Name: _____

Chapter 1, Sections 1-5
Chapter 2, Sections 1 ,2, 3, 5, 6
Chapter 3, Sections 1-2

Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. *(24 Points)* Tiffany, a commissioned sales-person earns $100 base pay plus $10 per item sold. She has a total of 36 items.

   a. Express her gross salary G as a linear function of the number of x items sold.

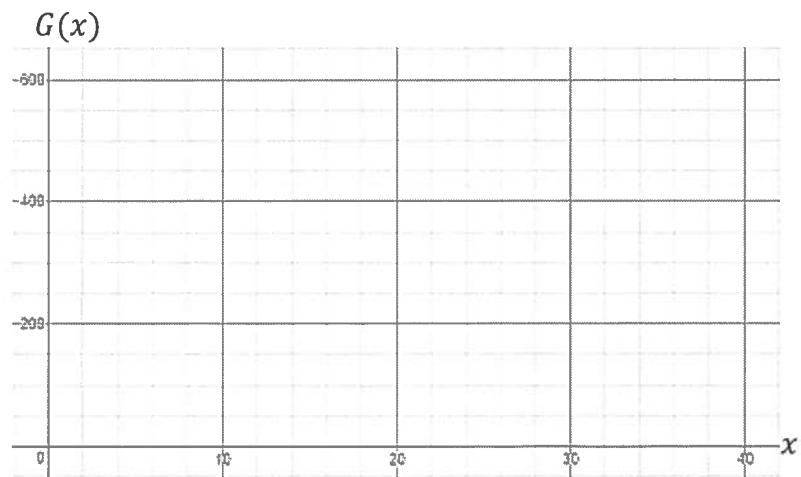   b. What is the domain and range?

   c. Evaluate G(24) describe what they represent in this context.

   d. Interpret $G^{-1}(175) = 7.5$ in this context.

   e. Solve G(x) = 250 and describe what it represents in this context.

   f. Graph the function on the axis provided.

2. *(20 Points)* Cell phones have produced a seismic cultural shift. No other recent invention has incited so much praise—and criticism. In 2000 there were 109.4 million cell phone subscriptions in the United States; since then subscriptions steadily increased to reach 395.9 million in 2016. (Note: Some people had more than one subscription.)

   a. What was the average rate of change in **millions of cell phone subscriptions per year** between 2000 and 2016?

   b. Construct a linear function C(t) for cell phone subscriptions (in millions) **from 2000**.

   c. What is the slope of the line? Explain what the value of the slope means in the context of this problem.

   d. What is the vertical intercept of the line? Explain what the value of the vertical intercept means in the context of this problem.

   e. If U.S. cell phone subscriptions continue to increase at the same rate, how many will there be in 2025?
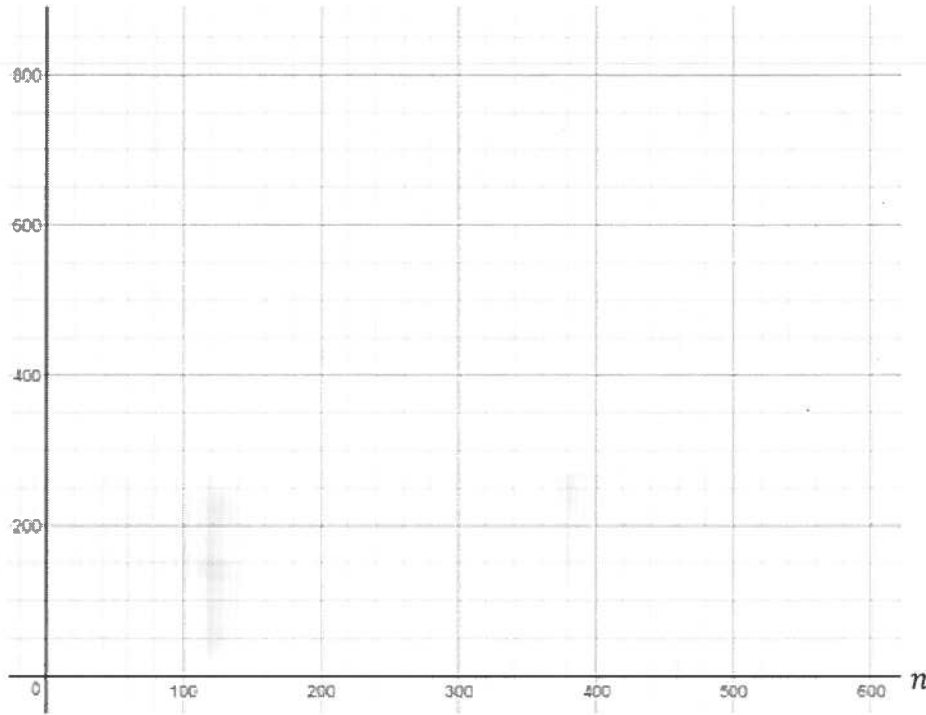
3. *(10 points)* *Doughy Delights* bakery has a fixed monthly cost of $550 with an additional cost of $0.20 per cookie, represented by the cost function

$$C(n) = 550 + .20n$$

where n is the number of cookies. The store manager sells each cookie for $1.40 represented by the revenue function

$$R(n) = 1.40n$$

a. Graph the cost and revenue on the axes below.

b. Based on your graph, estimate how many cookies the manager need to sell before making a profit (revenue exceeds costs)?

4. *(20 points)* A model rocket is launched from the roof of a building. Its flight path is modeled by

$$h(t) = -5t^2 + 30t + 10$$

where h is the height of the rocket above the ground in meters and t is the time after the launch is seconds.
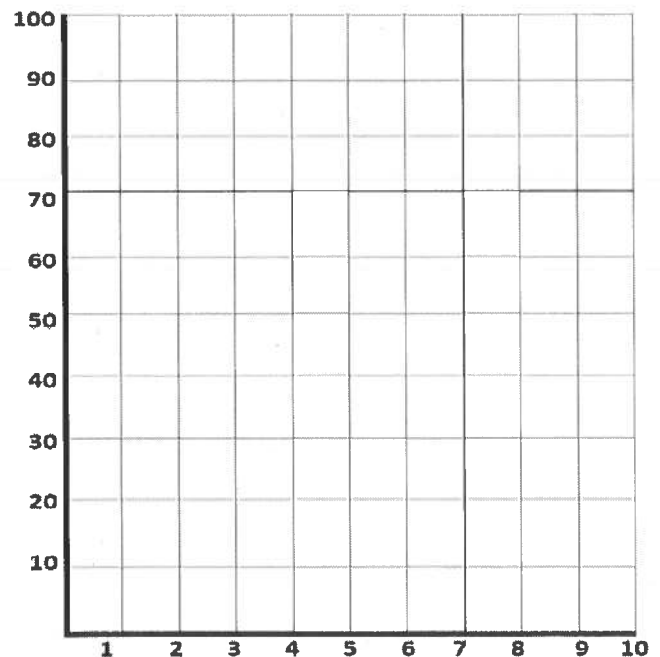
(a) Put this function in vertex form by completing the square.

(b) What was the maximum height of the model rocket?

(c) When did the model rocket reach its maximum height?

(d) If the model rocket does not get hit during flight, when does it hit the ground?

(e) Sketch a graph of the model rocket's path, make sure you correctly label your axis.

5. *(12 points)* Given line $L: 3x + 2y = 12$

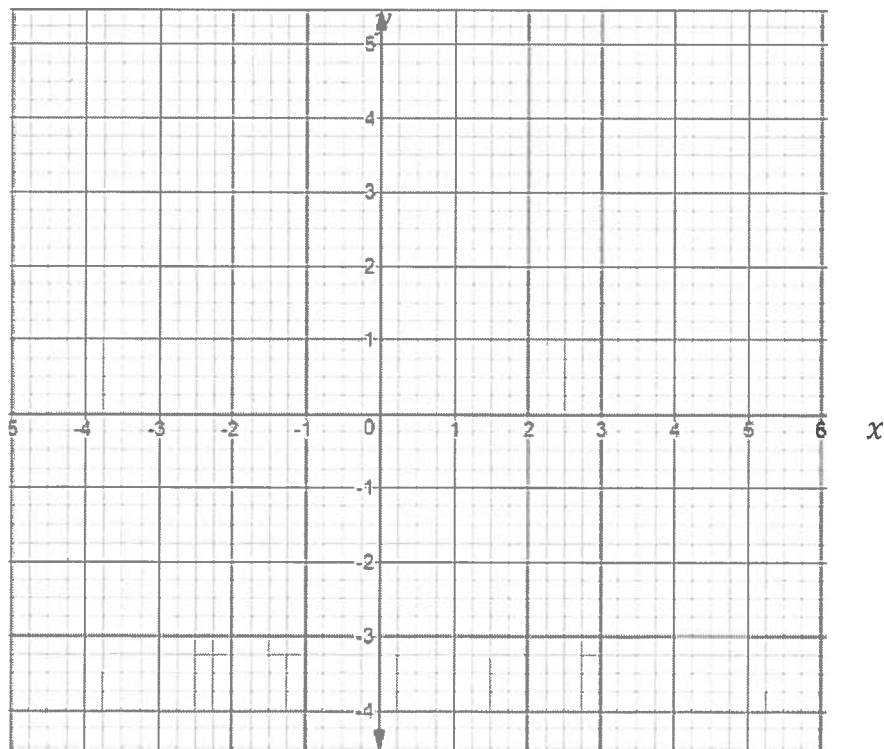   a. What is the slope of line L?

    b. Write the equation (in slope-intercept form) of the line **parallel to line L** through the point $(2, 1)$.

    c. Write the equation (in point-slope form) of the line **perpendicular to line L** through the point $(-3, 5)$.

6. *(14 points)* Graph the following piecewise function over the indicated domain.
$$f(x) = \begin{cases} -x + 3, & x < 1 \\ 2x - 3, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

Bonus Question:

Evaluate the difference quotient for the given function. Simplify your answer.
(you will need to simplify the complex fraction)

$$f(x) = \frac{x+3}{x+1}, \text{ and } \frac{f(x)-f(1)}{x-1}$$

Name: _____

Chapter 1, Sections 1-5
Chapter 2, Sections 1 ,2, 3, 5, 6
Chapter 3, Sections 1-2
Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. *(24 Points)* A plumber charges \$22 for a house call plus \$35 per hour while she is there, up to a maximum of 12 hours.

    a. Express her cost C as a function of the number of t hours.

    b. What is the domain and range?

    c. Evaluate C(6) describe what they represent in this context.

    d. Interpret $C^{-1}(389.5) = 10.5$ in this context.

    e. Solve C(t) = 372 and describe what it represents in this context.

$C(t)$

    f. Graph the function on
        the axis provided.

t
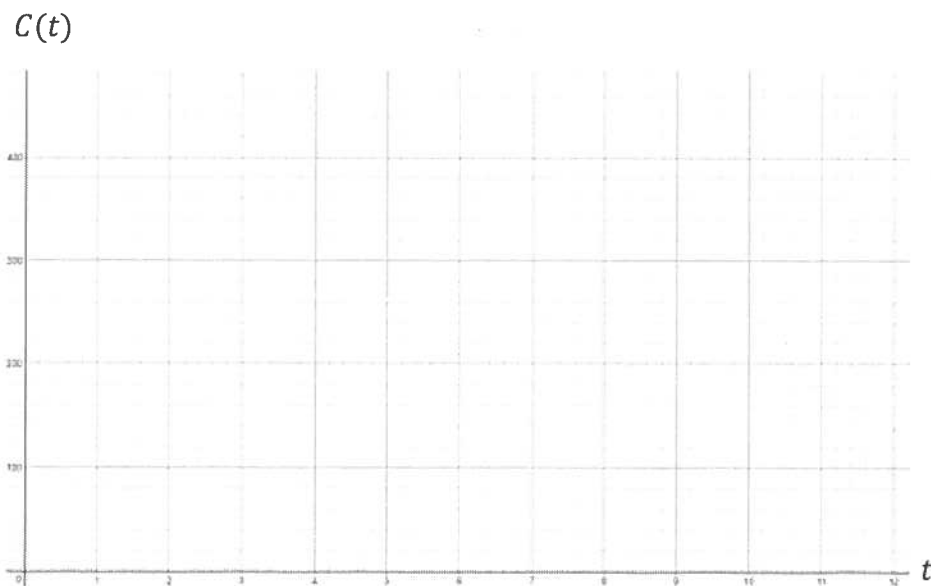
2. *(20 Points)* The monthly rent of a storage shed is a linear function. The size of the shed varies from $150 ft^2$ to $10,000\ ft^2$. A $300 ft^2$ rents for $490.00 where as a $150 ft^2$ rents for $320.00.

   a. What was the average rate of change in **dollars per ft^2**?

   b. Construct a linear function C(t) for the monthly rent in dollars where **t is additional square feet after $150 ft^2$.**

   c. What is the slope of the line? Explain what the value of the slope means in the context of this problem.

   d. What is the vertical intercept of the line? Explain what the value of the vertical intercept means in the context of this problem.

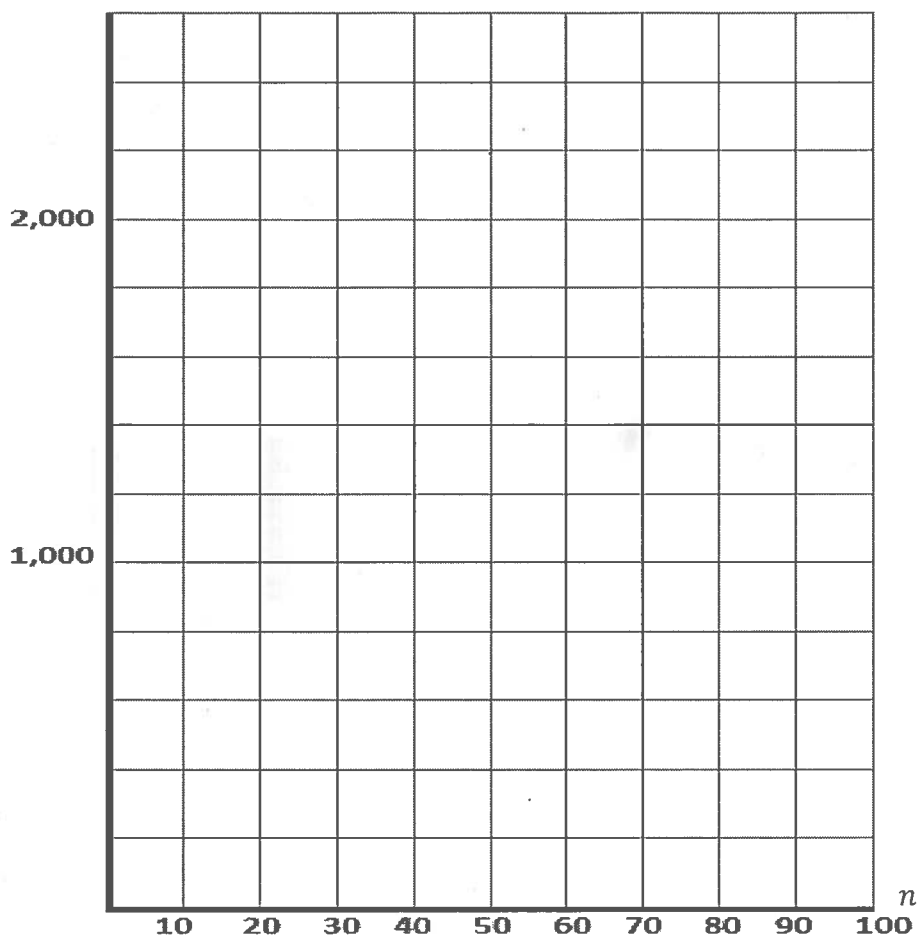   e. What would the cost be to rent a $4,200 ft^2$ shed?

3. *(10 points)* It costs the *Dragon Fly* punk band $480 to pay to rent a concert hall and additional $17 per ticket fee, represented by the cost function

$$C(n) = 480 + 17n$$

where n is the number of tickets. They sell each ticket for $25 represented by the revenue function

$$R(n) = 25n$$

a. Graph the cost and revenue on the axes below.

b. Based on your graph, estimate how many tickets the *Dragon Fly* need to sell before making a profit (revenue exceeds costs)?

4. *(20 points)* Abigail tosses a coin off a bridge into the stream below. The distance, in feet, the coin is above the water is modeled by the equation

$$f(x) = -16x^2 + 96x + 112$$

Where x represents time in seconds.

   a. Put this function in vertex form by completing the square.

   b. What was the maximum height of the coin?

   c. When did the coin reach its maximum height?

   d. If the coin does not get hit during flight, when does it hit the water?

   e. Sketch a graph of the coin's path, make sure you correctly label your axis.
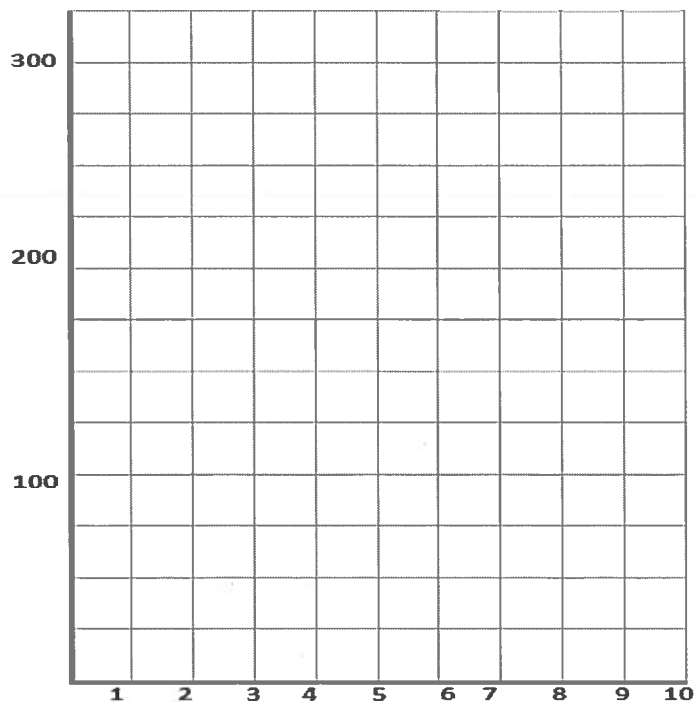
5. *(12 points)* Given line $L: 3x - 2y = 5$

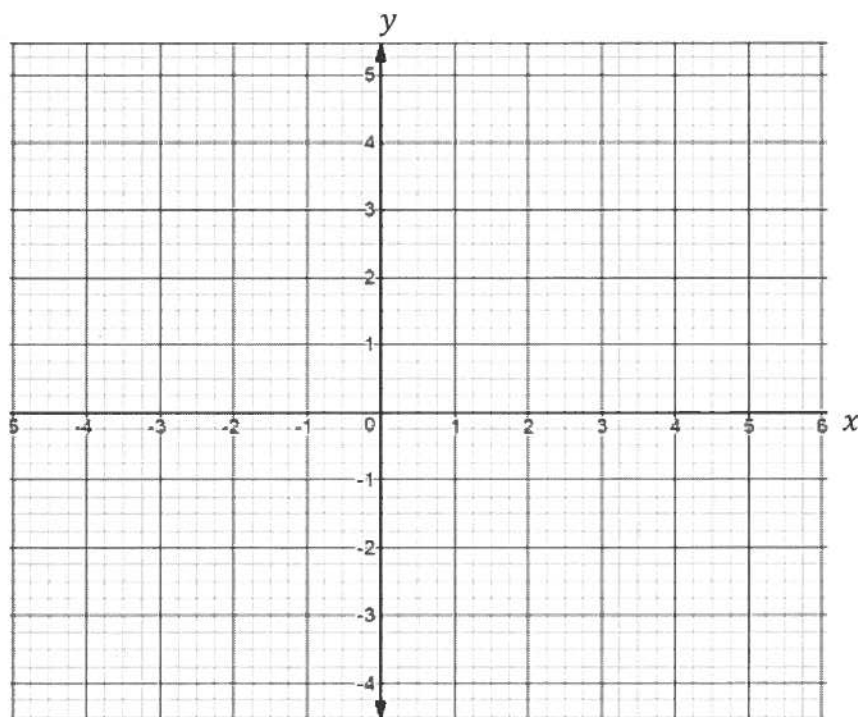   a. What is the slope of line L?

    b. Write the equation (in slope-intercept form) of the line **parallel to line L** through the point $(6, 3)$.

    c. Write the equation (in point-slope form) of the line **perpendicular to line L** through the point $(-3, 5)$.

6. *(14 points)* Graph the following piecewise function over the indicated domain.

$$f(x) = \begin{cases} x + 1, & x < 1 \\ 1, & 1 \leq x < 3 \\ x - 3, & x \geq 3 \end{cases}$$

Bonus Question:

Evaluate the difference quotient for the given function. Simplify your answer.
(you will need to simplify the complex fraction)

$$f(x) = \frac{x+3}{x+1}, \text{ and } \frac{f(x)-f(1)}{x-1}$$

Name: _____   Date: _____

Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. (18 Points) The population of a city is increasing exponentially. In 2000, the city had a population of 80,000. In 2016, the population was 143,558.

    (a) Write a formula, $P(t)$ for the population in **thousands people** of the town t years after 2000.

    (b) Graph the number P(t) over the interval $0 \leq t \leq 26$
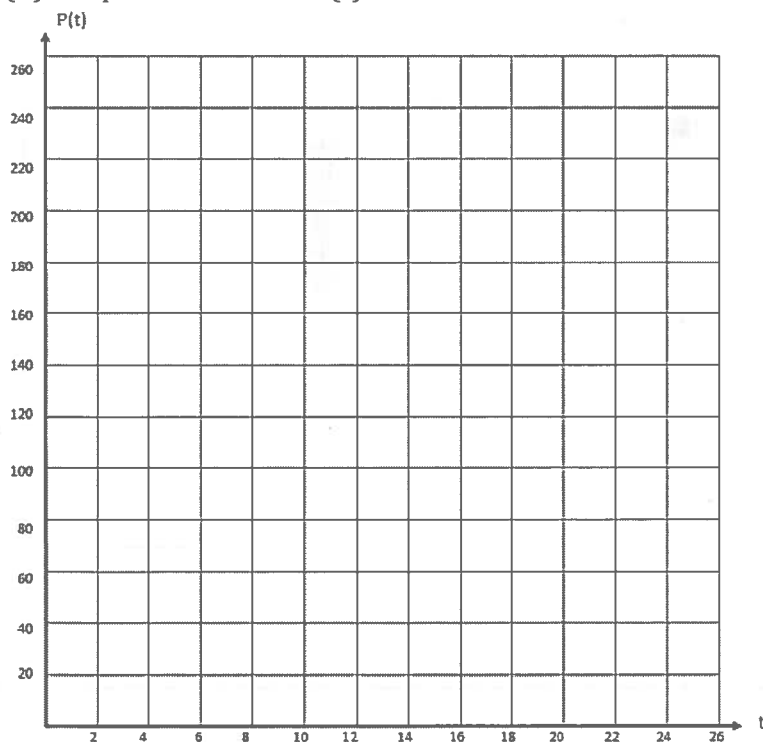
    (c) Us your function $P(t)$ to determine the year when the population will reach 220,000.

2. (12 points) Evaluate the following expressions exactly without using a calculator.

(a) Evaluate $100^{\log 16}$

(b) Evaluate $\dfrac{\log\left(10^{34}\right)}{\log\sqrt{10}}$

(c) Evaluate $e^{\ln 14}$

3. (12 Points) Complete the following tables and correctly write the function that represents each given situation. Make sure you show how you arrived at your solution.

(a) Let $f(x)$ be given in the table below. Find the value of k if $f(x)$ **is exponential.** Then write the function $f(x)$.

x	$f(x)$
0	1
1	k
2	25

(b) Let $g(x)$ be given in the table below. Find the value of k if $g(x)$ **is linear.** Then write the function $g(x)$.

x	$g(x)$
0	5
1	k
2	−7

4. (18 points) Suppose you have $2000 to invest. You have a choice of three accounts:

Bank 1 with a nominal rate of 6.45% compounded monthly.

Bank 2 with a nominal rate of 6.34% compounded yearly

Bank 3 with a nominal rate of 6.6% compounded continuously.

a. Write a formula for the value of your investment if you invest all $2000 in Bank 1. Determine the effective rate for Bank 1.

b. Write a formula for the value of your investment if you invest all $2000 in Bank 2. Determine the effective rate for Bank 2.

c. Write a formula for the value of your investment if you invest all $2000 in Bank 3. Determine the effective rate for Bank 3.

d. Which account is better (in terms of earning more interest)? Explain your reasoning.

5. (18 Points) Psychologists have found that the average walking speed, w, in feet per second, of a person living in a city of population P, in thousands of people, is given by the function

$$w = 0.37 \ln P + 0.05.$$

(a) The population of *Hartford, Connecticut,* is approximately 135,500. Find the average walking speed of people living in *Hartford*.

(b) A sociologist measures the average walking speed in a city to be approximately 2.0 feet/second. Use this information to estimate the population of the city.

(c) Let $w1$ and $w2$ be the average walking speeds in two different cities with populations $P1$ and $P2$, respectively. Using logarithm properties, find a simplified formula for the difference $w1 - w2$ in terms of $P1$ and $P2$.

6. (12 Points) Given a function $y = f(x)$

(a) Describe the effect of the transformation $f\left(\dfrac{x}{2}\right) + 8$

(b) The graph of $f(x)$ contains the point (5, 2).   What point must lie on the reflected graph if the graph is reflected about the x-axis?

(c) The point $(-4, 5)$ lies on the graph of f. What point must lie on the graph of $5f\left(\dfrac{x}{7}\right)$?

(d) The graph of $g(x)$ contains the point (-2, 3).   What is the corresponding point on the graph of $y = -2g(x) + 5$?

7. (10 Points) The graph of the function f(x) is shown below.
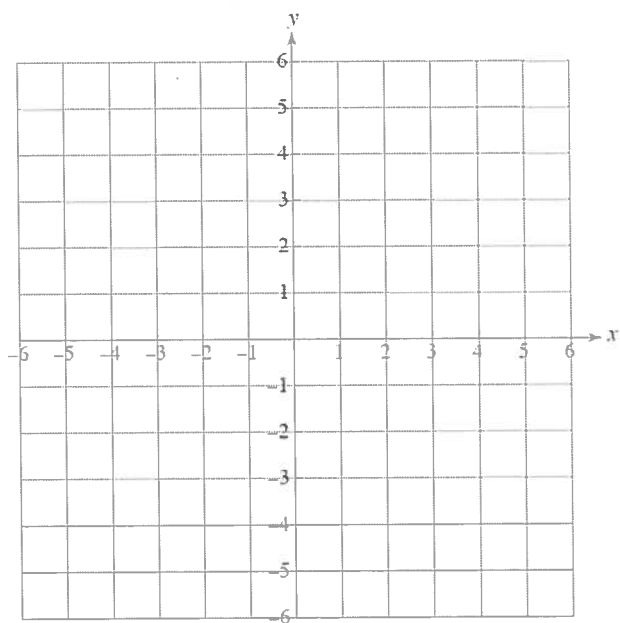   (a) Use this graph to sketch the graph of each of the following functions.
   (b) In each case, state what transformations are applied to obtain the graph from the graph of the original function.
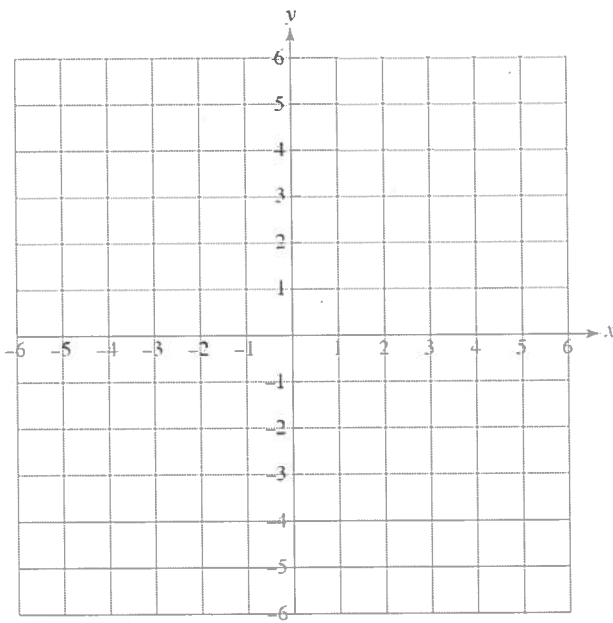
$f(x)$
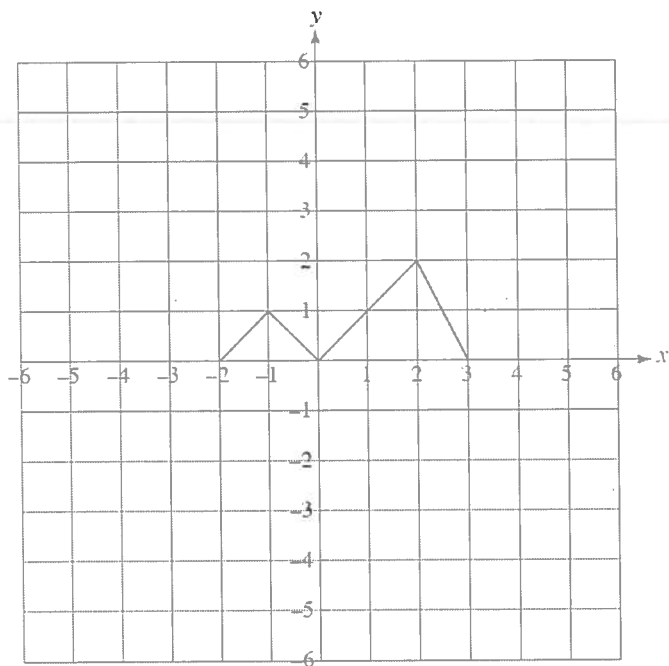
(a) $h(x) = -f(x + 3)$

(b) $k(x) = f\left(\frac{1}{2}x\right) - 1$

Bonus: Abigail tosses a coin off a bridge into the stream below.   The distance, in feet, the coin is above the water is modeled by the equation

$$f(x) = -16x^2 + 96x + 112$$

Where x represents time in seconds.

a. Put this function in vertex form by completing the square.

b. What was the maximum height of the coin?

c. When did the coin reach its maximum height?

d. If the coin does not get hit during flight, when does it hit the water?

Name: _____   Date: _____

Chapter 4, Sections 1, 2, 3, 4, 5
Chapter 5, Sections 1, 2, 3
Chapter 2, Section 4
Chapter 6, Sections 1, 2, 3

Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. (18 Points) The population of a city is increasing exponentially. In 2000, the city had a population of 65,000.   In 2012, the population was 115,694.

   (a) Write a formula, $P(t)$ for the population in **thousands people** of the town t years after 2000.

   (b) Graph the number P(t) over the interval $0 \le t \le 26$
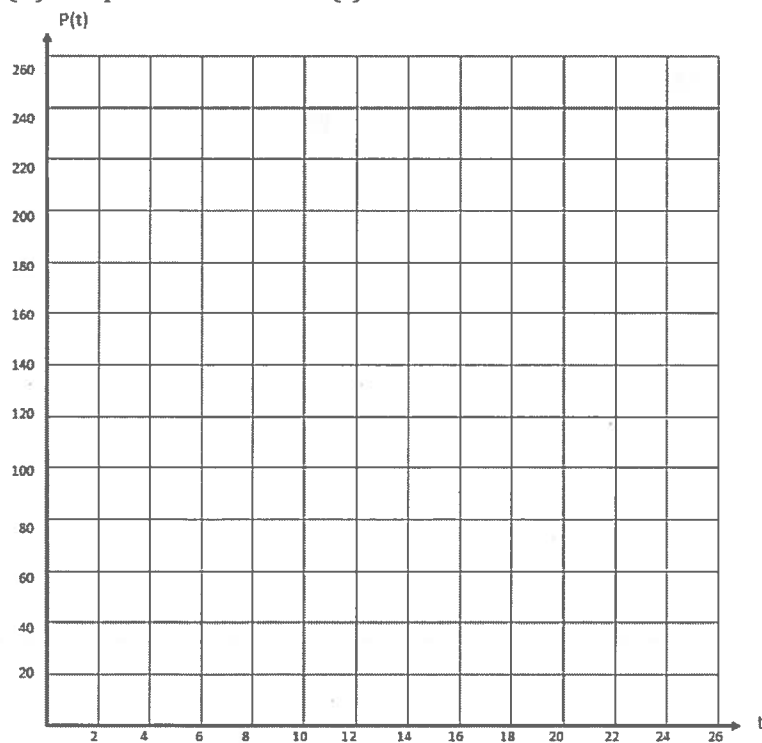
   (c) Us your function $P(t)$ to determine the year when the population will reach 220,000.

2. (12 points) Evaluate the following expressions exactly without using a calculator.

(a) Evaluate $100^{\log 15}$

(b) Evaluate $\dfrac{\log\left(10^{32}\right)}{\log\sqrt{10}}$

(c) Evaluate $e^{\ln 27}$

3. (12 Points) Complete the following tables and correctly write the function that represents each given situation. Make sure you show how you arrived at your solution.

(a) Let $f(x)$ be given in the table below.   Find the value of k if $f(x)$ **is exponential.** Then write the function $f(x)$.

x	$f(x)$
0	4
1	k
2	64

(a) Let $g(x)$ be given in the table below.   Find the value of k if $g(x)$ **is linear.** Then write the function $g(x)$.

x	$g(x)$
0	2
1	k
2	4

4. (18 points) Suppose you have $2000 to invest. You have a choice of three accounts:

Bank 1 with a nominal rate of 6.4% compounded monthly.

Bank 2 with a nominal rate of 6.33% compounded yearly

Bank 3 with a nominal rate of 6.55% compounded continuously.

a. Write a formula for the value of your investment if you invest all $2000 in Bank 1. Determine the effective rate for Bank 1.

b. Write a formula for the value of your investment if you invest all $2000 in Bank 2. Determine the effective rate for Bank 2.

c. Write a formula for the value of your investment if you invest all $2000 in Bank 3. Determine the effective rate for Bank 3.

d. Which account is better (in terms of earning more interest)? Explain your reasoning.

5. (18 Points) Psychologists have found that the average walking speed, w, in feet per second, of a person living in a city of population P, in thousands of people, is given by the function

$$w = 0.37 \ln P + 0.05.$$

(a) The population of *Hartford* is approximately 135,500. Find the average walking speed of people living in *Hartford*.

(b) A sociologist measures the average walking speed in a city to be approximately 2.0 feet/second. Use this information to estimate the population of the city.

(c) Let w_1 and w_2 be the average walking speeds in two different cities with populations P_1 and P_2, respectively. Using logarithm properties, find a simplified formula for the difference $w_1 - w_2$ in terms of P_1 and P_2.

6. (12 Points) Given a function $y = f(x)$

(a) Describe the effect of the transformation $f\left(\dfrac{x}{5}\right) + 9$

(b) The graph of $f(x)$ contains the point $(1, -2)$. What point must lie on the reflected graph if the graph is reflected about the x-axis?

(c) The point $(-4, 5)$ lies on the graph of f. What point must lie on the graph of $5f\left(\dfrac{x}{3}\right)$?

(d) The graph of $g(x)$ contains the point $(-4, -1)$. What is the corresponding point on the graph of $y = 4g(x) + 7$?
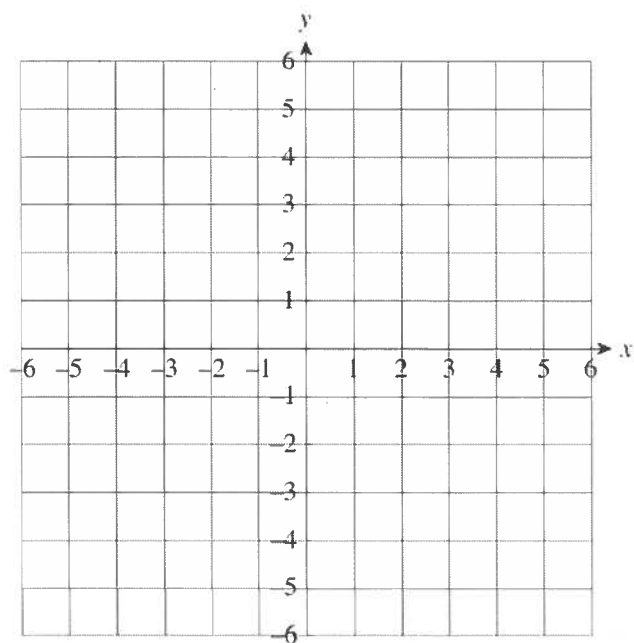
7. (10 Points) The graph of the function f(x) is shown below.
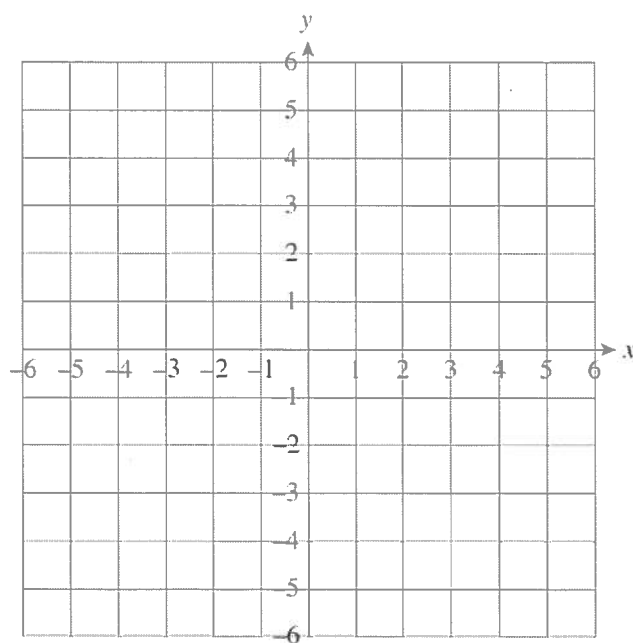   (a) Use this graph to sketch the graph of each of the following functions.
   (b) In each case, state what transformations are applied to obtain the graph from the graph of the original function.
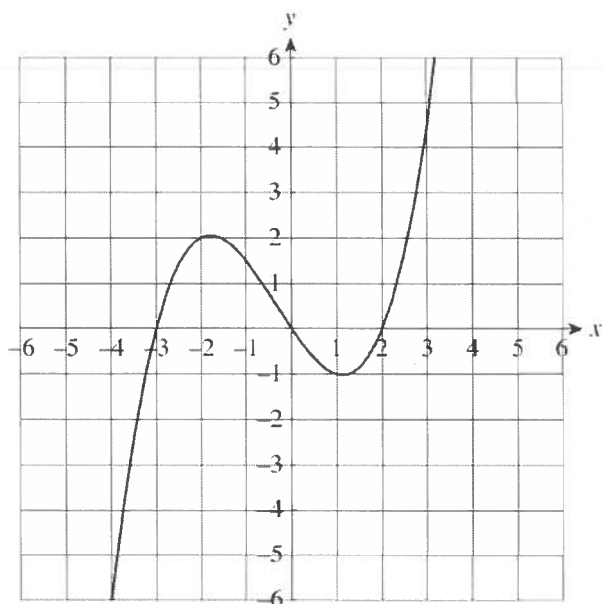
(a) $h(x) = -f(\frac{1}{2}x)$

(b) $k(x) = f(-x) + 2$

Bonus: Abigail tosses a coin off a bridge into the stream below.   The distance, in feet, the coin is above the water is modeled by the equation

$$f(x) = -16x^2 + 96x + 112$$

Where x represents time in seconds.

    a. Put this function in vertex form by completing the square.

    b. What was the maximum height of the coin?

    c. When did the coin reach its maximum height?

    d. If the coin does not get hit during flight, when does it hit the water?

Name: _____   Date: _____

Trigonometric Identities

- **Pythagorean Identities:**

$$\sin^2 t + \cos^2 t = 1$$
$$\tan^2 t + 1 = \sec^2 t \quad \text{and} \quad 1 + \cot^2 t = \csc^2 t$$

- **Double-Angle Formulas:**

$$\sin 2t = 2 \sin t \cos t$$
$$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2 \sin^2 t = 2 \cos^2 t - 1$$
$$\tan 2t = \frac{2 \tan t}{1 - \tan^2 t}$$

- **Negative Angle Identities:**

$$\sin(-t) = -\sin t, \qquad \cos(-t) = \cos t. \qquad \tan(-t) = -\tan t$$

- **Cofunction Identities:**

$$\sin t = \cos\left(t - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - t\right) \qquad \cos t = \sin\left(t + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - t\right)$$

- **Sum and Difference Identities:**

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \sin\phi\cos\theta \qquad \cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\sin(\theta - \phi) = \sin\theta\cos\phi - \sin\phi\cos\theta \qquad \cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

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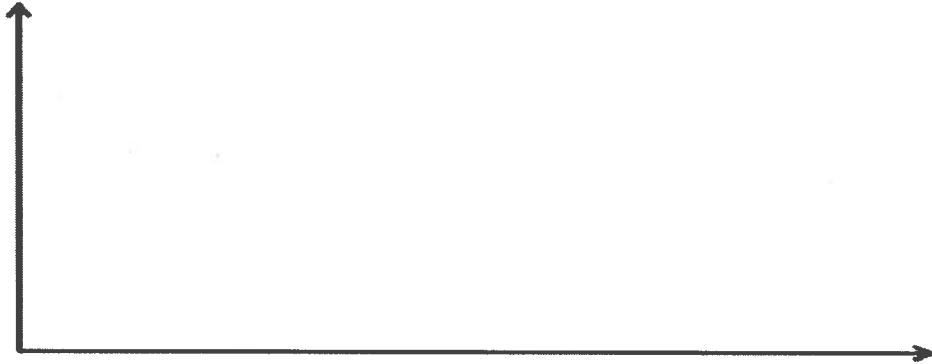
1. (10 points) Consider the function: $y = 2\sin(3t + 12) - 1$

   (a) Determine the maximum, minimum, amplitude, period, and horizontal shift.

   (b) Describe in words how to obtain the graph $y = 2\sin(3t + 12) - 1$ from the graph $y = \sin(3t)$?

2. (16 points) A caribou population in a national park dropped from a high of 132,000 in 1943 to a low of 63,000 in 1989, and has risen since then.   Scientists hypothesize that the population follows a sinusoidal cycle affected by predation and other environmental conditions, and that the caribou will again reach their previous high.

(a) Carefully sketch a graph that represents 2 periods of the change in the caribou population. Be sure to include all necessary information to best describe the situation.   Be sure to label the units of your axes.

(b) What are the values for the period, amplitude, midline, maximum and minimum?

(c) Predict the next year when the population will again be 132,000.

(d) Construct a formula to represent the sinusoidal function.

3. (12 points) Find the exact value of the following without a calculator.   If it is undefined, enter "undefined". Make sure you clearly show how you arrived at your solution.

   (a) $\cos 900°$

   (b) $\sin 765°$

4. (10 points) If $\dfrac{\pi}{2} \leq \theta \leq \pi$ and $\sin \theta = \dfrac{7}{9}$, find <u>exact values</u> for the other five trigonometric functions.

5. (12 points) Find the exact values of the following quantities without using a calculator. Indicate your reasoning. Use radian measure for any angles.

   (a) $\cos^{-1}\left(\dfrac{-1}{2}\right)$

   (b) $\cos\left[\cos^{-1}\left(\dfrac{-1}{2}\right)\right]$

   (c) $\cos^{-1}\left(\cos\left(\dfrac{4\pi}{3}\right)\right)$

6. (10 points) Prove the following identity algebraically:
$$\tan t = \frac{1 - \cos 2t}{\sin 2t}$$

7. (10 points) Solve for θ, an angle in a right triangle, if $5\cos(2\theta)+6=2\cos(2\theta)+7$.   Find the degree to 3 decimal places.

8. (10 points) A fire department's longest ladder is 111 feet long, and the safety regulation states that they can use it for rescues up to 104 feet off the ground. What is the maximum safe angle of elevation for the rescue ladder? Round to the nearest degree.

9. (10 points)   If $\dfrac{3\pi}{2}<\theta<2\pi$ and $\sin(\theta)=\dfrac{-4}{9}$,   find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ exactly.

Bonus: Psychologists have found that the average walking speed, w, in feet per second, of a person living in a city of population P, in thousands of people, is given by the function

$$w = 0.37 \ln P + 0.05.$$

(a) The population of *San Antonio, Texas is about 1,236,249.*   Find the average walking speed of people living in *San Antonio.*

(b) A sociologist measures the average walking speed in a city to be approximately 2.0 feet/second. Use this information to estimate the population of the city.

(c) Let $w1$ and $w2$ be the average walking speeds in two different cities with populations $P1$ and $P2$, respectively. Using logarithm properties, find a simplified formula for the difference $w1 - w2$ in terms of $P1$ and $P2$.

Name: _____   Date: _____

Trigonometric Identities

- **Pythagorean Identities:**
$$\sin^2 t + \cos^2 t = 1$$
$$\tan^2 t + 1 = \sec^2 t \qquad \text{and} \qquad 1 + \cot^2 t = \csc^2 t$$

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$$\sin 2t = 2 \sin t \cos t$$
$$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2 \sin^2 t = 2 \cos^2 t - 1$$
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$$\sin(-t) = -\sin t, \qquad \cos(-t) = \cos t, \qquad \tan(-t) = -\tan t$$

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- **Sum and Difference Identities:**
$$\sin(\theta + \phi) = \sin\theta\cos\phi + \sin\phi\cos\theta \qquad \cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\sin(\theta - \phi) = \sin\theta\cos\phi - \sin\phi\cos\theta \qquad \cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

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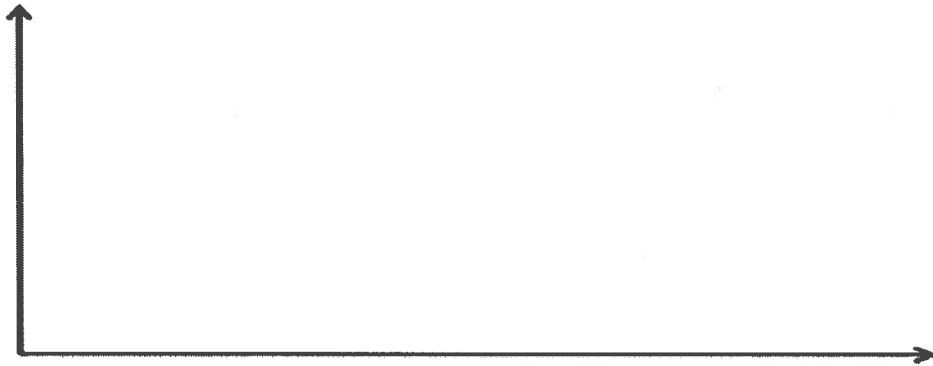
1. (10 points) Consider the function: $y = 2\sin(2t + 8) - 3$

   (a) Determine the maximum, minimum, amplitude, period, and horizontal shift.

   (b) Describe in words how to obtain the graph $y = 2\sin(2t + 8) - 3$ from the graph $y = \sin(2t)$?

2. (16 points) A caribou population in a national park dropped from a high of 132,000 in 1943 to a low of 63,000 in 1989, and has risen since then.   Scientists hypothesize that the population follows a sinusoidal cycle affected by predation and other environmental conditions, and that the caribou will again reach their previous high.

(a) Carefully sketch a graph that represents 2 periods of the change in the caribou population. Be sure to include all necessary information to best describe the situation.   Be sure to label the units of your axes.

(b) What are the values for the period, amplitude, midline, maximum and minimum?

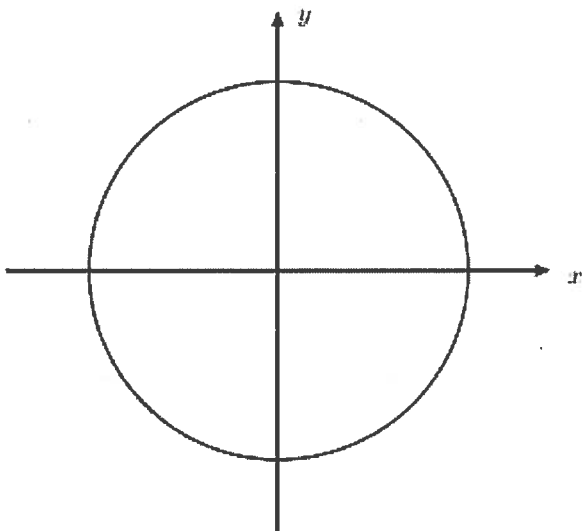(c) Predict the next year when the population will again be 132,000.

(d) Construct a formula to represent the sinusoidal function.

3. (12 points) Find the exact value of the following without a calculator.   If it is undefined, enter "undefined". Make sure you clearly show how you arrived at your solution.

(a) $\cos 990°$

(b) $\sin 765°$

4. (10 points) If $\dfrac{\pi}{2} \leq \theta \leq \pi$ and $\sin \theta = \dfrac{4}{7}$, find **exact values** for the other five trigonometric functions.

5. (12 points) Find the exact values of the following quantities without using a calculator. Indicate your reasoning. Use radian measure for any angles.

(a) $\cos^{-1}\left(\dfrac{-1}{2}\right)$

(b) $\cos\left[\cos^{-1}\left(\dfrac{-1}{2}\right)\right]$

(c) $\cos^{-1}\left(\cos\left(\dfrac{4\pi}{3}\right)\right)$

6. (10 points) Prove the following identity algebraically:
$$\tan t = \frac{1-\cos 2t}{\sin 2t}$$

7. (10 points) Solve for θ, an angle in a right triangle, if $5\cos(2\theta)+6=2\cos(2\theta)+7$. Find the degree to 3 decimal places.

8. (10 points) A fire department's longest ladder is 111 feet long, and the safety regulation states that they can use it for rescues up to 105 feet off the ground. What is the maximum safe angle of elevation for the rescue ladder? Round to the nearest degree.

9. (10 points) If $\dfrac{3\pi}{2}<\theta<2\pi$ and $\sin(\theta)=\dfrac{-4}{7}$, find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ exactly.

Bonus: Psychologists have found that the average walking speed, w, in feet per second, of a person living in a city of population P, in thousands of people, is given by the function

$$w = 0.37 \ln P + 0.05.$$

(a) The population of *San Antonio, Texas is about 1,236,249.*   Find the average walking speed of people living in *San Antonio.*

(b) A sociologist measures the average walking speed in a city to be approximately 2.0 feet/second. Use this information to estimate the population of the city.

(c) Let $w1$ and $w2$ be the average walking speeds in two different cities with populations $P1$ and $P2$, respectively. Using logarithm properties, find a simplified formula for the difference $w1 - w2$ in terms of $P1$ and $P2$.

Name: _____   Date: _____

Chapter 10, Sections 1,2
Chapter 11, Sections 1-5

Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. (12 Points)   The profit earned by a producer to manufacture and sell n units of a good is given by $P(n) = 13n - 2821$. The average profit for n units is given   by $A(n) = \dfrac{P(n)}{n}$.

   A)   Compute $A(1)$, $A(217)$, $A(284)$.

   B)   Interpret the economic significance of each the values in part (A).

   C)   What trend do you notice in the values of $A(n)$ as n gets large?   Explain this trend in economic terms.

2. (10 points) Let $P = 30\ln(t)$ give the annual profit of a company (in thousands of dollars) t years after its formation.

   What is $P^{-1}(38)$?   *Round to the nearest whole number and include units.*   Explain what this expression means in the context of this problem.

3. (10 points) List a set functions $(g(x), h(x), p(x))$ that is a decomposition of $f(x) = \cot^4(\ln x)$ in the form of $g(h(p(x)))$.

4. (10 points) Write a possible formula for a rational function, $f(x)$, with zeros at $x = -7$, $x = 2$, vertical asymptotes at $x = 11$, $x = -11$, and a horizontal asymptote at $y = 4$.

5.
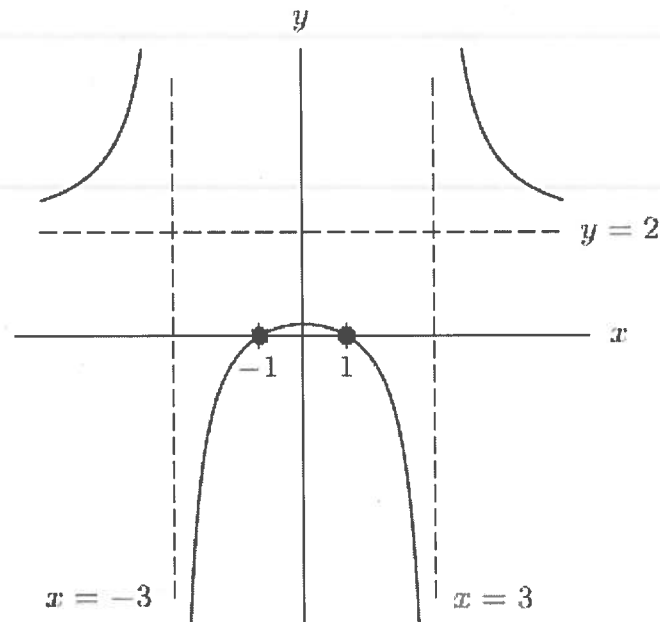(20 points) Given the function $f(x) = \dfrac{1}{x+7} - \dfrac{x}{x-6}$.

A) Rewrite the function $f(x) = \dfrac{p(x)}{q(x)}$, a ratio of polynomials *(Get a common denominator and subtract).*

B) Find any vertical asymptotes

C) Find any horizontal asymptotes.

D) Describe the long term behavior of the graph.

6.
(12 points) The graph of $f(x) = \dfrac{16}{x^2 - 9} + 2$ is shown below.

A) State the domain of f(x). What are the vertical asymptotes?

B) Does f(x) have an inverse over the domain you stated in part A?   Explain your reasoning.

C) Define (Restrict) a new domain and find the inverse of $f(x) = \dfrac{16}{x^2 - 9} + 2$.

7. (8 points) Suppose f and g are invertible functions such that $f(-9) = -5$, $f(-7) = -2$, $f(-5) = -6$, $g(-13) = -12$, $g(-7) = -13$, and $g(-2) = -5$.   Find $f^{-1}(g(f(-7)))$.

8. (18 points) The graph of a polynomial $f(x)$ is shown.

A) What is the y-intercept of $f(x)$?

B) What are the zeros of $f(x)$? State which of these are multiple zeros and whether their multiplicities are even or odd. Give reasons for your conclusions.

C) What is the long run behavior of $f(x)$?

D) Find a possible formula for f(x).   Do not multiply the factors.

Bonus   If $\dfrac{3\pi}{2} < \theta < 2\pi$ and $\sin(\theta) = \dfrac{-4}{7}$,   find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ exactly.

Double Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\qquad = 2\cos^2\theta - 1$

$\qquad = 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Name: _____   Date: _____

Chapter 10, Sections 1,2
Chapter 11, Sections 1-5

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1. (12 Points)   The profit earned by a producer to manufacture and sell n units of a good is given by $P(n) = 14n - 3038$. The average profit for n units is given   by $A(n) = \dfrac{P(n)}{n}$.

   A)   Compute $A(1)$, $A(217)$, $A(284)$.

   B)   Interpret the economic significance of each the values in part (A).

   C)   What trend do you notice in the values of $A(n)$ as n gets large?   Explain this trend in economic terms.

2. (10 points) Let $P = 30\ln(t)$ give the annual profit of a company (in thousands of dollars) t years after its formation.

   What is $P^{-1}(80)$?   *Round to the nearest whole number and include units.*   Explain what this expression means in the context of this problem.

3. (10 points) List a set functions $(g(x), h(x), p(x))$ that is a decomposition of $f(x) = \cos^6(\ln x)$ in the form of $g(h(p(x)))$.

4. (10 points) Write a possible formula for a rational function, $f(x)$, with zeros at $x = -5$, $x = 2$, vertical asymptotes at $x = 11$, $x = -13$, and a horizontal asymptote at $y = 2$.

5.
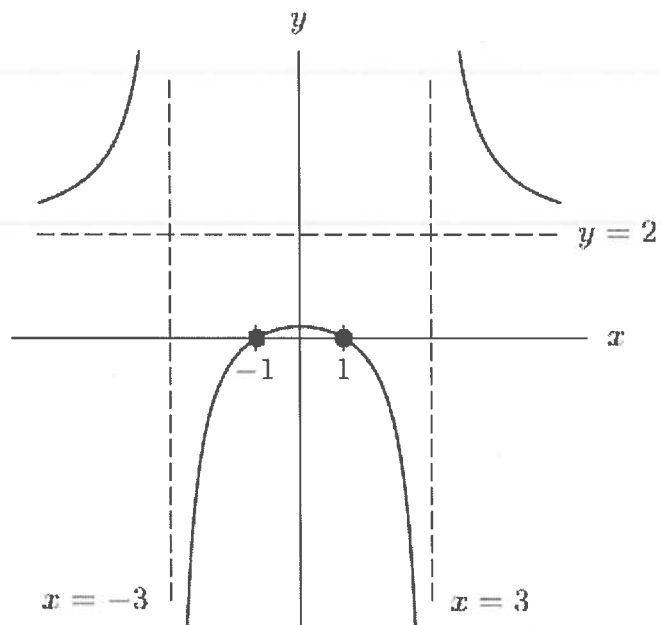(20 points) Given the function $f(x) = \dfrac{1}{x+6} - \dfrac{x}{x-3}$.

A) Rewrite the function $f(x) = \dfrac{p(x)}{q(x)}$, a ratio of polynomials *(Get a common denominator and subtract).*

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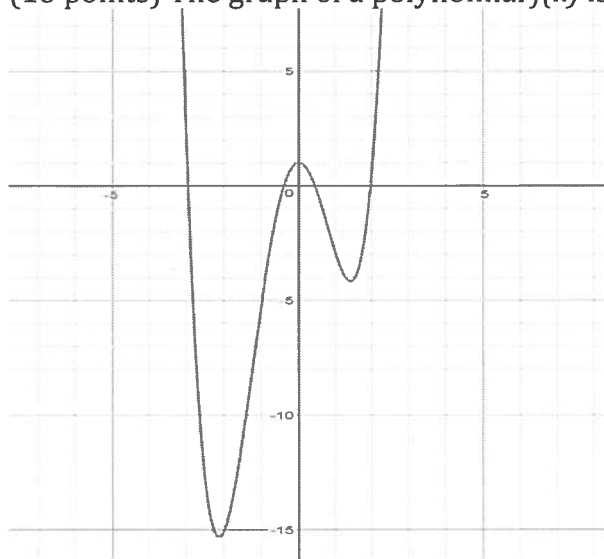(12 points) The graph of $f(x) = \dfrac{16}{x^2 - 9} + 2$ is shown below.

A) State the domain of f(x). What are the vertical asymptotes?

B) Does f(x) have an inverse over the domain you stated in part A?   Explain your reasoning.

C) Define (Restrict) a new domain and find the inverse of $f(x) = \dfrac{16}{x^2 - 9} + 2$.

7. (8 points) Suppose f and g are invertible functions such that $f(-12)=-9$, $f(-13)=-4$, $f(-14)=-7$, $g(-2)=-12$, $g(-7)=-4$, and $g(-4)=-9$.   Find $f^{-1}(g(f(-13)))$.

8. (18 points) The graph of a polynomial $f(x)$ is shown.

A) What is the y-intercept of $f(x)$?

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