Math 125 – Fall 2017
Exam 1B

Name: SOLUTIONS

Chapter 1, Sections 1-5
Chapter 2, Sections 1 ,2, 3, 5, 6
Chapter 3, Sections 1-2

Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. *(24 Points)* A plumber charges \$22 for a house call plus \$35 per hour while she is there, up to a maximum of 12 hours.

   a. Express her cost C as a function of the number of t hours.

+5 Sol: $C(t) = 22 + 35t$

   b. What is the domain and range?

+4 Sol DOMAIN: $[0, 12]$ or $0 \le t \le 12$
RANGE: $[22, 442]$ or $22 \le C(t) \le 442$

   c. Evaluate C(6) describe what they represent in this context.

+4 Sol $C(6) = 22 + 35(6) = \boxed{232}$ The plumber charges \$232 for working 6 hours.

   d. Interpret $C^{-1}(\,389.5\,) = 10.5$ in this context.

+4 Sol: If the cost of plumbing is \$389.50, then the plumber worked 10.5 hours.

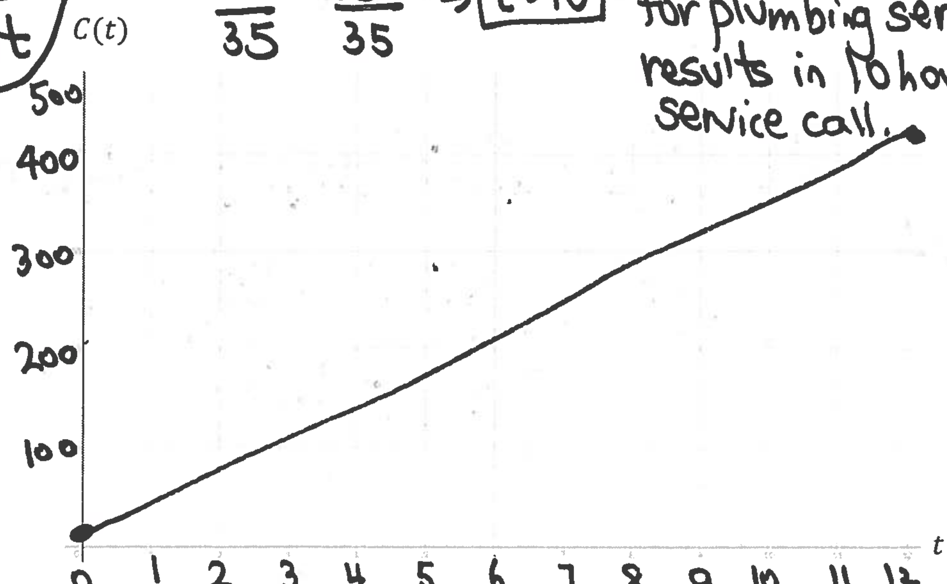
   e. Solve C(t) = 372 and describe what it represents in this context.

+3 Sol. $C(t) = 22 + 35t$
$372 = 22 + 35t$
$-22 \quad -22$
$\dfrac{350}{35} = \dfrac{35t}{35} \Rightarrow \boxed{t = 10}$

The cost of \$372 for plumbing service results in 10 hours of service call.

   f. Graph the function on the axis provided.

+4



2. *(20 Points)* The monthly rent of a storage shed is a linear function. The size of the shed varies from $150ft^2$ *to* $10{,}000\ ft^2$. A $300ft^2$ rents for \$490.00 where as a $150ft^2$ rents for \$320.00.

a. What was the average rate of change in **dollars per ft^2**?

Sol: a.v.r.c. $= \dfrac{320-490}{150-300} = \dfrac{-170}{-150} = \dfrac{17}{15} = \boxed{1.1333}$

+5

b. Construct a linear function C(t) for the monthly rent in dollars where **t is additional square feet after $150ft^2$.**

Sol: (0, 320) & (150, 490)

+4

$m = \dfrac{490-320}{150-0} = \dfrac{170}{150} = \dfrac{17}{15} = \boxed{1.1333}$

$C(t) = 1.1333t + 320$ where t is initially starting off with $150ft^2$.

c. What is the slope of the line? Explain what the value of the slope means in the context of this problem.

Sol: $m = \dfrac{1.1331}{1}$ or $\dfrac{17}{15}$ For every additional ft^2 over 150, the cost increases by \$17 for every 15 sq. ft.

+4

d. What is the vertical intercept of the line? Explain what the value of the vertical intercept means in the context of this problem.

Sol: (0,320). The initial cost of $150ft^2$ storage shed rents for \$320.00

+4

e. What would the cost be to rent a $4{,}200ft^2$ shed?

Sol: $C(t) = 1.1333t + 320$. $t = 4200 - 150 = 4050$

+3

$C(4050) = 1.1333(4050) + 320$

$\boxed{= \$4909.87}$

3. *(10 points)* It costs the *Dragon Fly* punk band \$480 to pay to rent a concert hall and additional \$17 per ticket fee, represented by the cost function

$$C(n) = 480 + 17n$$

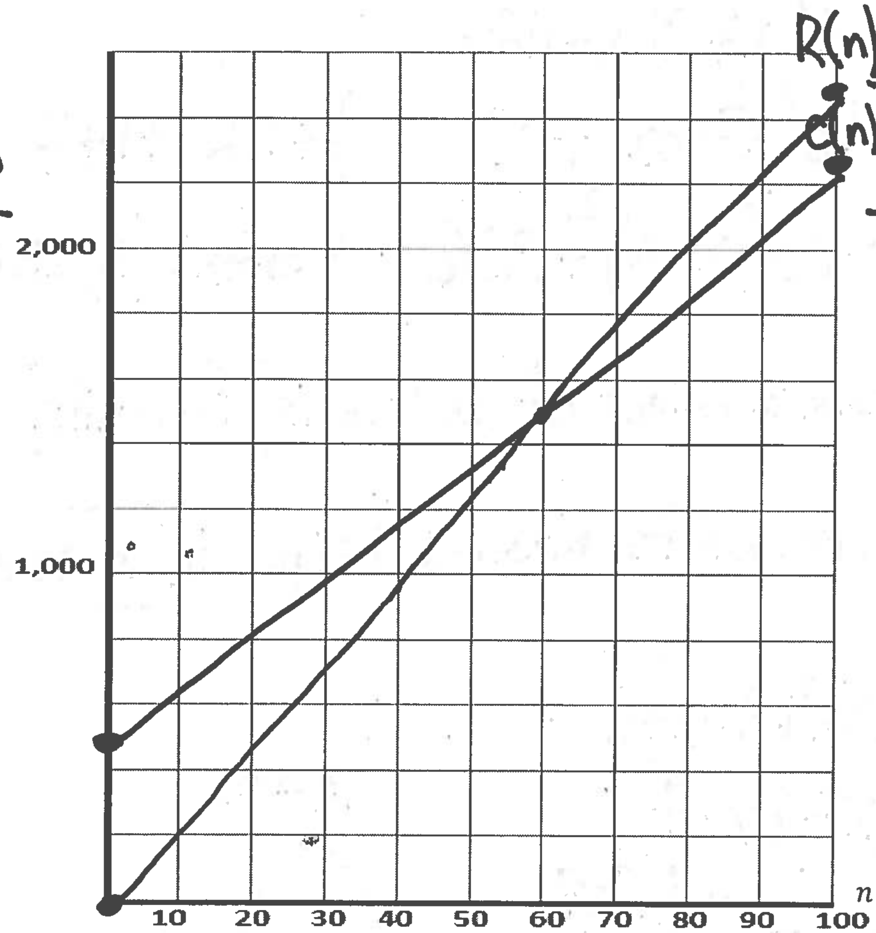
where n is the number of tickets. They sell each ticket for \$25 represented by the revenue function

$$R(n) = 25n$$

a. Graph the cost and revenue on the axes below.

n	C(n)
0	480
100	2180

+6



n	(R(n))
0	0
100	2500

b. Based on your graph, estimate how many tickets the *Dragon Fly* need to sell before making a profit (revenue exceeds costs)?

+4

Sol. Based on the graph, 60 tickets needs to sell before making a profit (revenue exceeds costs).

4. *(20 points)* Abigail tosses a coin off a bridge into the stream below. The distance, in feet, the coin is above the water is modeled by the equation

$$f(x) = -16x^2 + 96x + 112$$

Where x represents time in seconds.

a. Put this function in vertex form by completing the square.

Sol. $f(x) = -16x^2 + 96x + 112$

+6

$f(x) = -16(x^2 - 6x + 9 \quad) + 112 + 144$

$f(x) = -16(x-3)^2 + 256$; Vertex: $(3, 256)$

b. What was the maximum height of the coin?

+2

Sol. The maximum height of the coin is $\boxed{\text{256 units}}$

c. When did the coin reach its maximum height?

+2

The coin reaches its maximum height in $\boxed{\text{3 seconds.}}$

d. If the coin does not get hit during flight, when does it hit the water?

+6

Sol. $f(x) = -16x^2 + 96x + 112$

$0 = -16(x^2 - 6x - 7)$

$\overline{-16} \quad \overline{-16}$

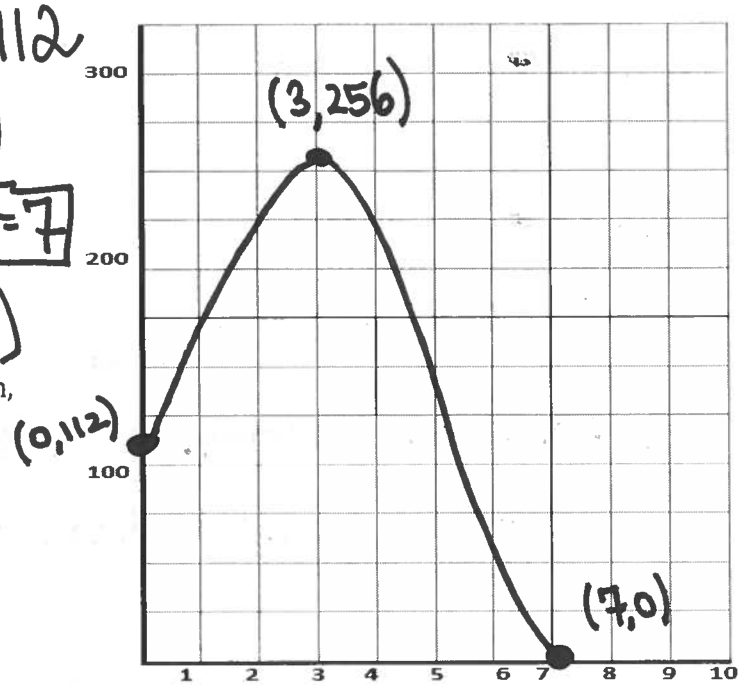
$0 = x^2 - 6x - 7$

$0 = (x-7)(x+1)$

$\boxed{x = 7}$

e. Sketch a graph of the coin's path, make sure you correctly label your axis.

+4



5. *(12 points)* Given line L: $3x - 2y = 5$

a. What is the slope of line L?

Sol. $3x-2y=5$; solve for y

$$\frac{\begin{array}{r}3x-2y=5\\ -3x\qquad -3x\end{array}}{-2y=-3x+5}$$

$$\frac{-2y}{-2}=\frac{-3x}{-2}+\frac{5}{-2}$$

$$y=\frac{3}{2}x-\frac{5}{2} \Rightarrow \boxed{\text{slope}=\frac{3}{2}}$$

+4

b. Write the equation (in slope-intercept form) of the line **parallel to line L** through the point $(6,3)$.

Sol. $m=\frac{3}{2}$; $(6,3)$

$$y-3=\frac{3}{2}(x-6)$$

$$y-3=\frac{3}{2}x-9$$

$$+3 \qquad\qquad +3$$

$$\boxed{y=\frac{3}{2}x-6}$$

+4

c. Write the equation (in point-slope form) of the line **perpendicular to line L** through the point $(-3,5)$.

+4 Sol. $m=-\frac{2}{3}$; $(-3,5)$

$$\boxed{y-5=-\frac{2}{3}(x+3)}$$

6. *(14 points)* Graph the following piecewise function over the indicated domain.

$$f(x)=\begin{cases} x+1, & x<1 \\ 1, & 1\le x<3 \\ x-3, & x\ge 3\end{cases}$$

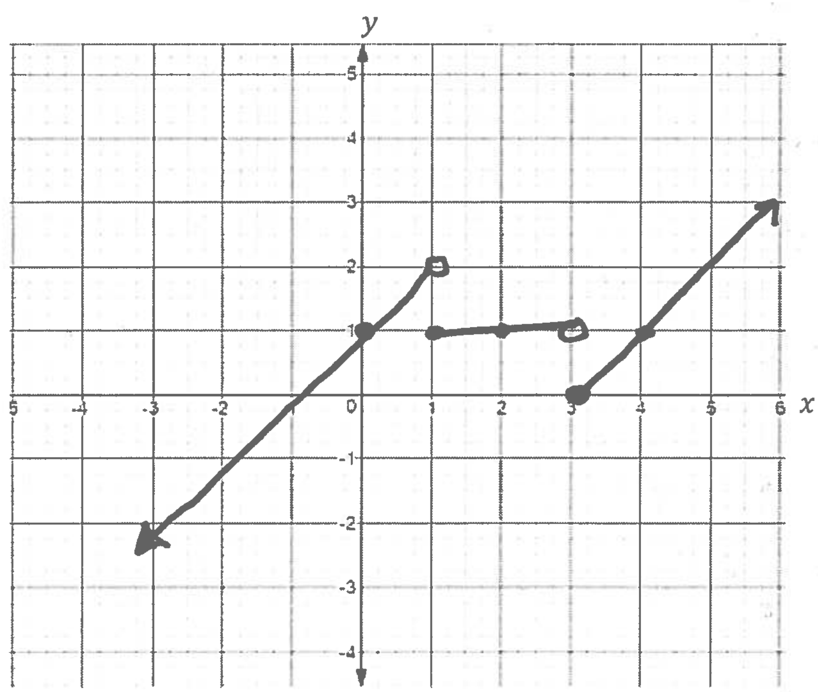
+14

Sol.

x	y=x+1
0	1
1	2

x	y=1
1	1
2	1
3	1

x	y=x-3
3	0
4	1



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Bonus Question:

Evaluate the difference quotient for the given function. Simplify your answer.
(you will need to simplify the complex fraction)

+5

$f(x) = \frac{x+3}{x+1}$, and $\frac{f(x)-f(1)}{x-1}$

$f(x) = \frac{x+3}{x+1}$

$f(1) = \frac{1+3}{1+1} = \frac{4}{2} = \boxed{2}$

Sol. $= \frac{f(x)-f(1)}{x-1}$

$= \frac{\frac{x+3}{x+1} - \frac{2(x+1)}{1(x+1)}}{x-1}$

$= \frac{\frac{x+3-2x-2}{x+1}}{x-1}$

$= \frac{\frac{1-x}{x+1}}{x-1}$

$= \frac{\overset{-1}{\cancel{1-x}}}{x+1} \cdot \frac{1}{\cancel{x-1}}$

$\boxed{= \frac{-1}{x+1}}$