Formula Cheat Sheet

Velocity

Rate at which a particle moves with per unit per second

One dimension

Formula: $V = V_{\text{o}} + at$

Usage: Find the velocity of a particle in one dimensional space

Example: A ball is thrown from the top of a six story building, find the velocity of the particle when it hits the ground after 10 seconds. Assume no air resistance and no terminal velocity.

- $a = 9.8m/s$
- $t = 10s$
- $v_{\text{o}} = 0$

$\therefore v = 0 + 9.8(10) = 98m/s^2$

Free Fall Acceleration

Formula: $y = y_{\text{o}} + v_{\text{o}}t + \frac{1}{2}at^2$

Usage:

Example: A ball is thrown from the top of a building 90 meters above the ground as a *projectile*. It takes 10 seconds for the ball to hit the ground. What was the initial velocity of the ball?

- $a = -g = -9.81m/s$
- $t = 10s$
- y = 90 m

$0 = 90 + (10)V_{\text{o}} - 490.5$

$0 = -400.5 + 10V_{\text{o}}$

$400.5 = 10V_{\text{o}}$

$V_{\text{o}} = 40.05$

Apply velocity formula: $V = 40.05 - 9.81(10)$

$V = -58.05m/s$

This is *downward speed*

Vectors

Addition and Subtraction

It is commutative and associative.

$\vec{S} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$

$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

Example:

$\vec{A} =< 1, 2, 3 >$

$\vec{B} =< 4, 5, 6 >$

$\vec{C} = \vec{A} + \vec{B} =< (1 + 4), (2 + 5), (3 + 6) >$

$\vec{C} =< 5, 7, 9 >$

Multiplication

Dot/Scalar Product

$\vec{A} \bullet \vec{B} = \cos\phi$

Result is a scalar and is commutative.

Example:

$\vec{A} =< 1, 2, 3 >$

$\vec{B} =< 4, 5, 6 >$

$\phi = 45$

$|\vec{A}| = \sqrt{13}$

$|\vec{B}| = \sqrt{77}$

$\vec{A} \bullet \vec{B} = \sqrt{13} * \sqrt{77} * \cos(45)$

Cross/Vector Product

$\vec{A} =< 1, 2, 3 >$

$\vec{B} =< 4, 5, 6 >$

$\vec{A} \times \vec{B}$

Produces an orthogonal (perpendicular) vector to both $\vec{A}$ and $\vec{B}$

Misc

$a_{\mathrm{x}} = a\cos\theta$

$a_{\mathrm{y}} = a\sin\theta$

$|a| = \sqrt{a((\cos\theta)^2 + (\sin\theta)^2)}$

$arctan(\frac{a_{\mathrm{x}}}{a_{\mathrm{y}}}) = \theta$

$\cos\phi = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}| * |\vec{B}|}$

Magnitude of $\vec{A} \times \vec{B} = |\vec{A}| * |\vec{B}| * \sin\phi$

Projectile Motion (Two Dimensions)

Tracking a particle that has parabolic motion. This is the positive half of angular motion.

Velocity

Components

- $V_{\mathrm{ox}} = V_{\mathrm{o}} \cos(\theta_{\mathrm{o}})$
- $V_{\mathrm{oy}} = V_{\mathrm{o}} \sin(\theta_{\mathrm{o}})$

Instantaneous

- $V_{\mathrm{x}} = x_{\mathrm{o}} + V_{\mathrm{ox}} t$
- $V_{\mathrm{y}} = V_{\mathrm{o}} y - gt$

Range

Horizontal

Description: track how far a particle will land when used as a projectile

Formula: $R = \frac{V_{\mathrm{o}}^2 \sin(2\theta_{\mathrm{o}})}{g}$

Vertical

Description: track how high a particle will achieve when used as a projectile

Formula: $VR = \frac{V_{\mathrm{o}}^2 \sin^2 \theta}{2g}$

Position in space

X Coordinate

Formula: $x = x_{\mathrm{o}} + V_{\mathrm{ox}} t$

Y Coordinate

Formula: $y = y_{\mathrm{o}} + V_{\mathrm{oy}} t - \frac{1}{2} g t^2$

Equation of the path of motion

Description: use this when the component of time is unknown

Formula: $\Delta y = \tan \theta_{\mathrm{o}} \Delta x - \frac{g \Delta x^2}{2 V_{\mathrm{o}}^2 \cos^2 \theta}$

Angular Motion

Polar Coordinates

Positions along the circle given a radius $|\vec{r}|$ and an angle denoted as θ

Formula(s):

- $x = r\cos\theta$
- $y = r\sin\theta$

The above assumes that $|\vec{r}|$ does **not** change

Uniform Circular Motion

Description: constant angular velocity (ω)

Formula: $\theta = \theta_\text{o} + \omega t$

Vector representing uniform circular motion: $\vec{r}(t) =< x(t), y(t) >$

Where the components are:

- $x(t) = r\cos(\theta_\text{o} + \omega t)$
- $y(t) = r\sin(\theta_\text{o} + \omega t)$

Misc information

- $\omega = 2\pi f$ (f being the cycles per second = Hertz)
- $f = \frac{1}{T}$ (rotation period)

Velocity for Uniform Circular Motion

Description: velocity in polar coordinates

Formula: $\frac{d\vec{r}}{dt}(\vec{r}) =< -\omega r\sin(\theta_\text{o} + \omega t), \omega r\cos(\theta_\text{o} + \omega t) >$

Magnitude of velocity: $|\vec{v}(t)| = \sqrt{{v_\text{x}}^2 + {v_\text{y}}^2} = \omega r$

Acceleration for Uniform Circular Motion

Description: acceleration in polar coordinates

Formula: $\frac{d^2\vec{r}(t)}{dt} =< -\omega^2 r\cos(\theta_\text{o} + \omega t), -\omega^2 r\sin(\theta_\text{o} + \omega t) >$

The above formula will decay to:

$\vec{a} = -\omega^2\vec{r}(t)$

Pull out the negative and the components of the vector are $\vec{r}(t)$

Magnitude of acceleration is: $|\vec{a}(t)| = \omega^2 r = \frac{V^2}{r}$