Contents

Day 22

Test is next Thursday 11/21.

Confidence Intervals

Neyman-Pearson Ideas:

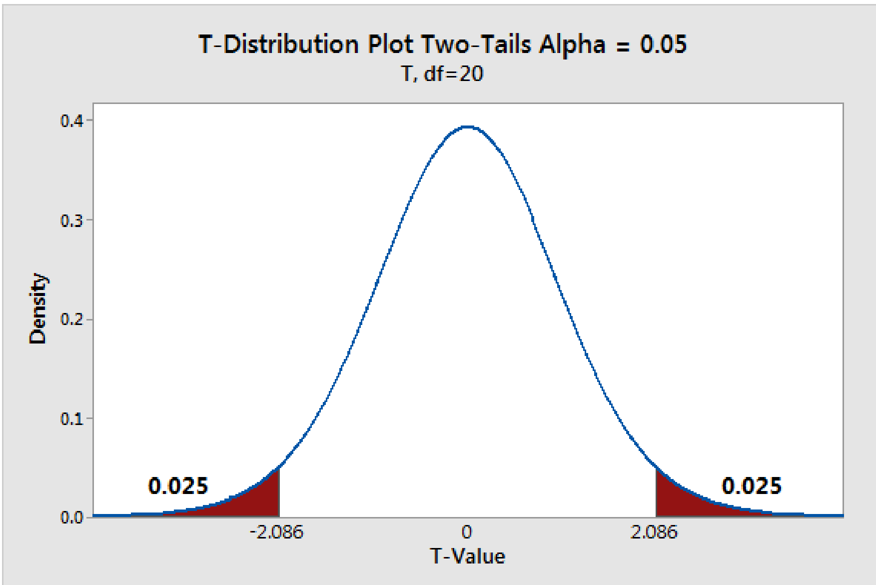

Figure 1: Two Sided Test

If t_{observed} is anywhere in the area $1-\alpha = C$, we accep H_0.

One-sample t-Test: For what values of μ_0 will we accept H_0: $\mu = \mu_0$?

$$-t^{**} < t_{\text{observed}} < t^{**}$$

$$-t^{**} < \frac{\bar{x}_{\text{observed}} - \mu_0}{\frac{s_{\text{observed}}}{\sqrt{n}}}$$

Any value of μ between

$$\bar{x}_{\text{observed}} - t^{**} \times \frac{s_{\text{observed}}}{\sqrt{n}}$$

and

$$\bar{x}_{\text{observed}} + t^{**} \times \frac{s_{\text{observed}}}{\sqrt{n}}$$

We will accept.

The interval$(\bar{x}_{\text{observed}} - t^{**} \times \frac{s_{\text{observed}}}{\sqrt{n}}, \bar{x}_{\text{observed}} + t^{**} \times \frac{s_{\text{observed}}}{\sqrt{n}})$, represents the range of values within which we resonably would believe μ to be. This interval is called a confidence interval for μ.

In many situations, we either don't know what μ_0 should be or don't care to make a decision - just want to estimate μ.

How confident are we?

We define confidence level as the proportion of samples for which we would accept H_0: $\mu = \mu_0$ when H_0 is true.

So confidence level $C = 1 - \alpha \leftarrow$ depends on H_0 is true.

- $\uparrow \alpha \implies \downarrow C$
- $\downarrow \alpha \implies \uparrow C$

Problem

We don't know μ_0!

Confidence is in our estimate of μ.

If μ is in our interval - "good" sample, correctly accept H_0 If μ is not in our interval - "bad" sample, make a Type 1 Error

We always assume we got a "good" sample.

Affecting Width

What affects the width of the confidence interval?

$$\bar{x} \pm t^{**} \times \frac{s}{\sqrt{n}}$$

- $\bar{x}$: center
- t^{**}: comes from $t(df)$ and is also dependent on α
  - $df \uparrow$, $t^{**} \downarrow$, width $\downarrow$
  - $\alpha \uparrow$, $t^{**} \downarrow$, width $\downarrow$
  - $C \uparrow$, $t^{**} \uparrow$, width $\uparrow$
- n: sample size $\uparrow$, width $\downarrow$
- s: sample standard deviation $\uparrow$, width $\uparrow$

Example

Suppose we take a simple random sample of 8 college students and ask how much time they spend per week watching broadcast TV. In the sample, $\bar{x} = 14.5$ hrs/week and $s = 14.884$ hrs/week. Use this information to estimate with 95% confidence the population mean time college students spend watching TV per week.

Is this data symmetric?

- This data is **not** because the sample standard deviation is quite large.

Solution

Step 1: Assume this is a good sample. So for any value in μ in:

$$y = mx + b$$

Step 2: Plug in for $\bar{x}, s, n$

$$y = ax^2 + bx + c$$

Step 3: Find t^{**}

$$df = 7, C = 0.95 \implies \alpha = 0.05$$

```
qt(0.025, df = 7, lower.tail = FALSE)
[1] 2.305
```

$\frac{\alpha}{2} = 0.025 \implies t^{**}$

Step 4:

$= (14.5 - 23.05(\frac{14.854}{\sqrt{8}}), 14.5 + 23.05(\frac{14.854}{\sqrt{8}}))$

$= (2.08, 26.92)$

Tying Example Back into Theory [Interpretation]

We are 95% confident (in our estimate) that the population mean (number of hours per week) is between 2.08 and 26.92.

Other Frameworks

Matched Pairs

- $t_{\text{observed}} = \frac{\bar{x}_{\text{d}} - \mu_{\text{d}}}{\frac{s_{\text{d}}}{\sqrt{n}}}$

  - Confidence Interval for μ_{d}:

$$\bar{x}_{\text{d}} \pm t^{**} \times \frac{s_{\text{d}}}{\sqrt{n}}$$

$$(\bar{x}_{\text{d}} - t^{**} \times \frac{s_{\text{d}}}{\sqrt{n}}, \bar{x}_{\text{d}} + t^{**} \times \frac{s_{\text{d}}}{\sqrt{n}})$$

Two-Sample

Terms

Point Estimate: statistic whose value is our "best guess" as to the value of a parameter

- $\bar{x}, \bar{x}_{\mathrm{d}}, \bar{x}_1 - \bar{x}_2, etc$

Margin of Error: how much to add/subtract to create an interval estimate we are C% confident in:

t critical value × standard error [for two sided N-P test]

Example : Book Exercise 7.71

202 "early" eaters [Population 1]

- $\bar{x}$ = 23.1 grams of fat
- s = 12.5 grams

200 "late" eaters [Population 2]

- $\bar{x}$ = 21.4 grams of fat
- s = 8.2 grams

Estimate with 95% confidence the difference in population mean fat consumption. $(\mu_1 - \mu_2)$: $(-0.4, 3.8)$

We are 95% confident in our estimate that the difference in population mean fat consumption between early & late eaters is between -0.4 and 3.8 grams.

Suppose H_0: $\mu_1 - \mu_2 = 0$. H_1: $\mu_1 - \mu_2 = \Delta$

Can I accept H_0 [in Neyman-Pearson Framework] using this sample.

- Yes because $-0.4 < 0 < 3.8$

Can I reject H_0: $\mu_1 - \mu_2 = 0$ [NHST] in favor of H_a: $\mu_1 = \mu_2 \neq 0$? (Using $\alpha = 0.05$)

- No because $-0.4 < 0 < 3.8$

Based on our sample:

1. μ_1 and μ_2 could be =
2. μ_1 could be as much as 3.8 bigger than μ_2
3. μ_1 could be as much as 0.4 smaller than μ_2

Interpretations

We are 95% confident in our estimate that, on average (in the population), early eaters consume between 0.4 grams less & 3.8 grams more fat compared to late eaters.

↑ SAME THING ↓

μ_1 = population mean late eaters, μ_2 = population mean early eater $(-3.8, 0.4)$

Hypothetical Scenarios

Suppose both bounds are positive:

$\mu_{\text{early}} - \mu_{\text{late}} \implies$ CI: $(0.4, 3.8)$

Only possibility: $\mu_{\text{early}} > \mu_{\text{late}}$

We are 95% confident in our estimate that, on average (in the population), early eaters consume between 0.4 grams more and 3.8 grams more than late eaters

Suppose both bounds are negative:

....

insert chart from picture

NOTE:

We can always always perform hypothesis testing by constructing a confidence interval with confidence level

$$C = 1 - \alpha$$

and seeing i the null value is in the confidence interval!

Prefer confidence interval over hypothesis testing:

- Additional information!
- Confidence interval screws up in the interpretation are much less costly than hypothesis testing screws ups.

Generally only use hypothesis for Fisher-type tests (χ^2, ANOVA)

H$_0$: $\mu_1 = \mu_2 = \mu_3$

Reject H$_0$: at least 1 μ is different.

$\rightarrow$ *Estimate*:

- $\mu_1 - \mu_2$
- $\mu_1 - \mu_3$
- $\mu_2 - \mu_3$

Knowing at least one of the differences exists.