MATH 338

MIDTERM 1

WED/THURS, MARCH 1/2, 2017

Your name: _____

Your scores (to be filled in by Dr. Wynne):

Problem 1: ____/9

Problem 2: ____/15

Problem 3: ____/11

Problem 4: ____/11

Problem 5: ____/9

Total: ____/55

You have 75 minutes to complete this exam. This exam is closed book and closed notes with the exception of your formula sheet.

For full credit, show all work except for final numerical calculations (which can be done using a scientific calculator).

1. For a class of 20 students, the following variables are recorded: student name, grade (A/B/C/D/F), score on final exam (out of 100), number of homework assignments completed (out of 10).

A) [1 pt] Is grade a categorical quantitative (circle one) variable?

categorical

B) [1 pt] How many cases are there in this data set?

20

C) [1 pt] Is there a label variable? If so, what is it?

Yes, student name

D) [2 pt] Can we model the number of homework assignments completed by a given student as a binomially distributed variable? Why or why not?

2 pts NO - Probability of completing homework assignments cannot be assumed constant for each assignment and/or cannot assume homework assignments are independent, or YES – with appropriate justification for each BINS assumption

E) [4 pt] Below are the final exam scores. Sketch an appropriate graph to summarize the scores.

48	50	58	64	70
74	74	75	79	80
82	85	86	86	88
90	91	93	97	99

1 pt draw a stem-and-leaf plot, histogram, or boxplot

3 pts for correctly drawing the graph

2. In a population, the variable *Rain* is normally distributed with mean 10 and standard deviation 2.

A) [1 pt] What is the median of *Rain*?

1 pt since symmetric, mean = median = 10

B) [6 pt] Compute the IQR of *Rain*.

2 pt find z-scores for Q1 and Q3 using table (cum. Prop. = 0.25 and 0.75 → z = -0.67 and 0.67)

2 pt Q1 = sigma z + mu = 2(-0.67) + 10 = 8.66 and Q3 = sigma z + mu = 2(0.67) = 11.34

2 pt IQR = Q3 – Q1 = 11.34 – 8.66 = 2.68

OR

2 pt find z-scores for Q1 and Q3 using table

1 pt IQR = Q3 – Q1

3 pts recognize that mu is irrelevant, so we just need to compute 2(0.67) – 2(-0.67) = 2.68

C) [3 pt] What proportion of the values of *Rain* are between 8 and 12?

1 pt use empirical rule or cumulative proportions

1 pt find z-scores of -1 and 1 (implicitly or explicitly)

1 pt answer: 0.68 or 68%, or 0.6826 or 68.26% if used cumulative proportions

D) [5 pt] Suppose that we perform the transformation, *Rain_new* = 1.5(*Rain)* – 4. Find the (population) distribution of the variable *Rain_new*.

1 pt the distribution is still normal under linear transformation

2 pt new mean is 1.5(old mean) – 4 = 1.5(10) – 4 = 11

2 pt new sd is 1.5(old sd) = 1.5(2) = 3

3. In the game of Twister, people spin a spinner to determine which of four limbs (left hand, right hand, left foot, or right foot) needs to be moved on a turn, and which of four colors (red, blue, yellow, or green) the limb needs to be moved to. Assume that each combination of limb and color is equally likely.

A) [1 pt] Are the events "right hand red" and "left foot blue" disjoint? Explain/show work.

Yes, since we cannot both move right hand to red and left foot to blue on the same turn.

B) [3 pt] What is the probability that you move your right foot OR move a limb to green?

1 pt Let A = "Right foot" and B = "Green" (or vice versa, or some way of indicating these are events)

1 pt P(A or B) = P(A) + P(B) – P(A and B)

1 pt plug in P(A or B) = 1/4 + 1/4 - 1/16 = 7/16

The probability you move your right foot OR move a limb to green is 7/16 or 0.4375

C) [7 pt] Suppose that a Twister variant is played in which you earn 1 point for successfully moving a hand and 2 points for successfully moving a foot. Let *X* be the number of points you earn on a successful move. Write the probability distribution of *X*, and find its mean and standard deviation.

2 pts: X takes value 1 with probability 0.5 and takes value 2 with probability 0.5

2 pts: E(X) = (0.5)(1) + (0.5)(2) = 1.5

2 pts: Var(X) = (0.5)(1 – 1.5)^2 + (0.5)(2 – 1.5)^2 = 0.25

1 pt: sd(X) = sqrt(0.25) = 0.5

4. The “Taxicab Problem” presented by Tversky and Kahneman (1982) is given below. It is notoriously impossible to solve using intuition, but can be solved using techniques learned in this module.

A) [6 pt] In a city, 85% of cabs in the city are Green and the other 15% are Blue. A cab is involved in a hit-and-run and the sole witness claims the cab was Blue. The witness is tested and shown to misidentify Green cabs as Blue 20% of the time and Blue cabs as Green 20% of the time. What is the probability that the actual cab in the accident was Blue (given that the witness claimed the cab was Blue)?

1 pt use Bayes’s rule and/or tree diagram

4 pt plug in probabilities correctly

1 pt get a final answer of about 0.414 or 41.4%

B) [5 pt] Suppose a coin is weighted to come up heads 90% of the time. You toss the coin 6 times (independently). Which outcome is more likely: that all 6 tosses come up heads, or that you get at least 1 tail? Use techniques from this module to prove your answer.

1 pt use binomial formula(s)

3 pt get P(6 heads) = (6!)/(6! 0!) (0.9)^6 (0.1)^0 = (0.9)^6 = 0.531 AND/OR P(at least 1 tail) = 0.469

1 pt recognize that “at least 1 tail” is the complement of “all heads,” so since P(all heads) > 0.5 or P(at least one tail) < 0.5, “all heads” is more likely OR since 0.531 > 0.469, “all heads” is more likely

5. A double-blind, randomized clinical study by Eli Lilly investigated the drug atomoxetine as a treatment for ADHD in autistic children. Every day for eight weeks, subjects took a pill containing either atomoxetine or a placebo.

A) [1 pt] Explain what is meant by the term "double-blind" in the context of this study.

Neither the people giving the drug nor the ones getting the drug know if it is atomoxetine or placebo.

B) [1 pt] Explain what is meant by the term "randomized" in the context of this study.

Subjects were randomly assigned to get either atomoxetine or placebo.

C) [1 pt] Was this study an observational study experiment (circle one)?

experiment

At the beginning of the study, scores on the Wechsler Intelligence Scale for Children (WISC) were measured. Scores in neurotypical children are normally distributed with mean 100 and standard deviation 15.

D) [6 pt] The mean score of the 49 participants in the control group was 94.6. What is the approximate probability of observing a mean score this low or lower, if all participants were neurotypical?

3 pt apply sampling distribution procedures to find that x-bar is distributed N(100, 2.14)

1 pt each for each of Steps 2-4 in cumulative proportion framework: z = -2.52, p = 0.006

The probability of observing a mean score this low or lower is approximately 0.006.

Extra Space. The tables below show a number of critical values *z* for the standard normal variable $Z \sim N(0, 1)$ and the corresponding cumulative proportions, corresponding to P(Z ≤ z).

z-score	Cumulative Proportion
-3.00	0.0013
-2.50	0.0062
-2.00	0.0228
-1.65	0.0495
-1.28	0.1003
-1.00	0.1587
-0.67	0.2514

z-score	Cumulative Proportion
0.67	0.7486
1.00	0.8413
1.28	0.8997
1.65	0.9505
2.00	0.9772
2.50	0.9938
3.00	0.9987

The rest of this space to be used for extra work: