Battle stats formula

Kivou [2000607]

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Starting from a formula that gives the battle stats gains for a certain amount of energy, we derive here a formula that gives the battle stats as a function of energy and, in turns, the energy needed to reach a certain battle stats level from a given one.

1 Theory

1.1 Gym gain formula

Vladar [1996140] gives a formula defining the battle stats gains as a function of the current stat, the energy spent on train and other parameters.

In this formula and for the rest we will need:

- 2 variables and their differentials
  - Energy E and dE
  - Battle stats S and dS
- 6 known coefficients
  - $a = 0.0000003480061091$
  - $b = 250$
  - $c = 0.000003091619094$
  - $d = 0.0000682775184551527$
  - $e = -0.0301431777$
  - $S_\mathrm{c} = 50\,000\,000$ (known as the battle stats cap)
- 3 states variables
  - Happy level $\mathcal{H}$
  - Gym coefficient $\mathcal{G}$
  - Gym gain bonus $\mathcal{B}$

The Vladar formula eq.(1) gives the stats gains dS as a function of the current stats S and the energy spent dE. It is usually written:

$$dS = \left[(a\ln(\mathcal{H}+b)+c)\bar{S} + d(\mathcal{H}+b)+e\right](1+\mathcal{B})\mathcal{G}dE \tag{1}$$

with $\bar{S} = \min(S_\mathrm{c}, S)$

By introducing 2 new parameters α and β that depend only on the 5 coefficients a, b, c, d, e and the state variables $\mathcal{H}, \mathcal{B}, \mathcal{G}$, eq.(1) can be written as follow:

$$\frac{dS}{dE} = \alpha\bar{S} + \beta \quad \Leftrightarrow \quad \frac{dS}{dE} - \alpha\bar{S} = \beta \tag{2}$$

with

$$\begin{cases} \alpha = (a\ln(\mathcal{H}+b)+c)(1+\mathcal{B})\mathcal{G} \\ \beta = (d(\mathcal{H}+b)+e)(1+\mathcal{B})\mathcal{G} \end{cases} \tag{3}$$

From now on we assume that $\alpha > 0$ and $\beta > 0$.

1.2 Battle stats formula

Assumption: $\mathcal{H}$, $\mathcal{G}$ and $\mathcal{B}$ remain constant (which is not realistic for a long term prediction at early stages).

From eq.(2) it can clearly be seen that $S(E)$ is driven by a simple ODE. Because of the piece wise definition of the equation the two cases $S < S_\mathrm{c}$ (before cap) and $S > S_\mathrm{c}$ (after cap) have to be treated separatly.

1.2.1 Before cap

With $S < S_\mathrm{c}$ eq.(2) can be written:

$$\frac{dS}{dE} - \alpha S = \beta \tag{4}$$

Which leads to the solution:

$$\forall k \in \mathbb{R}, \quad S(E) = ke^{\alpha E} - \frac{\beta}{\alpha} \tag{5}$$

From eq.(10) we can derive the inverse function:

$$\forall k' \in \mathbb{R}, \quad S^{-1} : E(S) = \frac{1}{\alpha}\ln\left(1 + \frac{\alpha S}{\beta}\right) + k' \tag{6}$$

giving the energy as a function of the battle stats.

If we assume a boundary coundition, we can determine k (and k'). For example with $S(0) = 0$ we have:

$$\begin{cases} S(E) = \dfrac{\beta}{\alpha}\left(e^{\alpha E} - 1\right) \\ E(S) = \dfrac{1}{\alpha}\ln\left(1 + \dfrac{\alpha S}{\beta}\right) \end{cases} \tag{7}$$

At this point it is important to put into perspective the domain of validity of this equation. Indeed, because the assumption of *constant states variables* the boundary condition $S(0) = 0$ will most of the time not be justified. As soon as a prediction is made with states variables different than the inital ones (for $E = 0$), another boundary counditions have to be stated. However even if most of the time these conditions are not known, it is not important to determine k (and k') since the value of interest here relies on differences of S (and E), thus cancelling out k (and k').

For sake of clarity and in order to ease the reading we have decided to keep eq.(7) as the predictive formulae. The reader can easily use eq.(10) in the follow to convince themself that the results are identical.

1.2.2 After cap

With $S < S_\mathrm{c}$ eq.(2) can be written:

$$\frac{dS}{dE} = \alpha S_\mathrm{c} + \beta \tag{8}$$

which directly yields

$$\forall k \in \mathbb{R}, \quad S(E) = (\alpha S_\mathrm{c} + \beta)E + k \tag{9}$$

Inverting this function directly gives:

$$\forall k' \in \mathbb{R}, \quad E(S) = \frac{S}{\alpha S_\mathrm{c} + \beta} + k' \tag{10}$$

In this case it is not even clearer to assum of boundary condition $S(E_\mathrm{c}) = S_\mathrm{c}$ we have:

$$\begin{aligned} S(E) &= (\alpha S_\mathrm{c} + \beta)E + S_\mathrm{c} - (\alpha S_\mathrm{c} + \beta)E_\mathrm{c} \\ &= (\alpha S_\mathrm{c} + \beta)(E - E_\mathrm{c}) + S_\mathrm{c} \end{aligned} \tag{11}$$

with E_c the energy to stats cap S_c, eq.(10) gives us:

$$E_\mathrm{c} = \frac{1}{\alpha} \ln \left(1 + \frac{\alpha S_\mathrm{c}}{\beta} \right) \tag{12}$$

Figure 1 shows $S(E)$ from 0 to 200 000 energy with $S(0) = 0$, $\mathcal{H} = 5000$, $\mathcal{G} = 7.3$ and $\mathcal{B} = 15\%$, giving $\alpha = 0.0000509798$ and $\beta = 2.7561943022$

1.3 Prediction formula

In this section we are now interested at making a prediction of the energy needed ΔE to reach S_f from S_i:

$$\Delta E = E(S_\mathrm{f}) - E(S_\mathrm{i}) \tag{13}$$

For this we need to anaylse the 3 cases:

1. Before cap: $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} < S_\mathrm{c}$
2. After cap: $S_\mathrm{i} > S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$
3. Passing cap: $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$

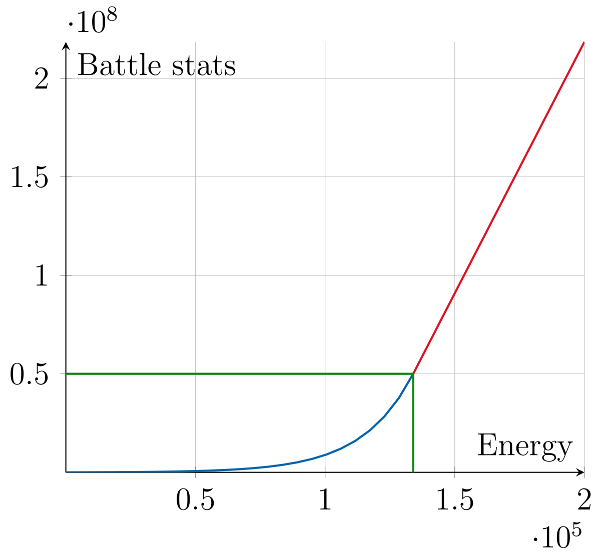

Figure 1: Battle stats as a function of energy for $\mathcal{H} = 5000$, $\mathcal{G} = 7.3$ and $\mathcal{B} = 15\%$

1.3.1 Before cap: $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} < S_\mathrm{c}$

Injecting eq.(7) in eq.(13) gives

$$\Delta E = \frac{1}{\alpha} \ln \left(\frac{1 + \alpha S_\mathrm{f}/\beta}{1 + \alpha S_\mathrm{i}/\beta} \right) \tag{14}$$

1.3.2 After cap: $S_\mathrm{i} > S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$

Injecting the linear relationship eq.(10) in eq.(13) gives

$$\Delta E = \frac{S_\mathrm{f} - S_\mathrm{i}}{\alpha S_\mathrm{c} + \beta} \tag{15}$$

1.3.3 Passing cap: $S_\mathrm{i} < S_\mathrm{c}$ and $S_\mathrm{f} > S_\mathrm{c}$

In this last case both exponential and linear regime have to be taken into account. We can decompose ΔE in two, the part needed to reach cap ΔE_bc (before cap) an the part in the linear regime ΔE_ac (after cap). S_c behing the frontiere between both, it plays the role of S_f for the exponential part and S_i for the linear one. It reads:

$$\Delta E = \Delta E_\mathrm{bc} + \Delta E_\mathrm{ac} = \frac{1}{\alpha} \ln \left(\frac{1 + \alpha S_\mathrm{c}/\beta}{1 + \alpha S_\mathrm{i}/\beta} \right) + \frac{S_\mathrm{f} - S_\mathrm{c}}{\alpha S_\mathrm{c} + \beta} \tag{16}$$

1.3.4 Generalized formula

We can account for the three cases above in a single generalized equation[1]:

$$\boxed{\Delta E = \underbrace{\frac{1}{\alpha}\ln\left(\frac{1+\alpha\min(S_{\mathrm{c}},S_{\mathrm{f}})/\beta}{1+\alpha\min(S_{\mathrm{c}},S_{\mathrm{i}})/\beta}\right)}_{\text{Before cap}} + \underbrace{\frac{\max(S_{\mathrm{c}},S_{\mathrm{f}})-\max(S_{\mathrm{c}},S_{\mathrm{i}})}{\alpha S_{\mathrm{c}}+\beta}}_{\text{After cap}}} \tag{17}$$

where ΔE is the energy needed to reach S_{f} battle stats from S_{i}.

2 Parametrical analysis

DRAFT...

2.1 The parameters

With eq. it is now possible to quantify the impact of each state variables: $\mathcal{H}$, $\mathcal{G}$, $\mathcal{B}$ and the energy needed to acheive a battle stats goal.

2.1.1 Effectiveness function

We define an effectiveness function $\eta(\mathcal{H},\mathcal{G},\mathcal{B})$ that will return a number between 0 and 1 quantifying the effectiveness of the state variables to reach a battle stats goal. It is computed with respect to the energy needed to reach this goal (ΔE) compared to the minimum possible ΔE^* (*i.e.* corresponding to $\mathcal{H}_{\max},\mathcal{G}_{\max},\mathcal{B}_{\max}$).

$$\eta(\mathcal{H},\mathcal{G},\mathcal{B}) = \frac{\Delta E^*}{\Delta E} \tag{18}$$

2.1.2 Reaching stats cap

In this case we set the goal to be reaching stats cap from 0 ($S_{\mathrm{f}} = S_{\mathrm{c}}$ and $S_{\mathrm{i}} = 0$). Thus we have:

$$\eta(\mathcal{H},\mathcal{G},\mathcal{B}) = \frac{\ln\left(1+\alpha_{\max}S_{\mathrm{c}}/\beta_{\max}\right)}{\ln\left(1+\alpha S_{\mathrm{c}}/\beta\right)} \tag{19}$$

where α and β depend on $\mathcal{H},\mathcal{G},\mathcal{B}$.

[1]Mathematicians will not see the point, programmers will love it.

3 TODO

- find out why to quantify which from $\mathcal{H}, \mathcal{G}$ and $\mathcal{B}$ in pacts the most and when
- add a time / energy conversion
- Happy jumps and books...
- find out what torn's community wants to know