Name: SOLUTIONS

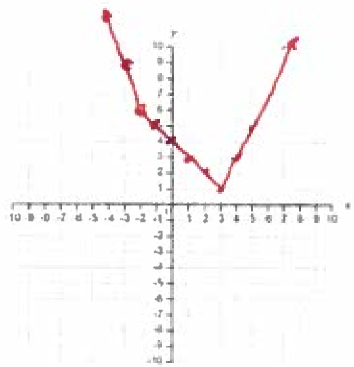
Chapter 2 – Section 2.3 Piecewise-Defined Functions

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. Create the graph for the function: $f(x)=\begin{cases}-3x & x<-2\\ 4-x & -2\le x\le 3\\ 2x-5 & x>3\end{cases}$

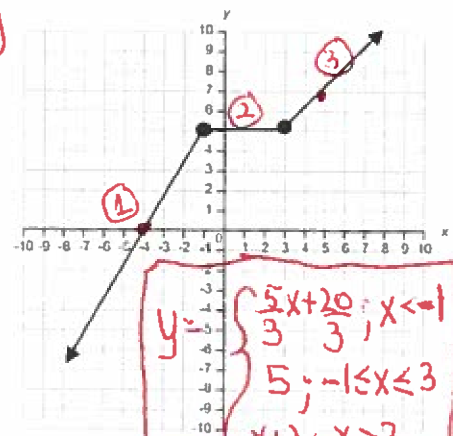


x	y
-4	12
-3	9
-2	6

x	y
-2	6
-1	5
0	4
1	3
2	2
3	1

x	y
3	1
4	3
5	5

2. Construct the piecewise linear function for the graph



Sol: $(-4,0)\ \&\ (-1,5)$

$m=\frac{5-0}{-1-(-4)}=\frac{5}{3}$

$y-0=\frac{5}{3}(x+4)$

$\boxed{y=\frac{5}{3}x+\frac{20}{3}}$ ①

$(-1,5)\ \&\ (3,5)$

$\boxed{y=5}$ ②

$(3,5)\ \&\ (5,7)$

$m=\frac{7-5}{5-3}=\frac{2}{2}=1$

$y-5=1(x-3)$

$y-5=x-3$

$+5 \qquad +5$

$\boxed{y=x+2}$ ③

$$\boxed{y=\begin{cases}\frac{5}{3}x+\frac{20}{3}; & x<-1\\ 5; & -1\le x\le 3\\ x+2; & x>3\end{cases}}$$

3. A long-distance calling plan charges 99 cents for any call up to 20 minutes in length and 7 cents for each additional minute.
   a. Use bracket notation to write a formula for the cost, C, of a call as a function of its length t in minutes.

Sol: $C=\begin{cases}0.99 & ;\ 0<x\le 20\\ 0.99+0.07(x-20) & ;\ x>20\end{cases}$

   b. State the domain and range of the function.

Sol: Domain: $(0,\infty)$

Range: $(0,\infty)$

4. Use the definition of absolute value to write a piecewise formula for $f(x)=|2x^2-8|$

Sol. $f(x)=|x|=\begin{cases}x; & \text{if } x\ge 0\\ -x; & \text{if } x<0\end{cases} \Rightarrow f(x)=\begin{cases}2x^2-8; & 2x^2-8\ge 0\\ -(2x^2-8); & 2x^2-8<0\end{cases}$

OR $f(x)=\begin{cases}2x^2-8; & x\le -2\\ 2x^2-8; & x\ge 2\\ 8-2x^2; & -2\le x\le 2\end{cases}$

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TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Given y = f(x) describe in words the transformation when k is a positive constant:

- y = f(x + k) HORIZontal shift to left by "K" units.
- y = f(x - k) HORIZontal shift to right by "K" units.
- y = f(x) + k VERTICAL shift upward by "K" units.
- y = f(x) - k VERTICAL Shift downward by "K" units.

Check your understanding:

1. Given the function $g(x) = \sqrt{4 - x^2}$. Write the function h(x) that will translate the graph of g 5 units to the right and 3 units up.

Sol. $\sqrt{4-(x-5)^2} + 3$

2. The graph of $f(x)$ contains the point (–4, –2). What is the corresponding point on the graph of $f(x+5)$?

Sol. $(-9, -2)$

3. If $f(x) = 2x^2 + 3x$ and $g(x) = f(x-4)$, what is $g(0)$? Sol $g(0) = f(0-4) = f(-4)$ $= 2(-4)^2 + 3(-4) = \boxed{20}$

4. The population P(t) gives the number of people in a certain population in year t. Interpret the following in terms of population.

   a. P(t) + 100 Sol. P(t)+100 represents the increase in population by 100 people @ given year of t.

   b. P(t+100) represents the population 100 years earlier than t.

5. The carpenter currently builds k chairs per week. What do the following expressions represent?

   f(k) represents the cost of building k chairs

   a. f(k+10): represents the cost of additional 10 chairs build per week.

   b. f(k) +10 represent an increase in cost by 10 dollars of building K chairs per week.

   c. f(2k) represent the cost of building twice the amount of original chairs per week.

   d. 2f(x) represent the cost which doubles for building "X" chairs.

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Chapter 2 – Section 2.5 Preview of Composite and Inverse Functions

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. The cost C in thousands of dollars of producing q kg of chemical is given by C = f(q) = 100 +0.2q find and interpret.
   a. f(10) $= 100+0.2(10) = 100+2 = \$102$ thousands

   Sol: The cost of producing 10 kg of chemicals equals \$102,000.

   b. $f^{-1}(200)$

   Sol: $200 = 100 + 0.2q \Rightarrow \frac{100}{0.2} = \frac{0.2q}{0.2} \Rightarrow q = 500$ (−100 −100) The cost of \$200,000 results in production of 500 kg.

2. Let $C(f(t))$ be a composite function, where $V = f(t)$ is the volume of punch, in liters, consumed at a party after t minutes, and $C(V)$ is the cost, in dollars, of V liters of punch. What are the units of $C(f(t))$?

Sol: Units of C(f(t)) represents the cost in dollars.

3. The gross domestic product (GDP) of the US is given by G(t) where t is the number of years since 1990 and the units are of G are billions of dollars.
   a. What is it meant by G(15) = 12,637.4?

   Sol: The GDP of U.S in 2005 was 12,637.4 billions of dollars.

   b. What is it meant by $G^{-1}(14{,}441.4) = 18$?

   Sol: The GDP of 14,441.4 billion of dollars resulted in the year of 1990+18 = 2008.

4. If $y = f(x) = x^5 + 8$, what is the inverse function $f^{-1}(y)$?

Sol: $y = x^5 + 8$; solve for x

$y - 8 = x^5 \Rightarrow \boxed{\sqrt[5]{y-8} = x = f^{-1}(y)}$

5. Given $f(x) = 4x^2 - 2$ and $g(x) = -3x + 1$, find $f(g(9))$ and $g(f(9))$.

Sol: $f(g(9)) = f(-3(9)+1) = f(-27+1) = f(-26) = 4(-26)^2 - 2 = \boxed{2702}$

$g(f(9)) = g(4(9)^2 - 2) = g(322) = -3(322)+1 = \boxed{-965}$

6. Let f and g be two invertible functions such that $f^{-1}(x) = (x-3)^3$ and $g(x) = 3x + 2$. Find $f(g^{-1}(4))$. Round your answer to two decimal places, if necessary.

Sol: $4 = 3x + 2$ (−2 −2)

$\frac{2}{3} = \frac{3x}{3} \Rightarrow x = \frac{2}{3}$

$f^{-1}(x) = (x-3)^3$

$\frac{2}{3} = (x-3)^3$

$\sqrt[3]{\frac{2}{3}} + 3 = x$

$\boxed{x = 3.87}$

Section 2.5 Preview of Composite and Inverse Functions

2.5

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TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. A model rocket is launched from the roof of a building. For height h, in meters, and time t, in seconds, after the rocket is launched, the height of the rocket above the ground is given by $h = f(t) = -4.9t^2 + 40t + 16$. Is the graph of $f(t)$ concave up or concave down?

Sol: Since the initial coefficient of $-4.9<0$, it opens down. Thus, the graph of $f(t)$ is concave down.

2. Calculate successive rates of change for the function $g(t)$ shown in the following table to determine if the graph is more likely concave up or concave down for $0 \le t \le 3$.

t	0	1	2	3
$g(t)$	0	5	9	12

Sol: a.v.r.c $_{[0,1]} = \frac{5-0}{1-0} = \boxed{5}$ a.v.r.c $_{[1,2]} = \frac{9-5}{2-1} = \boxed{4}$ a.v.r.c $_{[2,3]} = \frac{12-9}{3-2} = \boxed{3}$

Since the a.v.r.c over successive interval are decreasing $\boxed{\text{CONCAVE DOWN}}$

3. Does the graph of $y = -4x^2 - 4x$ appear to be concave up, concave down, or neither?

Sol: Since the initial coefficient of $-4<0$, it opens down. Thus, the graph of $y=-4x^2-4x$ appears to be $\boxed{\text{CONCAVE DOWN}}$

4. Determine the concavity of the graph of $f(x) = x^2 - x - 9$ between $x = -1$ and $x = 2$ by calculating average rates of change over intervals of length 1. Is the graph concave up or concave down?

$\boxed{\text{CONCAVE UP}}$

Sol: a.v.r.c $_{[-1,0]} = \frac{-9-(-7)}{0-(-1)} = \frac{-2}{1} = \boxed{-2}$ a.v.r.c $_{[0,1]} = \frac{-9+9}{1-0} = \boxed{0}$ a.v.r.c $_{[1,2]} = \frac{-7-(-9)}{2-1} \boxed{=2}$

5. When a rumor begins, the number of people who have heard the rumor increases slowly at first. As the rumor spreads, the rate of change increases (as more people continue to tell their friends the rumor), and then slows down again (once almost everyone has heard the rumor). Sketch a graph to represent this situation, what can you say about the concavity?

Sol