

Exam 4a

Name _____

Show your work and indicate your reasoning. You will not receive credit if you do not clearly show how you are obtaining your answers. Do all work on the exam.

1. (18 points) Airborne chemicals will disperse from their release point in a circular pattern. Suppose that a train crash results in the release of chlorine gas into the atmosphere. After t minutes, the radius of the circular area containing the gas plume is given by the function $r = f(t) = 0.17t$. The area of the gas plume as a function of the radius is $A = g(r) = \pi r^2$.

(a) Evaluate $g(f(30))$. What are its units? Explain what this expression means in the context of this problem.

(b) Evaluate $f^{-1}(4)$. What are its units? Explain what this expression means in the context of this problem.

(c) Evaluate $g^{-1}(100)$. What are its units? Explain what this expression means in the context of this problem.

2. (12 points) Give two different decompositions of the function $f(x) = \frac{1}{(1+2x)^3}$ into two new functions, u and v , where v is the inside function, and $u(x) \neq x$ and $v(x) \neq x$. Verify that $f(x) = u(v(x))$ in each case.

3. (12 points) Find the inverse of the function $f(x) = \ln(1 - 2x)$. Find the domain of f and the domain of f^{-1} . Justify your conclusions.

4. (12 points) The following table gives approximate values for two functions, $f(x)$ and $g(x)$. One is exponential and the other is a power function.

x	2	3	5
$f(x)$	32.8	68.9	303.9
$g(x)$	32.8	110.7	512.5

- (a) Determine which function is exponential and which is a power function. Explain your reasoning.

- (b) Find a possible formula for the power function.

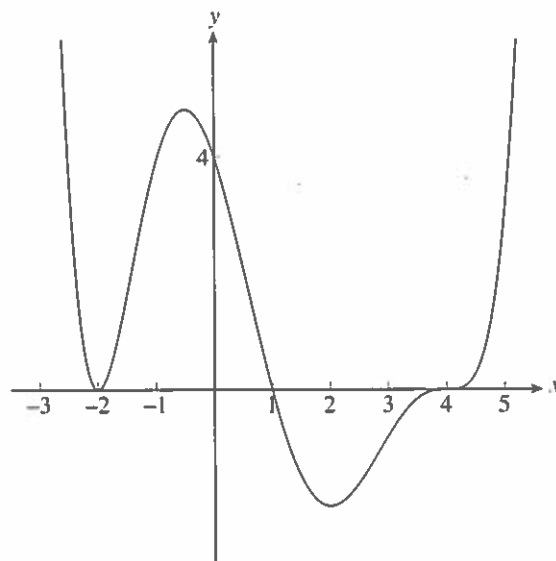
5. (12 points) According to Poiseuille's Law, the rate B at which blood flows through a blood vessel of radius R is directly proportional to R^4 . For medical reasons, we want to know how a reduction in the radius of the blood vessel affects the blood flow.

- (a) Write a formula for B in terms of R .

- (b) If the radius of the blood vessel is reduced in half from R_1 to $R_2 = \frac{1}{2}R_1$, how does the rate of blood flow change? Justify your conclusion.

6. (10 points) The graph of a polynomial function $f(x)$ is shown.

- (a) What are the zeros of $f(x)$? State which of these are multiple zeros and whether their multiplicities are even or odd. Give reasons for your conclusions.



- (b) Find a possible formula for $f(x)$. (Note that you do not have to multiply out the factors.)

7. (24 points) The volume of pollutants, in millions of cubic feet, in a water reservoir at time t , in years, is increasing and is given by

$$P(t) = 350 + 30t.$$

The total volume of the reservoir, including water and pollutants, is also gradually increasing and is given by the formula

$$V(t) = 12000 + 120t.$$

The fraction of the reservoir volume that consists of pollutants is then $C(t) = P(t)/V(t)$.

- (a) Evaluate and interpret $V(0)$, $P(0)$, and $C(0)$.

- (b) Evaluate and interpret $V(10)$, $P(10)$, and $C(10)$.

- (c) The reservoir will become unusable when the percentage of pollutants reaches 10%. After how many years will this occur?

- (d) After many, many years, about what percentage of the reservoir's total volume will consist of pollutants? Indicate your reasoning.

