# Outline

1. Recap of probability
2. Simulation
3. Random Variables

# Independent vs. Disjoint Events

- Independent events **can** happen at the same time, but knowing that event "A" occurred **does not** change P(B) and vice versa
- Disjoint events cannot happen at the same time.
  - Knowing that event "A" occurred changed P(B) = 0 and vice versa
- If A and B are independent, P(A cap B) = P(A) P(B)
- If A and B are disjoint, P(A cap B) = 0

## Example One

- Draw a tile from a bag of 100 scrabble tiles
- Event C = "the tile is a C"
- Event A = "the tile is an A"

P(C) = .02 P(A) = .09

Events "C" and "A" are disjoint Events "C" and "A" are <u>not</u> independent

## Example Two

Draw one tile and set it outside

Event C = "first tile is a C" Event A = "second tile is a A"

Event C and A are not disjoint Events "C" and "A" are <u>not</u> independent

**Sampling without replacement**

## Example Three

Draw a tile, put it back in the bag and then draw another tile

Event C = "first tile is a C" Event A = "second tile is an A"

Event C and A are <u>not</u> disjoint Event C and A <u>are</u> independent

**Sampling with replacement**

# Simulation

Trying to imitate in the real world where the outcome is uncertain but is random

- Specify our model for an uncertain situation/random event
- "Randomly" generate an outcome for the model
- Repeat step two many, many times

Why simulate? - Once we set up the model, the math maybe too difficult - Situation may be unique, or we only have ability to observe it once, due to physical/financial limitations - For fun and/or profit

Report assumptions of the model!

# Random Variables (RVs)

Random variable is a variable whose numerical values describe outcomes of a random event

Typically we map outcomes in our sample space denoted as "S" to numerical values of the random variable.

Discrete Random Variable: probability mass function (PMF) places positive probability at specific numbers on the number line - Only specific numbers - Example: all outcomes are real, positive numbers Continuous Random Variable: probability density function (PDF) - Places positive probability along a possibly infinite interval of the number line.

## Writing the PMF of a Discrete Random Variable

| X=x | P(X=x) |
|---|---|
| All possible values that X can take | |
| Corresponding probabilities of each value | |

Y represents a random variable y represents a "realization" of Y

## Example One

We can find P(Y=0)

Once we have observed the random event either y = 0 or y != 0

Let X = the point value of the chosen tile

*finish this later*

map = { 0: 0.02, 1: 0.68, 2: 0.07, 3: 0.08, 4: 0.10, 5: 0.01, 8: 0.02, 10: 0.02 }

values of map == 0

### Example Two

Use PMF & probability rules to find:

- P(X <= 3)
  - P(X = 0, 1, 2 or 3)
  - P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)
  - = 0.85
- P(X > 1)
  - P(X = 2, 3, 4, 5, 8, or 10)
  - 1 - P(X <= 1) = 1 - (X = 0 or 1)
  - 1 - [P(X = 0) + P(X = 1)] = 1 - [0.02 + 0.68] = 1 - .7 = 0.3
- P(X > 5)
  - P(X = {0..5})
  - 0.04
- P(3 < x <= 5)
  - 0.11
  - P(X = 4, or 5)

## Expected Value (Mean) of a Random Variable

**Read more about this section**

Called expectation, mean, all the same thing

On average, what value do we expect the random variable to be

Recall idea of "weighted average"

Summation notation

E[X] = mu_x = sum x P(X=x)

Expected value is a linear operator

For random variables X and Y, and constant C

E[X+Y] = E[X] + E[Y] -and- E[cX] = cE[X]

^ where "c" is a constant applied

This implies, for X, Y and arbitrary constants a,b E[aX + bY] = aE[X] + bE[Y]

Consequences: a = 1 , b = -1

E[X - Y] = E[X] - E[Y]

mu{x-y} = mu{x} - mu{y}