# Contents

# Chapter 1

# Introduction

In recent years, Unmanned Aerial Vehicles (UAVs) have been the subject of many research projects and public scrutiny. While fixed wing UAVs have been successfully deployed in several different scenarios [**citation needed**], rotor blade ones are still being actively researched and improved upon. Helicopters have long been the preferred type of rotor blade vehicle for manned transportation, mostly due to having a design that has better stability properties [**citation needed**].

Develop UAV history. Explain helicopters better. Introduce other rotorblade helicopters.

The concept of a quadcopter is not new [**citation needed**]. It was introduced in **??** and several models were built since then [**citation needed**]. It consists on a 'X' or plus shapped frame, with a rotor at each end. Each rotating blade produces air drag that counteracts its movement. To prevent the vehicle from spinning on the opposite direction of the blades direction of rotation, the rotors are assigned in pairs along the same axis of the quadcopter frame, with each pair rotating in opposite directions. The end effect is that, if the thrust produced by each rotor is the same, then the air drags created by one pair are compensated by the other's. Introducing a difference between the rotors thrust in one axis creates a torque around the other one, and creating a difference in each pair's total thrust originates a drag force that makes the frame rotate around itself.

These easy to understand concepts makes the quadcopter an interesting platform to study, as well as its high maneuverability that makes it an ideal platform for indoors flights [**citation needed**]. However, its inherently unstable nature prevented it to be used until the recent past [**citation needed**]. Nowadays, with the availability of inexpensive embedded computers, it is possible to equip these vehicles with onboard controllers that ensure that the system will behave in a desirable manner over a pilot (or autopilot) commands.

One major limitation of quadcopter systems relies in their autonomy [**citation needed**]. It is common to have a flight autonomy in the order of 10 to 20 minutes. This is due to the fact that the four rotor blades configuration is not the most power efficient one [**citation needed**]. If one wants to introduce an onboard attitude controller, the problem is magnified by the need to constantly having to compute a control signal to give to the motor controllers. One possible solution to reduce the impact of this problem is to try to minimize the number of control input updates by only computing a new control signal when some condition is fulfilled. This is called Event-Triggered control.

In this project, a saturating controller [1] is implemented in a quadcopter platform, and an event triggering technique is proposed and tested.

Develop EVT? More about the project when the project is done...

# Chapter 2

# Background

In this chapter, the background for this project will be given. Firstly, the quadcopter attitude mathematical model will be introduced, together with the quaternion attitude parametrization. Secondly, a quick survey of different approaches to the attitude control is presented, together with the saturating controller that was implemented. Finally, an introduction to Event-Triggered control can be found in section [**ref section**].

## 2.1 The quadcopter attitude model

The attitude control problem of a quadcopter can be reduced to the attitude control problem of a rigid-body in 3D space [**citation needed**]. We can then define a world frame, $E$, fixed in space, and a body frame, $B$, coincident with the center of mass of the system see figure ??. The system attitude is given by

$$\mathbf{x} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix},$$

where $[\phi \;\; \theta \;\; \psi]$ are, respectively, the rotations around the $x$, $y$ and $z$ axis of the quadcopter in its body frame (the Euler angles roll, pitch and yaw) and $\boldsymbol{\omega} = [\omega_x \;\; \omega_y \;\; \omega z]$ are the angular speeds around the same axis. The mapping from $E$ to $B$ can be done by a rotation matrix [**citation needed**], $R$, such that $B = RE$. By applying the Newton-Euler equations, an attitude model can be obtained:

$$\begin{cases} \dot{R} = R\boldsymbol{\omega}^\times \\ J\dot{\boldsymbol{\omega}} = J\boldsymbol{\omega} \times \boldsymbol{\omega} + \boldsymbol{\tau} \end{cases}, \tag{2.1}$$

where $J$ is a $3 \times 3$ moment of inertia matrix, $\boldsymbol{\omega}^\times$ denotes the skew symetric matrix **[citation needed]** and $\boldsymbol{\tau} = [\tau_x \ \ \tau_y \ \ \tau_z]^\top$ are the applied torques on the system, around the $x$, $y$ and $z$ axis, respectively. The system is fully actuated in the attitude, meaning that the system is controllable [2]. An attitude control system only needs to adjust the control torques in order to stabilize a quadcopter attitude.

### 2.1.1 Relationship between the control torques and rotors angular velocities

Each rotor produces a thrust, $T_i$, and the respective reaction force, from the drag, $D_i$ [3]. Those quantities can be expressed as proportional gain of the square of the angular velocities of the rotors, $\varpi_i$:

$$\begin{cases} T_i = c_T \varpi_i^2 \\ D_i = c_D \varpi_i^2 \end{cases}. \tag{2.2}$$

Assuming that the rotors are numbered and rotating as in figure insert_figure, and using the relationship (2.2), the control torques are given by

$$\begin{cases} \tau_x = T_3 - T_4 = c_T \left(\varpi_3^2 - \varpi_4^2\right) \\ \tau_y = T_2 - T_1 = c_T \left(\varpi_2^2 - \varpi_1^2\right) \\ \tau_z = D_3 + D_4 - D_1 - D_2 = c_D \left(\varpi_3^2 + \varpi_4^2 - \varpi_1^2 - \varpi_2^2\right) \end{cases} \tag{2.3}$$

The total thrust is given by the sum of each rotor thrust

$$T = \sum_{i=1}^{4} T_i = c_T \sum_{i=1}^{4} \varpi^2. \tag{2.4}$$

Together, equations (2.3) and (2.4) form a set of linear equations,

$$\begin{bmatrix} T \\ \boldsymbol{\tau} \end{bmatrix} = \underbrace{\begin{bmatrix} c_T & c_T & c_T & c_T \\ 0 & 0 & c_T & -c_T \\ -c_T & c_T & 0 & 0 \\ -c_D & -c_D & c_D & c_D \end{bmatrix}}_{\Gamma} \begin{bmatrix} \varpi_1 \\ \varpi_2 \\ \varpi_3 \\ \varpi_4 \end{bmatrix}, \tag{2.5}$$

and, by inverting $\Gamma$, one can obtain the desired rotor velocities.

## 2.2 Quaternion based model

Adapting the model (2.1) to an attitude representation given by quaternions has some advantages. This representation has no singularities, avoiding the gimbal lock problem, and it is unique [2] [4]. It is, though, a nonunique representation, which my give arise to the unwinding effect [2] Further exploit this.

A quaternion $\mathbf{q}$ is composed by a vector and a scalar part, $\mathbf{q} = [\mathbf{q}_v \;\; q_s]^\top$. A unit norm quaternion may be used to map a rotation between two coordinate frames, the same way as a rotation matrix. In that case, the quaternion can be viewed as representing a rotation around an axis

$$\mathbf{q} = \begin{bmatrix} \mathbf{e}\sin\left(\frac{\alpha}{2}\right) \\ \\ \cos\left(\frac{\alpha}{2}\right) \end{bmatrix} \tag{2.6}$$

where, following the notation in [1], $\mathbf{e}$ is the axis and $\alpha$ the angle of rotation.

Using quaternions, the attitude dynamics (2.1) becomes

$$\begin{cases} \dot{\mathbf{q}} = -\frac{1}{2}\mathbf{W}(\mathbf{q})\boldsymbol{\omega} \\ J\dot{\boldsymbol{\omega}} = J\boldsymbol{\omega} \times \boldsymbol{\omega} + \boldsymbol{\tau} \end{cases}, \tag{2.7}$$

where W definition goes here!.

## 2.3 Attitude control

## 2.4 Event triggered control

# Chapter 3

# Pratical Implementation

# Chapter 4

# Conclusions

# Bibliography


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