

## Statistics

A set of tools for understanding data and making decisions/conclusions/predictions under uncertainty

## Randomness

- in the short term, we don't know what will happen (flipping a coin)
- in the long term, we know the **distribution** of possibilities (what outcomes are possible and how often they occur)
  - the crux of randomness
  - as the amount of times we get a random variable approaches infinity, the clearer/less random the variable becomes

## Two definitions of probability

- **proportion** of times an outcome occurs or would occur over infinitely many repetitions of a random action
  - Frequentist
  - math is a lot nicer
  - there is a fixed outcome but we don't know it
- a number quantifying our **belief** that an outcome can/will occur
  - Bayesian
  - 2000 times more intuitive and math is just as hard
  - random outcome (not fixed)

Both are calculus based

## Probability Model

Consists of two parts:

- sample space: list (set/list of all unique values) of all possibilities and must be well defined.
- probability of each outcome

### **This in essence is a hash table**

An **event** is an arbitrary set of 0 or more outcomes in a sample space

## Axioms of Probability

- axioms : something is so obvious it does not need to be proven
- For events A and B in the same sample space denoted as “S”:
  - The probability of event A, denoted as  $P(A)$  is a number between 0 and 1 (inclusive).
    - \*  $[0, 1]$  notation as well.
    - \* **NOTE:**  $P(A) = 0$  means A is “impossible” and  $P(A) = 1$  means A is guaranteed
  - $P(S) = 1$ 
    - \* **Some outcome is bound to happen**
  - If A and B are disjoint (there are no common outcomes. A is not in B AND B is not in A), then  $P(A \text{ or } B) = P(A) + P(B)$

## Simple rules that follows from the Axioms

- Compliment Rule: Define  $A^C = A$  compliment, that is  $A^C$  is the event “A does not occur”
  - $P(A^C) = 1 - P(A)$
  - The summation of all events that **do not** occur minus the overall probability (100%)
- General addition rule: Suppose events A and B have at least one common outcome
  - Define  $A \cap B$  to be the set of outcomes common to A & B
  - Define  $A \cup B$  to be the set of outcomes in A, or in B or in both A & B
    - \* Then  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
    - \* This is the same as this:
      - a = [1, 2, 3]
      - b = [2, 4, 5]
      - c = a + b
      - # c = [1, 2, 2, 3, 4, 5]
      - c = set(a+b)
      - # c = {1, 2, 3, 4, 5}

## Example

Random phenomenon: Draw 1 tile from a standard Scrabble bag of 100 tiles

- Sample space 1 (option one):
  - $S$  = the 100 tiles in the bag
  - All tiles are equally likely to be drawn
  - $P(\text{draw particular tile}) = 1/100$  or 0.01 for all
- Sample space 2 (option two):
  - The 27 “letters” (26 letters and 1 blank)
- Let event  $C$  = “draw a letter in CAT”
- Let event  $D$  = “draw a letter in PET”

$$P(C) = P(C) + P(A) + P(T) = .02 + .09 + .06 = 0.17$$

$$P(D) = P(P) + P(E) + P(T) = 0.2 + 0.12 + 0.06 = 0.2$$

$$P(C^C) = P(\text{do not draw any of the letters in CAT}) = 1 - P(C) = 1 - .17 = .83$$

$$P(C \cap D) \Rightarrow$$

- =  $P(\text{letter in both CAT \& PET})$
- =  $P(T) = 0.06$

$$P(C \cup D) = P(\text{letter in CAT or PET or both words}) \Rightarrow$$

- =  $P(C) + P(D) - P(C \cap D)$
- =  $0.17 + 0.2 - 0.06 = 0.31$

## Python Code Representation

```
#!/usr/bin/env python3.5

# probability can be calculated by using a hash table in conjunction with a set
# hash tables are used when there are two different letters with the same probability
# using a bare list would result in incorrect calculations of probability
# they would be treated as non unique instances
# in turn allowing for it to filter out needed objects
# this boils down to a set of unique hash tables and summing up their values

class hashabledict(dict):
    def __hash__(self):
        return hash(tuple(sorted(self.items())))
value_mapping = {
    "c": 0.02,
    "a": 0.09,
    "t": 0.06,
    "p": 0.02,
    "e": 0.12
}

def get_probability(*args):
    # s has extra new line for code to fit
    s = set((hashabledict({letter: value_mapping[letter]})
              for argument in args for letter in argument))
    return sum([sum(dictionary.values()) for dictionary in s])

print(get_probability("cat", "pet"))
# this some times yields 0.3100000000000005 and 0.3099999999999994
# which is essentially the same number
```

## vim regex

```
# this replaces all caps in proper latex
:%s/cap/\$\$\\cap\$/g
```