Math 125 Pre-Calculus
Fall 2017, Kenny Shah, 10.1 – 11-2

NAME: SOLUTIONS
CLASS TIME: 7&8

QUIZ # 9

Please show all of your work for maximum credit. Good Luck!!!

1. **(2 points) Let $P = f(t) = 37.8(1.044)^t$ be the population of a town (in thousands) in year t.**

(a) **Find a formula for $f^{-1}(P)$ in terms of P.**

Sol. $P = 37.8(1.044)^t$, solve for t

$$\frac{P}{37.8} = (1.044)^t \Rightarrow \ln\left(\frac{P}{37.8}\right) = \ln(1.044)^t \Rightarrow t = \frac{\ln\left(\frac{P}{37.8}\right)}{\ln(1.044)}$$

Thus, $\boxed{t = f^{-1}(P) = \frac{\ln\left(\frac{P}{37.8}\right)}{\ln(1.044)}}$

(b) **Evaluate $f^{-1}(50)$. What does this quantity tell you about the population?**

Sol $t = f^{-1}(50) = \frac{\ln\left(\frac{50}{37.8}\right)}{\ln(1.044)} \approx 6.5$

Thus, it takes 6.5 years for the population to reach 50,000 people.

2. **(2 points) The air temperature, T, in °F, is given in terms of the chirp rate, R, in chirps per minute, of a snowy tree cricket by the function**

$$T = f(R) = \frac{1}{4}R + 40.$$

Suppose one night we record the chirp rate and find that it varies with time, x, according to the function

$$R = g(x) = 20 + x^2 \text{ where } x \text{ is in hours since midnight and } 0 \le x \le 10.$$

(a) **Find how temperature varies with time by obtaining a formula T = h(x) and simplify completely.**

Sol $T = f(R)$

$T = f(g(x))$

$T = f(20 + x^2)$

$T = \frac{1}{4}(20 + x^2) + 40$

$T = 5 + \frac{1}{4}x^2 + 40$

$\boxed{T = h(x) = \frac{1}{4}x^2 + 45}$

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3. (4 points) Geometry tells us that the radius of a sphere is directly proportional to the cube root of its volume.

(a) Use this proportionality relationship and the fact that a sphere of radius 18.2 cm has a volume of 25,252.4 cm^3 to find the equation for this relationship.

Sol $r = k \cdot \sqrt[3]{V}$

$18.2 = k \cdot \sqrt[3]{25252.4}$

$k = \dfrac{18.2}{\sqrt[3]{25252.4}} = 0.62035$

$\boxed{r = 0.62035 \cdot \sqrt[3]{V}}$

(b) What happens to the Volume if radius is doubled?

Sol $r = 0.62035 \cdot \sqrt[3]{V}$

$\dfrac{r}{0.62035} = \dfrac{0.62035 \cdot \sqrt[3]{V}}{0.62035}$

$\dfrac{r}{0.62035} = \sqrt[3]{V}$

$(\sqrt[3]{V})^3 = \left(\dfrac{r}{0.62035}\right)^3$

$V_{orig} = \left(\dfrac{r}{0.62035}\right)^3$

$V_{new} = \left(\dfrac{2r}{0.62035}\right)^3$

$V_{new} = 8 \cdot \left(\dfrac{r}{0.62035}\right)^3$

$V_{new} = \boxed{8} V_{orig}$

If radius is doubled, Volume increases by factor of 8.

4. (2 points) For the given polynomial function:

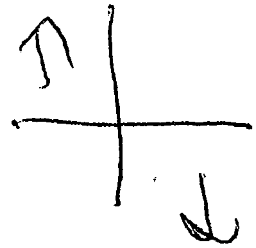
$$P(x) = -2x\,(x+3)^2(x-2)^2$$

State the degree, long-run behavior of P(x) and the end behavior of P(x).

Sol $P(x) = -2x \cdot x^2 \cdot x^2 = \boxed{-2x^5} + \ldots$

degree $= 5$

Long-run behavior $= -2x^5$



End-behavior:

as $x \to \infty$, $P(x) \to -\infty$

as $x \to -\infty$, $P(x) \to \infty$