TOWARDS AN ADVANCED NONLINEAR ROTORCRAFT FLIGHT CONTROL SYSTEM DESIGN

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Abstract

Traditional rotorcraft flight control designs rely heavily on plant models which have been linearized about various operating set points. These designs are only valid for operating conditions close to the original trim points and suffer performance degradations for significant deviations. Consequently, several linear controllers are designed and scheduled to cover the operational flight envelope. An alternate approach that uses the inherent nonlinearities of the plant model may provide improved performance. Described herein are the initial results of a rotorcraft control design methodology that uses an aircraft-model-based transformation to convert the input-output map of the original nonlinear plant into a linear time-invariant system. The validity of the design technique for attitude stabilization is evaluated in a high-fidelity computer simulation of the UH-60 Black Hawk helicopter.

INTRODUCTION

The severe nonlinear characteristics associated with rotorcraft dynamics pose serious challenges to traditional linear control design techniques. Rotorcraft control is especially demanding due primarily to the anomalies associated with attitude dynamics and severe inter-axis cross-coupling. Hence, any linear control design methodology hoping to consider significant portions of the flight envelope is forced to employ many different linearized plant models at various operating set points of interest. The resulting collection of linear controllers are then gain-scheduled to constitute the overall controller. Emerging nonlinear design methods exist, however, that make direct use of the inherent nonlinearities of the plant model to enable the design of a single controller which remains applicable over the full operational flight envelope. One such method is referred to in the literature as the feedback linearization technique [1].

Feedback linearization has enjoyed considerable success recently as a viable means of nonlinear control for aircraft, missiles, and rotorcraft [2, 3, 4, 5, 6]. The salient feature of this design methodology is the incorporation of an inverse model of the vehicle's force and moment generating process into the controller. The combination of the inverse model with the nonlinear plant transforms the input-output map of the original nonlinear system into a linear time-invariant form. The transformed system is then easily controlled using any well-known linear control design technique.

Further simplification of the control design process can be realized by dividing the rotorcraft dynamics into multiple time scales using the singular perturbation theory [7]. This technique has been used to produce a two-time-scale controller in which the fast dynamics of the rotational state components appear decoupled from the slower state components associated with translational dynamics [4, 5]. Controllers for the two resulting *reduced-order* dynamic systems are then designed separately, with commanded attitude output from the slow-time-scale system providing the necessary coupling between the controllers. This method has been shown to work well provided adequate eigenvalue separation is maintained [4]. In situations where the pilot commands the rotorcraft attitude, the fast-time-scale control law can be used to provide stability augmentation while imparting the desired handling qualities.

The ideas above are part of an initial design of an advanced nonlinear rotorcraft flight control system for

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the UH-60A Black Hawk helicopter (refer to Figure 1) being developed for the Automated Nap-of-the-Earth (NOE) Flight Program at NASA Ames Research Center [6]. The research is part of a cooperative effort

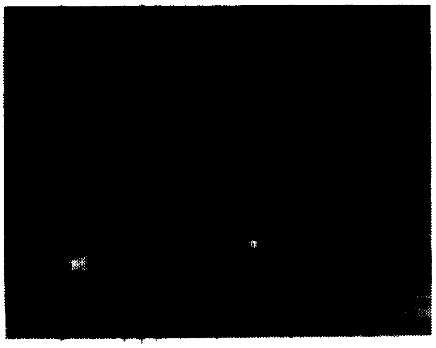

Figure 1: The ANOE flight research vehicle

between NASA and the Army to define and develop advanced technologies to substantially enhance piloted low-altitude and NOE rotorcraft flight [8]. The emphasis of the research is on reducing the rotorcraft pilot's work load during these flight modes, in which the aircrew seeks concealment while following mission objectives by maneuvering the rotorcraft as close to the ground as vegetation and ground-based obstacles will allow. The long-term objective of the automated NOE research program is to merge the required technologies in a piloted flight test to demonstrate the feasibility of automation and pilot aiding in NOE flight.

The critical nature of automated NOE flight, in which there is virtually no margin for error, places extremely high demands on the flight control system. Actual flight tests can occur only after extensive simulation of the guidance and control laws in the laboratory using an accurate model of the operational rotorcraft. Early research was based on the TMAN [9] simulation model and evaluated in a real-time graphical computer simulation [6]. However, the TMAN model differs significantly from the actual helicopter, thus the present research effort is to incorporate and test the proposed control system design with a much more realistic model of the actual system.

The results presented in this paper are part of an initial extension to the research reported in reference [6]. The real-time graphical simulation and control system design were first modified to include the high-fidelity GENHEL UH-60A rotorcraft model [10]. This model has been validated against flight test data obtained from the actual aircraft [11] and is being used at Ames Research Center for rotorcraft handling-qualities research. The dynamics of the new plant were then decoupled into fast and slow time scales, and a nonlinear control law incorporating an inverse model of the plant's forces and moments was then synthesized for the fast-time-scale rotational dynamics. This preliminary paper describes the development of the fast-time-scale nonlinear attitude control law for the GENHEL model, valid over most of the rotorcraft flight-envelope. The control law provides attitude stabilization and attitude command tracking while conforming to the Army's ADS-33C [12] handling-qualities requirements for military rotorcraft.

VEHICLE MODEL

The comprehensive mathematical model of the UH-60A is based on a set of generalized program modules developed by the manufacturer for flight dynamic analysis of total helicopter systems. These program modules make up the Sikorsky General Helicopter Flight Dynamics Simulation (GENHEL). A real-time simulation version of this mathematical model was provided to NASA under contract and is described in detail in reference [10]. NASA, under an internal research program, has increased the fidelity of the model, most notably in the areas of engine, drive train, and inflow modeling.

The model is a nonlinear representation of a single-main-rotor helicopter, applicable over the full range of angles of attack, sideslip, and the rotor inflow state. Six rigid-body degrees of freedom, as well as the main-rotor rigid-blade flapping and lead-lag, air mass, and hub rotational-speed degrees of freedom, are all modeled. Because of its modular nature, each major force and moment-producing component of the aircraft is treated as an independent entity. The interface between these components is provided by an executive module and framework. The module interfaces are physical quantities such as forces, moments, attitudes, body-fixed velocities, and downwash velocities. This allows interactions between the components to be modeled while maintaining the flexibility to modify or replace any one component without modification of the other components. Figure 2 gives a block diagram representation of the rotorcraft model components.

Blade-element theory [13] is used to model each main-rotor blade. Total rotor forces and moments are produced by summing the forces from each blade, these forces in turn are determined from aerodynamic, inertial, and gravitational forces. Aerodynamic forces are computed from angle of attack and dynamic pres-

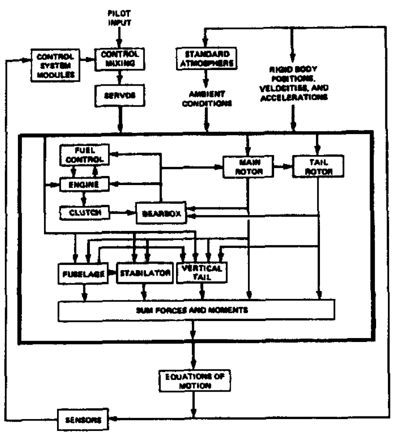

Figure 2: Components of the GENHEL model

sure at each blade segment based on the orthogonal velocity components. These velocity components are determined as functions of blade azimuth, lag and flap angles, local velocity of the blade segment, and local downwash. Rotor inflow is approximated to have a first harmonic distribution as a function of wake skew angle. Blade inertia and gravitational forces are computed from blade rotational velocity, lagging and flapping velocities and accelerations, and blade position. Nonlinear lead-lag damper characteristics are represented explicitly. No dynamic twisting or bending of the blades is modeled, although a pre-formed blade twist is represented through adjustment of geometric pitch of each blade segment. The summation of blade forces act on the airframe through the blade hinge and lag damper locations. Moments on the rotor shaft resulting from blade hinge and lag damper offsets from the main-rotor shaft are then computed.

Tail-rotor thrust is represented by linearized Bailey theory [14] and aerodynamic interference effects are either empirically determined or are derived from analysis-oriented simulations. Main-rotor downwash is used to modify the tail-rotor inflow. The aerodynamics of the fuselage, stabilator, and vertical tail pylon are each represented in separate modules so that nonlinear interference effects due to the main rotor and interference effects between the various components can be modeled separately. Aerodynamic function tables are used that were developed from wind tunnel test data, and supplemented with flight test data.

Rigid Body Dynamics

At each simulation cycle k within the rotorcraft real-time model, the forces and moments acting at the rotorcraft center of gravity are calculated for each GENHEL subcomponent i and summed to give the total force and moment vectors in the vehicle body-axis. These results are then integrated to obtain the instantaneous position, velocity, attitude, and attitude rate vectors to be used during the next cycle of the simulation.

An illustration of the vehicle's coordinate system is given in Figure 3.

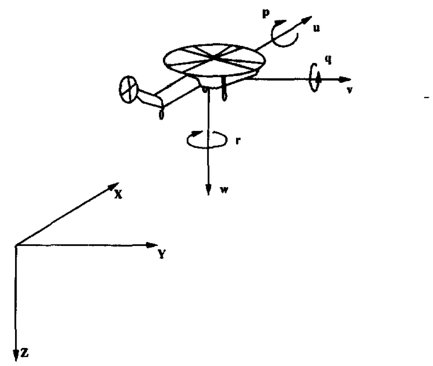

Figure 3: The rotorcraft coordinate system

The total force vector in the body-axis is given by

$$\mathcal{F}_k = \sum_i \begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix}_i . \tag{1}$$

The total inertial force vector F_k in the north-east-down inertial coordinate system is then obtained by coordinate transformation $F_k = T_{IB_k}\mathcal{F}_k$, where T_{IB} is the body to inertial transformation matrix. The instantaneous acceleration vector at time k with respect to the inertial coordinate system is now readily available from

$$\dot{V}_k = \frac{1}{m} F_k + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{2}$$

where m is the mass of the rotorcraft and g is the acceleration due to gravity.

The new velocity vector in inertial coordinates to be used during the next cycle $V_{k+1} = [V_n, V_e, V_d]'_{k+1}$ is obtained within the GENHEL simulation by integrating equation (2) using an Adams-Bashforth-Moulton predictor-corrector method [15].

The total moment vector in the body-axis is given by

$$\mathcal{T}_k = \sum_i \begin{bmatrix} L_k \\ M_k \\ N_k \end{bmatrix}_i - \omega_k \times H_{gyro} \tag{3}$$

where $\omega_k = [p, q, r]'_k$ are the instantaneous angular rates valid at time k and H_{gyro} is a constant vector in the GENHEL rotorcraft model body axis corresponding to the angular momentum due to internal rotating parts (i.e. the engine spool assembly).

The instantaneous angular accelerations in the body coordinate system are determined by

$$\dot{\omega}_k = J^{-1} [\mathcal{T}_k - \omega_k \times [J\omega_k]] \tag{4}$$

where the rotorcraft inertia matrix J is defined as

$$J = \begin{bmatrix} I_x & 0 & I_{xz} \\ 0 & I_y & 0 \\ I_{xz} & 0 & I_z \end{bmatrix}. \tag{5}$$

Note that the presence of zero entries in this matrix arise due to the assumption that the rotorcraft x-z plane is a plane of symmetry.

The angular velocities applicable during the next cycle ω_{k+1} are determined by integration of equation (4) using the same integration technique as that used for the translational velocities.

The Euler angle rates are then calculated using the kinematic relationship

$$\dot{\Theta} = \Omega = R\omega \tag{6}$$

where R is the transcendental transformation matrix from body angular rates to Euler rates. These Euler rates are then integrated to determine the Euler angles for the next cycle $\Theta_{k+1} = [\phi, \theta, \psi]'_{k+1}$.

The new inertial-to-body transformation matrix $T_{BI_{k+1}}$ is now computed using the updated Euler angles and used to obtain the new instantaneous translational body-axis velocities

$$\mathcal{V}_{k+1} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = T_{BI_{k+1}} V_{k+1} \tag{7}$$

and accelerations

$$\dot{\mathcal{V}}_{k+1} = T_{BI_{k+1}} \dot{V}_{k+1} - \omega_{k+1} \times \mathcal{V}_{k+1} \tag{8}$$

applicable at the beginning of the next cycle.

FLIGHT CONTROL SYSTEM DESIGN

The GENHEL model is used as the basis of the flight control system design. The model is a close approximation of the actual UH-60 Black Hawk helicopter, and has been shown to capture the essential nonlinear dynamical characteristics of the actual plant. In this section, the development of a nonlinear attitude control law for the GENHEL model will be described. First, the technique for time-scale separation of the rotorcraft dynamics into fast and slow time scales will be outlined. This is then followed by a detailed description of the nonlinear control law design process.

Time Scale Separation

The singular perturbation method is used to realize the effect of separating the rotorcraft dynamics into two reduced-order time scales, a slow time scale associated with translational dynamics, and a fast time scale for rotational dynamics. The resulting system approximates the behavior of the original system by combining the effects of the fast and slow time scale behaviors of the two reduced-order systems. The advantage of this technique is that the resulting controller will also be of reduced order. Additionally, the time-scale-separation process allows current helicopter flight control modes to be naturally preserved, while producing robust dynamical structures for the inversion procedure of feedback linearization.

The rotorcraft rigid body dynamics model is 12th-order and consists of three-dimensional translational and rotational displacement state vectors and their derivatives with respect to time. For the current analysis, it is convenient to split the state vector into two six-dimensional vectors, one representing the translational state, the other the rotational state.

The rotorcraft equations of motion can then be written in the form

$$\begin{bmatrix} \dot{X} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} V \\ f(X, V, \Theta, \Omega, col, ped) \end{bmatrix} \tag{9}$$

$$\epsilon \begin{bmatrix} \dot{\Theta} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} \Omega \\ g(X, V, \Omega, lat, lon, ped) \end{bmatrix} \tag{10}$$

where $f(\cdot)$ and $g(\cdot)$ are nonlinear vector functions of the rotorcraft state and control positions (lat, lon, ped, col). The interpolation parameter ϵ is introduced to motivate time scale separation, with $\epsilon = 0$ relating to the slow-time-scale system, and $\epsilon = 1$ corresponding to the full-order system [7].

By setting $\epsilon = 0$, one obtains the slow-time-scale system in which the rotational state dynamics appear instantaneous and satisfy the "quasi-steady-state" condition

$$0 = \Omega \quad , \quad 0 = g(X, V, 0, lat, lon, ped) \ . \tag{11}$$

These expressions imply that in the slow time scale, the aircraft control settings must be chosen to achieve moment equilibrium, and that the body rotational rates should be set to zero.

Thus

$$\dot{\overline{X}} = \overline{V} \quad , \quad \dot{\overline{V}} = f(\overline{X}, \overline{V}, \overline{\Theta}, 0, \overline{col}, \overline{ped}) \tag{12}$$

describes the new slow-time-scale translational dynamics. The primary control variable is the main rotor collective. In addition, the components of the attitude vector $\overline{\Theta}$ appear "control-like" as well, since their dynamics are instantaneous. Note that the tail rotor collective $\overline{ped}$ is chosen to ensure moment equilibrium.

Controllers can be designed in the slow time scale to generate $\overline{\Theta}$ and $\overline{col}$ in order to track commanded position/velocity commands. The commanded attitude components can then be tracked using a fast-time-scale attitude control system.

Attitude Controller

The attitude controller is designed to track commanded values for rotorcraft body attitudes. The commanded attitude reference input is either the result of a slow-time-scale outer-loop controller, or it is assumed to come directly from the pilot. This control law provides airframe stabilization while tracking commanded attitudes.

A main tenet of the feedback linearization technique used for this design is the incorporation of an inverse model of the rotorcraft's force and moment generating process within the structure of the controller. The inverse model uses the current state to transform the original plant into a linear time-invariant form. If one assumes at the outset that an inverse model is available, one can then start the control design by analyzing the series combination of the inverse model with the original plant.

For attitude tracking, an essential ingredient is the ability to generate a commanded moment in response to an attitude error signal. The commanded moment is then mapped by the inverse model into the corresponding commanded *control positions* which are then fed into the rotorcraft actuators. If the inverse model is exact, then the moment generated by the original system in response to these commanded control settings will be equal to the commanded moment. Ideally, because of the physical relationship of moments to angular displacement, the attitude error state of the transformed rotorcraft system should respond with the dynamics of a set of decoupled double-integrator systems.

The attitude errors are defined as follows

$$\Delta\Theta = \Theta - \overline{\Theta} = \begin{bmatrix} \phi - \bar{\phi} \\ \theta - \bar{\theta} \\ \psi - \bar{\psi} \end{bmatrix} . \tag{13}$$

From equation (11), the commanded attitudes are assumed to be in steady state (i.e. $\dot{\overline{\Theta}} = \ddot{\overline{\Omega}} = 0$). So $\Delta\dot{\Theta} = \dot{\Theta}$ and $\Delta\ddot{\Theta} = \ddot{\Theta}$. Thus, the total attitude error state is given by

$$\Delta\Phi = \begin{bmatrix} \Delta\Theta \\ --- \\ \dot{\Theta} \end{bmatrix} \tag{14}$$

and

$$\Delta\dot{\Phi} = \begin{bmatrix} \dot{\Theta} \\ --- \\ \ddot{\Theta} \end{bmatrix} . \tag{15}$$

Ultimately, the intention is to control the rotorcraft body moments directly in response to the attitude error. From equation (4) the instantaneous rotorcraft body moments are given by

$$\mathcal{T} = J\dot{\omega} + \omega \times [J\omega] \tag{16}$$

which can be expressed as a function of $\Delta\dot{\Phi}$ as seen from equations (6) and (15). Thus, to generate a commanded moment $\mathcal{T}_c$, one may choose $\mathbf{U}_c = \Delta\dot{\Phi}_c$ as the "pseudo-control" variable in the linearized system. A full-state feedback control law of the form

$$\mathbf{U}_c = -\mathbf{K}\Delta\Phi \tag{17}$$

is then designed. This control law may be expanded to yield

$$\ddot{\Theta}_c = -k_1\,(\Delta\Theta) - k_2\,\dot{\Theta} \tag{18}$$

(with $\dot{\Theta}_c = 0$) where k_1 and k_2 are chosen to realize some specified performance objective.

The resulting attitude control structure is shown in Figure 4. A commanded angular acceleration $\ddot{\Theta}_c$ is generated to counteract any GENHEL Euler angle state deviation from the reference input $\overline{\Theta}$. This results in a commanded moment $\mathcal{T}_c$ to the system via equation (16). The required control positions $u = [lat, lon, ped]'$ corresponding to the commanded moment are determined by the inverse GENHEL moment model which then drive the nonlinear GENHEL model. It should be noted that the $[\mathbf{J}]$ and $[\mathbf{J}^{-1}]$ blocks within the diagram include nonlinear effects associated with Euler-to-body-angle coordinate transformation.

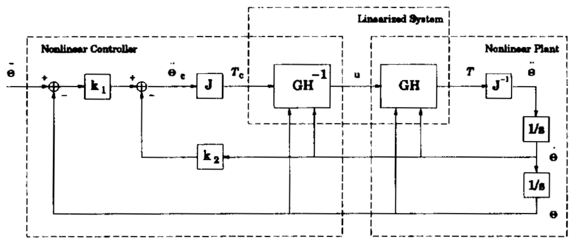

Figure 4: Detail of nonlinear attitude control structure

Model Transformation

The remaining task in the attitude controller design is the determination of the GENHEL inverse moment model. The inherent nonlinearities of the high-fidelity rotorcraft model are approximated by using the aerodynamic force-moment partial derivatives resulting from the available GENHEL trim logic. The trim logic determines the necessary control positions to satisfy the force-moment equilibrium condition ($\mathcal{F} = 0, \mathcal{T} = 0$) given input constraints (trim conditions) on the state variables. As a by-product, the logic produces a Jacobian matrix of partial derivatives corresponding to the rotorcraft state and control stability derivatives at the given trim condition. These stability derivatives can be used directly to determine the GENHEL inverse moment model.

In trim at some arbitrary rotorcraft state and control position, the trim logic assumes

$$\mathcal{T} = G(x_t, u_t, col_t) = 0 \tag{19}$$

where $G(\cdot)$ is a nonlinear function of the vehicle state and control positions, $x_t = [\mathcal{V}_t', \omega_t']'$ are the state trim constraints, $u_t = [lat, lon, ped]_t'$ are the control positions associated with moment generation, and col_t is the collective trim position. The first-order Taylor series approximation about this trim point incorporating the trim condition of equation (19) yields

$$\mathcal{T} \cong \left[\frac{\partial G}{\partial x}\right]_{x_t} \delta x + \left[\frac{\partial G}{\partial u}\right]_{u_t} \delta u + \left(\frac{\partial G}{\partial col}\right)_{col_t} \delta col \tag{20}$$

where $\delta(\cdot) \triangleq (\cdot) - (\cdot)_t$. Thus, given the desired moment $\mathcal{T}_c$, the control settings required to realize this moment are given by the expression

$$u = u_t + \left[\frac{\partial G}{\partial u}\right]_{u_t}^{-1} \left[\mathcal{T}_c - \left[\frac{\partial G}{\partial x}\right]_{x_t} \delta x - \left(\frac{\partial G}{\partial col}\right)_{col_t} \delta col\right] \tag{21}$$

where $\mathcal{T}_c$ is the commanded moment from our control law via equations (6), (16) and (18).

RESULTS

The attitude control system design was implemented and evaluated in a high-fidelity real-time graphical computer simulation of the UH-60 Black Hawk using the GENHEL rotorcraft model running at 100Hz. The feedback control gains were chosen to yield eigenvalues at $-2.7 \pm .8426i$ (i.e. $[k_1, k_2] = [8, 5.4]$). These eigenvalues correspond to Level 1 flying qualities for attitude-command/attitude-hold rotorcraft response types [12]. Trim points for the inverse moment model at a fixed NOE altitude were chosen to span a flight envelope of -20 to $+80$ ft/s in u-body velocities, ± 50 ft/s in v-body velocities, and -50 to $+30$ ft/s in w-body velocities. The stability derivatives and control positions at these trim points were stored in a three-dimensional array. Linear interpolation of the data with respect to the rotorcraft state velocity vector was then used to generate the instantaneous inverse transformation of equation (21).

To analyze the control law performance, the rotorcraft and flight control system model were commanded to track step inputs to commanded attitude about one body axis while commanding zero attitude about the other axes. Figure 5 shows the forced response of the nonlinear feedback control system to a step input of 10 degrees in commanded roll angle. The other attitude angles were commanded to zero. The plot shows a 30 second segment of the response. The test began with all attitudes commanded to zero degrees. After one second the step input was initiated and held for the remainder of the test.

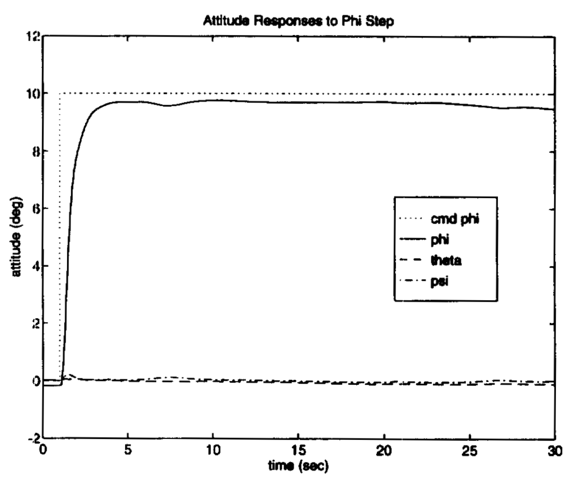

Figure 5: GENHEL attitude responses to 10 degree step in commanded roll angle

A critically damped response is obtained that tracks the commanded angle input well for the full 30 seconds. A steady-state offset can be observed that is present even in the one second zero-command-attitude time segment before the step. This steady-state error is present due to modeling error within the inverse GENHEL moment model and can be corrected by including an integrator into the linearized feedback control law of equation (18).

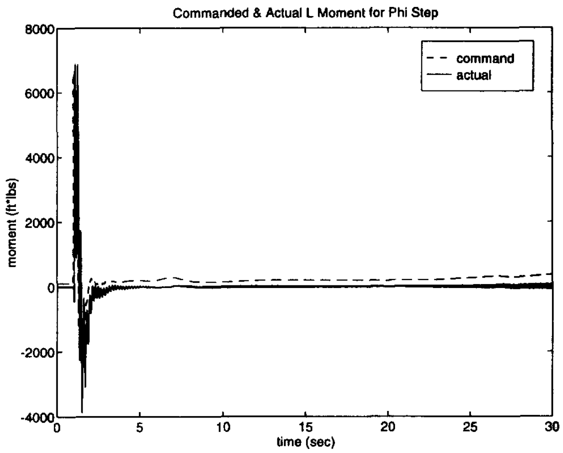

Figure 6: Commanded roll body moment in response to step in commanded roll angle

Figure 6 shows the commanded roll body moment from the control law due to the step input. Also shown is the actual GENHEL moment resulting from the commanded control position input. The small steady-state offset between the commanded and actual moments is related to the steady-state attitude error and is also a direct measure of the inverse modeling error. Ideally, the commanded-to-actual moment map should be identity.

The commanded rotorcraft control positions and body airspeed components resulting from the step in commanded roll attitude are shown in Figures 7 and 8 respectively. It should be noted that the control system was able to generate control positions well within the control limits to track the step input, even after significant buildup of translational velocities, with virtually no inter-axis cross-coupling.

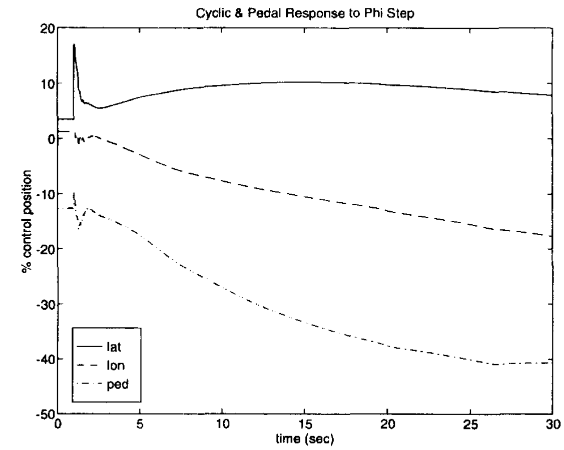

Figure 7: Control position response to step in commanded roll angle

CONCLUSION

A nonlinear attitude flight control system design incorporating feedback linearization and time-scale separation was described and implemented for a comprehensive flight-test-validated engineering model of the UH-60 Black Hawk helicopter. The effectiveness of the design for attitude stabilization was evaluated in a high-fidelity computer simulation and shown to provide good tracking response to a step input. In addition, excellent suppression of inter-axis cross-coupling was observed. Commanded attitude tracking remained valid for attitude hold over an extended time period even after significant buildup of translational velocities.

Work is currently underway to synthesize a completely automatic nonlinear autopilot for the GENHEL rotorcraft model for automated NOE applications by incorporating the control law design described in this report into a design for an outer-loop control law for the slow-time-scale translational dynamics. Robustness issues related to modeling errors within the inverse force and moment models are also topics of future research.

ACKNOWLEDGEMENTS

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References

[1] R.W. Brockett. "Nonlinear Systems and Differential Geometry". *Proceedings of The IEEE*, volume 64, number 1, pages 61–72, January 1976.

[2] G. Meyer and L. Cicolani. "Application of Non-

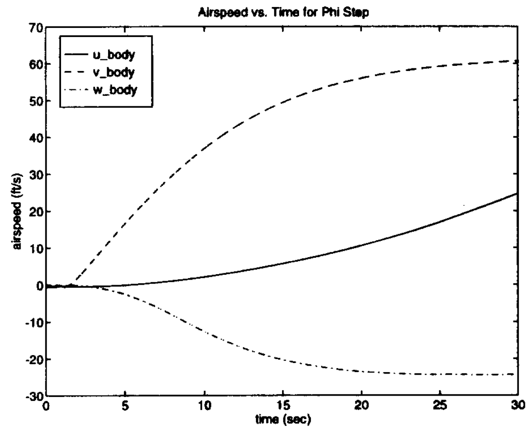

Figure 8: GENHEL airspeeds resulting from step in commanded roll angle

linear Systems Inverses to Automatic Flight Control Design: System Concepts and Flight Evaluations". *AGARDograph No. 251 on Theory and Applications of Optimal Control in Aerospace Systems*, pages 10.1–10.29. AGARD, July 1981.

[3] G.A. Smith and G. Meyer. "Aircraft Automatic Flight Control System With Model Inversion". *Advances in Aeronautical Systems*, volume 38 of *Control and Dynamic Systems*, pages 1–40. Academic Press, Inc., San Diego, CA, 1990.

[4] M.W. Heiges, P.K.A. Menon, and D.P. Schrage. "Synthesis of a Helicopter Full Authority Controller". *Proceedings of The AIAA Guidance, Navigation, and Control Conference*, Boston, MA, August 1989.

[5] P.K.A. Menon, G.B. Chatterji, and V.H.L. Cheng. "Two-Time Scale Autopilot for High-Performance Aircraft". *Proceedings of The AIAA Guidance, Navigation, and Control Conference*, New Orleans, LA, August 1991.

[6] V.H.L. Cheng and T. Lam. "Automatic Guidance and Control Laws for Helicopter Obstacle Avoidance". *Proceedings of The 1992 IEEE International Conference on Robotics and Automation*, Nice, France, May 1992.

[7] H.K. Khalil. *Nonlinear Systems*, chapter 8: Singular Perturbations. Macmillan Publishing Co., New York, NY, 1992.

[8] V.H.L. Cheng and B. Sridhar. "Considerations for Automated Nap-Of-The-Earth Flight". *Proceedings of American Control Conference*, Atlanta, GA, June 1988.

[9] M.S. Lewis and E.W. Aiken. "Piloted Simulation of One-on-One Helicopter Air Combat at NOE Flight Levels". Technical Memorandum 86686, NASA, Ames Research Center, Moffett Field, CA, April 1985.

[10] J.J. Howlett. "UH-60A Black Hawk Engineering Simulation Program: Volume I - Mathematical Model". Technical Memorandum 166309, NASA, Ames Research Center, Moffett Field, CA, December 1981.

[11] M.G. Ballin and M. Dalang-Secrétan. "Validation of the Dynamic Response of a Blade-Element UH-60 Simulation Model in Hovering Flight". *Proceedings of The 46th Annual Forum of the American Helicopter Society*, Washington, DC, May 1990.

[12] R.H. Hoh, D.G. Mitchell, B.L. Aponso, D.L. Key, and C.L. Blanken. "Handling Qualities Requirements for Military Rotorcraft". Aeronautical Design Standard ADS-33C, U.S. Army, Aviation Systems Command, St. Louis, MO, aug 1989.

[13] W.Z. Stepniewski and C.N. Keys. *Rotary-Wing Aerodynamics*, volume I: Basic Theories of Rotor Aerodynamics, chapter III: Blade Element Theory. Dover Publications, Inc., Mineola, NY, 1984.

[14] F.J. Bailey. "Simplified Theoretical Method of Determining the Characteristics of a Lifting Rotor in Forward Flight". Technical Report 716, NACA, Langley Research Center, Langley, VA, 1941.

[15] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. *Numerical Recipes in C: The Art of Scientific Computing*, chapter 16: Integration of Ordinary Differential Equations, pages 747–752. Cambridge University Press, New York, NY, 2nd edition, 1992.