Name: SOLUTIONS

Chapter 13 – Section 13.1 Sequences

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Define the following:

- Arithmetic Sequence SEQUENCE WITH COMMON DIFFERENCE for each CONS. terms
- Geometric Sequence SEQUENCE WITH COMMON RATIO for each CONS. terms

Check your understanding:

1. Is the following sequence arithmetic, geometric, or neither? Explain.

Sol: COMMON RATIO. $1, 3, 9, 27, \ldots$ $\frac{3}{1}=3;\ \frac{9}{3}=3;\ \frac{27}{9}=3$

2. Is the following sequence arithmetic, geometric, or neither? Explain.

Sol: COMMON RATIO. $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$ $\frac{\frac{1}{4}}{1}=\frac{1}{4};\ \frac{\frac{1}{16}}{\frac{1}{4}}=\frac{1}{4};\ \frac{\frac{1}{64}}{\frac{1}{16}}=\frac{1}{4}\ldots$

3. For the arithmetic sequence $a_1, a_2, \ldots, a_n$, suppose $a_1=1$, $a_2=3$. What is a_{20}?

$3-1=2$

Sol: $a_n=a_1+(n-1)d$

$a_n=1+(n-1)d$

$a_n=1+(n-1)2$

$a_{20}=1+(20-1)2=1+19(2)=1+38=\boxed{39}$

4. Let $a_1, a_2, a_3, \ldots, a_n$ be an arithmetic sequence with $a_1=6.4$ and $a_3=4.2$. What is a_8?

Sol: $d=\frac{4.2-6.4}{3-1}=\frac{-2.2}{2}=-1.1$ $\quad a_n=6.4+(n-1)[-1.1]$

$a_8=6.4+(8-1)[-1.1]=\boxed{-1.3}$

5. Let $a_1, a_2, a_3, \ldots, a_n$ be a geometric sequence with $a_2=12$ and $a_4=108$. What is a_{10}?

Sol: $r^2=\frac{108}{12} \Rightarrow r^2=9 \Rightarrow \boxed{r=3}$ $\quad a_n=a_1\cdot 3^{n-1}_3$ $\quad 108=a_1\cdot 3^3$ $\quad \boxed{a_1=4}$ $\quad a_n=4\cdot 3^{n-1}$

$a_{10}=4\cdot 3^9=\boxed{78732}$

6. A person decides to walk for 19 minutes one day, and then increases his walks by 3 minutes each day for a month. Let $a_1, a_2, \cdots, a_n$ be the sequence showing the length of time he walks each day, where a_n is the length of time he walks on day n. What is a_{29}?

Sol: $19, 22, 25, \ldots$

$a_n=a_1+(n-1)d$

$a_n=19+(n-1)3$

$a_{29}=19+(29-1)3=\boxed{103}$

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Chapter 13 – Section 13.2 Defining Functions Using Sums: Arithmetic Series

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Define the following:

- Arithmetic Series (state both forms)

$$S_n = \frac{1}{2}n\left[a_1 + a_n\right] = \frac{1}{2}n\left[a_1 + a_1 + (n-1)d\right] = \frac{1}{2}n\left[2a_1 + (n-1)d\right]$$

Check your understanding:

1. A person decides to walk for 11 minutes one day, and then increases his walks by 3 minutes each day for a month. Let $a_1, a_2, \cdots, a_n$ be the sequence showing the length of time he walks each day, where a_n is the length of time he walks on day n, and let $S_1, S_2, \cdots, S_n$ be the sequence of partial sums. What is S_{23}?

Sol. $11+14+17+\ldots+a_{23}$

$$S_{23} = \frac{1}{2}(23)\left[2(11)+(23-1)3\right] = \frac{1}{2}(23)\left[22+66\right] = \boxed{1012}$$

2. A child building a tower with blocks places 29 blocks in the first row, 26 blocks in the second row, 23 blocks in the third row, and so forth. How many blocks are in the tower if it has 8 rows total?

Sol. $29+26+23+\ldots$ $\quad S_8 = \frac{1}{2}(8)\left[2(29)+(8-1)-3\right]$

$$= \frac{1}{2}(8)\left[2(29)-21\right] = \frac{1}{2}(8)(37) = \boxed{148}$$

3. Find the sum of the first 400 odd integers.

Sol. $1+3+5+7+\ldots+799$: $S_{400} = \frac{1}{2}(400)\left[1+799\right] = \frac{1}{2}(400)(800)$

$$\boxed{= 160000}$$

4. What is $\sum_{k=1}^{25}(2k+4)$? $6+8+\ldots+54$

Sol. $S_{25} = \frac{1}{2}(25)\left[6+54\right] = \frac{1}{2}(25)\left[60\right] = (25)(30) = \boxed{750}$

5. Write $12+17+22+\ldots+57$ using sigma notation.

Sol. $a_n = 12+(n-1)5 = 12+5n-5 = 7+5n$.

$7+5n = 57$

$5n = 50$

$\boxed{n=10}$

$$\sum_{i=1}^{10} 7+5i$$

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Chapter 13 – Section 13.3 Finite Geometric Series

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Define the following:

- Finite Geometric Series

$$S_n = [a_1(1-r^n)]/[1-r]$$

Check your understanding:

1. If $1-3+9-27+81-\cdots+6561$ is a geometric series, give its ratio. If not, enter "not geometric"

Sol. GEOMETRIC; $\frac{-3}{1}=-3$; $\frac{9}{-3}=-3$; $\frac{-27}{9}=-3$; $\frac{81}{-27}=-3$ $\boxed{R=-3}$

2. If you were to fill out a pedigree chart, the first column would have one name (yours), the second column would have two names (your parents), the third column would have 4 names (your grandparents), etc. How many names would be on the chart if there were 8 columns total?

Sol. $1+2+4+\ldots$ $S_8 = \frac{1[1-2^8]}{1-2} = \boxed{255}$

3. Evaluate $7(0.7)^3 + 7(0.7)^4 + \cdots + 7(0.7)^{19}$ to 3 decimal places.

Sol. $S_{17} = 7(0.7)^3[1-(0.7)^{17}]/[1-0.7] = \boxed{7.985}$

4. To save for their child's college education, parents put \$200 at the beginning of each month into an account that pays 4.5% annual interest, compounded monthly. How much will they have saved if they do this for 18 years? Round to the nearest cent. $\frac{4.5}{12} = 0.375\%$

$S_{18} = 200[1-(1.00375)^{216}]/[1-1.00375] = \boxed{66373.60}$

5. Write the sum $32-16+8-4+2-1$ in sigma notation.

Sol. $r = \frac{-16}{32} = \frac{-1}{2}$; $\frac{8}{-16} = \frac{-1}{2}$; $\frac{2}{-4} = \frac{-1}{2}$; $\frac{-1}{2} = \frac{-1}{2}$ $a_n = 32(-\frac{1}{2})^{n-1}$ $\boxed{\sum_{i=1}^{6} 32(-\frac{1}{2})^{i-1}}$

6. Find $\sum_{n=0}^{7} 5\left(\frac{1}{7}\right)^n$ to 3 decimal places.

$5+5(\frac{1}{7})+5(\frac{1}{7})^2+\ldots+5(\frac{1}{7})^7 = S_8 = \frac{5(1-(\frac{1}{7})^8)}{(1-(\frac{1}{7}))} = \boxed{5.833}$

7. Does $\sum_{i=1}^{14} i^2 - \sum_{j=1}^{15} j^2 = -29$? Explain.

Sol. $[(1)^2+(2)^2+\cdots+(14)^2]-[(1)^2+(2)^2+\cdots+(14)^2+(15)^2] = -(15)^2 = -225$

Thus $\sum_{i=1}^{14} i^2 - \sum_{j=1}^{15} j^2 = -225 \neq -29$.

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Chapter 13 – Section 13.4 Infinite Geometric Series

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Define the following:

- Infinite Geometric Series

$$S = \frac{a}{1-r};\ |r| < 1$$

Check your understanding:

1. Does the infinite geometric series $60 - 54 + 48.6 - 43.74 + \cdots$ converge or diverge? $|r| = -0.9 < 1$

2. For what values of x does the sum $\sum_{i=1}^{\infty}(7x-5)^{3i}$ converge?

$$(7x-5)^3 + (7x-5)^6 + (7x-5)^9 + \cdots \Rightarrow r = (7x-5)^3 < 1$$

$$7x - 5 < 1 \Rightarrow 7x < 6 \Rightarrow \boxed{x < \frac{6}{7}}$$

3. Find the sum of the series: $5 + \frac{5}{3} + \frac{5}{9} + \ldots$

$$S = \frac{5}{1-\frac{1}{3}} = \frac{5}{\frac{2}{3}} = \boxed{\frac{15}{2}}$$

4. What is $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$? Round to 2 decimal places

$$S = \frac{1}{1+\frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4} = \boxed{0.75}$$

5. A patient takes 600 mg of a pain-killing drug twice a day. Every 12 hours, the patient's body metabolizes 20% of the drug present. If the patient continues to take the drug indefinitely, how many mg of the drug will remain in the patient's body in the long run? Round to 2 decimal places.

Sol: $600 + 600(0.80) + 600(0.20)^2 + \cdots \quad S = \frac{600}{1-0.80} = \boxed{3000}$

Super fun! We'll talk about this one in class.

6. The repeating decimal 0.212121. . . can be written as an infinite geometric series 0.21 + 0.0021 + 0.000021 + ... Use this fact to find a fraction that is equivalent to 0.212121. . . . Your answer should be of the form "A/B".

Sol

$$0.212121\ldots = 0.21 + 0.0021 + 0.000021 + \cdots$$

$$= \frac{21}{100} + \frac{21}{10000} + \frac{21}{1000000} + \cdots$$

$$S = \frac{\frac{21}{100}}{1-\frac{1}{100}} = \frac{21}{100} \Big/ \frac{99}{100} = \frac{21}{99} = \boxed{\frac{7}{33}}$$