Homework 03

Submission Notices:

- *Conduct your homework by filling answers into the placeholders given in this file (in Microsoft Word format). Questions are shown in black color, instructions/hints are shown in italic and blue color, and your content should use any color that is different from those.*
- *After completing your homework, prepare the file for submission by exporting the Word file (filled with answers) to a PDF file, whose filename follows the following format,*
  <StudentID-1>_<StudentID-2>_HW02.pdf (Student IDs are sorted in ascending order)
  E.g., ***1912001_1912002_HW02.pdf***
  and then submit the file to Moodle directly WITHOUT any kinds of compression (.zip, .rar, .tar, etc.).
- *Note that you will get zero credit for any careless mistake, including, but not limited to, the following things.*
  1. *Wrong file/filename format, e.g., not a pdf file, use "-" instead of "_" for separators, etc.*
  2. *Disorder format of problems and answers*
  3. *Conducted not in English*
  4. *Cheating, i.e., copy other students' works or let the other student(s) copy your work.*

Problem 1. (2pts) Consider the following sentence:

$[(Food \rightarrow Party) \lor (Drinks \rightarrow Party)] \rightarrow [(Food \land Drinks) \rightarrow Party]$

a) Use the truth table to verify whether the given sentence is valid (0.5pt)

Please fill your answer in the table below

Food	Drinks	Party	Sentence
True	True	True	True
False	True	True	True
True	True	False	True
False	True	False	True
True	False	True	True
False	False	True	True
True	False	False	True
False	False	False	True

b) Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a) (0.5pt)

Please fill your answer in the table below

Left-hand side	(Food → Party) ∨ (Drinks → Party) 1: (¬Food v Party) v (¬Drinks v Party) 2: Party v (¬Food v ¬Drinks) 3: Party v ¬Food v ¬Drinks
Right-hand side	[(Food ∧ Drinks) → Party] 1: ¬(Food ^ Drinks) v Party 2: ¬Food v ¬ Drinks v Party
Explanation	Because the CNF form of left hand side and right hand side are the same so the statement is valid (a => a is True no matter it is True or False).

c) Prove your answer to (a) using refutation resolution. (1pt)

Please fill your answer in the table below

¬[(Party v ¬Food v ¬Drinks) => (¬Food v ¬ Drinks v Party)] 1: ¬[¬(Party v ¬Food v ¬Drinks) v (¬Food v ¬ Drinks v Party)] 2: ¬[(¬Party ^ Food ^ Drinks) v (¬Food v ¬ Drinks v Party)] 3: ¬(¬Party ^ Food ^ Drinks) ^ ¬(¬Food v ¬ Drinks v Party) 4: (Party v ¬Food v ¬Drinks) ^ Food ^ Drinks ^ ¬Party From 4, the KB we have: (Party v -Food v –Drinks) (1) Food (2) Drinks (3) ¬Party (4) ¬Food v ¬Drinks (5) From (1), (4) ¬Drinks (6) From (5), (2) Empty (6), (3) Thus, the given statement is valid

Problem 2. (2pts) Given a knowledge base as follows

$$P \vee Q,\ Q \rightarrow (R \wedge S),\ (P \vee R) \rightarrow U$$

Check whether each of the following sentences is entailed by KB using PL-Resolution

a) U

b) ¬U

Please fill your answers in the table below

a) (1pt)

1) P v Q	7) ¬P v U (3)	
2)(¬Q v R) ^ (¬Q v S)	8) ¬R v U (3)	
3) (¬P v U) ^ (¬R v U)	9) ¬P (4,7)	
4) ¬U	10) Q (1, 9)	
5) ¬Q v R (2)	11) R (5, 10)	
6) ¬Q v S (2)	12) (empty) (8,11)	

b) (1pt)

1) P v Q	7) ¬P v U (3)	13) S (6, 10)
2)(¬Q v R) ^ (¬Q v S)	8) ¬R v U (3)	
3) (¬P v U) ^ (¬R v U)	9) ¬P (4,7)	
4) U	10) Q (1, 9)	
5) ¬Q v R (2)	11) R (5, 10)	
6) ¬Q v S (2)	12) U (11, 8)	

There is no contradiction of U when generate a new clauses. So KB does not entails ¬U

Problem 3. (2pts) Consider a vocabulary with the following symbols:

Occupation(p, o): Predicate. Person p has occupation o. [O(p, o)]

Customer (p1, p2): Predicate. Person p1 is a customer of person p2. [C(p1, p2)]

Boss(p1, p2): Predicate. Person p1 is a boss of person p2. [B(p1, p2)]

Doctor , Surgeon, Lawyer , Actor : Constants denoting occupations. [D, S, L, A]

Emily, Joe: Constants denoting people. [E, J]

Translate the following FOL sentences into English.

Please fill your answers in the table below

1	O(E, S) ∨ O(E, L) Emily has occupation Surgeon or Emily has occupation Lawyer
2	O(J, A) ∧ ∃p, p ≠ A ∧ O(J, p) Joe is an actor but he also has another jobs
3	∀p, O(p, S) → O(p, D) If everybody is a surgeon then they are also a doctor
4	-∃p, C(J, p) ∧ O(p, L) There does not exists a lawyer that Joe is a customer of them
5	∃p, B(p, E) ∧ O(p, L)

	Emily has a boss who is a lawyer
6	$\exists p, O(p, L) \wedge \forall q, C(q, p) \rightarrow O(q, D)$ There exists a lawyer that all of whose customers are doctor
7	$\forall p, O(p, S) \rightarrow \exists q, O(q, L) \wedge C(p, q)$ Every surgeon has a lawyer

Problem 4. (2pts) Given a chess board with 4 rows and 4 columns (4x4) as below.

			

Assign a Boolean variable to each cell of the board as below (*1, 2, 3, etc. are variable names*)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Write CNF clauses for each of the following cases when placing a queen in cell [6].

Please fill your answer in the table below

1.	There is a queen in cell [6] if and only if there is no queen in the same row $(\neg Q6 \vee \neg Q5) \wedge (\neg Q6 \vee \neg Q7) \wedge (\neg Q6 \vee \neg Q8) \wedge (Q6 \vee Q5 \vee Q7 \vee Q8)$
2.	There is a queen in cell [6] if and only if there is no queen in the same column $(\neg Q6 \vee \neg Q2) \wedge (\neg Q6 \vee \neg Q10) \wedge (\neg Q6 \vee \neg Q14) \wedge (Q6 \vee Q2 \vee Q10 \vee Q14)$
3.	There is a queen in cell [6] if and only if there is no queen in the same major diagonal $(\neg Q6 \vee \neg Q1) \wedge (\neg Q6 \vee \neg Q11) \wedge (\neg Q6 \vee \neg Q16) \wedge (Q6 \vee Q1 \vee Q11 \vee Q16)$
4.	There is a queen in cell [6] if and only if there is no queen in the same minor diagonal $(\neg Q6 \vee \neg Q3) \wedge (\neg Q6 \vee \neg Q9) \wedge (Q6 \vee Q3 \vee Q9)$

Question 5 (2pts) The following sentence states the rule for a two-player game that involves tossing a coin: "If it is the head then I win. If it is the tail then you lose". Use refutation resolution on propositional logic to prove that the situation "I win" will always hold.

1 Represent the above sentences with the following propositions, in both causual form and CNF form.

- Head: "It is the head of the coin"
- Tail: "It is the tail of the coin"
- IWin: "I win"
- YouLose: "YouLose".

Sentences	Casual form	CNF form
If it is the head then I win.	Head → IWin	¬Head v IWin
If it is the tail then you lose	Tail → YouLose	¬Tail v YouLose
I win.	IWin	IWin

2 The above clauses are still insufficient for your incoming proof. You may need to add some general knowledge axioms about coins, winning, and losing, yet the number of required axioms should be as few as possible.

1 IWin ↔ YouLose = (IWin → YouLose) ^ (YouLose → IWin) = (¬IWin v YouLose) ^ (¬YouLose v IWin)

2 ¬Head → Tail = Head v Tail

3 Perform the proof using refulation resolution.

1 ¬Head v IWin

2 ¬Tail v YouLose

3 ¬IWin v YouLose

4 ¬YouLose v IWin

5 Head v Tail

6 ¬IWin

7 IWin v Tail (5, 1)

8 YouLose v IWin (2, 7)

9 IWin (8, 4)

10 (empty) (9, 6)

KB entail IWin. So IWin will always hold