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Self-Triggered Predictive Control with Time-Dependent Activation Costs of Mixed Logical Dynamical Systems

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SUMMARY Many controllers are implemented on digital platforms as periodic control tasks. But, in embedded systems, an amount of resources are limited and the reduction of resource utilization of the control task is an important issue. Recently, much attention has been paid to a self-triggered controller, which updates control inputs aperiodically. A control task by which the self-triggered controller is implemented skips the release of jobs if the degradation of control performances by the skipping can be allowed. Each job computes not only the updated control inputs but also the next update instant and the control task is in the sleep state until the instant. Thus the resource utilization is reduced. In this paper, we consider self-triggered predictive control (stPC) of mixed logical dynamical (MLD) systems. We introduce a binary variable which determines whether the control inputs are updated or not. Then, we formulate an stPC problem of mixed logical dynamical systems, where activation costs are time-dependent to represent the preference of activations of the control task. Both the control inputs and the next update instant are computed by solving a mixed integer programming problem. The proposed stPC can reduce the number of updates with guaranteeing stability of the controlled system.

key words: *self-triggered control, hybrid systems, mixed logical dynamical system, predictive control, optimal control*

1. Introduction

Recent development of microprocessor technique can realize high performance digital control. A conventional digital controller is implemented as a periodic control task [1]. To reduce the resource utilization of the computing system on which the control task is executed, it is useful to adjust the activation instants based on the current state of the plant. If the state of the plant is sufficiently close to its target state, skipping several activations may not cause much degradation to its control performance.

Event-triggered and self-triggered control have been studied as approaches to adjust the activation instants of the task [2]. The event-triggered control requires continuous monitoring of the occurrence of a specific event to activate the control task [3]–[8]. On the other hand, self-triggered control computes not only the control input but also the next activation instant [5], [9], [10]. For, continuous-time systems, Mazo and Tabuada proposed a type of self-triggered control in which time intervals between successive activations are determined as large as possible, subject to the requirement that the input-to-state stability of the closed-loop system under the existence of additive disturbances is guaranteed [4], [5]. Wang and Lemmon proposed $\mathcal{L}_2$ Stabilization of linear systems by a self-triggered controller [13]. Kobayashi and Hiraishi proposed a self-triggered controller which determines both optimal inputs and the control activation times for networked systems [14].

On the other hand, Bemporad and Morari introduced a mixed logical dynamical (MLD) system, represented by both vector difference equations and inequalities, as a mathematical model of a discrete-time hybrid system, and proposed a predictive control method of the system [15]. Industrial applications of the model predictive control method have been studied [16]–[18]. Recently, several other self-triggered predictive control (stPC) methods of MLD systems have been proposed [19]–[21], where the update of the input and the next activation instant are determined by solving mixed integer programming (MIP). In [19], an integrated method of stPC and scheduling of networked control systems over limited bandwidth networks was proposed. In [21], an event-triggered sleep control method based on model predictive control for a wireless sensor network-based control system was proposed.

In this paper, we propose a novel stPC method of MLD systems, where the cost function includes costs of controller activations with time-dependent weight coefficients, by which a preferable activation pattern of the control tasks can be specified. We also introduce binary variables which represent whether the inputs are updated or not. In other words, we do not always update the inputs whenever the control tasks are activated. Thus, the energy and/or the load for the update of the input are reduced.

The rest of the paper is organized as follows. In Sect. 2, we formulate an MLD system controlled by a self-triggered controller. In Sect. 3, we consider a cost function with time-dependent costs of control activations and propose an algorithm for the stPC. Then, we show sufficient conditions for the self-triggered controlled MLD system to be stabilized. Since a Lyapunov function candidate includes discrete variables, the standard Lyapunov stability theorem [22] cannot be applied. Therefore, we consider another Lyapunov function candidate which has only continuous variables and switch them when the controlled trajectory becomes sufficiently close to the target state. In Sect. 4, an illustrative example is demonstrated. Finally, we summarize the paper in Sect. 6.

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2. Mixed Logical Dynamical System

We consider the following mixed logical dynamical (MLD) system [15].

$$x[k+1] = Ax[k] + B_1u[k] + B_2\delta[k] + B_3z[k], \tag{1a}$$
$$E_1\delta[k] + E_2z[k] \leq E_3u[k] + E_4x[k] + E_5, \tag{1b}$$

where $x[k] \in \Re^{n_c}\times\{0,\ 1\}^{\Re^{n_b}}$, $u \in \Re^{m_c}\times\{0,\ 1\}^{m_b}$, $\delta \in \{0,\ 1\}^{r_b}$, and $z \in \Re^{r_c}$ are a hybrid state, a hybrid control input, an auxiliary binary variable, and an auxiliary continuous variable at instant k. Let A, B_1, B_2, B_3, E_1, E_2, E_3, E_4, and E_5 be coefficient matrices with suitable sizes. Let $n := n_c + n_b$ and $m := m_c + m_b$. We assume that the control input u is bounded as follows:

$$u_i \in [\, u_i^{\min},\ u_i^{\max}\,], \tag{2}$$

where $u = [u_1,\ \ldots,\ u_m]$. Denoted by $(x_e,\ u_e,\ \delta_e,\ z_e)$ is an equilibrium point of Eq. (1). Let $\tau_i \in \mathbb{Z}_{\geq 0}$ be the instant when the i-th activation of the controller takes place, where $\mathbb{Z}_{\geq 0}$ is the set of all nonnegative integer. The key idea of self-triggered control is to determine the next controller activation instant τ_{i+1} at the current activation instant τ_i. Hence, in each inter-activation time interval $(\tau_i,\ \tau_{i+1})$, the need for time-triggered executions of control tasks is relaxed. Throughout this paper, we will refer to T_i as the i-th inter-activation time, that is, $T_i := \tau_{i+1} - \tau_i$. Several methods for computing the next activation instant have been proposed [5], [13], [20], [21]. For example, self-triggered predictive control (stPC) was recently studied as a way of self-triggered control [20], [21]. In the stPC, both the next activation instant and the control input are given by a solution of a finite-horizon optimal control problem formulated in terms of a mixed integer programming problem. Note that the control input is fixed in $[\tau_i,\ \tau_{i+1})$, that is, $u[k] = u[\tau_i]$, for each $k \in [\tau_i,\ \tau_{i+1})$. We introduce an auxiliary binary variable $\psi[k] \in \{0,\ 1\}$ defined as

$$\psi[k] = \begin{cases} 1 & \text{if the control input is updated at instant } k, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

We will call $\psi[k]$ a skip variable. It is clear that the control input is given by

$$u[k] = \psi[k]u[k] + (1-\psi[k])u[k-1]. \tag{4}$$

Then, if the sequence of skip variables is predicted at activation instant τ_i, then, the next activation instant τ_{i+1} is implicitly defined as $\tau_{i+1} = \tau_i + \min\{j \in \mathbb{Z}_{>0} | \psi[j] = 1\}$, where $\mathbb{Z}_{>0}$ is the set of all positive integers.

Let g and h be auxiliary variables:

$$g[k] = \psi[k]u[k], \tag{5}$$
$$h[k] = \psi[k]u[k-1]. \tag{6}$$

Then, using Eq. (2), $g[k]$ is rewritten by the following set of inequalities:

$$\begin{aligned} &\mathbf{u}^{min}\psi[k] - g[k] \leq 0, \\ &-\mathbf{u}^{min}\psi[k] + g[k] \leq u[k] - \mathbf{u}^{min}, \\ &-\mathbf{u}^{max}\psi[k] + g[k] \leq 0, \\ &\mathbf{u}^{max}\psi[k] - g[k] \leq -u[k] + \mathbf{u}^{max}, \end{aligned}$$

where

$$\begin{aligned} \mathbf{u}^{min} &= [u_1^{min}, u_2^{min}, \ldots, u_m^{min}]^T, \\ \mathbf{u}^{max} &= [u_1^{max}, u_2^{max}, \ldots, u_m^{max}]^T. \end{aligned}$$

Similarly, $h[k]$ is also rewritten by a set of inequalities. Thus, the control input (4) with the constraint (2) is rewritten as follows:

$$u[k] = u[k-1] + g[k] - h[k], \tag{7a}$$
$$F_1\psi[k] + F_2g[k] + F_3h[k] \leq F_4u[k] + F_5u[k-1] + F_6, \tag{7b}$$

where

$$F_1 = \begin{bmatrix} \mathbf{u}^{\min} \\ \mathbf{u}^{\min} \\ -\mathbf{u}^{\min} \\ -\mathbf{u}^{\min} \\ -\mathbf{u}^{\max} \\ -\mathbf{u}^{\max} \\ \mathbf{u}^{\max} \\ \mathbf{u}^{\max} \end{bmatrix}, \quad F_2 = \begin{bmatrix} -\mathbf{I} \\ \mathbf{0} \\ \mathbf{I} \\ \mathbf{0} \\ \mathbf{I} \\ \mathbf{0} \\ -\mathbf{I} \\ \mathbf{0} \end{bmatrix}, \quad F_3 = \begin{bmatrix} \mathbf{0} \\ -\mathbf{I} \\ \mathbf{0} \\ \mathbf{I} \\ \mathbf{0} \\ \mathbf{I} \\ \mathbf{0} \\ -\mathbf{I} \end{bmatrix},$$
$$F_4 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ -\mathbf{I} \\ \mathbf{0} \end{bmatrix}, \quad F_5 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ -\mathbf{1} \end{bmatrix}, \quad F_6 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ -\mathbf{u}^{\min} \\ -\mathbf{u}^{\min} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{u}^{\max} \\ \mathbf{u}^{\max} \end{bmatrix},$$

and $\mathbf{I}$ and $\mathbf{0}$ denote the $m \times m$ identity matrix and the $m \times m$ zero matrix, respectively. Thus, the controlled system is given by Eqs. (1) and (7).

3. Self-Triggered Predictive Control

3.1 Finite Horizon Optimal Control Problem

In this subsection, we formulate a finite-horizon optimal control problem.

Problem 1 (Problem(x_0, u_0)): Given an initial state x_0, an initial input u_0, and a prediction horizon H_n, then find a sequence of the optimal skip variables $\psi^*[0],\ \ldots,\ \psi^*[H_n-1]$ and the optimal control input $u^*[0],\ \ldots,\ u^*[H_n-1]$, which minimize a given cost function $J(X,\ U,\ \Psi)$ under the following constraints:

$$x[j+1] = Ax[j] + B_1u[j] + B_2\delta[j] + B_3z[j], \tag{8a}$$
$$E_1\delta[j] + E_2z[j] \leq E_1u[j] + E_4x[j] + E_5, \tag{8b}$$

$$u[j] = u[j-1] + g[j] - h[j], \tag{8c}$$

$$F_1\psi[j] + F_2 g[j] + F_3 h[j] \le F_4 u[j] + F_5 u[j-1] + F_6, \tag{8d}$$

$$x[0] = x_0,\ u[-1] = u_0, \tag{8e}$$

$$x[H_n] = x_e, \tag{8f}$$

where $X = [\, x[0],\ \ldots,\ x[H_n-1]\,]$, $U = [\, u[0],\ \ldots,\ u[H_n-1]\,]$, and $\Psi = [\, \psi[0],\ \ldots, \psi[H_n-1]\,]$ are sequences of the predicted states, control inputs, and skip variables in the finite time interval $[0,\ H_n-1]$, respectively. Constraint (8f) represents a terminal condition, that is, the state moves to the target state x_e when the prediction horizon is elapsed. Note that Problem(x_0, u_0) does not impose a constraint $\psi[0] = 1$, which implies that the control input is not always updated when the controller is activated.

Let $|v|$ be the ℓ_1-norm of a vector v. We consider the following cost function:

$$J(X,\ U,\ \Psi) := \sum_{j=0}^{H_n-1} \left(|P(x[j]-x_e)| + |Q(u[j]-u_e)| + R[j]\psi[j] \right), \tag{9}$$

where $P \in \Re^n_{>0}$, $Q \in \Re^m_{>0}$, and $R[j] \in \Re_{>0}$ are weight vectors. Let $R = [\, R[0],\ R[1],\ \ldots,\ R[H_n-1]\,]$.

In the conventional stPC [5], [20], [21], the weight vectors $R[j]$ is set to be constant, that is, $R[j] = R[j+1]$ for $j = 0,\ 1,\ \ldots,\ H_n - 2$. In this paper, however, they may be different each other. Thus, we can represent a preference for the skipping. For example, if we set $R[2j] < R[2j-1]$ for any $j = 1, 2, \ldots$, then the skips tend to occur at every even instant, which means that the sample period makes almost double.

3.2 Algorithm for stPC

We consider self-triggered predictive control using the finite horizon optimal problem Problem(x_0, u_0), where the problem is solved at every activation instant to determine both the next activation instant and the control input. When Problem(x_0, u_0) is solved at instant τ_i, the inter-activation time T_i is given by

$$T_i = \min\{ j \in \{1,\ 2,\ \ldots,\ H_n\} | \psi^*[j] = 1 \lor j = H_n \}, \tag{10}$$

where $x_0 = x[\tau_i]$ and $u_0 = u[\tau_i - 1]$. Then, the next activation instant τ_{i+1} is given by

$$\tau_{i+1} = \tau_i + T_i. \tag{11}$$

Note that the control input $u[k]$ in the time interval $k \in [\tau_i,\ \tau_{i+1})$ is fixed as follows:

$$u[k] = u^*[0],\quad k \in [\tau_i,\ \tau_{i+1}). \tag{12}$$

We summarize an algorithm for the stPC in Algorithm 1.

Algorithm 1: stPC's algorithm.

Let $\tau_0 = 0$.
While(1)
  if $k = \tau_i$.
    Set $x_0 = x[\tau_i]$ and $u_0 = u[\tau_i - 1]$.
    Solve Problem(x_0, u_0).
    Determine the next activation instant by Eqs. (10) and (11).
    if $\psi^*[0] = 1$
      Update the control input $u[\tau_i] = u^*[0]$.
    otherwise
      Do not update the control input.
    end if
  else if $k \in (\tau_i,\ \tau_{i+1})$
    Do not activate the controller.
  end if
end while

In the following, we make the following assumption to stabilize a target equilibrium point x_e asymptotically.

Assumption 1: Assume that, for the initial state $x[0]$, the initial control input $u[-1]$, and the prediction horizon H_n, a feasible solution of Problem(x_0, u_0) exists.

3.3 Stabilization by stPC

For the vector R, we introduce a bijective function *index* : $\{0, 1, \ldots, H_n - 1\} \to \{0, 1, \ldots, H_n - 1\}$ satisfying the following conditions:

- $index(0) = 0$,
- $index(1) = 1$, and
- for each i, j with $2 \le i < j \le H_n - 1$,

$$R[index(i)] \ge R[index(j)].$$

Then, for R, we define a vector $r = [r[0],\ r[1],\ \ldots,\ r[H_n-1]]$ such that $r[i] = R[index(i)]$ for each $i \in \{0,\ 1,\ \ldots,\ H_n-1\}$. In the following, the function $index(\cdot)$ and the vector r will be called an index function of the rearrangement and a rearrangement vector of R, respectively. We introduce the following technical notations used to represent sufficient conditions for the stabilization.

$$r_1[\,l\,] = \frac{1}{N - 2l + 2}\left(r[0] + \sum_{j=l}^{H_n-1} r[j] - \sum_{j=1}^{l-1} r[j] \right), \tag{13}$$

$$r_2[\,l\,] = \frac{1}{N - 2l + 3}\left(r[0] + \sum_{j=l}^{H_n-1} r[j] - \sum_{j=1}^{l-1} r[j] \right), \tag{14}$$

for each $l \in \{3,\ \ldots,\ H_n - 1\}$. Denoted by $X^*[k] = [\, x^*[k,0],\ \ldots,\ x^*[k,H_n-1]\,]$, $U^*[k] = [\, u^*[k,0],\ \ldots,\ u^*[k,H_n-1]\,]$, and $\Psi^*[k] = [\, \psi^*[k,0],\ \ldots,\ \psi^*[k,H_n-1]\,]$ are the solutions of Problem(x_0, u_0) at instant k. Then, we define $\tilde{X}[k]$, $\tilde{U}[k]$, and $\tilde{\Psi}[k]$ as follows:

$$\tilde{X}[k] = [\,\tilde{x}[k,\,0],\ \ldots,\ \tilde{x}[k,\,H_n-1]\,] = \begin{cases} X^*[k] & \text{if } k \in \{\tau_i\}_{i\in\mathbb{Z}_{\geq 0}}, \\ [\,x^*[k,\,k-\tau_i],\ \ldots,\ x^*[k,\,H_n-1],\ x_e,\ \ldots,\ x_e\,] & \\ & \text{if } k \in (\tau_i,\ \tau_{i+1}), \end{cases} \tag{15}$$

$$\tilde{U}[k] = [\,\tilde{u}[k,\,0],\ \ldots,\ \tilde{u}[k,\,H_n-1]\,] = \begin{cases} U^*[k] & \text{if } k \in \{\tau_i\}_{i\in\mathbb{Z}_{\geq 0}}, \\ [\,u^*[k,\,k-\tau_i],\ \ldots,\ u^*[k,\,H_n-1],\ u_e,\ \ldots,\ u_e\,] & \\ & \text{if } k \in (\tau_i,\ \tau_{i+1}), \end{cases} \tag{16}$$

$$\tilde{\Psi}[k] = [\,\tilde{\psi}[k,\,0],\ \ldots,\ \tilde{\psi}[k,\,H_n-1]\,] = \begin{cases} \Psi^*[k] & \text{if } k \in \{\tau_i\}_{i\in\mathbb{Z}_{\geq 0}}, \\ [\,\psi^*[k,\,k-\tau_i],\ \ldots,\ \psi^*[k,\,H_n-1],\ 0,\ \ldots,\ 0\,] & \\ & \text{if } k \in (\tau_i,\ \tau_{i+1}), \end{cases} \tag{17}$$

We consider the following scalar function $V(X,\,U,\,\Psi)$ as a Lyapunov function candidate to prove the stabilization of the target state:

$$V(X,\,U,\,\Psi) := \sum_{j=0}^{H_n-1} \left(|P(x[j]-x_e)| + |Q(u[j]-u_e)| + R_L\psi[j]\right), \tag{18}$$

where $R_L \in \Re_{>0}$ represents a weight vector. Obviously, $V(X_e,\,U_e,\,\Psi_e) = 0$ and $V(X,\,U,\,\Psi) > 0$ for all $(X,\,U,\,\Psi) \neq (X_e,\,U_e,\,\Psi_e)$, where $X_e = [x_e,\ \ldots,\ x_e]$, $U_e = [u_e,\ \ldots,\ u_e]$, and $\Psi_e = [0,\ \ldots,\ 0]$. However, note that, if n is greater than one, there exists $x \in U(x_e)$ such that at least two variables in the optimal skip variables $\tilde{\psi}[j] (j = 0, 1, \ldots, H_{n-1})$ takes one, that is, $V(\tilde{X},\,\tilde{U},\,\tilde{\Psi}) \geq R_L$ for the optimal solution $\tilde{X}$, $\tilde{U}$, and $\tilde{\Psi}$ of Problem$(x_0,\,u_0)$, which means that V does not always converge to 0 even if the controlled state converges to the target state. Moreover, Eq. (18) has discrete variables. Thus, the standard Lyapunov stability theorem [22] cannot be applied directly.

Let $\Theta[k] := (\tilde{X}[k],\,\tilde{U}[k],\,\tilde{\Psi}[k])$, and consider V along a state trajectory of the controlled system. We introduce the following Lemma that we will use in the proof of Theorem 1.

Lemma 1: We consider the MLD system (1), the cost function (9), and the scalar function (18). Let $\{\tau_i\}_{i\in\mathbb{Z}_{\geq 0}}$ be a sequence of activation instants. Assume that the rearrangement vector r of the weight vector R satisfies one of the conditions in Table 1. Then, Eq. (18) satisfies

$$V(\Theta[k+1]) - V(\Theta[k]) \leq -|P(\tilde{x}[k,\,0]-x_e)|, \tag{19}$$

if R_L satisfies one of the following conditions:

- Condition (C-1) holds and

$$r[2] \leq R_L \leq \min\left\{r[1],\ \frac{1}{H_n-2}\left(r[0] + \sum_{j=2}^{H_n-1} r[j] - r[1]\right)\right\}. \tag{20a}$$

- Condition (C-2) holds and

$$\max\{r[1],\ r[2]\} \leq R_L \leq \frac{1}{N-1}\left(r[0] + \sum_{j=2}^{H_n-1} r[j]\right). \tag{20b}$$

- For l satisfying Condition (C-3),

$$r[\,l\,] \leq R_L \leq \min\{r[1],\ r[l-1],\ r_1[\,l\,]\}. \tag{20c}$$

- For l satisfying Condition (C-4),

$$\max\{r[\,l\,],\ r_1[\,l\,]\} \leq R_L \leq \min\{r[1],\ r[l-1]\}. \tag{20d}$$

- For l satisfying Condition (C-5),

$$\max\{r[1],\ r[\,l\,]\} \leq R_L \leq \min\{r[l-1],\ r_2[\,l\,]\}. \tag{20e}$$

- For l satisfying Condition (C-6),

$$\max\{r[1],\ r[\,l\,],\ r_2[\,l\,]\} \leq R_L \leq r[l-1]. \tag{20f}$$

- Condition (C-7) holds and

$$0 < \frac{1}{H_n-2}\left(\sum_{j=1}^{H_n-1} r[j] - r[0]\right) \leq R_L \leq \min\{r[1],\ r[H_n-1]\}. \tag{20g}$$

- Condition (C-8) holds

Table 1 Sufficient conditions for stabilization (r is the rearrangement vector of R.).

(C-1)	$r[2] \leq \min\left\{r[1],\ \frac{1}{H_n-2}\left(r[0] + \sum_{j=2}^{H_n-1} r[j] - r[1]\right)\right\}$
(C-2)	$\max\{r[1],\ r[2]\} \leq \frac{1}{N-1}\left(r[0] + \sum_{j=2}^{H_n-1} r[j]\right)$
(C-3)	$\exists\, l \in \{3, 4, \ldots, H_n-1\}$ such that $l \leq \frac{H_n+1}{2} \ \wedge\ r[\,l\,] \leq \min\{r[1],\ r[l-1],\ r_1[\,l\,]\}$
(C-4)	$\exists\, l \in \{3, 4, \ldots, H_n-1\}$ such that $l \leq \frac{H_n+3}{2} \ \wedge\ \max\{r[\,l\,],\ r_1[\,l\,]\} \leq \min\{r[1],\ r[l-1]\}$
(C-5)	$\exists\, l \in \{3, 4, \ldots, H_n-1\}$ such that $l \leq \frac{H_n+2}{2} \ \wedge\ \max\{r[1],\ r[\,l\,]\} \leq \min\{r[l-1],\ r_2[\,l\,]\}$
(C-6)	$\exists\, l \in \{3, 4, \ldots, H_n-1\}$ such that $l \leq \frac{H_n+4}{2} \ \wedge\ \max\{r[1],\ r[\,l\,],\ r_2[\,l\,]\} \leq r[l-1]$
(C-7)	$0 < \frac{1}{H_n-2}\left(\sum_{j=1}^{H_n-1} r[j] - r[0]\right) \leq \min\{r[1],\ r[H_n-1]\}$
(C-8)	$\max\left\{r[1],\ \frac{1}{H_n-3}\left(\sum_{j=2}^{H_n-1} r[j] - r[0]\right)\right\} \leq r[H_n-1]$

$$\max\left\{r[1],\ \frac{1}{H_n-3}\left(\sum_{j=2}^{H_n-1} r[j]-r[0]\right)\right\}$$
$$\leq R_L \leq r[H_n-1]. \tag{20h}$$

See the Appendix for a proof of Lemma 1. The coefficients $R[j]$ ($j = 0, 1, \ldots, H_n - 1$) of the skip variables in Eq. (9) may be different each other. But, the coefficient R_L in Eq. (18) is constant. For Eq. (19) to hold, we consider 8 cases shown in Table 1 depending on the $R[j]$ and show an assignment of R_L for each case in Lemma 1. It is future work to show the existence of R_L such that Eq. (19) holds for $R[j]$ satisfying no case of Table 1.

The following theorem shows sufficient conditions for the stabilization of an equilibrium point of the controlled system by the self-triggered predictive controller.

Theorem 1: We consider the MLD system (1) and the self-triggered predictive controller given by Algorithm 1 with the cost function (9). Let $\{\tau_i\}_{i\in\mathbb{Z}_{\geq 0}}$ be a sequences of the activation instants. The equilibrium point x_e of the controlled system is asymptotically stable if the rearrangement vector r of the weight vector R satisfies one of the conditions in Table 1.

Proof: Since $x[k] = \tilde{x}[k, 0]$, by Lemma 1, it is shown that there exists a Lyapunov function candidate V such that $V(\Theta[k+1]) - V(\Theta[k]) \leq -|P(x[k]-x_e)|$. Since $V(\Theta[k]) \geq 0$, there exists a constant $c \geq 0$ such that $\lim_{k\to\infty} V(\Theta) = c$. Therefore, there exists a K such that, for any $k \leq K$, the sum of the 3rd term of $V(\Theta[k])$ takes the same value, that is, there exists R' such that

$$R' = \sum_{j=0}^{H_n-1} R_L\psi(k,\ j) \quad \text{for any } k > K. \tag{21}$$

We define the following scalar function V' as another Lyapunov function candidate:

$$V'(X,\ U) := \sum_{j=0}^{H_n-1} \left(|P(x[j]-x_e)| + |Q(u[j]-u_e)|\right). \tag{22}$$

Obviously, we have $V(X_e, u_e) = 0$ and $V(X, U) > 0$ for all $(X, U, \Psi) \neq (X_e, U_e, \Psi_e)$. Note that V' is continuous at $(\tilde{X}, \tilde{U})$. Then, for any $k > K$, we have

$$V'(\tilde{X}[k],\ \tilde{U}[k]) = V(\Theta[k]) - c - R'. \tag{23}$$

From Lemma 1, it follows that:

$$V'(\tilde{X}[k+1],\ \tilde{U}[k+1]) - V'(\tilde{X}[k],\ \tilde{U}[k]) \leq -|P(x[k]-x_e)|. \tag{24}$$

Hence, for any $j = 0,\ 1,\ \ldots,\ H_n - 1$, we have

$$\lim_{k\to\infty} \tilde{x}[k,\ j] = x_e,\ \lim_{k\to\infty} \tilde{u}[k,\ j] = u_e.$$

Thus, if one of the conditions in Table 1 holds, the equilibrium point x_e of the controlled system is asymptotically stable. ■

It is noted that, if all coefficients $R[j]$'s are equal, all conditions of Table 1 hold and the target state is always stabilized by Algorithm 1. In other words, Table 1 shows how they can be deviated from the case that they are equal in order to maintain the stabilization.

4. Example

As a simple example, we consider the following discrete-time unstable system with the input constraint.

$$x[k+1] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2.5 & -5 & 3.5 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u[k], \tag{25a}$$
$$u \in [-10,\ 10], \tag{25b}$$

where x and u are the continuous state and the continuous control input. The initial conditions are $x[0] = [-8,\ -1,\ 1]^T$ and $u[-1] = 0$. Set $(x_e,\ u_e) = (0,\ 0,\ 0,\ 0)$ as a target equilibrium pair. We consider a self-triggered predictive controller given by Algorithm 1 with $H_n = 10$, $P = [1,\ 1,\ 1]$, $Q = 5$, and $R = [30,\ 25,\ 5,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1]$. Then, the condition (C-7) of Table 1 holds and the target state is stabilized. We use the solver Gurobi 4.0 [23] to solve Problem(x_0, u_0) in Algorithm 1. Simulation was done in a personal computer (CPU:Intel i5 660@3.3GHz, RAM:3GB). The average of the computation time per one activation of Algorithm 1 was approximately 0.8[s] (*resp.* 0.07[s]) in the transient state (*resp.* the steady state). The average in the transient state is much larger than that after the state converges to the target equilibrium.

Shown in Figs. 1, 2, and 3 are time responses of the controller activations, the state, and the control input. In Fig. 1, the symbol "•" (*resp.* "×") means that activation of the controller with (*resp.* without) update of the control input. At instant 4, the controller is activated, but the control input is not updated so that it is not sent to the actuator and its transmission cost is reduced. Especially, after the state $x[k]$ converges to the origin, the controller is activated but the control input is not updated. Therefore, the introduction of the cost for the update of the control input is useful for the reduction of the transmission of the control input.

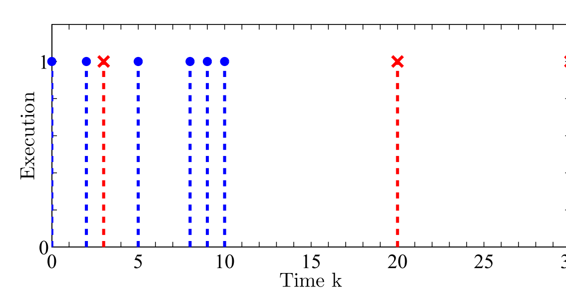

Fig. 1 Activations of the controller and updates of the input in example. The symbol "•" (*resp.* "×") means that activation of the controller with (*resp.* without) update of the control input.

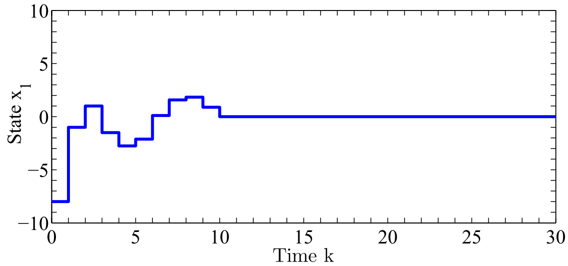

Fig. 2 State $x_1[k]$ in example.

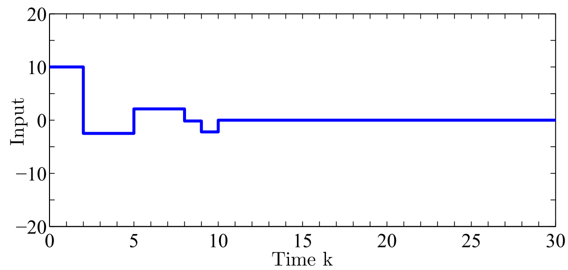

Fig. 3 Control input $u[k]$ in example.

5. Conclusion

We have considered a mixed logical dynamical system and have proposed a self-triggered predictive control with control activation costs, which determines both the control inputs and the next activation instant of the control task. The control activation costs are represented by time-dependent weight coefficients by which a desired activation pattern of the control tasks is specified. Moreover, the cost of the update of the input is taken into a consideration so that they may not be always updated when the control tasks are executed. We have also proved sufficient conditions for a target state to be stabilized. The proof is done by using two Lyapunov function candidates since the skip variable is a binary variable.

In general, the larger the horizon of the optimal control problem is, the larger the inter-activation time may be. But, the computation time of Algorithm 1 is large if there are many discrete variables in the MLD systems. So, our future work is to develop an efficient method to compute Algorithm 1. It is also our future work to extend the proposed method to decentralized control systems and networked control systems.

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Appendix: Proof of Lemma 1

We consider the differences ΔJ and ΔV of the functions J and V along a trajectory of the controlled system, respectively, given by

$$\Delta J(k,\, k+1) := J(\Theta[k+1]) - J(\Theta[k]), \qquad \text{(A· 1)}$$
$$\Delta V(k,\, k+1) := V(\Theta[k+1]) - V(\Theta[k]). \qquad \text{(A· 2)}$$

Let r and $index(\cdot)$ be the rearrangement vector of R and its index function. We define a rearrangement vector $\tilde{\Psi}_r[k]$ of $\tilde{\Psi}[k]$ as follows:

$$\tilde{\Psi}_r[k] = [\, \tilde{\psi}_r[k,\, 0],\ \tilde{\psi}_r[k,\, 1],\ \ldots,\ \tilde{\psi}_r[k,\, H_n - 1]\,],$$

where $\tilde{\psi}_r[k,\ j] = \tilde{\psi}[k,\ index(j)]$ for each $j \in \{0, \ldots, H_n - 1\}$. There are the following four cases for controller activations at instants k and $k+1$.

(C1) The controller is only activated at instant k.
(C2) The controller is activated at neither instant k nor $k+1$.
(C3) The controller is only activated at instant $k+1$.
(C4) The controller is activated at both instants k and $k+1$.

First, we consider the difference ΔV in Cases (C1) and (C2). The controller is not activated at instant $k+1$, that is, $\tilde{x}[k,\ j+1] = \tilde{x}[k+1,\ j]$, $\tilde{u}[k,\ j+1] = \tilde{u}[k+1,\ j]$, and $\tilde{\psi}[k,\ j+1] = \tilde{\psi}[k+1,\ j]$ for $j = 1, \ldots,\ H_n - 2$. Moreover, by the terminal condition (8f), we also have $\tilde{x}[k+1,\ H_n - 1] = x_e$, $\tilde{u}[k+1,\ H_n - 1] = u_e$, and $\tilde{\psi}[k+1,\ H_n - 1] = 0$. Hence, it follows that

$$\Delta V(k,\, k+1) \le -|P(\tilde{x}[k,\, 0] - x_e)| - |Q(\tilde{u}[k,\, 0] - u_e)|,$$

which implies that $\Delta V(k,\, k+1) \le -|P(\tilde{x}[k,\, 0] - x_e)|$.

Next, we consider ΔV in Cases (C3) and (C4). Assume that the rearrangement vector r satisfies one of the conditions in Table 1. From Eqs. (9) and (18), we have

$$\Delta V(k,\, k+1) = \Delta J(k,\, k+1) + \Delta\Psi_r[k,\, k+1], \qquad \text{(A· 3)}$$

where

$$\Delta\Psi_r[k,\, k+1] = \sum_{j=0}^{H_n-1} \left\{ (r[j] - R_L) \left(\tilde{\psi}_r[k,\ j] - \tilde{\psi}_r[k+1,\ j] \right) \right\}.$$

In Case (C3), since the controller is only activated at instant $k+1$, we have

$$\Delta\Psi_r[k,\, k+1] = r[1] - r[0] - (r[1] - R_L)\tilde{\psi}_r[k+1,\, 1] + \sum_{j=2}^{H_n-1} \left\{ (r[j] - R_L) \left(\tilde{\psi}_r[k,\ j] - \tilde{\psi}_r[k+1,\ j] \right) \right\}. \qquad \text{(A· 4)}$$

Because of the optimality, we also have

$$\Delta J(k,\, k+1) \le -|P(\tilde{x}(k,\, 0) - x_e)| - |Q(\tilde{u}[k,\, 0) - u_e)|. \qquad \text{(A· 5)}$$

Therefore, it is sufficient to prove that

$$\Delta\Psi_r[k,\, k+1] \le 0. \qquad \text{(A· 6)}$$

- Assume that r satisfies Condition (C-1) of Table 1. Then, we consider a weight vector R_L satisfying Eq. (20a). Since $r[2] \le R_L \le r[1]$, Eq. (A· 4) becomes

$$\Delta\Psi_r[k,\, k+1] \le r[1] - r[0] - \sum_{j=2}^{H_n-1} (r[j] - R_L) = (H_n - 2)R_L - r[0] - \sum_{j=2}^{H_n-1} r[j] + r[1],$$

  which implies together with Eq. (20a) that Eq. (A· 6) holds.
- Assume that r satisfies Condition (C-2) of Table 1. We consider R_L satisfying Eq. (20b). Then, we have

$$\Delta\Psi_r[k,\, k+1] \le r[1] - r[0] - \sum_{j=1}^{H_n-1} (r[j] - R_L) = (H_n - 1)R_L - r[0] - \sum_{j=2}^{H_n-1} r[j],$$

  which implies that Eq. (A· 6) holds.

Similarly, we can prove that Eq. (A· 6) holds if one of Conditions (C-3)–(C-8) given by Table 1 holds.

Next, we consider Case (C4). Then, $\Delta\Psi_r[k,\, k+1]$ becomes as follows:

$$\Delta\Psi_r[k,\, k+1] = r[1] - R_L - (r[1] - R_L)\tilde{\psi}_r[k+1,\, 1] + \sum_{j=2}^{H_n-1} \left\{ (r[j] - R_L) \left(\tilde{\psi}_r[k,\ j] - \tilde{\psi}_r[k+1,\ j] \right) \right\}. \qquad \text{(A· 7)}$$

On the other hand, we have

$$\Delta J(k,\, k+1) \le -|P(\tilde{x}(k,\, 0) - x_e)| - |Q(\tilde{u}[k,\, 0) - u_e)| - r[0]. \qquad \text{(A· 8)}$$

Thus, it is sufficient to prove that

$$\Delta\Psi_r[k,\, k+1] - r[0] \le 0. \qquad \text{(A· 9)}$$

- Assume that r satisfies Condition (C-1) of Table 1. Then, we consider a weight vector R_L satisfying Eqs. (20a) and (A· 4) becomes

$$\Delta\Psi_r[k,\, k+1] \le r[1] - R_L - \sum_{j=2}^{H_n-1} (r[j] - R_L) < (H_n - 2)R_L - \sum_{j=2}^{H_n-1} r[j] + r[1],$$

  which implies together with Eq. (20a) that Eq. (A· 9) holds.
- Assume that r satisfies Condition (C-2) of Table 1. Then, we consider R_L satisfying Eq. (20b). Then, using Eq. (A· 7), we have

$$\begin{aligned}\Delta\Psi_r[k,\, k+1] &\le r[1] - R_L - \sum_{j=1}^{H_n-1}(r[j] - R_L)\\ &< (H_n - 1)R_L - \sum_{j=2}^{H_n-1} r[j].\end{aligned}$$

Thus, Eq. (A· 9) holds.

Similarly, we can prove that Eq. (A· 9) holds if one of Conditions (C-3)–(C-8) given by Table 1 holds.

Therefore, if r satisfies one of Conditions (C-1)–(C-8) and choose R_L such that Eq. (20) holds, then we have

$$\begin{aligned}\Delta V(k,\, k+1) &\le -|P(\tilde{x}[k,\, 0] - x_e)| - |Q(\tilde{u}[k,\, 0] - u_e)|\\ &\le -|(\tilde{x}[k,\, 0] - x_e)|, \qquad \text{(A· 10)}\end{aligned}$$

which completes the proof of the lemma.

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