Contents

NOTE: all equations containing the $\cdot$ symbol denote a DOT PRODUCT. Multiplication in formulas will be denoted with the $\times$ symbol to avoid confusion.

Kinetic Energy and Work

Kinetic: energy of motion.

Work: Force (along the direction of motion) · Displacement (two vectors dotted will always result in a scalar)

It is important to be aware of the work done by what force on what object.

If an applied force is referenced, it is an applied force - For example: If work done by "you" as opposed to work done by a spring force, etc

Free Body Diagram

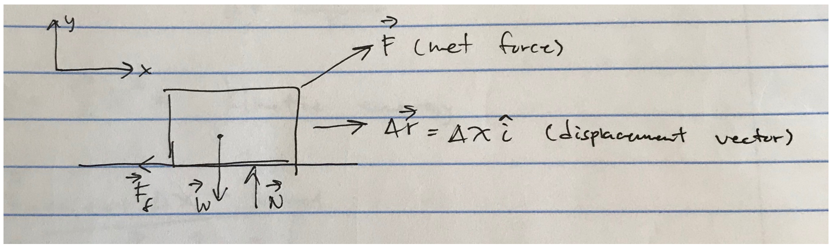

Figure 1: Kinetic Energy

- Work done by $\vec{F}$ (pulling force)
- Work done by a specified force or combination of forces
  - Vector sum is denoted by Σ
  - $\vec{W}_{\text{net}} = (\Sigma \vec{F}_{\text{i}}) \cdot \Delta\vec{r}$
  - $\vec{F}_{\text{net}} = \vec{W}_{\text{friction}} + \vec{V}_{\text{normal}} + \vec{W}_{\text{applied force}} + \vec{W}_{\text{gravity}}$
  - $\vec{W}_{\text{friction}} < 0$ and will always act to decrease the kinetic motion.

Work Formula

Constant

$\vec{W} = \vec{F} \cdot \Delta\vec{r} = \vec{F} \cdot \Delta x \hat{i}$

This formula works when there is a [constant] force

Variable

$\vec{W} = \vec{F} \cdot d\vec{r}$

$\vec{W} = \int_i^f \vec{F} \cdot d\vec{r} dx$

This applies when the force is variable

Note: $\vec{F} \cdot \Delta\vec{r} = |\vec{F}| \times |\Delta\vec{r}| cos(\theta)$

General

$W = \int_{x_{\text{i}}}^{x_{\text{f}}} F(x) dx$

1D Constant Acceleration

$((\vec{V}_{\mathrm{f}})^2 - (\vec{V}_{\mathrm{o}})^2) = 2\vec{a}\Delta x$

This is derived from Newton's 2^{nd} Law

$\vec{a} = \frac{\vec{F}_{\text{net, ext}}}{m}$

Kinetic Energy Formula

$$K = \frac{m}{2} \times v^2$$

1-D Work Energy Theorem

$$\vec{F}_{\text{net, ext}}\Delta x = \vec{W}_{\text{net}}$$

$$\frac{m}{2} \times ((\vec{V}_{\mathrm{f}})^2 - (\vec{V}_{\mathrm{o}})^2) = \vec{W}_{\text{net}}$$

Units for Work

$\vec{W} \sim \vec{F} \times \Delta x \sim 1N \times 1\,meter = \ Joule$

Kinetic energy has the same units as work

Displacement Vector

$\vec{r}_{\mathrm{i}} + \Delta\vec{r} = \vec{r}_{\mathrm{f}}$

$\Delta\vec{r} = \vec{r}_{\mathrm{f}} - \vec{r}_{\mathrm{i}}$

Work done by a Spring Force

$\Delta\vec{K} = \vec{W}_{\text{net}} = \vec{K}_{\mathrm{f}} - \vec{K}_{\mathrm{i}}$

Above is the kinetic-energy theorem

$$\vec{W}_{\text{net}} = \int_a^b \vec{F}_n \cdot d\vec{r} = \vec{F}_{\text{net}} \cdot \Delta\vec{r}$$

Work done by a Gravitational Force

Tension does positive work, gravity does negative work

Spring Force

A variable force from a spring

Relaxed state: neither compressed or extended

Restoring force: attempting to put spring into a relaxed state

Displacement is the opposite of the spring force

Hooke's Law

$F_\mathrm{s} = -k\Delta x$

- $\vec{W}_\mathrm{net} > 0 \to \Delta k > 0 \to KE \uparrow$
- $\vec{W}_\mathrm{net} < 0 \to \Delta k < 0 \to KE \downarrow$

By integrating the function $\vec{F}_\mathrm{s}$ with respect to x, we can develop a formula that looks like this:

$$\vec{W}_\mathrm{s} = \frac{1}{2}k((x_\mathrm{i})^2 - (x_\mathrm{f})^2)$$

If starting from **relaxed state**, then $\vec{W}_\mathrm{s} = -\frac{1}{2}kx^2$

Power

Symbol "P" is used to denote power and it is the time rate of doing work.

$\frac{dW}{dt} \rightarrow$ this is instantaneous power (power at any given place in time)

Think acceleration; how fast the velocity changes per second of time.

$P_{\text{average}} = \frac{W}{\Delta t} = \int_i^f P(t)dt$

Potential Energy

Symbol "U"._

$\Delta U = -W$

Change in potential energy is negative work.

$\vec{F}_{\text{net}} = \vec{F}_{\text{c}} + \vec{F}_{\text{nc}}$

Total force is the combination of conservative forces (forces that aim to keep the system in equilibrium) and non-conservative forces (forces that appose conservative forces)

- For conservative forces we can define/associate a potential energy "U" with that force{s}

General Form of Work

$\Delta U = -\int_{x_{\text{i}}}^{x_{\text{f}}} F(x)dx$

Change in Potential Energy

Only depends on the initial and final points, not the path taken between them.

$U + C = (U_\mathrm{f} + C) - (U_\mathrm{i} + C) = \Delta U$

$\Delta U_\mathrm{g} = -Wg = mg\Delta y = mgh$

Potential energy is dependent on how high the object is and how massive it is.

Work Energy Theorem : Potential

$W_\mathrm{c} + W_\mathrm{nc} = W_\mathrm{net} = \Delta K = -\Delta U - W_\mathrm{nc}$

- Valid for both $\vec{F}_\mathrm{c}$ and $\vec{F}_\mathrm{nc}$ forces

$\therefore\ \Delta K + \Delta U = W_\mathrm{nc}$ These are considered external “agents”

Work energy theorem is present when conservative and non-conservative forces are present

Mechanical Energy

All energy in a given system.

It is denoted by the letter “E”.

$E = K + U$

Kinetic plus potential energy

$\therefore\ \Delta K + \Delta U$

Change in total work energy is work done by non-conservative forces

When no work is done by non-conservative forces ($\mathrm{W_{nc}} = 0$):

$\Delta E = \Delta K + \Delta K = 0$

$\implies E_\mathrm{f} = E_\mathrm{i}$

This is the conservation of mechanical energy, where energy is not lost in the system. Small amounts of energy can be used to trigger a large energy release.

Reading a Potential Energy Curve

$F(x) = \frac{dU(x)}{dx}$

$F\Delta x = \Delta W = -\Delta U$

- for "small enough" Δx, the following can be inferred $\rightarrow \Delta U = -\vec{F}\Delta x$

Reimann Sum Equivalent: $\lim_{\Delta x \to \infty} \frac{\Delta U}{\Delta x} = F = -\frac{du}{dx}$

The above is the same as saying:

Infinite sum: $U = -\int F(x)dx \implies F = -\frac{du}{dx}$

Both are the same and can be used interchangeably.

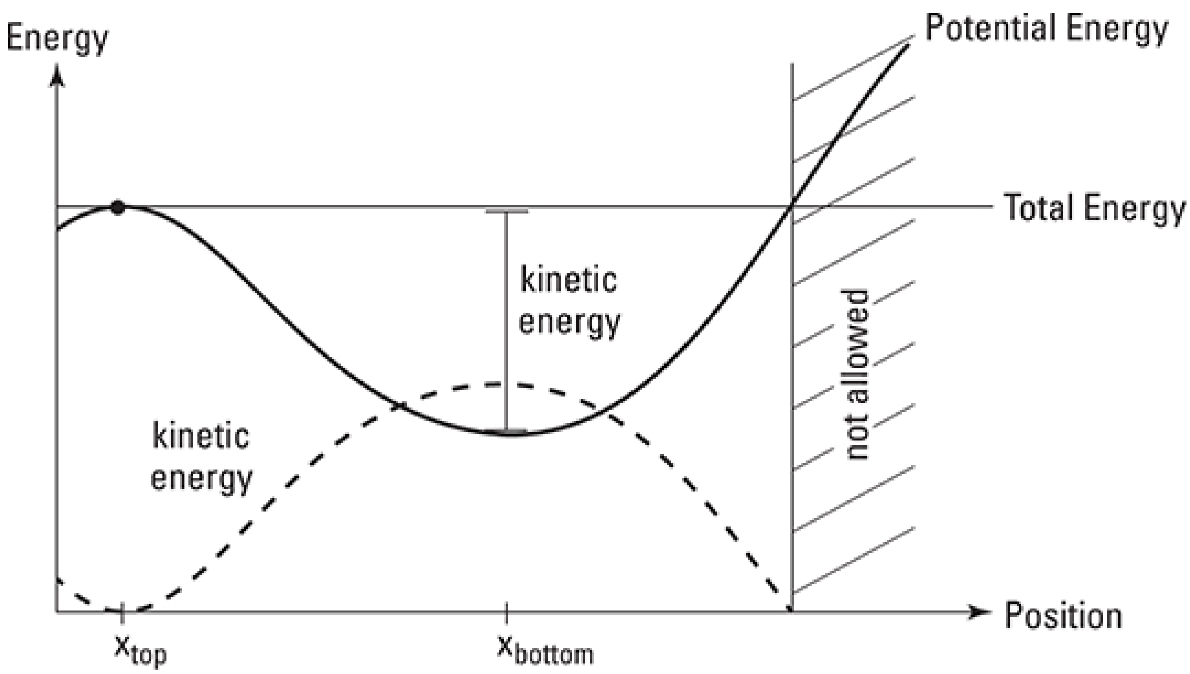

Figure 2: Potential Energy Curve

The following curve can be interpreted as:

- Low points represent the "restoring force", which is stable equilibrium.
- Peaks represent unstable equilibrium and can be activated with little force.

Equilibrium merely states that the sum (Σ) of the forces is zero. It does not mean however it is stable to stay there for a prolonged period of time. Since it is it requires a given amount of force $\vec{F}$

The above curve is a function of potential energy with respect to which is $U(x)$. These are called parametric functions, where the x position is a *parameter* of the function of potential energy. This allows us to find instantaneous potential energy **anywhere** along the x axis.

Inherently Unstable

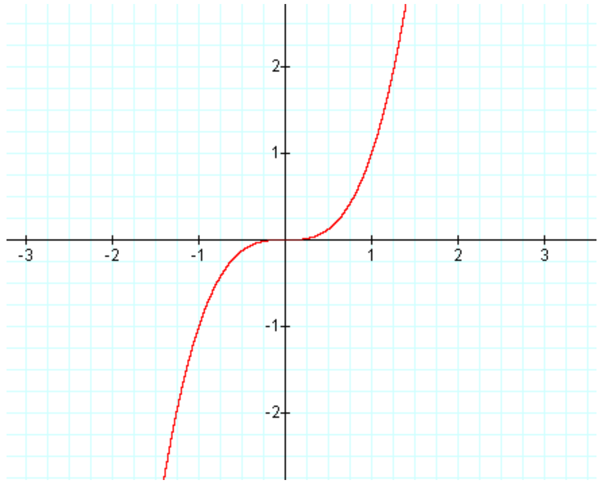

Figure 3: Cubic Graph

Any given force will a not be able to overcome the steep curve and will either rest in the recess (0, 0) or continue to drop off on the -x direction.

Conservation of Mechanical Energy

Recall that:

$E_{\text{mechanical}} = K + U$

To conserve mechanical energy, this means that no amount of energy in a given system can leave. Therefore, this can only be implemented in a universe where friction is non-existent and thermodynamics does not work. These are examples of how energy can leave a given system.