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Model-Following Control of a Helicopter in Hover

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Abstract

Tracking with model-following behavior for a Bell 205 helicopter is investigated using linear-quadratic optimal control. The objective of the design is to produce the desired performance in simulation for a nominal plant model of the Bell 205 in hovering flight. Two fundamentally different types of model-following control designs, explicit and implicit, are studied and discussed in this paper. The two designs result in controllers of different structures. The primary objective of the paper is to compare the merits of the resulting closed-loop systems.

1. Introduction

The use of model-following control system design for helicopter flight control is natural in situations where the closed-loop performance objectives are expressed in terms of target transfer functions [1]. Current military handling qualities specifications for the Bell 205 helicopter provide the necessary transfer functions for the various aircraft modes. They are viewed as constituting an ideal model with desired performance from which it is possible to construct a state-space realization of a linear time-invariant dynamic target model. Finally, it is a designer goal to reproduce this ideal performance in the actual aircraft and synthesize a control system that ensures this behavior.

The objective of this paper is to present the results of a study where model-following controller design techniques are used. The design may be based on explicit model-following (EMF), in which the target model explicitly appears in the controller [1], or by way of implicit model-following (IMF), for which a description of the model is used only in the synthesis process [2].

2. Aircraft Modeling

The aircraft considered in this paper is a Bell 205 helicopter, fully instrumented so that all relevant aircraft states are measurable. The aircraft can be configured to allow direct manual pilot control via hydraulic actuators or full-authority fly-by-wire actuation [3]. Six degree of freedom linear models of the Bell 205 are available at a variety of trim points from [4] and these models are to be used as nominal flight conditions. The steady hover condition is used as the trimmed operating condition as this is the starting point for a variety of important helicopter tasks such as precision hovering.

The state space model in (1) represents the aircraft trimmed at a nominal +5° pitch attitude, with a mid-range weight, a mid-position center of gravity and operating in-ground effect at near sea level. The model is described by

$$\dot{x} = Ax + Bu \tag{1}$$

where the matrices (A,B) are in [4]. The aircraft states are forward, lateral and vertical velocities, pitch, roll and yaw rates, and pitch and roll attitudes. The control variables are collective, longitudinal and lateral cyclic and tail collective.

3. Handling Qualities Specifications

The desired performance of the closed-loop aircraft is based on standard rotorcraft handling qualities specifications found in [5]. The report quantifies the minimum acceptable parameters defining the dynamics of an aircraft dependent upon the task to be performed (such as precision hover) and the flying environment. The flying environment is ranked on a Usable Cue Environment (UCE) scale, ranging from 1 (normal daylight visual environment) to 3 (extremely poor visual environment).

The specification details the type of response appropriate for the desired aircraft task and UCE. For example, to perform the precision hover task in a UCE of 2, an attitude command attitude hold (ACAH) response in pitch and roll axes with a rate command heading hold (RCHH) in vertical and yaw axes is specified. This specification implies that in pitch and roll a unit pilot input will command a unit attitude change in the aircraft. A unit input in vertical or yaw results in a unit rate response in these axes.

For level one handling qualities, the small amplitude pitch and roll responses are to be of an ACAH type with a bandwidth of at least 3.0 rad/sec. Coupling in the pitch and roll axes is to be less than 10%. The yaw axis response specifications are similar to those for pitch and roll except that yaw rate as opposed to attitude is the command output. For small amplitude maneuvers for level one handling qualities, the bandwidth is to be at least 3 rad/sec. The disturbance response in yaw to exogenous wind gusts shall not be objectionable to the pilot while the cross-coupling collective to yaw shall be below 15 %. The time constant of this first order vertical rate response shall not be less than 0.2 sec^{-1}. Coupling in the vertical and yaw axes should also be less than 10%.

4. Ideal Target Model

In this section a state-variable realization of an ideal target model is developed. According to the handling qualities specification, the desired closed-loop system is selected as low order with zero cross-coupling. Low order on-axis responses are specified as first order equations in forward, lateral, vertical and yaw respectively as

$$\dot{U} = -\lambda_U U - \lambda_U \theta, \quad \dot{V} = -\lambda_V V + \lambda_V \phi$$
$$\dot{W} = -\lambda_W W + \lambda_W r_W, \quad \dot{R} = -\lambda_R R + \lambda_R r_R.$$

The pitch and roll attitude responses must take into account the pitch and roll rates thus giving second order response characteristics

$$\dot{Q} = -2\zeta_\theta \omega_\theta Q - \omega_\theta^2 \theta - \omega_\theta^2 r_\theta, \quad \dot{\theta} = Q$$
$$\dot{P} = -2\zeta_\phi \omega_\phi P - \omega_\phi^2 \phi - \omega_\phi^2 r_\phi, \quad \dot{\phi} = P.$$

Combining these equations into a state space model yields the ideal target model

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}r. \tag{2}$$

The model is driven by the reference input, *r(t)*, which includes the vertical velocity, pitch and roll attitudes, and yaw rate commands. The target model states are identical to those of the actual plant. The state transition and input matrices for the ideal target model are denoted by the matrices ($\underline{A}$,$\underline{B}$). Values for the model parameters are presented below and slightly exceed those required by the handling qualities specifications

$$\lambda_U = 4.0, \quad \lambda_V = 4.0, \quad \lambda_W = 3.0, \quad \lambda_R = 5.0$$
$$\omega_\theta = 4.0, \quad \omega_\phi = 4.0, \quad \zeta_\theta = 0.7, \quad \zeta_\phi = 0.7.$$

5. Model-Following Control System Design

The following techniques use linear quadratic regulator theory in an attempt to minimize the error transients between the responses of the actual plant and those of the desired target model. An H_2 or linear quadratic (LQ) minimization criterion is selected for this design problem which will devise a controller that minimizes the area in the frequency domain between the system and target model responses over all frequencies for the steady-state optimal control solution. The system to be controlled is assumed, for purposes of control system design, to be exactly modeled by (1).

EMF Design: Command Generator Tracker Technique

The helicopter is fully instrumented to permit use of full state feedback

$$y = Cx, \quad C = I_8 .$$

Control law design objectives are directly related to the aircraft flight task from Section 3, resulting in the performance outputs

$$z = Hx = \begin{bmatrix} vertical\ \ velocity \\ pitch\ \ attitude \\ roll\ \ attitude \\ yaw\ \ rate \end{bmatrix}.$$

To produce the same forced response in the plant as in the target model, model states for feedback and model performance outputs are chosen as

$$\underline{y} = \underline{C}\underline{x} = C\underline{x}, \quad \underline{z} = \underline{H}\underline{x} = H\underline{x}.$$

Proper selection of the plant control *u(t)* will attempt to minimize the model mismatch error

$$e_e = \underline{z} - z = \underline{H}\underline{x} - Hx \tag{3}$$

such that the helicopter will demonstrate the desired time responses. Collecting the plant and ideal target model dynamics into one augmented system yields

$$\begin{bmatrix} \dot{x} \\ \dot{\underline{x}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & \underline{A} \end{bmatrix} \begin{bmatrix} x \\ \underline{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ \underline{B} \end{bmatrix} r \tag{4}$$
$$\dot{x}' = A'x' + B'u + G'r, \quad x' = \left[x^T \quad \underline{x}^T \right]^T.$$

The tracking problem is modified using the command-generator tracker (CGT) technique, whereby (4) is converted into a regulator problem [1]. The method is based on the fact that many reference commands *r(t)* of practical interest satisfy the differential equation

$$r^d + a_1 r^{d-1} + \ \dots \ + a_d r = 0 \tag{5}$$

for a given differential order d and coefficient set $\{a_i\}$. The above equation is called the *command generator* system. For example, the unit step of magnitude r_o satisfies

$$\dot{r} = 0, \quad r(0) = r_o . \tag{6}$$

Section 3 specifies a unit step reference input which satisfies (5) when the coefficients d and a_i are set to 1 and 0, respectively. The command generator characteristic differential equation is defined as

$$\Delta(m) = m^d + a_1 m^{d-1} + \ \dots \ + a_d m . \tag{7}$$

Performing this operation on the augmented system dynamics (4) with degree d and coefficient set $\{a_i\}$ satisfying (6) results in the modified system

$$\dot{\xi} = A'\xi + B'\mu. \tag{8}$$

The modified states and control inputs are

$$\xi = \Delta(x') = (x')^1 = \dot{x}'$$
$$\mu = \Delta(u) = (u)^1 = \dot{u}.$$

Most importantly the reference input $r(t)$ is known to satisfy (6) and so it disappears from the modified system dynamics. To clarify, the state ξ is partitioned as

$$\xi = \begin{bmatrix}\xi_p^T & \xi_m^T\end{bmatrix}^T$$

where the subscripts p and m denote plant and model, respectively. Applying (7) to the model mismatch error (3) results in

$$\begin{aligned}\Delta(e_e) &= \dot{e}_e \\ &= [-H \quad \underline{H}]\xi \\ &= H'\xi.\end{aligned}$$

In state variable form the errors are represented as

$$\begin{aligned}\dot{e}_e &= 0_4 e_e + [-H \quad \underline{H}]\xi \\ \dot{e}_e &= Fe_e + H'\xi.\end{aligned} \tag{9}$$

Finally, collecting the dynamics of (8) and (9) into one system produces

$$\dot{\begin{bmatrix} e_e \\ \xi \end{bmatrix}} = \begin{bmatrix} F & H' \\ 0 & A' \end{bmatrix}\begin{bmatrix} e_e \\ \xi \end{bmatrix} + \begin{bmatrix} 0 \\ B' \end{bmatrix}\mu \tag{10}$$

$$\dot{\xi}' = \tilde{A}\xi' + \tilde{B}\mu, \quad \xi' = \begin{bmatrix} e_e^T & \xi^T \end{bmatrix}^T.$$

Through use of (7) the dynamic system in (4) has been prefiltered to produce the modified system above that is not driven by the reference input, $r(t)$. Using (10) it is possible to perform an LQ regulator design, which will drive the states, including the model mismatch error $e_e(t)$ to zero. As such, we will attempt to design a control system that causes the helicopter to follow the reference command with performance like that of the target model.

Using full state feedback in (10) the objective is to minimize the error $e_e(t)$ without using excessive control energy $u(t)$. The performance index required to achieve this control objective is

$$J = \tfrac{1}{2}\int_0^\infty e_e^T Q e_e + u^T R u \; dt. \tag{11}$$

with $\mu(t)$ replacing $u(t)$ to permit an LQ regulator design. This indirectly achieves the original objective by attempting to minimize the model mismatch error $e_e(t)$ without using excessive change in control input with respect to time.

The augmented system (10) has many states and thus a large number of undetermined control gains. The design methodology however, makes selection of these gains clear and concise. According to the design objective, the weighting matrices Q and R are set as identity, I_4. This ensures equal weighting between the errors constituting $e_e(t)$ and the control variables, respectively. The relative weighting of the two matrices is then adjusted in order to meet the design objective. The error in terms of the state is written as

$$\begin{aligned} e_e &= [I_4 \quad 0_{4\times16}]\xi' \\ &= C_e\xi' \end{aligned}.$$

such that the state of (10) is weighted in the performance index using

$$\tilde{Q} = C_e^T Q C_e.$$

The design is summarized in Table 5.1.

System Model:	$\dot{\xi}' = \tilde{A}\xi' + \tilde{B}\mu$
Full State Feedback:	$\mu = -[K_e \quad K_p \quad K_m]\xi'$
Performance Index:	$J = \frac{1}{2}\int_0^\infty \xi'^T\tilde{Q}\xi' + \mu^T R\mu \; dt$

Table 5.1: Command Generator Tracker Technique

Performing an LQ regulator design to the above setup we may compute the control gains in the pseudo-control law

$$\mu = -K_e e_e - K_p\xi_p - K_m\xi_m.$$

Integrating the above equation yields the true control input

$$u = \int \mu \; dt = -K_e \int e_e dt - K_p x - K_m \underline{x}. \tag{12}$$

The control structure of (12) consists of state feedback, feedforward compensation of the target model, and a PI controller as the additional feedforward compensator in the error channel. The result is a dynamic controller that guarantees asymptotic tracking.

EMF Design: Reference Feedforward Technique

This section approaches the previous problem from a new and slightly different perspective. The objective remains to modify the tracker problem into one that can be handled by LQ regulator theory. Begin by defining

$$\dot{r} = Lr + e_e = Lr - Hx + \underline{Hx}. \tag{13}$$

For L = 0, solution to (13) yields

$$r(t) = r(0) + \int_0^t e_e \, dt.$$

This revised form of a reference signal is used as input to the closed-loop system to generate tracking behavior. The dynamics of the plant and target model are collected under one system in addition to the dynamics of (13) to produce the augmented system

$$\begin{bmatrix} \dot{x} \\ \dot{\underline{x}} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ 0 & \underline{A} & \underline{B} \\ -H & \underline{H} & L \end{bmatrix} \begin{bmatrix} x \\ \underline{x} \\ r \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u$$

$$\dot{\hat{x}} = A'\hat{x} + B'u, \quad \hat{x} = \begin{bmatrix} x^T & \underline{x}^T & r^T \end{bmatrix}.$$

This methodology creates an alternative system for which LQR theory is suitable. Once again the LQ regulator will drive the states and thus the model mismatch error to zero. The objective remains to minimize the model mismatch error $e_e(t)$ without using excessive control energy $u(t)$. Thus the performance index and weighting matrices from (11) are also used here. Replacing the plant state for the error $e_e(t)$ in (11) yields

$$J = \frac{1}{2}\int_0^\infty \hat{x}^T Q'\hat{x} + u^T Ru \ \ dt$$

$$Q' = \begin{bmatrix} H^T QH & -H^T Q\underline{H} & 0 \\ -\underline{H}^T QH & \underline{H}^T Q\underline{H} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

This methodology also converts the original tracking problem into one that may be handled by LQ regulator theory. The modified problem is summarized in Table 5.2.

System Model:	$\dot{\hat{x}} = A'\hat{x} + B'u$
Full State Feedback:	$u = -[K_p \quad K_m \quad K_r]\hat{x}$
Performance Index:	$J = \frac{1}{2}\int_0^\infty \hat{x}^T Q'\hat{x} + u^T Ru \ \ dt$

Table 5.2: Reference Feedforward Technique

The resulting control is

$$\begin{aligned} u &= -K_p x - K_m \underline{x} - K_r r \\ &= -K_p x - K_m \underline{x} - K_r r(0) - K_r \int e_e \, dt \, . \end{aligned} \tag{14}$$

The control structure of (14) is similar to (12) with the addition of $r(0)$ as a feedforward compensator. This model-following design likewise renders a dynamic controller guaranteeing asymptotic tracking.

IMF Design

The control system structure is assumed made up of state feedback with an added command feedforward compensator defined by

$$u = Kx + Fr = u_{fb} + u_{ff} \tag{15}$$

where $r(t)$ contains the command variables. The control system design problem is to select the matrices K and F to achieve the desired model behavior in the aircraft. The controller development is adapted from [1] and [2] and as used in [6].

The ideal target model is assumed described by (2). If the closed-loop system were to follow the target model perfectly with zero input then

$$\dot{x} = \underline{A}x \, .$$

This condition will not hold in general and so a model-following error $e_i(t)$ is defined as

$$\begin{aligned} e_i &= \dot{x} - \underline{A}x \\ &= (A - \underline{A})x + Bu. \end{aligned} \tag{16}$$

To make the model-following error $e_i(t)$ small without using excessive control energy, it is necessary to minimize

$$J = \tfrac{1}{2}\int_0^\infty e_i^T Q_i e_i + u^T Ru \ \ dt. \tag{17}$$

Substituting (16) into the performance index gives

$$\begin{aligned} J &= \tfrac{1}{2}\int_0^\infty x^T \hat{Q} x + 2x^T \hat{S} u + u^T \hat{R} u \ \ dt \\ \hat{Q} &= (A - \underline{A})^T Q_i (A - \underline{A}) \\ \hat{S} &= (A - \underline{A})^T Q_i B \\ \hat{R} &= R + B^T Q_i B. \end{aligned} \tag{18}$$

The state feedback which minimizes J from (18) is derived from

$$\begin{aligned} &P\hat{A} + \hat{A}^T P - PB\hat{R}^{-1}B^T P + Q^* = 0 \\ &Q^* = \hat{Q} - \hat{S}R^{-1}\hat{S}^T \\ &\hat{A} = A - BR^{-T}\hat{S}^T. \end{aligned}$$

The feedback portion of the applied control is given by

$$u_{fb} = -(\hat{R}^{-1}B^T P + \hat{S}\hat{R}^{-1})x = Kx \, .$$

The command feedforward processing is accomplished via

$$F = \hat{R}^{-1}B^T (A + BK)^{-T} \underline{A}^T Q_i \underline{B} \tag{19}$$

as specified in [7] to give u_{ff} in (15). Table 5.3 outlines the design.

System Model:	$\dot{x} = Ax + Bu$
Control law:	$u = Kx + Fr$
Performance Index:	$J = \frac{1}{2}\int_0^\infty e_i^T Q_i e_i + u^T Ru \ \ dt$

Table 5.3: Implicit Design Setup

The weighting matrices (Q_i, R) in (17) are chosen based on an inverse square rule [8]. These matrices are selected as diagonal with elements chosen to normalize contributions based on maximum deviations in the corresponding state or control. All control movements are in units of centimeters and a reasonable maximum expected deviation is 15 cm. Thus the control weighting matrix is selected as

$$R = \tfrac{1}{15^2} \times I_4 \, .$$

The forward and lateral velocities are not directly controlled. Therefore, the elements of Q_i corresponding to these terms in the first and fourth rows are set to zero. The pitch and roll rates are similarly not directly controlled. However, to preserve stabilizability in the LQR problem the third and fifth diagonal elements are selected as 0.01. For vertical velocity, a model tracking error of 1 m/s is a reasonable maximum. Therefore, the second diagonal element is selected as unity. A maximum yaw rate deviation of 0.1 radians per second is considered a reasonable goal. Thus the sixth diagonal element is selected as 100. Lastly the pitch and roll attitudes are to be kept as close as possible to the target model states. They are both weighted 400 corresponding to a deviation of 0.05 radians. The state weighting matrix can then be expressed as

$$Q_i = diag(0, 1, 0.01, 0, 0.01, 100, 400, 400).$$

6. Control Design Comparison

One of the objectives of the paper remains to present the results of controller simulation and compare the relative merits of the various designs. Four criteria are used to determine the effectiveness and practicality of the control system designs: closed-loop performance of the nominal plant, controller effort, controller complexity, and the expertise and tuning required of the system design.

Nominal Performance

The dynamics of the actual aircraft are compared to those of the ideal target model. A closed-loop system whose time responses more closely resemble the responses of the ideal target model exhibits preferred performance.

The EMF design techniques provide the desired time responses in each of the four channels. Figure 6.1 depicts the desired pitch attitude response to a 0.1 radian step input supplied by the CGT controller.

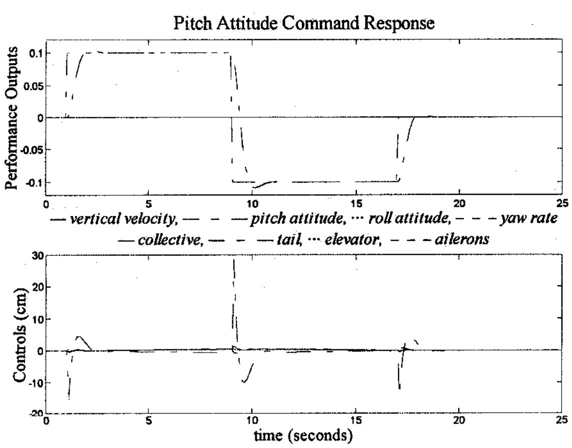

Fig. 6.1. EMF Pitch Attitude Response for the CGT Controller

Virtually no coupling is exhibited in the time responses. Conversely, the IMF technique only provides desired time responses for vertical and yaw rate commands. Its pitch attitude command response is poor, evident in Figure 6.2.

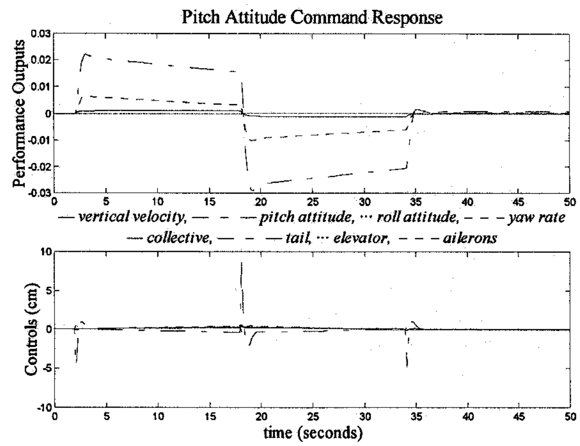

Fig. 6.2. IMF Pitch Attitude Response

The roll attitude command response is better, reaching a value of 0.07 radians for the same input. Pitch and roll attitudes are slightly coupled in each of these responses and do not actually reach steady-state in the allotted simulation time.

Control Effort

Assessment of the control effort is necessary to guarantee that the controllers do not require actuator movements of the helicopter exceeding the reasonable maximum expected deviation. If one control design yields a controller that drives the actuators with large control movements, then it should be discarded in light of a second design that requires smaller control movements of the actuators [9].

Comparing the pitch attitude response of the CGT controller in Figure 6.1 to that of the reference feedforward controller depicted in Figure 6.3

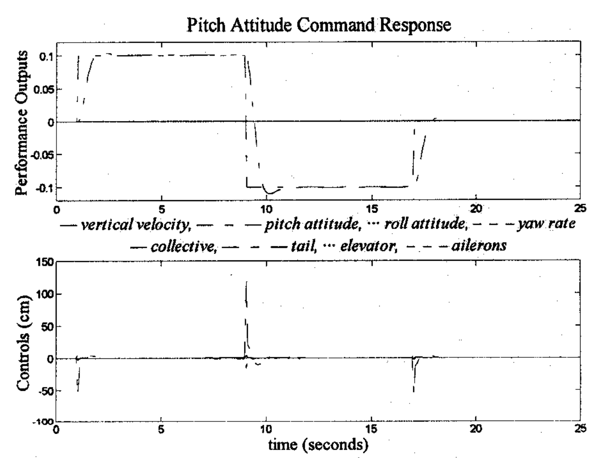

Fig. 6.3. EMF Pitch Attitude Response for the Reference Feedforward Controller

it is evident that control movements of dissimilar magnitudes are required for similar nominal performance. The size of control movements required of the CGT controller is smaller than those required of the reference feedforward controller. In some cases the size of these movements is very much smaller. The implicit control design on the other hand yields a controller that drives the actuators with smaller sized movements for the pitch attitude response. The CGT controller uses slightly smaller sized movements in the remaining channels.

Controller Complexity

Control design methodologies yield controllers of different structures of which some are more complex than others. As an example, one design methodology may yield a controller that requires dynamic compensation as opposed to static feedback [9]. In these terms, the controller that proves easiest to implement should be chosen.

The two fundamentally different types of model-following control, explicit and implicit, produce controllers of different structures. The IMF design merely uses the target model in defining the performance index. The resulting controller uses static feedback and reference command feedforward only. The model states do not appear in the control input $u(t)$. It is evident from (12) and (14) that simulation of the model is required as part of the feedforward compensators in the two EMF controllers. The EMF controllers developed here are much like standard PI controllers and so are not very complex. Controller complexity is therefore not a significant factor in choosing between the two designs.

Expertise in Selecting and Tuning of Design Parameters

In LQ regulator theory it is necessary to define the performance index weighting matrices Q and R. Experience and expertise are often required of some design methodologies to provide the desired performance objectives. Selection of the weighting matrices is imprecise and so may require modification [10]. Design methodologies that simplify the selection of the weighting matrices are preferable. They may reduce the number of design parameters, even for those systems with many states and numerous undetermined control gains. This reduces the design time required to achieve performance specifications.

The selection of the weighting matrices in the EMF design is simple and iteration is needed only to determine their relative weighting. The performance index in the IMF design is structured such that selection of the diagonal elements requires experience and ingenuity. The EMF design methodology simplifies the desired task and is preferable to the IMF design where advanced expertise and significant tuning are needed to achieve like goals.

7. Conclusion

This paper concerns itself with flight control system design for a helicopter in hover. The appropriateness of the model-following technique is recognized and used to achieve flight control with good results. All methods incorporate the ideal model yet produce controllers of different structures.

The EMF design yields distinct controllers which provide the desired target model performance. Arguments can be made to suggest that in-flight simulation of the model for the EMF controllers is not difficult and that the performance provided outweighs the deficiencies. Lastly, the clear and concise manner in which the design parameters are determined in the EMF design might make it preferable to the implicit design.

Considering the criteria the CGT controller is best suited for the application. Its control effort is acceptable with performance like that of the ideal target model. The control effort of the reference feedforward controller and performance of the IMF controller make them unlikely choices. The CGT controller is much like a standard PI controller and so is not very complex. The result is a dynamic controller that guarantees asymptotic tracking.

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