

Day 9

Outline

1. Conditional probability example
2. Power analysis example

Please reference the attached sheets for full examples

Conditional Probability Examples

Example 1

In a lecture class of 150 students, 110 students are freshmen, 50 own a dog, and 25 are freshmen who own a dog. Suppose a student is selected at random.

Finish filling in from picture

Tree Diagram Version

Root

- Freshman ($\frac{110}{150}$)
 - Own dog : $\frac{25}{110}$ [Freshman AND own dog = ($\frac{110}{150} \times \frac{25}{110}$)]
 - No dog : $\frac{85}{110}$ [Freshman AND no dog = $\frac{110}{150} \times \frac{85}{110}$]
 - Not Freshman ($\frac{40}{150}$)
 - Own dog : $\frac{25}{40}$
 - No dog : $\frac{15}{40}$
- a. What is the probability of being a freshman, given that the student owns a dog?
- $P(\text{Freshman}|\text{Dog}) = \frac{P(\text{Freshman AND Dog})}{P(\text{Dog})} = \frac{\frac{25}{150}}{\frac{25}{150} + \frac{25}{150}} = \frac{25}{50} = \frac{1}{2}$
- b. What is the probability of owning a dog, given that the student is a freshman?
- $P(\text{Dog}|\text{Freshman}) = \frac{P(\text{Dog AND Freshman})}{P(\text{Freshman})} = \frac{\frac{25}{150}}{\frac{110}{150}} = \frac{25}{110} = \frac{5}{22}$
- $P(\text{Freshman} \& \text{Dog}) = P(\text{Freshman}) P(\text{Dog}|\text{Freshman})$
 - $P(\text{Freshman} \& \text{Dog}) = P(\text{Freshman}) P(\text{Dog}) \leftarrow \text{Independence}$

Table Diagram Version

	Freshman	Not Freshman	
Dog	25	25	50
No Dog	85	15	100
	110	40	150

Figure 1: Freshman Table Example

Power Analysis Examples

Example 1

It is believed that about 10% of the population is left-handed. However, China has claimed that less than one percent of its students are left-handed. Suppose we are interested in evaluating whether there is something special about Chinese people, or whether the Chinese government is lying. Suppose further that we have devised a scientifically perfect test to measure a person's dominant hand. Would a random sample of 50 Chinese students be large enough to detect a population difference of 10% vs. 1%?

We want low α and high power

- $H_0 : p = 0.1$ [Null]
- $H_1 : p = 0.01$ [Alternate]
- $N : 50$
- $\alpha : 0.05$
 - If α is not given, please assume $\alpha = 0.05$
- Define p = proportion of left handed students
- For midterm one, define X = number of (successes) left-handed students in our sample.
- Decision rule:
 - Critical region: $X \leq x$
 - If X is in critical region, accept H_0 , else accept H_1 .
 - Only problem is we do not know what x is.
 - Defining our critical region to be $X \leq 5$
 - * Under the null hypothesis H_0 , $X \sim B(50, 0.1)$
 - $P(X \leq 5 | p = 0.1) = 0.616$
 - $P(\text{Type 1 Error}) = 0.616$
 - * Under the alternative hypothesis H_1 , $X \sim B(50, 0.01)$
 - $\beta = P(X > 5 | p = 0.01) = 0$
 - $\text{Power} = P(X \leq 5 | p = 0.01) = 1$
 - Defining our critical region to be $X \leq 1$
 - * When $p = 0.1$
 - 3.4% false positive
 - 96.6% true negative
 - $\alpha = 0.034$
 - * When $p = 0.01$
 - 91.1% true positive
 - 8.9% false negative
 - $\text{Power} = 0.911$
 - $\beta = 0.089$

Example 2

Is this sample large enough to detect something \rightarrow power rule!!!!!!

We want low α and high power

- $H_0 : p = 0.26$ [Null]
- $H_1 : p = 0.52$ [Alternate]
- $N : 14$
- $\alpha : 0.05$
 - If α is not given, please assume $\alpha = 0.05$
- Define $p =$ % of patients progression free after 6 months

Using R:

- Critical region is $X > 6$ or $X \geq 6$
- `lower.tail = TRUE` includes \leq
- `lower.tail = FALSE` includes $>$
- When $P = 0.26$
 - 4.7% false positive
 - 95.3% true negative
 - $\alpha = 0.047$
- When $P = 0.52$
 - 66.2% true positive
 - 33.8% false negative
 - Power = 0.662
 - $\beta = 0.338$