic equations are now expressed. Using the Newton's laws about the center of mass one obtains the dynamic equations for the miniature four rotors helicopter[3]

$$\begin{aligned} m\dot{V}_0 &= \sum F_{ext} \\ J\dot{\omega} &= -\omega \times J\omega + \sum T_{ext} \end{aligned} \tag{3}$$

m is the mass, J is the inertia matrix given by

$$J = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

and $\sum F_{ext}$, $\sum T_{ext}$ represent the vector of external forces and external torques respectively. They contain the helicopter's weight, the aerodynamic forces vector, the thrust and the torque developed by the four rotors. Some calculations yield the following form for theses two vectors

$$\begin{aligned} \sum F_{ext} &= \begin{bmatrix} A_x - (C\phi C\psi S\theta + S\phi S\psi)u_1 \\ A_y - (C\phi S\theta S\psi - C\psi S\phi)u_1 \\ A_z + mg - (C\theta C\phi)u_1 \end{bmatrix} \\ \sum T_{ext} &= \begin{bmatrix} A_p + u_2 d \\ A_q + u_3 d \\ A_r + u_4 d \end{bmatrix} \end{aligned} \tag{4}$$

in which

- $col(A_x, A_y, A_z)$ and $col(A_p, A_q, A_r)$ are the resulting aerodynamic forces and moments acting on the UAV and are computed from the aerodynamic coefficients C_i as $A_i = \frac{1}{2}\rho_{air}C_i W^2$ (ρ_{air} is the air density, W is the velocity of the UAV with respect to the air) [10];
- g is the gravity constant ($g = 9.81ms^{-2}$);

[2] $C\theta$, $S\theta$ and $T\theta$ denote respectively $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$
[3] $\times$ denotes the usual "vector" product

- d is the distance from the center of mass to the rotors;
- u_1 is the resulting thrust of the four rotors;
- u_2 is the difference of thrust between the left rotor and the right rotor;
- u_3 is the difference of thrust between the front rotor and the back rotor;
- u_4 is the difference of torque between the two clockwise turning rotors and the two counter-clockwise turning rotors.

Each rotor undergoes thrust and torque and of course leaves a wake behind as it moves. If the velocity induced by the wake is omitted, it can be shown that they are proportional to the square of the angular speed of the rotor shaft [12]. Assuming that the electric motors are velocity controlled, then (u_1, u_2, u_3, u_4) may be considered directly as control inputs.

Using equations (1), (2), (3) and (4), one get a system of non linear differential equations which is described in state space form by

$$\dot{x} = f(x) + \sum_{i=1}^{4} g_i(x) u_i \tag{5}$$

where

$$x = col(x_0, y_0, z_0, \psi, \theta, \phi, u_0, v_0, w_0, p, q, r)$$

$$f(x) = \begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ q\sin\phi\sec\theta + r\cos\phi\sec\theta \\ q\cos\phi - r\sin\phi \\ p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ \frac{A_x}{m} \\ \frac{A_y}{m} \\ \frac{A_z}{m} + g \\ \frac{I_y - I_z}{I_x} qr + \frac{A_p}{I_x} \\ \frac{I_z - I_x}{I_y} pr + \frac{A_q}{I_y} \\ \frac{I_x - I_y}{I_z} pq + \frac{A_r}{I_z} \end{bmatrix}$$

and

$$\begin{aligned} g_1(x) &= col(0,0,0,0,0,0,g_1^7, g_1^8, g_1^9, 0,0,0) \\ g_2(x) &= col(0,0,0,0,0,0,0,0,0,\frac{d}{I_x},0,0) \\ g_3(x) &= col(0,0,0,0,0,0,0,0,0,0,\frac{d}{I_y},0) \\ g_4(x) &= col(0,0,0,0,0,0,0,0,0,0,0,\frac{1}{I_z}) \end{aligned}$$

with

$$\begin{aligned} g_1^7 &= -\frac{1}{m}(\cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi) \\ g_1^8 &= -\frac{1}{m}(\cos\phi\sin\theta\sin\psi - \cos\psi\sin\phi) \\ g_1^9 &= -\frac{1}{m}(\cos\theta\cos\phi) \end{aligned}$$

The mathematical model (5) is assumed to be sufficiently accurate in representing all UAV functional motions [3] but is not suitable for control design because it depends upon the aerodynamic forces and moments, which are unknown in the presence of unpredictable winds and turbulences. Thus, these terms are neglected during the control design and are considered as external disturbances. The purpose of the next section is to design a feedback controller for the four rotor miniature helicopter which presents robustness properties against neglected effects and parametric uncertainties. This is the purpose of the feedback loop, which has the potential to counteract model uncertainties as it's verified in the simulated examples of Section IV.

3 Exact linearization and non-interacting control

This section deals with the design of a feedback control law (and a change of coordinates in the state-space) to the purpose of transforming the nonlinear system (5) into a linear and controllable one. This problem is known as the *exact linearization problem* in the literature [13][11][14]. Moreover, we would like to reduce the system from an input-output point of view, to an aggregate of independent single-input single-output channels: This is the *noninteracting control problem* [13] or *input-output decoupling problem* [11].

It will be shown that none of these two problems are solvable for the nonlinear system (5) by means of a *static state feedback control law* but by means of a *dynamic feedback control law*.

First, it's necessary to define the control objective by choosing an output function for the system (5). To avoid unnecessary complications, we set the number of input channels equal to the number of output channels. We would like to control the absolute position of the UAV (x_0, y_0, z_0) and the yaw angle ψ therefore the output function is chosen as

$$y = h(x) = col(x_0, y_0, z_0, \psi)$$

We assume in this paper the state x of the system being fully available for measurements and we seek a static state feedback control law of the form

$$u = \alpha(x) + \beta(x)v \tag{6}$$

where v is an external reference input to be defined later, $\alpha(x) = col(\alpha_1(x), \alpha_2(x), \alpha_3(x), \alpha_4(x))$ and $\beta(x)$ is a 4x4 matrix.

Let $\{r_1, r_2, r_3, r_4\}$ be the vector relative degree of the system (5). Recall that the relative degree r_i is exactly the number of times one has to differentiate the i^{th} output in order to have at least one component of the input vector u explicitly appearing. Thus, we have[4]

$$r_i = (\inf\ k,\ \exists j, 1 \leq j \leq 4,\ L_{g_j} L_f^{k-1} h_i \neq 0)$$

and

$$col(y_1^{(r_1)}, y_2^{(r_2)}, y_3^{(r_3)}, y_4^{(r_4)}) = b(x) + \Delta(x)u$$

where

$$\Delta(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \dots & L_{g_4} L_f^{r_1-1} h_1(x) \\ \dots & \ddots & \dots \\ L_{g_1} L_f^{r_4-1} h_4(x) & \cdots & L_{g_4} L_f^{r_4-1} h_4(x) \end{bmatrix} \tag{7}$$

$$b(x) = \begin{bmatrix} L_f^{r_1} h_1(x) \\ \vdots \\ L_f^{r_4} h_4(x) \end{bmatrix} \tag{8}$$

The main result about the input-output decoupling problem is that this problem is solvable if and only if the matrix $\Delta(x)$ is nonsingular. In this case, the static state feedback (6) with

$$\begin{aligned} \alpha(x) &= -\Delta^{-1}(x)b(x) \\ \beta(x) &= \Delta^{-1}(x) \end{aligned} \tag{9}$$

renders the closed loop system linear and decoupled from an input-output point of view. More precisely, we have

$$y^{(r_i)} = v_i \quad \text{for all } i,\ 1 \leq i \leq 4$$

[4] L_f denotes the Lie derivative along the vectorfield f

But, for the nonlinear system (5), we have

$$r_1 = r_2 = r_3 = r_4 = 2$$

and

$$\Delta(x) = \begin{bmatrix} \Delta_{1,1} & 0 & 0 & 0 \\ \Delta_{2,1} & 0 & 0 & 0 \\ \Delta_{3,1} & 0 & 0 & 0 \\ 0 & 0 & \Delta_{4,3} & \Delta_{4,4} \end{bmatrix}$$

with

$$\begin{aligned} \Delta_{1,1} &= g_1^7 \\ \Delta_{2,1} &= g_1^8 \\ \Delta_{3,1} &= g_1^9 \\ \Delta_{4,3} &= \frac{d}{I_y}(\sin\phi \sec\theta) \\ \Delta_{4,4} &= \frac{1}{I_z}(\cos\phi \sec\theta) \end{aligned}$$

Obviously, $\Delta(x)$ is singular for all x therefore, the input-output decoupling problem is not solvable for the system (5) by means of a static state feedback control law.

We seek explanations why the matrix $\Delta(x)$ is always singular. The reason is that the derivatives $y_1^{(2)}$, $y_2^{(2)}$ and $y_3^{(2)}$ are affected all by the input u_1 and none by u_2, u_3, u_4. Thus, in order to get $\Delta(x)$ nonsingular, we could try to render $y_1^{(2)}$, $y_2^{(2)}$ and $y_3^{(2)}$ independent of u_1, that is to delay the appearance of u_1 to higher order derivatives of y_1, y_2 and y_3 and hope that the others inputs show up [13]. In order to achieve this result, we set u_1 equal to the output of a double integrator driven by $\bar{u}_1$, i.e.

$$\begin{aligned} u_1 &= \zeta \\ \dot{\zeta} &= \xi \\ \dot{\xi} &= \bar{u}_1 \end{aligned} \tag{10}$$

For consistency of notation we also set, for the other input channels which have been left unchanged

$$\begin{aligned} u_2 &= \bar{u}_2 \\ u_3 &= \bar{u}_3 \\ u_4 &= \bar{u}_4 \end{aligned} \tag{11}$$

Note that u_1 is not anymore an input for the system (5) but becomes the internal state ζ for the new dynamical system (10). The extended system obtained is described by equations of the form

$$\dot{\bar{x}} = \bar{f}(\bar{x}) + \sum_{i=1}^{4} \bar{g}_i(\bar{x})\bar{u}_i \qquad (12)$$

in which

$$\bar{x} = col(x_0, y_0, z_0, \psi, \theta, \phi, u_0, v_0, w_0, \zeta, \xi, p, q, r)$$

and

$$\bar{f}(\bar{x}) = \begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ q\sin\phi\sec\theta + r\cos\phi\sec\theta \\ q\cos\phi - r\sin\phi \\ p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ \frac{A_x}{m} + g_1^7(\psi,\theta,\phi)\zeta \\ \frac{A_y}{m} + g_1^8(\psi,\theta,\phi)\zeta \\ \frac{A_z}{m} + g + g_1^9(\psi,\theta,\phi)\zeta \\ \xi \\ 0 \\ \frac{I_y - I_z}{I_x}qr + \frac{A_p}{I_x} \\ \frac{I_z - I_x}{I_y}pr + \frac{A_q}{I_y} \\ \frac{I_x - I_y}{I_z}pq + \frac{A_r}{I_z} \end{bmatrix}$$

$$\begin{aligned} \bar{g}_1(\bar{x}) &= col(0,0,0,0,0,0,0,0,0,0,1,0,0,0) \\ \bar{g}_2(\bar{x}) &= col(0,0,0,0,0,0,0,0,0,0,0,\frac{d}{I_x},0,0) \\ \bar{g}_3(\bar{x}) &= col(0,0,0,0,0,0,0,0,0,0,0,0,\frac{d}{I_y},0) \\ \bar{g}_4(\bar{x}) &= col(0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{I_z}) \end{aligned}$$

Now, the input-output decoupling problem is solvable for the nonlinear system (5) by means of a dynamic feedback control law if it is solvable via static feedback for the extended system (12).

For the nonlinear system (12), the vector relative degree $\{r_1, r_2, r_3, r_4\}$ is given by

$$r_1 = r_2 = r_3 = 4; \quad r_4 = 2$$

and we have

$$col(y_1^{(r_1)}, y_2^{(r_2)}, y_3^{(r_3)}, y_4^{(r_4)}) = b(\bar{x}) + \Delta(\bar{x})u \qquad (13)$$

where $\Delta(\bar{x})$ and $b(\bar{x})$ are computed using equations (7) and (8).

The matrix $\Delta(\bar{x})$ is nonsingular at any point characterized by $\zeta \neq 0$, $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Therefore, the input-output decoupling problem is solvable for the system (5) by means of a dynamic feedback control law of the form:

$$\bar{u} = \alpha(\bar{x}) + \beta(\bar{x})v$$

where $\alpha(\bar{x})$ and $\beta(\bar{x})$ are computed using (9). Recall the relation between u and $\bar{u}$ (equations 10 and 11), we get the structure (Figure 2) for the control law of the original system (5):

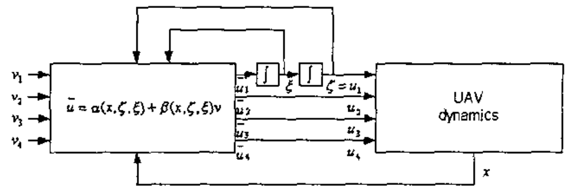

Figure 2: Block diagram of the control law

Moreover, since the extended system (12) has dimension $n = 14$, the condition

$$r_1 + r_2 + r_3 + r_4 = n$$

is fulfilled and therefore, the system can be transformed via dynamic feedback into a system which, in suitable coordinates, is fully linear and controllable. The change of coordinates $z = \Phi(\bar{x})$ is given by

$$\begin{array}{ll} z_1 = h_1(x) = x_0 & z_8 = L_f^3\, h_2(x) = y_0^{(3)} \\ z_2 = L_f\, h_1(x) = \dot{x}_0 & z_9 = h_3(x) = z_0 \\ z_3 = L_f^2\, h_1(x) = \ddot{x}_0 & z_{10} = L_f\, h_3(x) = \dot{z}_0 \\ z_4 = L_f^3\, h_1(x) = x_0^{(3)} & z_{11} = L_f^2\, h_3(x) = \ddot{z}_0 \\ z_5 = h_2(x) = y_0 & z_{12} = L_f^3\, h_3(x) = z_0^{(3)} \\ z_6 = L_f\, h_2(x) = \dot{y}_0 & z_{13} = h_4(x) = \psi \\ z_7 = L_f^2\, h_2(x) = \ddot{y}_0 & z_{14} = L_f\, h_4(x) = \dot{\psi} \end{array}$$

In the new coordinates, the system appears as

$$\begin{aligned} \dot{z} &= Az + Bv \\ y &= Cz \end{aligned} \qquad (14)$$

in which[5]

$$\begin{aligned} z &= col(z_1, z_2, \ldots, z_{14}) \\ v &= col(v_1, v_2, v_3, v_4) \end{aligned}$$

[5] 0 denotes a matrix of zeros of appropriate dimensions

$$
\begin{aligned}
A &= \begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_1 & 0 & 0 \\ 0 & 0 & A_1 & 0 \\ 0 & 0 & 0 & A_2 \end{bmatrix} \\
B &= \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \qquad C = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_1 & 0 \\ 0 & 0 & 0 & C_2 \end{bmatrix} \\
A_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\
B_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \\
B_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad B_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
C_1 &= (1,0,0,0) \\
C_2 &= (1,0)
\end{aligned}
$$

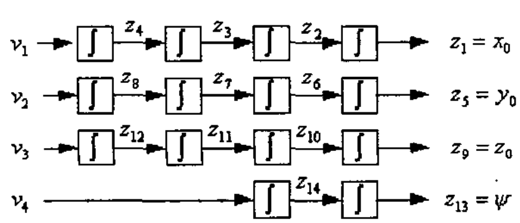

Figure 3: Block diagram of the closed loop system

On the linear system (14) one can impose new feedback controls, like for example

$$
\begin{aligned}
v_1 &= x_d^{(4)} - c_3(x_0^{(3)} - x_d^{(3)}) - c_2(\ddot{x}_0 - \ddot{x}_d) \quad (15) \\
&\quad -c_1(\dot{x}_0 - \dot{x}_d) - c_0(x_0 - x_d) \\
v_2 &= y_d^{(4)} - c_3(y_0^{(3)} - y_d^{(3)}) - c_2(\ddot{y}_0 - \ddot{y}_d) \\
&\quad -c_1(\dot{y}_0 - \dot{y}_d) - c_0(y_0 - y_d) \\
v_3 &= x_d^{(4)} - c_3(z_0^{(3)} - z_d^{(3)}) - c_2(\ddot{z}_0 - \ddot{z}_d) \\
&\quad -c_1(\dot{z}_0 - \dot{z}_d) - c_0(z_0 - z_d) \\
v_4 &= \ddot{\psi}_d - c_5(\dot{\psi} - \dot{\psi}_d) - c_4(\psi - \psi_d)
\end{aligned}
$$

where the coefficients c_i's are chosen in order to assign a specific set of eigenvalues and (x_d, y_d, z_d, ψ_d) is the desired trajectory.

This achieves the design of a dynamic feedback controller for the miniature four rotor helicopter. In the next section, simulations are carried out to verify the robustness of the proposed controller in the presence of uncertainties in the dynamic model.

4 Simulation results

Extensive simulations were made considering different wind conditions and parametric uncertainties. Some of the obtained results are presented in the following to illustrate the performance of the proposed controller. In this simulation, the reference trajectory chosen is a vertical helix (Figure 4) whose equations are given by

$$
\begin{aligned}
x_d &= \frac{1}{2}\cos\left(\frac{t}{2}\right) \\
y_d &= \frac{1}{2}\sin\left(\frac{t}{2}\right) \\
z_d &= -1 - \frac{t}{10} \\
\psi_d &= \frac{\pi}{3}
\end{aligned}
$$

The helicopter is initially in hover flight and the initial position is $x_0 = 0$, $y_0 = 0$, $z_0 = 0$, $\psi = 0$. The parameters used for the helicopter are given by $m = 0.7$, $Ix = Iy = Iz = 1.2416$, $d = 0.3$, $g = 9.81$. We have used the following control gains: $c_0 = 625$, $c_1 = 500$, $c_2 = 150$, $c_3 = 20$, $c_4 = c_5 = 4$.

4.1 Flight without wind

Figures (4) and (5) show the tracking errors with no wind. The parameters m, I_x, I_y, I_z are assumed exactly known. In these conditions, the helicopter follows the desired trajectory after a short transient.

4.2 Flight with wind and parametric uncertainties

Figures (6) and (7) show the tracking results with a constant wind blowing from North, when there is a -20% error in the parameters m, I_x, I_y, I_z. The wind exerts an unmodelled 10 Newton force on the helicopter. It is seen from the figures that the tracking errors still converge towards zero although the actual

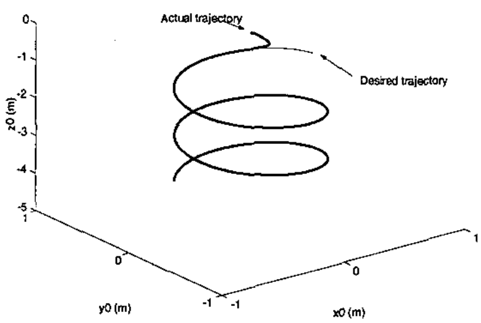

Figure 4: Reference trajectory and actual trajectory with no wind

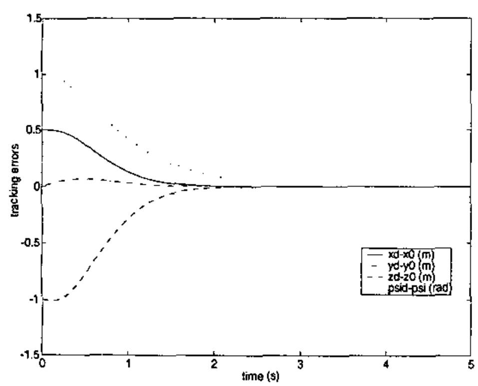

Figure 5: Tracking errors with no wind

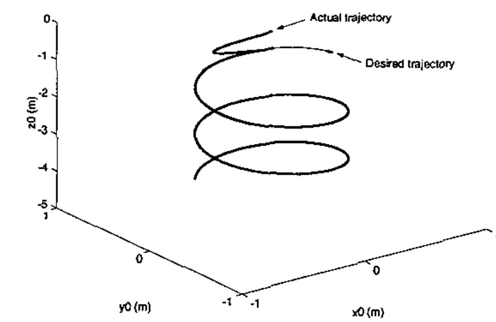

Figure 6: Reference trajectory and desired trajectory with wind

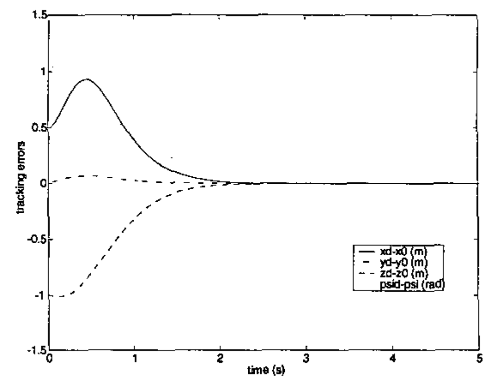

Figure 7: Tracking errors with wind

trajectory is altered by the wind during a short transient. The helicopter fairly follows the desired trajectory with little influence from the presence of wind and parameter uncertainties.

5 Conclusion

This paper has presented a dynamic model for a miniature four rotors helicopter. It has been shown that this nonlinear model cannot be transformed into a linear and controllable one by means of a static feedback control law. Then a dynamic feedback controller has been developed, which renders the system linear, controllable and noninteractive from an input-output point of view. Finally, some simulations have been carried out and the stability - robustness of the control law in the presence of wind, turbulences and parametric uncertainties have been validated.

The requirement to measure all the states for feedback is the most important limitation of the proposed controller. Hence, future research includes the design of a nonlinear state observer for feedback and the stability analysis of the closed-loop system. Another drawback of the linearization techniques is the wasteful cancellation of beneficial nonlinearities. Other techniques based on control Lyapunov functions are more flexible and do not force the closed-loop system to appear linear. They can avoid cancellations of useful nonlinearities, leading to a dramatic reduction in the control effort [4][15]. These techniques need to be investigated and compared with the dynamic controller presented in this paper. Finally, it's planned to validate the proposed control schemes on an experimental four rotors helicopter that is under construction in our laboratory.

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