# Outline

- 1. Recap of probability
- 2. Simulation
- 3. Random Variables

# Independent vs. Disjoint Events

- Independent events **can** happen at the same time, but knowing that event "A" occurred **does not** change P(B) and vice versa
- Disjoint events cannot happen at the same time.
  - Knowing that event "A" occurred changed P(B) = 0 and vice versa
- If A and B are independent,  $P(A \cap B) = P(A) P(B)$
- If A and B are disjoint,  $P(A \cap B) = 0$

### Example One

- Draw a tile from a bag of 100 scrabble tiles
- Event C = "the tile is a C"
- Event A = "the tile is an A"

P(C) = .02

P(A) = .09

Events "C" and "A" are disjoint

Events "C" and "A" are not independent

#### Example Two

Draw one tile and set it outside

Event C = "first tile is a C"

Event A = "second tile is a A"

Event C and A are not disjoint

Events "C" and "A" are <u>not</u> independent

Sampling without replacement

### Example Three

Draw a tile, put it back in the bag and then draw another tile

Event C = "first tile is a C" Event A = "second tile is an A"

Event C and A are <u>not</u> disjoint Event C and A <u>are</u> independent

Sampling with replacement

### Simulation

Trying to imitate in the real world where the outcome is uncertain but is random

- Specify our model for an uncertain situation/random event
- "Randomly" generate an outcome for the model
- Repeat step two many, many times

#### Why simulate?

- Once we set up the model, the math maybe too difficult
- Situation may be unique, or we only have ability to observe it once, due to physical/financial limitations
- For fun and/or profit

Report assumptions of the model!

# Random Variables (RVs)

Random variable is a variable whose numerical values describe outcomes of a random event

Typically we map outcomes in our sample space denoted as "S" to numerical values of the random variable.

 $\underline{\text{Discrete Random Variable}}$  : probability mass function (PMF) places positive probability at specific numbers on the number line

- Only specific numbers
- Example: all outcomes are real, positive numbers

<u>Continuous Random Variable</u>: probability density function (PDF)

 Places positive probability along a possibly infinite interval of the number line.

# Writing the PMF of a Discrete Random Variable

```
Each unique key value X=x is mapped to an non unique value P(X=x)
example_hash_map = {
    key: value
}
A hash table is another way to represent data mapping.
```

- value represents a random variable
- key represents a "realization" of value

### Example One

Sum of values in map == 0

```
We can find P(Y=0)
Once we have observed the random event either y=0 or y!=0
Let X= the point value of the chosen tile map = {
0: 0.02,
1: 0.68,
2: 0.07,
3: 0.08,
4: 0.10,
5: 0.01,
8: 0.02,
10: 0.02
}
```

# Example Two

Use PMF & probability rules to find:

• 
$$P(X <= 3)$$
  
•  $P(X = 0, 1, 2 \text{ or } 3)$   
•  $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$   
•  $= 0.85$   
•  $P(X > 1)$   
•  $P(X = 2, 3, 4, 5, 8, \text{ or } 10)$   
•  $= 1 - P(X <= 1) = 1 - (X = 0 \text{ or } 1)$   
•  $= 1 - [P(X = 0) + P(X = 1)] = 1 - [0.02 + 0.68] = 1 - .7 = 0.3$   
•  $= P(X > 5)$   
•  $= P(X = \{0..5\})$   
•  $= 0.04$   
•  $= P(X = 4, \text{ or } 5)$ 

# Expected Value (Mean) of a Random Variable

- Called expectation, mean, all the same thing
- On average, what value do we expect the random variable to be

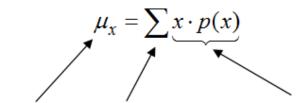
Recall idea of "weighted average"

#### Mean of a Probability Distribution

μ

Denotes the average of all events, which in turn gives us an expected value for a given function.

#### **Summation notation**



The mean equals the sum of all the values of x times their probabilities.

Figure 1: discrete random variable formula

Expected value is a <u>linear</u> operator (can take in sum and give back a result in the form of a sum of the applied operators)

### For random variables X and Y, and constant C

- E[X+Y] = E[X] + E[Y]
- E[cX] = cE[X]
- ^ where "c" is a constant applied

This implies, for X, Y and arbitrary constants a,b E[aX + bY] = aE[X] + bE[Y]

Consequences: a = 1, b = -1

$$E[X - Y] = E[X] - E[Y]$$

 $\mu_{x-y} = \mu_x - \mu_y$