Name: Solutions



TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Write the general formula for an Exponential function

$f(x) = a \cdot b^x$ or $f(t) = a \cdot b^t$

Check your understanding:

1. What is the growth factor for the following
   a. Water usage in increasing by 3% per year $b = 1 + r \Rightarrow b = 1 + 0.03 \Rightarrow \boxed{b = 1.03}$
   b. A diamond mine is depleted by 1% per day $b = 1 - r \Rightarrow b = 1 - 0.01 \Rightarrow \boxed{b = 0.99}$
   c. A forest shrinks by 80% per century $b = 1 - r \Rightarrow b = 1 - 0.80 \Rightarrow \boxed{b = 0.20}$

2. Given the following initial values (at year t = 0) and rates, write the formula for Q as a function of t.
   a. Initial amount 35, decreased by 8% per year $b = 1 - r = 1 - 0.08 = \boxed{0.92}$
      Sol $Q = 35(0.92)^t$
   b. Initial amount 112.8, decreased by 23.4% per year $b = 1 - r = 1 - 0.234 = \boxed{0.766}$
      Sol $Q = 112.8(0.766)^t$
   c. Initial amount 5.35, increased by 0.8% per year $b = 1 + r = 1 + 0.008 = \boxed{1.008}$
      Sol $Q = 5.35(1.008)^t$

3. The amount in milligrams of a drug in the body t hours after taking a pill is given by $A(t) = 25(0.85)^t$
   a. What is the initial dose given? Sol $A(0) = 25(0.85)^0 = \boxed{25 \text{ mg}}$
   b. What percent of the drug leaves the body each hour?
      Sol $b = 1 - r \Rightarrow 0.85 = 1 - r \Rightarrow r = 0.15 = \boxed{15\%}$
   c. What is the amount of drug left after 10 hours?
      Sol $A(10) = 25(0.85)^{10} = \boxed{4.92 \text{ mg}}$

4. In 2010, the population of a country was 70 million and growing at a rate of 1.9% per year. Assuming the percentage growth rate remains constant; express the population *P* in millions as a function of *t* the number of years after 2010. $b = 1 + r = 1 + 0.019 = 1.019$
   Sol $\boxed{P(t) = 70(1.019)^t}$

5. A typical cup of coffee contains about 100mg of caffeine and every hour approximately 16% of the amount of caffeine in the body is metabolized and eliminated. $b = 1 - r = 1 - 0.16 = \boxed{0.84}$
   a. Write C the amount of caffeine in the body in mg, as a function of t, the number of hours since the coffee was consumed
      Sol $C(t) = 100(0.84)^t$
   b. How much caffeine is in the body after 5 hours?
      Sol $C(5) = 100(0.84)^5 = \boxed{41.82 \text{ mg}}$

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TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Write the general formula for a linear function (slope-intercept form) $y = mx+b$

Write the general formula for an Exponential function $y = a \cdot b^t$

Check your understanding:

1. Determine whether the data is linear or exponential

| $x$ | $y$ |
|---|---|
| 3 | 21.10 |
| 7 | 36.70 |
| 8 | 40.60 |
| 12 | 56.20 |

a.v.r.c $[3,7]$ $= \dfrac{36.70-21.10}{7-3} = \boxed{3.9}$

a.v.r.c $[7,8]$ $= \dfrac{40.60-36.70}{8-7} = \boxed{3.9}$

a.v.r.c $[8,12]$ $= \dfrac{56.20-40.60}{12-8} = \boxed{3.9}$

Since the a.v.r.c is Constant, the data is $\boxed{\text{LINEAR}}$

2. Write a function in the form $f(x) = a \cdot b^x$ for an exponential function where $f(0) = 2.5$ and $f(1) = 6.75$. Sol $(0,2.5)$ & $(1,6.75) \Rightarrow b = \dfrac{6.75}{2.5} = 2.7 \Rightarrow \boxed{f(x) = 2.5(2.7)^x}$

3. Write an equation for a function that satisfies these two points: $(0,2)$ and $(1,17)$ assuming that it is

   a) a linear function, in the form $y = b + mx$ and $m = \dfrac{17-2}{1-0} = \dfrac{15}{1} = \boxed{15}$

   Sol $\boxed{y = 2+15x}$

   b) an exponential function in the form $y = a \cdot b^x$. $b^1 = \dfrac{17}{2} \Rightarrow \boxed{b = 8.5}$

   Sol $\boxed{y = 2(8.5)^x}$

4. In an effort to control John's disease in dairy cattle, the state of Minnesota set a goal to reduce the number of cattle with the disease from 16,555 in 1990 to 1,830 in by the year 2000.

   a) Assuming a linear model, what would the average rate of change be per year?

   Sol a.v.r.c $[1990, 2000]$ $= \dfrac{1830-16555}{2000-1990} = \boxed{-1472.5}$

   b) Assuming an exponential model, what would be the decay factor needed?

   Sol $b^{10} = \dfrac{1830}{16555} \Rightarrow b = \left(\dfrac{1830}{16555}\right)^{\frac{1}{10}} \Rightarrow \boxed{b = 0.8023}$

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TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

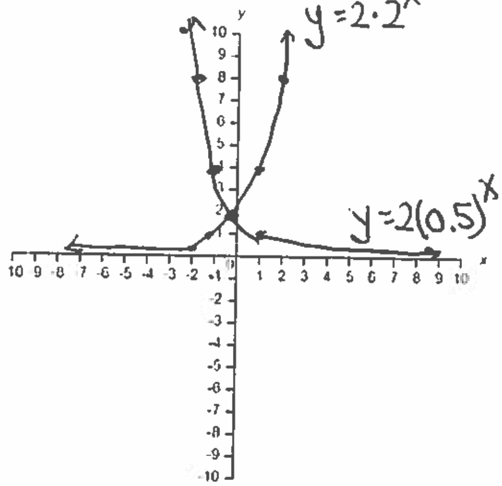
1. For which value(s) of a and b is $y = ab^x$ and increasing function? A decreasing function? Concave up?

Increasing: $a>0, b>1$; $a<0, 0<b<1$ Decreasing: $a>0, 0<b<1$; $a<0, b>1$ Concave up: $a>0, b>1$; $a>0, 0<b<1$

2. Construct graph of $y = 2\cdot 2^x$ and $y = 2\cdot(0.5)^x$

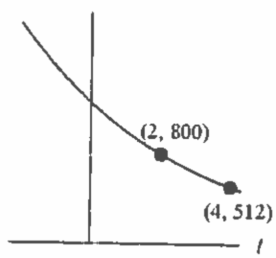
Sol

| x | $y=2\cdot 2^x$ |
|---|---|
| -2 | 0.5 |
| -1 | 1 |
| 0 | 2 |
| 1 | 4 |
| 2 | 8 |




| x | $y=2(0.5)^x$ |
|---|---|
| -2 | 8 |
| -1 | 4 |
| 0 | 2 |
| 1 | 1 |
| 2 | 0.5 |

3. Write the formula of the exponential function $P(t)$ is shown below.




Sol. $b^2 = \frac{512}{800}$

$b = \sqrt{\frac{512}{800}}$

$\boxed{b = 0.8}$

$y = a\cdot b^t$

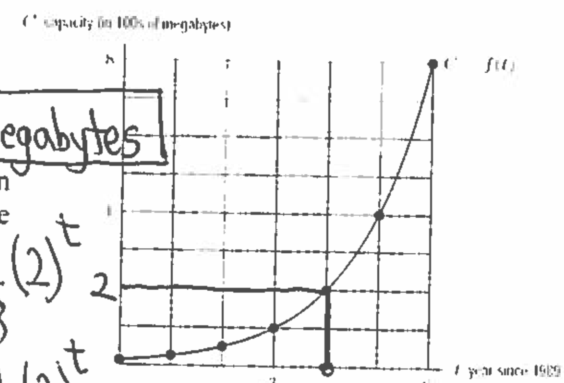
$y = a(0.8)^t$

$\frac{800}{(0.8)^2} = \frac{a(0.8)^2}{(0.8)^2}$

$1250 = a$

$\boxed{P(t) = 1250(0.8)^t}$

4. The following figure gives the graph of $C = f(t)$, where C is the computer hard disk capacity (in hundreds of megabytes) that could be bought for \$500 $t$ years past 1989.




a. What was the capacity in the year 1993?

Sol $1993-1989 = 4^{th}$ year $\Rightarrow$ $\boxed{200 \text{ megabytes}}$

b. If the trend displayed in the graph continued, in what year would the capacity that can be bought for \$500 be 7500?

Sol. $(3,1)\&(4,2)$   $y = a\cdot 2^t$   $y = \frac{1}{8}(2)^t$

$b^1 = \frac{2}{1}$   $1 = a\cdot 2^3$   $75 = \frac{1}{8}(2)^t$

$\boxed{b=2}$   $\frac{1}{8} = a$

$600 = 2^t$ $\begin{cases} t=9 \Rightarrow 2^9 = 512 \\ t=10 \Rightarrow 2^{10} = 1024 \end{cases}$ $\Rightarrow \approx 9\text{-}10$ years past 1989.

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Chapter 4 – Section 4.4 Applications to Compound Interest

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. Write a formula that gives the value of an investment, which is initially worth \$124,000 and loses value at a rate of 2.8% per year.

Sol. $b = 1 - r = 1 - 0.028 = 0.972 \Rightarrow V(t) = 124000(0.972)^t$

2. Kathleen opens a savings account with \$1500. The account earns 3.5% annual interest compounded monthly. How much will be in the account after 12 years?

Sol $A = 1500\left(1 + \frac{0.035}{12}\right)^{12(12)} = \boxed{2281.55}$

3. For an account paying 5% annual interest, compounded monthly, what is the
   a) the nominal rate 5%
   b) the effective rate $APY = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{0.05}{12}\right)^{12} - 1 = 0.0512 = \boxed{5.12\%}$

4. What is the balance after 7 years in an account containing \$600 that earns 7% interest compounded monthly? Round to the nearest cent.

Sol. $A = 600\left(1 + \frac{0.07}{12}\right)^{12(7)} = \boxed{978.00}$

5. Suppose Taylor win \$10,000 in a lottery. If she invests half in a CD earning 4.2% annual interest compounded quarterly and the rest in a savings account earning 3.8% annual interest compounded monthly. How much money does she have after 10 years?

Sol. $A = 5000\left(1 + \frac{0.042}{4}\right)^{4(10)} + 5000\left(1 + \frac{0.038}{12}\right)^{12(10)} = 7593.16 + 7307.03$

$\boxed{14900.19}$

6. Which bank has the best effective annual yield?
   a) Bank 1 with a nominal rate of 6.45% compounded monthly.

   Sol. $APY = \left(1 + \frac{0.0645}{12}\right)^{12} - 1 = 0.0664 = 6.64\%$

   b) Bank 2 with a nominal rate of 6.33% compounded weekly.

   $APY = \left(1 + \frac{0.0633}{52}\right)^{52} - 1 = 0.0653 = 6.53\%$

   c) Bank 3 with a nominal rate of 6.55% compounded yearly.

   $APY = (1 + 0.0655)^1 - 1 = 0.0655 = 6.55\%$

   $\boxed{\text{BANK 1}}$

7. You place \$10,000 in an account. You hope to have \$20,000 in the account after 15 years. What effective annual yield is needed to accomplish this? Give your answer correct to four decimal places.

Sol. $20{,}000 = 10{,}000 \cdot b^{15}$

$2 = b^{15}$

$\sqrt[15]{2} = b$

$1.047294 = b$

$b = 1 + r$

$1.047294 = 1 + r$

$-1 \qquad -1$

$r = 0.047294 = \boxed{4.7294\%}$