Assignment 3 solutions

Sol. 1) Let, x_1 = Number of model-1

x_2 = Number of model-2

Objective function:

$$\text{Maximize } Z = 30x_1 + 40x_2$$

Subject to:

$$2x_1 + 3x_2 \leq 1200$$

$$2x_1 + x_2 \leq 1000$$

$$4x_2 \leq 800$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Augmented problem: -

$$\text{Maximize } Z = 30x_1 + 40x_2 + 0S_1 + 0S_2 + 0S_3$$

S.T:

$$2x_1 + 3x_2 + S_1 = 1200$$

$$2x_1 + x_2 + S_2 = 1000$$

$$4x_2 + S_3 = 800$$

$$x_i \geq 0\ \cancel{\forall}\ i = 1 \ldots 5.$$

Iteration 0

		C_j	30	40	0	0	0
C_j	Basis	Value	x_1	x_2	S_1	S_2	S_3
0	S_1	1200	2	3	1	0	0
0	S_2	1000	2	1	0	1	0
0	S_3	800	0	4	0	0	1
		C_j-Z_j	30	40	0	0	0

Iteration 1

		C_j	30	40	0	0	0
C_j	Basis	Value	x_1	x_2	S_1	S_2	S_3
0	S_1	600	2	0	1	0	-3/4
0	S_2	800	2	0	0	1	-1/4
40	x_2	200	0	1	0	0	1/4
		C_j-Z_j	30	0	0	0	-10

Iteration 2

		C_j	30	40	0	0	0
C_j	Basis	Value	x_1	x_2	S_1	S_2	S_3
30	x_1	450	1	0	-1/4	3/4	0
0	S_2	400	0	0	-2	2	1
40	x_2	100	0	1	1/2	-1/2	0
		C_j-Z_j	0	0	-25/2	-5/2	0

Optimal solution: $x_1^*= 450$; $x_2^* =100$; Z=17,500

(b) Basis matrix (B): - This is the matrix formed by the variables in the basis of the final table. Its values, however, will be obtained from the initial table.

$$B=\begin{bmatrix} 2 & 0 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

Inverse of the basis matrix (B^{-1}): This matrix is formed by the variables in the basis of initial table and values from final table.

$$B^{-1}=\begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}$$

Resource (b_1) of milling machine changes from 1200 to 1300.

Changes in resource matrix affect the optimal solution. Therefore, new optimal solution will be: -

$$B^{-1}b=\begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}\begin{bmatrix} 1300 \\ 1000 \\ 800 \end{bmatrix}=\begin{bmatrix} 425 \\ 200 \\ 150 \end{bmatrix}$$

$x_1^*= 425$; $x_2^*=150$; Z=18,750

(C) Here, Resource (b_2) of Grinding machine changes from 800 to 350

$$B^{-1}b=\begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}\begin{bmatrix} 1200 \\ 1000 \\ 350 \end{bmatrix}=\begin{bmatrix} 450 \\ -50 \\ 100 \end{bmatrix}$$

No, we cannot determine the new optimal solution directly from the given information. Because $S_3 \leq 0$ i.e. infeasible.

Sol 2) Dual problem

Minimize $z = 5y_1+3y_2+8y_3$

Subject to $y_1 - y_2+4y_3 =5$

$2y_1+5y_2+7y_3 \geq 6$

y_1 unrestricted, $y_2 \leq 0$, $y_3 \geq 0$

Sol 3) Maximize $2x_1+x_2$

subject to $x_1 \leq 0$

let, $x_1 = -x_1^{'}$; $x_2 = x_2^{+} - x_2^{-}$

Now the problem transforms into:

Maximize $-2x_1^{'} + x_2^{+} - x_2^{-}$

Subject to: $x_1^{'} \geq 0$

$x_2^{+} \geq 0$

$x_2^{-} \geq 0$

(a) The dual form does not exist.
(b) Dual solution is infeasible.
(c) Primal optimal solution cannot be obtained because dual form is infeasible.

Sol 4) From the table, the starting primal variable x_4 and R uniquely correspond to the dual variables y_1 and y_2, respectively. Thus, we determine the optimum dual solution as follows:

Staring primal basic variables	x_4	R
Z- equation coefficients	29/5	-2/5+M
Original objective coefficient	0	-M
Dual variables	y_1	y_2
Optimal dual values	29/5+0=29/5	-2/5+M+(-M) =-2/5

5) Minimize $Z = 600x_1 + 500x_2$

s.t. $2x_1 + x_2 \geq 80$

$x_1 + 2x_2 \geq 60$

$x_1, x_2 \geq 0$

Dual:

Maximize $w = 80y_1 + 60y_2$.

s.t. $2x_1 + x_2 \leq 600$

$x_1 + 2x_2 \leq 500$.

	y_1	y_2	y_3	y_4	b	ratio
y_3	2	1	1	0	600	300
y_4	1	2	0	1	500	500
$(z_j - c_j)$	-80	-60	0	0	0	
y_1	1	1/2	1/2	0	300	600
y_4	0	3/2	-1/2	1	200	400/3
$(z_j - c_j)$	0	-20	40	0	24000	
y_1	1	0	2/3	-1/3	700/3.	
y_2	0	1	-1/3	2/3	400/3	
	.0	0	100/3	40/3	80000/3	

$y_1 = \frac{700}{3}$, $y_2 = \frac{400}{3}$, $w = \frac{80{,}000}{3}$.

6) Minimize $Z = x_1 + 2x_2 + 3x_3$

s.t. $3x_1 + 4x_2 \leq 5$

$5x_1 + x_2 + 6x_3 = 7$

$8x_1 + 9x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

Dual:

Maximize $W = 5y_1 + 7y_2 + 2y_3$

s.t., $3y_1 + 5y_2 + 8y_3 \geq 1$

$4y_1 + y_2 = 2$

$6y_2 + 9y_3 \leq 3$.

	y_1	y_2	y_3	y_4	y_5	s_1	s_2	b	ratio
y_5	0	6	9	0	1	0	0	3	1/3
s_1	3	5	8	-1	0	1	0	1	1/8
s_2	4	1	0	0	0	0	1	2	–
Z	-5	-7	-2	0	0	0	0	0	
z'	-7	-6	-8	1	0	0	0	-3	

	y_1	y_2	y_3	y_4	y_5	s_1	s_2	b	ratio
y_5	0	6	9	0	1	0	0	3	–
y_1	1	5/3	8/3	-1/3	0	1/3	0	1/3	–
s_2	0	-17/3	-32/3	4/3	0	-4/3	1	2/3	1/2
Z	0	4/3	34/3	-5/3	0	5/3	0	5/3	
z'	0	17/3	32/3	-4/3	0	7/3	0	-2/3	

	y_1	y_2	y_3	y_4	y_5	s_1	s_2	b	ratio
y_5	0	6	9	0	1	0	0	3	1/2
y_1	1	1/4	0	0	0	0	1/4	1/2	2
y_4	0	-17/4	-8	1	0	-1	3/4	1/2	–
Z	0	-23/4	-2	0	0	0	5/4	5/2	
z'	0	0	0	0	0	1	1	0	

	y_1	y_2	y_3	y_4	y_5	b
y_2	0	1	3/2	0	1/6	1/2
y_1	1	0	-3/8	0	-1/24	3/8
y_4	0	0	-13/8	1	17/24	21/8
Z	0	0	53/8	0	23/24	43/8

$y_1 = 3/8$

$y_2 = 1/2$

$y_4 = 21/8$

$W = 43/8$