Day 3

Outline

1. Expected value of the random variable
2. Variance and standard deviation of random variable
3. [If time allows]

Expected values (mean)

Please refer to day_two.pdf

Law of Large Numbers

Suppose we have "N" independent and identically distributed (IID) realization of "X".

That is, we observe out random event "N" times independently and record the value of "X".

Then, as "N" increases, the sample mean of the "N" independent observations converges to μ_x

We can get arbitrarily close to µby simply observing values of "X" enough times.

Variance of a Random Variable

Average squared deviance from mean (distance away from the middle)

Variance Formula:

$$\sigma^2{}_x = \sum[x^2 * P(x)] - \mu_x^2$$

Figure 1: variance formula

- Variance is non-negative
- Variance is not a linear operator

In general, Var(X+Y) != Var(X) + Var(Y)

However, if X and Y are independent

Var(X+Y) == Var(X) + Var(Y)

Var(cX) != cVar(X)

However! –> Var(cX) == C^2Var(X)

When X and Y are independent,

Var(aX+bY) = $a^2\sigma^2{}_\mathrm{x} + b^2\sigma^2{}_\mathrm{y}$

Standard Deviation of Random Variable

$\sigma_\mathrm{x} = \sqrt{\sigma_\mathrm{x}}$

Standard deviation is not linear

$\sigma_\mathrm{x+y} = \sqrt{\sigma_\mathrm{x} + \sigma_\mathrm{y}}$

If X and Y are independent

$\sigma_\mathrm{cx} = |C|\sigma_\mathrm{x}$

Adding a constant

Consider W = X + c (where c is an arbitrary constant)

Σ[W] = Σ[X+c] = Σ[X] + Σ[c]

Σ(W) = Σ(X) + c

Var(W) = Var(X+c)

= Var(X) + Var(c)

Var(c) = 0

Var(W) = Var(x)

SD(W) = SD(X)

Example One

- Toss two fair coins.
- Let X be the number of heads observed
- Find the PMF, expected value, variance and standard deviation of X.

Each win is independent

P(Heads) = 1/2

Independence: P(A ∩ B) = P(A) * P(B) 0 heads: TT => P(TT) = $P(T_1)$ * $P(T_2)$ = 1/2 * 1/2 = 1/4 1 heads: HT TH 2 heads: HH

Easy Way

1. Find the PMF and write as a table
2. Expand our table by adding columns
3. Add down each column

```
# key is X=x
# value is P(X=x)
# --> points to xP(X=x)
# ---> points to variance
map = {
    0: 0.25 --> 0 ---> 0.25
    1: 0.50 --> 0.5 ---> 0
    2: 0.25 --> 0.5 ---> 0.25
}
```

summation of P(X=x) = 1

summation of xP(X=x) μ_x = 1

summation of variance sigma$^2{}_x$ = 0.5

summation of standard deviation of X ~= 0.7

Example Two

You enter a lottery in which there is a 1 in 1000 chance of winning. If you win, you get $500 and if you don't you get nothing Let Y be the amount of money you win.

Find the PMF, expected value, variance and standard deviation

```
map = {
    0:   0.999 -------> 0 ----> 0
    500: 0.001 ---> 0.5 --> 250 ---> 15.811
}
```

expected value: 0 and 500

Example Two (With a twist)

You enter a lottery in which there is a 1 in 1000 chance of winning. If you win, you get $500 and if you don't you get nothing Let Y be the amount of money you win.

Let V = amount of money you have after the lottery

Find the PMF, expected value, variance and standard deviation

Σ[V] = Σ[Y-1] = Σ[Y] - 1 = 0.5 - 1 = -0.5 Var(V) = Var(Y-1) = Var(Y) = 249.75 SD(V) = SD(Y) = 15.8

Relationship Between Probability and Statistics

Let our random event be:

Pick one person at random and record some characteristics of the individual

The individual we record is the <u>case</u> or <u>unit</u>

The characteristics we record are called <u>variables</u>

The set of all cases of interest: <u>population</u>