Math 125 – Fall 2017
Exam 1A

Name: SOLUTIONS

Chapter 1, Sections 1-5
Chapter 2, Sections 1 ,2, 3, 5, 6
Chapter 3, Sections 1-2

Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. *(24 Points)* Tiffany, a commissioned sales-person earns \$100 base pay plus \$10 per item sold. She has a total of 36 items.

   a. Express her gross salary G as a linear function of the number of x items sold.

5 Sol. $G(x) = 100 + 10x$

   b. What is the domain and range?

4 Sol. DOMAIN: $[0, 36]$ or $0 \leq x \leq 36$
RANGE: $[100, 460]$ or $100 \leq G \leq 460$

   c. Evaluate G(24) describe what they represent in this context.

4 Sol $G(24) = 100 + 10(24) = 340$ Tiffany earns \$340 for selling 24 items.

   d. Interpret $G^{-1}(175) = 7.5$ in this context.

4 Sol. The salary of \$175 earned by Tiffany results in her selling 7.5 items.

   e. Solve G(x) = 250 and describe what it represents in this context.

3 Sol. $250 = 100 + 10x$
$150 = 10x$
$\boxed{x = 15}$ Salary of 250 dollars results in her selling 15 items

   f. Graph the function on the axis provided.

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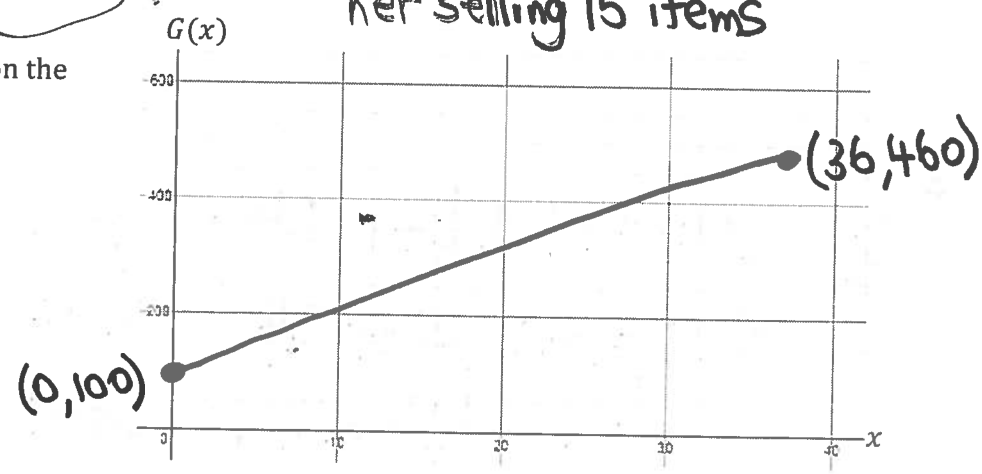


2. *(20 Points)* Cell phones have produced a seismic cultural shift. No other recent invention has incited so much praise—and criticism. In 2000 there were 109.4 million cell phone subscriptions in the United States; since then subscriptions steadily increased to reach 395.9 million in 2016. (Note: Some people had more than one subscription.)

   a. What was the average rate of change in **millions of cell phone subscriptions per year** between 2000 and 2016?

5

Sol: (2000, 109.4) & (2016, 395.9)

$$\underset{[2000,2016]}{\text{a.v.r.c.}} = \frac{395.9-109.4}{2016-2000} = \frac{286.5}{16} = \boxed{17.91}$$

   b. Construct a linear function C(t) for cell phone subscriptions (in millions) **from 2000**.

4 Sol: $m = 17.91$; $(0, 109.4)$

$$\boxed{C(t) = 17.91t + 109.4}$$

   c. What is the slope of the line? Explain what the value of the slope means in the context of this problem.

4 Sol: $m = \frac{17.91}{1}$; Every year starting 2000, the cell phone subscription in U.S. increases by 17.91 million per year.

   d. What is the vertical intercept of the line? Explain what the value of the vertical intercept means in the context of this problem.

4 Sol: (0, 109.4). The number of cell phone subscription in the starting year of 2000 was 109.4 million.

   e. If U.S. cell phone subscriptions continue to increase at the same rate, how many will there be in 2025?

3 Sol $C(t) = 17.91t + 109.4$; $t = 2025 - 2000$

$t = 25$

$C(25) = 17.91(25) + 109.4$

$\boxed{= 557.15 \text{ million subscriptions}}$

3. *(10 points) Doughy Delights* bakery has a fixed monthly cost of $550 with an additional cost of $0.20 per cookie, represented by the cost function

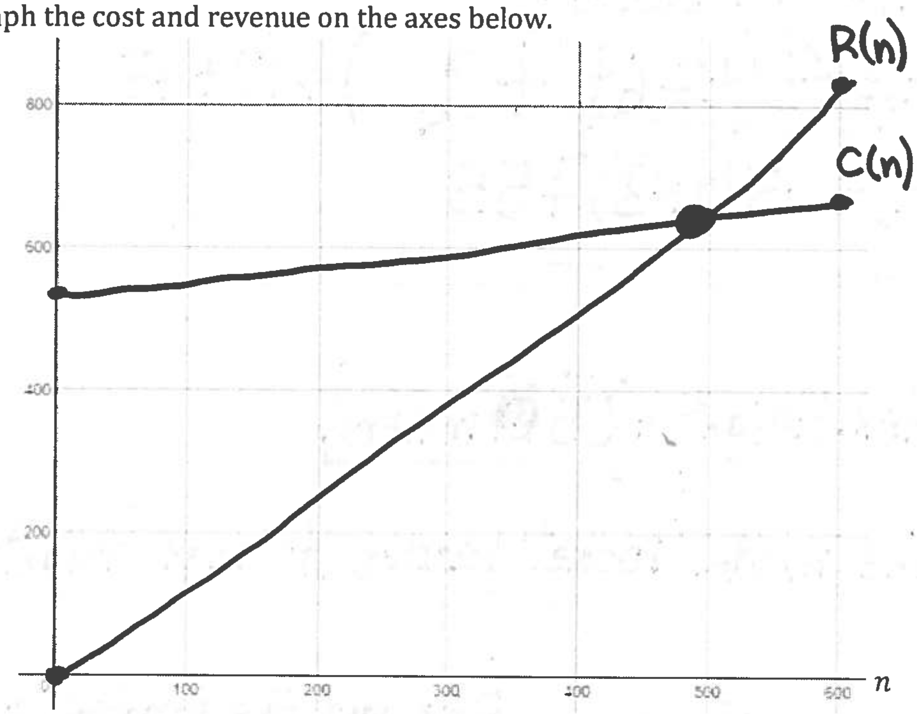
$$C(n) = 550 + .20n$$

where n is the number of cookies. The store manager sells each cookie for $1.40 represented by the revenue function

$$R(n) = 1.40n$$

6 a. Graph the cost and revenue on the axes below.

n	C(n)
0	550
600	670



n	R(n)
0	0
600	840

4 b. Based on your graph, estimate how many cookies the manager need to sell before making a profit (revenue exceeds costs)?

Sol: Based on the graph, the manager needs to sell $\boxed{\text{approx. } 460}$ cookies before making profit.

Algebraically: $R(x) > C(x)$

$1.40n > 550 + 0.20n$

$1.20n > 550$

$\boxed{n > 458 \text{ cookies}}$

4. *(20 points)* A model rocket is launched from the roof of a building. Its flight path is modeled by

$$h(t) = -5t^2 + 30t + 10$$

where h is the height of the rocket above the ground in meters and t is the time after the launch is seconds.

6 (a) Put this function in vertex form by completing the square.

Sol. $h(t) = -5t^2 + 30t + 10$

$h(t) = -5(t^2 - 6t + 9\quad) + 10 + 45$

$\boxed{h(t) = -5(t-3)^2 + 55}$

2 (b) What was the maximum height of the model rocket?

Sol. Max. Height $= \boxed{55 \text{ meters}}$

2 (c) When did the model rocket reach its maximum height?

Sol. The model rocket reaches its max height @ $\boxed{t = 3\text{sec}}$

(d) If the model rocket does not get hit during flight, when does it hit the ground?

6 Sol $0 = -5(t-3)^2 + 55$

Disregard the negative solution

$\frac{-55}{-5} = (t-3)^2$

$11 = (t-3)^2$

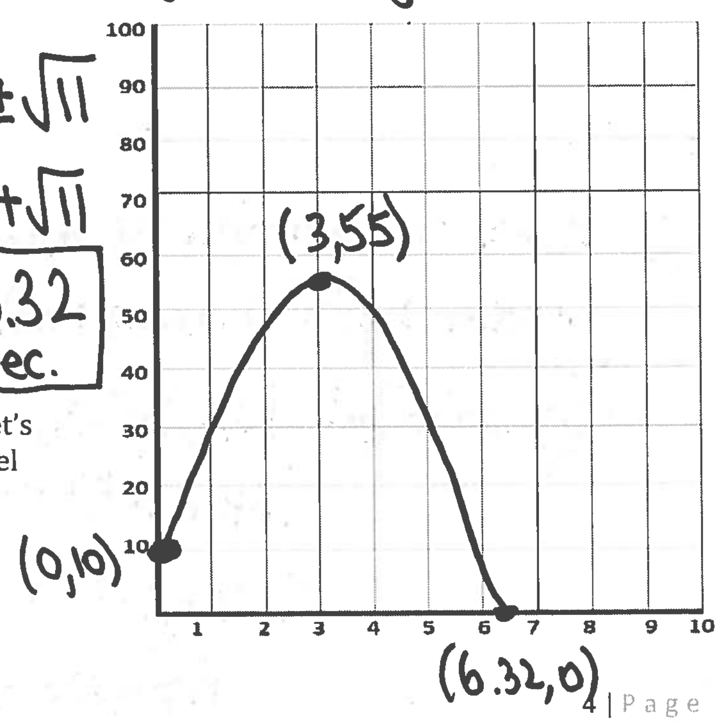
$t - 3 = \pm\sqrt{11}$

$t = 3 \pm \sqrt{11}$

$t = 3 + \sqrt{11}$

$\boxed{t = 6.32 \text{ sec.}}$

4 (e) Sketch a graph of the model rocket's path, make sure you correctly label your axis.



y-int. (0,10)

Vertex: (3,55)

X-int: (6.32,0)

5. *(12 points)* Given line L: $3x + 2y = 12$

a. What is the slope of line L?

Sol $3x+2y=12$; solve for y

4 $\frac{2y}{2} = \frac{-3x}{2} + \frac{12}{2} \Rightarrow y = -\frac{3}{2}x + 6$; $\boxed{m = -\frac{3}{2}}$

b. Write the equation (in slope-intercept form) of the line **parallel to line L** through the point $(2, 1)$.

4 Sol $m = -\frac{3}{2}$; $(2,1)$ $y - 1 = -\frac{3}{2}(x-2)$

$y - 1 = -\frac{3}{2}x + 3$

$+1 \qquad +1$

$\boxed{y = -\frac{3}{2}x + 4}$

c. Write the equation (in point-slope form) of the line **perpendicular to line L** through the point $(-3, 5)$.

4

Sol: $m = \frac{2}{3}$; $(-3,5)$ $\boxed{y - 5 = \frac{2}{3}(x+3)}$

$y - 5 = \frac{2}{3}x + 2$

$\boxed{y = \frac{2}{3}x + 7}$

6. *(14 points)* Graph the following piecewise function over the indicated domain.

14

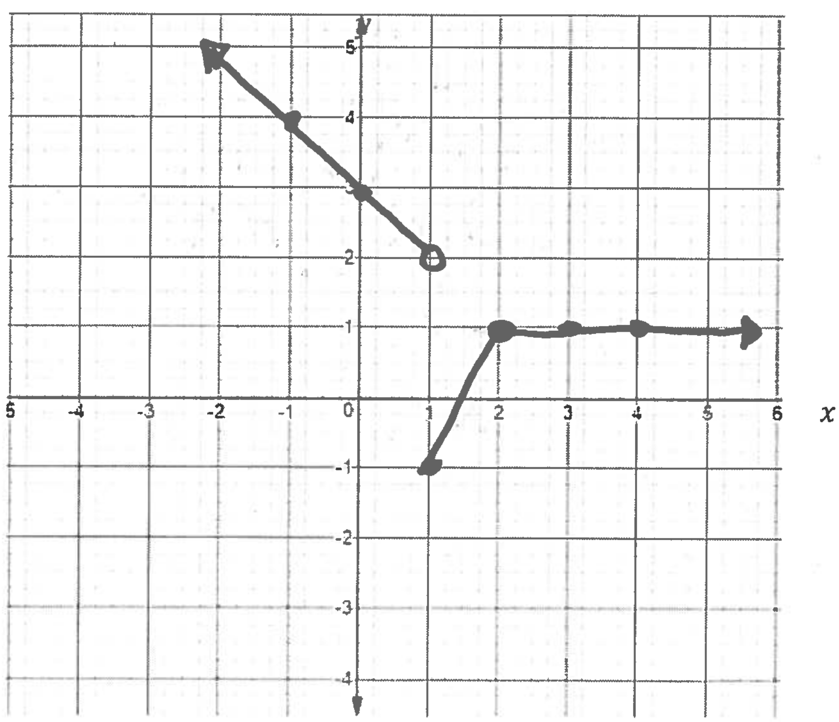
$$f(x) = \begin{cases} -x + 3, & x < 1 \\ 2x - 3, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

$x<1$

x	$y=-x+3$
-1	4
0	3
1	2

x	$y=2x-3$
1	-1
2	1

x	$y=1$
2	1
3	1
4	1



Bonus Question:

5 Evaluate the difference quotient for the given function. Simplify your answer.
(you will need to simplify the complex fraction)

$$f(x) = \frac{x+3}{x+1}, \text{ and } \frac{f(x)-f(1)}{x-1}$$

$f(1) = \frac{1+3}{1+1} = \frac{4}{2} = 2$

Sol.

$$\frac{f(x)-f(1)}{x-1} = \frac{\frac{x+3}{x+1} - 2}{x-1}$$

$$= \frac{\frac{x+3}{x+1} - \frac{2(x+1)}{x+1}}{x-1}$$

$$= \frac{\frac{x+3-2x-2}{x+1}}{x-1}$$

$$= \frac{\overset{-1}{\cancel{-x+1}}}{x+1} \cdot \frac{1}{\cancel{x-1}}$$

$$\boxed{= \frac{-1}{x+1}}$$