Name: SOLUTIONS

Chapter 9 – Section 9.1 Trigonometric Equations

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

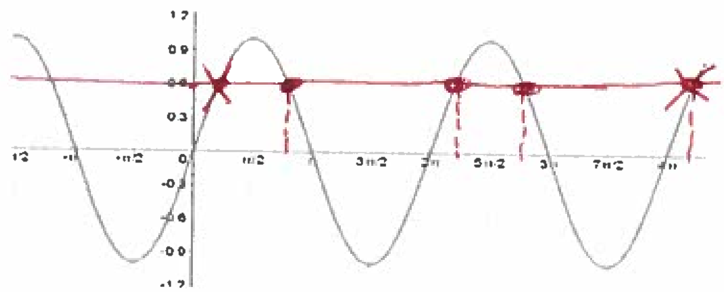
Check your understanding:

1. Use a graph of $y = \sin t$ to estimate the solution to the equation $\sin t = 0.6$ for $\pi/2 \le t \le 4\pi$.

$t \approx 2.498$

$t \approx 6.927$

$t \approx 8.781$



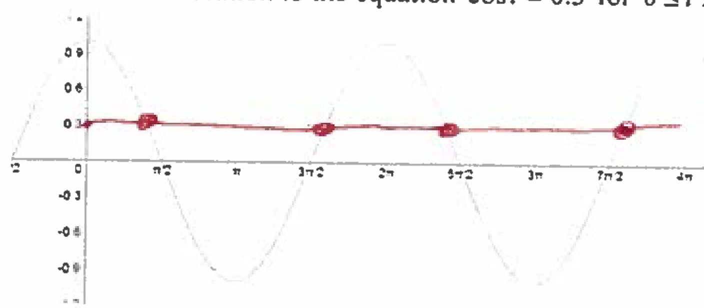
2. Use a graph of $y = \cos t$ to estimate the solution to the equation $\cos t = 0.3$ for $0 \le t \le 4\pi$

$t \approx 1.27$

$t \approx 5.01$

$t \approx 7.55$

$t \approx 11.30$



3. Find the exact solutions to the equation cos x = ½ when $-2\pi \le x \le 2\pi$

Sol $\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right)$ → $x = -\frac{\pi}{3}, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

4. Find the exact solutions to the equation sin 2x = ~~0.3~~ 0.5 when $0 \le x \le 2\pi$

Sol $\sin 2x = 0.5$

$2x = \sin^{-1}(0.5)$ → $\frac{1}{2} \cdot 2x = \frac{\pi}{6} \cdot \frac{1}{2}$ or $\frac{1}{2} \cdot 2x = \frac{5\pi}{6} \cdot \frac{1}{2}$ → $\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}}$

5. Find the exact solutions to $\sin\theta = \frac{-\sqrt{2}}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$

Sol $\sin\theta = -\frac{\sqrt{2}}{2}$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\boxed{\theta = -\frac{\pi}{4}}$

$\cos\theta = \frac{\sqrt{3}}{2}$; $0 \le \theta \le \pi$

$\boxed{\theta = \frac{\pi}{6}}$

Name: SOLUTIONS

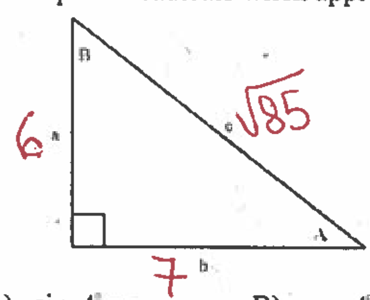
Chapter 9 – Section 9.2 Identities, Expressions and Equations

TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. Review:
   Find the following exactly using the figure given if $a = 6$ and $b = 7$. Express your answers as unsimplified radicals when appropriate.



$(a)^2 + (b)^2 = (c)^2$
$(6)^2 + (7)^2 = c^2$
$36 + 49 = c^2$
$85 = c^2$
$\boxed{c = \sqrt{85}}$

   A) $\sin A^\circ$ $\frac{6}{\sqrt{85}}$ B) $\cos A^\circ$ $\frac{7}{\sqrt{85}}$ C) $\tan A^\circ$ $\frac{6}{7}$

   D) $\sin B^\circ$ $\frac{7}{\sqrt{85}}$ E) $\cos B^\circ$ $\frac{6}{\sqrt{85}}$ F) $\tan B^\circ$ $\frac{7}{6}$

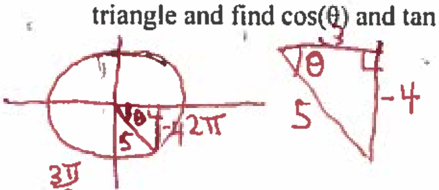
2. Find all possible solutions to $4\cos t - \sin t \cos t = 0$ for $0 \le t \le 2\pi$. Give your answer in radians.

see back page.

3. Use trigonometric identities to solve the following exactly over $0 \le \theta \le 2\pi$. Give your answer in radians.
   a. $\sin^2\theta - \cos^2\theta = \sin\theta$
   b. $\tan(2\theta) + \tan\theta = 0$

see back page.

4. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\sin(\theta) = \frac{-4}{5}$, find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$. Hint (use your right triangle and find cos(θ) and tan (θ) then use the ***double angle***)

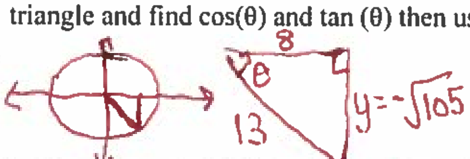


$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = \boxed{-\frac{24}{25}}$

$\cos(2\theta) = 1 - 2\sin^2\theta = 1 - 2\left(-\frac{4}{5}\right)^2 = 1 - \frac{32}{25} = \boxed{-\frac{7}{25}}$

5. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\cos(\theta) = \frac{8}{13}$, find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$. Hint (use your right triangle and find cos(θ) and tan (θ) then use the ***double angle***)

Sol:



$(8)^2 + y^2 = (13)^2$
$64 + y^2 = 169$
$y^2 = 105$
$y = \sqrt{105}$

$\sin(2\theta) = 2\left(-\frac{\sqrt{105}}{13}\right)\left(\frac{8}{13}\right) = \boxed{-\frac{16\sqrt{105}}{169}}$

$\cos(2\theta) = 2\left(\frac{8}{13}\right)^2 - 1 = \frac{128}{169} - \frac{169}{169} = \boxed{-41/169}$

$\tan(2\theta) = \boxed{\frac{16\sqrt{105}}{41}}$

#2) $4\cos t - \sin t\cos t = 0$; $0 \le t \le 2\pi$

Sol $\cos t(4 - \sin t) = 0$

$\cos t = 0$ OR $4 - \sin t = 0$

$\boxed{t = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\sin t = 4$

$\boxed{N|A}$

#3) (a) $\sin^2\theta - \cos^2\theta = \sin\theta$

Sol $\sin^2\theta - (1 - \sin^2\theta) = \sin\theta$

$\sin^2\theta - 1 + \sin^2\theta = \sin\theta$

$2\sin^2\theta - \sin\theta - 1 = 0$

Let $u = \sin\theta$

$2u^2 - u - 1 = 0$

$(2u + 1)(u - 1) = 0$

$2u + 1 = 0$ or $u - 1 = 0$

$u = \frac{-1}{2}$ $\quad u = 1$

$\sin\theta = \frac{-1}{2}$ $\quad \sin\theta = 1$

$\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}}$ $\quad \boxed{\theta = \frac{\pi}{2}}$

(b) $\tan(2\theta) + \tan\theta = 0$

Sol $\frac{\sin 2\theta}{\cos 2\theta} + \frac{\sin\theta}{\cos\theta} = 0$

$\frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1} + \frac{\sin\theta}{\cos\theta} = 0$

$0 = \frac{2\sin\theta\cos^2\theta + 2\sin\theta\cos^2\theta - \sin\theta}{(2\cos^2\theta - 1)(\cos\theta)}$

$0 = 4\sin\theta\cos^2\theta - \sin\theta$

$\sin\theta(4\cos^2\theta - 1) = 0$

$\sin\theta = 0$ or $4\cos^2\theta = 1$

$\boxed{\theta = 0, \pi, 2\pi}$

$\cos\theta = \pm\frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

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TICKET-IN-THE-DOOR

In order to be prepared for class you must watch the module and complete the following activity. This is due first thing when you get to class.

Check your understanding:

1. Find the exact values of
   a. $\sin 345° = \sin(300+45) = \sin 300 \cos 45 + \sin 45 \cos 300$
      $= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

   b. $\sin 105°$
      $\sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45$
      $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

   c. $\cos 285°$
      $\cos(240+45) = \cos 240 \cos 45 - \sin 240 \sin 45 = -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$
      $= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$
      $\boxed{= \frac{-\sqrt{2}+\sqrt{6}}{4}}$

2. Using the sum or difference formulas, $2\sin t + 6\cos t =$ ______ $\sin(t +$ ______ $)$.

   N/A

3. Does $\cos(3t) = 4\cos^3 t - 3\cos t$? Explain.

Sol $\cos(3t) = \cos(2t+t)$
$= \cos 2t \cos t - \sin 2t \sin t$
$= (2\cos^2 t - 1)\cos t - 2\sin t \cos t \cdot \sin t$
$= 2\cos^3 t - \cos t - 2\cos t \sin^2 t$
$= 2\cos^3 t - \cos t - 2\cos t(1 - \cos^2 t)$
$= 2\cos^3 t - \cos t - 2\cos t + 2\cos^3 t$
$= \boxed{4\cos^3 t - 3\cos t}$