

$$p(A) = p(B) = p(C) = 1/3$$

$$p(X_1=x_1, X_2=x_2 | Y=A) = c_1 * \max(x_1, x_2)$$

$x_1 \backslash x_2$	0	1	2
0	0	$c_1$	$2c_1$
1	$c_1$	$c_1$	$2c_1$
2	$2c_1$	$2c_1$	$2c_1$

Adding up, we find  $13c_1 = 1$ , i.e.  $c_1 = 1/13$

$$p(X_1=x_1, X_2=x_2 | Y=B) = c_2 * \min(x_1, x_2)$$

$x_1 \backslash x_2$	0	1	2
0	0	0	0
1	0	$c_2$	$c_2$
2	0	$c_2$	$2c_2$

Adding up, we find  $5c_2 = 1$ , i.e.  $c_2 = 1/5$

$$p(X_1=x_1, X_2=x_2 | Y=C) = c_3 * |x_1 - x_2|$$

$x_1 \backslash x_2$	0	1	2
0	0	$c_3$	$2c_3$
1	$c_3$	0	$c_3$
2	$2c_3$	$c_3$	0

Adding up, we find  $8c_3 = 1$ , i.e.  $c_3 = 1/8$

Now the question is about  $p(Y | X_1=x_1, X_2=x_2) = c * p(X_1=x_1, X_2=x_2 | Y) * p(Y)$

For  $(x_1=0, x_2=0)$ :  $p(Y=A | 0,0) = c * 0 * 1/3 = 0$ ,  $p(Y=B | 0,0) = c * 0 * 1/3 = 0$ ,  $p(Y=C | 0,0) = 0$

So, this point has 0 probability under all three classes

For  $(x_1=1, x_2=2)$ ,  $p(Y=A | 1,2) = c * 2/13 * 1/3 = 2c/39$ ,  $p(Y=B | 1,2) = c * 1/5 * 1/3 = c/15$ ,  $p(Y=C | 1,2) = c * 1/8 * 1/3 = c/24$ . Prediction: B (largest)

Repeat for all points, and find the predicted class label for each.

Next, for Naïve Bayes assumption:  $p(X_1=x_1, X_2=x_2 | Y) = p(X_1=x_1 | Y) * p(X_2=x_2 | Y)$

For example,  $p(X_1=1 | Y=B) = p(X_1=1, X_2=0 | Y=B) + p(X_1=1, X_2=1 | Y=B) + p(X_1=1, X_2=2 | Y=B) = 0 + 1/5 + 1/5 = 2/5$

$p(X_2=2 | Y=B) = p(X_1=0, X_2=2 | Y=B) + p(X_1=1, X_2=2 | Y=B) + p(X_1=2, X_2=2 | Y=B) = 0 + 1/5 + 2/5 = 3/5$

Therefore  $p(Y=B | X_1=1, X_2=2) = c * p(X_1=1 | Y=B) * p(X_2=2 | Y=B) * p(Y=B)$   
 $= c * (2/5) * (3/5) * (1/3) = 2c/25$

Similarly calculate  $p(Y=A | X_1=1, X_2=2)$  and  $p(Y=C | X_1=1, X_2=2)$  and predict the label which has the highest probability value