MATH 338

MIDTERM 2

THURSDAY, NOVEMBER 3, 2016

Your name: ______________________

Your scores (to be filled in by Dr. Wynne):

Problem 1:	____/10
Problem 2:	____/9
Problem 3:	____/8
Problem 4:	____/16
Problem 5:	____/12
Total:	____/55

You have 75 minutes to complete this exam. This exam is closed book and closed notes with the exception of your single sheet of notes (front and back).

For full credit, show all work except for final numerical calculations (which can be done using a scientific calculator).

Problem 1. For each of the following scenarios, determine whether it is appropriate to do inference of the types we have learned in Chapters 6-8. If it is appropriate, identify the design (one sample, two independent samples, or matched pairs) and type of critical value (z* or t*) and that should be used. If it is not appropriate, explain why.

1 pt yes/no, 1 pt explanation or 0.5 pt design and 0.5 pt critical value type

A) [2 pt] The state of California wants to know how long it takes people to drive to work. They design their study so that people from large cities are disproportionately likely to be included in the sample.

No, it is not appropriate to do inference here. We do not have a simple random sample.

B) [2 pt] A group of 100 gamblers wants to know whether gamblers win more money in Las Vegas or Atlantic City. Half the group is randomly sent to Las Vegas and the other half goes to Atlantic City.

Yes, two independent samples using a t* critical value (it's a two-sample t CI or HT for difference of means since neither population standard deviation is known)

C) [2 pt] A company wants to know if its salaries are competitive. It compares the mean starting salary of a simple random sample of 5 recent hires to the estimated mean salary of all employees in its industry.

No, we have a small sample size from a highly skewed population distribution, and/or we are not comparing two population means for the same variable (salary of new hires vs. all workers)

D) [2 pt] An integrated circuit manufacturer randomly selects 200 silicon wafers from a shipment and accepts the shipment if it believes at least 90% of the wafers in the shipment are acceptable.

Yes, one sample using a z* critical value (it's a one-proportion z CI or HT)

E) [2 pt] A researcher wants to know whether the picture or sound of a television program is more distracting. The researcher measures how long it takes the same set of 45 subjects to complete a series of tasks when the picture is on and sound is off, compared to when the sound is on and picture is off.

Yes, matched pairs using a t* critical value (it's a matched pairs CI or HT, which almost always uses t*)

Problem 2. Katie wants to make some money tutoring high school students. She asks 10 of her sister's friends' moms what they would be willing to pay per hour of statistics tutoring. She computes a 95% confidence interval for the mean amount parents are willing to pay per hour as (22.47, 27.53).

A) [1 pt] Katie claims that she is 95% confident that the true mean amount parents will pay is between \$22.47 and \$27.53. Circle the letter corresponding to the most correct interpretation of her statement.

(a) 95% of population means will be between 22.47 and 27.53

(b) The probability that the population mean is between 22.47 and 27.53 is 95% (or 0.95)

(c) 95% of samples will produce a confidence interval containing the population mean, which may or may not be between 22.47 and 27.53

c: 95% of samples will produce a confidence interval containing the population mean

B) [4 pt] Katie's confidence interval was computed from a sample mean of 25 dollars and a sample standard deviation of 4.08 dollars. Did Katie use a z* or a t* critical value? How do you know?

1 pt identify margin of error = 25 – 22.47 = 2.53

1 pt use margin of error formula = (critical value)(standard deviation/error of estimate)

2 pt solve 2.53 = (crit. value) (4.08/sqrt(10)) and get crit. value = 1.96 → she used a z* critical value

OR

2 pt compute margin of error using z* = (1.96)(4.08/sqrt(10)) = 2.53 and using t* = (2.262)(4.08/sqrt(10))

= 2.92 → she used a z* critical value

C) [2 pt] What concerns, if any, do you have any concerns about Katie's method of determining the amount parents will pay per hour for statistics tutoring?

sister's friends' moms are unlikely to be a simple random sample AND/OR: small sample size from a population distribution that cannot be assumed normal AND/OR: should have used t* since we don't know population standard deviation (1 pt each up to a maximum of 2 pts)

D) [2 pt] If Katie does a new study to satisfy your concerns, which of these steps could she take in order to most likely <u>decrease</u> the variability of her CI? Circle the letter corresponding to each correct step.

(a) Increase the confidence level to 99%

(b) Decrease the confidence level to 90%

(c) Increase the sample size to 20 parents

(d) Decrease the sample size to 5 parents

1 pt selecting b, but not a; 1 pt selecting c, but not d

Problem 3. In 2012, Microsoft commissioned the “Bing it On” Challenge. In one test of the challenge, a simple random sample of 1,000 Internet users indicated whether they preferred the left or right screen of search results, without knowing which set of results came from Bing and which came from Google. Of the roughly 880 who had a preference, 60% preferred Bing’s results and 40% preferred Google’s results.

A) [5 pt] Assuming that the assumptions for inference are met, construct a 95% confidence interval for the true proportion of Internet users (with a search engine preference) who prefer Bing’s search results. Do you agree with Microsoft’s claim that people who take the challenge prefer Bing over Google?

1 pt use CI = point estimate +/- critical value * standard deviation of point estimate

1 pt use the correct version: CI = $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

1 pt plug in correctly: CI = 0.6 +/- 1.96 (sqrt(0.6*0.4/880)) OR CI = 0.52 +/- 1.96 (sqrt(0.52*0524/1000))

1 pt either way get CI = 0.6 +/- 0.03 or (0.57, 0.63); or 0.52 +/- 0.03 or (0.49, 0.55)

1 pt we are 95% confident that between 57% and 63% (49% and 55%) of Internet users with a preference will prefer Bing. (Or, 60% /52% of challenge takers preferred Bing, with a 95% margin of error of 3 percentage points.) Therefore our statistical evidence supports (does not support if using point estimate of 52%) Microsoft’s claims.

B) [2 pt] For this study, explain what would be a Type I Error and a Type II Error.

1 pt Type I Error: claiming that Bing is preferred to Google when it is not

1 pt Type II Error: not claiming that Bing is preferred to Google when it is

Accept reasonable decisions as Type I Error (e.g., launching an advertising campaign touting Bing’s superiority when people don’t prefer it to Google) or Type II Error (shutting down Bing because Microsoft believes it doesn’t beat Google, even though it does)

C) [1 pt] In response to a competing study that showed a different result, a Bing researcher asked, “Which has greater statistical power: [a sample of] 1,000 people or 333 people?” Answer the question.

Sample of 1000 people has greater statistical power

Problem 4. It is generally accepted that Intelligence Quotient (IQ) test scores are normally distributed with a mean of 100 and population standard deviation of 15. You suspect that an online IQ test is giving people higher scores than their true IQ. You obtain a simple random sample of 100 subjects and find that their mean test score is an IQ of 103. Assume that the population standard deviation does not change among different IQ tests, and that the assumptions for inference are met.

A) [7 pt] At the 1% significance level, can you conclude that the online test is giving higher IQ scores?

1 pt work in a one-sample z HT framework

2 pts H_0: scores are not inflated (mu = 100 or mu $\leq$ 100) and H_a: scores are inflated (mu > 100)

1 pt test statistic: z = (103 – 100)/(15/sqrt(100)) = 2

1 pt p-value = 1 – 0.9772 = 0.0228 OR Critical Region: z > 2.326

1 pt Fail to Reject H_0 since p > alpha and/or z is not in the rejection region

1 pt We cannot conclude at the 1% level that the online test is giving higher IQ scores

B) [1 pt] What would happen to the value of the test statistic if the significance level increased? (circle one answer below)

increase decrease stay the same unable to determine

stay the same

C) [1 pt] What would happen to the probability of Type I Error if the significance level increased? (circle one answer below)

increase decrease stay the same unable to determine

increase

D) [5 pt] What is the power of this test under the alternative $\mu_a = 105$, that is, that the online IQ test truly inflates scores by, on average, 5 points? (Keep the 1% significance level)

1 pt rejection region is $Z > 2.326$

1.5 pt rejection region in x-bar scale is x-bar > (2.326)(15/sqrt(100))+100 = 103.489

1.5 pt P(x-bar in rejection region under H_a) = P (x-bar > 103.489)

= P(Z > (103.489 – 105)/(15/sqrt(100))) = P(Z > -1.00)

1 pt P(Z > -1.00) = 1 – 0.1587 = 0.8413 → Power of this test is roughly 84%

E) [2 pt] What is the probability of Type II Error under this alternative?

1 pt P(Type II Error) = 1 - power

1 pt plug in answer to part (C) for power → Probability of Type II error is about 16%

Problem 5. In a 2011 study, researchers compared nitrogen oxide emissions of a Volkswagen Passat under EPA testing conditions and under real-world conditions driving along the 10 Freeway between Los Angeles and Ontario. Assume that nitrogen oxide emissions, in grams per kilometer driven (g/km), are normally distributed across multiple repetitions of the same test, and that each repetition is independent, so we can perform statistical inference with even a small number of emissions tests.

In 3 repetitions under EPA test conditions, the Passat averaged 0.016 g/km of nitrogen oxide emissions with a standard deviation of 0.002 g/km. In 2 repetitions of the drive along the 10 Freeway, the Passat averaged 0.344 g/km of nitrogen oxide emissions with a standard deviation of 0.096.

A) [3 pt] Compute the standard error of the mean nitrogen oxide emissions under real-world conditions.

1 pt SE = s/sqrt(n)

1 pt plug in correctly: s = 0.096, n = 2

1 pt SE = 0.096/sqrt(2) = 0.068

B) [8 pt] Do you believe, at the 5% significance level, that this Passat performs differently under EPA test conditions compared to real-world driving conditions? (Hint: this is not a matched pairs design)

CI OR HT:

1 pt work in two-sample t framework

1 pt min(3 – 1, 2 – 1) = 1 degrees of freedom; 1 pt t* = 12.71

HT:

2 pt H_0: no difference in emissions between EPA test conditions and real-world driving conditions, mu1 = mu2 where Population 1 is EPA test emissions and Population 2 is real world emissions (or other way around), H_a: difference in emissions between the two conditions, mu1 not equal to mu2

1 pt test statistic t = (0.016 – 0.344)/sqrt(0.002^2/3 + 0.096^2/2) = -4.83

1 pt critical region is |t| > 12.71; 1 pt since |-4.83| < 12.71, we fail to reject H_0

1 pt We do not have evidence, at the 5% level, that this Passat performs differently under the two conditions. (Or, even though we do not have sufficient statistical evidence, this is a large effect size and is worth investigating with a larger study)

CI:

2 pt CI = (xbar1 – xbar2) +/- t* sqrt(s1^2/n1 + s2^2/n2)

2 pt CI = (0.344 – 0.016) +/- (12.71)(0.068) = (-0.535, 1.191)

1 pt since we have 0 in the CI → not sufficient evidence to conclude this Passat performs differently

C) [1 pt] Do you believe you can generalize your findings to all Volkswagen Passats? Why or why not?

No – this is a single Passat and we don't know if there is something wrong with it that is different from other Passats (or: we don't have a large enough SRS of Passats to make that generalization)

Extra Space. The tables below show a number of critical values *z* for the standard normal variable $Z \sim N(0, 1)$ and the corresponding cumulative proportions, corresponding to P(Z $\leq$ z).

z-score	Cumulative Proportion
-3.00	0.0013
-2.00	0.0228
-1.65	0.0495
-1.28	0.1003
-1.00	0.1587
-0.43	0.3336

z-score	Cumulative Proportion
0.43	0.6664
1.00	0.8413
1.28	0.8997
1.65	0.9505
2.00	0.9772
3.00	0.9987

Refer to the following tables for t* and z* critical values for confidence intervals:

Degrees of freedom	C = 0.90 (90%)	C = 0.95 (95%)	C = 0.98 (98%)	C = 0.99 (99%)
1	6.314	12.71	31.82	63.66
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	2.764
≈100	1.660	1.984	2.364	2.626
≈1000	1.646	1.962	2.330	2.581

	C = 0.90 (90%)	C = 0.95 (95%)	C = 0.98 (98%)	C = 0.99 (99%)
z* values	1.645	1.960	2.326	2.576

For a two-sided hypothesis test, use the column corresponding to C = 1 – α

For a one-sided hypothesis test, use the column corresponding to C = 1 - 2α

The rest of this space to be used for extra work: