

Statistics

A set of tools for understanding data and making decisions/conclusions/predictions under uncertainty

Randomness

- in the short term, we don't know what will happen (flipping a coin)
- in the long term, we know the **distribution** of possibilities (what outcomes are possible and how often they occur)
 - the crux of randomness
 - as the amount of times we get a random variable approaches infinity, the clearer/less random the variable becomes

Two definitions of probability

- **proportion** of times an outcome occurs or would occur over infinitely many repetitions of a random action
 - **Frequentist**
 - math is a lot nicer
 - there is a fixed outcome but we don't know it
- a number quantifying our **belief** that an outcome can/will occur
 - **Bayesian**
 - 2000 times more intuitive and math is just as hard
 - random outcome (not fixed)

Both are calculus based

Probability Model

Consists of two parts:

- sample space: list (set/list of all unique values) of all possibilities
- probability of each outcome

This in essence is a hash table

An **event** is an arbitrary set of 0 or more outcomes in a sample space

Axioms of Probability

- axioms : something is so obvious it does not need to be proven
- sample space must be well defined
- For events A and B in the same sample space denoted as “S”:
 - The probability of event A, denoted as $P(A)$ is a number between 0 and 1 (inclusive).
 - * $[0, 1]$ notation as well.
 - * **NOTE:** $P(A) = 0$ means A is “impossible” and $P(A) = 1$ means A is guaranteed
 - $P(S) = 1$
 - * **Some outcome is bound to happen**
 - If A and B are disjoint (there are no common outcomes. A is not in B AND B is not in A), then $P(A \text{ or } B) = P(A) + P(B)$

Simple rules that follows from the Axioms

- Compliment Rule: Define $A^C = A$ compliment, that is A^C is the event “A does not occur”
 - $P(A^C) = 1 - P(A)$
- General addition rule: Suppose events A and B have at least one common outcome
 - Define $A \cap B$ to be the set of outcomes common to A & B
 - Define $A \cup B$ to be the set of outcomes in A, or in B or in both A & B
 - * Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example

Random phenomenon: Draw 1 tile from a standard Scrabble bag of 100 tiles

- Sample space 1 (option one):
 - S = the 100 tiles in the bag
 - All tiles are equally likely to be drawn
 - $P(\text{draw particular tile}) = 1/100$ or 0.01 for all
- Sample space 2 (option two):
 - The 27 “letters” (26 letters and 1 blank)
- Let event C = “draw a letter in CAT”
- Let event D = “draw a letter in PET”

$$P(C) = P(\text{draw a C}) + P(\text{draw an A}) + P(\text{draw a T}) = .02 + .09 + .06 = 0.17$$

$$P(D) = P(\text{draw a P}) + P(\text{draw an E}) + P(\text{draw a T}) = 0.2 + 0.12 + 0.06 = 0.2$$

$$P(C^c) = P(\text{do not draw any of the letters in CAT}) = 1 - P(C) = 1 - .17 = .83$$

$$P(C \cap D) = P(\text{draw a letter in **both** CAT & PET}) = P(\text{draw a T}) = 0.06$$
$$P(C \cup D) = P(\text{draw a letter in CAT or PET or both words}) = P(C) + P(D) - P(C \cap D) = 0.17 + 0.2 - 0.06 = 0.31$$

Python Code Representation

```
#!/usr/bin/env python3.5

# probability can be calculated by using a hash table in conjunction with a set
# hash tables are used when there are two different letters with the same probability
# using a bare list would result in incorrect calculations of probability
# they would be treated as non unique instances
# in turn allowing for it to filter out needed objects
# this boils down to a set of unique hash tables and summing up

class hashabledict(dict):
    def __hash__(self):
        return hash(tuple(sorted(self.items())))
value_mapping = {
    "c": 0.02,
    "a": 0.09,
    "t": 0.06,
    "p": 0.02,
    "e": 0.12
}

def get_probability(*args):
    # s has extra new line for code to fit
    s = set((hashabledict({letter: value_mapping[letter]})
              for argument in args for letter in argument))
    return sum([sum(dictionary.values()) for dictionary in s])

print(get_probability("cat", "pet"))
# this some times yields 0.3100000000000005 and 0.3099999999999994
# which is essentially the same number
```

vim regex

```
# this replaces all caps in proper latex
:%s/cap/\$\$\\cap\$/g
```