Contents

Day 14

Continuous Random Variables

Can take any real number ($\mathbb{R}$) value within any given interval.

We cannot use a probability mass function so we will instead use a probability density function (PDF) denoted as $f(x)$

Properties

- The probability of being in an interval (a, b] is:

$$\int_a^b f(x)dx = \int_{-\infty}^b f(x) - \int_{-\infty}^a f(x)dx$$

  - This is considered the area under the curve between a and b
- $P(X = x) = 0 \ \forall x$
  - $P(X \leq x) = P(X < x)$
  - $P(X \geq x) = P(X > x)$

$f(x)$ is displayed graphically as a density curve

Properties of $f(x)$

- $\forall \ x \in \mathbb{R}, \ f(x) \geq 0$
  - Density never goes below x-axis
- $\int_{-\infty}^{\infty} f(x)dx = 1$

Mean of continous random variable: $\mu_x = \int_{-\infty}^{\infty} x \times f(x)dx$

Variance of continous random variable is $\sigma^2 = \int_{-\infty}^{\infty}(X = \mu_x)^2 f(x)dx$

Uniform Random Variable

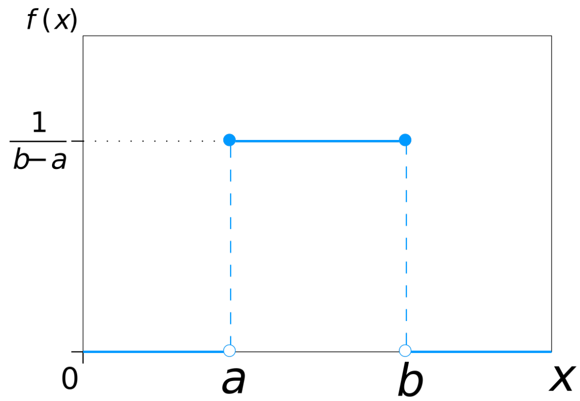

Figure 1: Graphical Representation

$X \sim U(a, b)$

Example : Standard Uniform Random Variable

$X \sim U(0, 1)$

Find

- P(X $\geq$ 0.3)
- P(X $=$ 0.3)
- P(0.3 $<$ X $\leq$ 1.3)
- P(0.2 $\leq$ X $\leq$ **or** 0.7 $\leq$ X $\leq$ 0.9)
- P(X is not in the interval (0.4, 0.7))

Answers

- $\sqcap = (0.7) \times (1) = 0.7$
- $\sqcap = 0$
  - The probability of being exactly on a point in the infinite sum will **always** be 0.
- $\sqcap = (0.7) \times (1) = 0.7$
  - Do not keep shading when there is no density curve, meaning it is a hard stop at $X = 1$
- $\sqcap = ((0.25 - 0.2) \times \frac{1}{0.25-0.2}) + ((0.91 - 0.7) \times \frac{1}{0.91-0.7}) = 0.25$
- $\sqcap = 0.4 + 0.3 = 0.7$

Normal Random Variable

$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Density curve is also a "bell curve"

Empirical (68-95-99.7) Rule

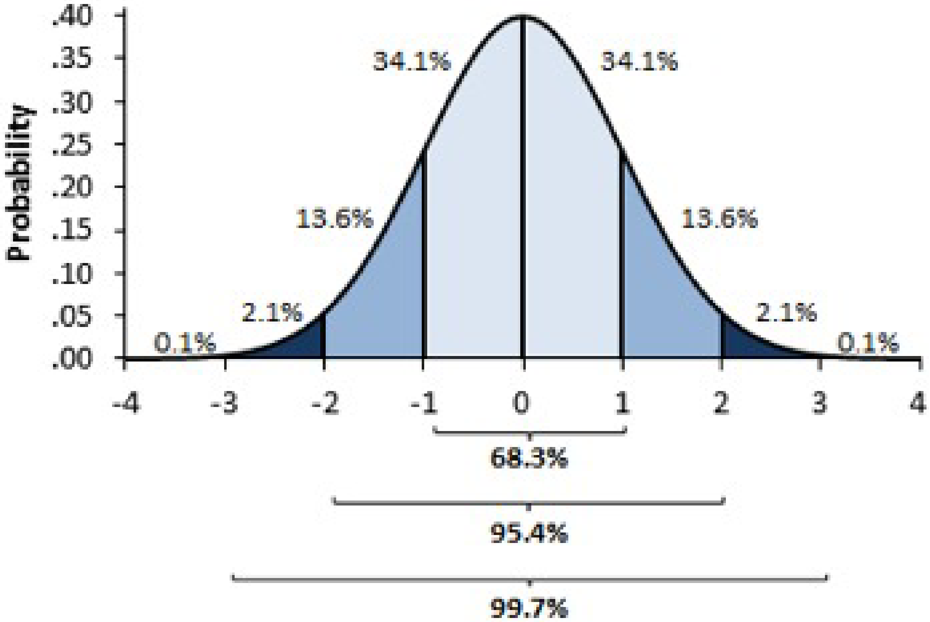

Figure 2: Bell Curve

$X \sim N(\mu\,, \sigma)$

Standardization

It may be useful to standardize distributions to compare 2 variables with same density curve shape but different scales. For normal distributions $X \sim N(\mu, \sigma)$, we convert to Z-Scores $Z \sim N(0, 1)$

$$Z = \frac{x - \mu}{\sigma} = \frac{value\ -\ mean\, of\, distribution}{standard\, deviation}$$

$$P(Z \leq z) = P(X \leq x)$$

"Cumulative proportion"/"Cumulative probability"