Event-Triggered Saturating Attitude Stabilization of a Quadcopter

DIOGO ALMEIDA

KTH Electrical Engineering

Master's Degree Project
Stockholm, Sweden 2014

IR-EE-SB 2000:099

Contents

Chapter 1

Introduction

The development of control techniques for Unmanned Aerial Vehicles (UAVs) has been an active research topic over the past years. Furthermore, Event-Triggering strategies have been recurrently studied and applied with encouraging results on reducing the number of computations required for the successful execution of control algorithms. In this thesis, the attitude control of a Quadrotor Helicopter is discussed, with a recently proposed nonlinear controller being implemented on a real system. An Event-Triggering rule for this controller is proposed and tested experimentally.

1.1 Quadcopter control

In recent years, UAVs have been the subject of many research projects and public scrutiny. While fixed wing UAVs have been successfully deployed in several different scenarios [**citation needed**], rotor blade ones are still being actively researched and improved upon. Helicopters have long been the preferred type of rotor blade vehicle for manned transportation. This is due to the fact that single and dual blade helicopters achieve a high level of thrust while maintaining a decent level of power consumption [1, pp 3-20]. The stability concerns can be addressed mostly through the mechanical design of the rotor blades and associated mechanical systems [2][3]. Check this out.

The concept of a quadcopter is not new. The first reported working system was developed in 1907 [4], and one of the most successful designs for early helicopter vehicles had precisely a quadcopter-like design [5]. Add more info.

A quadcopter helicopter is controlled by changes in the rotation speeds of its four rotors. By increasing the difference in speeds on rotors along the same axis, a torque around the other axis is generated, allowing for changes in the roll and pitch angles of the system. See figure... Each rotor produces a counteracting torque along its plane of rotation. By making the rotors along the same axis spin in the oposite direction of the rotors in the other axis, those torques are canceled and no torque along the yaw axis is created. To rotate the quad, one needs to increase the difference in speed of the rotors in different axis Figure....

These easy to understand concepts and high maneuverability makes the quadcopter an interesting platform to study, from the attitude stabilization of this inherently unstable system [6][7] to flight in formation even over complex environments [8]. This became possible over the last decade, with the increasing availability of appropriate inertial measurement units. These, together with onboard control units, allows for attitude control algorithms to run and stabilize the system.

1.2 Event Triggering Framework

Today's control systems are mostly digital, meaning that the control signal is not evolving continuously in time, but instead it needs to be computed at discrete times. Digital control has been extensively studied over the last century [9][10], but solid results were present only for time-triggered systems, where the control is updated at fixed time intervals. Nevertheless, it is known that event-triggered approaches may outperform time-triggered ones, although the analysis becomes more complex [11][12].

Some results for event-triggering control of dynamical systems have been derived in recent years, where the event-triggering and feedback rule are derived from a Control Lyapunov Function [13][14], while others create an event-triggering rule after defining the controller [15][16].

Mais info, melhor comparação entre abordagens -¿ requer mais leitura...

1.3 Problem Formulation

In this thesis, the Fast and Saturating Attitude controller [17] is discussed, and an event-triggering rule to work with it is derivated. The main pratical work consists in the implementation of the controller in an Arducopter platform.

The first problem considered will then be how to create an event-triggering rule that ensures the stability of the system with the chosen control rule.

Afterwards, considerations about the implementation and performance of the real system need to be addressed.

1.4 Thesis Outline

Chapter 2

Background

In this chapter, the project's background will be given. Firstly, the quadcopter mathematical model will be introduced, together with the quaternion attitude parametrization. Secondly, the proposed saturating controller is discussed. Finally, an introduction to event-triggered control is given.

2.1 The quadcopter attitude model

The attitude control problem of a quadcopter can be reduced to the attitude control problem of a rigid-body in 3D space [**citation needed**]. We can then define a world frame, E, fixed in space, and a body frame, B, coincident with the center of mass of the system see figure.... The system attitude is given by

$$\mathbf{x} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix},$$

where $[\phi \;\; \theta \;\; \psi]$ are, respectively, the rotations around the x, y and z axis of the quadcopter in its body frame (the Euler angles roll, pitch and yaw) and $\boldsymbol{\omega} = [\omega_x \;\; \omega_y \;\; \omega z]$ are the angular speeds around the same axis. The mapping from E to B can be done by a rotation matrix [**citation needed**], R, such that $B = RE$. By applying the Newton-Euler equations, an attitude model can be obtained:

$$\begin{cases} \dot{R} = R\boldsymbol{\omega}^{\times} \\ J\dot{\boldsymbol{\omega}} = J\boldsymbol{\omega} \times \boldsymbol{\omega} + \boldsymbol{\tau} \end{cases}, \tag{2.1}$$

where J is a 3×3 moment of inertia matrix, $\boldsymbol{\omega}^{\times}$ denotes the skew symetric matrix **[citation needed]** and $\boldsymbol{\tau} = \begin{bmatrix}\tau_x & \tau_y & \tau_z\end{bmatrix}^{\top}$ are the applied torques on the system, around the x, y and z axis, respectively. The system is fully actuated in the attitude, meaning that the system is controllable [18]. An attitude control system only needs to adjust the control torques in order to stabilize a quadcopter attitude.

2.1.1 Relationship between the control torques and rotors angular velocities

Each rotor produces a thrust, T_i, and the respective reaction force, from the drag, D_i [19]. Those quantities can be expressed as proportional gain of the square of the angular velocities of the rotors, ϖ_i:

$$\begin{cases} T_i = c_T \varpi_i^2 \\ D_i = c_D \varpi_i^2 \end{cases} . \tag{2.2}$$

Assuming that the rotors are numbered and rotating as in figure insert_figure, and using the relationship (2.2), the control torques are given by

$$\begin{cases} \tau_x = T_3 - T_4 = c_T \left(\varpi_3^2 - \varpi_4^2\right) \\ \tau_y = T_2 - T_1 = c_T \left(\varpi_2^2 - \varpi_1^2\right) \\ \tau_z = D_3 + D_4 - D_1 - D_2 = c_D \left(\varpi_3^2 + \varpi_4^2 - \varpi_1^2 - \varpi_2^2\right) \end{cases} \tag{2.3}$$

The total thrust is given by the sum of each rotor thrust

$$T = \sum_{i=1}^{4} T_i = c_T \sum_{i=1}^{4} \varpi^2. \tag{2.4}$$

Together, equations (2.3) and (2.4) form a set of linear equations,

$$\begin{bmatrix} T \\ \boldsymbol{\tau} \end{bmatrix} = \underbrace{\begin{bmatrix} c_T & c_T & c_T & c_T \\ 0 & 0 & c_T & -c_T \\ -c_T & c_T & 0 & 0 \\ -c_D & -c_D & c_D & c_D \end{bmatrix}}_{\Gamma} \begin{bmatrix} \varpi_1 \\ \varpi_2 \\ \varpi_3 \\ \varpi_4 \end{bmatrix}, \tag{2.5}$$

and, by inverting Γ, one can obtain the desired rotor velocities.

2.2 Quaternion based model

Adapting the model (2.1) to an attitude representation given by quaternions has some advantages. This representation has no singularities, avoiding the gimbal lock problem, and it is global [18] [20]. It is, though, a nonunique representation, which may give rise to the unwinding effect [18] Further exploit this.

A quaternion $\mathbf{q}$ is composed by a vector and a scalar part, $\mathbf{q} = [\mathbf{q}_v \;\; q_s]^\top$. A unit norm quaternion may be used to map a rotation between two coordinate frames, the same way as a rotation matrix. In that case, the quaternion can be viewed as representing a rotation around an axis

$$\mathbf{q} = \begin{bmatrix} \mathbf{e}\sin\left(\frac{\alpha}{2}\right) \\ \\ \cos\left(\frac{\alpha}{2}\right) \end{bmatrix} \tag{2.6}$$

where, following the notation in [17], $\mathbf{e}$ is the axis and α the angle of rotation.

Using quaternions, the attitude dynamics (2.1) becomes

$$\begin{cases} \dot{\mathbf{q}} = -\frac{1}{2}\mathbf{W}(\mathbf{q})\boldsymbol{\omega} \\ J\dot{\boldsymbol{\omega}} = J\boldsymbol{\omega} \times \boldsymbol{\omega} + \boldsymbol{\tau} \end{cases}, \tag{2.7}$$

where W definition goes here!.

2.3 Attitude control

2.4 Event triggered control

Chapter 3

Event-Triggered Rule

Chapter 4

System Implementation

Chapter 5

Conclusions

Bibliography

[1] W. Johnson. *Helicopter theory*. Dover Books on Aeronautical Engineering Series. Dover Publications, 1994.

[2] David A. Lee. “Rotary-Wing Static Stability”. In: *Encyclopedia of Aerospace Engineering*. John Wiley & Sons, Ltd, 2010.

[3] G.D. Padfield. *Helicopter Flight Dynamics: The Theory and Application of Flying Qualities and Simulation Modelling*. AIAA education series. American Institute of Aeronautics and Astronautics, 1996.

[4] G.P. Cyril. *The Bréguet-Richet Quad-Rotor Helicopter of 1907*.

[5] *Oemichen 1922*. URL: `http://www.aviastar.org/helicopters_eng/oemichen.php`.

[6] Samir Bouabdallah, Pierpaolo Murrieri, and Roland Siegwart. “2004-b, “Design and Control of an Indoor Micro Quadrotor”. In: *In Proc. of Int. Conf. on Robotics and Automation*. 2004.

[7] Tarek Hamel et al. “Dynamic Modelling And Configuration Stabilization For An X4-Flyer.” In: (2002).

[8] Aleksandr Kushleyev, Vijay Kumar, and Daniel Mellinger. “Towards A Swarm of Agile Micro Quadrotors”. In: *Proceedings of Robotics: Science and Systems*. Sydney, Australia, July 2012.

[9] J.R. Ragazzini and G.F. Franklin. *Sampled-data control systems*. McGraw-Hill series in control systems engineering. McGraw-Hill, 1958.

[10] K.J. Åström and B. Wittenmark. *Computer controlled systems: theory and design*. Prentice-Hall information and system sciences series. Prentice-Hall, 1984.

[11] Karl Johan Åström and Bo Bernhardsson. “Comparison of periodic and event based sampling for firstorder stochastic systems”. In: *Proceedings of the 14th IFAC World Congress*. 1999.

[12] Karl J. Aström. “Event Based Control”. In: *Analysis and Design of Nonlinear Control Systems*. Ed. by Alessandro Astolfi and Lorenzo Marconi. Springer Berlin Heidelberg, 2008, pp. 127–147.

[13] Nicolas Marchand, Sylvain Durand, and Jose-Fermi Guerrero-Castellanos. “A general formula for the stabilization of event-based controlled systems.” In: IEEE, 2011.

[14] Fermi Guerrero Castellanos et al. “Event-triggered nonlinear control for attitude stabilization of a quadrotor”. In: *Journal of Intelligent and Robotic Systems* (2013).

[15] Daniel Lehmann and Karl Henrik Johansson. “Event-triggered PI control subject to actuator saturation”. In: 2012.

[16] Paulo Tabuada. “Event-Triggered Real-Time Scheduling of Stabilizing Control Tasks”. In: ().

[17] Oliver Fritsch, Bernd Henze, and Boris Lohmann. “Fast and Saturating Attitude Control for a Quadrotor Helicopter”. In: (2013).

[18] N.A. Chaturvedi, A.K. Sanyal, and N.H. McClamroch. “Rigid-Body Attitude Control”. In: *Control Systems, IEEE* 31.3 (June 2011), pp. 30–51.

[19] Robert E. Mahony, Vijay Kumar, and Peter Corke. “Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor.” In: *IEEE Robot. Automat. Mag.* (2012), pp. 20–32.

[20] M. D. Shuster. “Survey of attitude representations”. In: *Journal of the Astronautical Sciences* 41 (Oct. 1993), pp. 439–517.