

Chapter 10 Section 10.1 Composition of Functions

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. Given $f(x) = \frac{2}{x}$ and $g(x) = x^4 - 1$, what is $g(f(x))$?

Sol: $g(f(x)) = g\left(\frac{2}{x}\right) = \left(\frac{2}{x}\right)^4 - 1 = \boxed{\frac{16}{x^4} - 1}$

2. Let $m(x) = e^x$ and $n(x) = \frac{x^6}{x+1}$. Does $n(m(x)) = \frac{e^{6x}}{e^x + 1}$?

Sol: $n(m(x)) = n(e^x) = \frac{(e^x)^6}{e^x + 1} = \boxed{\frac{e^{6x}}{e^x + 1}}$

3. $p(q(x)) = \frac{5}{5+x}$ and $q(x) = 3+x$, what is $p(x)$?

Sol: $p(x) = \frac{5}{2+x}$; $q(x) = 3+x \Rightarrow p(q(x)) = p(3+x) = \frac{5}{2+3+x}$

4. Write two functions $v(x)$ and $u(x)$ such that $y = \frac{4x^2 + 1}{12x^2} = u(v(x))$

Sol: $v(x) = 4x^2$, $u(x) = \frac{x+1}{3x} \Rightarrow u(v(x)) = \frac{4x^2+1}{12x^2}$

$= \frac{5}{5+x}$

5. Give a practical interpretation in words of the function.

a. $g(t) = f(h(t))$, where $A = f(r)$ is the area of a circle of radius r and $r = h(t)$ is radius of the circle time is a function of time, t .

$g(t)$ represents the Area of a circle at given time t .

b. $W(q) = R(Y(q))$, where R gives a farmer's revenue as a function of corn yield per acre, and Y give the corn yield as a function of quantity, Q of fertilizer.

Sol: $W(q)$ represents farmer's revenue at given quantity of fertilizer " Q ".

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. Given $f^{-1}(x) = 1500(1.03)^x$, what is $f(x)$?

Sol $x = 1500(1.03)^y$ solve for y
 $\frac{x}{1500} = (1.03)^y$

$$\ln\left(\frac{x}{1500}\right) = \ln(1.03)^y$$

$$\ln\left(\frac{x}{1500}\right) / \ln(1.03) = y = f(x)$$

2. Let $f(x) = 3^x$ and $g(x) = f(f(x))$. Evaluate $g^{-1}(27)$.

Sol $g(x) = f(f(x))$

$$g(x) = 3^{3^x} = 27^x$$

$$27 = 27^x \Rightarrow x = 1$$

3. For positive numbers x , what is the inverse of $h(x) = e^{\sqrt{x}-4}$?

Sol $h(x) = e^{\sqrt{x}-4}$

$$x = e^{\sqrt{y}-4} - 4$$

$$x+4 = e^{\sqrt{y}}$$

$$\ln(x+4) = \sqrt{y}$$

$$y = [\ln(x+4)]^2$$

$$h^{-1}(x) = (\ln(x+4))^2$$

4. Determine whether or not $y = 6x^2 + 1$ is invertible. Explain.

Sol $y = 6x^2 + 1$ is not invertible because it fails H.L.T.



5. Let $P = 25 \ln(t)$ give the annual profit of a company (in thousands of dollars) t years after its formation.

What is $P^{-1}(37)$? Round to the nearest whole number and include units.

Sol $\frac{37}{25} = \frac{25 \ln(t)}{25} \Rightarrow \frac{37}{25} = \ln t \Rightarrow t = e^{\frac{37}{25}} \approx 4 \text{ years}$

6. If $P = f(t)$ gives the population of a city in thousands, as a function of time, t in years what does $f^{-1}(P)$ represent?

Sol $f^{-1}(P)$ represents time in years @ given pop. of a city in thousands.

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. Let $f(x) = x^2 + 3$ and $g(x) = \frac{1}{x} + 5$. What is $f(5) + g(1)$? $= 28 + 6 = \boxed{34}$

Sol $f(5) = (5)^2 + 3 = 28$ $g(1) = \frac{1}{1} + 5 = 6$

2. Let $f(x) = x + 5$ and $g(x) = x^2$. What is $3f(x) - g(x)$?

Sol $3(x+5) - (x^2) = \boxed{-x^2 + 3x + 15}$

3. Let $f(x) = x + 1$ and $g(x) = x^4$. What is $f(x)g(x)$?

Sol $(x+1) \cdot x^4 = \boxed{x^5 + x^4}$

4. Let $f(x) = x + 7$, $g(x) = x^3$, and $h(x) = \sqrt{x-3}$. What is $\frac{f(x)}{g(h(x))}$? $= \frac{x+7}{(\sqrt{x-3})^3}$

Sol $g(h(x)) = g(\sqrt{x-3}) = (\sqrt{x-3})^3$

5. Find $f(x)(g(x))^2$ if $f(x) = 2e^{2x} + 1$ and $g(x) = \frac{1}{5}e^{-x}$.

Sol $(2e^{2x} + 1) \left(\frac{1}{5}e^{-x} \right)^2 = (2e^{2x} + 1) \left(\frac{1}{25}e^{-2x} \right) = \frac{2}{25}e^0 + \frac{1}{25}e^{-2x} = \frac{2}{25} + \frac{1}{25e^{2x}}$

6. Let $f(x) = \frac{5}{x+1}$. Find and simplify $f\left(\frac{1}{x}\right) - \frac{1}{f(x)}$.

Sol $\frac{5}{\frac{1}{x}+1} - \frac{1}{\frac{5}{x+1}} = \frac{5x}{1+x} - \frac{x+1}{5} = \frac{25x - (x+1)^2}{5(1+x)} = \frac{25x - x^2 - 2x - 1}{5(1+x)} = \frac{-x^2 + 23x - 1}{5(1+x)}$

7. Let $f(t)$ be the number of men and let $g(t)$ be the number of women residing in a certain town in year t . Let $h(t)$ be the average income, in dollars, of residents of that town in year t . If $f(t) = 250 + 5t$, $g(t) = 275 + 9t$, and $h(t) = 32,000 + 400t$, find a simplified formula for the total amount of money earned by all adult residents of the town in year 3.

Sol $[f(t) + g(t)] \cdot h(t)$

$= [(250 + 5t) + (275 + 9t)] \cdot (32,000 + 400t)$

$= (525 + 14t)(32,000 + 400t)$

$= (525 + 14(3))(32,000 + 400(3)) = \boxed{18,824,400}$