Name_____

Show your work and indicate your reasoning. You will not receive credit if you do not clearly show how you are obtaining your answers. Do all work on the exam.

1. (12 points) (12 points) In 2016, the inflation rate in Venezuela was approximately 4% per <u>week</u>, so that each week, prices were increasing at a rate of 4%.

    (a) Suppose an item's inital price is P_0. Write a function for the item's price, P, after n weeks, assuming this rate of inflation. If a dozen eggs cost 450 bolivars on January 1, 2016, how much would it have cost 4 weeks later? (A bolivar is the Venezuelan unit of currency.)

    (b) What is the <u>annual</u> rate of inflation corresponding to this weekly rate? Why is this greater than the rate $52(4) = 208\%$ obtained by multiplying the weekly rate by 52, the number of weeks in a year?

2. (14 points) In 1999, the United Nations Department of Economic and Social Affairs reported that the world's population was 6.0 billion and that there was an annual addition of 78 million people per year to the world's population.

(a) Is this growth linear or exponential? Explain how you are deciding. Using this information, construct a function for the world's population, P, as a function of time, t, with $t = 0$ corresponding to the year 1999.

(b) The same United Nations report stated to the world's population was growing at 1.3% per year. Does this correspond to linear or exponential growth? Explain how you are deciding. Using this information, construct a function for the the world's population, P, as a function of time, t, with $t = 0$ corresponding to the year 1999.

3. (16 points) According to a medical study, men's bone mass decreases exponentially beginning at age 30. A 30-year-old man who weighs 180 pounds has a bone mass of approximately 17 kilograms. At age 33, his bone mass is approximately 16 kilograms.

(a) Write a function for his bone mass, B, in kilograms, as a function of time, t, measured in years past age 30.

(b) Graph the function B on the axes below over the interval $0 \leq t \leq 40$.
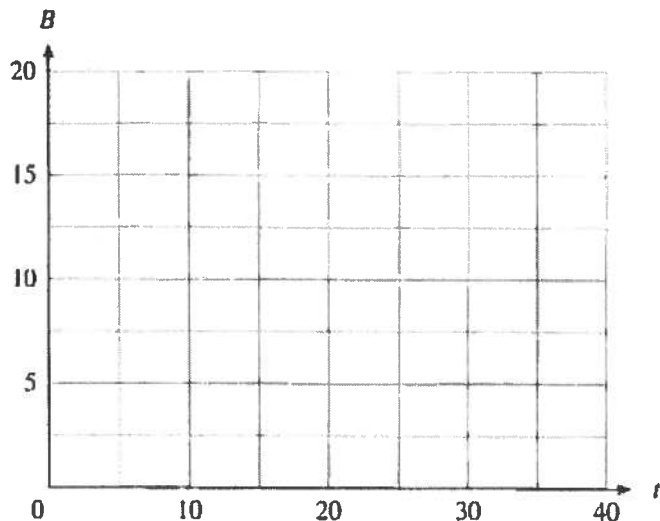
(c) Solve for the age at which the man will have lost half his bone mass.

4. (12 points) Given the function $f(x) = e^x$.

   (a) Find a formula for $g(x)$ where the graph of $g(x)$ is obtained from the graph of $f(x)$ by shifting up 5 units and then reflecting about the x-axis.

   (b) Find a formula for $h(x)$ where the graph of $h(x)$ is obtained from the graph of $f(x)$ by reflecting about the x-axis and then shifting up 5 units.

   (c) Are the functions $g(x)$ and $h(x)$ the same? If not, how are their graphs related?

5. (10 points) Evaluate the following expressions exactly without using a calculator.

   (a) $\left(\dfrac{1}{e^5}\right)^{\ln 2}$
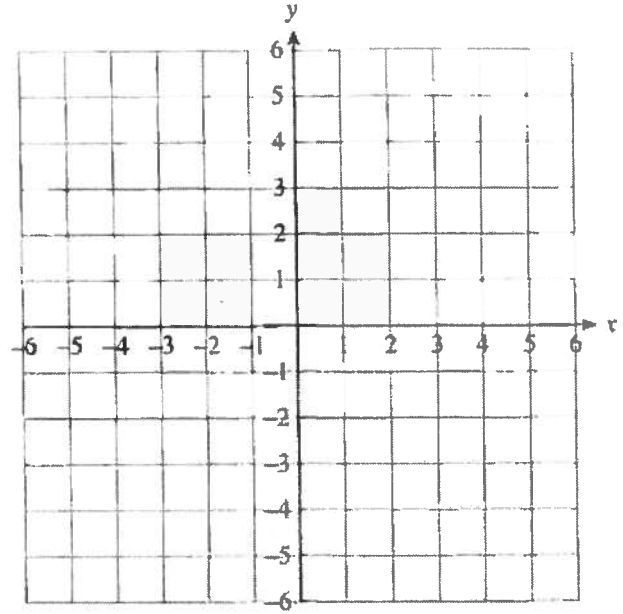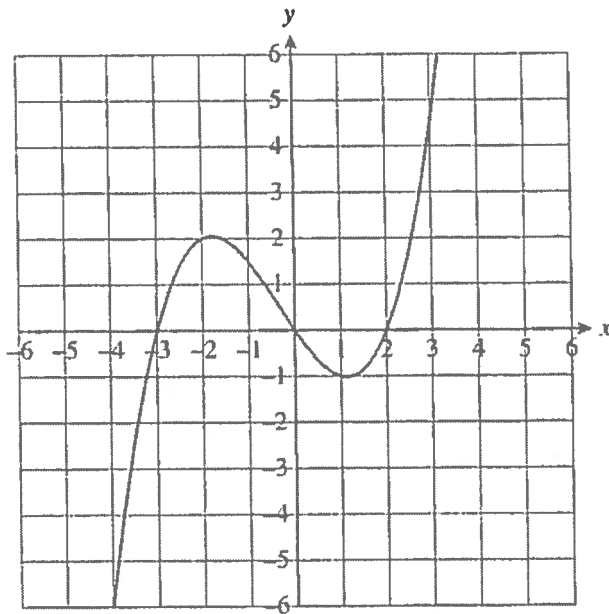
   (b) $100^{\frac{1}{2}\log 5}$

6. (18 points) The Richter magnitude, M, of an earthquake on the Richter scale is related to the energy released by the earthquake, E, measured in ergs, by the formula
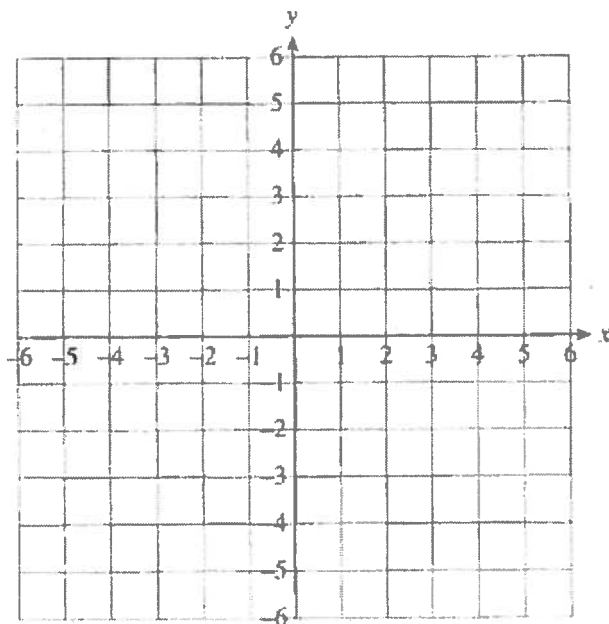
$$M = 0.67 \log E - 11.8.$$

(a) The explosion of 1 ton of dynamite releases 3.4×10^{16} ergs of energy. What would be the magnitude of an earthquake that released an equivalent amount of energy?

(b) The largest earthquake ever recorded struck Chile on May 22, 1960 and had a Richter magnitude of 9.5. What was the energy released by this earthquake?

(c) Let M_1 and M_2 be the Richter magnitudes of two earthquakes whose energy releases are E_1 and E_2, respectively. Using logarithm properties, find a simplified formula for the difference $M_1 - M_2$ in terms of E_1 and E_2.

(d) On March 7 of this year, a magnitude 1.4 earthquake was recorded near Placentia. How many times more powerful, as measured by energy released, was the 9.5 magnitude Chilean earthquake than the March 7 quake?

7. (18 points) The graph of the function $f(x)$ is shown below. Use this graph to sketch the graph of each of the following functions. In each case, state what transformations are applied to obtain the graph from the graph of the original function.

(a) $g(x) = 2f(x-3)$

(b) $h(x) = -f(\frac{1}{2}x)$

(c) $k(x) = f(-x) + 2$