Battle stats formula

Kivou [2000607]

April 27, 2020

Starting from a formula that gives the battle stats gains for a certain amount of energy, we derive here a formula that gives the battle stats as a function of energy and, in turns, the energy needed to reach a certain battle stats level from a given one.

1 Gym gain formula

Vladar [1996140] gives a formula defining the battle stats gains as a function of the current stat, the energy spent on train and other parameters.

In this formula and for the rest we will need:

- 2 variables and their increments
  - Energy E and dE
  - Battle stats S and dS
- 6 known coefficients
  - $a = 0.0000003480061091$
  - $b = 250$
  - $c = 0.000003091619094$
  - $d = 0.0000682775184551527$
  - $e = -0.0301431777$
  - $S_{\mathrm{c}} = 50000000$ (known as the battle stats cap)
- 3 states variables
  - Happy level $\mathcal{H}$
  - Gym coefficient $\mathcal{G}$
  - Gym gain bonus $\mathcal{B}$

The Valdar formula eq.(1) gives the stats gains dS as a function of the current stats S and the energy spent dE. It is usually written:

$$dS = \left[(a\ln(\mathcal{H}+b)+c)\bar{S} + d(\mathcal{H}+b)+e\right](1+\mathcal{B})\mathcal{G}dE \tag{1}$$

with $\bar{S} = \min(S_\mathrm{c}, S)$

By introducing 2 new parameters α and β that depend only on the 5 coefficients a, b, c, d, e and the state variables $\mathcal{H}, \mathcal{B}, \mathcal{G}$, eq.(1) can be written as follow:

$$\frac{dS}{dE} = \alpha\bar{S} + \beta \quad \Leftrightarrow \quad \frac{dS}{dE} - \alpha\bar{S} = \beta \tag{2}$$

with

$$\begin{cases} \alpha = (a\ln(\mathcal{H}+b)+c)(1+\mathcal{B})\mathcal{G} \\ \beta = (d(\mathcal{H}+b)+e)(1+\mathcal{B})\mathcal{G} \end{cases} \tag{3}$$

From now on we assume that $\alpha > 0$ and $\beta > 0$.

2 Battle stats formula: $S(E)$

Assumption: $\mathcal{H}$, $\mathcal{G}$ and $\mathcal{B}$ remain constant (which is not realistic for a long term prediction at early stages).

From eq.(2) it can clearly be seen that $S(E)$ is driven by a simple ODE. Because of the piece wise definition of the equation the two cases $S < S_\mathrm{c}$ (before cap) and $S > S_\mathrm{c}$ (after cap) have to be treated separatly.

2.1 Before cap

With $S < S_\mathrm{c}$ eq.(2) can be written:

$$\frac{dS}{dE} - \alpha S = \beta \tag{4}$$

Which leads to the solution:

$$\forall k \in \mathbb{R}, \quad S(E) = ke^{\alpha E} - \frac{\beta}{\alpha} \tag{5}$$

With the initial condition $S(0) = 0$ we have

$$S(E) = \frac{\beta}{\alpha}\left(e^{\alpha E} - 1\right) \tag{6}$$

It can be interesting to compute $E = E_\mathrm{c}$ such that $S(E_\mathrm{c}) = S_\mathrm{c}$ which can be done by inverting eq.(6). It gives:

$$E_\mathrm{c} = \frac{1}{\alpha}\ln\left(\frac{\alpha S_\mathrm{c}}{\beta} + 1\right) \tag{7}$$

2.2 After cap

With $S < S_\mathrm{c}$ eq.(2) can be written:

$$\frac{dS}{dE} = \alpha S_\mathrm{c} + \beta \tag{8}$$

which directly yields

$$\forall k \in \mathbb{R}, \quad S(E) = (\alpha S_\mathrm{c} + \beta)E + k \tag{9}$$

With the condition $S(E_\mathrm{c}) = S_\mathrm{c}$ we have:

$$\begin{aligned} S &= (\alpha S_\mathrm{c} + \beta)E + S_\mathrm{c} - (\alpha S_\mathrm{c} + \beta)E_\mathrm{c} \\ &= (\alpha S_\mathrm{c} + \beta)(E - E_\mathrm{c}) + S_\mathrm{c} \end{aligned} \tag{10}$$

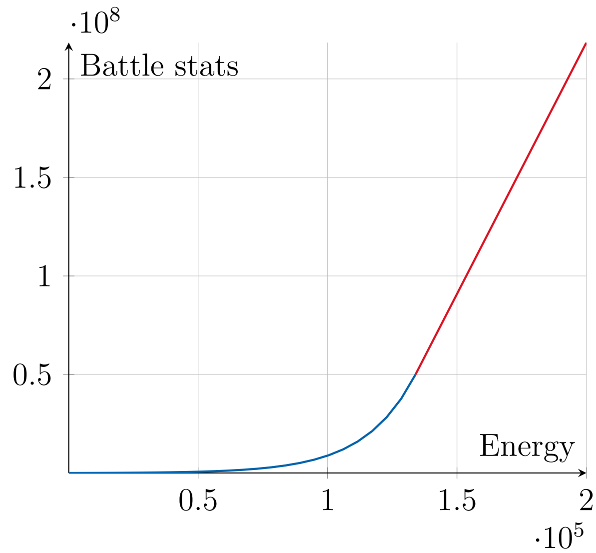

Figure 1: Battle stats as a function of energy for $\mathcal{H} = 5000$, $\mathcal{G} = 7.3$ and $\mathcal{B} = 15\%$

3 Prediction

In this section we are now interested at making a prediction of the energy needed ΔE to reach S_f from S_i. For this we need to anaylse the 3 cases:

- $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} < S_\mathrm{c}$
- $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$
- $S_\mathrm{i} > S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$

3.1 Before cap: $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} < S_\mathrm{c}$

From eq.(6) we can determine k with the condition $S(0) = S_\mathrm{i}$ which gives:

$$k = S_\mathrm{i} + \frac{\beta}{\alpha} \tag{11}$$

leading to

$$S(E) = \left(S_\mathrm{i} + \frac{\beta}{\alpha}\right) e^{\alpha E} - \frac{\beta}{\alpha} \tag{12}$$

In our context we can rewrite this equation with S_f and ΔE as:

$$S_\mathrm{f} = \left(S_\mathrm{i} + \frac{\beta}{\alpha}\right) e^{\alpha \Delta E} - \frac{\beta}{\alpha} \tag{13}$$

which can be inverted giving:

$$\Delta E = \frac{1}{\alpha} \ln \left(\frac{S_\mathrm{f} + \frac{\beta}{\alpha}}{S_\mathrm{i} + \frac{\beta}{\alpha}} \right) \tag{14}$$

ΔE being the energy needed to reach S_f stats from S_i.

3.2 After cap: $S_\mathrm{i} > S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$

With the linear relationship between S and E after cap we directly have

$$S_\mathrm{f} - S_\mathrm{i} = (\alpha S_\mathrm{c} + \beta)\Delta E \tag{15}$$

which gives:

$$\Delta E = \frac{S_\mathrm{f} - S_\mathrm{i}}{\alpha S_\mathrm{c} + \beta} \tag{16}$$

3.3 Passing cap: $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$

In this last case both exponential and linear regime have to be taken into account. We can decompose ΔE in two, the part needed to reach cap ΔE_bc (before cap) an the part in the linear regime ΔE_ac (after cap). S_c behing the frontiere between both, it plays the role of S_f for the exponential par and S_i in the linear one. It reads:

$$\Delta E = \Delta E_\mathrm{bc} + \Delta E_\mathrm{ac} = \frac{1}{\alpha} \ln \left(\frac{S_\mathrm{c} + \frac{\beta}{\alpha}}{S_\mathrm{i} + \frac{\beta}{\alpha}} \right) + \frac{S_\mathrm{f} - S_\mathrm{c}}{\alpha S_\mathrm{c} + \beta} \tag{17}$$

3.4 Generalized formula

We can account for the three cases above in a single generalized equation:

$$\boxed{\Delta E = \underbrace{\frac{1}{\alpha} \ln \left(\frac{\min(S_\mathrm{c}, S_\mathrm{f}) + \frac{\beta}{\alpha}}{\min(S_\mathrm{c}, S_\mathrm{i}) + \frac{\beta}{\alpha}} \right)}_{\text{Before cap}} + \underbrace{\frac{\max(S_\mathrm{c}, S_\mathrm{f}) - \max(S_\mathrm{c}, S_\mathrm{i})}{\alpha S_\mathrm{c} + \beta}}_{\text{After cap}}} \tag{18}$$

where ΔE is the energy needed to reach S_f battle stats from S_i.

4 Impact of the parameters

With eq. is now possible to quantify the impact of each state variables: $\mathcal{H}$, $\mathcal{G}$, $\mathcal{B}$ and the energy needed to acheive a battle stats goal.

4.1 Effectiveness function

We define a effectiveness function $\eta(\mathcal{H}, \mathcal{G}, \mathcal{B})$ that will return a number between 0 and 1 quantifying the efficiency of the state variables to reach a battle stats goal. It as computed with respect to the energy needed to reach this goal (ΔE) compared to the minimum possible ΔE^* (*i.e.* corresponding to $\mathcal{H}_\mathrm{max}, \mathcal{G}_\mathrm{max}, \mathcal{B}_\mathrm{max}$).

$$\eta(\mathcal{H}, \mathcal{G}, \mathcal{B}) = \frac{\Delta E}{\Delta E^*} \tag{19}$$

4.2 Reaching stats cap

In this case we set the goal to be reaching stats cap from 0 ($S_\mathrm{f} = S_\mathrm{c}$ and $S_\mathrm{i} = 0$). Thus we have:

$$\eta(\mathcal{H}, \mathcal{G}, \mathcal{B}) = \frac{\ln \left(\frac{\alpha}{\beta} S_\mathrm{c} + 1 \right)}{\ln \left(\frac{\alpha_\mathrm{max}}{\beta_\mathrm{max}} S_\mathrm{c} + 1 \right)} \tag{20}$$

where α and β depend on $\mathcal{H}, \mathcal{G}, \mathcal{B}$.

TODO: parametrical analysis... Pretty straitforward at this stage...