Name: SOLUTIONS

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Define a rational function

ANY FUNCTION that can be written as a quotient of 2 polynomials is called "Rational function". i.e., $\frac{P}{Q}$; $Q \neq 0$

Check your understanding:

1. Which of the following are rational functions:

   A) $y = \frac{x^2-2}{x^5} - \frac{1}{2x^2}$

   B) $y = \frac{2^x - 5}{4^x}$

   C) $y = \frac{2\sqrt{x}+5}{x^4-2}$

   D) $y = \frac{2x^{-5}}{4+x^2} + \frac{x^{-2}}{1-x^{-4}}$

2. Find the long-run behavior of the function $y = \frac{x^2-3}{x^3} - \frac{1}{5x^2}$. $= \frac{5(x^2-3)}{5x^3} - \frac{1(x)}{5x^2 \cdot x} = \frac{5x^2 - x - 15}{5x^3} \Rightarrow \frac{5x^3}{5x^3}$ $\boxed{y=1}$

3. Determine the vertical and horizontal asymptotes, if they exist, of the function

(a) V.A. = set the denominator to zero.

$$y = 6 - \frac{14}{4x+36} + \frac{1}{5x^4}$$

$4x+36=0$; $5x^4=0$

$\boxed{x=-9}$, $\boxed{x=0}$

$y = \frac{6(20)(x^4)(x+9) - 14(5)(x^4) + 1(4)(x+36)}{20x^4(x+36)} = \frac{120x^5 + 1080x^4 - 70x^4 + 4x + 144}{20x^5 + 720x^4}$

H.A. $\Rightarrow \frac{120x^5}{20x^5}$ $\boxed{\Rightarrow y=6}$

4. The profit earned by a producer to manufacture and sell n units of a good is given by $P(n) = 11n - 2343$. The average profit for n units is given by $A(n) = \frac{P(n)}{n}$.

   a. Compute $A(1)$, $A(213)$, $A(280)$. $A(1) = \frac{11(1) - 2343}{1} = \boxed{-2332}$

   $A(213) = \boxed{0}$

   $A(280) = \boxed{2.63}$

   b. In practical terms what of the values in part (A).

   Average profit per unit.

   c. What trend do you notice in the values of $A(n)$ as n gets large?

   as $n \to \infty$, $A(n) \to 11.00$

Name: Solutions.

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class. $R(x) = \frac{P(x)}{Q(x)}$

Describe in words how to find:

- Zeros of the rational function Set the f(x) or y=0 and solve for x.
- Vertical asymptote: Set the denominator to zero.
- Horizontal asymptote Long-run behavior = $\frac{\text{leading co-eff. of } P(x)}{\text{leading co-eff. of } Q(x)}$

Check your understanding:

1. For the given rational function $f(x) = \dfrac{x^2-9}{x^2+6x}$ find, if possible the following.
   a. x-intercept(s) $0 = \frac{x^2-9}{x^2+6x} \Rightarrow x^2-9=0 \Rightarrow x=\pm 3;$ $\boxed{(3,0) \& (-3,0)}$
   b. y-intercept $f(0) = \frac{(0)^2-9}{(0)^2+6(0)}$; undefined
   c. vertical asymptotes $x^2+6x=0 \Rightarrow x(x+6) \Rightarrow \boxed{x=0}$ or $\boxed{x=-6}$
   d. horizontal asymptotes $f(x) = \frac{\text{leading coeff. of } P(x)}{\text{leading coeff of } Q(x)} = \frac{x^2}{x^2} \Rightarrow \boxed{y=1}$
2. For the given rational function $f(x) = \dfrac{x}{x^2-3x+2}$ find, if possible the following.
   a. x-intercept(s) $0 = \frac{x}{x^2-3x+2} \Rightarrow \boxed{x=0}$, $\boxed{(0,0)}$
   b. y-intercept $f(0) = \frac{0}{(0)^2-3(0)+2} = \frac{0}{2} = \boxed{0}$, $\boxed{(0,0)}$
   c. vertical asymptotes $x^2-3x+2=0 \Rightarrow (x-2)(x-1)=0 \Rightarrow \boxed{x=2}$ or $\boxed{x=1}$
   d. horizontal asymptotes $f(x) = \frac{x}{x^2}$ as $x \to \infty$, $f(x) \to 0 \Rightarrow \boxed{y=0}$
3. Write a function that has a graph with vertical asymptotes at $x = 3$ and $x = 5$, a horizontal asymptote at $y = 1$, and touches the x-axis at $x = 2$?

Sol. $f(x) = K \cdot \frac{(x-2)^2}{(x-3)(x-5)} = \boxed{\frac{(x-2)^2}{(x-3)(x-5)}}$