

MATH-338 Midterm 1 Study Guide

THEORY

Day 2: Independent events happen same time, not affecting one another ($P(A \cap B) = P(A)P(B)$). Disjoint is the opposite ($P(A \cap B) = 0$). Probability Mass Function (PMF) is a dictionary mapping of events to positive probabilities. Over an infinite amount of iterations, RVs converge to a number.

Day 3: Law of Large Numbers: more times means more precise result.

Day 4: Parameter: any numerical quantity that characterizes a given population. Population proportion: a percentage value associated with a population. Sample proportion: the proportion of individuals in a sample sharing a certain trait (\hat{p}). Sample Mean(\bar{X}). Sampling distribution: probability distribution of statistic obtained through a large number of samples drawn (sample **must** be know).

Day 5: We want low bias and high variability. Bias bad. Variability \downarrow as the sample size \uparrow . Binomial Probability Distribution Conditions: Binary outcome (TF), Independent (previous outcomes do **not** affect next.), Number of outcomes, Success is equally likely. 'X' denotes the number of successes and 'n' is the number of elements in your sample. \hat{P} does **NOT** have a binomial distribution.

Day 6: Interacting variables: one variable can affect the another variable (non-independent). Confounding variable: a factor that influences the results of an experiment. Block design: split sample initially based on traits (possibly confounding) then randomly assign in those groups. Matched Pairs Design: blocks sizes of two (only looking with two levels). Repeated Measures Design two similar subjects have the same tests and those results are compared. Hawthorne Effect: individuals know they are being experimented on.

Day 7: Sensitivity: proportion of actual positive. Specificity: proportion of actual negative. Positive Predictive Value: proportion of positive tests that were actually positive. Negative Predictive Value: same as above but for negative. Prevalence: base rate.

Day 8: Neyman-Pearson Testing: This test will allow us to make preemptive decisions based on conditions presented before the study is conducted. These are the theoretical outcomes WITHOUT taking any sample data. Null Hypothesis: nothing unexpected (original hypothesis, H_0). Alternate Hypothesis: "something is happening and we should change our minds" (H_a). Critical region: range of values that corresponds to the rejection of H_0 at some chosen probability level. Type I Error: occurs when a significance test results in the rejection of a true null hypothesis. Type II Error: the data do not provide strong evidence that the null hypothesis is false.

FORMULAS

- Mean of Probability Dist. : $\mu_x = \sum x \times p(x)$
- Variance : $\sigma^2_x = \sum [x^2 \times P(x)] - \mu^2_x$
- Standard Deviation : $\sigma_x = \sqrt{\sigma_x^2}$ and $\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$
- Number successes : $X \sim B(n, p)$
- Mean of binomial RV: nP
- Variance of Bernoulli RV: $P(1 - P)$
- Variance of binomial RV: $nP(1 - P)$
- Standard deviation of binomial RV: $\sqrt{nP(1 - P)}$
- Bayes' Rule: $\frac{P(B|A)P(A)}{P(B)}$
- $P(B|A) = \frac{\text{number of outcomes in } A \cap B}{\text{number of outcomes in } A} = \frac{P(A \cap B)}{P(A)} > 0$

- Population proportion: $\hat{P} = \frac{X}{n}$
- Variance(\hat{P}) = $\frac{P(1-P)}{n}$
- Standard Deviation(\hat{P}) = $\sqrt{\frac{P(1-P)}{n}}$
- Sensitivity: $\frac{TP}{TP+FN}$
- Specificity: $\frac{TN}{TN+FP}$
- PPV: $\frac{TP}{TP+FP}$
- NPV: $\frac{TN}{TN+FN}$
- Prevalence: $\frac{\text{Actual Positive}}{\text{Actual Positive} + \text{Actual Negative}}$
- $\alpha = P(1) - P(\text{Concluded } H_a \mid H_0 \text{ is true})$
- Baseline $\alpha = 0.05$
- $\beta = P(2) - P(\text{Concluded } H_0 \mid H_a)$
- Power: $1 - \beta$

ABBREVIATIONS AND MISC.

- TP: True Positive
- TN: True Negative
- FP: False Positive
- FN: False Negative
- FTP: File Transfer Protocol
- Independent events: $P(A \cap B) = P(A) \times P(B)$
- Conditional probability: $P(A \cap B) = P(A) \times P(B|A)$ [Tree Mapping]