

Name _____

Show your work and indicate your reasoning. You will not receive credit if you do not clearly show how you are obtaining your answers. Do all work on the exam.

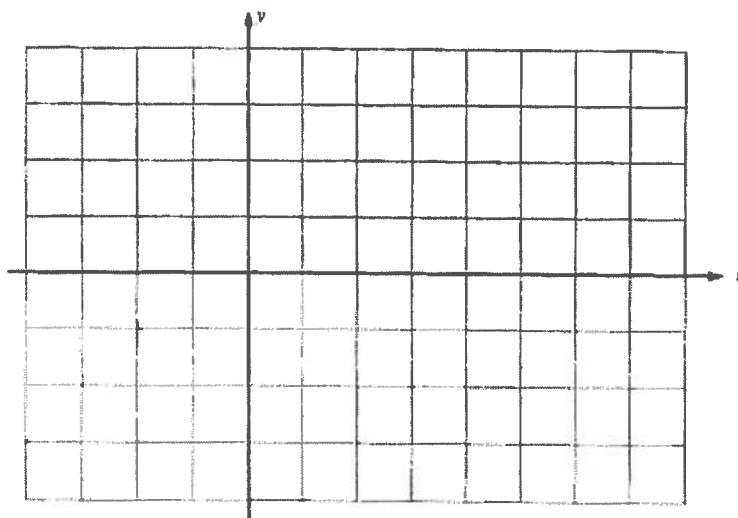
1. (16 points) Consider the function

$$y = -2 \cos\left(3t - \frac{\pi}{4}\right)$$

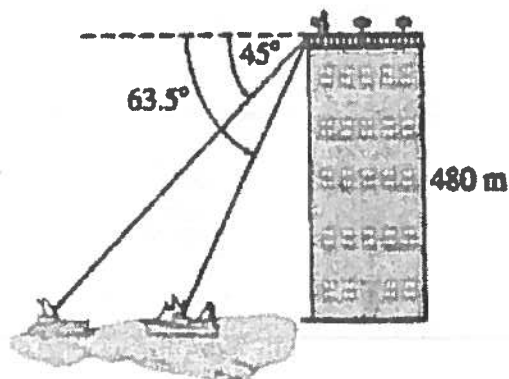
- (a) Determine the amplitude, period, and horizontal shift.

- (b) Describe in words how to obtain the graph of $y = -2 \cos\left(3t - \frac{\pi}{4}\right)$ from the graph of $y = \cos(3t)$.

- (c) Without using a calculator, graph $y = -2 \cos\left(3t - \frac{\pi}{4}\right)$ over two full periods. Be sure to label the units on your axes.



2. (16 points) While watching the sunset from the observation deck of an oceanfront building 480 meters high, you see two fishing boats in the distance. The angle of depression to the nearer boat is 63.5° , while for the farther boat, the angle is 45° .
- (a) How far out to sea is the nearer boat?
- (b) How far apart are the two boats?



3. (8 points) A person on the earth is looking up at the sun, which is 92,900,000 miles away. The angle spanned by the sun at the person's eye is approximately 0.5° . Find an estimate for the diameter of the sun.

4. (10 points)

(a) Find the exact values for all solutions to the equation $\cos x = -1/\sqrt{2}$ on the interval $0 \leq x \leq 2\pi$.

(b) Are the solutions to $\cos x = -1/\sqrt{2}$ the same as $\cos^{-1}(-1/\sqrt{2})$? Explain the differences, if any.

5. (14 points) Suppose θ is an angle in the fourth quadrant with $\cos \theta = x/4$. Find expressions for the other five trigonometric functions in terms of x .

6. (10 points) Use identities to find exact values of $\sin 105^\circ$ and $\cos 105^\circ$.

7. (12 points) Prove the following identity algebraically: $\sin(2t) = \frac{2 \tan t}{1 + (\tan t)^2}$

8. (14 points) Solve the following equation exactly on the interval $0 \leq \theta \leq 2\pi$.

$$\cos 2\theta + \sin \theta = 0$$

Trigonometric Identities

- Pythagorean Identities:

$$\sin^2 t + \cos^2 t = 1$$

$$\tan^2 t + 1 = \sec^2 t \quad \text{and} \quad 1 + \cot^2 t = \csc^2 t$$

- Double-Angle Formulas:

$$\sin 2t = 2 \sin t \cos t$$

$$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2 \sin^2 t = 2 \cos^2 t - 1$$

$$\tan 2t = \frac{2 \tan t}{1 - \tan^2 t}$$

- Negative Angle Identities:

$$\sin(-t) = -\sin t, \quad \cos(-t) = \cos t, \quad \tan(-t) = -\tan t$$

- Cofunction Identities:

$$\sin t = \cos\left(t - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - t\right) \quad \cos t = \sin\left(t + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - t\right)$$

- Sum and Difference Identities:

$$\begin{aligned} \sin(\theta + \phi) &= \sin \theta \cos \phi + \sin \phi \cos \theta & \cos(\theta + \phi) &= \cos \theta \cos \phi - \sin \theta \sin \phi \\ \sin(\theta - \phi) &= \sin \theta \cos \phi - \sin \phi \cos \theta & \cos(\theta - \phi) &= \cos \theta \cos \phi + \sin \theta \sin \phi \end{aligned}$$

