Math 125 - Fall 2017 Version B

Name: SOLUTIONS Date: Fall 2017

Chapter 4, Sections 1, 2, 3, 4, 5
Chapter 5, Sections 1, 2, 3
Chapter 2, Section 4
Chapter 6, Sections 1, 2, 3

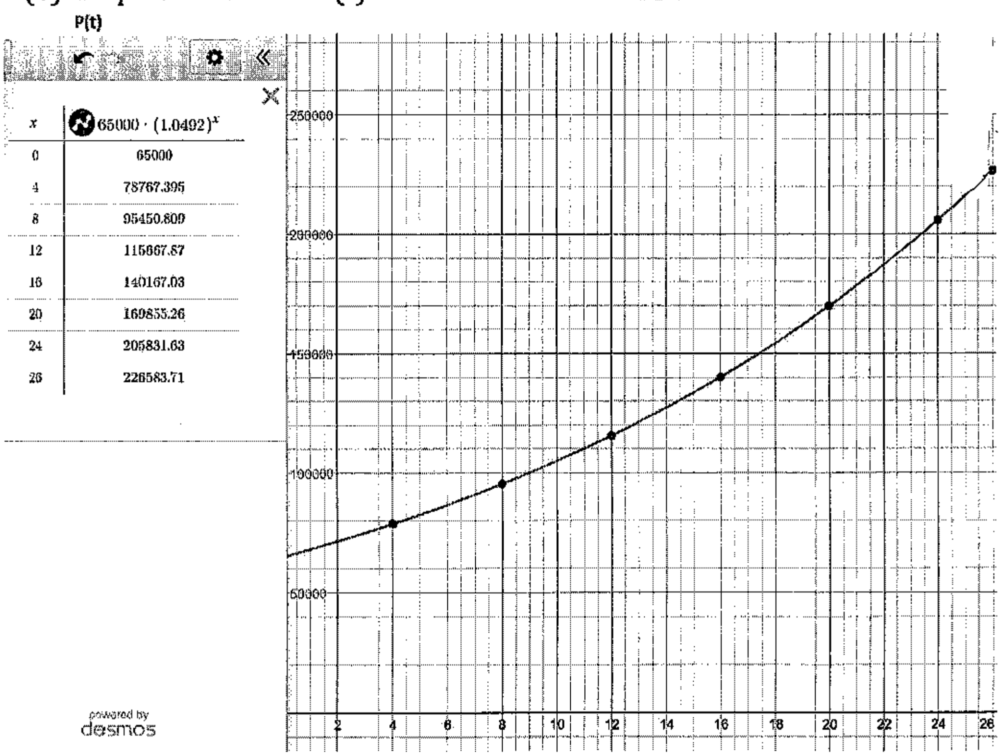
Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. (18 Points) The population of a city is increasing exponentially. In 2000, the city had a population of 65,000. In 2012, the population was 115,694.

(a) Write a formula, $P(t)$ for the population in **thousands people** of the town t years after 2000.

$$b = \left(\frac{115{,}694}{65{,}000}\right)^{\frac{1}{12}} \approx 1.0492 \quad \therefore P(t) = 65{,}000(1.0492)^t$$

(b) Graph the number P(t) over the interval $0 \le t \le 26$



(c) Us your function $P(t)$ to determine the year when the population will reach 220,000.

$$220{,}000 = 65000(1.0492)^t$$

$$\frac{220{,}000}{65{,}000} = (1.0492)^t$$

$$\log\left(\frac{220000}{65000}\right) = t \log(1.0492)$$

$$t = \log\left(\frac{220000}{65000}\right) / \log(1.0492) \approx \boxed{25.39 \text{ yrs}}$$

$2000 + 25.39$

$\approx \boxed{2025}$

Exact: $t \approx 25.38$

2. (12 points) Evaluate the following expressions exactly without using a calculator.

(a) Evaluate $100^{\log 15}$

$= 10^{2\log(15)}$

$= 10^{\log(15^2)}$

$= 15^2 = \boxed{225}$

(b) Evaluate $\frac{\log(10^{32})}{\log\sqrt{10}} \approx \frac{32}{\log(10^{1/2})} = \frac{32}{1/2} = \boxed{64}$

(c) Evaluate $e^{\ln 27}$ $= \boxed{27}$

3. (12 Points) Complete the following tables and correctly write the function that represents each given situation. Make sure you show how you arrived at your solution. More space!

(a) Let $f(x)$ be given in the table below. Find the value of k if $f(x)$ **is exponential.** Then write the function $f(x)$.

x	$f(x)$
0	4
1	k
2	64

$b = \frac{k}{4}$

$b = \frac{64}{k}$

$\frac{k}{4} = \frac{64}{k}$

$k^2 = 256$

$k = 16 \longrightarrow b = \frac{64}{16} = 4$

$\boxed{f(x) = 4(4)^x}$

(a) Let $g(x)$ be given in the table below. Find the value of k if $g(x)$ **is linear.** Then write the function $g(x)$.

x	$g(x)$
0	2
1	k
2	4

$\left.\begin{array}{l} m = k-2 \\ m = 4-k \end{array}\right\}$ $k-2 = 4-k$

$2k = 6$

$k = 3$

$\therefore m = 3-2 = 1$

$\boxed{g(x) = x+2}$

4. (18 points) Suppose you have $2000 to invest. You have a choice of three accounts:

Bank 1 with a nominal rate of 6.4% compounded monthly.

Bank 2 with a nominal rate of 6.33% compounded yearly

Bank 3 with a nominal rate of 6.55% compounded continuously.

a. Write a formula for the value of your investment if you invest all $2000 in Bank 1. Determine the effective rate for Bank 1.

$$V(t) = 2000\left(1 + \frac{.064}{12}\right)^{12t}$$

$$= \boxed{2000(1.0659)^t}$$

$$EIR = 1.0659 - 1 \rightarrow \boxed{6.59\%}$$

b. Write a formula for the value of your investment if you invest all $2000 in Bank 2. Determine the effective rate for Bank 2.

$$\boxed{V(t) = 2000\,(1.0633)^t}$$

$$\boxed{EIR = 6.33\%}$$

c. Write a formula for the value of your investment if you invest all $2000 in Bank 3. Determine the effective rate for Bank 3.

$$\boxed{V(t) = 2000\, e^{0.0655t}}$$

$$EIR = e^{0.0655} - 1 \approx 0.0677 \rightarrow \boxed{6.77\%}$$

d. Which account is better (in terms of earning more interest)? Explain your reasoning.

The third account is best as the EIR is higher, which means more money is earned.

5. (18 Points) Psychologists have found that the average walking speed, w, in feet per second, of a person living in a city of population P, in thousands of people, is given by the function

$$w = 0.37 \ln P + 0.05.$$

(a) The population of *Hartford* is approximately 135,500. Find the average walking speed of people living in *Hartford*.

$$W = 0.37 \ln(135.5) + 0.05$$
$$\approx \boxed{1.86 \text{ ft/sec}}$$

(b) A sociologist measures the average walking speed in a city to be approximately 2.0 feet/second. Use this information to estimate the population of the city.

$$2 = 0.37 \ln(P) + 0.05$$
$$1.95 = 0.37 \ln(P)$$
$$\frac{1.95}{0.37} = \ln(P)$$
$$P = e^{1.95/0.37} \approx \boxed{194.468 \text{ thousand } (194,469) \text{ people}}$$

(c) Let w_1 and w_2 be the average walking speeds in two different cities with populations P_1 and P_2, respectively. Using logarithm properties, find a simplified formula for the difference $w_1 - w_2$ in terms of P_1 and P_2.

$$W_1 = 0.37 \ln(P_1) + 0.05$$
$$W_2 = 0.37 \ln(P_2) + 0.05$$

$$W_1 - W_2 = 0.37 \ln(P_1) + 0.05 - (0.37 \ln(P_2) + 0.05)$$
$$= 0.37 \ln(P_1) \cancel{+0.05} - 0.37 \ln(P_2) \cancel{-0.05}$$
$$= 0.37 \ln(P_1) - 0.37 \ln(P_2)$$
$$= 0.37 [\ln(P_1) - \ln(P_2)]$$

$$\boxed{W_1 - W_2 = 0.37 \ln\left(\frac{P_1}{P_2}\right)}$$

OR

$$\boxed{e^{\frac{W_1 - W_2}{0.37}} = P_1/P_2}$$

6. (12 Points) Given a function $y = f(x)$

(a) Describe the effect of the transformation $f\left(\frac{x}{5}\right)+9$

- h. stretch by a factor of 5
- shift up 9 units

(b) The graph of $f(x)$ contains the point (1, −2). What point must lie on the reflected graph if the graph is reflected about the x-axis?

$\boxed{(1,2)}$

(c) The point $(-4,5)$ lies on the graph of f. What point must lie on the graph of $5f\left(\frac{x}{3}\right)$?

$\boxed{(-12,25)}$

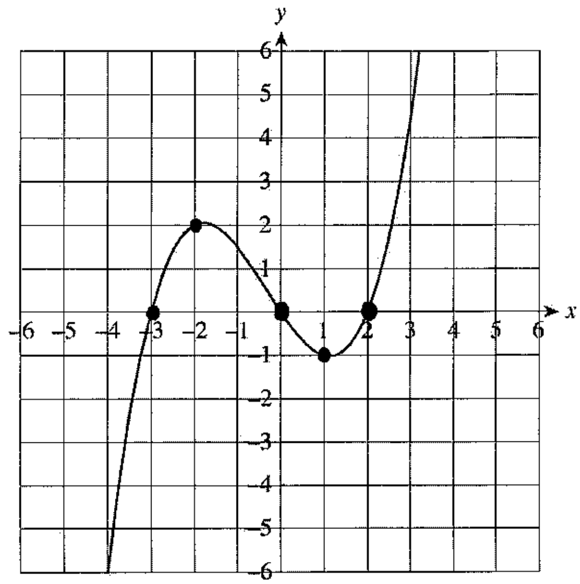
(d) The graph of $g(x)$ contains the point (−4, −1). What is the corresponding point on the graph of $y = 4g(x)+7$?

$(-4,-1) \longrightarrow (-4,-4) \longrightarrow \boxed{(-4,3)}$

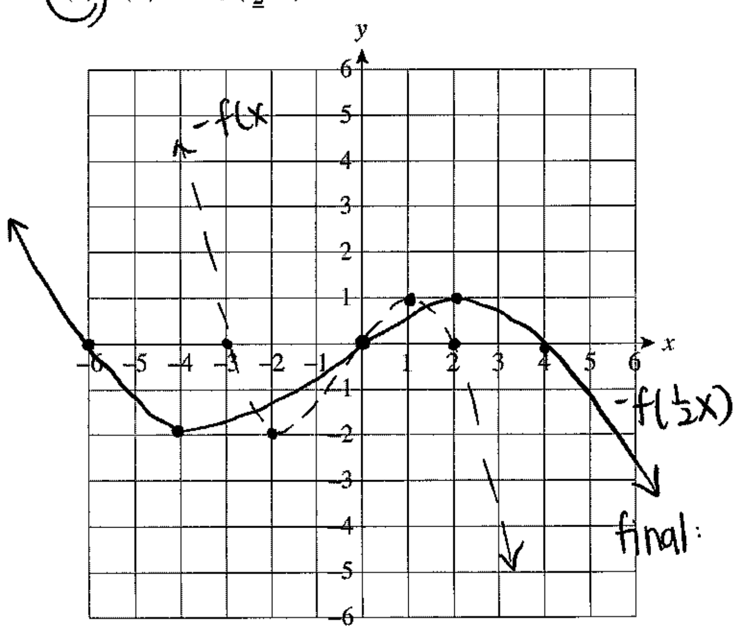
7. (10 Points) The graph of the function f(x) is shown below.

(a) Use this graph to sketch the graph of each of the following functions.

(b) In each case, state what transformations are applied to obtain the graph from the graph of the original function.

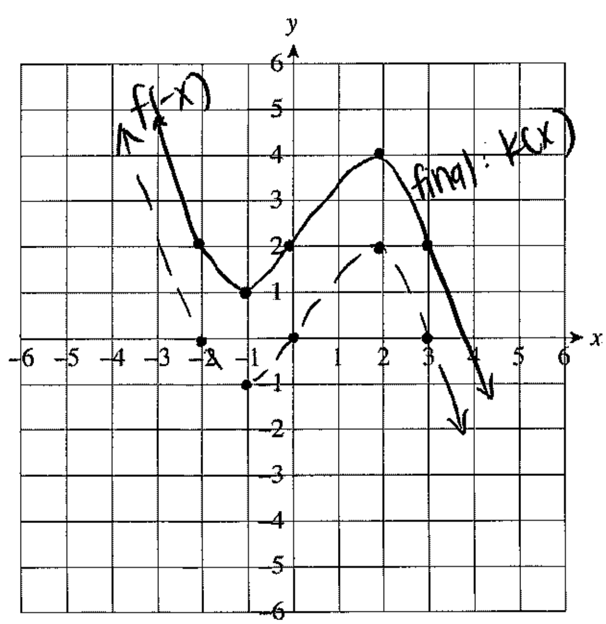


(a) $h(x) = -f(\frac{1}{2}\, x)$



- reflect across x-axis
- h. stretch by a factor of 2

(b) $k(x) = f(-x) + 2$



- reflect across y-axis
- shift up 2 units

Bonus: Abigail tosses a coin off a bridge into the stream below. The distance, in feet, the coin is above the water is modeled by the equation

$$f(x) = -16x^2 + 96x + 112$$

Where x represents time in seconds.

a. Put this function in vertex form by completing the square.

$$f(x) = -16(x^2 - 6x + 9 - 9) + 112$$
$$= -16((x-3)^2 - 9) + 112$$
$$= -16(x-3)^2 + 144 + 112 = -16(x-3)^2 + 256$$

b. What was the maximum height of the coin?

256 ft

c. When did the coin reach its maximum height?

3secs

d. If the coin does not get hit during flight, when does it hit the water?

$$0 = -16(x-3)^2 + 256$$
$$\sqrt{\frac{256}{16}} = \sqrt{(x-3)^2}$$
$$\pm 4 = (x-3)^2$$
$$x = 3 \pm 4 \rightarrow \boxed{7\text{secs}}$$
$$\rightarrow \times$$