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Position Control by Feedback Linearization for a Simplified Helicopter Model

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Abstract

In this paper we describe a method for angular pitch control with feedback linearization for a laboratory helicopter model. Linear angular control can only be applied in a small angle interval. Better control can be obtained by using a linearizating transformation. This transformation tends to be very complex. For the application in this paper it can be shown that a simplified transformation can be sufficient for achieving good control results.

1 Introduction

The laboratory helicopter model in figure 1 is basically a pendulum with two degrees of freedom, which is driven by two dc motors with propellers. The arm of the system is mounted a few centimeters from the center of mass. Its position is described by the pitch angle ϑ and the yaw angle φ. For steering the pitch angle, the main propeller velocity ω_1 must be controlled by the armature current i_1 of the main motor M_1. With the second motor M_2 in the rear of the model the yaw angle φ can be controlled over the armature current i_2. Due to the weak coupling between the two motor systems both angles can be controlled separately. In this paper we consider the yaw angle to be perfectly controlled or slowly moving.

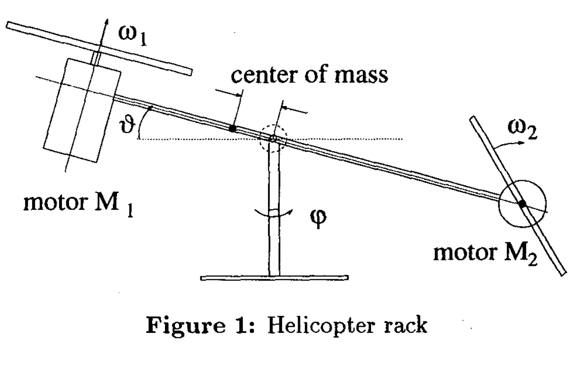

Figure 1: Helicopter rack

2 Pendulum Angular Control Problem

For simplicity, we will first consider the system as a single physical pendulum with equation:

$$\Theta\ddot{\gamma} = -M_s g \sin\gamma - c_\mu\dot{\gamma} + u \quad , \tag{1}$$

where Θ is the moment of inertia, $M_s g$ the torque of the center of mass, c_μ the friction coefficient, and u the control input torque.

The equation can be linearized at a chosen operating point angle γ_a. For deflection angles γ less than 90 degrees, the linearized system is stable. At angles equal or greater to 90 degrees the system is unstable. The family of eigenvalues of the system at different operating point angles forms a cross in the complex plane. Thus, the linear description is not sufficient for control, if the system is moving in a large angular interval.

The differential system fits in the canonical form:

$$\begin{aligned} y &= x_1 = \gamma \\ \dot{x}_1 &= x_2 = \dot{\gamma} \\ \dot{x}_2 &= \alpha(\mathbf{x}) + \beta(\mathbf{x})u \quad , \end{aligned} \tag{2}$$

with y as output and $\mathbf{x} = [x_1, x_2]'$ as the state vector. Those systems can be transformed into the Brunovsky form by the use of:

$$u = \frac{1}{\beta(\mathbf{x})}\left(-\alpha(\mathbf{x}) + v\right) \quad , \tag{3}$$

where v is a linear control input ([1]).

For example, with the use of a state space controller $(\mathbf{k}, \rho)$ with $v = -\mathbf{k}\,\mathbf{x} + \rho r$, where r is the input of the closed-loop control, the eigenvalues of the controlled system can be placed to a given pair of eigenvalues λ_1, λ_2.

This leads to the control law:

$$\begin{aligned} u = {} & M_s g \sin\gamma + (\Theta(\lambda_1 + \lambda_2) + c_\mu)\dot{\gamma} \\ & -\Theta\lambda_1\lambda_2\gamma + \Theta\lambda_1\lambda_2 r \quad . \end{aligned} \tag{4}$$

The nonlinear part of the control law is nothing but the static equilibrium torque of the current angle γ.

Normally the torque of the pendulum cannot be driven directly. There is an actuator motor system with an input $\tilde{u}$ which introduces a delay. The equation of motion must be expanded by the states of the motor system or augmentated with the transfer function $U(s) = T(s)\tilde{U}(s)$, in the best case a first order linear system. The use of equation 3 then leads to an unstable system. For a feedback linearization, the system can be transformed into a canonical form with differential geometric methods by exact state linearization ([1],[4]).

As a simpler way for linearization, one can try to use a filter $G_{inv}(s)$ inverse to the motor dynamics $T(s)$.

$$\begin{aligned}\tilde{U}(s) &= U_{feedback}(s) + U_{linear}(s) \\ U_{feedback}(s) &= G_{inv}(s) \quad \mathcal{L}\left\{\frac{-\alpha(\mathbf{x})}{\beta(\mathbf{x})}\right\} \\ G_{inv}(i\omega) &\approx T^{-1}(i\omega) \quad for \quad \omega < \omega_b\ , \end{aligned} \tag{5}$$

where ω_b is the bandwidth of the closed-loop control circuit and $\mathcal{L}$ the laplacian. The inverting filter $G_{inv}(s)$ shall compensate the dynamics of the motor system. The following application will show that this can also be done when the driving system is difficult to describe and the inversion is imperfect.

3 Mathematical Model of the Helicopter Rack

The helicopter system in figure 1 can be considered as divided into the two motor systems M_1 and M_2, which produce the driving torques $M_\vartheta(\omega_1)$ and $M_\varphi(\omega_2)$ by the propeller velocities ω_1 and ω_2, and a mechanical system.

The mechanical equation of motion for the angles can be described by:

$$\begin{aligned}\ddot{\vartheta} &= \Theta_x^{-1}\Big[-M_s g\cos\vartheta + \frac{1}{2}(\Theta_y - \Theta_z)\,\dot{\varphi}^2\sin(2\,\vartheta) \\ &\quad -\dot{\varphi}\Theta_{M1}\omega_1\sin\vartheta - c_{\mu\vartheta}\dot{\vartheta} - c_{m2}i_2 + M_\vartheta(\omega_1)\Big] \\ \ddot{\varphi} &= \Big[-(\Theta_y - \Theta_z)\,\dot{\vartheta}\dot{\varphi}\sin(2\,\vartheta) + \dot{\vartheta}\Theta_{M1}\omega_1\sin(\vartheta) \\ &\quad -c_{\mu\varphi}\dot{\varphi} - \cos(\vartheta)c_{m1}i_1 + M_\varphi(\omega_2)\cos(\vartheta)\Big] \\ &\quad \left(\Theta_y\sin^2\vartheta + \Theta_z\cos^2\vartheta\right)^{-1} \end{aligned} \tag{6}$$

where $\Theta_x \approx \Theta_z > \Theta_y$ stand for the moments of inertia of the helicopter model and Θ_{M1},Θ_{M2} are the moments of inertia of the motors. c_{m1}, c_{m2} are the motor constants and $c_{\mu\vartheta}, c_{\mu\varphi}$ the friction coefficients ([3]).

The Coriolis term affects only the φ angle. For small angle velocities of φ the effect the centrifugal acceleration and propeller gyroscopic influences are inferior against the gravity term for the ϑ- equation. The driving torque $M_\vartheta(\omega_1)$, as an input for the mechanical subsystem, is nearly a quadratic function of the angular propeller velocity ω_1 of the main motor, but in the velocity range used the torque can be described by the linear function:

$$M_\vartheta(\omega_1) \approx c\,\omega_1 + d \quad . \tag{7}$$

Due to air turbulence effects, the torque is not constant even for static propeller velocities. Small disturbances have to be taken in account.

In our application, we use an underlying time-discrete controller for the velocity ω_1 to improve the transient response of the motor. This controller drives an underlying analog control loop for the armature current i_1. The dynamic of this closed-loop control circuit is well described by the second order time-discrete transfer function $T(z)$.

4 Applied Simplified Feedback Linearization

The angular position control including the nonlinear feedback linearization has to be implemented by time-discrete control in a microcontroller. Thus, the control law has to be kept as simple as possible.

The system is linearized if the part of the control torque which holds the pendulum at its current angle is provided by the feedback linearization $u_{feedback}$ all the time. Additional torque u_{linear} from a linear controller will then move the system like a double integral system with friction.

For this reason the static torque for each angle can be computed by equation (8) or experimentally measured:

$$M_{\vartheta\ static} \approx M_s g\cos\vartheta + c_{m2}i_2 \quad . \tag{8}$$

This torque must be generated by the propeller velocity:

$$\omega_{1static} = (M_{\vartheta\ static} - d)/c \quad .$$

For the velocity ω_1, we have built an underlying time-discrete control loop, with an identified transfer function $T(z)$. Additionally there is a transfer function $T_?(z)$ which describes the generation delay for the torque by the changing propeller velocity ω_1. This transfer function is unknown and is thought to be changing, but it has only minor influence on the driving torque delay.

By augmentation of $\omega_{1static}$ with the inverting filter $G_{inv}(z) \approx T^{-1}(z)\,T_?^{-1}(z)$, the reference magnitude r_{ω_1} for the propeller velocity can be constructed by:

$$r_{\omega_1} = \omega_{1\,feedback} + \omega_{1\,linear} = G_{inv}(z)\omega_{1\,static} + \omega_{1\,linear}.$$

In our application, we use a simple time-discrete lead filter for $G_{inv}(z)$. Although it is quite far away from the real driving dynamic for the torque, it is sufficient to produce a stable linearization.

5 Linear Controller Synthesis

The linearized plant for the pitch angle ϑ is represented by a time-discrete state space model, where u_ν represents the input $\omega_{1\,linear}$ for the underlying propeller velocity control loop:

$$\begin{aligned} \mathbf{x}_{\nu+1} &= \mathbf{\Phi x}_\nu + \mathbf{H} u_\nu \\ y_\nu &= \vartheta_\nu = \mathbf{C x}_\nu \quad . \end{aligned} \tag{9}$$

Let $\mathbf{\Phi}$ be the transition matrix of the system, $\mathbf{H}$ the input vector, and $\mathbf{C}$ the constant output vector.
Because of the uncertain parameters of the model dynamics, a controller with integral part is needed. Therefore, the model is expanded by an integral state $x_{i\,\nu}$:

$$x_{i\,\nu+1} = x_{i\,\nu} + (r_\nu - \vartheta_\nu) \quad , \tag{10}$$

with reference magnitude r_ν for the closed-loop control. The controller is specified by the following structure:

$$u_\nu = -\mathbf{k x}_\nu - k_i x_{i\,\nu} + \rho r_\nu \quad . \tag{11}$$

The integral gain k_i is placed together with the gain vektor $\mathbf{k}$ by chosing the eigenvalues of the controlled system. Since the input gain ρ does not change the eigenvalues, it can be tuned for the response of the reference transfer function. For minimizing the overshooting real eigenvalues for the closed-loop control circuit are chosen.

6 Experimental Results

The controller was implemented on a 16-bit microcontroller "SAB 80C166". Operation is done under MATLAB on a PC with the use of the toolbox "MIRCOS" ([2]).

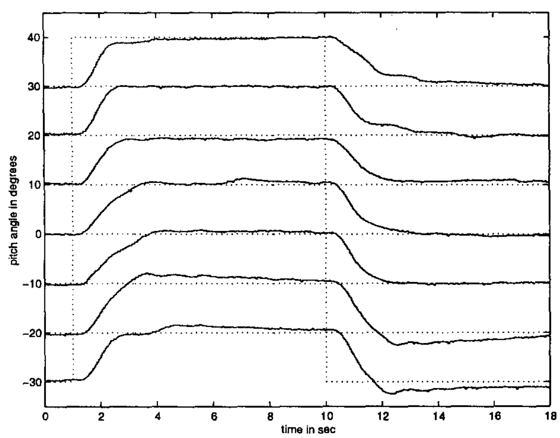

Figure 2: Equal steps starting at different pitch angles

With the linearization the step response of equal steps remains similar at different start angles (figure 2). Without linearization, there will be overshooting for low and high start angles.
Although the linearization filter $G_{inv}(z)$ is far from being perfect, the system remains stable even for large angle steps (figure 3).

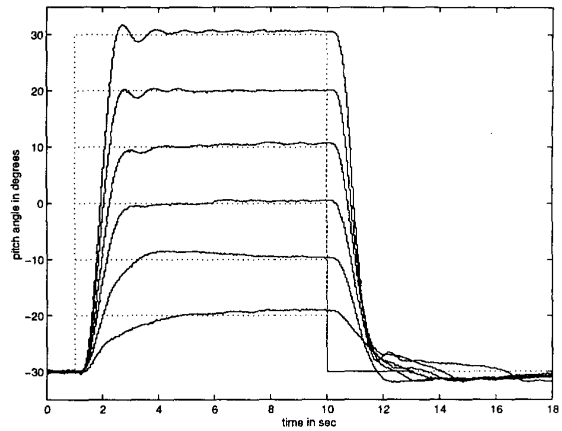

Figure 3: Steps with different amounts starting from the same pitch angle

7 Conclusions

The helicopter pitch angle can be controlled by dividing the control input u into two parts. The first nonlinear part $u_{feedback}$ holds the current angle, whereas the second linear part u_{linear} moves the system and controls the acceleration. To compute the static part of the control variable for the current angle an inverting filter must be installed because of the delay of the driving motor system. Compared with possible exact linearization this method is simpler and the implementation even on a relatively slow microcontroller is easy. The experimental results are good and prove the viability of the simplified method described here.

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