H^{∞}-OPTIMAL DESIGN FOR HELICOPTER CONTROL

Andrew Yue
Ian Postlethwaite
University of Oxford, Department of Engineering Science,
Oxford, OX1 3PJ, England.

ABSTRACT

This paper presents the results of a study into the use of H^{∞} - optimization for the design of robust feedback control laws for improving the handling qualities of a battlefield helicopter. Control laws are designed for precise control of pitch and roll attitude, yaw rate and heave velocity in in the hover flight condition.

1. Introduction

An unaugmented helicopter exhibits unacceptable responses in the hover. The responses to the collective, longitudinal and lateral cyclic and pedals are highly coupled and unstable in the hover. Pilot workload is high and precise control is difficult without augmentation.

Robustness is a primary issue in the design, because of model uncertainty. The major approximations associated with the linearized helicopter model are due to both neglected rotor dynamics and neglected nonlinearities in the blade angle actuation system as well as changes in flight condition. Attention will be focused primarily on errors due to neglected rotor dynamics and at a single flight condition in the hover. The maximum bandwidth is determined by the high frequency dynamic characteristics. Frequency domain techniques such as H^{∞} are able to expose high frequency details more easily and integrate with current proposed handling quality specifications which are based on frequency domain criteria.

The importance of the H^{∞} norm stems from its natural characterization of uncertainty (1). The H^{∞} norm of a transfer matrix is the maximum over all frequencies of its largest singular value. Singular values provide information in terms of guaranteed bounds on system performance or tolerable perturbations (2). The norm is used to place an upper bound on an appropriately weighted closed-loop transfer function matrix. The minimization of such a norm is now well understood, and all the computations can be carried out using state space representations of the transfer function matrices (3).

2. Control system designs as H^{∞}-optimizations

The Hardy space H^{∞} consists of all complex-valued functions $G(s)$ of a complex variable s which are analytic and bounded in the open right half-plane. This can be thought of as the space of all stable transfer matrices. The systems are modelled as linear, time invariant and finite dimensional and they operate in continuous time with a state-space transfer function representation in the Laplacian s given by:

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \quad and \quad G(s) = C(sI - A)^{-1}B + D \quad (1)$$

Suppose G is stable; then the H^{∞} norm of G is defined as

$$\|G\|_{\infty} = \sup_{\omega} \bar{\sigma}[G(j\omega)] \quad (2)$$

where, $\bar{\sigma}[G(j\omega)]$ denotes the largest singular value of G at frequency ω.

The compensation configuration depicted in fig. 1 will be referred to as the Standard Compensator Configuration (SCC). The objective is to design a controller K, for the plant P such that the input/output transfer characteristics from the external input vector v to the external output vector e is desirable, according to some engineering specification. The exogenous input vector v, typically consists of command signals, disturbances, and sensor noises; u is the control signal; e is the output to be controlled, its components typically being tracking errors, filtered actuator signals, etc.; and y is the measured output.

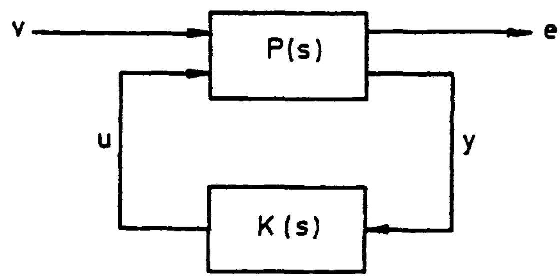

Fig. 1 Standard compensator configuration

Let $F_l(P,K)$ denote the closed-loop transfer function matrix mapping external input v to external output e. This operation of forming $F_l(P,K)$ is referred to as a *linear fractional transformation*. The H^{∞} approach is to design a stabilizing controller K such that the H^∞ norm of $F_l(P,K)$ is minimized ie. the objective is to solve the following optimization problem

$$\min_{K \text{ stabilizing}} \|F_l(P,K)\|_\infty \qquad (3)$$

where the minimization is over the whole set of stabilizing controllers.

2.1 Weight Selections in H^∞ Design

Weighting functions are used to emphasize (de-emphasize) maximum singular values of $F_l(P,K)$ at various frequencies so that engineering objectives can be incorporated into the optimization procedure (3) as:

$$C := \min_{K \text{ stabilizing}} \|W_o F_l(P,K) W_i\|_\infty \qquad (4)$$

where W_o and W_i are weights to be chosen, and C is the minimum cost. An optimal controller achieving the minimum cost C can be computed automatically within a Computer Aided Design (CAD) environment such as Stable-H (4). In general the weights used are diagonal transfer function matrices whose elements are constants, low pass or high pass filters.

2.2 Classes of H^∞ Design Problems

Consider the feedback configuration depicted in fig. 2. The transfer function mapping d to y and r to e is called the *sensitivity* matrix and is represented by the symbol S, where $S = (I+GK)^{-1}$. The transfer function mapping r to y and η to y is called the *complementary sensitivity* matrix, and is denoted by T, where $T = I - S$. Finally, the transfer function mapping r to u is simply KS. The minimization of S can be considered as a performance requirement, and the minimization of T and KS, as robust stability requirements (5). A bound on the gain of KS can be thought of as a design constraint reflecting the actuator saturation limits.

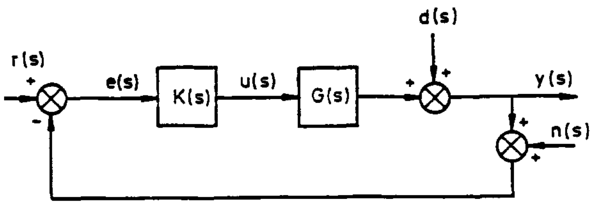

Fig. 2 Unity feedback system

3. Helicopter design

To illustrate the technique outlined in this paper, a controller has been designed for a single main rotor helicopter. The basic linear model of the helicopter has 8 states and 4 inputs, and is unstable and non-minimum phase. If the rotor flapping dynamics are included a 14 state model is obtained, but the rotor dynamics will be treated as being uncertain and will be left out of the nominal plant description. The first step in the design is to estimate the errors due to the neglected rotor dynamics. Since the collective, longitudinal and lateral cyclic controls are all implemented with the main rotor, model uncertainty can be parameterized as a multiplicative perturbation at the four inputs (fig. 3). From this figure it can be seen that the uncertainty level will limit the bandwidth of the closed loop system to about 10 rad/s.

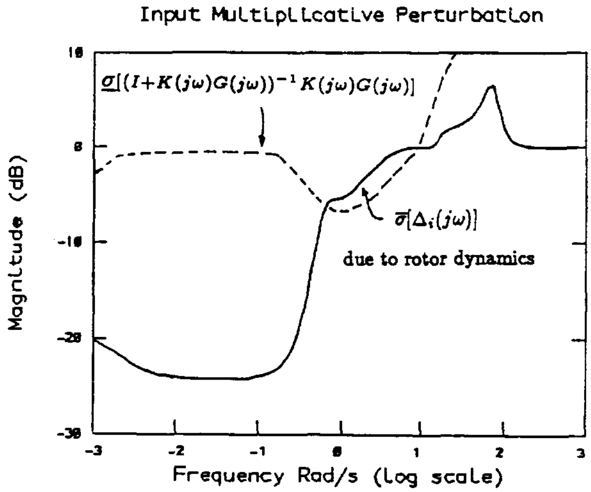

Fig. 3 Uncertainty due to unmodelled rotor dynamics

The effect of increases in system order due to unmodelled rotor dynamics is an important area for helicopters, since the rotor system can introduce a significant number of modes into the vehicle dynamics and these can lead to vibration problems or place constraints on sensor positions. The task of identifying regions for which stability is assured is complex, since each element of the open loop transfer function is frequency dependent.

In addition to the rotor dynamics, another major source of model uncertainty is the variation of the state matrix A and control matrix B of the linearised model with changes in flight condition. A direct way to compute a bound on the uncertainty due to coefficient variations is to evaluate the loop transfer matrix for a number of representative flight conditions and then compute the maximum (singular value) deviation. Results of this process are shown in fig. 4, where the flight conditions were taken to be from 0 to 80 knts straight and level flight. Also shown in fig. 4 are the perturbations due to a ± 0.5 rads/s turn rate from the hover flight condition. The unmodelled dynamics due to changes in speed and turn rate have been represented as an unstructured additive perturbation. These flight conditions were chosen as being typical worst case deviations from the hover. The change in dynamics of the helicopter is most severe from this flight condition. From fig. 4 it can be seen that $\overline{\sigma}[E(s)]$ becomes quite large at low frequencies. This is because the helicopter's low frequency modes vary greatly with speed.

Each control channel of the helicopter also exhibits various nonlinearities which are neglected in the nominal design model. Of these, the rate limit nonlinearity imposes the greatest dynamic constraints on performance. Bounds on this effect can be developed by treating the saturation element as an uncertain component. These bounds will depend on the magnitudes of the arguments of the saturating function and in general will lead to conservative designs because every actuator is unlikely to saturate at the same magnitude.

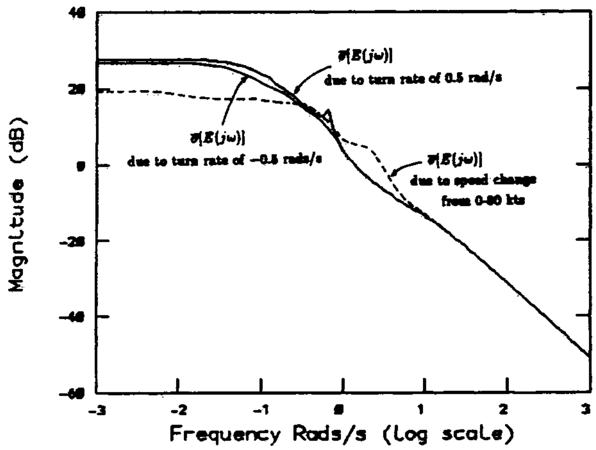

Fig. 4 Perturbation in helicopter dynamics due to speed and turn rate

The following 6 outputs are selected for control: height rate ($\dot{h}$), roll rate (p), pitch rate (q), heading rate ($\dot{\psi}$), pitch attitude (θ) and roll attitude (ϕ). The plant has only 4 inputs and therefore it is only possible to independently control 4 outputs, however the availability of measured variables in excess of the number of commanded outputs should lead to a design which makes more efficient use of gain. In general it is better to provide the controller with as much information as possible, provided of course that it is reliable. Hence the outputs, roll rate and pitch rate are not to be controlled directly but are included to improve control.

The state space description of the linearized rigid body equations of motion are expressed in the standard form as:-

$$\dot{x} = Ax + Bu \qquad (6)$$

$$y = Cx + Du$$

The state variables and inputs are described in table 1, it is assumed that all these states are available from appropriate sensors. The A, B and C matrices were obtained by numerical linearization of the nonlinear helicopter model (6). The specification for the closed loop system is to design a compensator for the controlled outputs in such a way as to satisfy the handling qualities criteria outlined in the proposed airworthiness design standard (7).

Table 1 - State variable and plant input description

State	Description	Input	Description
θ	Pitch attitude	θ_o	Collective
ϕ	Roll attitude	θ_{1s}	Longitudinal cyclic
p	Roll rate	θ_{1c}	Lateral cyclic
q	Pitch rate	θ_{ot}	Tail rotor collective
r	Yaw rate		
u	Forward velocity		
v	Lateral velocity		
w	Vertical velocity		

3.1 Problem Formulation and Weighting Function Selection

It is proposed to find a stabilizing controller K that minimizes:

$$\left\| \begin{matrix} W_1 S W_3 \\ W_2 K S W_3 \end{matrix} \right\|_\infty \qquad (7)$$

where the diagonal weights W_1, W_2, W_3 are chosen to meet the design objectives.

Selection of W_1 A plot of W_1 is shown in fig. 5. First order high gain low-pass filters are used on $\dot{h}$, $\dot{\psi}$, θ and ϕ to ensure that these outputs can be controlled accurately (d.c. sensitivity of 0.2%) with good disturbance attenuation up to about 6 rad/s. With only 4 plant inputs, no attempt is made to control directly the extra outputs p and q at low frequencies, but second order band-pass filters are used on each of these variables to reject disturbances and cross coupling effects in the frequency range 5 to 8 rad/s. The frequency response of W_1 approximates integral action in each of the four controlled loops; hence it forces the resulting closed loop system to have almost perfect steady-state disturbance rejection. The low-pass filters on $\dot{h}, \dot{\psi}, \theta$ and ϕ are given a finite attenuation which has the effect of reducing overshoot in these channels.

Selection of W_2 First-order high pass filters are used on each of the plant inputs and are shown in fig. 6. A cut off frequency of about 10 rad/s is used on each of the heave, pitch and roll loops to limit the system bandwidth, and also to limit the magnitude of the poles of the controller. A low frequency gain of -100db was used so that $W_1 S W_3$ clearly dominates the cost function at low frequencies.

Selection of W_3 The weighting function W_3 is chosen to be a constant matrix with a weight of 0.1 on each of the rates and 1 on each of the other output demands. The reduced weighting on the rates (which are not directly controlled) is chosen so that some disturbance rejection is obtained on these outputs without them significantly affecting the cost.

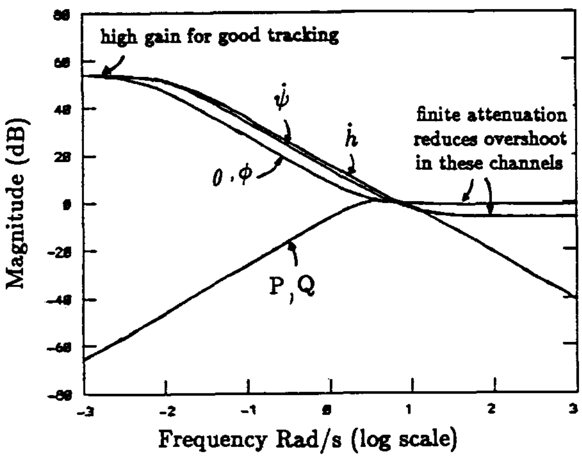

Fig. 5 Singular values of weight W_1

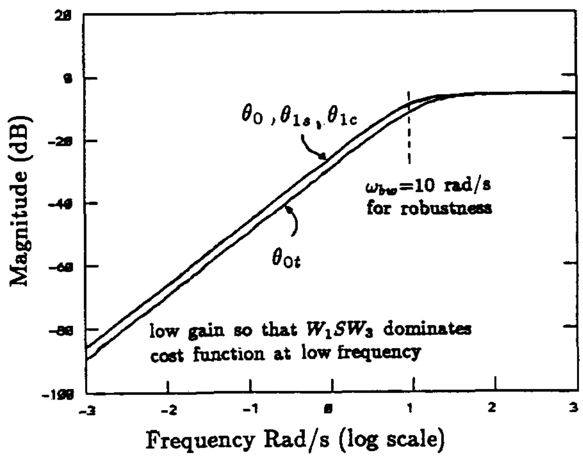

Fig. 6 Singular values of weight W_2

3.2 Analysis

This section will summarize the analysis of the helicopter control system and will concentrate on linear analysis, although extensive nonlinear simulation was also used to verify the control laws.

Sensitivity A plot of the singular values of S is shown in fig. 7. These singular values correspond to the sensitivity between the four inputs, collective, longitudinal and lateral cyclic and tail rotor collective to the four directly controlled outputs $\dot{h}, \theta, \phi$ and $\dot{\psi}$ respectively. Note that the sensitivity is small up a frequency of 6 rad/s consistent with the choice of W_1

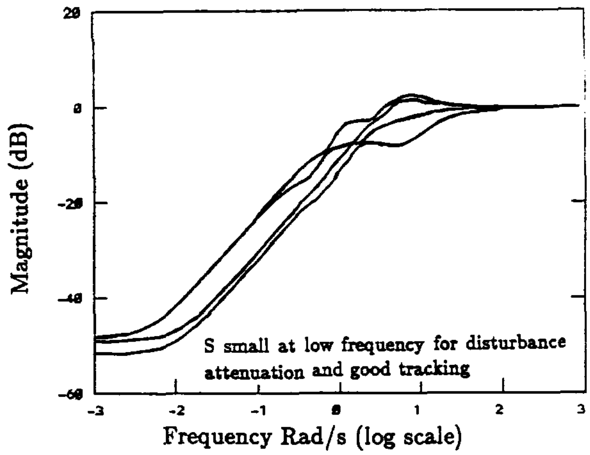

Fig. 7 Sensitivity function

Complementary Sensitivity A plot of the singular values of T is shown in fig. 8. From this figure it is seen that the desired bandwidth of 10 rad/s for stability robustness is achieved.

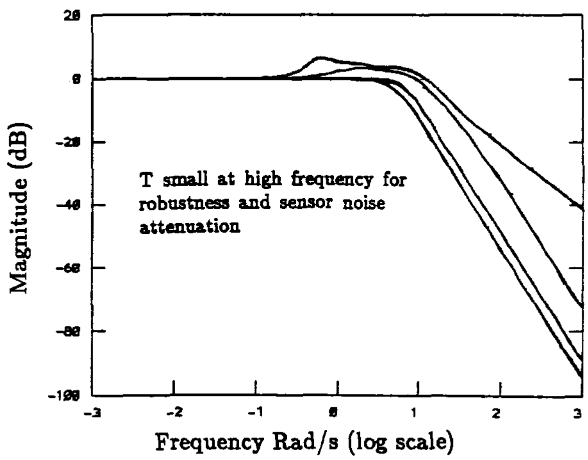

Fig. 8 Complementary sensitivity function

Robustness to unmodelled rotor dynamics Fig. 3 shows the uncertainty Δ_i modelled as a *unstructured* input multiplicative perturbation and superimposed is the frequency response of $\underline{\sigma}[(I + KG)^{-1}KG]$. Using the standard robustness tests for unstructured uncertainties (3), it can be seen that the robust stability test fails in the intermediate frequency range from 1 to 10 rad/s. However, one should be careful when apply such tests as they are in general conservative. The uncertainty can have structure that is not accounted for in comparing the maximum singular value to the desired upper bound. Structured perturbations can be dealt with by the small μ test of Doyle (8).

Time Simulations The response of the helicopter to the four pilot commands was simulated in the TSIM environment (9), using the nonlinear model HELISIM3

(6), provided by RAE, Bedford. This nonlinear model contained blade flapping dynamics and actuator dynamics modelled as first order lags. These dynamics where initially omitted from the nominal linearized description of the plant, Simulations showed that the responses to the pilot commands were significantly more decoupled compared to the unaugmented system and were not noticeably affected by the unmodelled dynamics. The actuator dynamics proved significant however, when the time constants of the filters were of similar magnitude to the desired closed loop response.

Fig. 9 shows a step command of 30° of bank angle. It can be seen from the plot that the desired response is achieved in 1.5 sec consistent with the requirement given in (7). A slight drift in the roll angle is seen due to the directional stability of the helicopter trying to turn the craft from its original heading and causing the nose to drop. The increase in speed from 0 to 40 knts represents a considerable change in the dynamic characteristics of the helicopter. This change represents unmodelled dynamics with variation in flight envelope. It has been shown that the controller is still stabilizing and is able to maintain performance throughout.

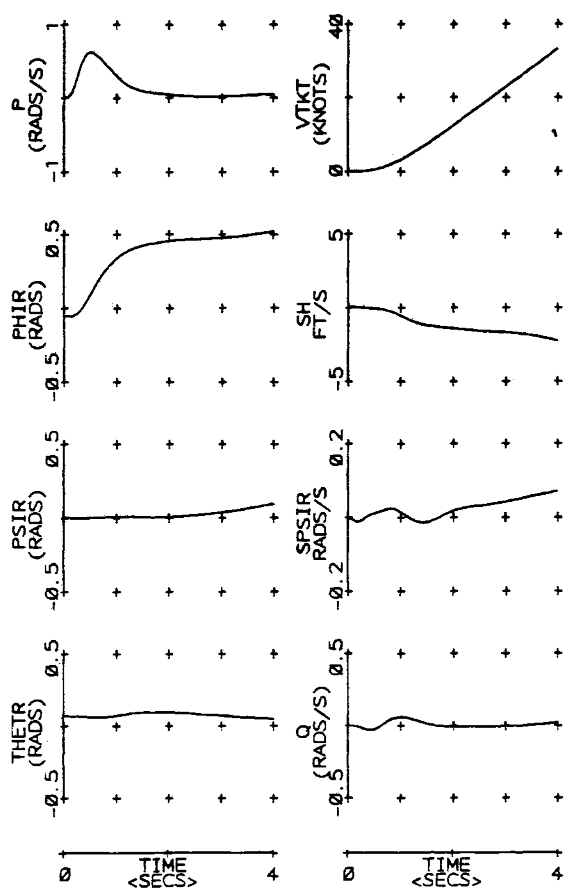

Fig. 9 Roll response to a 30° step command in bank angle

Fig. 10 shows a corresponding 15° step command in pitch angle. The desired response is achieved in less than 1.5 secs with little coupling in the other channnels.

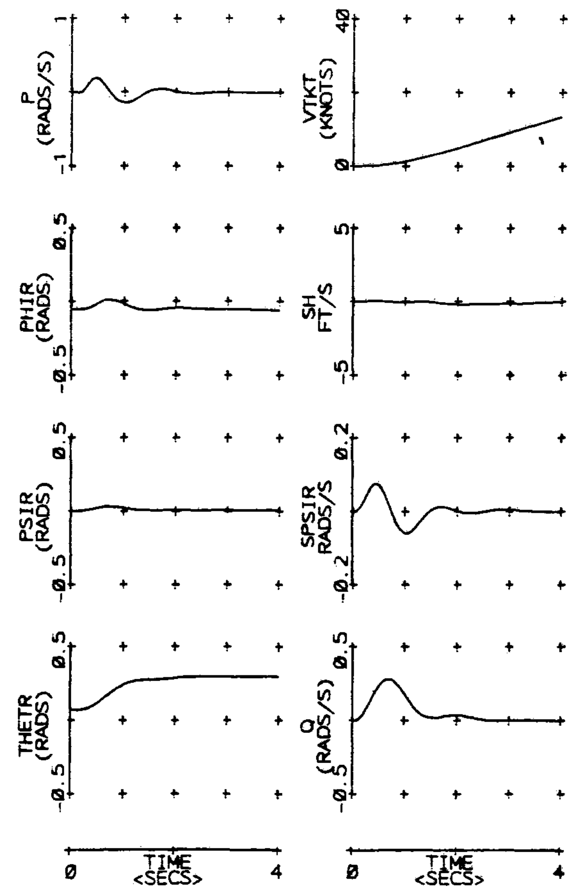

Fig. 10 Pitch response to a 15° step command in pitch angle

4. Conclusions

Control laws for a typical battlefield helicopter in the hover flight condition have been presented. The control laws form the stabilization loop of a control structure for tailoring pilot commands to meet desired criteria. The performance and stability robustness of the control laws were presented in terms of singular values of specific frequency responses. The loop shaping procedure for designing the control laws using H^{∞} optimization was presented and then analyzed with respect to the singular values of the sensitivity and complementary sensitivity functions.

It is well known that control in the hover is difficult due to the rapid and significant changes in dynamics when manoeuvring from this position. Throughout the design procedure a nominal model of the helicopter trimmed about the hover has been used. The controller designed has fixed gains relative to the nominal model. The nonlinear time simulations show the design to be robustly stable to the unmodelled rotor and actuator dyanmics and to the unmodelled dynamics due to changes in the flight envelope.

5. Acknowledgements

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6. References

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