

QUIZ # 4

Please show all of your work for maximum credit. Good Luck!!!

1. (4 points) Let $P(t)$ be the population of a country, in millions, t years after 1990, with

$P(7) = 3.21$ and $P(13) = 3.75$.

The initial population is 2.58 millions and the pop. grows by 0.09 million per year.

(a) Find a formula for $P(t)$ assuming it is linear. Interpret the slope and y-intercept.

Sol $(7, 3.21)$ & $(13, 3.75)$

$$m = \frac{3.75 - 3.21}{13 - 7} = 0.09$$

$$m = 0.09; (7, 3.21)$$

$$y - 3.21 = 0.09(x - 7)$$

$$y - 3.21 = 0.09x - 0.63$$

$$\begin{array}{r} +3.21 \qquad \qquad +3.21 \end{array}$$

$$y = 0.09x + 2.58$$

$$P(t) = 0.09t + 2.58$$

(b) Find a formula for $P(t)$ assuming it is exponential. Interpret the factor and the y-intercept.

Sol $(7, 3.21)$ & $(13, 3.75)$

$$b^6 = \frac{3.75}{3.21}$$

$$b = \left(\frac{3.75}{3.21} \right)^{\frac{1}{6}} = 1.0263$$

$$y = a \cdot b^t$$

$$3.21 = a(1.0263)^7$$

$$\frac{3.21}{(1.0263)^7} = \frac{a(1.0263)^7}{(1.0263)^7}$$

$$a = 2.68$$

The initial pop @ $t=0$ is 2.68 millions & it grows by factor of 1.0263 every year.

2. (2 points) An investment decreases by 5% per year for 4 year period. By what percent does it decrease over the 4-year period?

Sol $b = (1 - 0.05)^4$

$$b = 0.8145$$

$$b = 1 - r$$

$$0.8145 = 1 - r$$

$$r = 1 - 0.8145$$

$$r = 0.1855$$

$$r = 18.55\%$$

3. (2 points) The GDP of Chile was 145.8 billion dollars in 2007 and was growing at a continuous rate of 5.1% per year.

(a) Find a formula for G , the GDP of Chile in billion dollars, as a function of t , the number of years since 2007.

Sol. $G(t) = a \cdot b^t$

$$G(t) = 145.8(e^{0.051})^t$$

$$\boxed{G(t) = 145.8e^{0.051t}}$$
 , $G(t)$ is in billions.

(b) By what percent does the GDP increase each year? (r)

Sol. $b = e^k$

$$b = e^{0.051}$$

$$b = 1.05232$$

$$b = 1 + r$$

$$\begin{array}{r} 1.05232 = 1 + r \\ -1 \quad \quad -1 \\ \hline \end{array}$$

$$0.05232 = r$$

$$\boxed{r = 5.232\%}$$

4. (2 points) A bank pays interest at the nominal rate of 1.3% per year. What is the effective annual rate if compounding is weekly?

Sol. $APY = \left(1 + \frac{0.013}{52}\right)^{52} - 1$

$$= 1.013083 - 1$$

$$= 0.013083$$

$$\boxed{= 1.3083\%}$$

The eff. rate is the actual rate earned at the end of the investment period.