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**NOTE:** all equations containing the  $\cdot$  symbol denote a DOT PRODUCT. Multiplication in formulas will be denoted with the  $\times$  symbol to avoid confusion.

# Kinetic Energy and Work

Kinetic: energy of motion.

Work: Force (along the direction of motion) · Displacement (two vectors dotted will always result in a scalar)

It is important to be aware of the work done by what force on what object.

If an applied force is referenced, it is an applied force - For example: If work done by “you” as opposed to work done by a spring force, etc

## Free Body Diagram



Figure 1: Kinetic Energy

- Work done by  $\vec{F}$  (pulling force)
- Work done by a specified force or combination of forces
  - Vector sum is denoted by  $\Sigma$
  - $\vec{W}_{\text{net}} = (\Sigma \vec{F}_i) \cdot \Delta \vec{r}$
  - $\vec{F}_{\text{net}} = \vec{W}_{\text{friction}} + \vec{V}_{\text{normal}} + \vec{W}_{\text{applied force}} + \vec{W}_{\text{gravity}}$
  - $\vec{W}_{\text{friction}} < 0$  and will always act to decrease the kinetic motion.

## Work Formula

### Constant

$$\vec{W} = \vec{F} \cdot \Delta \vec{r} = \vec{F} \cdot \Delta x \hat{i}$$

This formula works when there is a [constant] force

### Variable

$$\vec{W} = \vec{F} \cdot d\vec{r}$$

$$\vec{W} = \int_i^f \vec{F} \cdot d\vec{r} dx$$

This applies when the force is variable

**Note:**  $\vec{F} \cdot \Delta \vec{r} = |\vec{F}| \times |\Delta \vec{r}| \cos(\theta)$

### General

$$W = \int_{x_i}^{x_f} F(x) dx$$

## 1D Constant Acceleration

$$((\vec{V}_f)^2 - (\vec{V}_o)^2) = 2\vec{a}\Delta x$$

This is derived from Newton's 2<sup>nd</sup> Law

$$\vec{a} = \frac{\vec{F}_{\text{net, ext}}}{m}$$

## Kinetic Energy Formula

$$K = \frac{m}{2} \times v^2$$

## 1-D Work Energy Theorem

$$\vec{F}_{\text{net, ext}}\Delta x = \vec{W}_{\text{net}}$$

$$\frac{m}{2} \times ((\vec{V}_f)^2 - (\vec{V}_o)^2) = \vec{W}_{\text{net}}$$

## Units for Work

$$\vec{W} \sim \vec{F} \times \Delta x \sim 1N \times 1\text{ meter} = \text{Joule}$$

Kinetic energy has the same units as work

## Displacement Vector

$$\vec{r}_i + \Delta\vec{r} = \vec{r}_f$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

## Work done by a Spring Force

$$\Delta\vec{K} = \vec{W}_{\text{net}} = \vec{K}_f - \vec{K}_i$$

Above is the kinetic-energy theorem

$$\vec{W}_{\text{net}} = \int_a^b \vec{F}_n \cdot d\vec{r} = \vec{F}_{\text{net}} \cdot \Delta\vec{r}$$

## Work done by a Gravitational Force

Tension does positive work, gravity does negative work

## Spring Force

A variable force from a spring

Relaxed state: neither compressed or extended

Restoring force: attempting to put spring into a relaxed state

Displacement is the opposite of the spring force

### Hooke's Law

$$F_s = -k\Delta x$$

- $\vec{W}_{\text{net}} > 0 \rightarrow \Delta k > 0 \rightarrow KE \uparrow$
- $\vec{W}_{\text{net}} < 0 \rightarrow \Delta k < 0 \rightarrow KE \downarrow$

By integrating the function  $\vec{F}_s$  with respect to  $x$ , we can develop a formula that looks like this:

$$\vec{W}_s = \frac{1}{2}k((x_i)^2 - (x_f)^2)$$

If starting from **relaxed state**, then  $\vec{W}_s = -\frac{1}{2}kx^2$

## Power

Symbol “P” is used to denote power and it is the time rate of doing work.

$\frac{dW}{dt} \rightarrow$  this is instantaneous power (power at any given place in time)

Think acceleration; how fast the velocity changes per second of time.

$$P_{\text{average}} = \frac{W}{\Delta t} = \int_i^f P(t)dt$$

## Potential Energy

Symbol “U”.

$$\Delta U = -W$$

Change in potential energy is negative work.

$$\vec{F}_{\text{net}} = \vec{F}_{\text{c}} + \vec{F}_{\text{nc}}$$

Total force is the combination of conservative forces (forces that aim to keep the system in equilibrium) and non-conservative forces (forces that appose conservative forces)

- For conservative forces we can define/associate a potential energy “U” with that force{s}

## General Form of Work

$$\Delta U = - \int_{x_i}^{x_f} F(x)dx$$

## Change in Potential Energy

Only depends on the initial and final points, not the path taken between them.

$$U + C = (U_f + C) - (U_i + C) = \Delta U$$

$$\Delta U_g = -W_g = mg\Delta y = mgh$$

Potential energy is dependent on how high the object is and how massive it is.

## Work Energy Theorem : Potential

$$W_c + W_{nc} = W_{net} = \Delta K = -\Delta U - W_{nc}$$

- Valid for both  $\vec{F}_c$  and  $\vec{F}_{nc}$  forces

$\therefore \Delta K + \Delta U = W_{nc}$  These are considered external “agents”

Work energy theorem is present when conservative and non-conservative forces are present

## Mechanical Energy

All energy in a given system.

It is denoted by the letter “E”.

$$E = K + U$$

Kinetic plus potential energy

$$\therefore \Delta K + \Delta U$$

Change in total work energy is work done by non-conservative forces

When no work is done by non-conservative forces ( $W_{nc} = 0$ ):

$$\Delta E = \Delta K + \Delta U = 0$$

$$\implies E_f = E_i$$

This is the conservation of mechanical energy, where energy is not lost in the system. Small amounts of energy can be used to trigger a large energy release.

## Reading a Potential Energy Curve

$$F(x) = \frac{dU(x)}{dx}$$

$$F\Delta x = \Delta W = -\Delta U$$

- for “small enough”  $\Delta x$ , the following can be inferred  $\rightarrow \Delta U = -\vec{F}\Delta x$

**Riemann Sum Equivalent:**  $\lim_{\Delta x \rightarrow \infty} \frac{\Delta U}{\Delta x} = F = -\frac{du}{dx}$

The above is the same as saying:

**Infinite sum:**  $U = -\int F(x)dx \implies F = -\frac{du}{dx}$

Both are the same and can be used interchangeably.

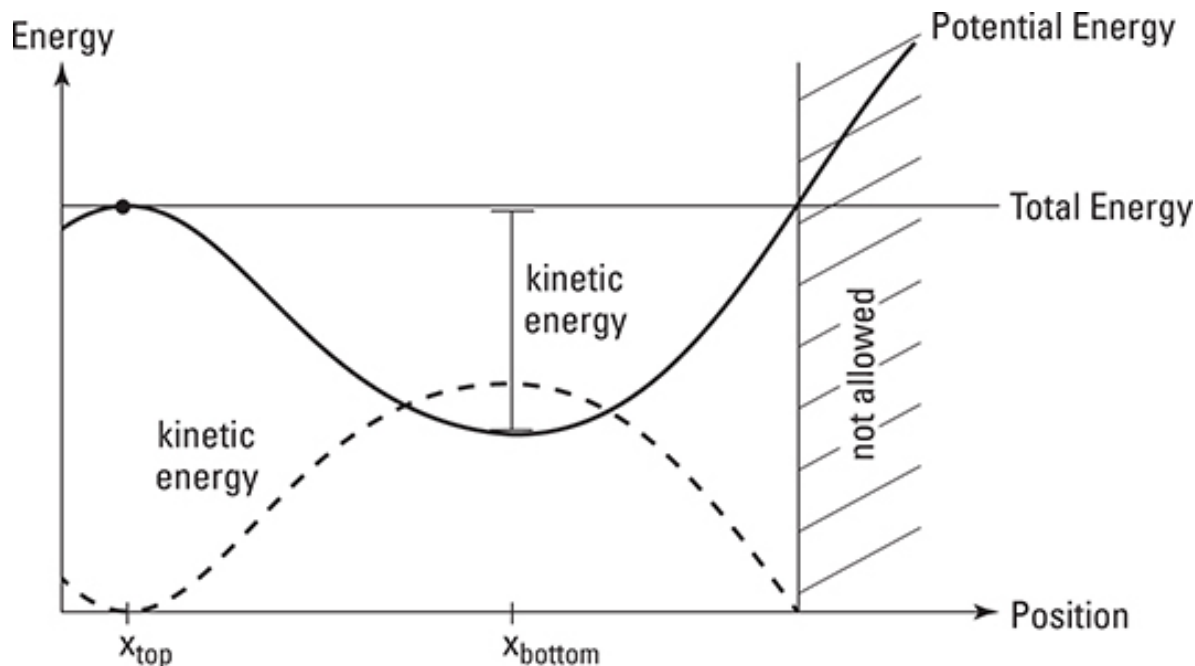


Figure 2: Potential Energy Curve

The following curve can be interpreted as:

- Low points represent the “restoring force”, which is stable equilibrium.
- Peaks represent unstable equilibrium and can be activated with little force.

Equilibrium merely states that the sum ( $\Sigma$ ) of the forces is zero. It does not mean however it is stable to stay there for a prolonged period of time. Since it is it requires a given amount of force  $\vec{F}$

The above curve is a function of potential energy with respect to which is  $U(x)$ . These are called parametric functions, where the  $x$  position is a *parameter* of the function of potential energy. This allows us to find instantaneous potential energy **anywhere** along the  $x$  axis.

## Inherently Unstable



Figure 3: Cubic Graph

Any given force will not be able to overcome the steep curve and will either rest in the recess  $(0, 0)$  or continue to drop off on the  $-x$  direction.



## Conservation of Mechanical Energy

Recall that:

$$E_{\text{mechanical}} = K + U$$

To conserve mechanical energy, this means that no amount of energy in a given system can leave. Therefore, this can only be implemented in a universe where friction is non-existent and thermodynamics does not work. These are examples of how energy can leave a given system.