Improvement of helicopter handling qualities using H^∞-optimisation

A. Yue, MSc, DIC, DPhil
Prof. I. Postlethwaite, BSc(Eng), PhD, CEng, MIEE

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Abstract: In the paper, we present the results of a study into the use of H^∞-optimisation for the design of robust feedback control laws for improving the handling qualities of high-performance helicopters. A low-speed controller was designed to give accurate control of pitch and roll attitude, yaw rate and heave velocity for a typical combat helicopter. The design was based on a linearised model obtained from a six-degree-of-freedom nonlinear model of a helicopter provided by the Royal Aerospace Establishment (RAE), Bedford, UK. Robustness was a major issue because rotor dynamics were not included in the design model. Numerical compliance of the design with handling qualities requirements was assessed by testing simulated responses against specifications from a proposed airworthiness design standard. Robust stability was assessed from singular-value plots. In addition, the qualitative features of the design were assessed in a piloted flight simulation trial held at RAE, Bedford.

1 Introduction

Handling qualities is the name given to the total set of characteristics that make an aircraft suitable or unsuitable for a particular flying task. The most significant change affecting handling qualities of fixed-wing aircraft has been the adoption of digital FBW (fly-by-wire) control systems with full authority control. Future helicopters will also include FBW full authority control systems providing the opportunity for considerable improvement in handling qualities.

An unaugmented (open-loop except for the pilot) helicopter exhibits unacceptable responses in the hover. The responses to the collective, longitudinal and lateral cyclic, and pedals are highly coupled and unstable in the hover. Pilot workload is high and precise control is difficult without augmentation [1]. The helicopter, therefore, represents a challenge to any method of control system design. Furthermore, robustness is a primary issue because of model uncertainty. In this paper, we will examine the usefulness of H^∞ design for helicopter control.

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Dr. Yue is with the Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, United Kingdom

Prof. Postlethwaite is with the Department of Engineering, University of Leicester, Leicester LE1 7RH, United Kingdom

Various appproaches exist for the design of multivariable flight-control systems, the most popular methods use classical single-input/single-output (SISO) techniques to design one control loop at a time. For example, the Apache AV05 YAH-64 flight-control system [2], was designed using single-loop techniques, with a proportional-plus-integral (PI) configuration. This approach can be useful for certain problems, but its capability is severely limited for highly coupled multivariable systems. Furthermore, analysis of multivariable feedback systems with SISO techniques can give misleading results.

Alternatives to the frequency-domain methods include modal control and linear optimal control. Both of these approaches deal with state-space formulations which permit nonsquare systems. Garrard and Liebst, [3], and Parry [4], have proposed eigenspace assignment methods for the design of multivariable helicopter flight-control systems. Eigenvalue placement is used for stability enhancement and eigenvector shaping is used for modal decoupling. This approach can be useful when design requirements can be easily expressed in terms of desired closed-loop modal characteristics, but, in many cases, system design requirements cannot be so simply expressed. This is especially true for design requirements associated with tolerance to model uncertainty. Eigenspace assignment methods can also be sensitive to small perturbations.

The linear quadratic approach has been used by Broussard *et al.* [5] and Townsend [6], in the design of the Chinoock CH-47 flight-control system and an example of the LQG/LTR approach can be found in Reference 7.

Despite the growing activity in the development of advanced control laws, there are currently only a few FBW helicopters flying. These include a Bell 205 of Canada's National Aeronautical Establishment, an MBB BO105 of West Germany's DFVLR research agency and NASA's Bell UH-1 and Boeing-Vertol CH-47 rotorcraft. Each is a research machine having a full authority automatic flight-control system (AFCs) interfacing with the existing mechanical flight controls.

A brief review of H^∞ controller design will be given in the remainder of this introduction. In Section 2, the helicopter control problem is formulated and a low-speed controller is designed. The controller is evaluated in Section 3; quantitatively against a proposed airworthiness standard and qualitatively in a piloted flight simulation trial. Conclusions are given in Section 4.

What is H^∞ controller design? Mathematically, it is a frequency-dependent optimisation problem. Therefore all design specifications need to be expressed in the frequency domain. This is not always straightforward, particularly for time-domain specifications, but the approach is ideally suited for robustness considerations. In the optimisation procedure, a controller is selected which stabilises a nominal plant model and minimises the energy gain (the H^∞-norm) of a closed-loop transfer function which describes the design objectives. This closed-loop transfer function may be a combination of closed-loop functions and will include weights to tailor the design to meet the specifications; more on this later.

The H^∞-norm of a stable transfer function matrix $G(s)$, denoted $\|G\|_\infty$, is the maximum over all frequencies of the largest singular value of the frequency response $G(j\omega)$. Its power stems from two important results:

(i) A sufficient condition for closed-loop stability to be robust against a set of plant perturbations is given by a bound on the H^∞-norm of a stable closed-loop transfer function

(ii) The H^∞-norm of a stable transfer function matrix represents a bound on the maximum energy gain from the input signals to the output.

To be more precise about (i), consider the feedback configuration in Fig. 1, where the uncertainty in the nominal plant model $G(s)$ is represented by an additive perturbation $\Delta(s)$. Suppose, for simplicity, that $\Delta(s)$ is stable and that $\|\Delta(s)W(s)\|_\infty \leqslant 1$ where $W(s)$ is a weight which represents the variation of uncertainty with frequency and also normalises the H^∞-norm of the uncertainty to a maximum of 1. Then the perturbed feedback system is stable if the nominal feedback system ($\Delta(s) = 0$) is stable and $\|W^{-1}K(I + GK)^{-1}\|_\infty < 1$. Therefore, minimising $\|W^{-1}K(I + GK)^{-1}\|_\infty$ over the set of all stabilising controllers for $G(s)$ maximises the margin of stability.

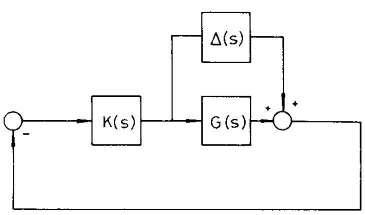

Fig. 1 *Additive perturbation*

To say more about (ii), consider the feedback configuration shown in Fig. 2, and suppose that we want to minimise the energy from the disturbance $d(t)$ to the output $y(t)$, the transfer function from d to y is $(I + G(s)K(s))^{-1}$ and therefore we want to find a stabilising controller $K(s)$ to minimise $\|(I + GK)^{-1}\|_\infty$ subject to internal stability. If the disturbance is known to have its energy concentrated over a given frequency band, then this can be specified by a weighting function W, and the problem is then to minimise $\|W(I + GK)^{-1}\|_\infty$.

There are a variety of closed-loop transfer functions which are useful in determining the behaviour of a feedback system. In the preceding discussion, the importance of the two functions $S(s) = (I + G(s)K(s))^{-1}$ and $K(s)S(s)$ has been demonstrated. Note that $K(s)S(s)$ is also the transfer function from r to u in Fig. 2, and therefore this

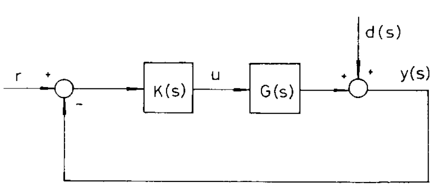

Fig. 2 *Disturbance rejection at the plant output*

function also provides information about the energy in the control signals.

In H^∞ design it is usual to combine several weighted closed-loop tranfer functions (of the type already shown) into a single composite transfer function whose H^∞-norm is minimised. The standard approach is to consider the general feedback configuration of Fig. 3, where the inputs

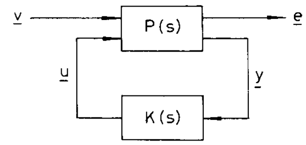

Fig. 3 *Standard compensator configuration*

and outputs v and e are selected so that the transfer function from v to e is the one whose H^∞-norm is to be minimised. $P(s)$ is an interconnection of the plant $G(s)$ and weights specified by the designer. If $P(s)$ is partitioned as

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

to conform with the

$$\text{inputs} \begin{bmatrix} v \\ u \end{bmatrix} \text{ and outputs} \begin{bmatrix} e \\ y \end{bmatrix}$$

then the closed-loop transfer function from v to e is

$$P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

This is called a linear fractional transformation of P and K and is denoted LFT(P, K). The standard H^∞ design problem is then to minimise $\|\text{LFT}(P, K)\|_\infty$ over the set of all stabilising controllers. The solution to this problem is now well understood [8–11] and can be computed automatically within a computer-aided design package such as Stable-H [12].

In this paper we are interested in the application of the H^∞ technique (not its solution) and the choice of weights to meet the design specifications. The selection of weights is an important aspect of H^∞ design and it is hoped that the presentation to follow on the helicopter controller design will assist future workers in this sometimes difficult task.

2 H^∞-optimal design for the helicopter

This section presents an application of H^∞-optimisation to the design of a control system for a typical combat helicopter and illustrates how practical problems may be formulated in the H^∞-framework. Particular attention is paid to the presence of model uncertainty and the motivation behind the selection of H^∞-weights. Attention will be restricted to designing a fixed-gain linear controller which is able to both stabilise the closed-loop system and maintain some nominal performance in the hover/low-speed flight envelope.

A theoretical mathematical model of a single main rotor helicopter has been developed at RAE, Bedford, primarily for flight mechanics studies with application to handling qualities evaluation [13]. This model is used for both control law design and ground-based simulator studies. Particular emphasis has been given to modelling the Puma and Lynx helicopters which are research vehicles at RAE, Bedford.

The nonlinear model, called HELISIM, has 6 fuselage degrees of freedom with optional quasi-steady or blade-flapping rotor dynamics. The rotor-flapping dynamics are difficult to model accurately owing to aerodynamic non-linearities. Actuators are modelled as first-order lags with rate and amplitude limits. Validation of the model has been made with flight test data in both steady and manoeuvring conditions. Although the model is very complex, some degree of uncertainty remains.

HELISIM is configured to run in conjunction with TSIM [14], an interactive computer-aided simulation package for use in flight-control-system design. The package allows simulation, analysis and design of continuous and discrete control systems, based on either linear or nonlinear models of the aircraft dynamics, and provides a means of linearising the equations of motion for any flight condition.

The uncertainties in the model were represented as unstructured norm-bounded perturbations. The linear controllers resulting from the design were then implemented in a full nonlinear simulation of the helicopter, including both rotor and actuator dynamics.

The dynamic modes of motion of the rotor have often been assumed to be of sufficiently high frequency so as not to couple with the airframe dynamics. The lowest-frequency mode in the multiblade co-ordinate system is the regressing flap mode which responds at about 1 Hz. This mode may couple with the fuselage roll subsidence mode or with some control system modes. Therefore it is important that the modes of the controller be kept small.

Input/output scaling of the nominal model is an important aspect in the design process. This is particularly so for multivariable systems where signals have a directional as well as a frequency distribution. Certain actuators might saturate at lower signal levels, some sensors are noisier than others, so it is important to emphasise the relative importance of each signal.

To illustrate the H^∞-design technique, a so-called 2-block model-matching formulation is used. The control laws were developed to provide decoupling, desensitisation and stability augmentation in the face of aircraft modelling uncertainty. The control laws are analysed with frequency and time responses, and stability robustness is assessed using singular value techniques.

Because the helicopter control system is intended to demonstrate advanced rotorcraft system performance and handling, it is a good test to use the proposed standards defined in Reference 15 as a guide to assess the response characteristics in this case study. An attitude-command/attitude-hold (ACAH) system was chosen, because in simulation studies, it was considered to be a requirement for satisfactory handling qualities, especially in the hover/low-speed flight region. For an ACAH system it is desired that pilot longitudinal stick commands correspond to pitch attitude, lateral stick, to roll attitude, collective to heave velocity and pedal position to yaw rate.

2.1 Hover/low-speed controller

The basic 6-degrees-of-freedom linearised model of the helicopter has 8 states and 4 inputs, and is unstable and nonminimum phase. The nonminimum-phase characteristic of the helicopter, limits the achievable performance [16–17]. If the rotor-flapping dynamics are included, a 14-state model is obtained, but the rotor dynamics will be treated as uncertain and left out of the nominal plant description. The actuator dynamics will also be neglected in the linear description of the model, to reduce the complexity of the controller.

The effect of the unmodelled actuator dynamics on the closed-loop system will become significant when the time constants of the filters are of similar magnitude to the desired closed-loop responses. They have the effect of making the response more sluggish. To eliminate this effect, it is necessary to include the actuator dynamics in the original plant description.

The state-space description of the linearised rigid-body equations of motion are expressed in the standard form as:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \tag{1}$$

where the state vector x and input vector u are:

$$x = \begin{bmatrix} \theta \\ \phi \\ p \\ q \\ r \\ u \\ v \\ w \end{bmatrix} \qquad u = \begin{bmatrix} \theta_0 \\ \theta_{1s} \\ \theta_{1c} \\ \theta_{0t} \end{bmatrix} \tag{2}$$

The state variables and inputs are described in Table 1 and the measured outputs are shown in Table 2; it is assumed that all these outputs are available from appropriate sensors.

Table 1: State variable and plant input description

State	Description	Input	Description
θ	pitch attitude	θ_0	collective
ϕ	roll attitude	θ_{1s}	longitudinal cyclic
p	roll rate	θ_{1c}	lateral cyclic
q	pitch rate	θ_{0t}	tail rotor collective
r	yaw rate		
u	forward velocity		
v	lateral velocity		
w	vertical velocity		

Table 2: Measured outputs

Outputs for controller	Description
$\dot{h}$	heave velocity
p	roll rate
q	pitch rate
$\dot{\psi}$	heading rate
θ	pitch attitude
ϕ	roll attitude

Robustness is a primary issue in the design because of model uncertainty (especially due to the omission of high-frequency rotor dynamics and actuator dynamics), sensor noise and the uncertain response to wind gusts. The maximum achievable bandwidth is determined by the high-frequency dynamic characteristics, especially those which are due to the rotor dynamics, and high-gain feedback may aggravate these. This problem has been reported by Tischler [18], in the implementation of the ADOCS system, in which a 40% reduction of feedback gains was necessary because of unmodelled rotor lead-lag dynamics. Tischler also reports problems in implementing high-bandwidth systems whose designs are based on time-domain methods such as LQG. This was a result of the compensators being too highly tuned to the assumed high frequency dynamics, so that, when the compensators were used on the actual system with additional unmodelled dynamics, unacceptable system sensitivity and instability resulted. Frequency-domain techniques such as H^∞ are able to expose the high-frequency details more easily and integrate with current proposed handling-quality specifications which are based on frequency-domain criteria.

By examining the open-loop poles of the linearised helicopter model about the trimmed hover position, it is possible to see that the rotor dynamics are significant at frequencies above 10 rad/s, indicating that a bandwidth of less than 10 rad/s is desirable if these dynamics are to have little effect.

2.2 Design of H^∞-optimal controller

H^∞-performance specifications are given in terms of bounded energy. As the $\|\cdot\|_\infty$ norm is defined as a supremum over frequency, it is possible to include frequency-dependent weights at the inputs and outputs to appropriately specify performance. The selection of the weights determines the characteristics of the controller.

Care must be taken to ensure that the problem is well posed. In particular, in Fig. 3 the transfer functions between $\boldsymbol{v}$ and $\boldsymbol{y}$ ($\boldsymbol{P}_{12}$) and between $\boldsymbol{u}$ and $\boldsymbol{e}$ ($\boldsymbol{P}_{21}$) should be invertible at all frequencies. The physical interpretation of these constraints is that, if $\boldsymbol{P}_{12}$ loses rank, this would lead to unconstrained controller action, and, if $\boldsymbol{P}_{21}$ loses rank, this would correspond to an error being uncontrollable.

The H^∞-design procedure will produce a controller $\boldsymbol{K}$, which fits the feedback structure shown in Fig. 4. The H^∞-synthesis framework requires that outputs be quantities that are kept small. For example, tracking errors and actuator movement. Therefore the problem in Fig. 4 is reformulated to get the required structure shown in Fig. 5.

The linearised model of the helicopter was taken about the hover flight condition. The outputs chosen for control

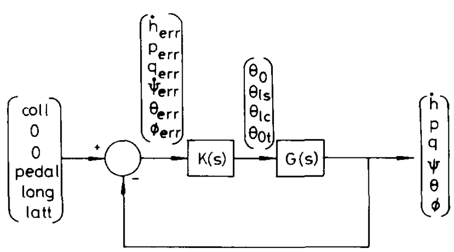

Fig. 4 *Attitude feedback loop*

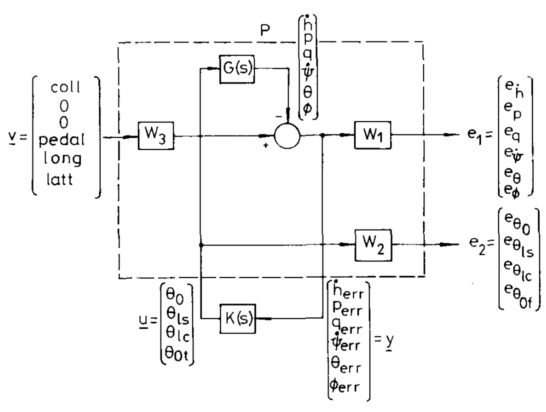

Fig. 5 *Interconnection structure*

were scaled as in Table 3. The effect of these scalings is such that a unit step input on each of the pilot inceptors would result in the responses given in Table 3. For example, a unit step command in collective would result in a response of 10 ft/s in heave velocity. These scalings ensure that each output channel is of equal importance. The choice of scaling requires some engineering insight into what is physically possible in the real system.

Table 3: Output scalings

Outputs	Scaling
$\dot{h}$	10 ft/s
p	0.5 rad/s
q	0.2 rad/s
$\dot{\psi}$	0.5 rad/s
θ	0.2 rads
ϕ	0.5 rads

The open-loop frequency response of the linearised helicopter model is shown in Fig. 6. From this Figure, it can be seen that there are singular values at -45 dB and -47 dB which mean that closed-loop tracking will be very poor in these channels. Also, the difference in magnitude between the smallest and largest singular value is about 65 dB, resulting in a condition number of 1778. Therefore, the plant is close to being singular at low frequency. The design implication of this is that any attempt to provide compensation at low frequency by inverting the plant may result in erroneous results.

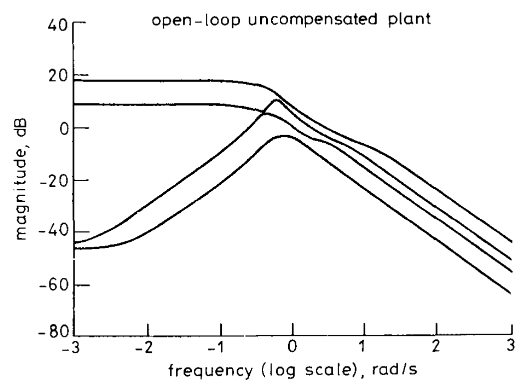

Fig. 6 *Singular-value plot of uncompensated helicopter at the hover*

In particular, any loop-shaping approach which inverts the nominal plant and then multiplies by the desired loop shape, to obtain the controller, will lead to a very complicated controller and one which is not very robust. This is because, if the controller is based on the inverse of the plant, it must essentially have the same condition number. A high-condition-number controller will perform badly in the presence of any plant input uncertainty. In practice, pseudo-inverse precompensation cannot be exactly realised without unstable pole-zero cancellation, but it may be possible to use a constant approximation to it.

The peformance criterion chosen in the design was

$$\left\| \begin{bmatrix} \boldsymbol{W}_1(\boldsymbol{I}+\boldsymbol{GK})^{-1}\boldsymbol{W}_3 \\ \boldsymbol{W}_2\boldsymbol{K}(\boldsymbol{I}+\boldsymbol{GK})^{-1}\boldsymbol{W}_3 \end{bmatrix} \right\|_\infty \tag{1}$$

$\boldsymbol{W}_1$, $\boldsymbol{W}_2$ and $\boldsymbol{W}_3$ are weighting functions to be chosen. There is no loss of generality in assuming that the weighting functions are both stable and minimum-phase, as it is the magnitude of the frequency response which is to be shaped. A key feature of the design procedure is that the largest singular value of the frequency response of eqn. 1 is independent of frequency. In the attitude-control design the sensitivity term $(\boldsymbol{I} + \boldsymbol{GK})^{-1}$ is used to reduce the tracking deviations and also to attenuate any system disturbances. The term $\boldsymbol{K}(\boldsymbol{I} + \boldsymbol{GK})^{-1}$ is used to improve the system robustness to additive perturbations [19] and also to reduce the amplitude of the control signals to the actuators.

Initial studies showed that it was not possible to obtain good control of the outputs $\dot{h}$, $\dot{\psi}$, θ and ϕ without information from q and p, the pitch-rate and roll-rate sensors, therefore these outputs were added.

2.2.1 Weighting function selection: In choosing the weighting functions, it is convenient to assume the problem is normalised to a value of 1. The weights are then obtained iteratively. In practice, the procedure is to find weights so that the optimal cost is approximately 1. The rationale for this is that, if the optimal cost is greater than 1, then performance is not achieved, and, if the optimal cost is less than 1, then the performance specified by the weights is, in some sense, conservative.

The design problem and specifications give a general shape for the weighting functions. The desired frequency responses should be in inverse proportion to the magnitudes of the weights, if they are chosen to have disjoint frequency ranges.

(*a*) *Selection of $W_1(s)$:* To ensure good tracking accuracy in each of the controlled outputs $\dot{h}$, $\dot{\psi}$, θ and ϕ, integral action is required and this specifies a $1/s$ shape for the weights. There is no point, however, in controlling the rates and angular position to less than the resolution of the sensors; this allows the weights to be levelled off at low frequencies. In tuning the weights, it was found that a finite attenuation in these channels was useful in reducing overshoot. Therefore, high-gain lowpass filters were used to ensure that these outputs could be controlled accurately with good disturbance attenuation up to 6 rad/s (see Fig. 7). $\boldsymbol{W}_1(s)$ will be dominant in the frequency range from 0 to 10 rad/s. The presence of unmodelled rotor dynamics beyond 10 rad/s makes it impractical to widen the frequency range of $\boldsymbol{W}_1$.

With only 4 plant inputs, no attempt was made to control directly the extra rate outputs p and q at low frequencies, but second-order bandpass filters were used on each of these variables to reject disturbances and crosscoupling effects in the frequency range from 4 to 7 rad/s.

(*b*) *Selection of $W_2(s)$:* First-order highpass filters are used with a cutoff frequency of 10 rad/s to limit the system bandwidth and also to limit the magnitudes of the poles of the controller. The actuator penalty weight $\boldsymbol{W}_2$ is increased at higher frequencies to inhibit fast actuator movement. This is done in order that any digital implementation of the controller will not be an issue. Simulations showed that allowing the actuators to be unconstrained resulted in the controller having some poles situated in the extreme left half-plane, resulting in a very large dynamic range over which the controller must operate. This is clearly not physically realisable.

A low-frequency gain of -100 db was used so that $\boldsymbol{W}_1$ dominates at low frequencies.

(*c*) *Selection of $W_3(s)$:* The diagonal weighting function $\boldsymbol{W}_3$ was chosen to be a constant matrix with a weight of 0.1 on each of the (fictitious) rate input demands and 1 on each of the remaining inputs. The reduced weighting on the rates (which are not directly controlled) is chosen so that some disturbance rejection is obtained on these outputs, without them significantly affecting the cost function (6.1). That is, the primary aim of $\boldsymbol{W}_3$ is to force good tracking in $\dot{h}$, $\dot{\psi}$, θ and ϕ.

The design weights $\boldsymbol{W}_1$, $\boldsymbol{W}_2$ and $\boldsymbol{W}_3$ are given by

$$\boldsymbol{W}_1 = \mathrm{diag}\left\{0.5\,\frac{(s+12)}{(s+0.012)}, \frac{2s}{(s+4)(s+4.5)}, \frac{2s}{(s+4)(s+4.5)}, 0.5\,\frac{(s+10)}{(s+0.01)}, 0.89\,\frac{(s+2.81)}{(s+0.005)}, 0.89\,\frac{(s+2.81)}{(s+0.005)}\right\}$$

$$\boldsymbol{W}_2 = \mathrm{diag}\left\{0.5\,\frac{(s+0.0001)}{(s+10)}, 0.5\,\frac{(s+0.0001)}{(s+10)}, 0.5\,\frac{s+0.0001)}{(s+10)}, 0.5\,\frac{(s+0.0001)}{(s+10)}\right\}$$

$$\boldsymbol{W}_3 = \mathrm{diag}\,\{1.0, 0.1, 0.1, 1.0, 1.0, 1.0\}$$

$\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are plotted in Fig. 7.

2.2.2 Analysis of the controller: The design process leads to a controller with as many states as the interconnection structure of Fig. 5. In this case 20 states arising from the following: plant (8) and performance weights (12). The controller initially had 39 states, but balancing and truncation of the small Hankel singular values [20], gives an 18-state controller without any significant change in the singular-value plots of the optimal cost function.

The frequency response of the controller is shown in Fig. 8. It can be seen that the controller has high gain at

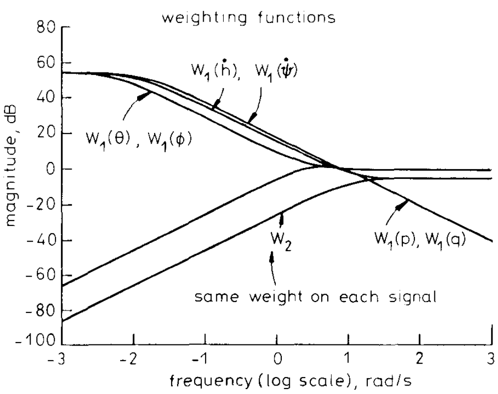

Fig. 7 *Performance weightings W_1 and W_2*

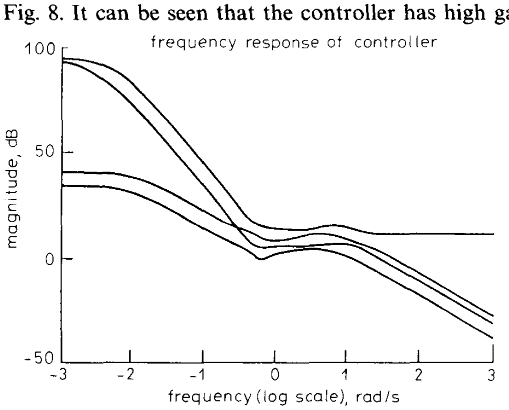

Fig. 8 *Singular values of controller*

low frequency, for good tracking, and low gain at high frequency, for robustness. This is consistent with the specification of the performance weight $W_1(s)$. An interesting characteristic of H^∞-controllers is that they are always proper (for well defined problems), and so there is at least one singular value of the controller which remains constant at high frequency.

The following Subsections will summarise the analysis of the helicopter control system and will concentrate on linear techniques, although extensive nonlinear simulations were used to verify the control laws.

2.2.3 Sensitivity: A plot of the singular values of $(I + GK)^{-1}$ is shown in Fig. 9. For disturbance attenuation at the plant output and for tracking performance, this function is required to be small over the operating bandwidth. The four singular values which are small at low frequency correspond to the controlled outputs and the other two are a result of the inherent interaction between the aircraft attitudes and angular rates.

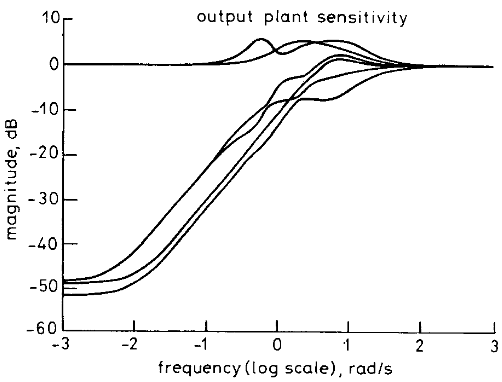

Fig. 9 *Singular values of $(I + GK)^{-1}$*

The largest singular value of $(I + GK)^{-1}$ also determines the size (maximum singular value) of the smallest unstructured output inverse multiplicative perturbation that could destabilise the system [19, 21, 22]. Let a bound on the maximum singular value of the perturbation be $\tilde{\delta}_0$. For robust stability at the plant output, $\bar{\sigma}[(I + GK)^{-1}]$ should be less than $1/\tilde{\delta}_0$. From Fig. 9 it can be seen that $\bar{\sigma}[(I + GK)^{-1}] = 6\,\text{dB} = 1/\tilde{\delta}_0$, at $\omega = 0.75$ rad/s. This implies that the smallest perturbation that will destabilise the system equals 0.5. This value of $\tilde{\delta}_0$ guarantees a gain margin of

$$\left(\frac{1}{1 - \tilde{\delta}_0}, \frac{1}{1 + \tilde{\delta}_0}\right)$$

and a phase margin of

$$\pm\cos^{-1}\left(1 - \frac{\tilde{\delta}_0^2}{2}\right)$$

[23]. Thus there is a guaranteed gain margin of (2.00, 0.67) and a phase margin of $\pm 29°$ in every output channel.

However, one normally associates gain and phase margins at the input to the plant. These can be determined by considering the largest singular value of $(I + KG)^{-1}$ which can also be used to calculate the smallest unstructured input inverse multiplicative perturbation $\tilde{\Delta}_i$ that could destablise the system. A plot of the singular values of $(I + KG)^{-1}$ is shown in Fig. 10. From this Figure, it can be seen that $\bar{\sigma}[(I + KG)^{-1}] = 4\,\text{dB}$ at $\omega = 0.8$ rad/s. This implies that the smallest perturbation that will destabilise the system is given by $\bar{\sigma}(\tilde{\Delta}_i) = 0.63$. This gives rise to a guaranteed gain margin of (2.71, 0.61) and phase margin of $\pm 36.8°$.

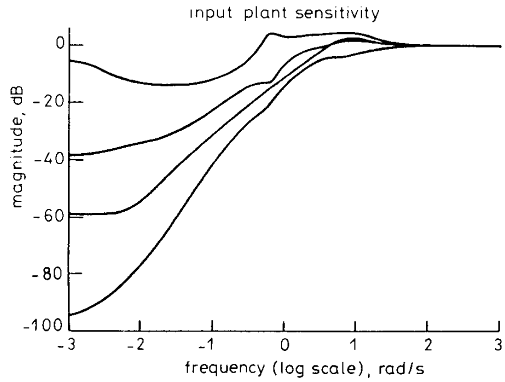

Fig. 10 *Singular values of $(I + KG)^{-1}$*

At first sight, the gain margins given above seem to be poor, this is because the gain margins calculated using singular values are generally conservative. Any peaks in the sensitivity function will cause $(I + GK)^{-1}$ or $(I + KG)^{-1}$ to be greater than 1. It is hard to remove these peaks without losing performance and is a consequence of the system being nonminimum-phase. For nonminimum-phase systems, it is not possible to attain good sensitivity at some frequencies without the sensitivity being poor at other frequencies [17].

2.2.4 Complementary sensitivity: A plot of the singular values of $(I + KG)^{-1}KG$ is shown in Fig. 11. $\bar{\sigma}[(I + KG)^{-1}KG]$ determines the size of the smallest unstructured input multiplicative perturbation Δ_i that could destabilise the system. Fig. 13 suggests that the closed-loop system is stable for all Δ_i such that $\bar{\sigma}(\Delta_i) <$ 0.45. The unity gain crossover frequency is 8 rad/s, which is within the desired value of 10 rad/s.

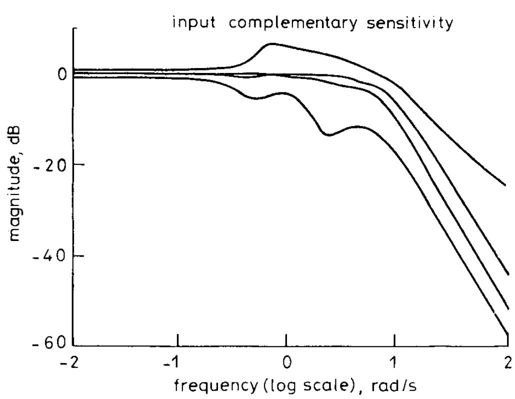

Fig. 11 *Singular values of $(I + KG)^{-1}KG$*

Fig. 12 shows the rotor dynamics modelled as an unstructured input multiplicative perturbation and superimposed is $(\bar{\sigma}[(I + KG)^{-1}KG])^{-1}$. From this Figure, it can be seen that the robustness test is satisfied for all frequencies and, therefore, the closed-loop system should be stable with the introduction of the extra rotor degrees of freedom.

$\bar{\sigma}[KS(s)]$ is useful for determining the smallest unstructured additive perturbation that could destabilise the system. The uncertainty of the helicopter dynamics due to changes in flight condition can be represented as an unstructured additive perturbation (Fig. 13). From this Figure, $\bar{\sigma}[E(s)] = 25$, which suggests that, for stability, $\bar{\sigma}[KS(s)] = 0.04$, which represents virtually no control movement; i.e. the most robust control is open-loop control! This stability robustness test is clearly very conservative and is of no practical use.

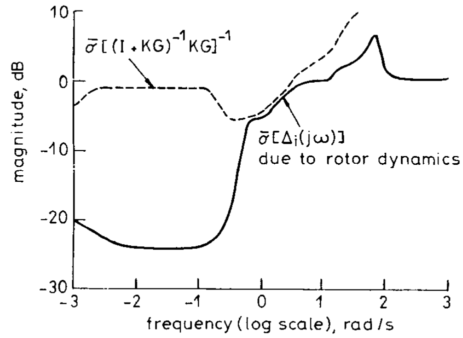

Fig. 12 *Singular value robustness test*

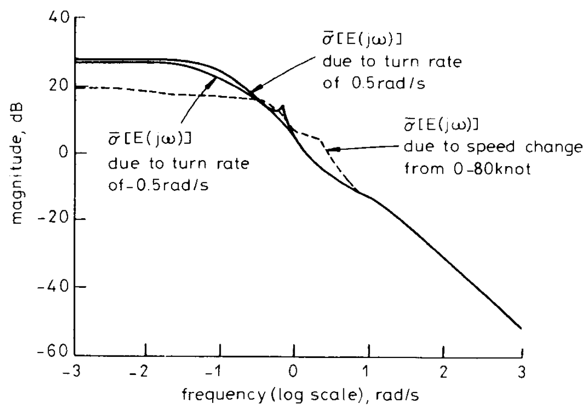

Fig. 13 *Perturbations in helicopter dynamics due to speed and turn rate*

2.2.5 Time simulations: The H^∞-controller was then simulated in the TSIM environment [14], using the nonlinear model HELISIM [13]. This nonlinear model contained rotor-flapping dynamics and actuator dynamics, which were initially omitted from the nominal linearised description of the plant. The following effects were observed as a result of including rotor-tip path plane dynamics into the helicopter model:

(i) the time constant associated with the roll subsidence mode is increased

(ii) neither the dutch roll mode nor the phugoid oscillation is changed significantly

(iii) longitudinal and lateral flapping contribute significantly to the pitch-rate and roll-rate equations, respectively

(iv) the aircraft is now very sensitive to roll gain.

Models of the actuators included blade-angle deflection and rate limits [24].

Figs. 14 and 15 show nonlinear time responses to the longitudinal and lateral cyclic inputs, together with the actuator-blade responses. These time responses are not significantly different from the linear responses, even with the addition of the extra rotor dynamics, which indicates the robustness of the design to these unmodelled dynamics.

The greatest concern is the change in dynamics of the helicopter with manoeuvering flight. Recall that the nominal design model is linearised about the hover flight condition and, therefore, any change in the characteristics of the helicopter with speed represents unmodelled dynamics. This is seen quite readily in Fig. 15, which shows a step command of 30° of bank angle. As the helicopter banks, the tilt of the thrust vector causes the heli-

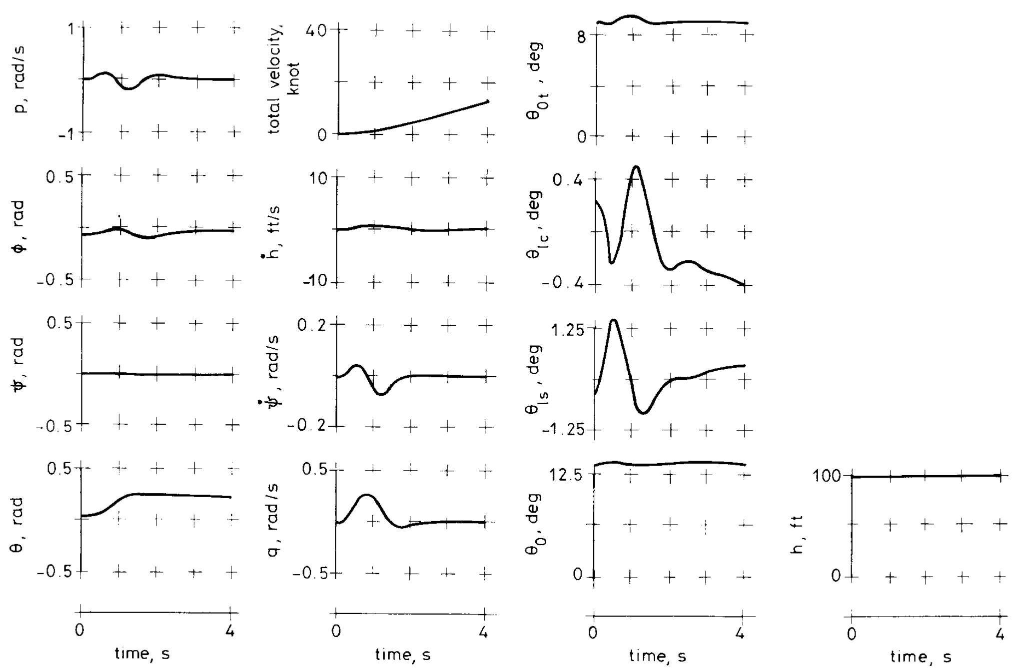

Fig. 14 *Step command of 0.2 rad in pitch attitude*

copter to have a resultant lateral acceleration. The change in speed from 0 to 40 knot represents a considerable change in the dynamic characteristics of the helicopter.

scales is discussed and a summary of the existing techniques is presented.

A piloted simulation study to assess the effectiveness of advanced control laws was initiated as part of an RAE/Oxford University research programme to study robust control laws for helicopters. This paper describes the first phase of the study which took place at RAE, Bedford, during August to September 1988. During this study, handling characteristics were assessed at the hover and low-speed flight region. The next stage of the work, which is being carried out at Leicester University, is the design of a control system to operate over the helicopter's full flight envelope.

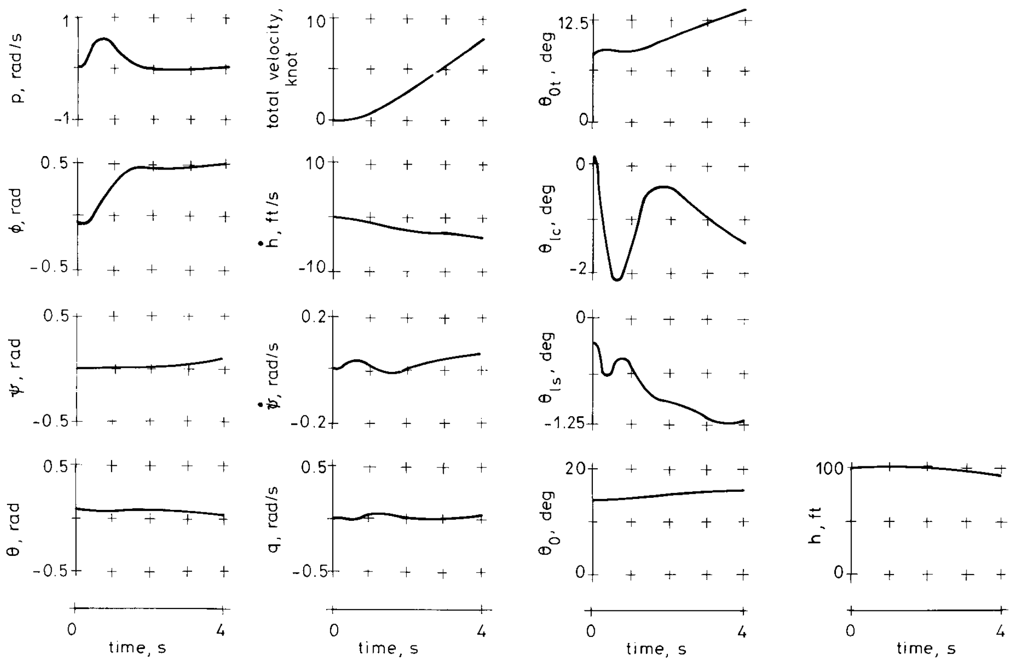

Fig. 15 *Step command of 0.5 rads in roll attitude*

The difficulty experienced by the pilot, in achieving co-ordinated control, can be illustrated by considering the helicopter's response to a step command in longitudinal cyclic. To achieve a transition from hover to forward flight requires forward cyclic with a corresponding increase in collective, or height is lost owing to the change in the thrust vector. As more collective is applied, there needs to be a change in pedal position to offset the extra main rotor torque. As the helicopter starts to move, there is an asymmetric increase in the blade lift (greater on the advancing side of the disc) so the helicopter tends to pitch nose up and possibly roll to one side. This needs further forward cyclic and lateral cyclic to compensate for the pitching and rolling. Also, variations in the engine torque will require further pedal action. It is not difficult to see then why pilots find helicopters fatiguing to fly without some form of augmentation.

No actuator limits were reached with the manoeuvres simulated, indicating that the design had been successful in limiting the actuator power needed. However, if these limits are reduced, to force limiting to occur, then instability results owing to integral wind-up. To offset this, some form of antiwind-up device is necessary. It is hard to encapsulate this nonlinearity in the design of the compensator, so that performance is maintained even when actuator limits are reached. The best that can be achieved is to constrain the actuator power needed to perform a manoeuvre, but is is accepted that this may lead to a possible deterioration in performance.

3 Handling qualities evaluation

This section discusses the problem of how to determine whether a flight vehicle exhibits satisfactory handling qualities. The difficulty of trying to define suitable rating

The controller designed in Section 2 was assessed against specifications from the proposed airworthiness design standard [15]. Having established that the controlled responses were within the 'right ball park', this controller was further evaluated under simulated flight on a motion simulator. The aim of the exercise was to obtain an assessment of the control law by performing a series of tasks. The control law was developed with the aim of being able to realise it practically with current hardware technology. Practical considerations in the scheduling of the trials restricted the amount of time individual pilots flew the simulator.

3.1 Pilot rating techniques

The suitability of a pilot-controlled vehicle to serve its intended purpose is ultimately assessed by a series of judgments. Perhaps the most difficult is the evaluation of the vehicle's handling qualities, which play such a key role in the overall suitability of the vehicle, and yet have been, in the past, perplexing even to define satisfactorily. Unlike for example performance, reliability, maintainability, durability and structural integrity, for which targets can be set, specifications defined and compliance measured by appropriate testing, handling qualities have traditionally belonged to a subjective discipline, with pilots differing in some cases quite widely in their opinion of the helicopter's merits. Very often, the pilot's comments are signficantly biased by previous experience of the aircraft type, role flown or simply piloting skill. With the absence of any quantitative assessment, pilots frequently resort to adjectives such as 'sluggish', 'responsive' or 'crisp' to describe their pleasure or displeasure with a particular machine, comments which, if of a negative nature and without quantitive detail, can make improvement of the design difficult.

In the process of measuring and evaluating pilot/vehicle performance, it is necessary, as one facet of the investigation, to measure pilot opinion. These subjective measures are, in fact, the ultimate evaluation of the system and, consequently, are foremost in the designer's mind throughout vehicle development. Unfortunately, the current connections between pilot ratings, pilot behaviour and vehicle characteristics are, at best, highly qualitative. This situation has not improved as vehicles and associated pilot/vehicle handling qualities considerations have steadily increased in complexity, for then the difficulties with existing rating scales and subjective measures become still more obscure.

The problem is centered on transforming the subjective assessment into engineering parameters. The most successful method to date has been the Cooper-Harper pilot rating scale (Fig. 16), in which the pilot can classify the desirable and unsatisfactory handling aspects on a points system, scaled from 1 to 10, where 1 represents the most satisfactory qualities. A rating of 10 represents major and unacceptable system deficiencies, where control may be lost during part of the flight envelope [25].

3.2 Basis for rating scales

Pilot evaluation is intended to meet two objectives:

(i) to provide an overall assessment of the suitability of the vehicle for its intended use

(ii) to provide information on deficiencies that may exist in the vehicle's characteristics, which interfere with its intended use.

The first objective requires that the pilot be able to express a subjective impression of the handling qualities of the vehicle, in performing the required manoeuvres. This 'impression' is the sum total of all the physical factors which contribute to the handling qualities of the vehicle. As there is no common physical measure which integrates all of the factors, a scale must in part be in subjective terms.

The second objective requires that the pilot be able to provide information on specific problem areas, to aid the experimenter or designer in solving the problems. Thus, a language is required which is unambiguous to as large a population as possible, to minimise training requirements and to maximise repeatability.

3.3 Comparison of the design with proposed handling qualitites requirements

The specifications outlined in the proposed airworthiness design standard are designed to define the minimum response that will allow accomplishment of the mission-task element with level 1 handling qualities [15]. (A task is defined to be level 1 if it is given a Cooper-Harper rating from 1 to 3 and level 2 if it is given a rating from 4 to 6.) Experience has shown that there are a large number of response characteristics that are important for good handling. However, it is not practical to attempt to formulate every conceivable response to show compliance with the dynamic requirements. It is proposed, instead, to specify certain fundamental response characteristics which are important in each axis and to show compliance with the necessary specification. Those specifications which are obviously not yet applicable have been ignored, such as those dependent on the aircraft flight control computer, hardware and software failure states and specifications defining control forces or breakout forces.

The information and data used to compare the controlled helicopter with the proposed MIL-H-8501 [26] revision was derived by computer simulation of flight manoeuvres.

Use of a mathematical model introduces the limitation that results are dependent on the accuracy of the model and ultimately require confirmation by flight tests. For the majority of handling qualities requirements, compliance could be demonstrated through time responses and frequency- response analysis.

This Section demonstrates that the basic response characteristics required for the ACAH system described in Section 2 are met. To illustrate this, it is necessary to quote the relevant sections from the design specification.

3.3.1 Attitude hold: 'If an attitude hold is required, the pitch (roll) attitude response to a pulse longitudinal (lateral) controller input shall return to within $\pm 10\%$ of the peak in less than 10 s for level 1. For level 2, there shall be no tendency for pitch (roll) attitude to diverge following the pulse input.' $t_{10\%}$ denotes the time taken for the response to reach 10% of its final value.

Figs. 17 and 18 show the responses for the augmented helicopter. It can be seen that the requirements are easily met for level 1 handling qualities. These Figures also suggest that the pitch and roll attitude response to a pulse input are almost linear.

3.3.2 Heading hold: 'If heading hold is required, the heading response to a pulse input of the control actuator shall return to within $\pm 10\%$ of peak in less than 10 s for level 1. For level 2, there shall be no tendency for heading to diverge following the pulse input.'

The heading hold response is shown in Fig. 19 and, once again, it can be seen that this requirement is met.

3.3.3 Height response to collective controller: 'The rise time ($t_{R_{50}}$) in altitude rate following an abrupt step input of the collective controller shall be within 2.8 s for level 1 and 4 s for level 2. The time delay ($t_{R_{25}}$) and response shaping parameter

$$\frac{(t_{R_{50}} - t_{R_{25}})}{(t_{R_{75}} - t_{R_{50}})}$$

shall meet the boundaries specified in Fig. 20.' $t_{R_{50}}$ denotes the time required for the response to a step input to reach 50% of its final value. Similarly, for $t_{R_{25}}$ and $t_{R_{75}}$. The above times assume a good useable cue environment.

The altitude rate response is linear with step collective input. The resultant response for the augmented system is superimposed on Fig. 20 and is within the level 1 region.

3.3.4 Large-amplitude heading changes: 'The ratio of peak yaw rate to change in heading $r_{pk}/\Delta\psi$ shall exceed the limit specified in Fig. 21. This requirement applies for control inputs large enough to produce at least a $\pm 10°$ heading change, up to a heading change of 40°.'

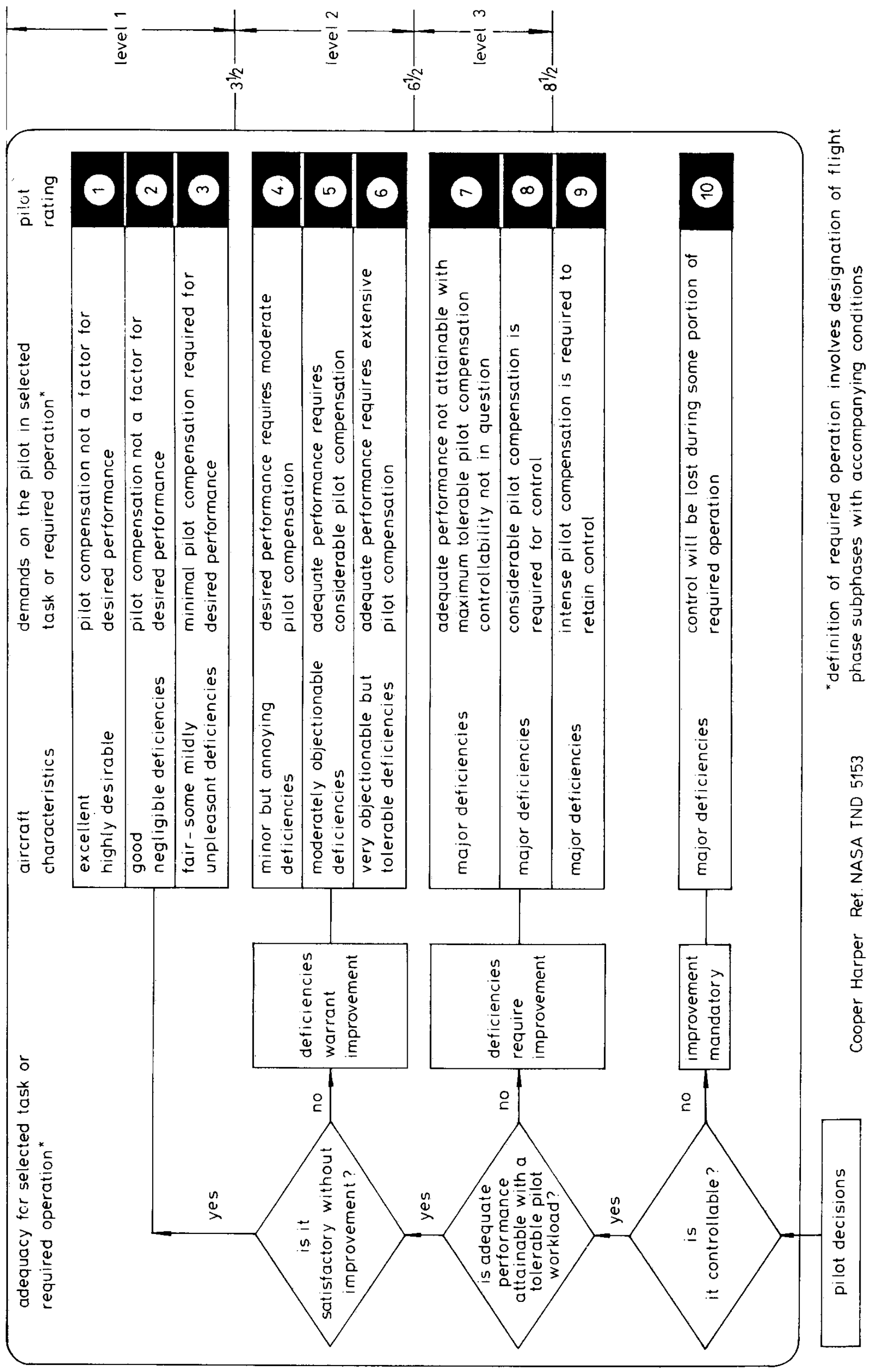

Fig. 16 *Cooper-Harper handling qualities rating scales*

It can be seen from Fig. 21 that more angular rate per change in heading is required for small changes in heading. This is a translation of the pilot requiring faster responses during tight manoeuvres and slower responses during more gentle manoeuvres. As the controller has not been designed to provide nonlinear responses, the handling qualities rating degrades to level 2 for attitude angles less than 20°. It is easy to push the line drawn on Fig. 21 higher, so that it lies above the level 1 curve for all attitude changes. However, this would mean that the controlled response would be unnecessarily great for large heading changes. To achieve the specification would require a scheduling of controllers able to provide fast responses for small angular changes and slower responses for large angular changes.

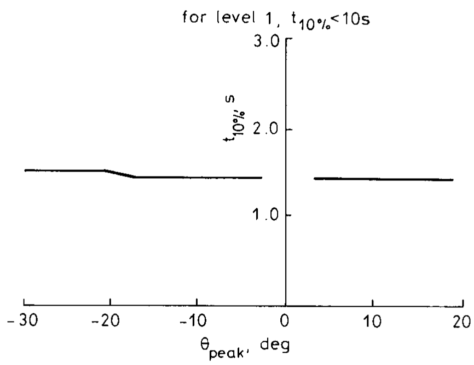

Fig. 17 *Pitch attitude response to a pulse longitudinal controller input*

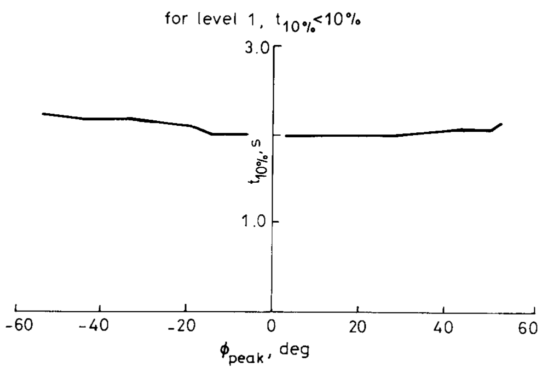

Fig. 18 *Roll attitude response to a pulse lateral controller input*

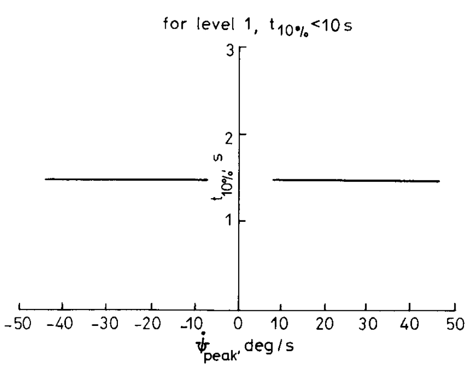

Fig. 19 *Heading response to a pulse pedal input*

3.3.5 Interaxis coupling: Fig. 22 shows the collective-to-yaw coupling. It can be seen that this coupling is nonlinear and the level 1 boundary is exceeded for altitude rate demands of greater than 30 ft/s.

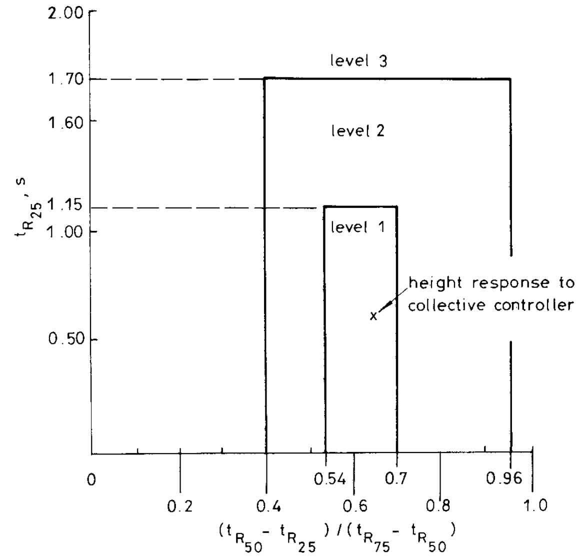

Fig. 20 *Limits on shape of vertical-axis response*

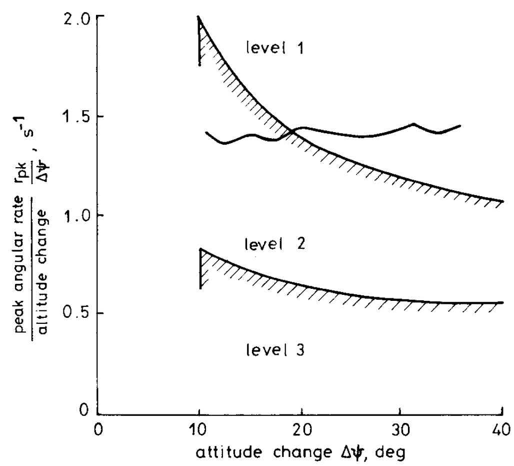

Fig. 21 *Heading response limits for large control inputs*

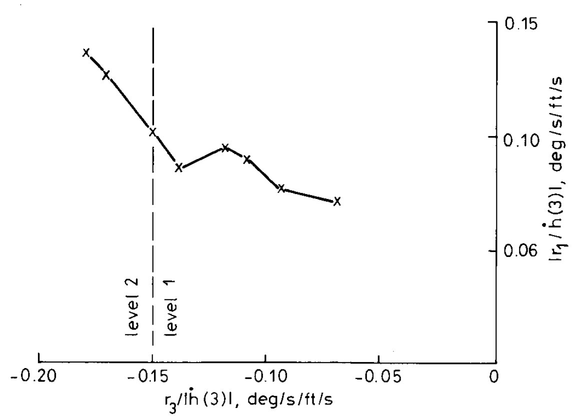

Fig. 22 *Collective-to-yaw coupling*

The pitch-to-roll and roll-to-pitch couplings are shown in Fig. 23 and 24. These responses all lie within the level 1 region.

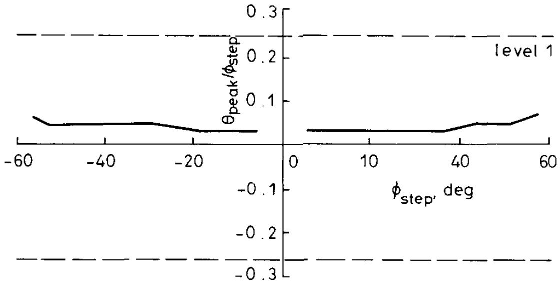

Fig. 23 *Pitch-to-roll coupling*

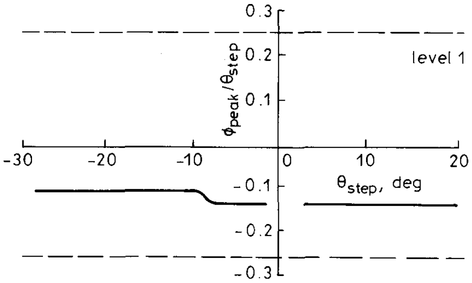

Fig. 24 *Roll-to-pitch coupling*

3.4 Flight test requirements

In addition to showing numerical compliance with parameter limits and boundaries, it is also necessary to conduct a series of qualitative inflight-handling qualities tests [27]. The need for precisely defined tasks to promote aggressive closed-loop flight tracking has led to the creation of a set of mission task elements at RAE, Bedford, for evaluating helicopter handling qualities. These include a series of concentric circular tasks, a spiral and triple-bend course. Some of the tasks are illustrated in Fig. 25.

Detailed questionaires have been designed to help the pilot describe the interaction between task cues, control strategy and task performance. The validity of the simulations are tested qualitatively, through general pilot impressions and quantatively by comparison with flight test data. The pilot will fly the task three times (or more if desired) for each evaluation and, then, the handling qualities are evaluated using the Cooper-Harper rating scale and a pilot comment card. The procedure is to assign a pilot rating immediately after the task is completed, and to record the comments using the comment card.

3.5 Simulator characteristics

The trials were carried out in the single-seat cockpit flight simulator, at the Managment Department, RAE, Bedford. A view of the outside world was produced by a closed-circuit television system, in which a TV camera tracked across a 700 : 1 scale model landscape, carried on a continuous belt, in response to position and attitude demands from the computer. Motion cues were provided to the pilot in pitch, roll and heave, and vibration was simulated as a function of g and speed. Audio cues included engine roar, turbine whine and rotor flapping. A number of specific features exist on the terrain belt to define courses for pilots to fly as part of the assessment task. The collimated terrain image gave the pilot an effective field of view of 47° horizontally and 35° vertically.

Aircraft models for the AFS (Advanced Flight Simulator) are operated in a modelling environment known as SESAME (System of Equations for the Simulation of Aircraft in a Modular Environment) [28]. Essentially, SESAME requires an aircraft model in the form of software modules to calculate the total forces and moments on the aircraft, as functions of aircraft states and control inputs. From such a model, SESAME provides a real-time simulation by computing linear and angular accelerations and integrating to give velocities and displacements. SESAME is also used for communicating with the simulator cueing systems (motion, visual, sound, cockpit displays), to provide simulation control and operation.

At present, the AFS is tied to a single, fixed, frame rate of 50 Hz (20 ms frame rate). To integrate from acceler-

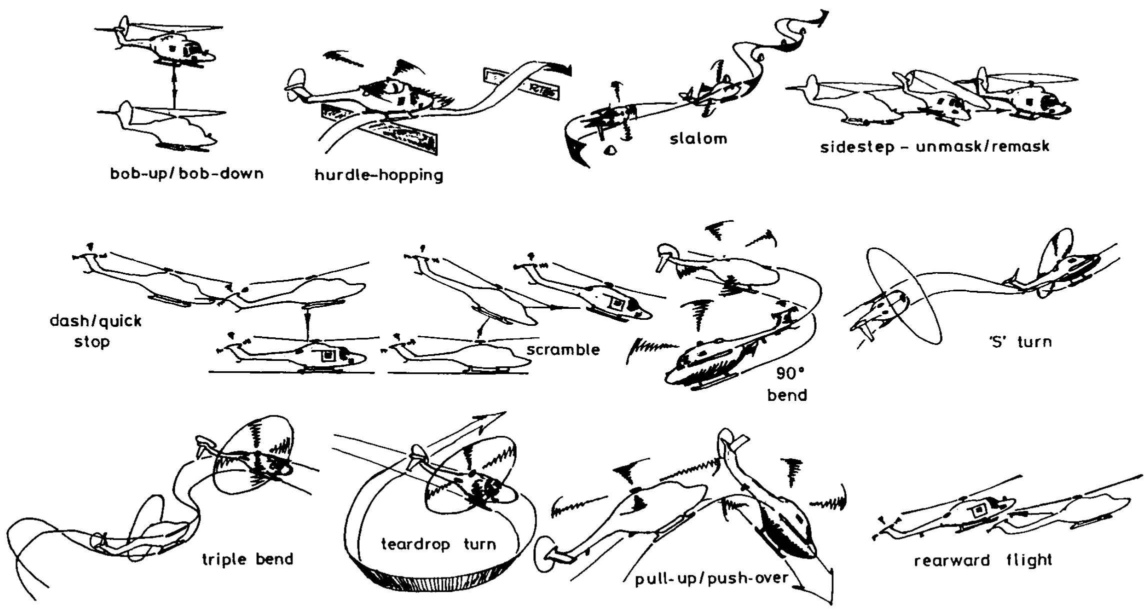

Fig. 25 *Sketch of agility manoeuvres*

ations to velocities, the Adams-Bashforth second-order algorithm was used, and, to integrate from velocities to positions, the trapezoidal algorithm was used.

In the design of the controls laws, no assumptions have been made regarding the physical aspects of the inceptors, e.g. whether the cyclic inceptor should be a central stick or a side-arm controller. It is assumed that a means of independent input into each of the four axes (collective, roll, pitch and yaw) is available. In each of the pitch, roll and yaw inceptors there is a central deadband of 2.5% of the total stick travel. The collective inceptor is a displacement stick with linear scaling, there is no deadband. The three pilot inceptors were scaled so that a maximum displacement in either the collective, pitch, roll or yaw axes demanded 30 ft/s heave velocity, 30° of pitch attitude, 60° of roll attitude or 60° of roll attitude or 60° per second of heading rate, respectively.

The three angular rate gyros (roll, pitch and yaw) are modelled as first order lags with a very small time constant and the three accelerometers (longitudinal, lateral and normal) are each modelled by second-order dynamics. Attitude information is derived by integrating the outputs of the angular rate gyros through an attitude transformation. Height information is derived from the normal, lateral and longitudinal accelerometers, using estimates of attitude to transform the body axis accelerations into an earth referenced set. A head-up display (HUD) was used during the simulations to compensate for the lack of visual cues.

3.6 Simulation trials

Before any trials work on the motion simulator could be conducted, it was necessary, first, to test the controller offline. All control law design had been carried out using the helicopter mathematical model HELISIM, however, this software was not configured for real-time use. A subset of the HELISIM code was used to generate a rationalised helicopter model (RHM), which is the model used for real-time helicopter simulation. Two versions of this model exist at RAE, Bedford; one which runs under the TSIM environment on a VAX 11/785 computer and the other on the Concept Gould-32 computer. The former is used to verify control law design for the RHM and the latter for real-time use.

One of the major concerns in implementing H^∞-controllers in real-time, was whether there would be sufficient frame time available to execute all the differential equations. Previous to this exercise, the controller had always used double-precision arithmetic and 4th-order Runge-Kutta integration with a 15 ms step size. This choice of frame time was to facilitate the inclusion of high-frequency rotor-flapping dynamics in the HELISIM model. Rotor-flapping dynamics are not included in the RHM because of the limited frame time of 20 ms.

The user available integration routine for the RHM was rectangular. Unfortunately, this proved to be inadequate, as drifting in the states could readily be seen. Instead, a 2nd-order Runge-Kutta routine was used with single-precision arithmetic and this proved successful. To give an indication of the computation times, the helicopter model took about 8 ms of processing time and the controller 6 ms. Thus, the controller still required quite a high percentage of processing time relative to the model.

At this stage of the study, it was established that the low-speed controller was stabilising over a very wide speed range. Stability was maintained for speeds up to about 150 knots. Performance, however, degraded especially for large amplitude inputs outside the intended operating range of the controller.

3.7 Piloted trials

Lt. Cmdr R.I. Horton of the Royal Navy was the test pilot who flew the sorties during the piloted trials. Owing to constraints on the simulator facility and pilot availability, it was only possible to conduct piloted trials on one day. Prior to any flying, the pilot was briefed on the control strategy and the purpose of the trial.

The trial was monitored via the control desk and instructions were passed to the pilot via an intercom. The trial was conducted with motion and sound; and forces on the control inceptors were generated by a feel force system. The pedals were centre-sprung so that 'feet off' would result in a zero heading rate demand. This facility was not available for the collective. The centre-spring feature was to play a large roll in the trials. All conversation over the intercom was recorded and the simulator trial was also videod. A limited amount of chart-recorder information was obtained.

The pilot flew four sorties over a period of 3 h. The first sortie was devoted to familiarisation with the simulator and control strategy, and for practising a series of flying tasks. In the other sorties, the pilot was asked to assess the handling qualities using ratings from the Cooper-Harper scale. During the familiarisation sortie, the pilot found the control strategy quite natural, and so it took only a short time before he was confident enough to perform tasks.

3.7.1 Tasks: A series of tasks requiring the pilot to fly in the hover/low-speed flight envelope and based on existing features of the terrain model were devised. The tasks were as follows:

(*a*) task 1: sidesteps
(*b*) task 2: quick hops
(*c*) task 3: free-flight, low-level, low-speed, general manoeuvring.

(*a*) *Results from task 1:* The sidestep manoeuvre is intended to test the lateral-axis response and to highlight any roll-to-pitch cross couplings. The sidesteps were performed from the hover with a maximum bank angle of 10° or 20°, over lateral distances of 50 ft, 100 ft, 150 ft and 300 ft. The distances were chosen to test the helicopter's response to bank angle hold, with changes in lateral velocity. It was found that roll-to-pitch coupling became significant when the lateral velocity exceeded about 40 knots, below this speed the couplings were quite acceptable. Heading also started to wander as the length of the step increased and problems were also experienced during roll reversals at the end of the sidesteps. Compared with the real aircraft, the sidestep task was easier to perform, typical tolerances for the real aircraft in pitch attitude would be $\pm 5°$ to $\pm 10°$ and $\pm 10°$ in heading; the augmented helicopter was below these tolerances. The main criticism was the pitch-axis coupling and the inability to maintain heading accurately.

The difficulty experienced with heading hold was attributed to the pedals not being centred properly, so that there was always a small demand in heading rate as a result of leaving the pedals free. This, of course, is more prominent in a rate system as opposed to an attitude system and perhaps some form of heading hold is required.

The sidestep manoeuvre is difficult to perform well in flight and the pilot commented that the H^∞ controller tested in the simulator was one of the few control systems he had flown that allowed him to perform this task well. Overall he rated the short sidesteps with level 1 handling qualities, with a handling qualities rating (HQR) of 3, and Level 2 for the longer steps with an HQR of 4. The workload was significantly reduced with the H^∞-controller.

Fig. 26 shows the amount of control activity required to perform a typical sidestep manoeuvre with the H^∞-controller. It can be seen that the sidesteps have been performed with only movement from the lateral cyclic stick and very little need for any adjustments from the other inceptors.

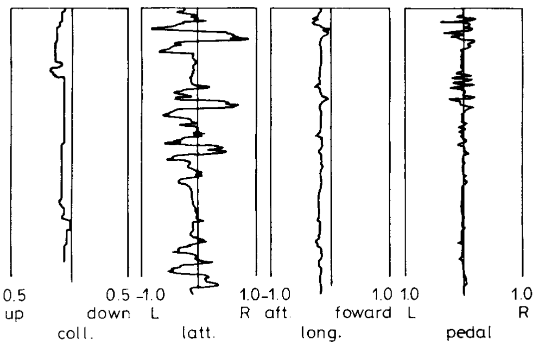

Fig. 26 *Control activity for sidestep manoeuvres with H^∞ controller*

(*b*) *Results from task 2:* The quick-hop manoeuvre is intended to test the longitudinal-axis response. Starting from the hover flight position the pilot would pitch the nose of the helicopter down, to fly forward a certain distance and then pitch the nose of the helicopter up to decelerate back to the hover. The pilot was assigned tasks of 5°, 10° and 15° pitch-down attitude, which he flew between two hedges roughly 1000 ft apart. The 5° and 10° attitude changes were relatively easy, with the helicopter dropping back into the hover after decelerating. The helicopter started to yaw, the greater the pitch attitude demand, nevertheless the pilot gave these tasks level 1 handling qualities with an HQR of 3. At 15° nose-down pitch attitude, the rating degraded to an HQR of 4, because the workload was slightly increased. At attitude changes greater than this, the pilot could not see, when pitching up, owing to the lack of visual cues. The pilot felt that the basic handling in the primary axis was good, apart from the off-axis coupling in yaw. The above tasks could be performed to within $\pm 1°$ tolerance of the pitch angle required. At the 15° nose-down attiude task, the helicopter was attaining speeds of up to 40 knots.

To test the ability of the controller to maintain pitch attitude at higher speed, the quick-hop manoevres were performed over a distance of 2000 ft. For this task, distance markers on the runway were used. For a 5° nose-down pitch attitude over 2000 ft, height control proved not to be a problem and heading was maintained within $\pm 1°$. However, pilot workload was increased a little, so a CHR of 4 was given for this task. For a 10° pitch-attitude demand, the high nose-up position required to decelerate the helicopter caused the pilot to lose all visual cues. It was, therefore, not possible to rate this task.

(*c*) *Results form task 3:* As a test for the controller, the pilot flew the serpent course, which is probably the most difficult task on the terrain belt. This course is used to test vehicles manoeuvrability through 'aggressive' flying. Usually, a pilot requires considerable practice with the control system before being able to complete the serpent course. Lt. Cmdr Horton flew this course successfully first time. During the task, speeds of up to 60 knots were attained. It has to be said, however, that the pilot had to work quite hard during tight turns.

The pilot then flew a nap of the earth (NOE) course to assess the overall manoeuvrability of the augmented helicopter. Height proved to be the hardest to control during the NOE course, the pitch-axis response felt rather underdamped and flying at 60–70 knot introduced large cross-axis couplings in yaw. The difficulty in controlling height was attributed to the lack of cues in the vertical axis.

4 Conclusions

This paper has presented an application of H^∞-optimisation to the design of a helicopter control system. An ACAH controller has been designed for the hover. It is shown that this controller conforms to the military handling qualities specifications. This controller is further evaluated on the AFS at RAE, Bedford. This is the first time that any real-time piloted simulation has been conducted using an H^∞-controller.

In the context of robustness, among the contraints to which control systems must be designed are those associated with the mathematical model. Singular-value analysis is used to identify an acceptable variation in aircraft dynamics for which stability is assured. The helicopter was represented by the nonlinear model HELISIM. The inclusion of rotor dynamics in the nonlinear model had previously compromised other control law designs; this proved not to be the case for the H^∞-design.

Comparison of the performance of the controlled helicopter with the handling qualities design specifications gave generally level 1 handling qualities using computer simulation. Where requirements were quantified, comparisons were drawn at the hover and at a number of airspeeds within the applicable flight region.

Although only one day was available for piloted simulation studies, the results from that trial were encouraging. They showed that it was feasible to use an H^∞-controller for improving the handling qualities of a typical high-performance helicopter.

Overall, the pilot commented that there were not many systems he had flown which would allow sidesteps or quick hops to be performed with so much aggression or accuracy. There were very good responses to turns in the hover, and height control was easy, especially during accelerations with nose-down pitch. This allowed more accurate tracking, because the pilot did not need to compensate for height excursions, which, in turn allowed him to fly more accurately. The pilot felt that it was a 'good control system', he felt confident when flying the NOE course at low level. He commented that he had never taken any system so low or so fast round that particular course. At high speed the controller was still more pleasant to fly than the uncontrolled helicopter.

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