Math 125 Exam #4 - Fall 2017 Version F

Name: SOLUTIONS Date: 12/1/2017

Chapter 10, Sections 1,2
Chapter 11, Sections 1-5

Instructions: You must solve each question completely, explaining your reasoning. Partial credit will be awarded for answers that are incorrect, but show progress towards a correct solution. You will not receive credit if you do not clearly show how you are obtaining your answers. Grading will be based on the solution and your write-up. Do all the work on the exam.

1. (12 Points) The profit earned by a producer to manufacture and sell n units of a good is given by $P(n) = 13n - 2821$. The average profit for n units is given by $A(n) = \frac{P(n)}{n}$.

   A) Compute $A(1)$, $A(217)$, $A(284)$. $A(n) = \frac{13n-2821}{n}$

   Sol. $A(1) = \frac{13(1)-2821}{1} = \boxed{-2808}$; $A(217) = \frac{13(217)-2821}{217} = \boxed{0}$ (BREAK-EVEN)

   $A(284) = [13(284)-2821]/284 = \boxed{3.07}$

   B) Interpret the economic significance of each the values in part (A).

   Sol. The average profit earned to manufacture & sell 1 unit is \$-2808 (loss of 2808).
   The average profit earned to manufacture & sell 217 units is \$0.
   The average profit earned to manufacture & sell 284 units is \$3.07.

   C) What trend do you notice in the values of $A(n)$ as n gets large? Explain this trend in economic terms.

   Sol. as $n \to \infty$, $A(n) \to 13$ because the long-run behavior of $A(n)$ is: leading coeff. term of numerator / leading coeff. term of denominator $= \frac{13n}{n} = \boxed{13}$

2. (10 points) Let $P = 30\ln(t)$ give the annual profit of a company (in thousands of dollars) t years after its formation.

   What is $P^{-1}(38)$? *Round to the nearest whole number and include units.* Explain what this expression means in the context of this problem.

   Sol. $P = 30\ln(t)$

   $\frac{38}{30} = \frac{30\ln(t)}{30}$

   $\frac{38}{30} = \ln(t)$

   $t = e^{\frac{38}{30}}$

   $t = 3.549$

   $\boxed{t = 4 \text{ years}}$

   The annual profit of a Company is \$38000 in 4 years after its formation.

3. (10 points) List a set functions (*g(x), h(x), p(x)*) that is a decomposition of $f(x)=\cot^4(\ln x)$ in the form of $g(h(p(x)))$.

Sol.

$p(x) = \ln x$

$h(x) = \cot x$ ⇒ VERIFY:

$g(x) = x^4$

$g(h(p(x))) = g(h(\ln x))$

$= g(\cot(\ln x))$

$= \cot^4(\ln x)$ ✓

NUMERATOR

4. (10 points) Write a possible formula for a rational function, $f(x)$, $\boxed{\text{with zeros at } x=-7,\ x=2,}$ $\boxed{\text{vertical asymptotes at } x=11,\ x=-11}$ and a horizontal asymptote at $y=4$.

DENOMINATOR

Sol.

$$f(x) = \frac{P(x)}{Q(x)}$$

$$f(x) = K \cdot \frac{(x+7)(x-2)}{(x-11)(x+11)}$$

NOTE: Since the horizontal asymptote at $y=4 \Rightarrow \boxed{K=4}$

$$\boxed{f(x) = 4\,\frac{(x+7)(x-2)}{(x-11)(x+11)}}$$

5. (20 points) Given the function $f(x)=\frac{1}{x+7}-\frac{x}{x-6}$.

A) Rewrite the function $f(x)=\frac{p(x)}{q(x)}$, a ratio of polynomials *(Get a common denominator and subtract).*

Sol. $f(x)=\frac{1(x-6)-x(x+7)}{(x+7)(x-6)}=\frac{x-6-x^2-7x}{(x+7)(x-6)}$

$$\boxed{f(x)=\frac{-x^2-6x-6}{x^2+x-42}}$$

B) Find any vertical asymptotes

Sol. Set the denominator to zero.

$(x+7)(x-6)=0$

$\boxed{x=-7}$ or $\boxed{x=6}$

C) Find any horizontal asymptotes.

Sol. $f(x)=\frac{-x^2-6x-6}{x^2+x-42}=\frac{\text{degree}=2}{\text{degree}=2}=\frac{-x^2}{x^2}=-1$

$\boxed{\text{Thus, } y=-1 \text{ is the H.A.}}$

D) Describe the long term behavior of the graph.

Sol. $f(x)=\frac{-x^2-6x-6}{x^2+x-42}$

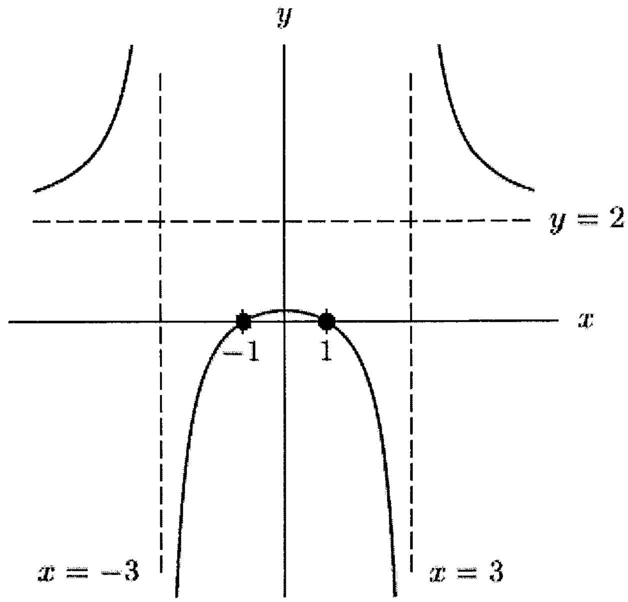
Long-run behavior of $f(x)=\frac{\text{leading coeff. term of } P(x)}{\text{leading coeff. term of } Q(x)}$

$=\frac{-x^2}{x^2}=\boxed{-1}$

as $x\to\infty$, $f(x)\to-1$

as $x\to-\infty$, $f(x)\to-1$.

6. (12 points) The graph of $f(x)=\dfrac{16}{x^2-9}+2$ is shown below.



A) State the domain of f(x). What are the vertical asymptotes?

Sol. Domain of f(x): $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

Vertical Asymptotes: $x=-3$; $x=3$

B) Does f(x) have an inverse over the domain you stated in part A? Explain your reasoning.

Sol. No, f(x) doesn't have an inverse over the stated domain in part (a) because it fails H.L.T.

$$\boxed{f^{-1}(x)=\sqrt{\frac{16}{x-2}+9}}$$

C) Define (Restrict) a new domain and find the inverse of $f(x)=\dfrac{16}{x^2-9}+2$.

Sol New Domain: $[0,3)\cup(3,\infty)$

$y=\dfrac{16}{x^2-9}+2$

$x=\dfrac{16}{y^2-9}+\cancel{2}$; solve for y

$-2 \qquad\qquad \cancel{-2}$

$x-2=\dfrac{16}{y^2-9}$

$y^2-9=\dfrac{16}{x-2}$

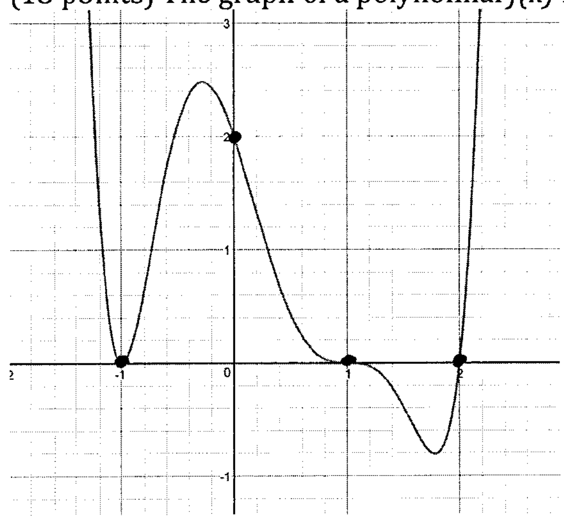
$y^2=\dfrac{16}{x-2}+9$

$y=\sqrt{\dfrac{16}{x-2}+9}$

7. (8 points) Suppose f and g are invertible functions such that $f(-9)=-5$, $f(-7)=-2$, $f(-5)=-6$, $g(-13)=-12$, $g(-7)=-13$, and $g(-2)=-5$. Find $f^{-1}(g(f(-7)))$.

Sol. $f^{-1}(g(f(-7))) = f^{-1}(g(-2)) = f^{-1}(-5) = \boxed{-9}$

8. (18 points) The graph of a polynomial *f(x)* is shown.



A) What is the y-intercept of *f(x)*?

Sol. $(0,2)$

B) What are the zeros of *f(x)*? State which of these are multiple zeros and whether their multiplicities are even or odd. Give reasons for your conclusions.

Sol. (i) $(-1,0)$ = double zero (graph touches & turns around or bounces) (even multiplicity)

(ii) $(1,0)$ = triple zero (graph flattens & crosses) odd multiplicity.

(iii) $(2,0)$ = single zero

C) What is the long run behavior of *f(x)?*

Sol Long-run behavior: leading coeff. term = x^6

as $x \to \infty$, $f(x) \to \infty$ & as $x \to -\infty$, $f(x) \to \infty$.

D) Find a possible formula for f(x). Do not multiply the factors.

Sol. $f(x) = k(x+1)^2(x-1)^3(x-2)$

$2 = k(0+1)^2(0-1)^3(0-2)$

$2 = k(1)(-1)(-2)$

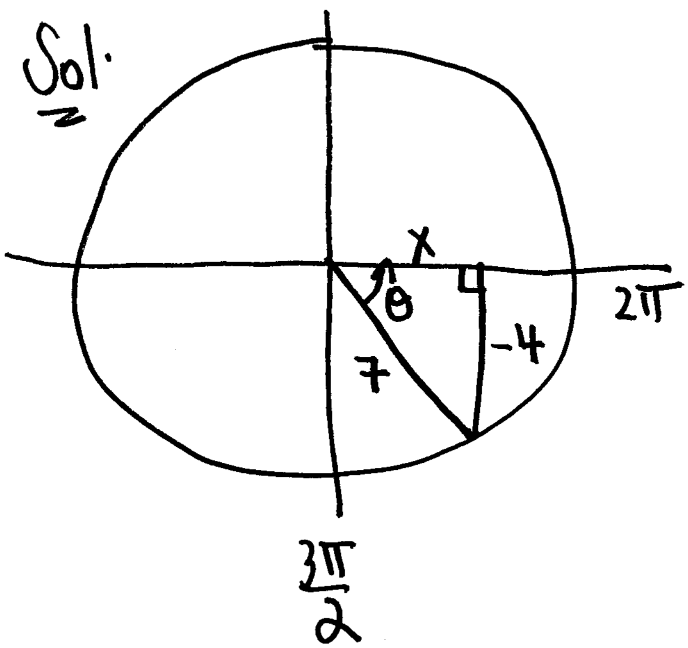
$\frac{2}{2} = \frac{2k}{2} \Rightarrow \boxed{k=1}$

$\boxed{f(x) = (x+1)^2(x-1)^3(x-2)}$

Bonus If $\frac{3\pi}{2} < \theta < 2\pi$ and $\sin(\theta) = \frac{-4}{7}$, find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ exactly.

Double Angle Formulas
$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$
$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Sol.



$$(x)^2 + (-4)^2 = (7)^2$$

$$x^2 + 16 = 49$$
$$-16 \quad -16$$

$$x^2 = 33$$

$$\boxed{x = \sqrt{33}}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= 2\left(-\frac{4}{7}\right)\left(\frac{\sqrt{33}}{7}\right)$$
$$\boxed{= -\frac{8\sqrt{33}}{49}}$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$
$$= 1 - 2\left(-\frac{4}{7}\right)^2$$
$$= 1 - 2\left(\frac{16}{49}\right)$$
$$= 1 - \frac{32}{49} = \frac{49}{49} - \frac{32}{49} = \boxed{\frac{17}{49}}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{\frac{-8\sqrt{33}}{\cancel{49}}}{\frac{17}{\cancel{49}}}$$
$$= \boxed{\frac{-8\sqrt{33}}{17}}$$