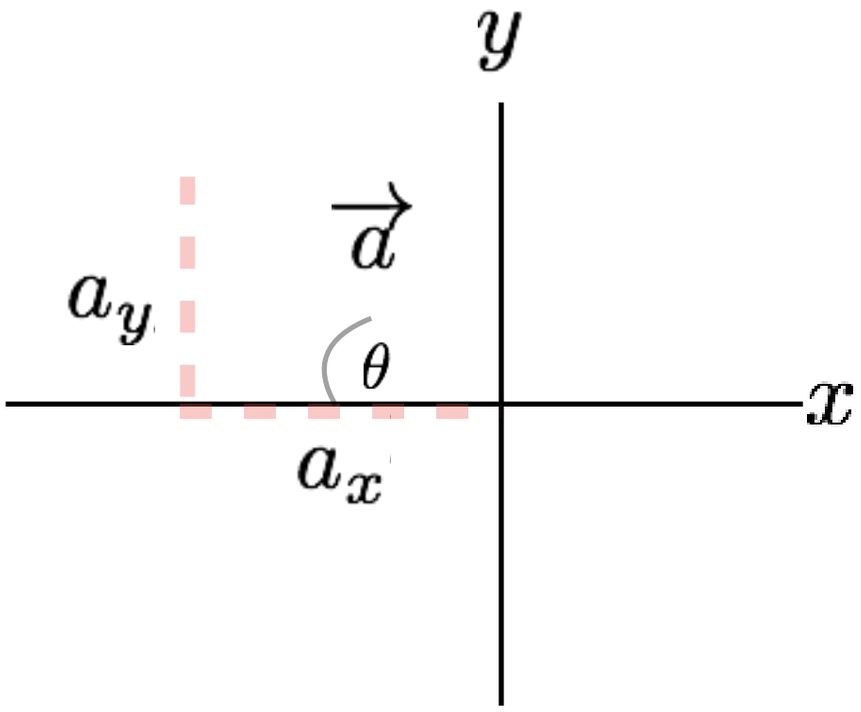
Physics 225

Section 2,
Fall 2018
Lecture 5

Unit vectors and components



$$\vec{d} = a_x\hat{i} + a_y\hat{j}$$

vector scalar vector scalar vector

Unit vectors:

$$x, y, z \rightarrow \hat{i}, \hat{j}, \hat{k}$$

Scalar:

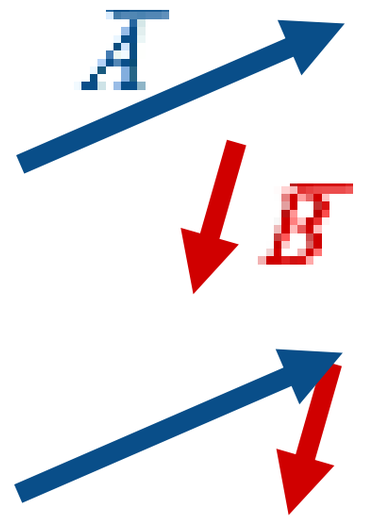
$$a_x = a\cos\theta$$

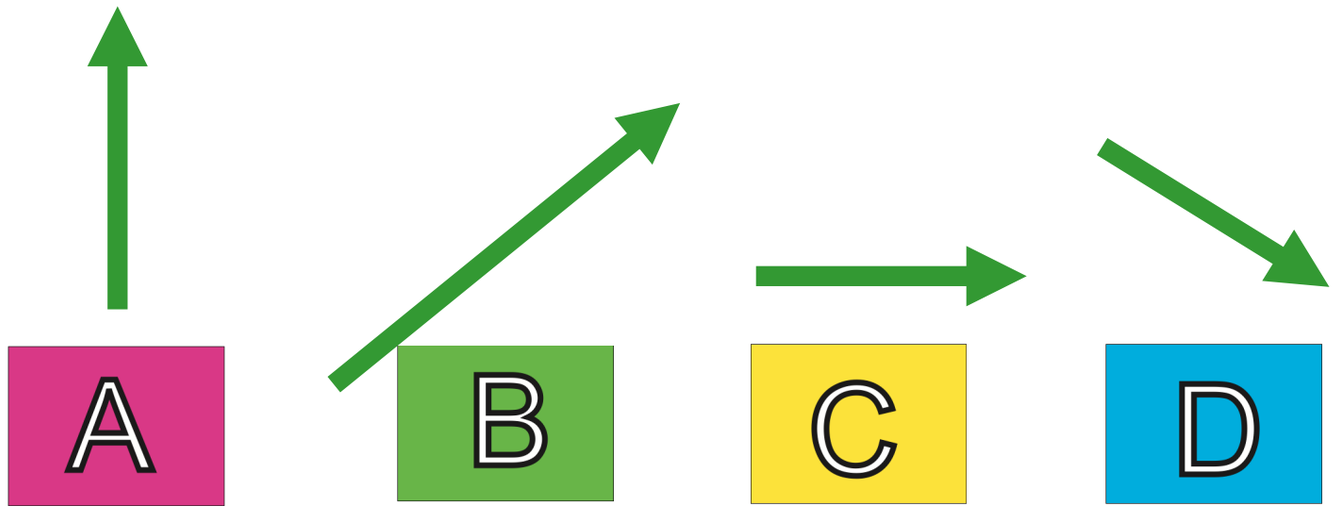
$$a_y = a\sin\theta$$

- A **unit** vector has a magnitude of 1
  - Its sole purpose is to point in a direction
- A vector **component** includes a scalar value and a unit vector

Clicker Question 4a

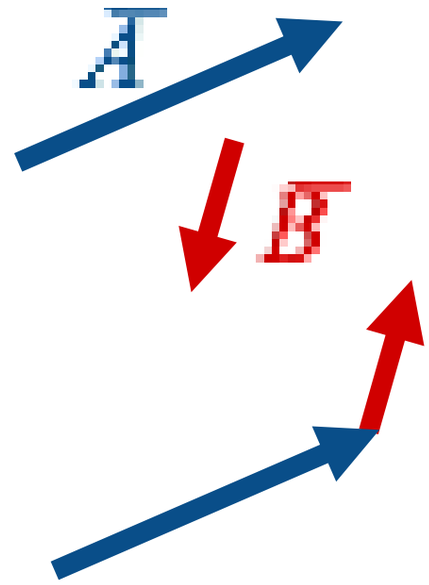
Vectors $\vec{A}$ and $\vec{B}$ are shown to the right.
Which of the following best describes $\vec{A} + \vec{B}$?

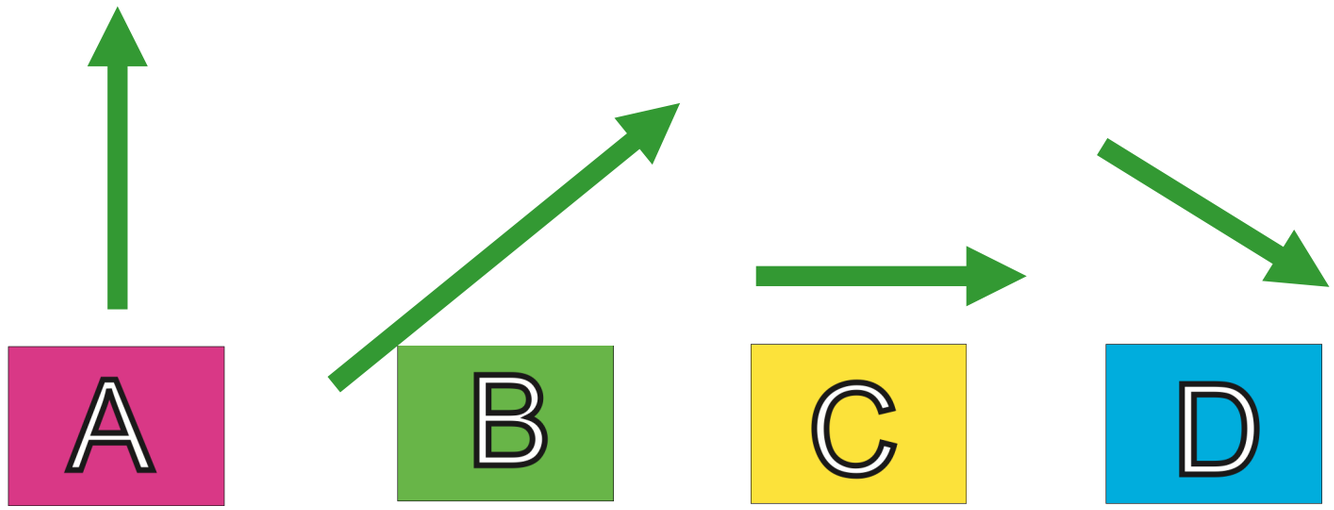




Clicker Question 4b

Vectors $\vec{A}$ and $\vec{B}$ are shown to the right.
Which of the following best describes $\vec{A} - \vec{B}$?





Clicker question #5

Question 3.1a Vectors I

If two vectors are given such that A + B = 0, what can you say about the magnitude and direction of vectors A and B?

A same magnitude, but can be in any direction

B same magnitude, but must be in the same ection

C different magnitudes, but must be in the me direction

D same magnitude, but must be in opposite ections

different magnitudes, but must be in opposite directions

Clicker question #6

Question 3.3 Vector Addition

You are adding vectors of length 20 and 40 units. What is the only possible resultant magnitude that you can obtain out of the following choices?

A **0**

B **18**

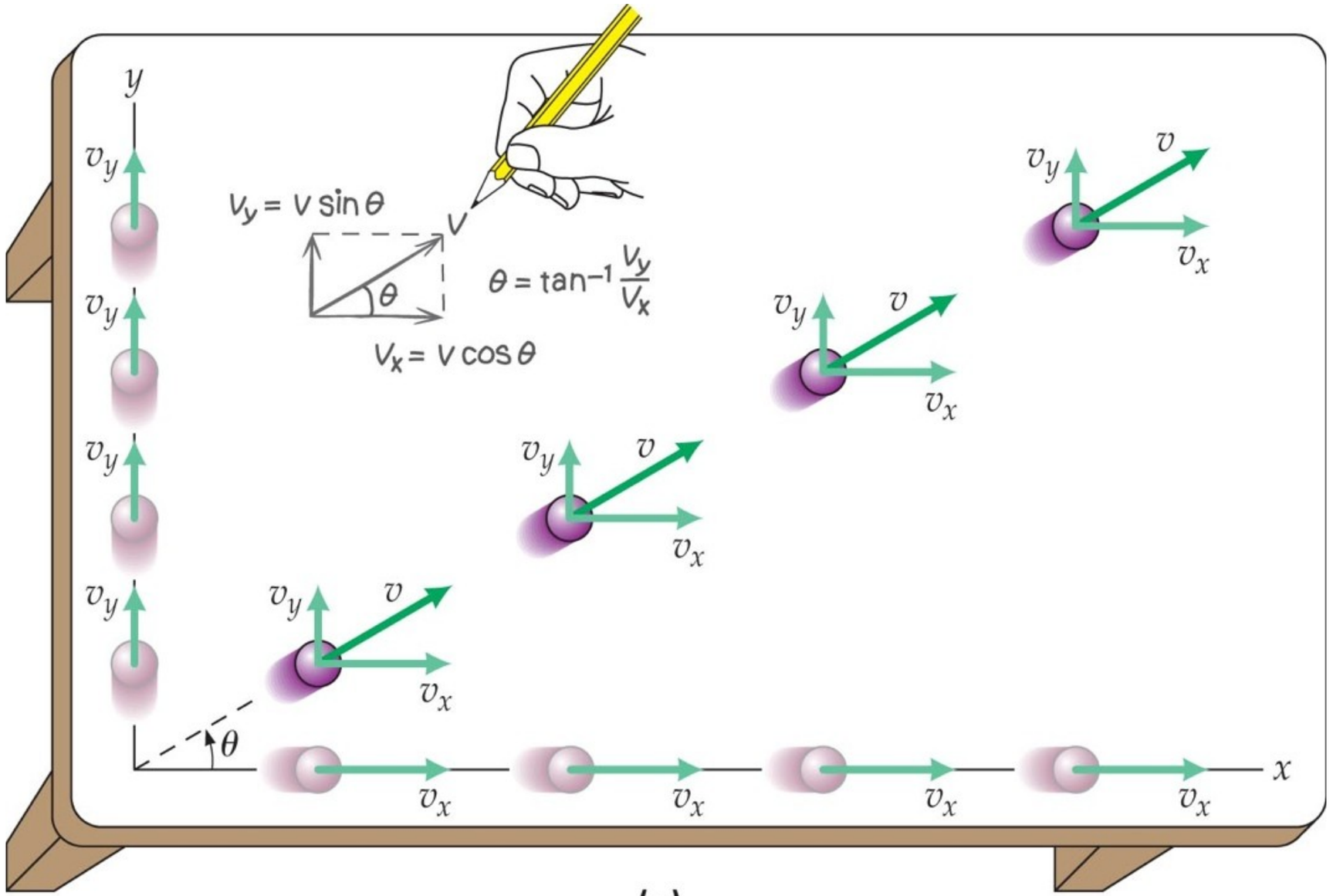
C **37**

D **64**

100

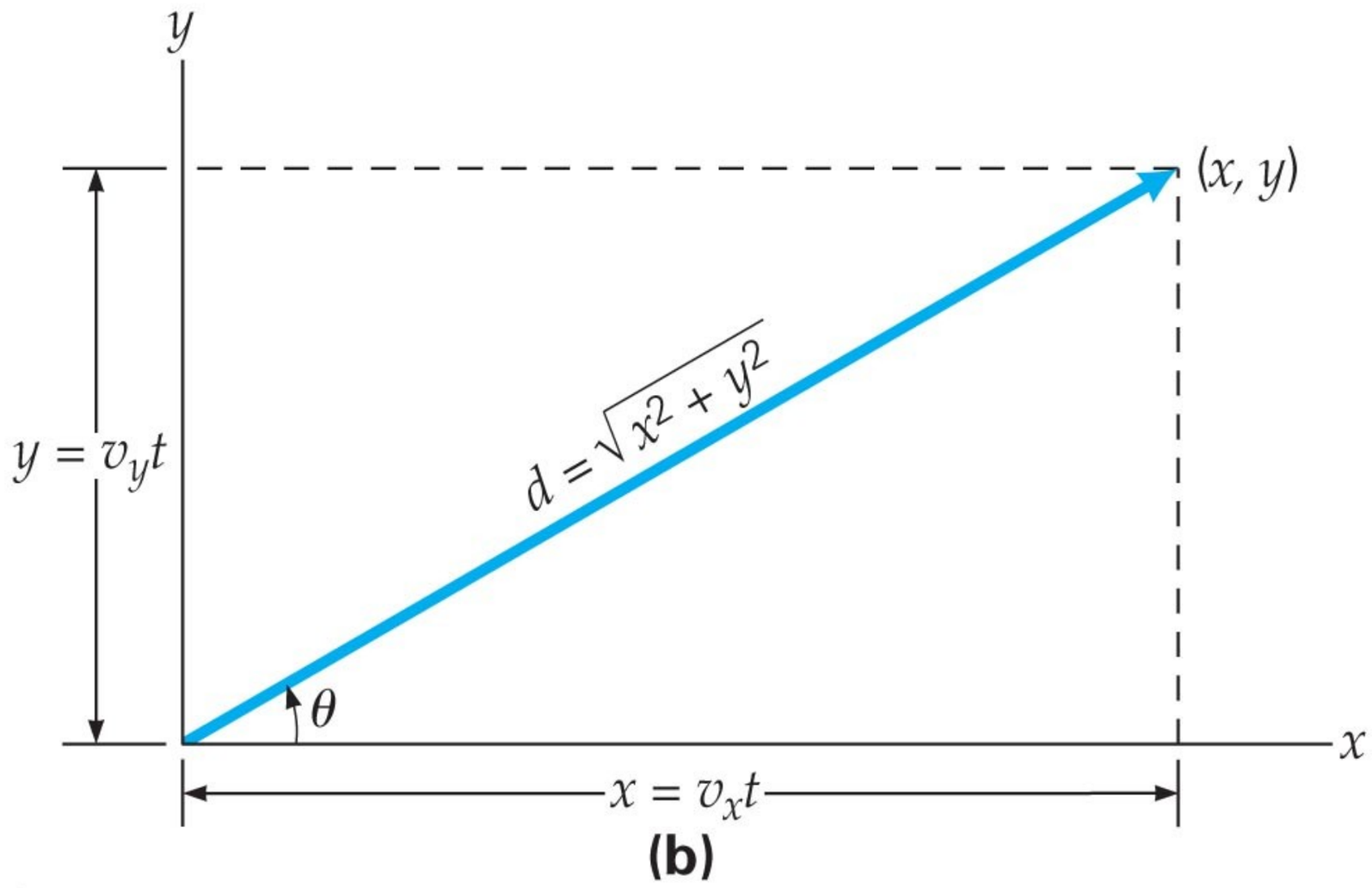
Motion in 2 dimensions

- Horizontal & vertical motion *independent*
  - Break vectors into components
  - Treat each component separately



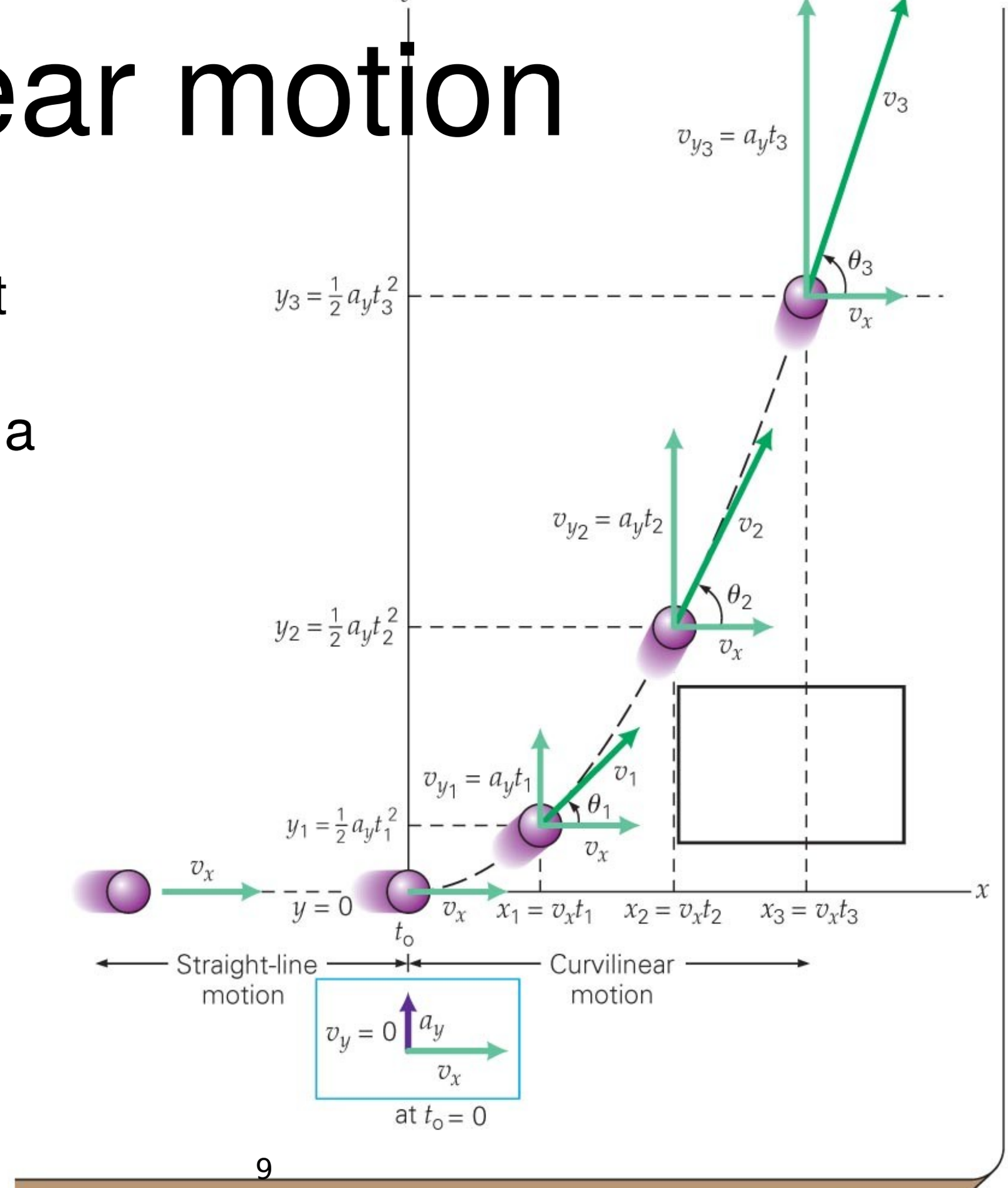
Motion in 2 dimensions

- Horizontal & vertical motion *independent*
  - Break vectors into components
  - Treat each component separately



Curvilinear motion

- If velocity, acceleration not parallel, then motion is along a curve ("curvilinear motion")

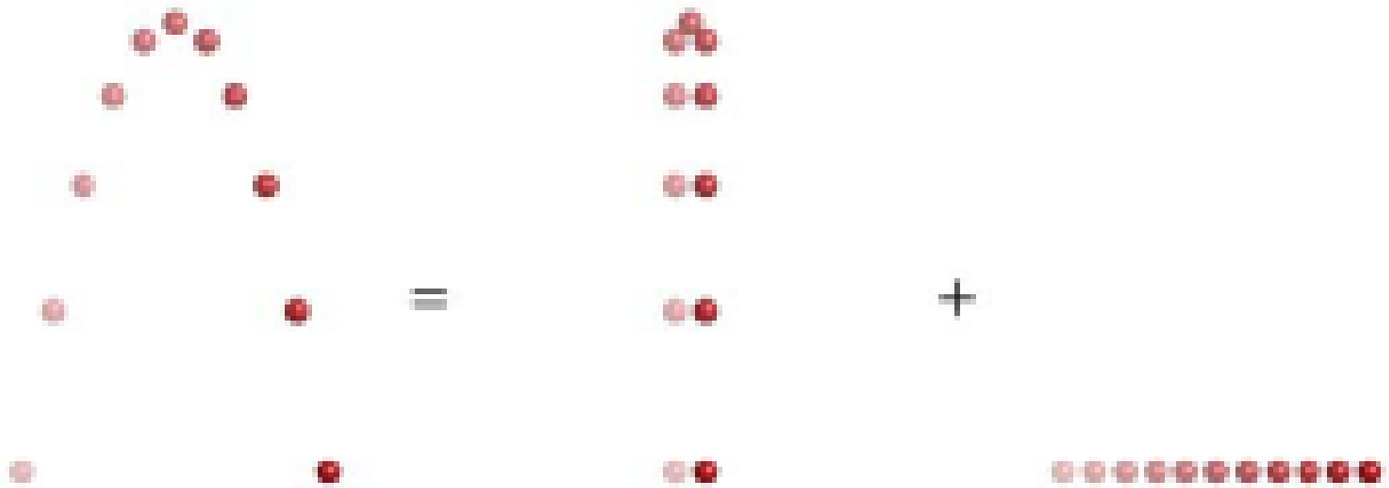


Main Points – Projectile Motion

Projectile motion is the superposition of two independent motions:
1) **Horizontal**: constant velocity
2) **Vertical**: constant acceleration

Projectile motion can be understood simply as free fall viewed from a moving reference frame.

Projectile Motion = Vertical Motion with Constant Acceleration + Horizontal Motion with Constant Velocity



Acceleration of Gravity

$$g \equiv +9.8\frac{\text{m}}{\text{s}^2}$$

$$a_y = -g \qquad a_x = 0$$

$$v_y = v_{oy} - gt \qquad v_x = v_{ox}$$

$$y = y_o + v_{oy}t - \frac{1}{2}gt^2 \qquad x = x_o + v_{ox}t$$

Projectile Motion

Horizontal

$$a_x = 0$$

$$v_x = v_{o_x}$$

$$x = x_o + v_{o_x} t$$

Vertical

$$a_y = -g$$

$$v_y = v_{o_y} - gt$$

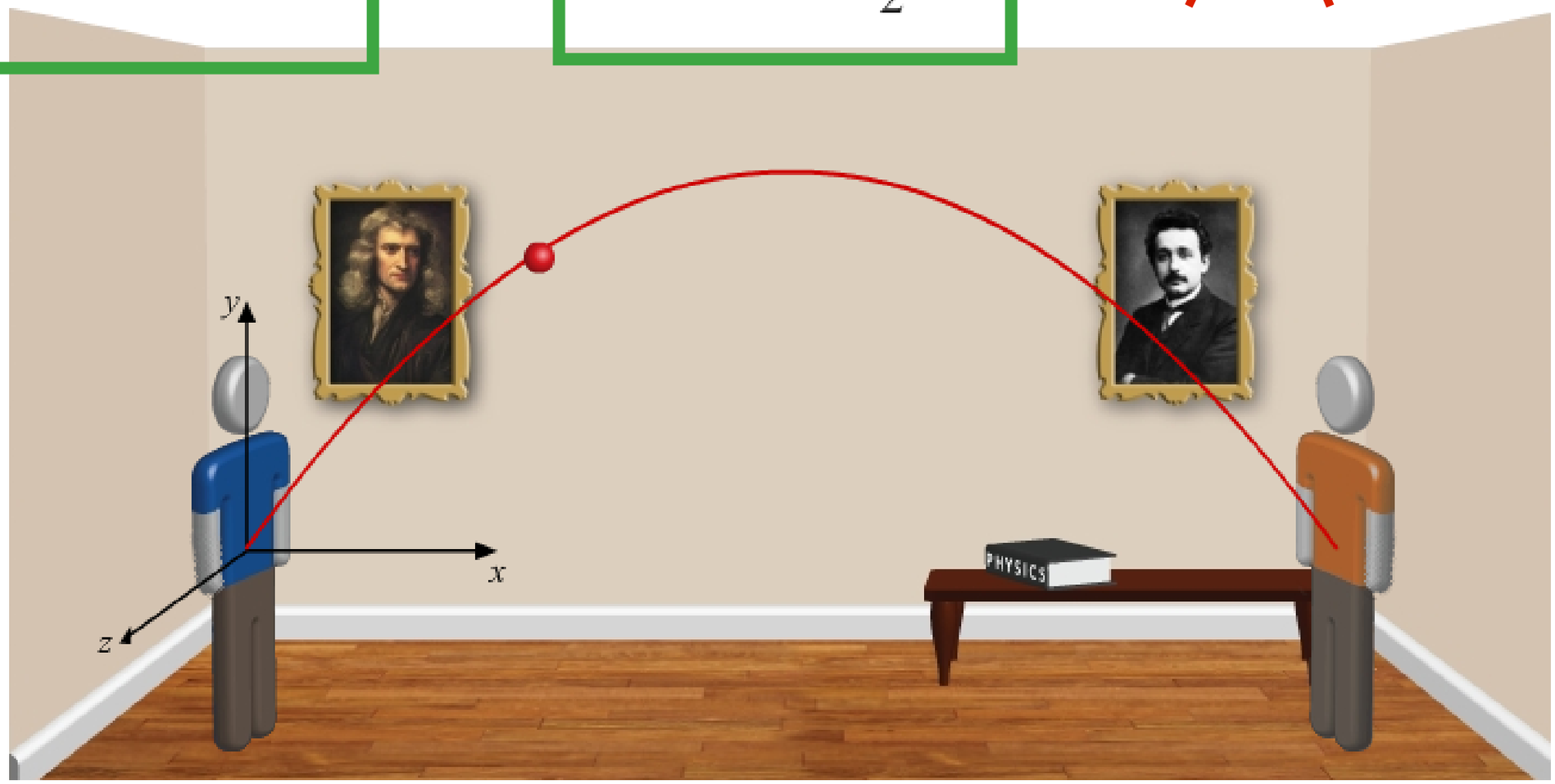
$$y = y_o + v_{o_y} t - \frac{1}{2} g t^2$$

Boring

~~$a_z = 0$~~

~~$v_z = 0$~~

~~$z = z_o$~~



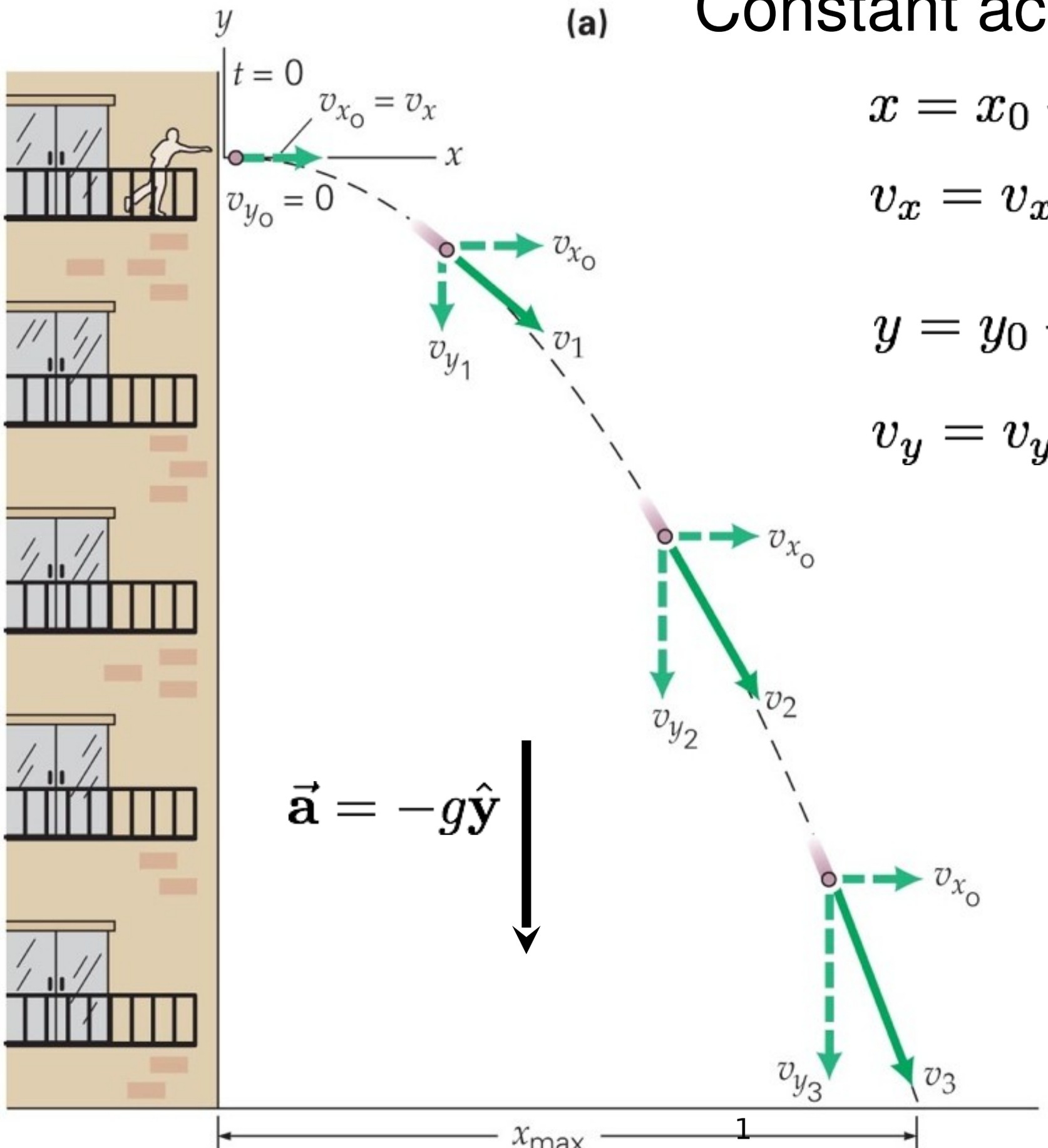
Constant acceleration in 2D:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

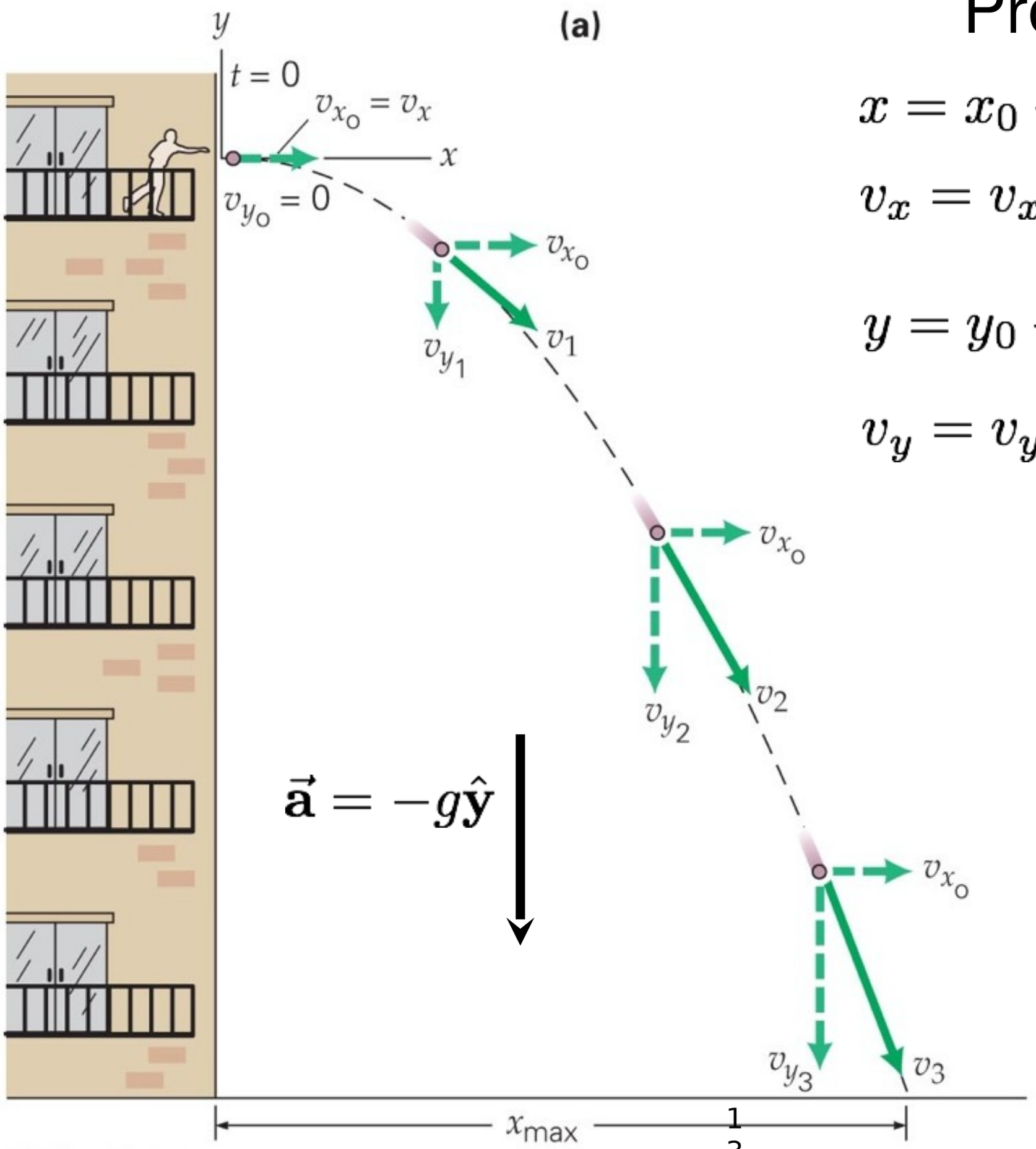
$$v_x = v_{x0} + a_x t$$

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

$$v_y = v_{y0} + a_y t$$



Projectile



$$x = x_0 + v_{x0}t$$

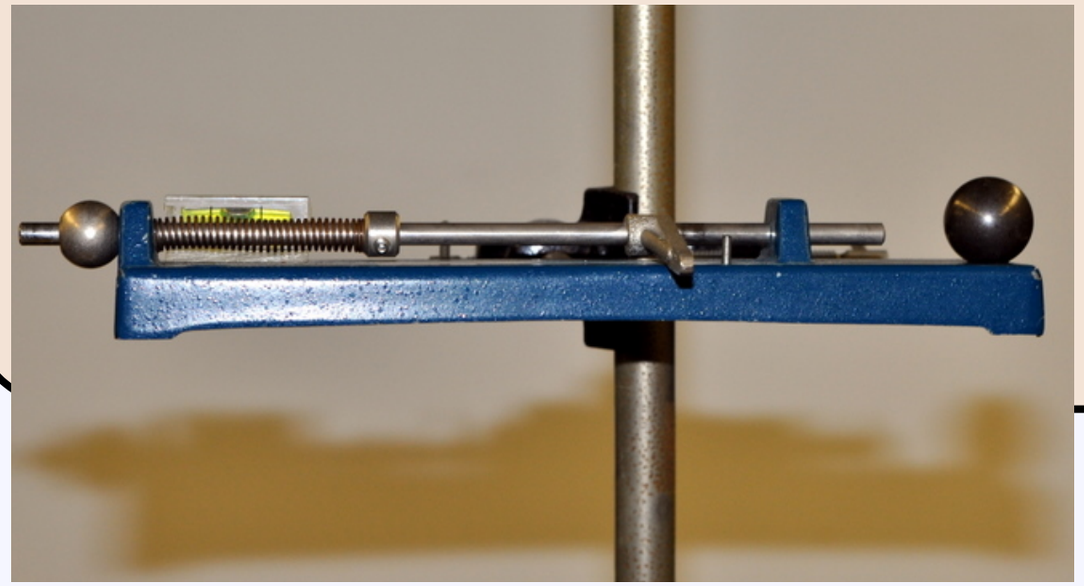
$$v_x = v_{x0}$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$v_y = v_{y0} - gt$$

Clicker question #1

From the *same* height (and at the *same* time), the left ball is dropped and the right ball is fired horizontally. Which one will hit the ground first?



A — the left, "dropped" ball

B — the right, "fired" ball

C — they both hit at the same time

D — it depends on how hard the right ball was fired

☐ — it depends on the initial height

$\mathbf{v}_0^L = \mathbf{0}$

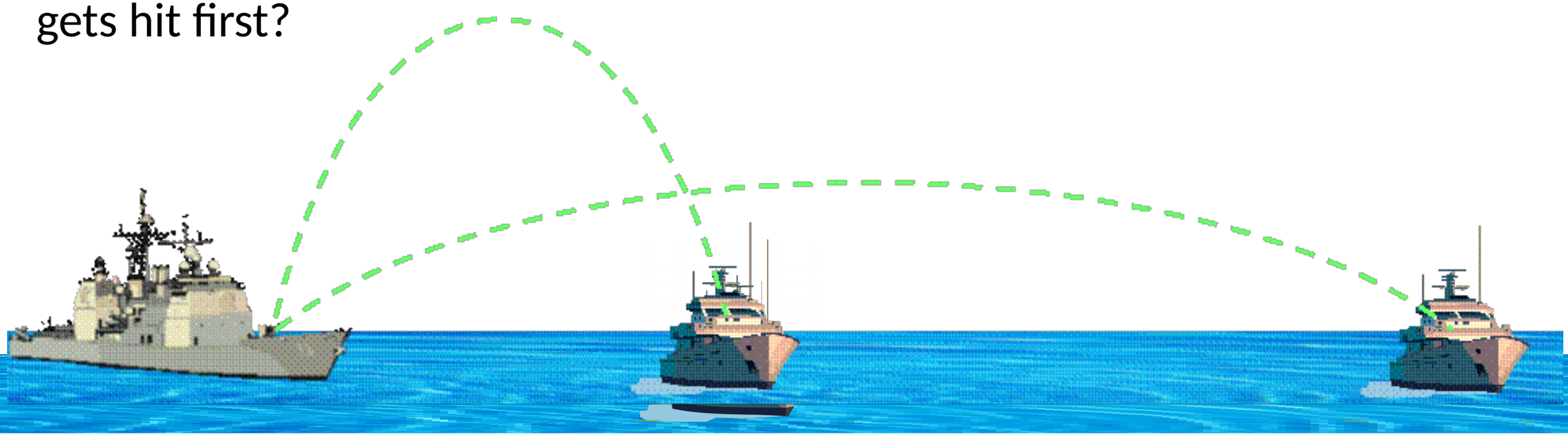
$\mathbf{v}_0^R \longrightarrow$

Clicker question #1

https://www.youtube.com/watch?v=uzaxTDx1KgE

Clicker Question 4

A destroyer fires two shells with the **same** initial speeds at two different enemy ships. The shells follow the trajectories shown. Which enemy ship gets hit first?



Destroyer Enemy 1 Enemy 2

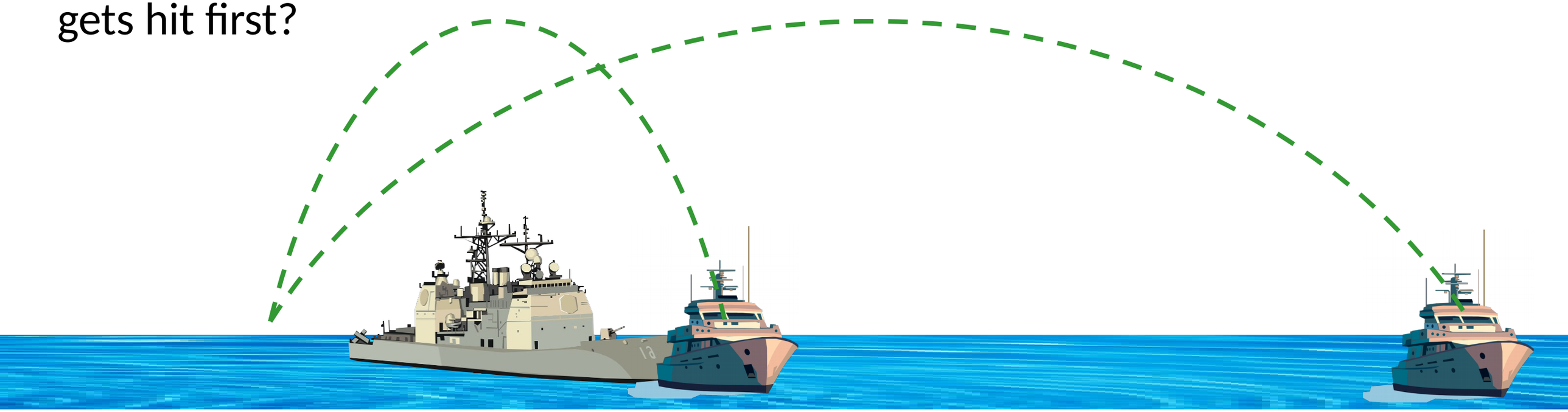
A) Enemy 1

B) Enemy 2

C) They are both hit at the same time

Clicker Question 5

A destroyer fires two shells with **different** initial speeds at two different enemy ships. The shells follow the trajectories shown. Which enemy ship gets hit first?



Destroyer Enemy 1 Enemy 2

A) Enemy 1

B) Enemy 2

C) They are both hit at the same time

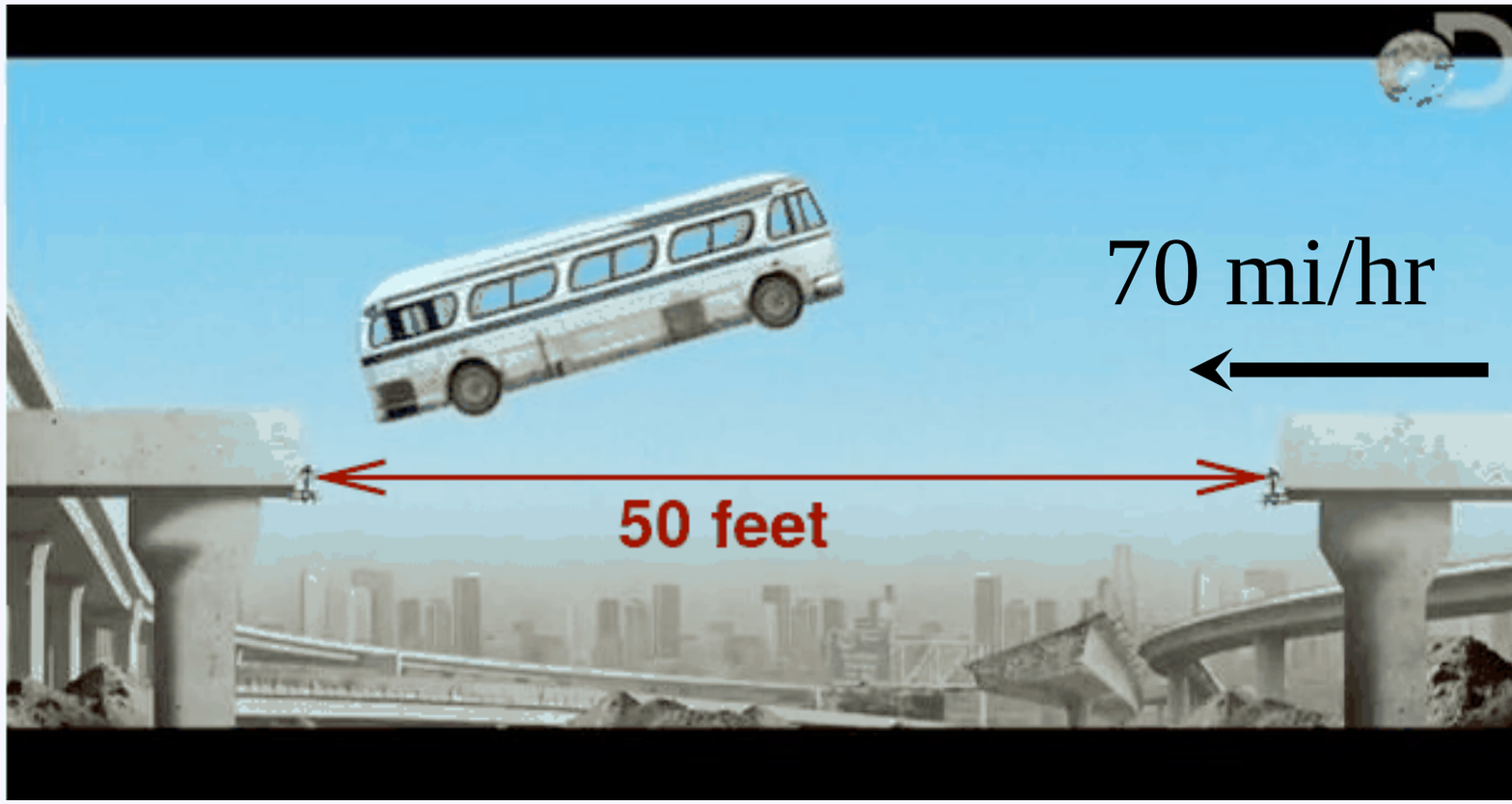
Clicker question #6

"Speed bus jump:" A bus moving at 70 mi/hr jumps over a gap of 50 feet between two level stretches of freeway. In real life…

A The bus makes the jump

B The bus misses the jump

C It depends on how heavy the bus is



http://www.youtube.com/watch?v=2b28hYqtbR8

http://www.youtube.com/watch?v=NPhNlS69ayQ

Clicker question #31

Angle for bus to jump the farthest?

$$d = \frac{v_0^2}{g} \sin 2\theta$$

A 0°

B 45°

C 30°

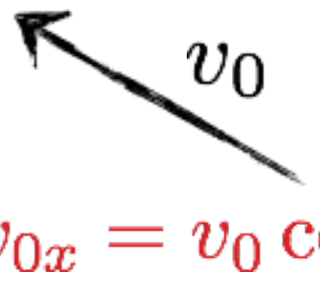
D 60°

Clicker question #7a

4. Vector components?

A

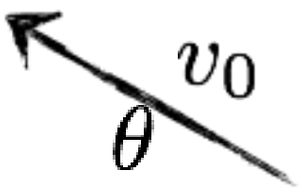
$v_{0y} = v_0 \sin\theta$



$v_{0x} = v_0 \cos\theta$

C

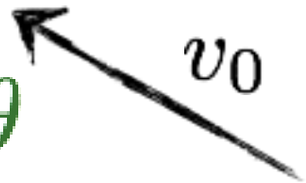
$v_{0y} = v_0 \sin\theta$



$v_{0x} = -v_0 \cos\theta$

B

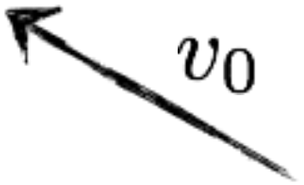
$v_{0x} = v_0 \cos\theta$



$v_{0x} = v_0 \sin\theta$

D

$v_{0y} = v_0 \cos\theta$



$v_{0y} = -v_0 \sin\theta$

E None of ABCD are correct

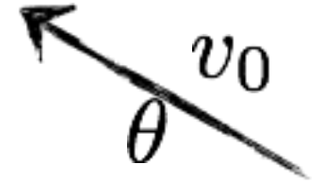
1. Read carefully
2. Draw a sketch
3. Given? Goal?
4. Principles & equations?
5. Calculate
   - Choose axes & origin
   - Resolve vector components
   - Solve equations

$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$-d = -v_0 t\cos\theta \qquad 0 = v_0 t\sin\theta - \frac{1}{2}gt^2$$

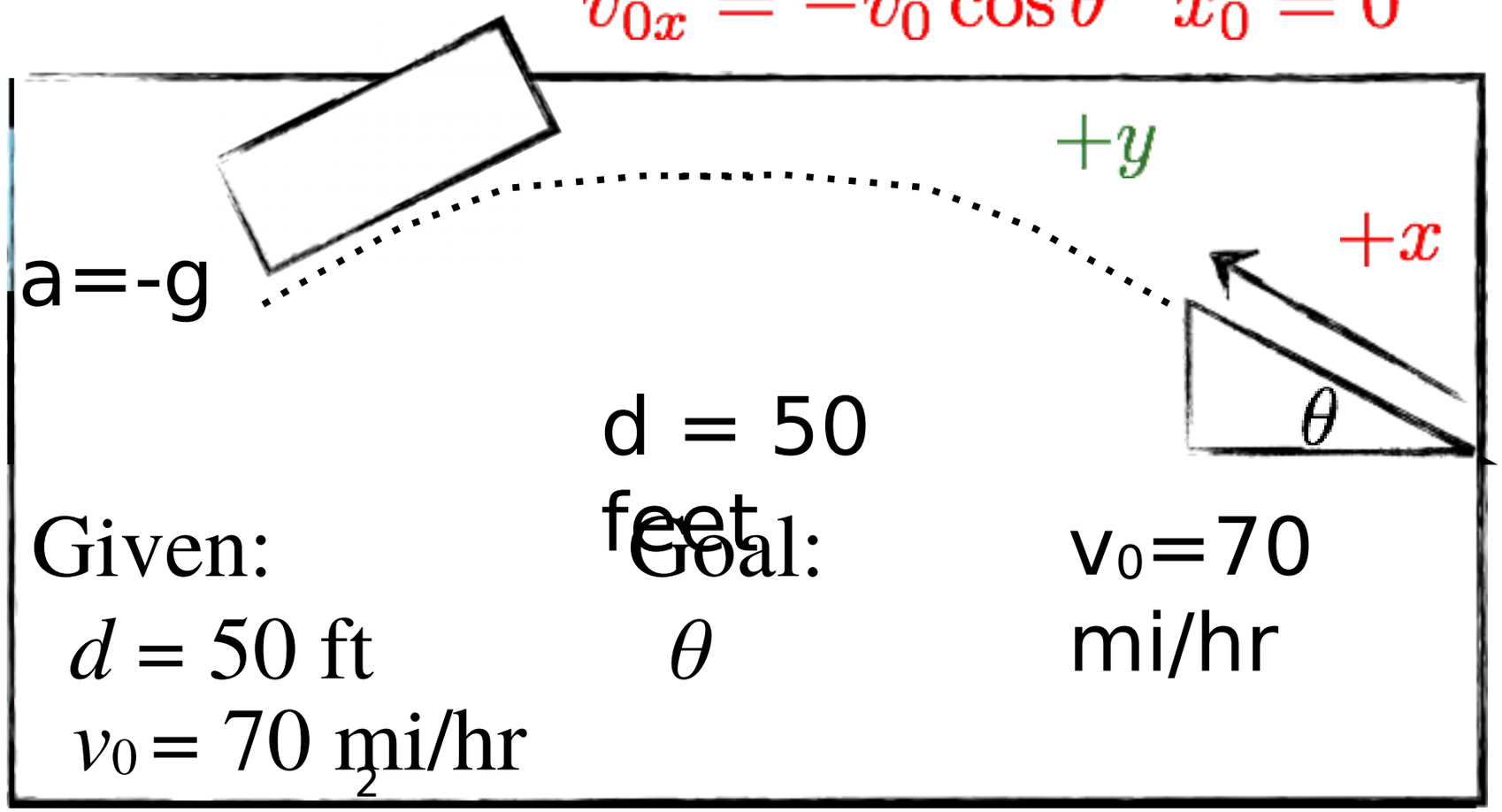
$y_0 = 0$

$y = 0 \quad v_{0y} = v_0\sin\theta$



$x = -d$

$v_{0x} = -v_0\cos\theta \quad x_0 = 0$



1.Read carefully
2.Draw a sketch
3.Given? Goal?
4.Principles & equations?
5. Calculate

Note: range equation

$$d = \frac{v_0^2}{g} \sin 2\theta$$

$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$-d = -v_0 t \cos\theta \qquad 0 = v_0 t \sin\theta - \frac{1}{2}gt^2$$

$$t = \frac{d}{v_0 \cos\theta} \qquad v_0 t \sin\theta = \frac{1}{2}gt^2$$

$$t = \frac{2v_0}{g}\sin\theta$$

$$\frac{d}{v_0 \cos\theta} = \frac{2v_0}{g}\sin\theta$$

$$\boxed{\theta = \frac{1}{2}\sin^{-1}\left(\frac{dg}{v_0^2}\right)}$$

$$\frac{dg}{v_0^2} = 2\sin\theta\cos\theta$$

$$= \sin 2\theta$$

1. Read carefully
2. Draw a sketch
3. Given? Goal?
4. Principles & equations?
5. Calculate
6. Plug in numbers
7. Is answer reasonable?

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{dg}{v_0^2}\right)$$

$d = 50$ ft

$v_0 = 70$ mi/hr

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

$$\theta = \frac{1}{2}\sin^{-1}\left[(50\,\text{ft})\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1}{70}\frac{\text{hr}}{\text{mi}}\right)^2\left(\frac{\text{mi}}{5280\,\text{ft}}\right)^2\left(\frac{3600\,\text{s}}{\text{hr}}\right)^2\right]$$

$$\theta = \frac{1}{2}\sin^{-1}[0.15] \qquad \theta = 4.4°$$

Next time…

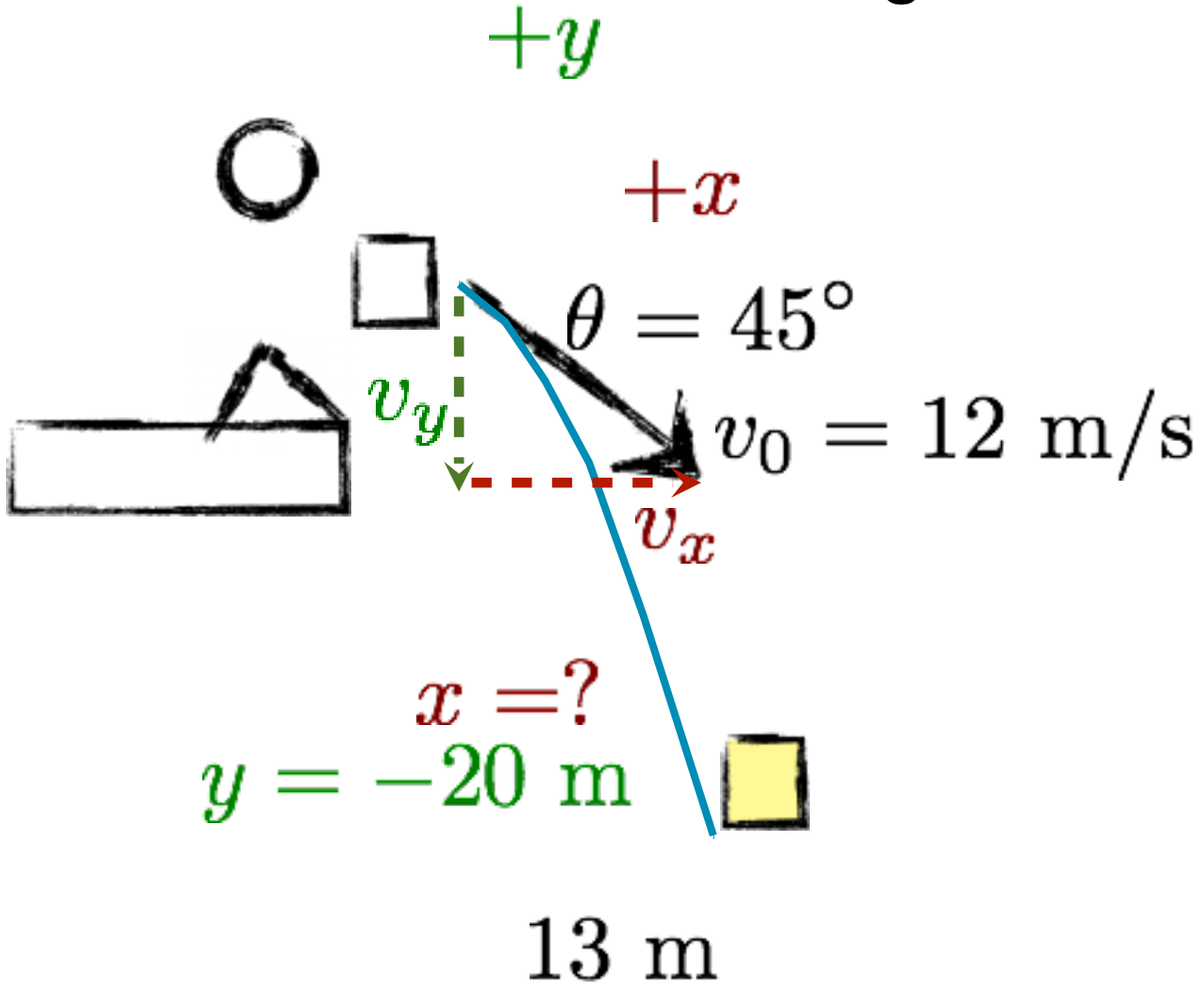
- Vectors and 2D Motion

Due dates

- **Assignments**
  - Should have already read **Ch.3** of the book
  - Next read **Ch. 4**
  - **HW2** due Friday at 11:45PM

Ex. 3.7: stone toss

- A girl on a bridge (height 20 m) throws a stone at 12 m/s, 45° below horizontal. Does the block hit a target on the water that is 13 m from where it goes under the bridge?



Given: $x_0 = y_0 = 0$ $y = -20$ m
$v_0 = 12$ m/s $\theta = 45°$

Goal: $x =?$

Concept: Projectile

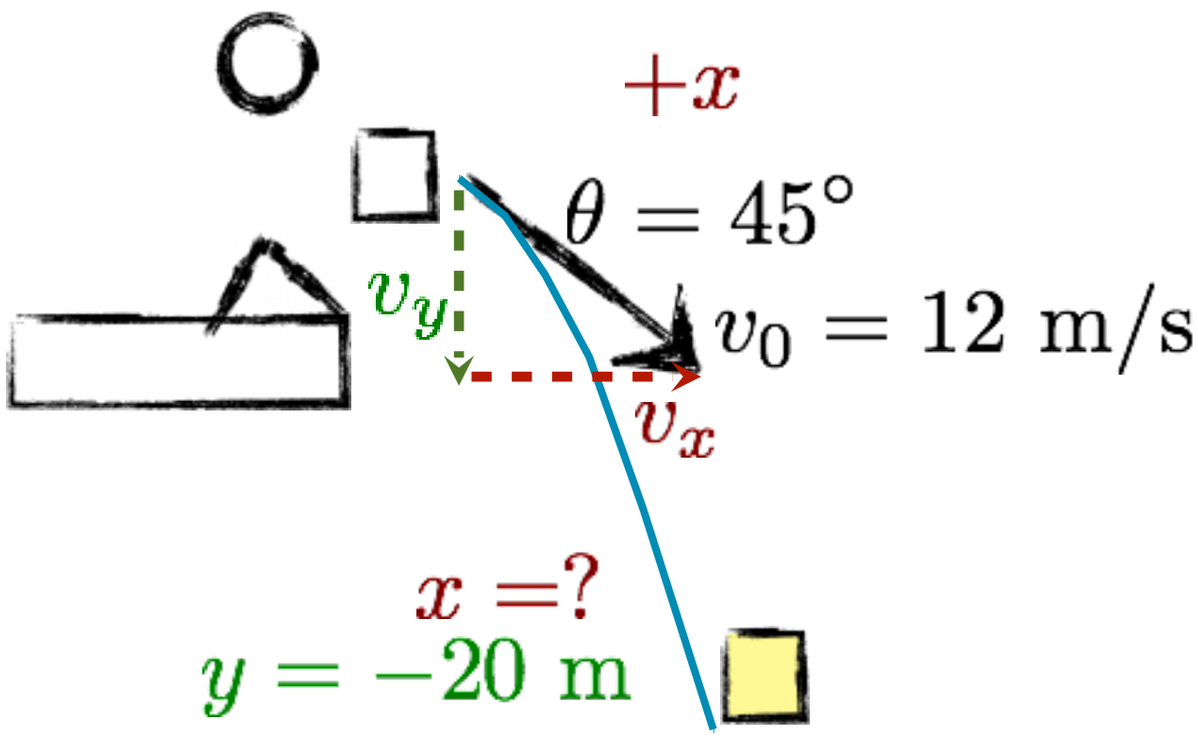
1. Read carefully
2. Draw a sketch
3. Given? Goal?
4. Brainstorm

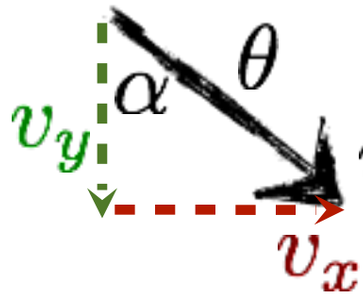
Ex. 3.7

Given: $x_0 = y_0 = 0 \; y = -20 \text{ m}$

$v_0 = 12 \text{ m/s} \quad \theta = 45°$

Goal: $x = ?$ **Concept:** Projectile





$$|v_{0x}|/|v_0| = \sin\alpha$$

$$|v_{0x}| = |v_0|\sin\alpha$$

$$v_{0x} = +v_0\sin\alpha$$

$$|v_{0y}|/|v_0| = \cos\alpha$$

$$|v_{0y}| = |v_0|\cos\alpha$$

$$v_{0y} = -v_0\cos\alpha$$

1. Read carefully
2. Draw a sketch
3. Given? Goal?
4. Brainstorm
5. Calculate

Ex. 3.7

Given: $x_0 = y_0 = 0 \qquad y = -20\ \text{m} \qquad v_{0x} = +v_0 \sin\alpha$

$v_0 = 12\ \text{m/s} \qquad \theta = 45° \qquad v_{0y} = -v_0 \cos\alpha$

Goal: $x = ?$ **Concept:** Projectile

$$x = x_0 + v_{0x}t \qquad t = \frac{x}{v_{0x}} \qquad t = \frac{x}{v_0 \sin\alpha}$$

$$\sin^2 45° = \cos^2 45° = \frac{1}{2}$$

$$\sin^2\alpha = \cos^2\alpha = \frac{1}{2}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$y = y_0 - (v_0 \cos\alpha)\left(\frac{x}{v_0 \sin\alpha}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \sin\alpha}\right)^2$$

$$0 = \left(\frac{g}{v_0^2}\right)x^2 + x + y$$

1. Read carefully
2. Draw a sketch
3. Given? Goal?
4. Brainstorm
5. Calculate

Ex. 3.7

Given: $x_0 = y_0 = 0 \; y = -20 \text{ m}$

$v_0 = 12 \text{ m/s} \quad \theta = 45°$

Goal: $x = ?$ **Concept:** Projectile

$$0 = \left(\frac{g}{v_0^2}\right) x^2 + x + y$$

$$x = \frac{-1 + \sqrt{1 - 4\frac{gy}{v_0^2}}}{2\left(\frac{g}{v_0^2}\right)}$$

$$x = \frac{-1 + \sqrt{1 - 4\left[9.8 \text{ m/s}^2\right]\left[-20 \text{ m}\right] / \left[12 \text{ m/s}\right]^2}}{2\left(\left[9.8 \text{ m/s}^2\right] / \left[12 \text{ m/s}\right]^2\right)} = 11 \text{ m}$$

Target at *x* = 13 m. Stone does not hit target.

Note: book uses different method, rounds differently, gets 12 m (though I get 11 m)

1. Read carefully
2. Draw a sketch
3. Given? Goal?
4. Brainstorm
5. Calculate
6. Plug in numbers
7. Reasonable?

In-Class Example (solution) Name_____

$a = (-3,-4,5) \quad b = (-1,4,-2) \quad c = (3,2,-5)$

① $\vec{a}\cdot(\vec{b}+\vec{c})$

start $(\vec{b}+\vec{c}) = (-1+3, 4+2, -2-5) = (2,6,-7)$

$\vec{a}\cdot(\vec{b}+\vec{c}) = (-3,-4,5)\cdot(2,6,-7) = -6-24-35 = \boxed{-65}$

② $\vec{a}\cdot(\vec{b}\times\vec{c})$

start $(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & -2 \\ 3 & 2 & -5 \end{vmatrix} \begin{matrix} i & j \\ -1 & 4 \\ 3 & 2 \end{matrix}$

$= (-20)\hat{i} + (-6)\hat{j} + (-2)\hat{k} - (5)\hat{j} - (-4)\hat{i} - (12)\hat{k}$

$= (-20+4)\hat{i} + (-6-5)\hat{j} + (-2-12)\hat{k}$

$= (-16, -11, -14)$

$\vec{a}\cdot(\vec{b}\times\vec{c}) = (-3,-4,5)\cdot(-16,-11,-14)$

$= 48 + 44 + -70 = \boxed{22}$