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Day 14

Continuous Random Variables

Can take any real number (\mathbb{R}) value within any given interval.

We cannot use a probability mass function so we will instead use a probability **density** function (PDF) denoted as $f(x)$

Properties

- The probability of being in an interval $(a, b]$ is:

$$\int_a^b f(x)dx = \int_{-\infty}^b f(x) - \int_{-\infty}^a f(x)dx$$

- This is considered the area under the curve between a and b
- $P(X = x) = 0 \forall x$
 - $P(X \leq x) = P(X < x)$
 - $P(X \geq x) = P(X > x)$

$f(x)$ is displayed graphically as a density curve

Properties of $f(x)$

- $\forall x \in \mathbb{R}, f(x) \geq 0$
 - Density never goes below x-axis
- $\int_{-\infty}^{\infty} f(x)dx = 1$

Mean of continuous random variable: $\mu_x = \int_{-\infty}^{\infty} x \times f(x)dx$

Variance of continuous random variable is $\sigma^2 = \int_{-\infty}^{\infty} (X - \mu_x)^2 f(x)dx$

Uniform Random Variable

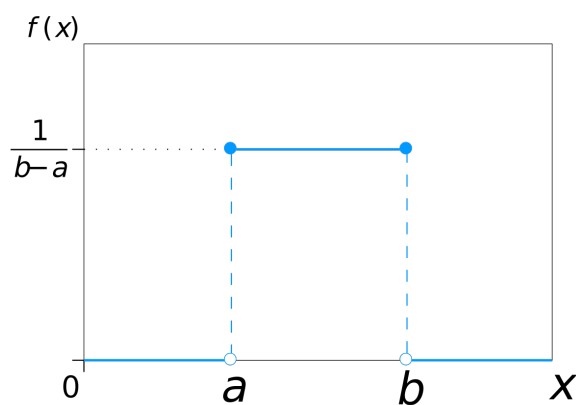


Figure 1: Graphical Representation

$$X \sim U(a, b)$$

Example : Standard Uniform Random Variable

$$X \sim U(0, 1)$$

Find

- $P(X \geq 0.3)$
- $P(X = 0.3)$
- $P(0.3 < X \leq 1.3)$
- $P(0.2 \leq X \leq 0.7 \text{ or } 0.7 \leq X \leq 0.9)$
- $P(X \text{ is not in the interval } (0.4, 0.7))$

Answers

- $\square = (0.7) \times (1) = 0.7$
- $\square = 0$
 - The probability of being exactly on a point in the infinite sum will ****always**** be 0.
- $\square = (0.7) \times (1) = 0.7$
 - Do not keep shading when there is no density curve, meaning it is a hard stop at $X = 1$
- $\square = ((0.25 - 0.2) \times \frac{1}{0.25 - 0.2}) + ((0.91 - 0.7) \times \frac{1}{0.91 - 0.7}) = 0.25$
- $\square = 0.4 + 0.3 = 0.7$

Normal Random Variable

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad X \sim N(\mu, \sigma)$$

Density curve is also a “bell curve”

Empirical (68-95-99.7) Rule

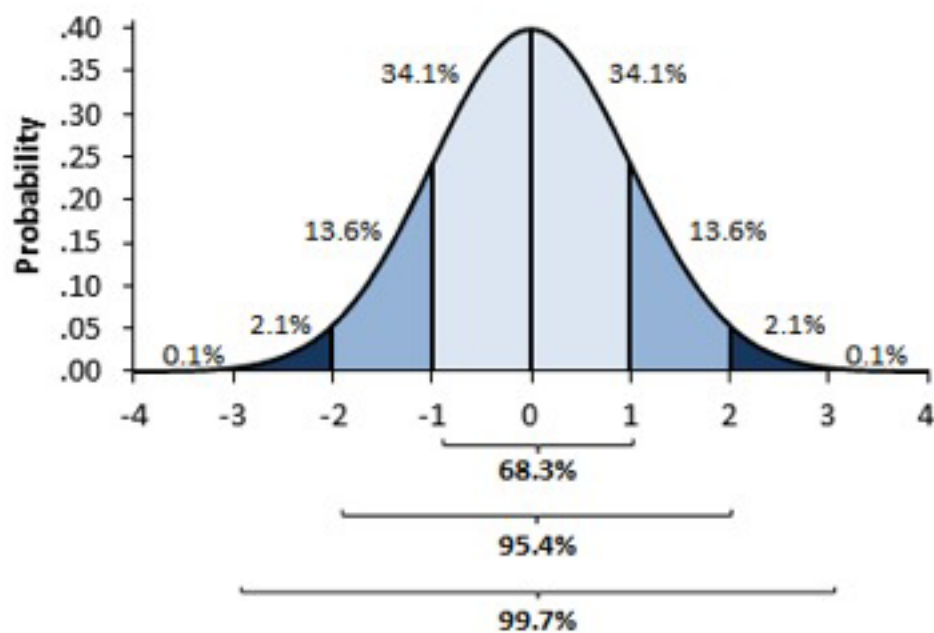


Figure 2: Bell Curve

$$X \sim N(\mu, \sigma)$$

Standardization

It may be useful to standardize distributions to compare 2 variables with same density curve shape but different scales. For normal distributions $X \sim N(\mu, \sigma)$, we convert to Z-Scores $Z \sim N(0, 1)$

$$Z = \frac{x - \mu}{\sigma} = \frac{\text{value} - \text{mean of distribution}}{\text{standard deviation}}$$

$$P(Z \leq z) = P(X \leq x)$$

“Cumulative proportion”/“Cumulative probability”