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TETLEY

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Excellent!

Instructions: Show all work. Partial credit will be awarded only for correct work consistent with the requirement. For full credit state your answer in the space provided.

NO CALCULATORS. NO SMARTPHONES. NO TABLETS.

1. (6 points) (Similar to 19/page 542) Find the equation of the sphere with center $(5, 2, -1)$ that touches the xz−plane.

6

Answer: $(x-5)^2 + (y-2)^2 + (z+1)^2 = 4$ ✓

2. (6 points) (Similar to 15 /section 10.5) Find the parametric equations for the line segment from $P(2, 1, 0)$ to $R(1, 2, 1)$.

$\vec{PR} = \langle -1, 1, 1 \rangle$

6

Answer: $x = 2 - t$
$y = 1 + t$
$z = 0 + t$
, $t \in [0, 1]$ ✓

3. (6 points) Write the parametric equations of the line through the point $(5, -1, 6)$ and perpendicular to the plane $x - 3y + z = 5$.

$\vec{n} = \langle 1, -3, 1 \rangle$

6

Answer: $x = 5 + t$
$y = -1 - 3t$
$z = 6 + t$
, $t \in \mathbb{R}$ ✓

18

4. (3 points) (Similar to 3/pg.564) Find the cross product $\mathbf{a} \times \mathbf{b}$ for $\mathbf{a} = 6\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, and $\mathbf{b} = -\mathbf{i} + 5\mathbf{k}$.

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 1 & -2 \\ -1 & 0 & 5 \end{vmatrix}$ $\vec{i}(5) - \vec{j}(30 - 2) + \vec{k}(1)$

$5\vec{i} - 28\vec{j} + \vec{k}$

3

Answer: $\langle 5, -28, 1 \rangle$ ✓

5. (3 points) Compute the dot product of the vectors $\mathbf{a} = 4\mathbf{i} - \mathbf{j} - 4\mathbf{k}$, and $\mathbf{b} = -\mathbf{i} + 5\mathbf{k}$.

$(4)(-1) + (-1)(0) + (-4)(5)$

3

Answer: -24 ✓

6. (6 points) (19/page 549) What is the angle between the vector $\mathbf{i} + \sqrt{3}\,\mathbf{j}$ and the positive direction of the x−axis?

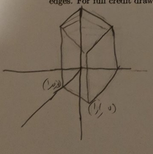
$\vec{i}$

$\cos\theta = \frac{\vec{i} \cdot (\vec{i} + \sqrt{3}\vec{j})}{|\vec{i}| \cdot |\vec{i} + \sqrt{3}\vec{j}|} = \frac{1}{1 \cdot 2}$ $\sqrt{1^2 + \sqrt{3}^2} = \sqrt{1+3} = 2$

6

Answer: $\cos^{-1}\left(\frac{1}{2}\right) = \theta = 60°$ ✓

7. (6 points) (43/page 557) Find the angle between a diagonal of a cube and one of its edges. For full credit draw a correct figure.



$\vec{d}_{1,1,1} = \langle 1, 1, 1 \rangle$

$\vec{e}_{1,0,0} = \langle 1, 0, 0 \rangle - \langle 0, 0, 0 \rangle = \langle 0, 1, 0 \rangle$

$\frac{\langle 1,1,1 \rangle \cdot \langle 0,1,0 \rangle}{\sqrt{3} \cdot 1} = \cos\theta$

$\Rightarrow \frac{1}{\sqrt{3}} = \cos\theta$

Answer: $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ ✓

18

8. (7 points) (23/ section 10.5) Write the equation of the plane through the point $(1, -1, -1)$ and parallel to the plane $5x - y - z = 6$.

$n_1 = \langle 5, -1, -1 \rangle = n_2$

$5(x-1) - 1(y+1) - 1(z+1) = 0$

7

Answer: $5(x-1) - (y+1) - (z+1) = 0$ ✓

9. (7 points) Write the parametric equations of the line of intersection of the planes $x + y - z = 1$ and $x + y = 0$.

$x + y - z = 1$

$x + y = 0$

$0 - z = 1$

$z = -1$

$x = -y$

$\therefore \langle 0, 0, -1 \rangle$

7

Answer: $x = t$
$y = -t$
$z = -1$
, $t \in \mathbb{R}$ ✓

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