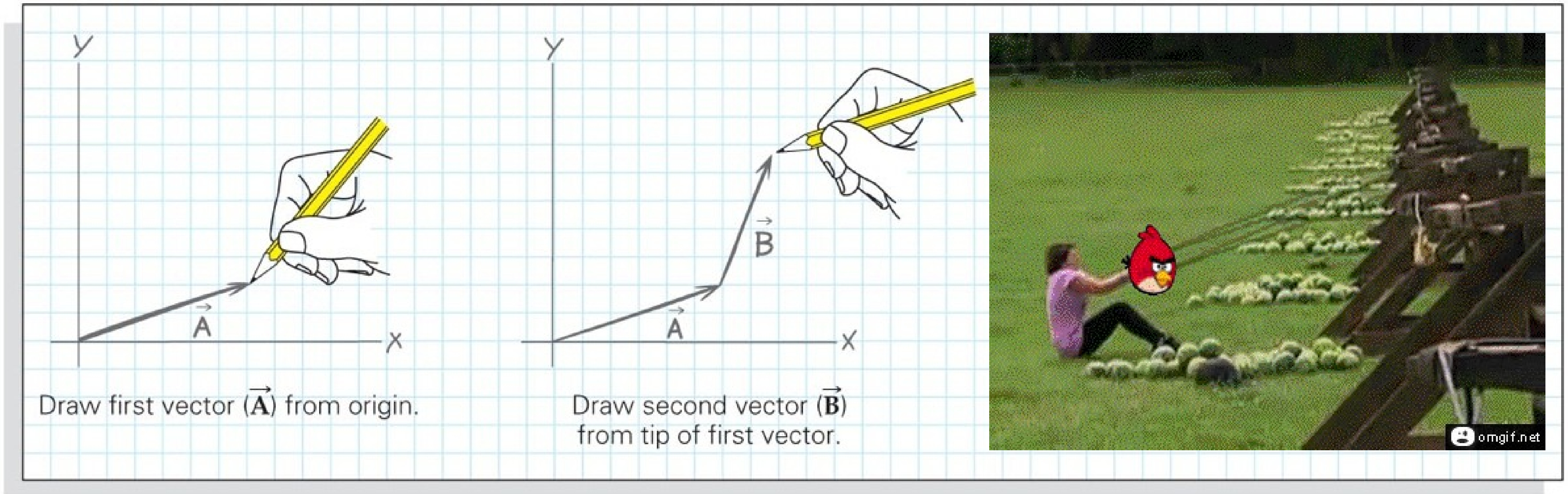
Physics 225

Section 2,
Spring Fall 2018
Lecture 4

Today: Vectors

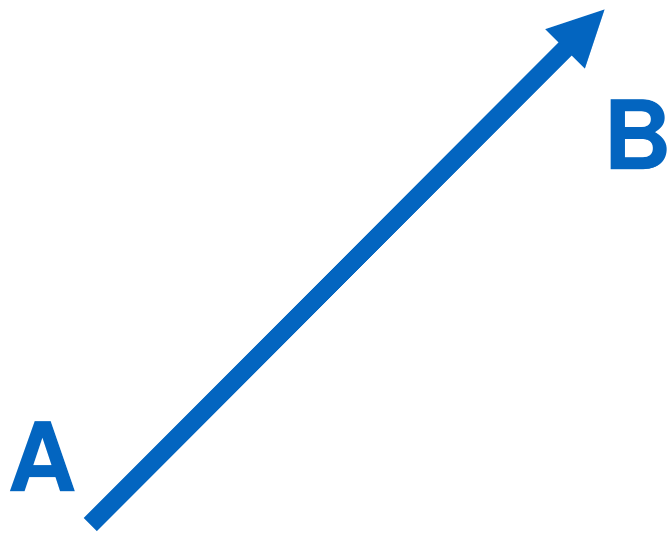
$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$



- **Take Home Message**
  - Vectors have a magnitude **and** direction
  - It "carries" a point from A to B
  - Vectors require special care during math operations

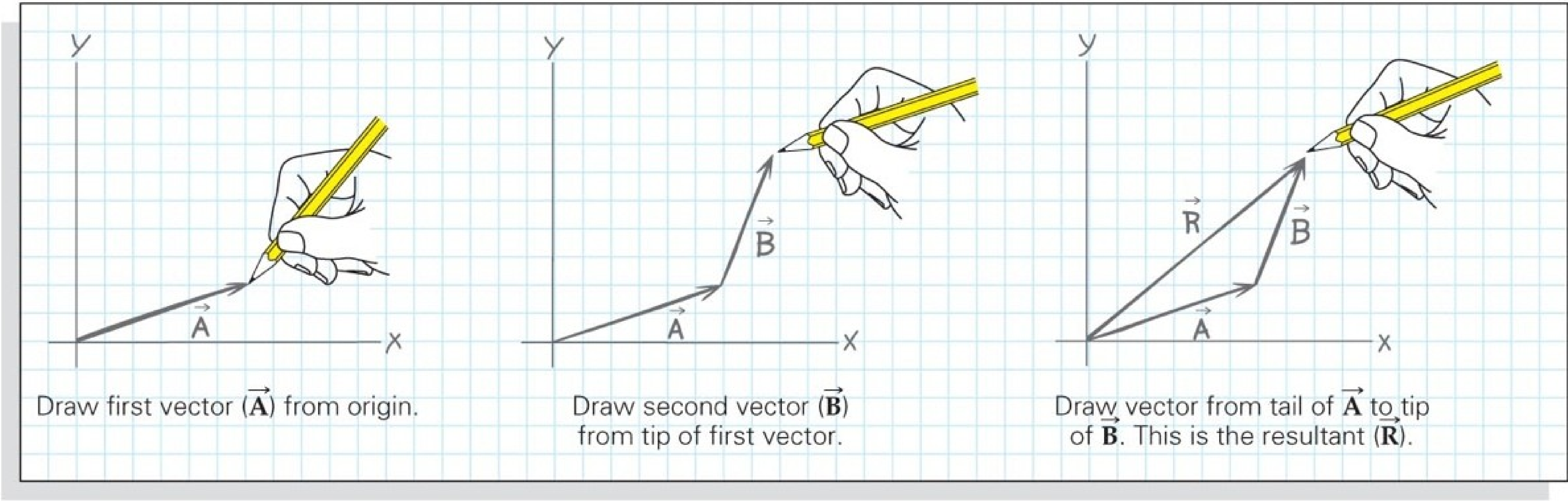
Vectors

- Magnitude + direction
- Represent as arrows
- New kind of object
- Operations
  - Addition +, subtraction –
  - Scaling
  - Later: multiplying two vectors (•, x)

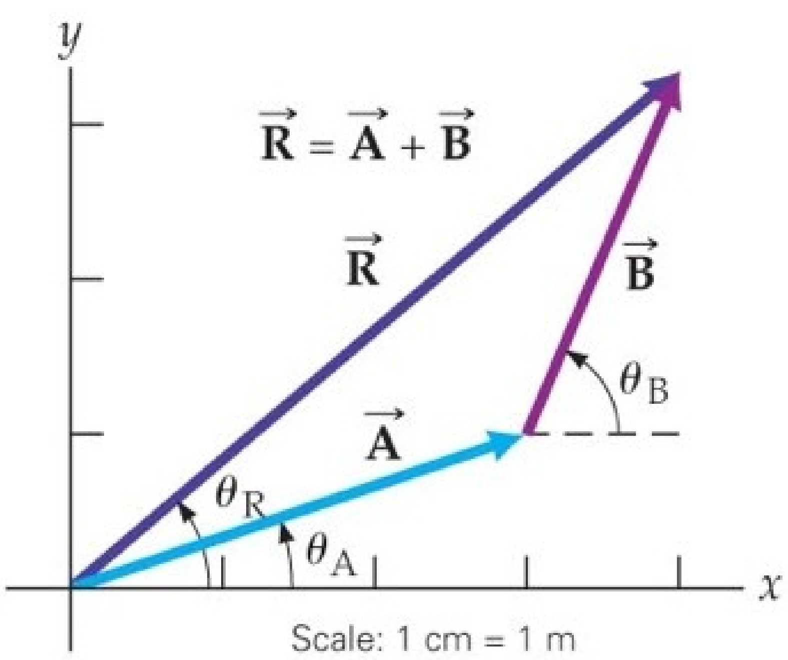


Vector addition

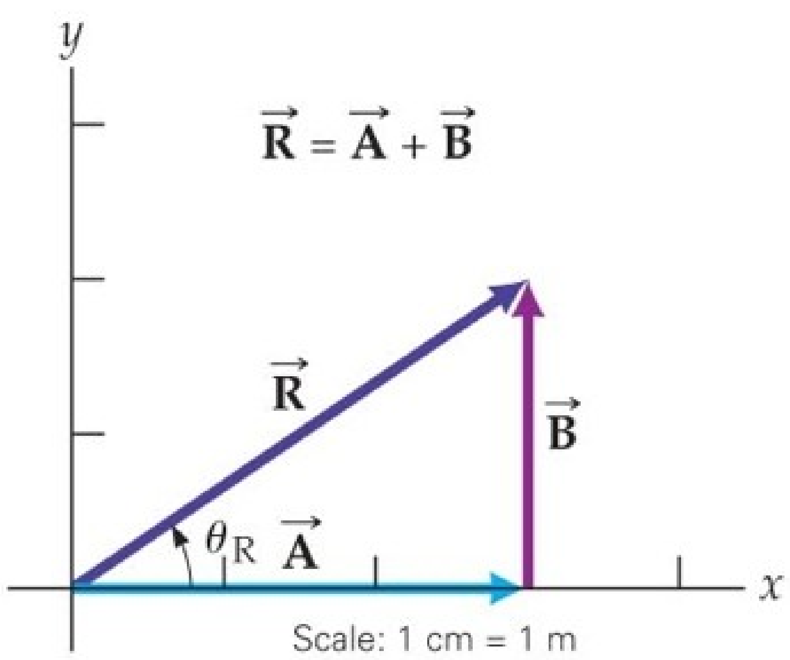
$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$



(a)

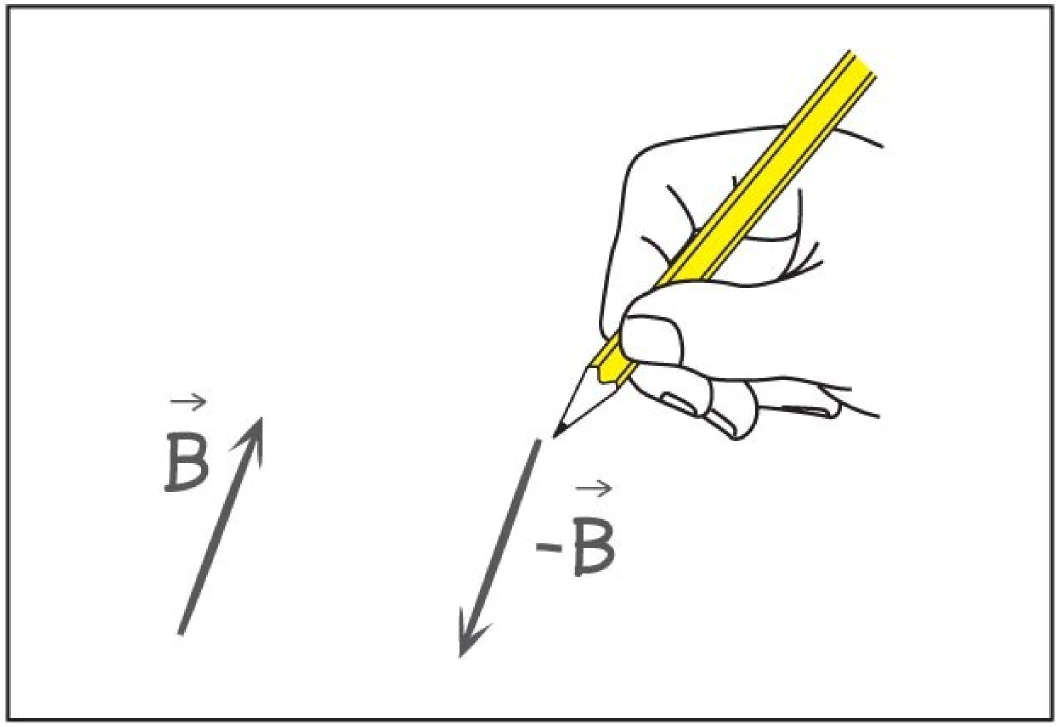


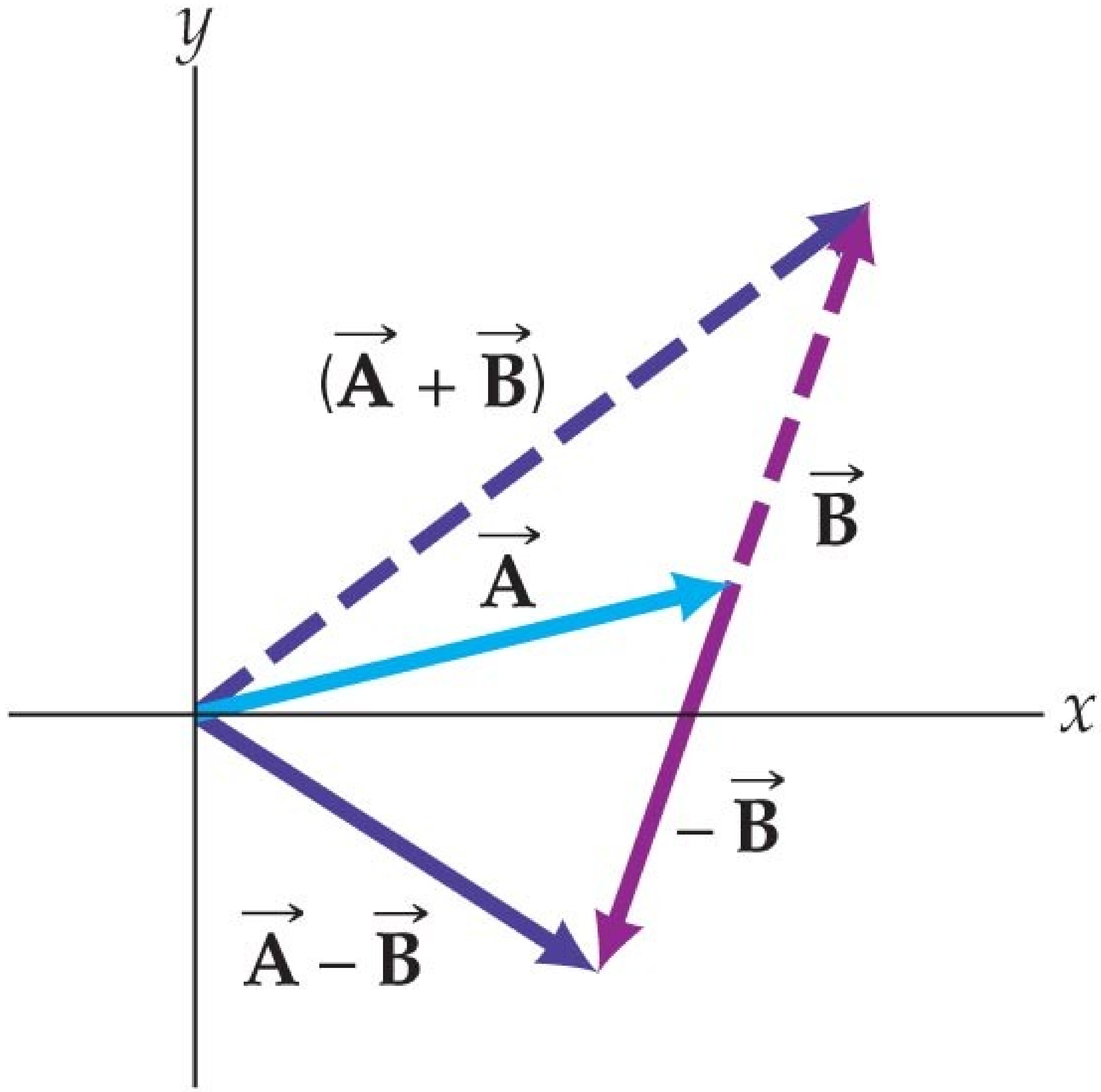
(b)



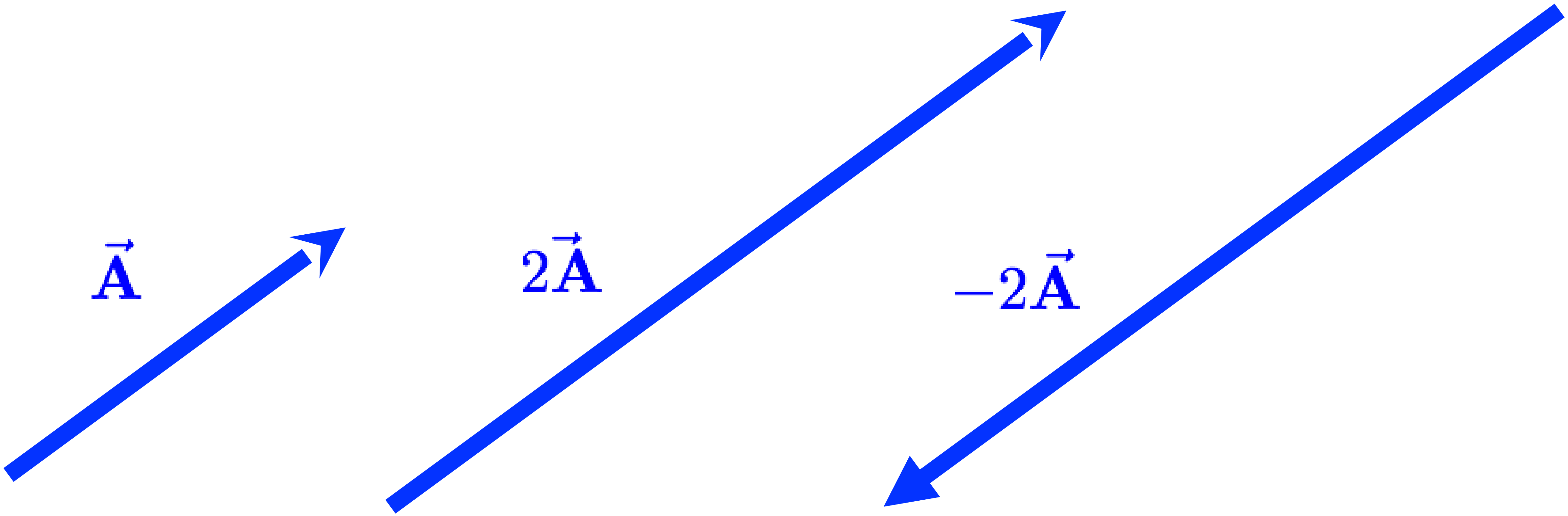
(c)

Vector subtraction





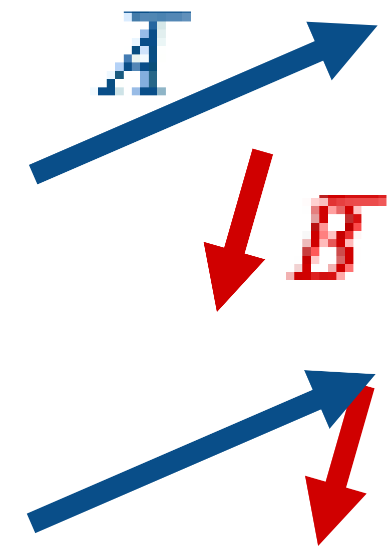
Vector scaling

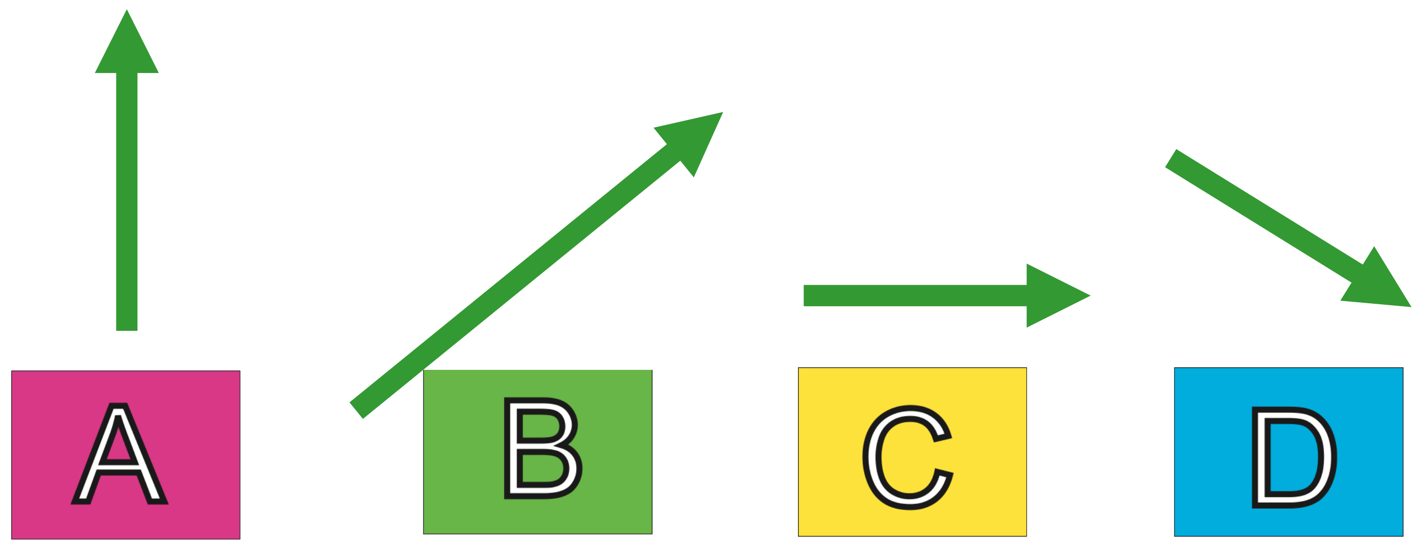


Scaling a vector by a scalar changes the length

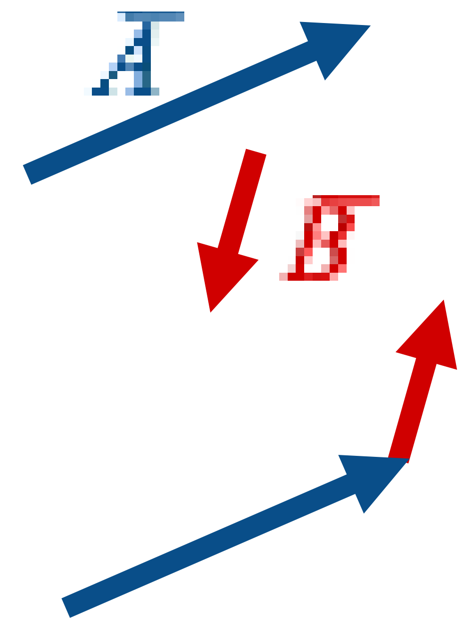
The direction reverses also, if the scalar is negative

Vectors $\vec{A}$ and $\vec{B}$ are shown to the right.
Which of the following best describes $\boldsymbol{A} + \boldsymbol{B}$?

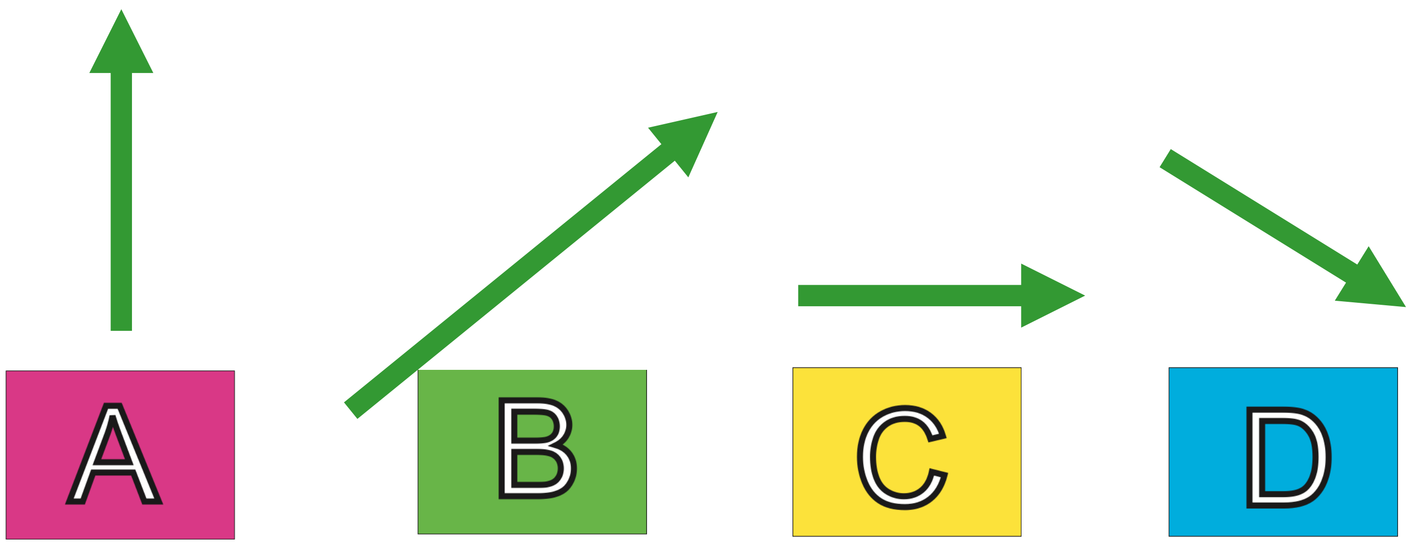




Clicker Question 1b



Vectors $\vec{A}$ and $\vec{B}$ are shown to the right.
Which of the following best describes ***A*** - ***B***?



Clicker question #2

Question 3.1a Vectors I

If two vectors are given such that A + B = 0, what can you say about the magnitude and direction of vectors A and B?

same magnitude, but can be in any direction

A same magnitude, but must be in the same direction

B different magnitudes, but must be in the same direction

C same magnitude, but must be in opposite directions

D different magnitudes, but must be in opposite directions

Clicker question #3

Question 3.1b Vectors II

Given that **A** + **B** = **C**, and that $|\mathbf{A}|^2 + |\mathbf{B}|^2 = |\mathbf{C}|^2$, how are vectors **A** and **B** oriented with respect to each other?

- A) they are perpendicular to each other
- B) they are parallel and in the same direction
- C) they are parallel but in the opposite direction
- D) they are at 45° to each other
- they can be at any angle to each other

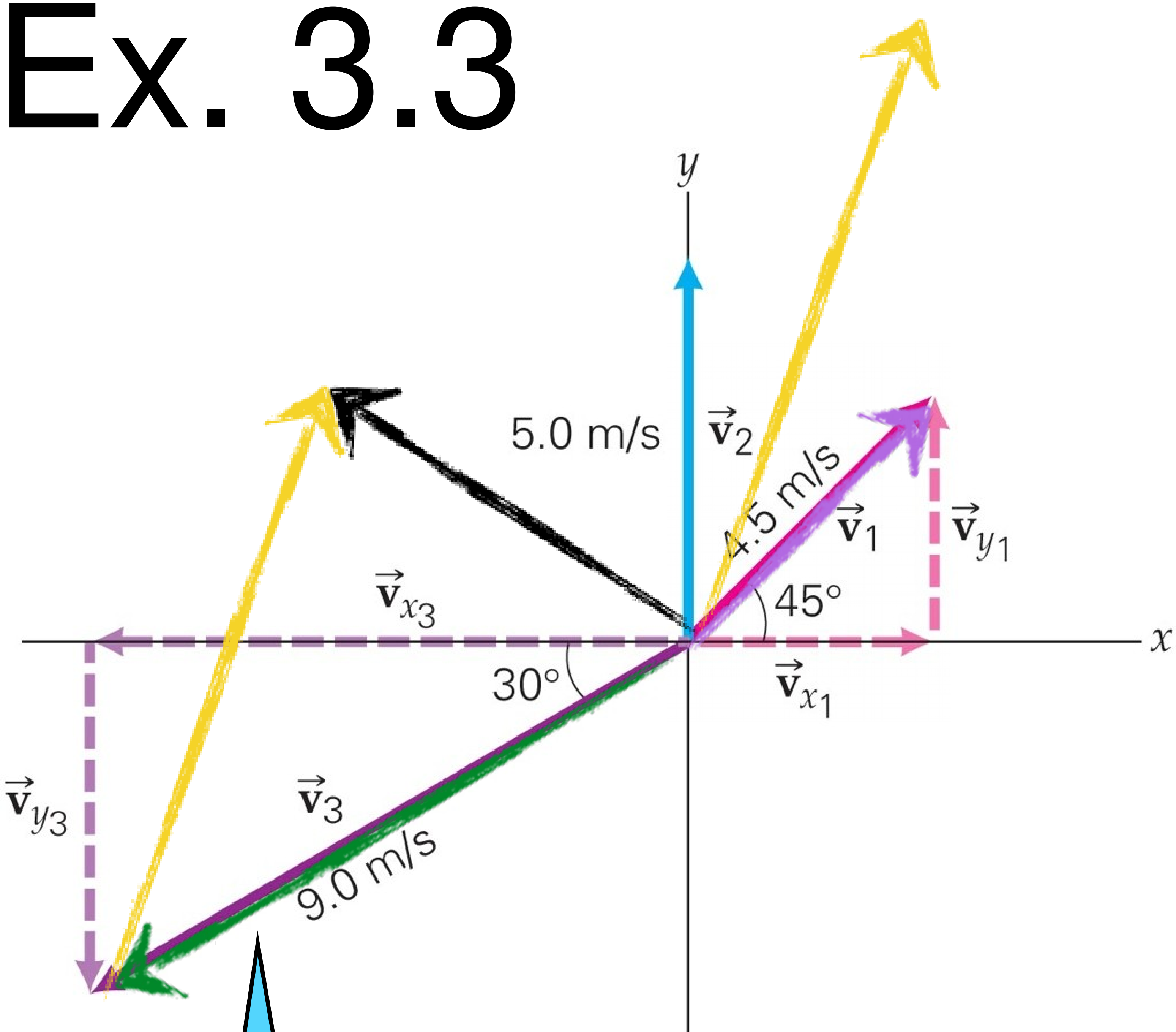
Clicker question #4

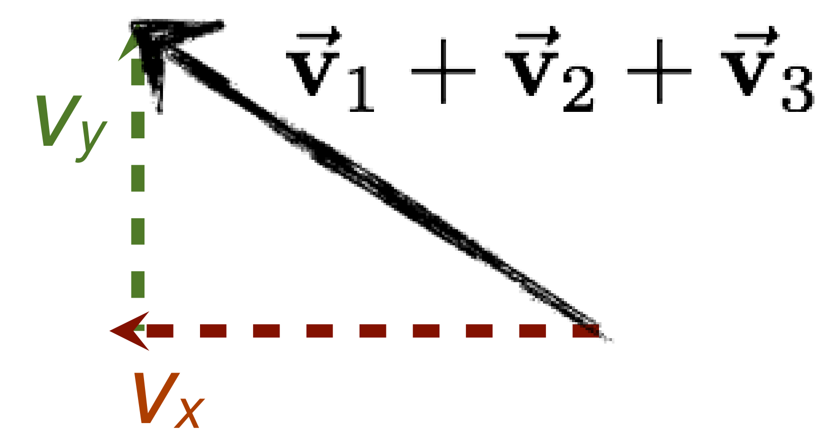
Question 3.3 Vector Addition

You are adding vectors of length 20 and 40 units. What is the only possible resultant magnitude that you can obtain out of the following choices?

- A **0**
- B **18**
- C **37**
- D **64**
- **100**

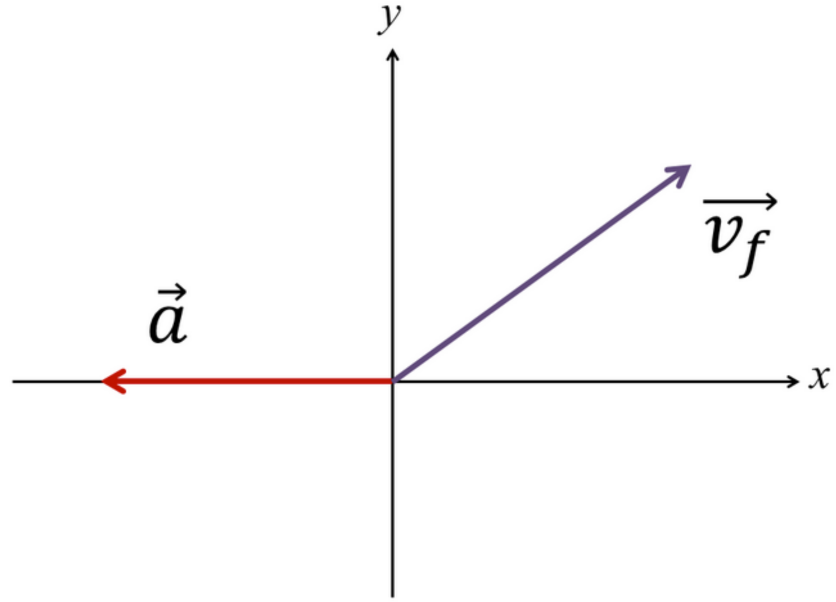
Ex. 3.3



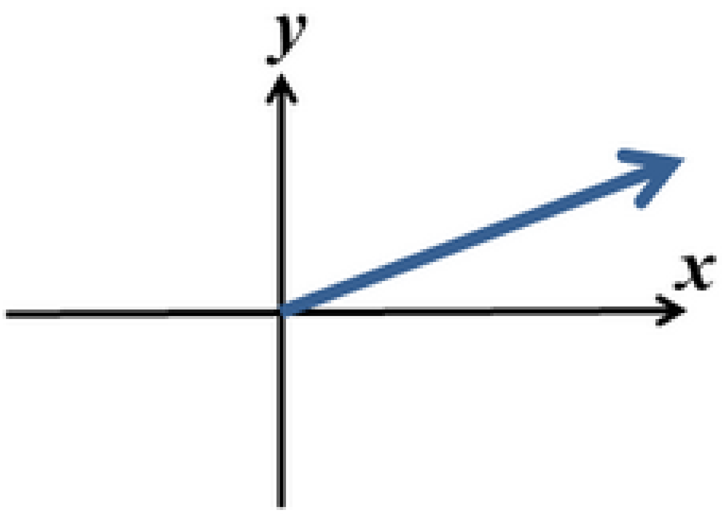


Graphically add the yellow and green vectors, compare with a partner next to you

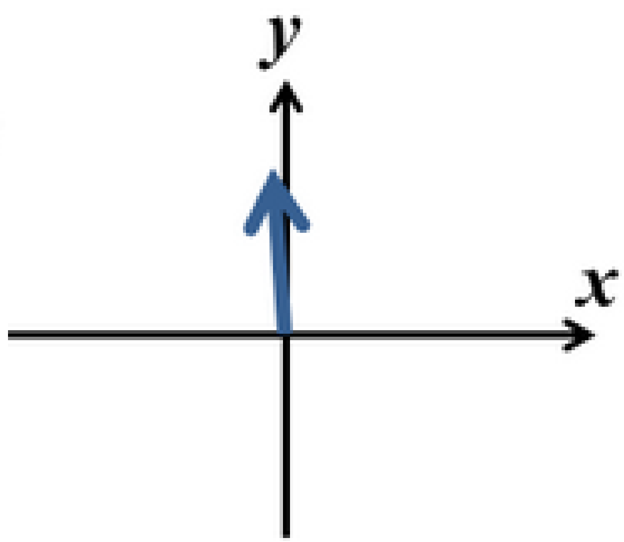
1. An object experiences a constant acceleration and attains a velocity as shown in the figure (right). Which of the following vectors best corresponds to the initial velocity of the object?



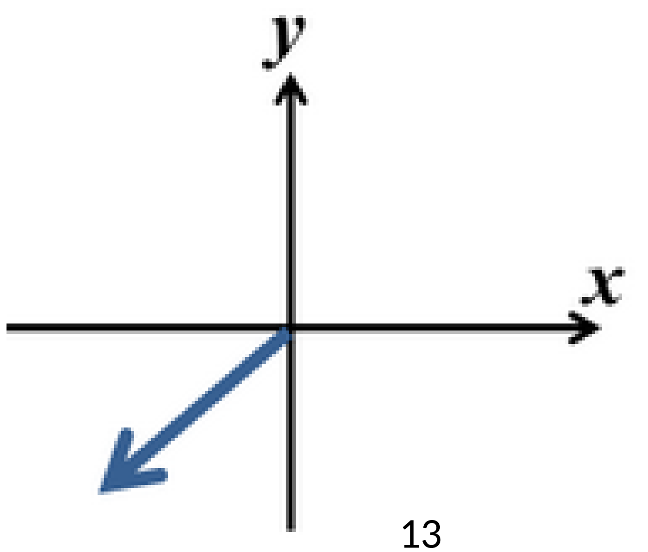
A



B

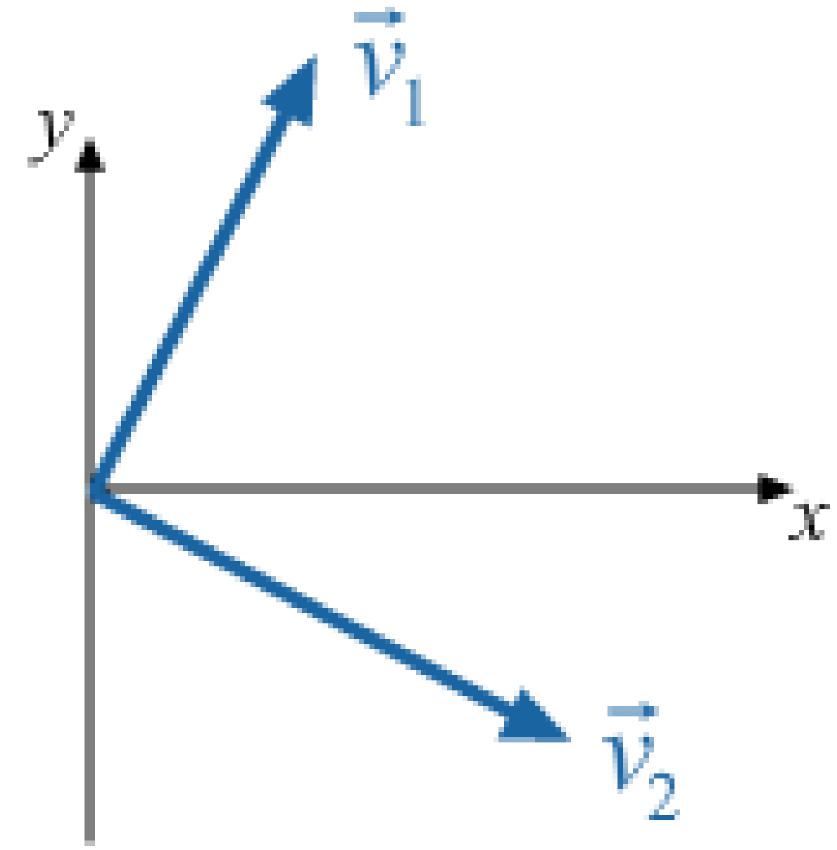


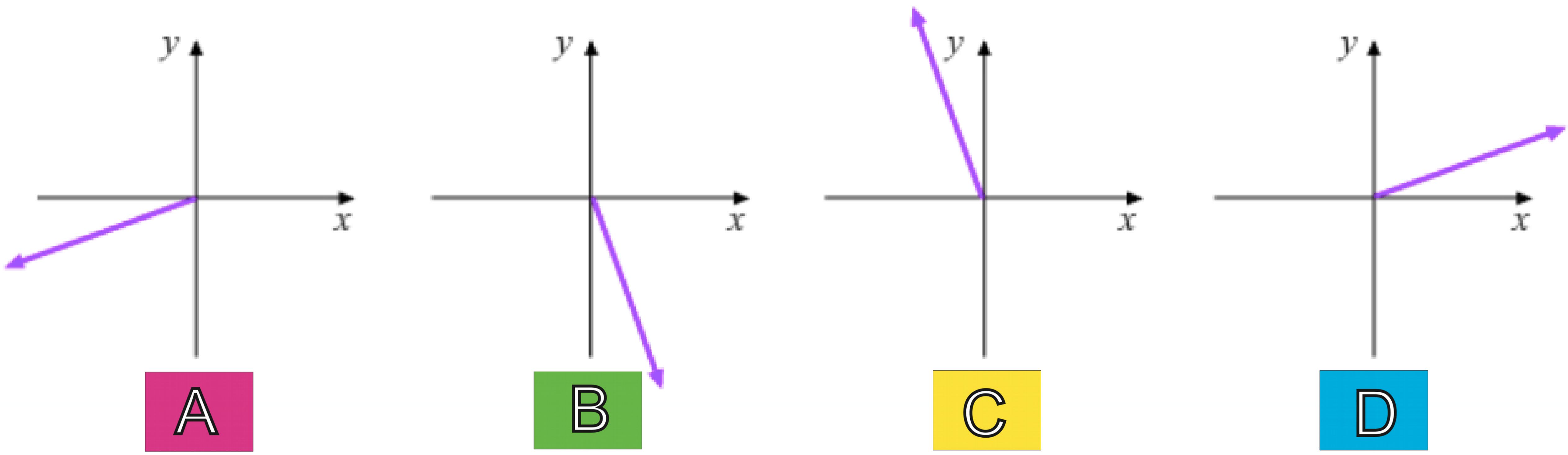
C



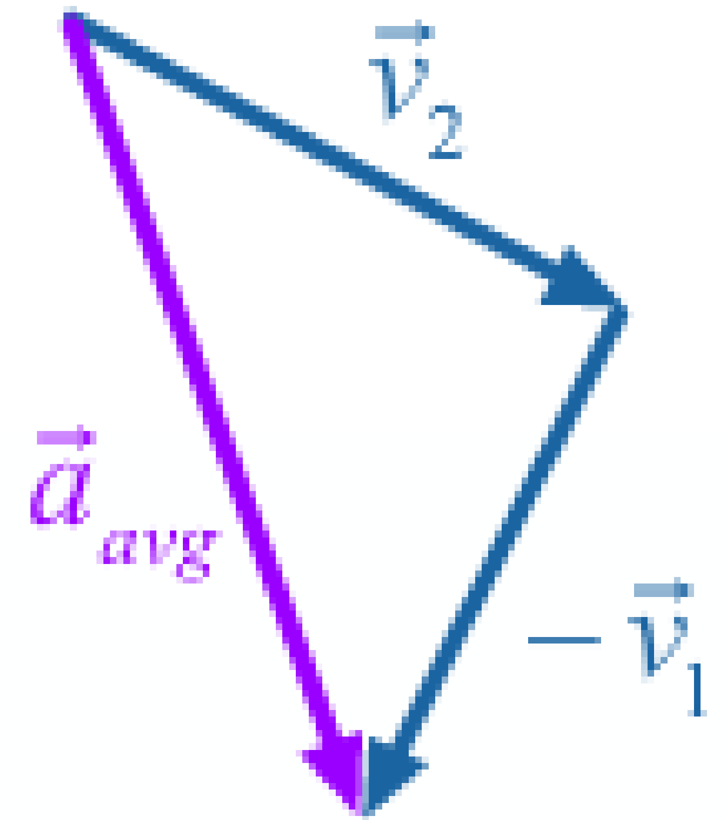
Clicker 6

1. An object has velocity v_1 at time t=0 and v_2 at time t=1 s. Which of the following are the average acceleration?

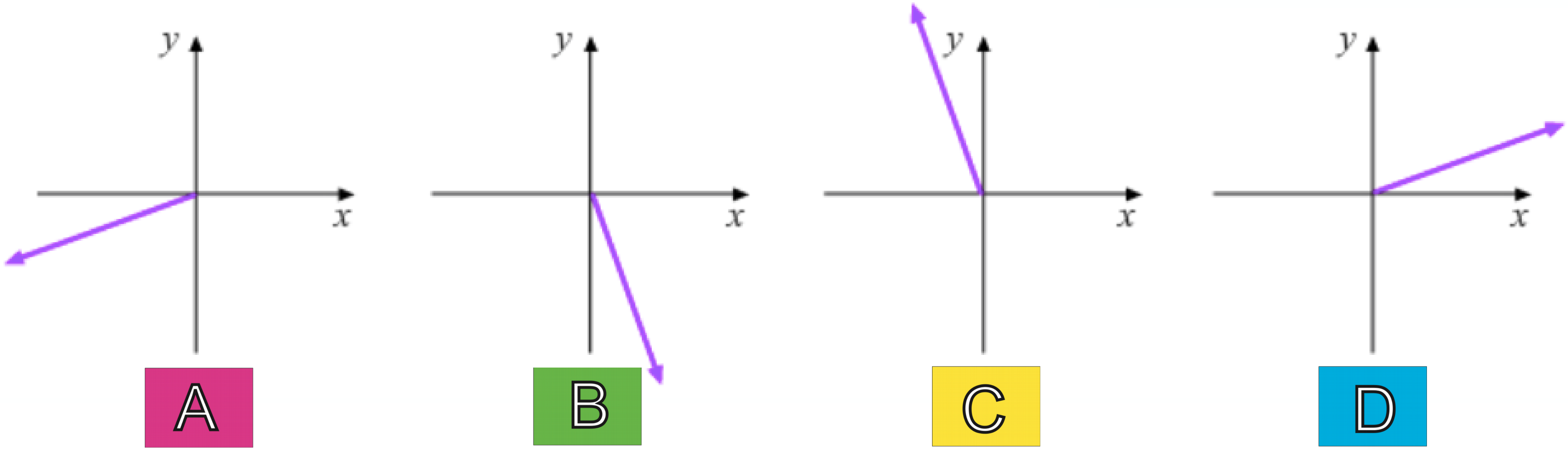




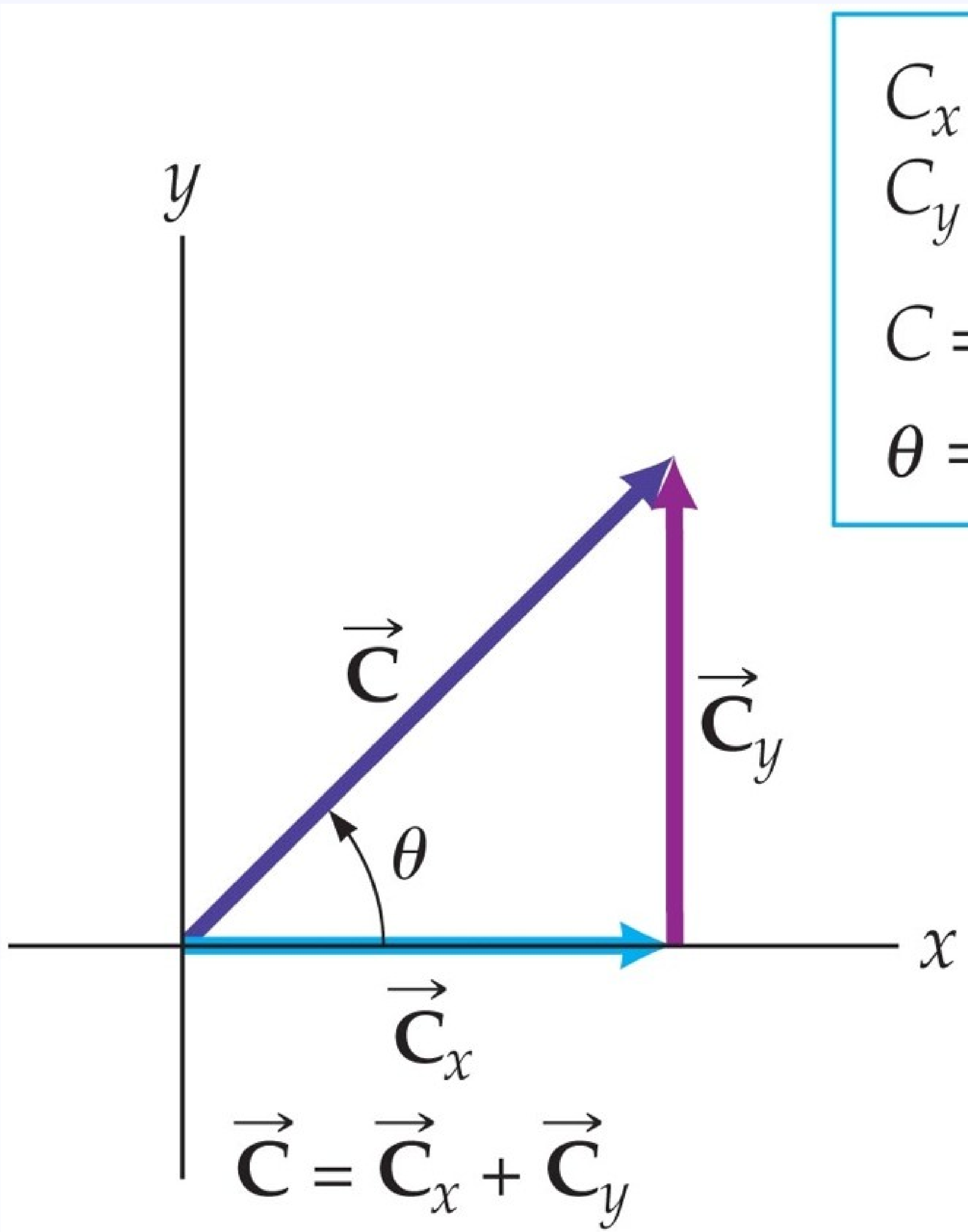
Clicker 6



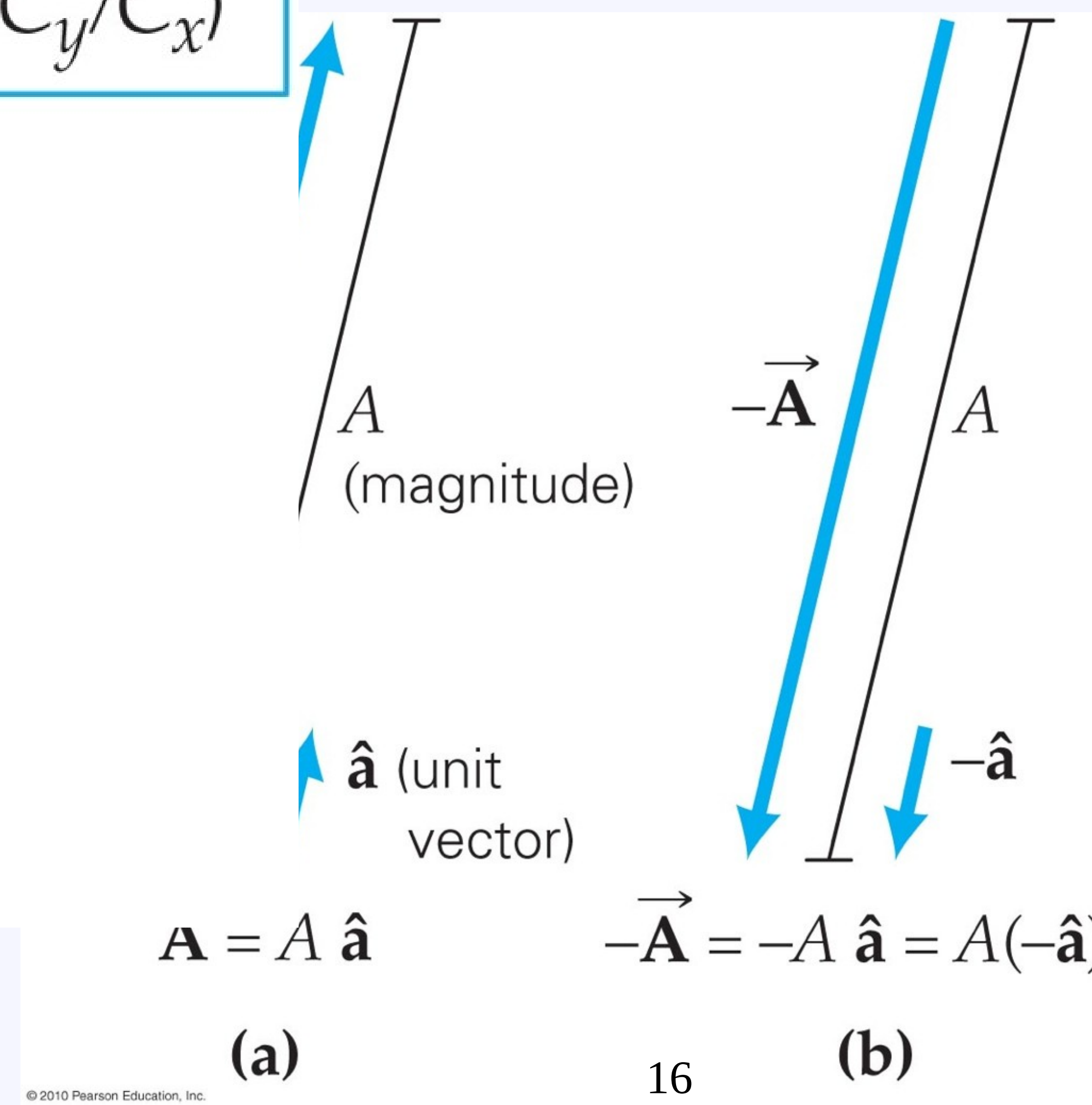
1. An object has velocity v_1 at time t=0 and v_2 at time t=1 s. Which of the following are the average acceleration?



Vector components



$$C_x = C\cos\theta$$
$$C_y = C\sin\theta$$
$$C = \sqrt{C_x^{\,2} + C_y^{\,2}}$$
$$\theta = \tan^{-1}(C_y/C_x)$$



Clicker question #7

Question 3.2a **Vector Components I**

If each component of a vector is doubled, what happens to the angle of that vector?

A **it doubles**

B **it increases, but by less than double**

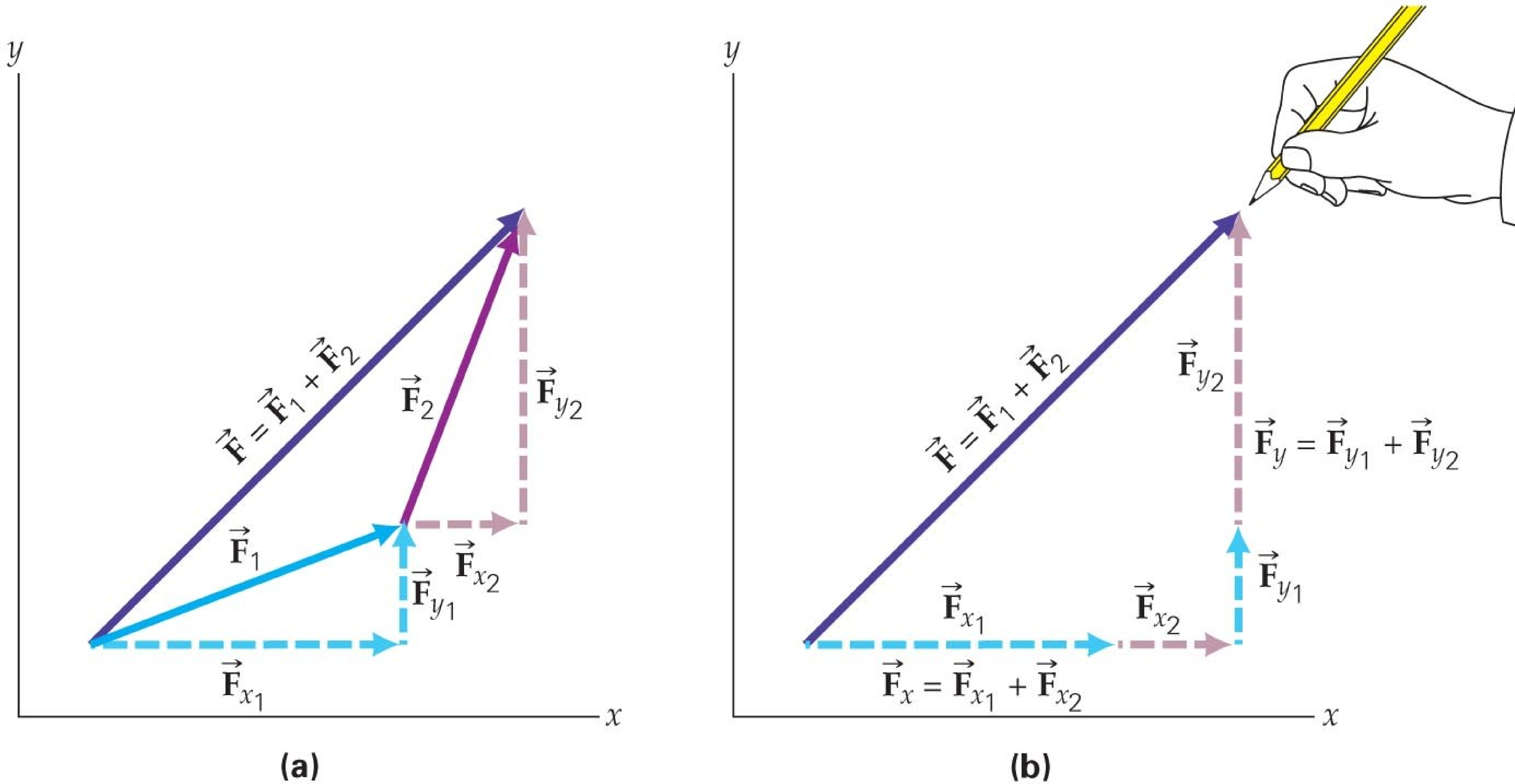
C **it does not change**

D **it is reduced by half**

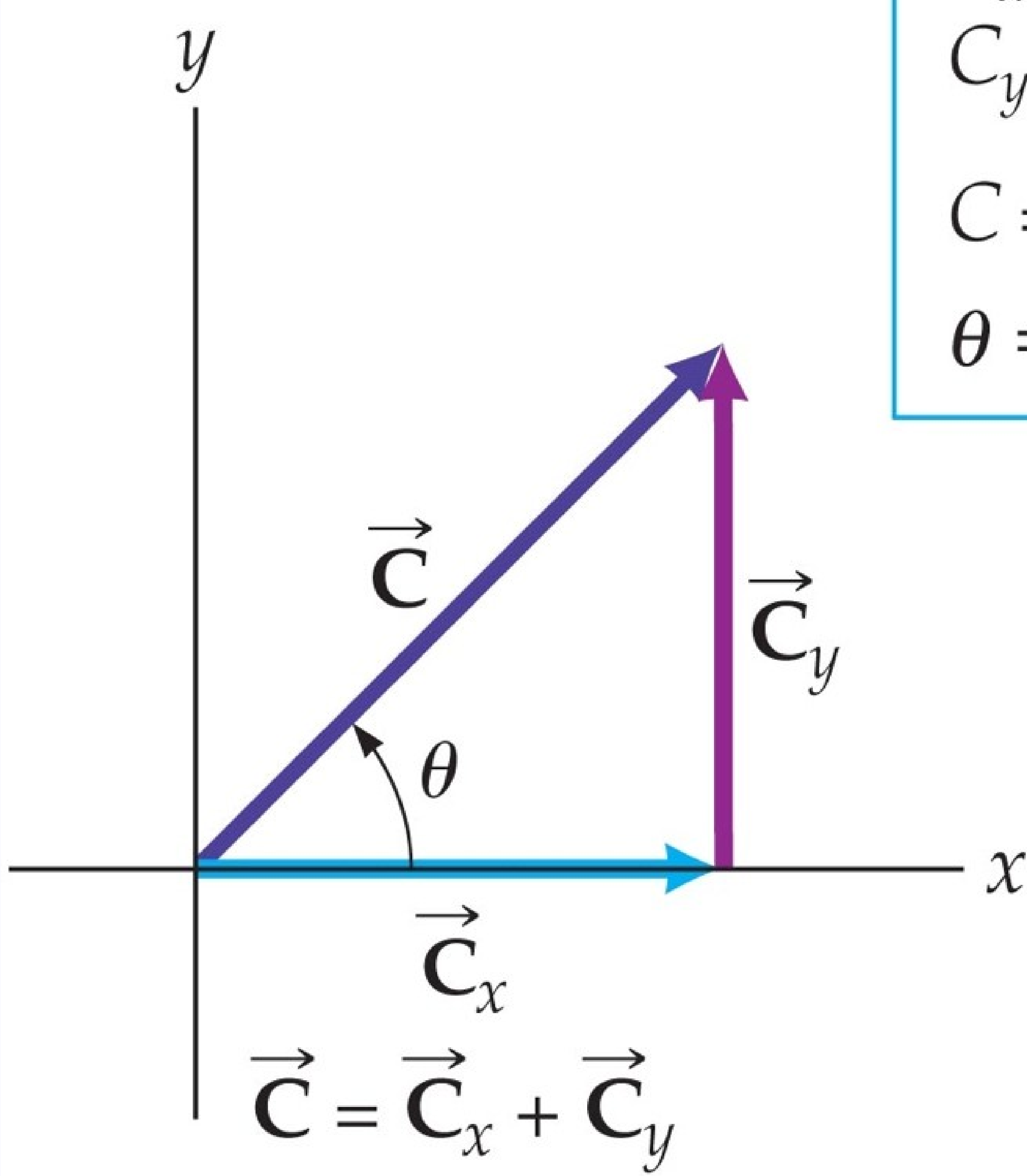
it decreases, but not as much as half

Add vectors by components

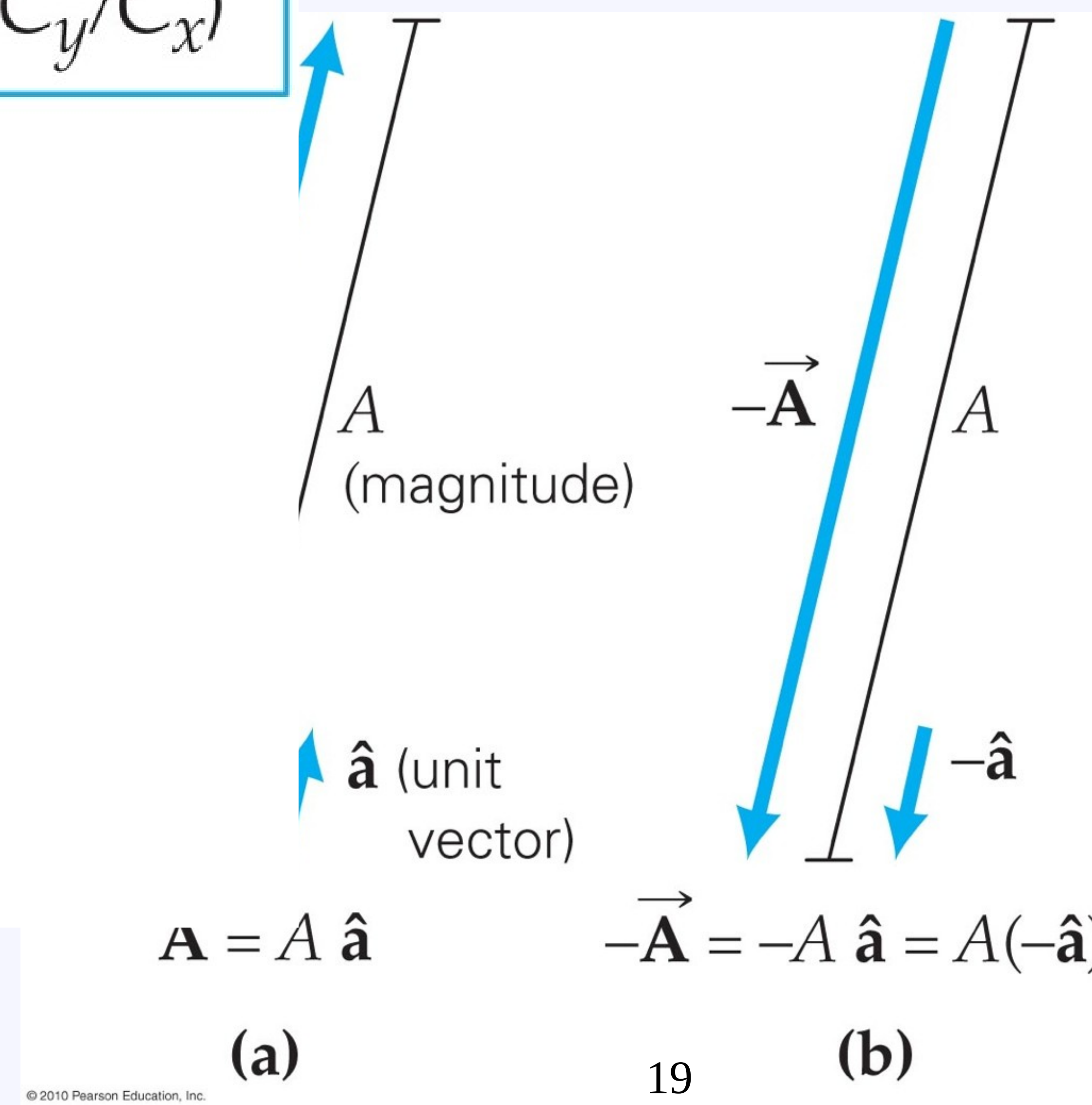
$$\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2 = ([v_{x1} + v_{x2}])\hat{\mathbf{x}} + ([v_{y1} + v_{y2}])\hat{\mathbf{y}}$$



Vector components

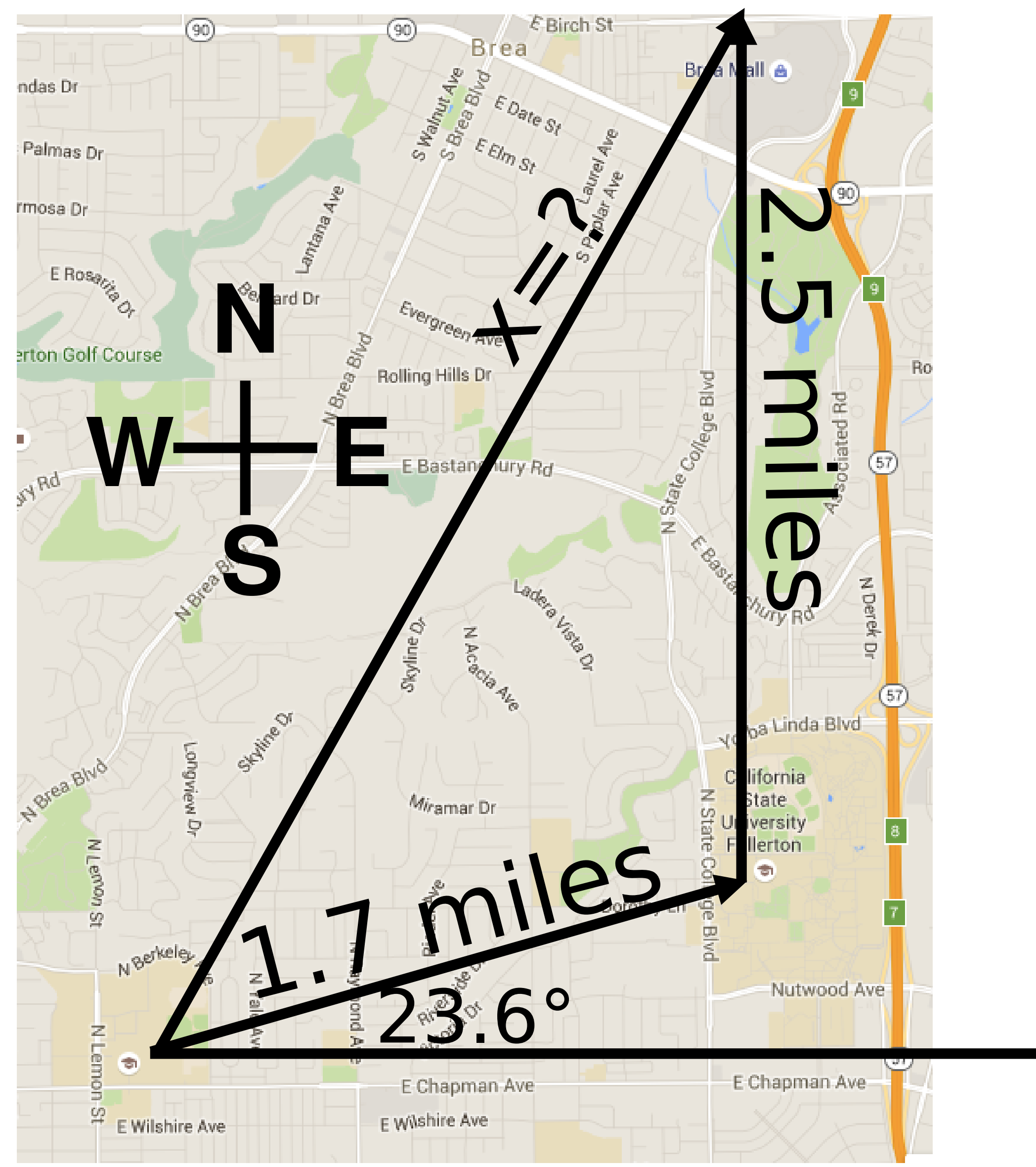


$$C_x = C \cos\theta$$
$$C_y = C \sin\theta$$
$$C = \sqrt{C_x{}^2 + C_y{}^2}$$
$$\theta = \tan^{-1}(C_y/C_x)$$



Vector addition example

- Cal State Fullerton is 1.7 miles away from Fullerton College, at an angle 23.6° north of east. The Apple Store is 2.5 miles due north of Cal State Fullerton.
  - A bird flies straight from Fullerton College to Cal State Fullerton, and then straight from Cal State Fullerton to the Apple Store. What is the bird's displacement?



Vector addition example

- The black vector is the sum of a red vector (horizontal) and a green vector (vertical). Which way should the red vector point?

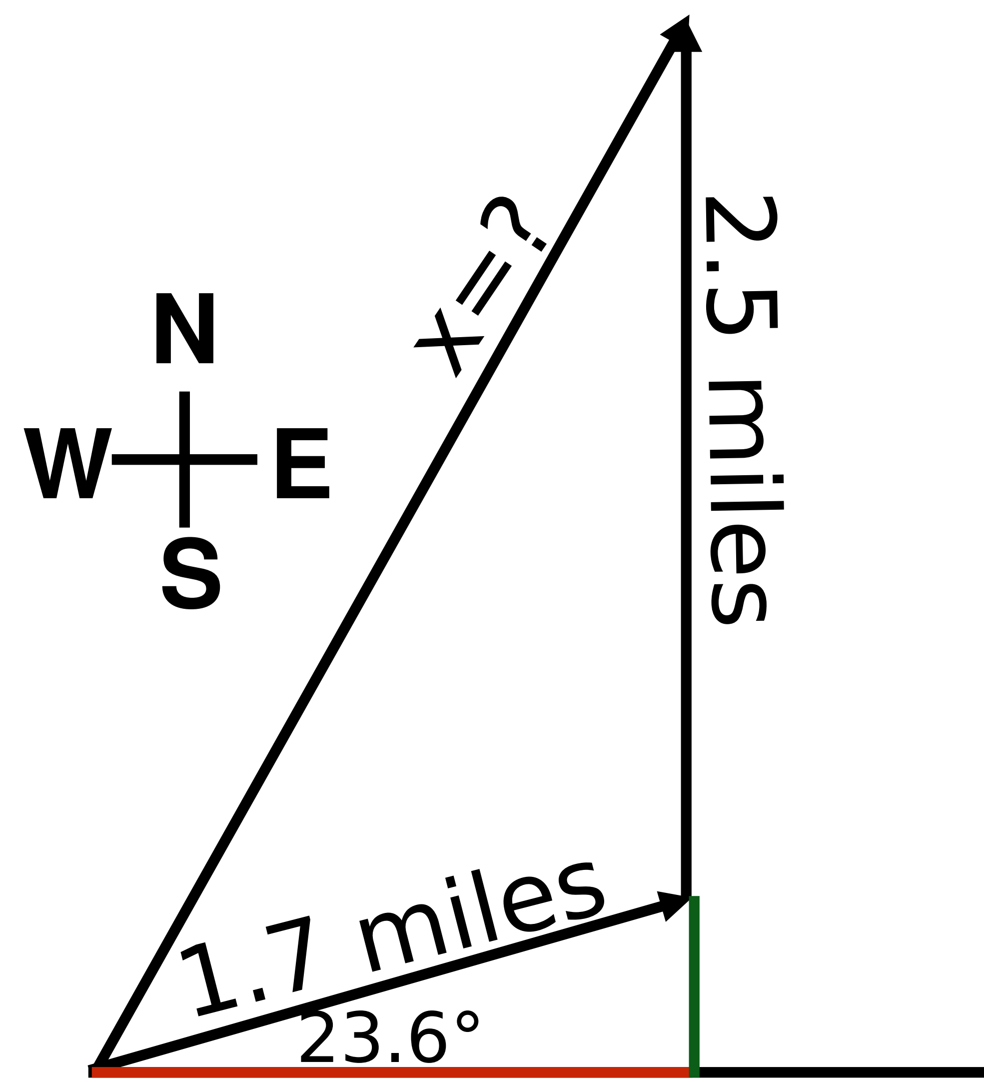
A Left

B Right

C Up

D Down

None of ABCD



Vector addition example

- The black vector is the sum of a red vector (horizontal) and a green vector (vertical). Which way should the green vector point?

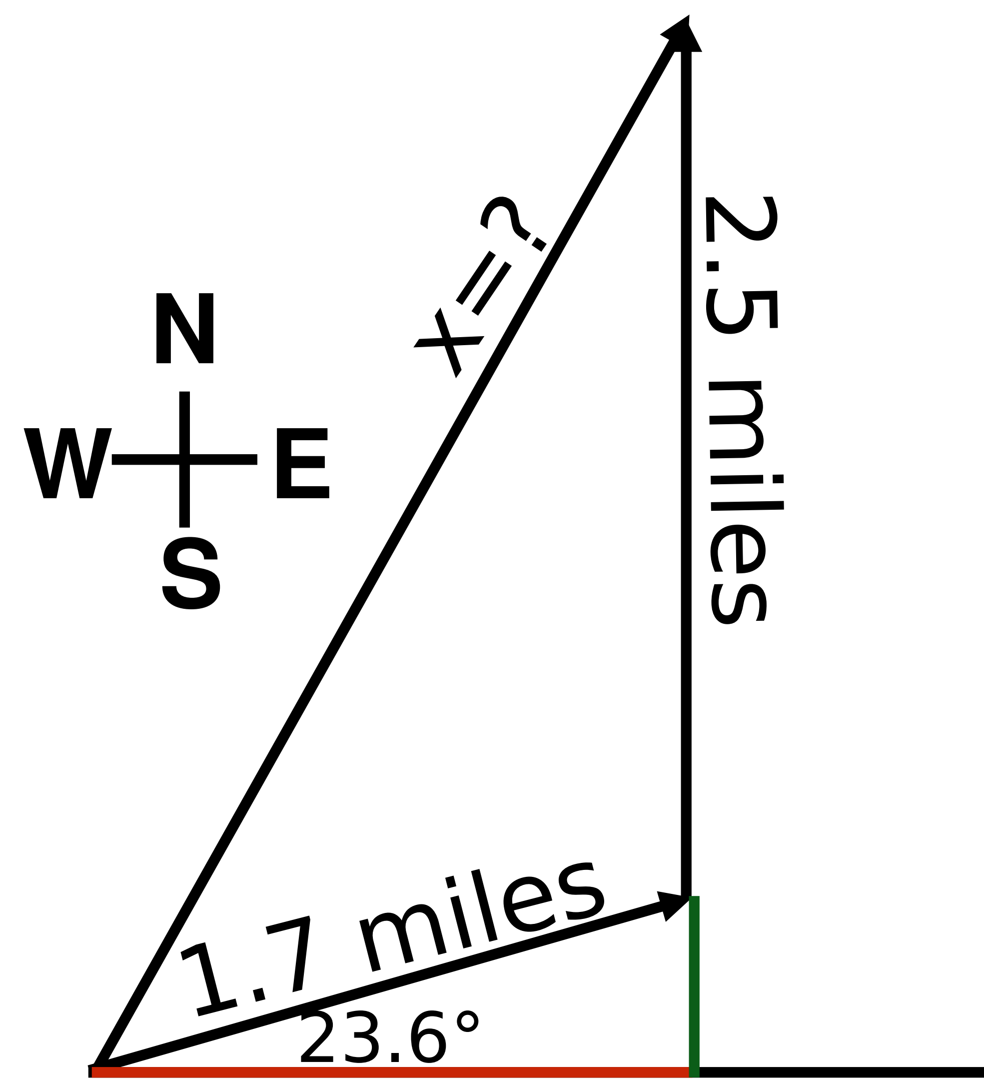
A Left

B Right

C Up

D Down

None of ABCD



Vector addition example

- How long is the red arrow?

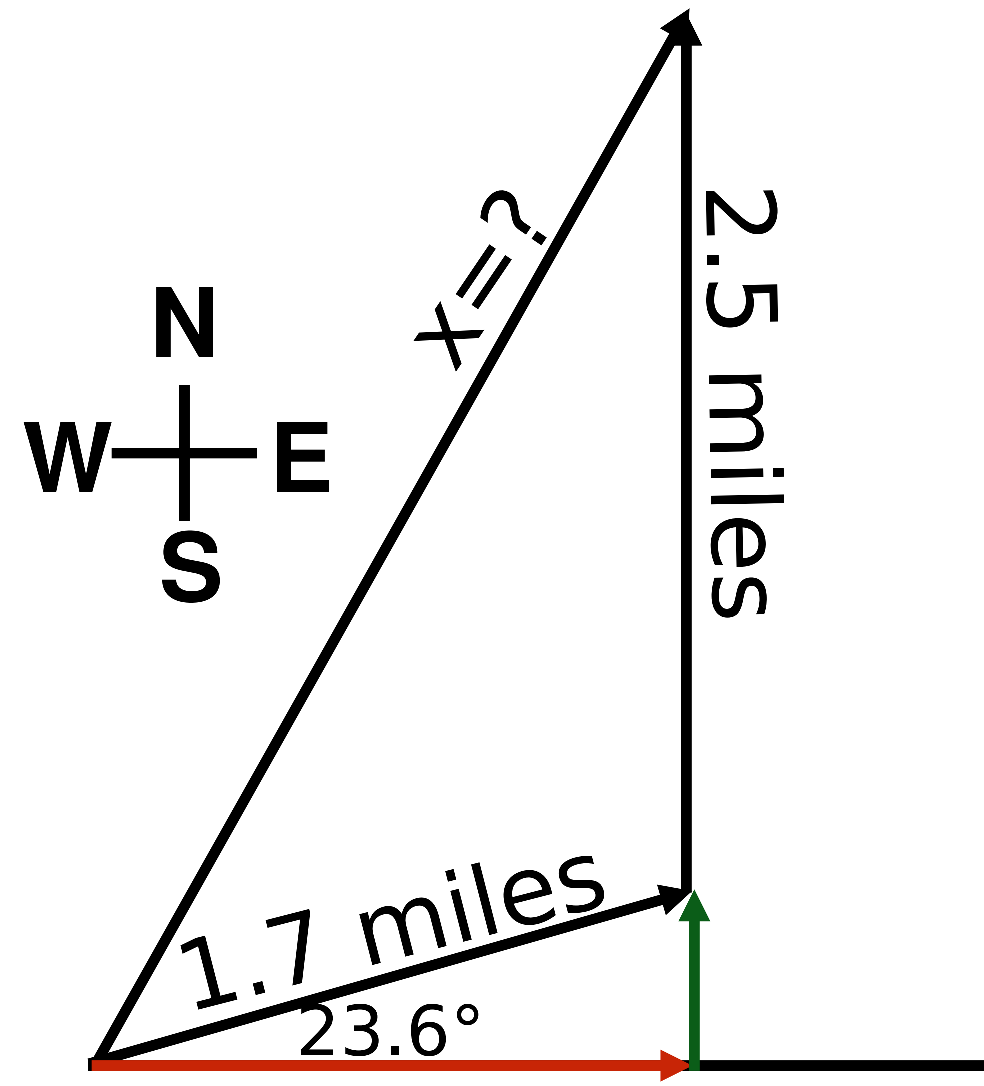
A 1.7 miles x sin 23.6°

B 1.7 miles x cos 23.6°

C 1.7 miles x tan 23.6°

D 1.7 miles

None of ABCD



Vector addition example

- How long is the green arrow?

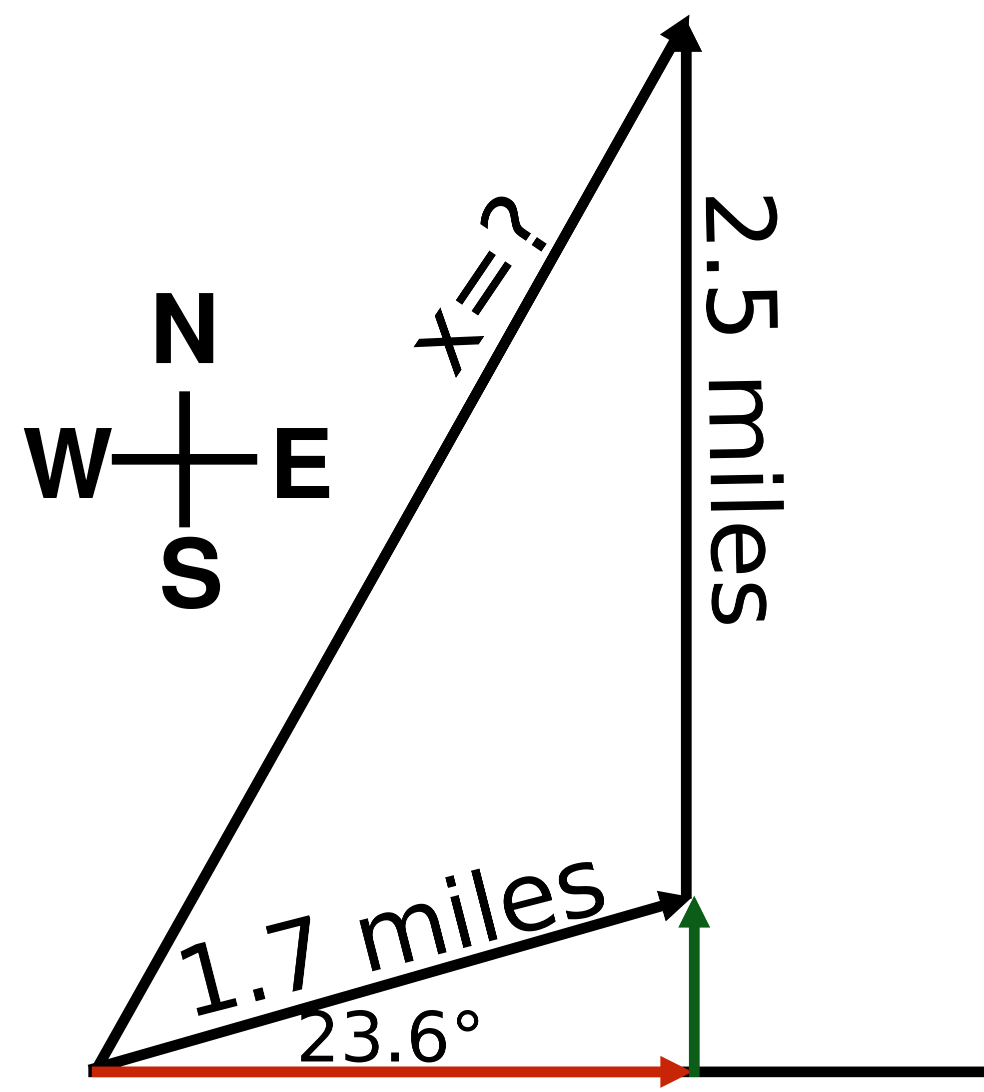
A 1.7 miles x sin 23.6°

B 1.7 miles x cos 23.6°

C 1.7 miles x tan 23.6°

D 1.7 miles

None of ABCD



Vector addition example

$$\mathbf{x} = (1.7\text{mi}\cos 23.6°)\hat{\mathbf{x}} + (1.7\text{mi}\sin 23.6° + 2.5\text{ mi})\hat{\mathbf{y}}$$

Magnitude of displacement:

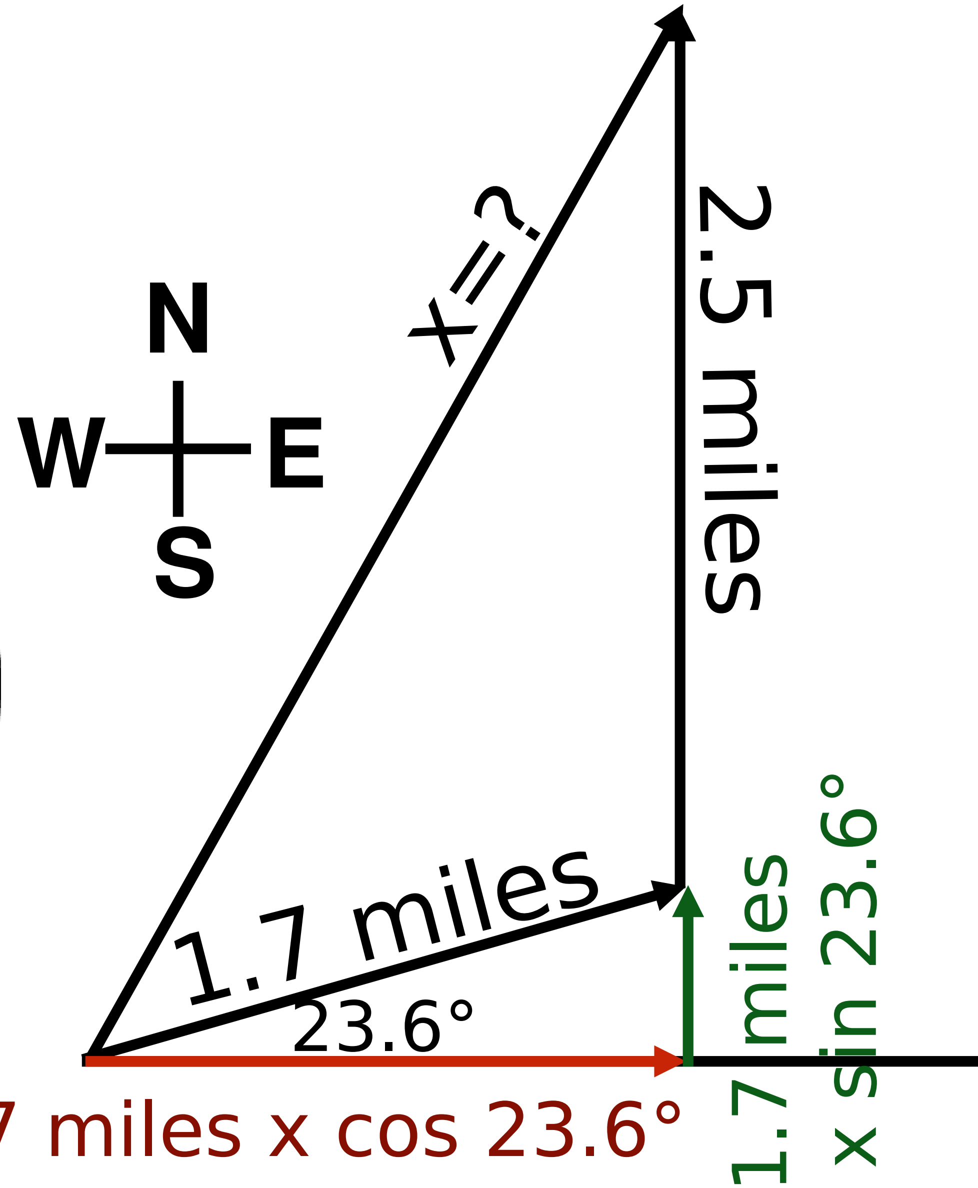
$$|\mathbf{x}| = \sqrt{(1.7\text{mi}\cos 23.6°)^2 + (1.7\text{mi}\sin 23.6° + 2.5\text{ mi})^2}$$

= 3.5 mi

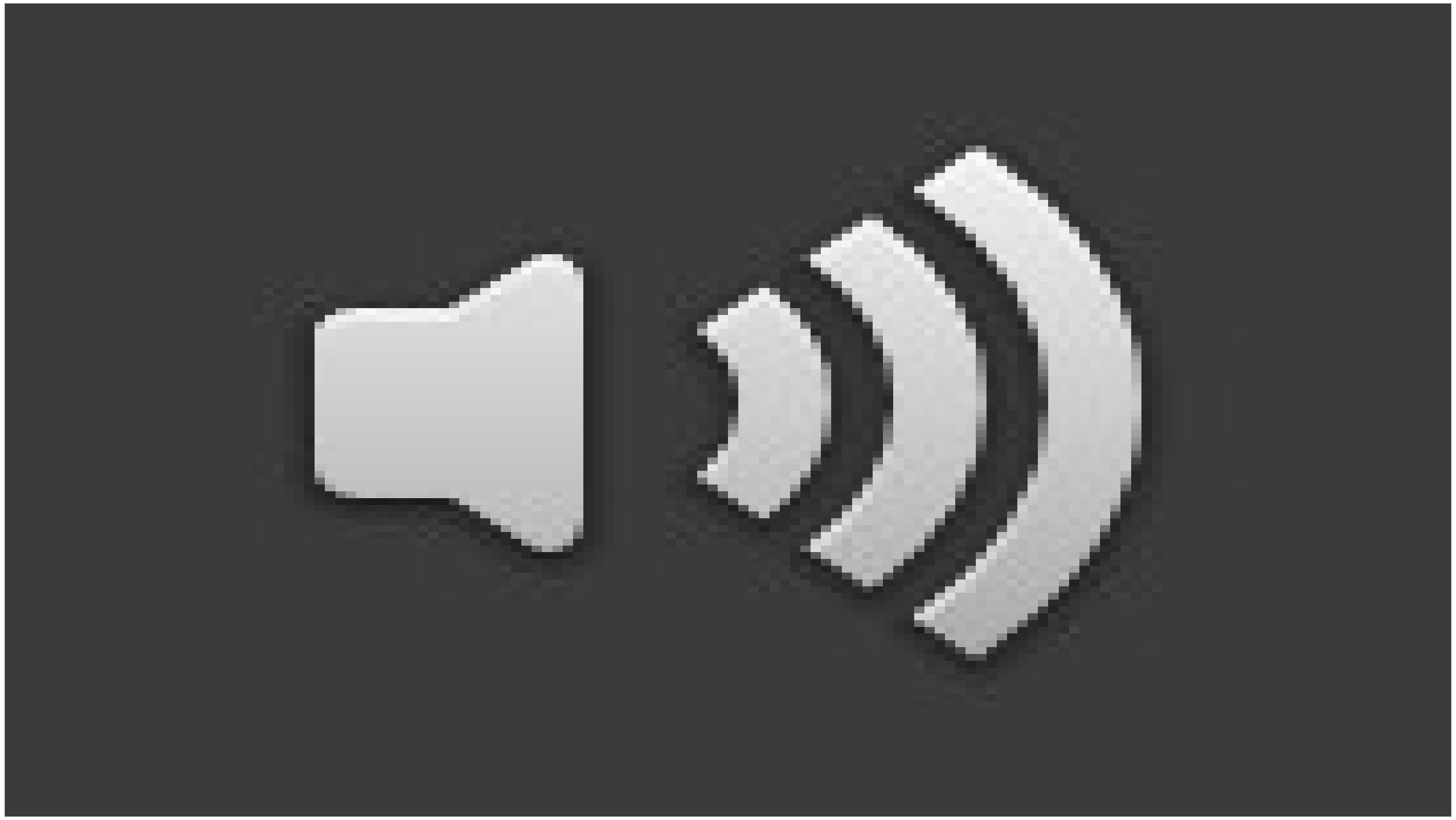
Direction of displacement (angle counter-clockwise = angle "north of east":

$$\theta = \tan^{-1}\left(\frac{1.7\text{ mi}\sin 23.6° + 2.5\text{ mi}}{1.7\text{ mi}\cos 23.6°}\right)$$

= 64°



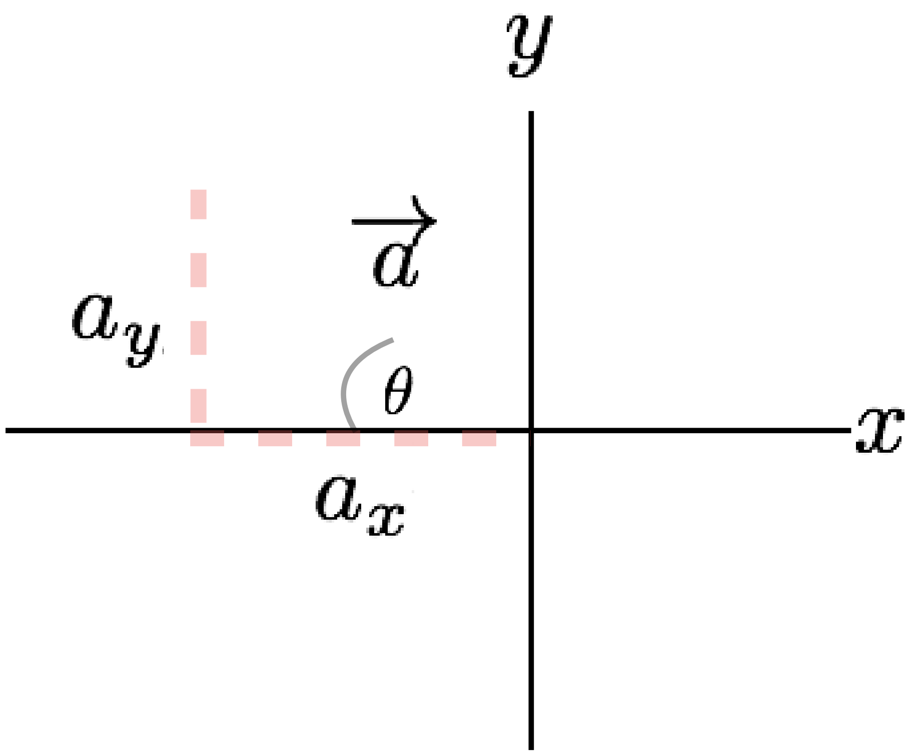
Next: Vector Multiplication & 2D Motion



reddit.com

- **Take Home Message**
  - There are **two ways** to multiply vectors!
    - Dot-product and Cross-product
  - Vector components allow you to simplify 2D and 3D motion

Unit vectors and components



$$\underbrace{\vec{d}}_{\text{vector}} = \underbrace{a_x}_{\text{scalar}}\underbrace{\hat{i}}_{\text{vector}} + \underbrace{a_y}_{\text{scalar}}\underbrace{\hat{j}}_{\text{vector}}$$

Unit vectors: $$x, y, z \rightarrow \hat{i}, \hat{j}, \hat{k}$$

Components:
$$a_x = a\cos\theta$$
$$a_y = a\sin\theta$$

- A **unit** vector has a magnitude of 1
  - Its sole purpose is to point in a direction
- A vector **component** includes a scalar value and a unit vector

Vector multiplication

- Dot Product - creates a new **scalar**

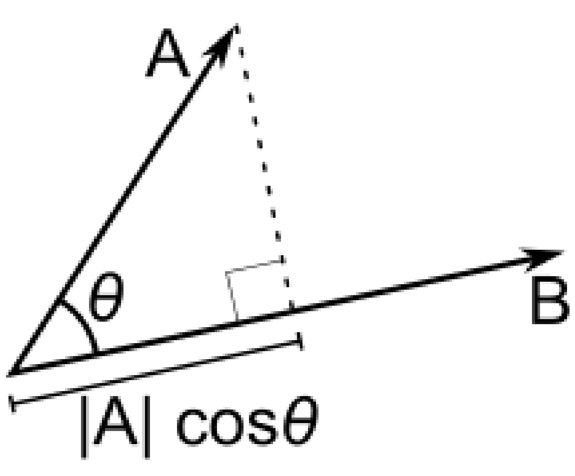
$$\vec{a} \cdot \vec{b} = (a_x\hat{i} + a_y\hat{y} + a_z\hat{k}) \cdot (b_x\hat{i} + b_y\hat{y} + b_z\hat{k})$$

vector vector

$$= a_x b_x + a_y b_y + a_z b_z$$

scalar scalar scalar

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$



wikipedia.org

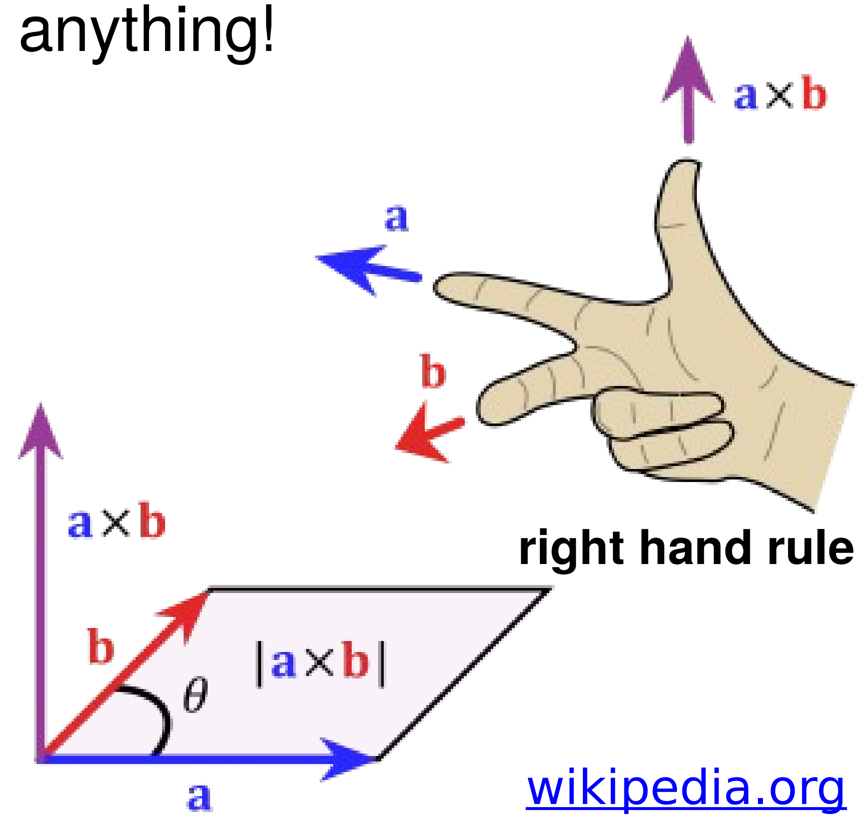
Vector multiplication

- Cross Product - creates a new **vector**

$$
\begin{aligned}
\underset{\text{vector}}{\vec{a}} \times \underset{\text{vector}}{\vec{b}} &= (a_x\hat{i} + a_y\hat{y} + a_z\hat{k}) \cdot (b_x\hat{i} + b_y\hat{y} + b_z\hat{k}) \\
&= \underbrace{(a_yb_z - a_zb_y)\hat{i}}_{\text{vector component}} + \underbrace{(a_zb_x - a_xb_z)\hat{j}}_{\text{vector component}} + \underbrace{(a_xb_y - a_yb_x)\hat{k}}_{\text{vector component}}
\end{aligned}
$$

- umm… WHAT?!?! I thought we didn't have to memorize anything!
- you don't

$$
\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}
$$

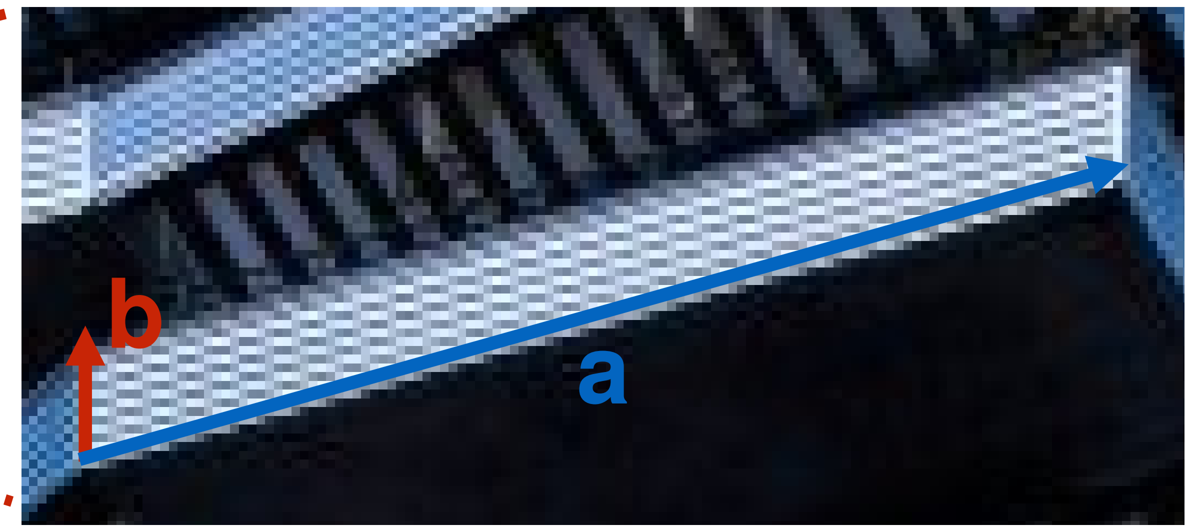


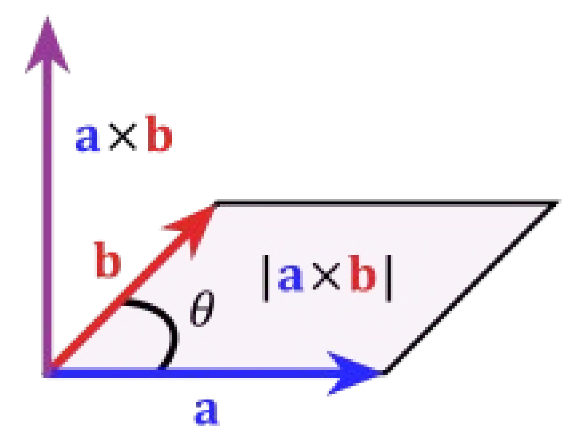
wikipedia.org

Vector multiplication

Why is the cross-product useful?

https://static.dezeen.com/





it help us define surfaces

In-Class Example (graded) Name______________________

$$\overrightarrow{a} = -3\hat{i} - 4\hat{j} + 5\hat{k} \text{ and } \overrightarrow{b} = -1\hat{i} + 4\hat{j} - 2\hat{k} \text{ and } \overrightarrow{c} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

1. Find: $\overrightarrow{a} \cdot (\overrightarrow{b} + \overrightarrow{c})$

2. Find: $\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c})$

very similar to a HW problem

Next time…

- Vectors and 2D Motion

Due dates

- **Assignments**
  - Should have already read **Ch.3** of the book
  - **HW1** due friday at 11:45PM

extra slides

In-Class Example Name____________________

$a = (-3, -4, 5) \quad b = (-1, 4, -2) \quad c = (3, 2, -5)$

① $\vec{a} \cdot (\vec{b} + \vec{c})$

start w/ $(\vec{b} + \vec{c}) = (-1+3, 4+2, -2-5) = (2, 6, -7)$

$\vec{a} \cdot (\vec{b} + \vec{c}) = (-3, -4, 5) \cdot (2, 6, -7) = -6 - 24 - 35 = \boxed{-65}$

② $\vec{a} \cdot (\vec{b} \times \vec{c})$

start w/ $(\vec{b} \times \vec{c}) = \left|\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & -2 \\ 3 & 2 & -5 \end{array}\right| \begin{array}{cc} i & j \\ -1 & 4 \\ 3 & 2 \end{array}$

$= (-20)\hat{i} + (-6)\hat{j} + (-2)\hat{k} - (5)\hat{j} - (-4)\hat{i} - (12)\hat{k}$

$= (-20 + 4)\hat{i} + (-6 - 5)\hat{j} + (-2 - 12)\hat{k}$

$= (-16, -11, -14)$

$\vec{a} \cdot (\vec{b} \times \vec{c}) = (-3, -4, 5) \cdot (-16, -11, -14)$

$= 48 + 44 + -70 = \boxed{22}$

Reminder - Vector multiplication

- Dot Product - creates a new **scalar**

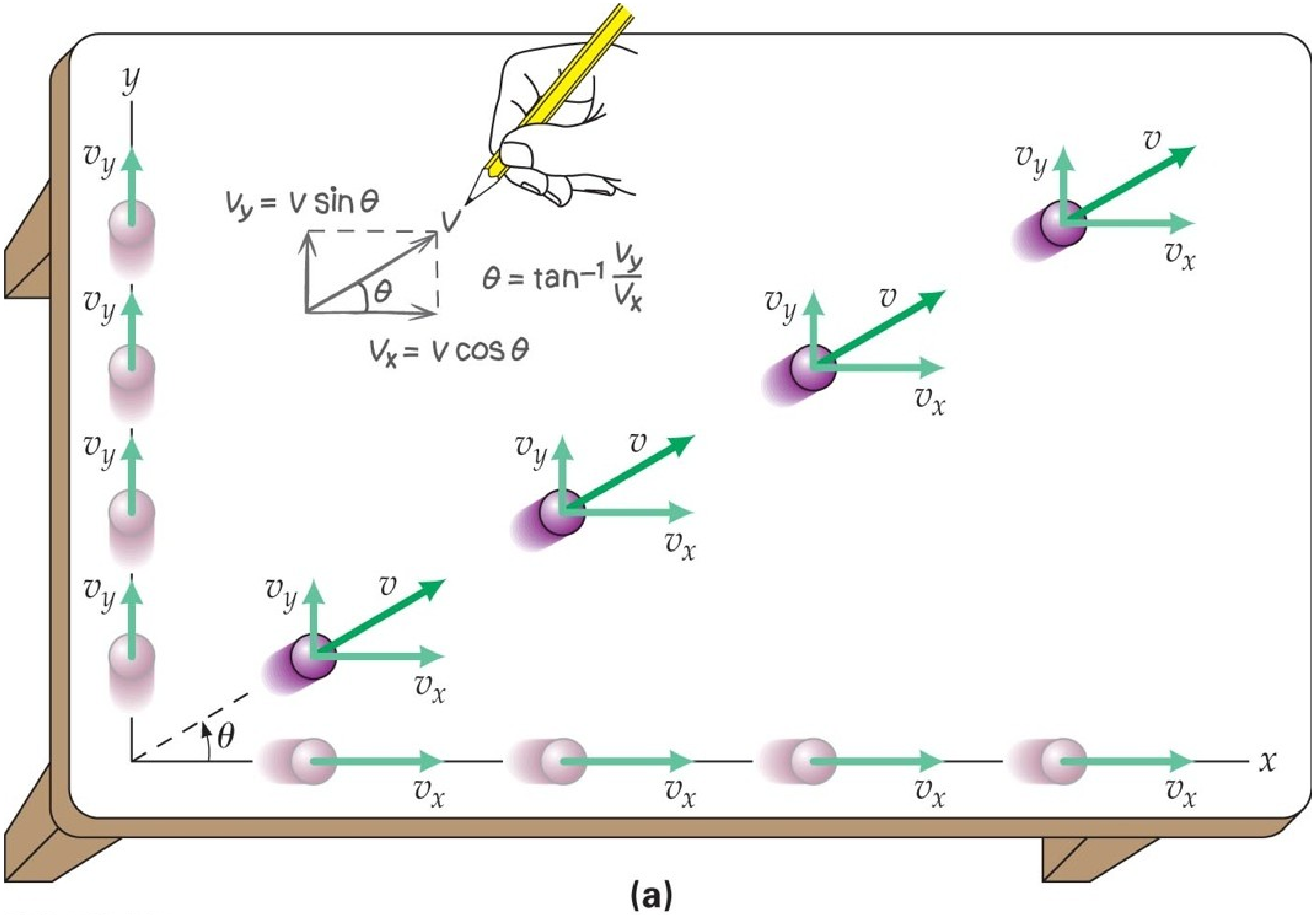
$$
\begin{aligned}
\underset{\text{vector}}{\overrightarrow{a}} \cdot \underset{\text{vector}}{\overrightarrow{b}} &= (a_x\hat{i} + a_y\hat{y} + a_z\hat{k}) \cdot (b_x\hat{i} + b_y\hat{y} + b_z\hat{k}) \\
&= \underset{\text{scalar}}{a_x b_x} + \underset{\text{scalar}}{a_y b_y} + \underset{\text{scalar}}{a_z b_z}
\end{aligned}
$$

- Cross Product - creates a new **vector**

$$
\begin{aligned}
\underset{\text{vector}}{\overrightarrow{a}} \times \underset{\text{vector}}{\overrightarrow{b}} &= (a_x\hat{i} + a_y\hat{y} + a_z\hat{k}) \cdot (b_x\hat{i} + b_y\hat{y} + b_z\hat{k}) \\
&= \underbrace{(a_y b_z - a_z b_y)\hat{i}}_{\text{vector component}} + \underbrace{(a_z b_x - a_x b_z)\hat{j}}_{\text{vector component}} + \underbrace{(a_x b_y - a_y b_x)\hat{k}}_{\text{vector component}}
\end{aligned}
$$

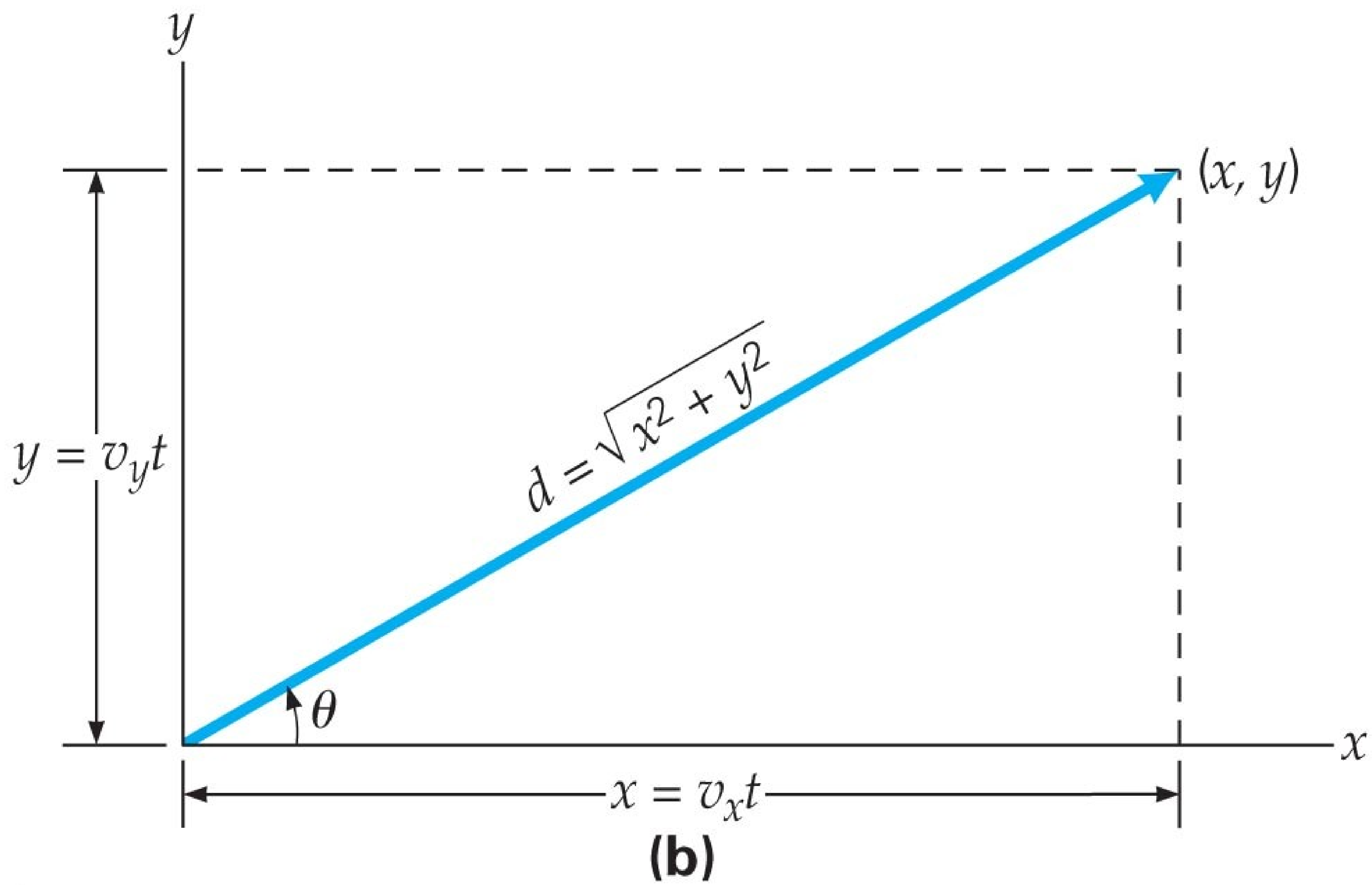
Motion in 2 dimensions

- Horizontal & vertical motion *independent*
  - Break vectors into components
  - Treat each component separately



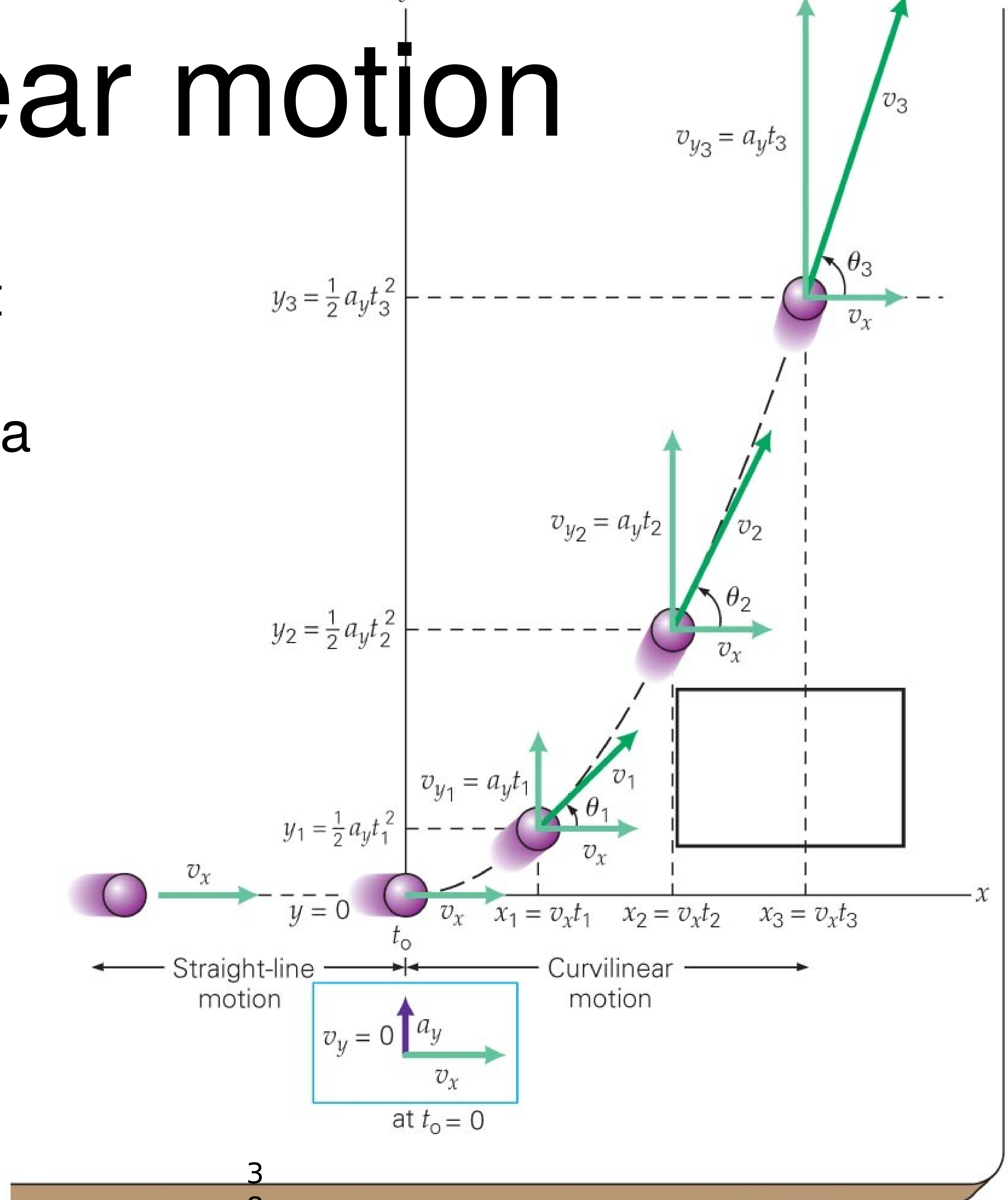
Motion in 2 dimensions

- Horizontal & vertical motion *independent*
  - Break vectors into components
  - Treat each component separately



Curvilinear motion

- If velocity, acceleration not parallel, then motion is along a curve ("curvilinear motion")

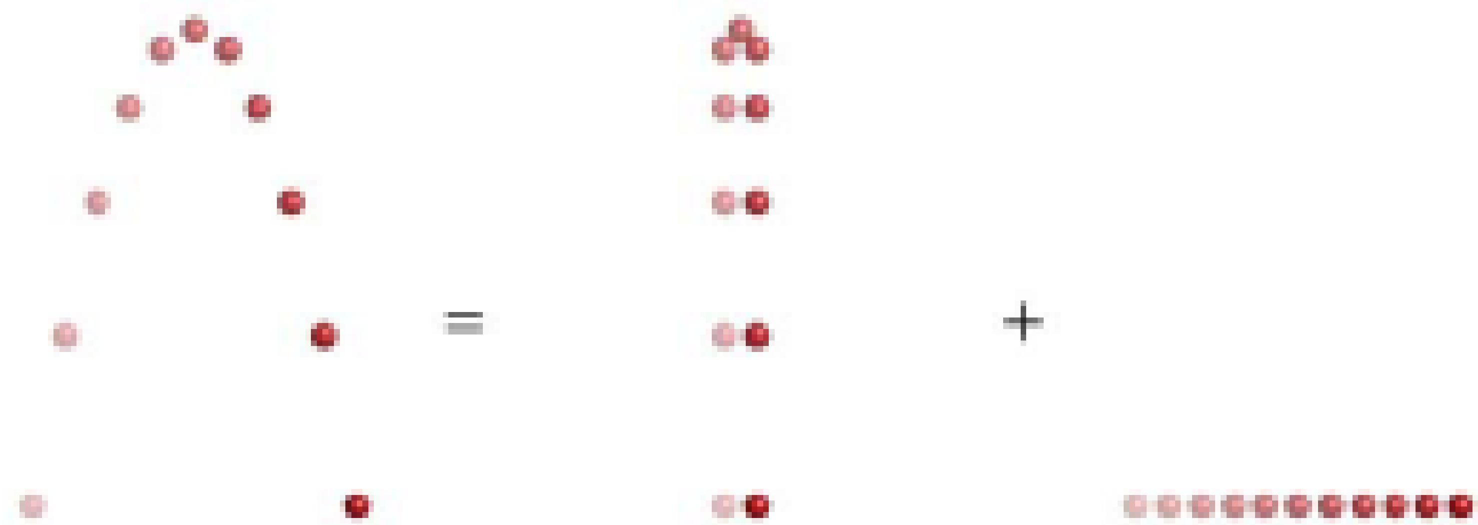


Main Points – Projectile Motion

Projectile motion is the superposition of two independent motions:
1) **Horizontal**: constant velocity
2) **Vertical**: constant acceleration

Projectile motion can be understood simply as free fall viewed from a moving reference frame.

Projectile Motion = Vertical Motion with Constant Acceleration + Horizontal Motion with Constant Velocity



Acceleration of Gravity	Vertical	Horizontal
$g \equiv +9.8\,\frac{m}{s^2}$	$a_y = -g$	$a_x = 0$
	$v_y = v_{oy} - gt$	$v_x = v_{ox}$
	$y = y_o + v_{oy}t - \frac{1}{2}gt^2$	$x = x_o + v_{ox}t$

Projectile Motion

Horizontal

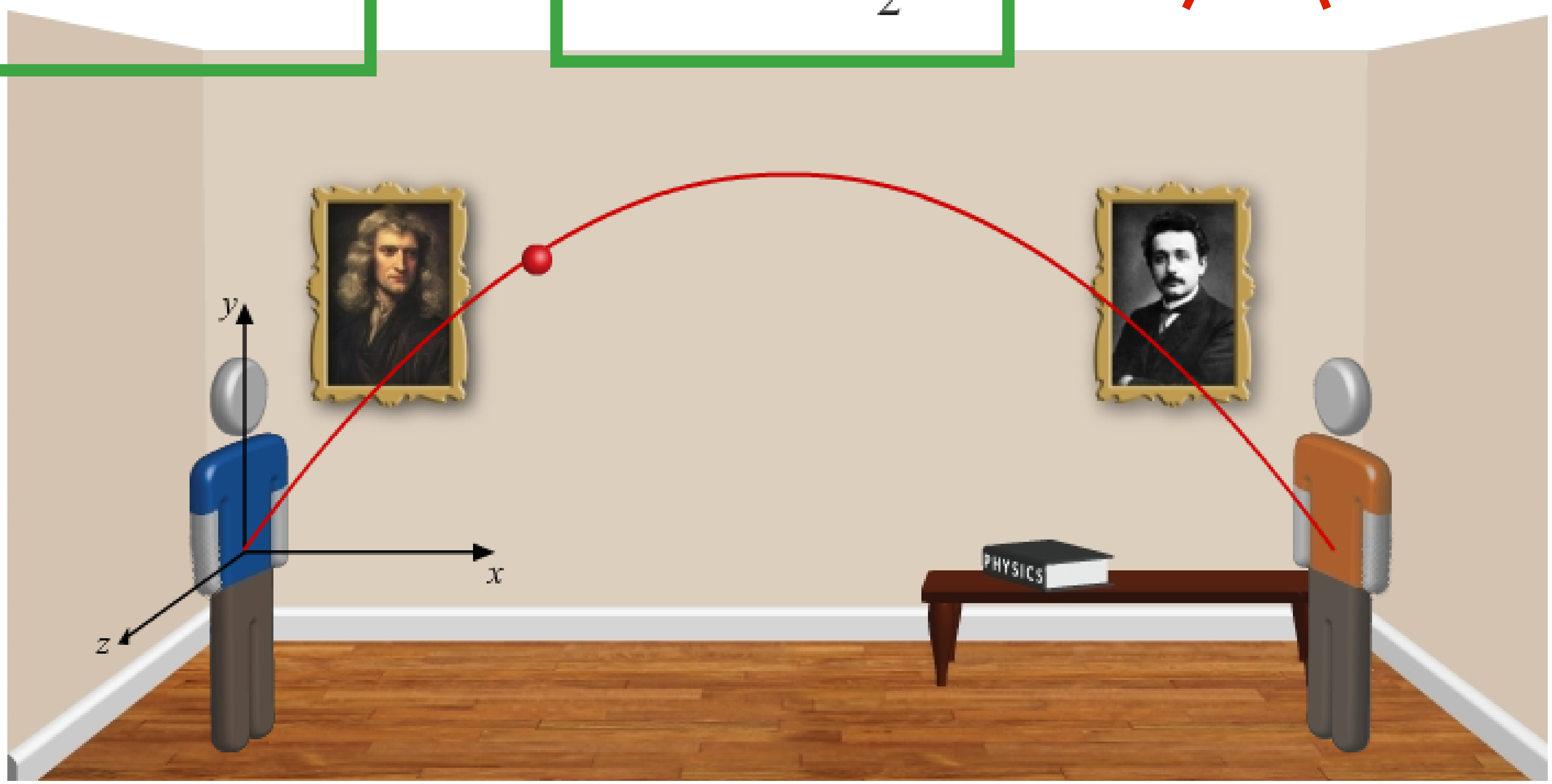
$$a_x = 0$$
$$v_x = v_{o_x}$$
$$x = x_o + v_{o_x} t$$

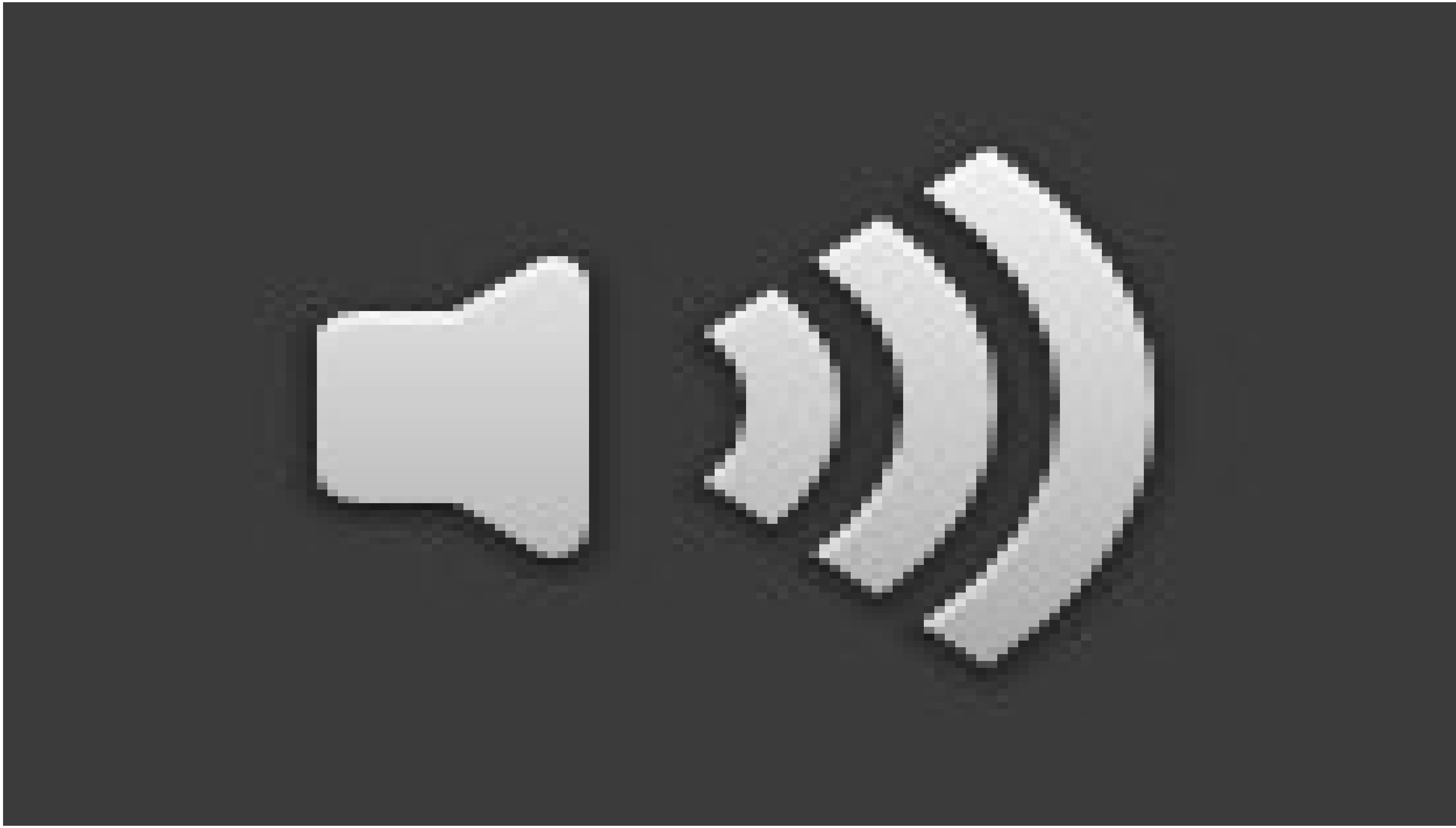
Vertical

$$a_y = -g$$
$$v_y = v_{o_y} - gt$$
$$y = y_o + v_{o_y} t - \frac{1}{2} g t^2$$

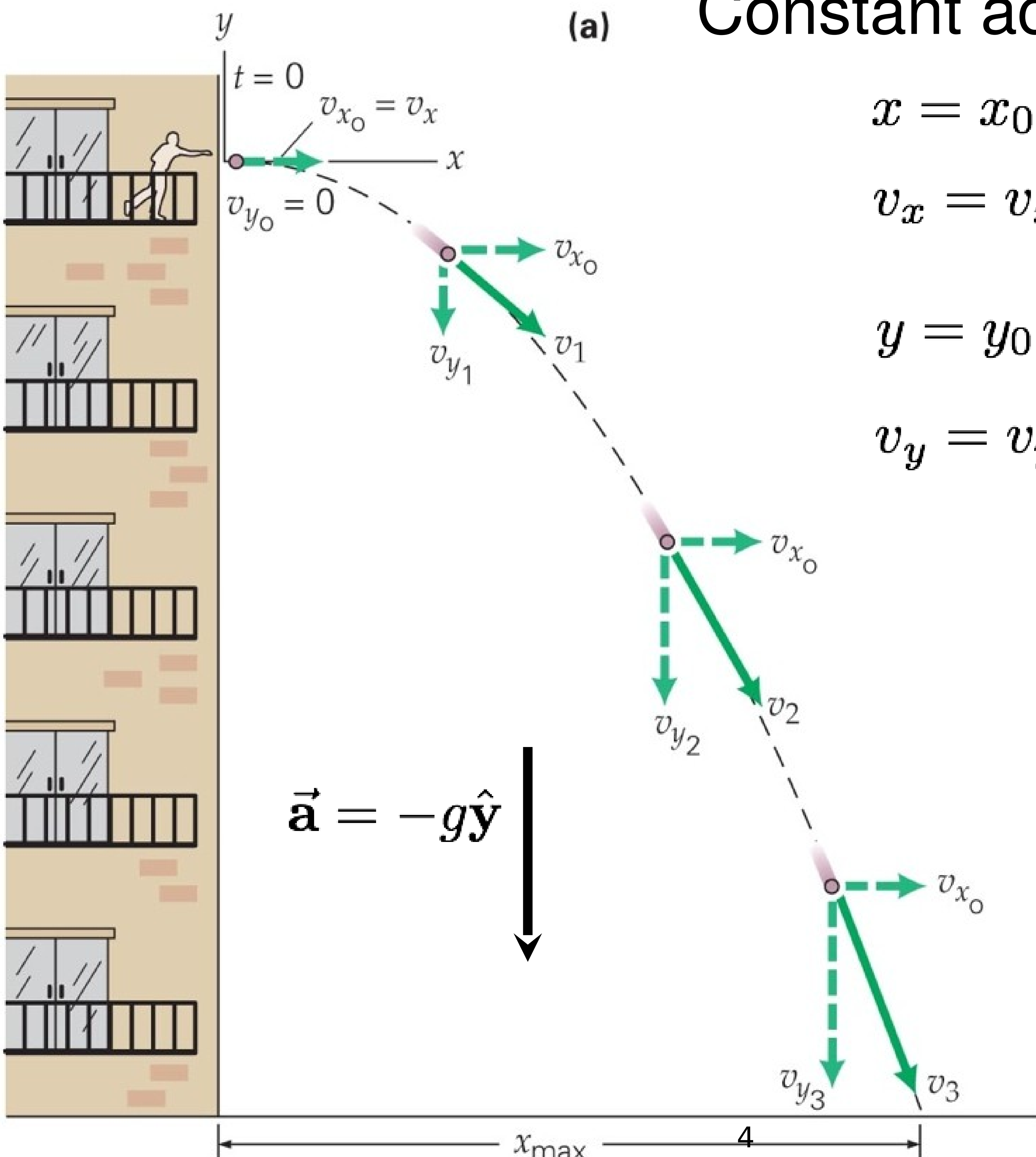
Boring

$$\cancel{a_z = 0}$$
$$\cancel{v_z = 0}$$
$$\cancel{z = z_o}$$





Constant acceleration in 2D:



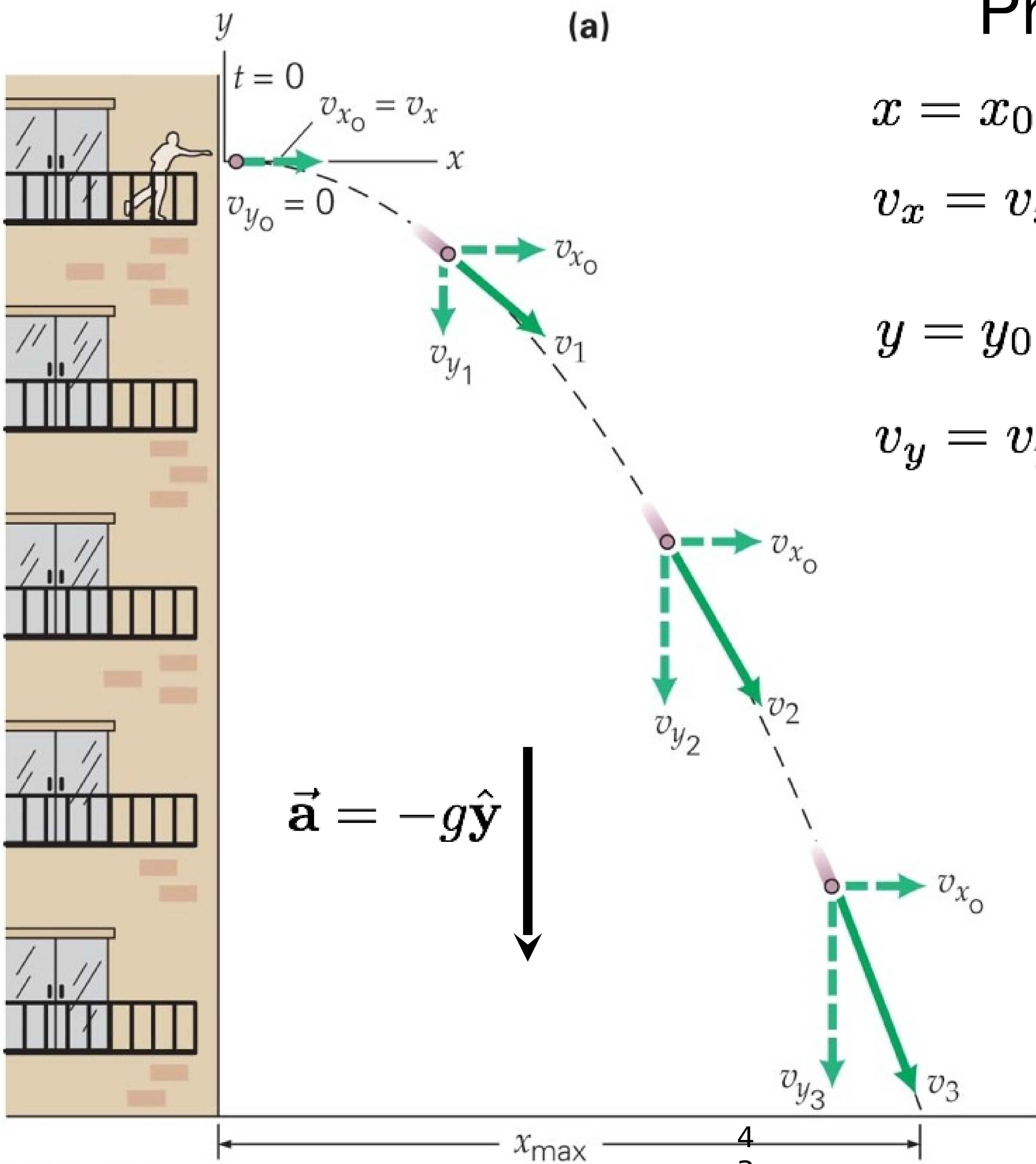
$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_x = v_{x0} + a_x t$$

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

$$v_y = v_{y0} + a_y t$$

Projectile



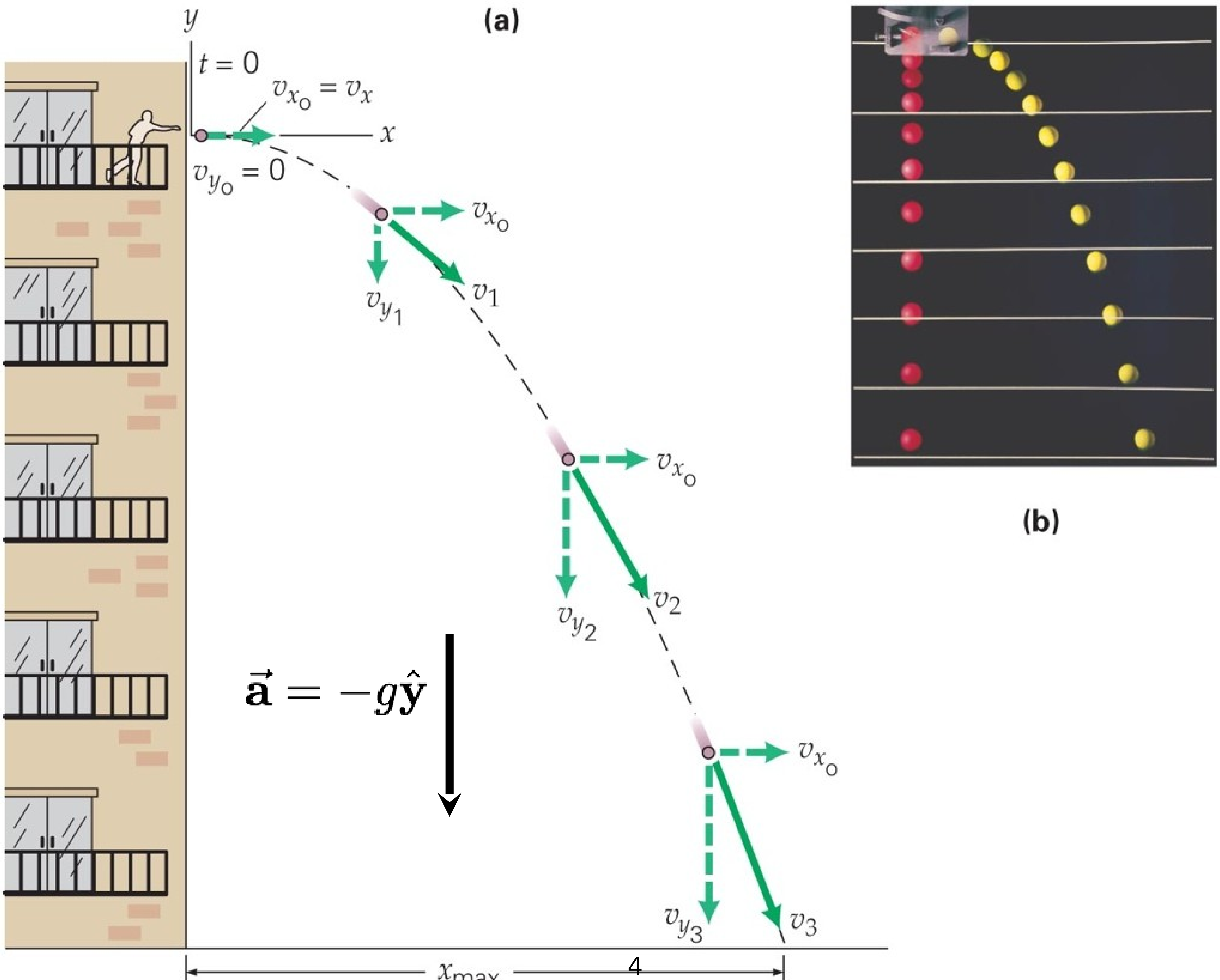
$$\vec{\mathbf{a}} = -g\hat{\mathbf{y}}$$

$$x = x_0 + v_{x0}t$$

$$v_x = v_{x0}$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$v_y = v_{y0} - gt$$


(a)
y
$t = 0$
$v_{x_o} = v_x$
x
$v_{y_o} = 0$
v_{x_o}
v_{y_1}
v_1
v_{x_o}
v_{y_2}
v_2
v_{x_o}
v_{y_3}
v_3
$\vec{\mathbf{a}} = -g\hat{\mathbf{y}}$
x_{max}
(b)