0.2. Найти полные дифференциалы первого и второго порядков функции: $u = f(\xi, \eta, \theta), \xi = x^2 + y^2, \eta = x^2 - y^2, \theta = 2xy$

$du = \frac{\partial f}{\partial \xi} d\xi + \frac{\partial f}{\partial \eta} d\eta + \frac{\partial f}{\partial \theta} d\theta = \frac{\partial f}{\partial \xi} d(x^2 + y^2) + \frac{\partial f}{\partial \eta} d(x^2 - y^2) + \frac{\partial f}{\partial \theta} d(2xy) =$
$= [\frac{\partial f}{\partial \xi} 2xdx + \frac{\partial f}{\partial \xi} 2ydy] + [\frac{\partial f}{\partial \eta} 2xdx - \frac{\partial f}{\partial \eta} 2ydy] + 2[\frac{\partial f}{\partial \theta} ydx + \frac{\partial f}{\partial \theta} xdy] =$
$= 2(x\frac{\partial f}{\partial \xi} + x\frac{\partial f}{\partial \eta} + y\frac{\partial f}{\partial \theta})dx + 2(y\frac{\partial f}{\partial \xi} - y\frac{\partial f}{\partial \eta} + x\frac{\partial f}{\partial \theta})dy.$

$d(du) = d(2(x\frac{\partial f}{\partial \xi} + x\frac{\partial f}{\partial \eta} + y\frac{\partial f}{\partial \theta})dx + 2(y\frac{\partial f}{\partial \xi} - y\frac{\partial f}{\partial \eta} + x\frac{\partial f}{\partial \theta})dy) =$
$= 2d(x\frac{\partial f}{\partial \xi} + x\frac{\partial f}{\partial \eta} + y\frac{\partial f}{\partial \theta})dx + 2d(y\frac{\partial f}{\partial \xi} - y\frac{\partial f}{\partial \eta} + x\frac{\partial f}{\partial \theta})dy =$
$= 2d(x\frac{\partial f}{\partial \xi})dx + 2d(x\frac{\partial f}{\partial \eta})dx + 2d(y\frac{\partial f}{\partial \theta})dx + 2d(y\frac{\partial f}{\partial \xi})dy - 2d(y\frac{\partial f}{\partial \eta})dy + 2d(x\frac{\partial f}{\partial \theta})dy =$
$= 2[(\frac{\partial f}{\partial \xi})dx^2 + xd(\frac{\partial f}{\partial \xi})dx] + 2[(\frac{\partial f}{\partial \eta})dx^2 + xd(\frac{\partial f}{\partial \eta})dx] + 2[(\frac{\partial f}{\partial \theta})dxdy + yd(\frac{\partial f}{\partial \eta})dx] +$
$+2[(\frac{\partial f}{\partial \xi})dy^2 + yd(\frac{\partial f}{\partial \xi})dy] - 2[(\frac{\partial f}{\partial \eta})dy^2 + yd(\frac{\partial f}{\partial \eta})dy] + 2[(\frac{\partial f}{\partial \theta})dxdy + xd(\frac{\partial f}{\partial \eta})dy] =$
$= 2[\frac{\partial f}{\partial \xi} + \frac{\partial f}{\partial \eta}]dx^2 + 2[\frac{\partial f}{\partial \xi} - \frac{\partial f}{\partial \eta}]dy^2 + 4[\frac{\partial f}{\partial \theta}]dxdy +$
$+2[xd(\frac{\partial f}{\partial \xi}) + xd(\frac{\partial f}{\partial \eta}) + yd(\frac{\partial f}{\partial \eta})]dx +$
$+2[yd(\frac{\partial f}{\partial \xi}) - yd(\frac{\partial f}{\partial \eta}) + xd(\frac{\partial f}{\partial \eta})]dy.$

Осталось подставить зачения:
$d(\frac{\partial f}{\partial \xi}) = 2(x\frac{\partial^2 f}{\partial \xi^2} + x\frac{\partial^2 f}{\partial \eta \partial \xi} + y\frac{\partial^2 f}{\partial \theta \partial \xi})dx + 2(y\frac{\partial^2 f}{\partial \xi^2} - y\frac{\partial^2 f}{\partial \eta \partial \xi} + x\frac{\partial^2 f}{\partial \theta \partial \xi})dy;$
$d(\frac{\partial f}{\partial \eta}) = 2(x\frac{\partial^2 f}{\partial \xi \partial \eta} + x\frac{\partial^2 f}{\partial \eta^2} + y\frac{\partial^2 f}{\partial \theta \partial \eta})dx + 2(y\frac{\partial^2 f}{\partial \xi \partial \eta} - y\frac{\partial^2 f}{\partial \eta^2} + x\frac{\partial^2 f}{\partial \theta \partial \eta})dy;$
$d(\frac{\partial f}{\partial \theta}) = 2(x\frac{\partial^2 f}{\partial \xi \partial \theta} + x\frac{\partial^2 f}{\partial \eta \partial \theta} + y\frac{\partial^2 f}{\partial \theta^2})dx + 2(y\frac{\partial^2 f}{\partial \xi \partial \theta} - y\frac{\partial^2 f}{\partial \eta \partial \theta} + x\frac{\partial^2 f}{\partial \theta^2})dy.$

4.2. Найти первую и вторую производные функции y(x), определяемойиз уравнения: $\ln(\sqrt{x^2 + y^2}) = \mathrm{arctg}(\frac{y}{x})$

Дифференцируем исходную функцию по $x, y = y(x)$:
$\frac{1}{\sqrt{x^2+y^2}} \frac{2x+2yy'}{2\sqrt{x^2+y^2}} = \frac{1}{1+(\frac{y}{x})^2} \frac{xy'-y}{x^2}$
$\frac{x+yy'}{x^2+y^2} = \frac{xy'-y}{x^2+y^2}$
$x + yy' = xy' - y$
$y'(y - x) = -y - x$
$y' = \frac{x+y}{x-y}$
$y'' = (\frac{x+y}{x-y})' = \frac{(x-y)(1+y')-(x+y)(1-y')}{(x-y)^2} = \frac{x-y+xy'-yy'-x-y+xy'+yy'}{(x-y)^2} = \frac{2xy'-2y}{(x-y)^2} =$
$= \frac{2x(\frac{x+y}{x-y})-2y}{(x-y)^2} = \frac{2x^2+2xy-2xy+2y^2}{(x-y)^3} = \frac{2y^2+2x^2}{(x-y)^3}.$

5.2. Найти дифференциалы первого и второго порядков функции $u = u(x, y):$
$x + y + u = e^{-(x+y+u)}$
$e^{\ln(x+y+u)} = e^{-(x+y+u)}$

$\ln(x+y+u) = -(x+y+u)$
$d(\ln(x+y+u)) + dx + dy + du = 0$
$\frac{dx+dy+du}{x+y+u} + dx + dy + du = 0$
$(dx+dy+du)(\frac{1}{x+y+u} + 1) = 0$
$(dx+dy+du)(\frac{1+x+y+u}{x+y+u}) = 0$
$du = -\frac{x+y+u}{1+x+y+u}\frac{dx+dy+xdx+xdy+ydx+ydy+udx+udy}{x+y+u}$
$du = -dx - dy$
$d(du) = -d(dx) - d(dy)$
Т.к. $y = y(x)$:
$du^2 = 0$