Changing the order of integration

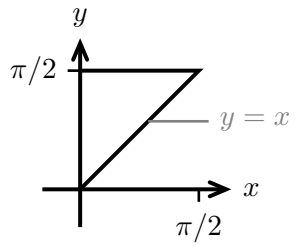
1. Evaluate

$$I = \int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y}\, dy\, dx$$

by changing the order of integration.

Answer:

The given limits are (inner) y from x to $\pi/2$; (outer) x from 0 to $\pi/2$.
We use these to sketch the region of integration.



The given limits have inner variable y. To reverse the order of integration we use horizontal stripes. The limits in this order are

(inner) x from 0 to y; (outer) y from 0 to $\pi/2$.

So the integral becomes

$$I = \int_0^{\pi/2} \int_0^{y} \frac{\sin y}{y}\, dx\, dy$$

We compute the inner, then the outer integrals.

Inner: $\dfrac{\sin y}{y} x \Big|_0^y = \sin y.$ Outer: $-\cos y|_0^{\pi/2} = 1.$