Theory of Production

References

1. Pindyck & Rubinfeld
2. Henderson & Quandt
3. Gravelle & Rees

Theory of production

is the study of technology or technological constraint that a firm faces.

Firm decides how much to produce => depends on technology and input price

Technological constraint: certain feasible ways of producing a good from combination of inputs.
It depends on the state of scientific knowledge.

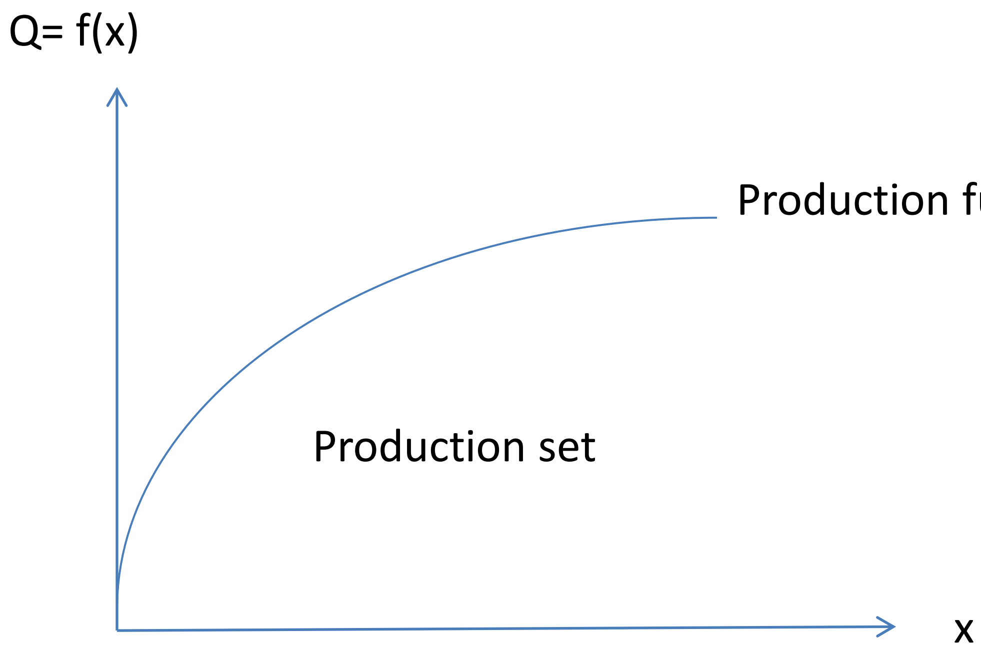
Let Q (output) depends on vector of inputs $x = (x_1, x_2, \ldots\ldots x_n)$

A technology permits output levels such as $0 \leq Q \leq f(x)$

Suppose only one input x_1.

Let Q = f(x) => mapping from input space to output space.

A production function is the set of inputs and technologically feasible maximum levels of output.



Production set: set of input levels and output levels that the given state of technology makes feasible.

$S = \{Q \mid 0 \leq Q \leq f(x)\}$

Restrictions on the production function:

i. Essentiality of inputs: $f(0,0)=0$;

ii. Strict essentiality of inputs: $f(0,L)= f(K,0)=0$;

iii. MPs are positive but diminishing;

iv. MP_i/MP_j diminishes as use of ith input increases.

Isoquant: set of input combinations yielding same level of output.

$I(Q_0)= \{x| \ f(x)= Q_0\}$

Input requirement set: set of input x that yields at least Q_0 level of output.

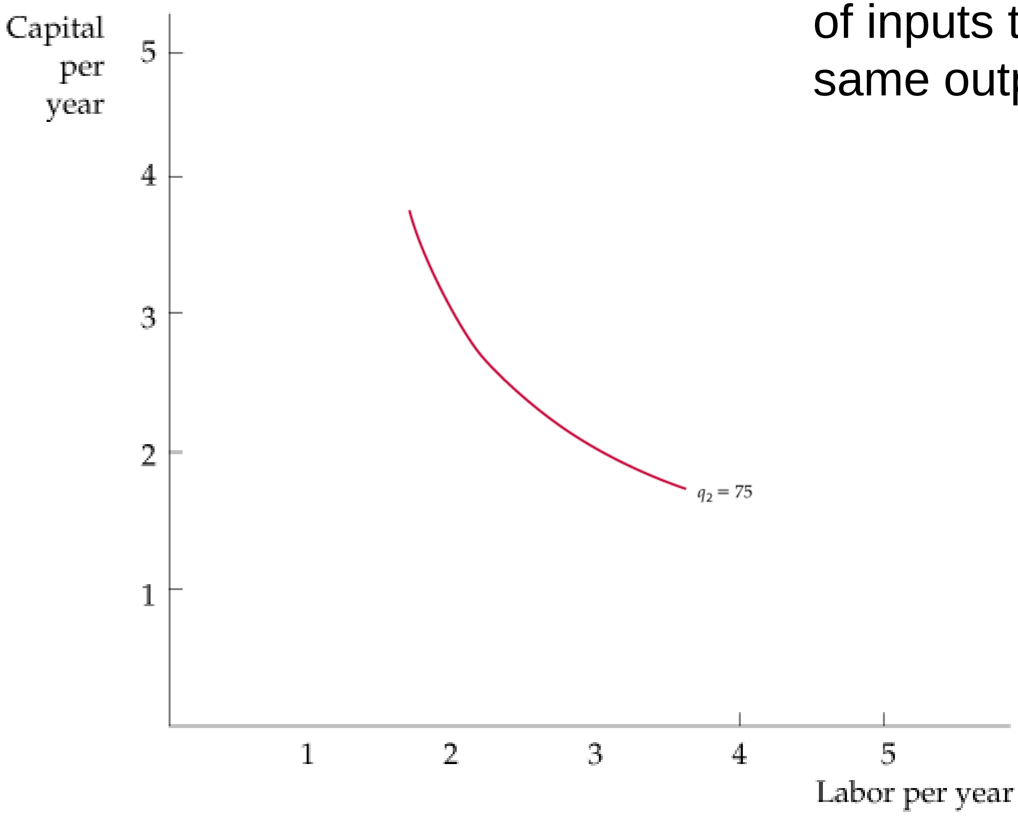
Let the production function be: $Q= f(x_1, x_2)$

Isoquant: $\bar{Q} = f(x_1, x_2)$

Slope of IQ: $\left.\dfrac{dx_2}{dx_1}\right|_{\bar{Q}} = -\dfrac{MP_1}{MP_2}$

PRODUCTION WITH TWO VARIABLE INPUTS

- **isoquant** Curve showing all possible combinations of inputs that yield the same output.

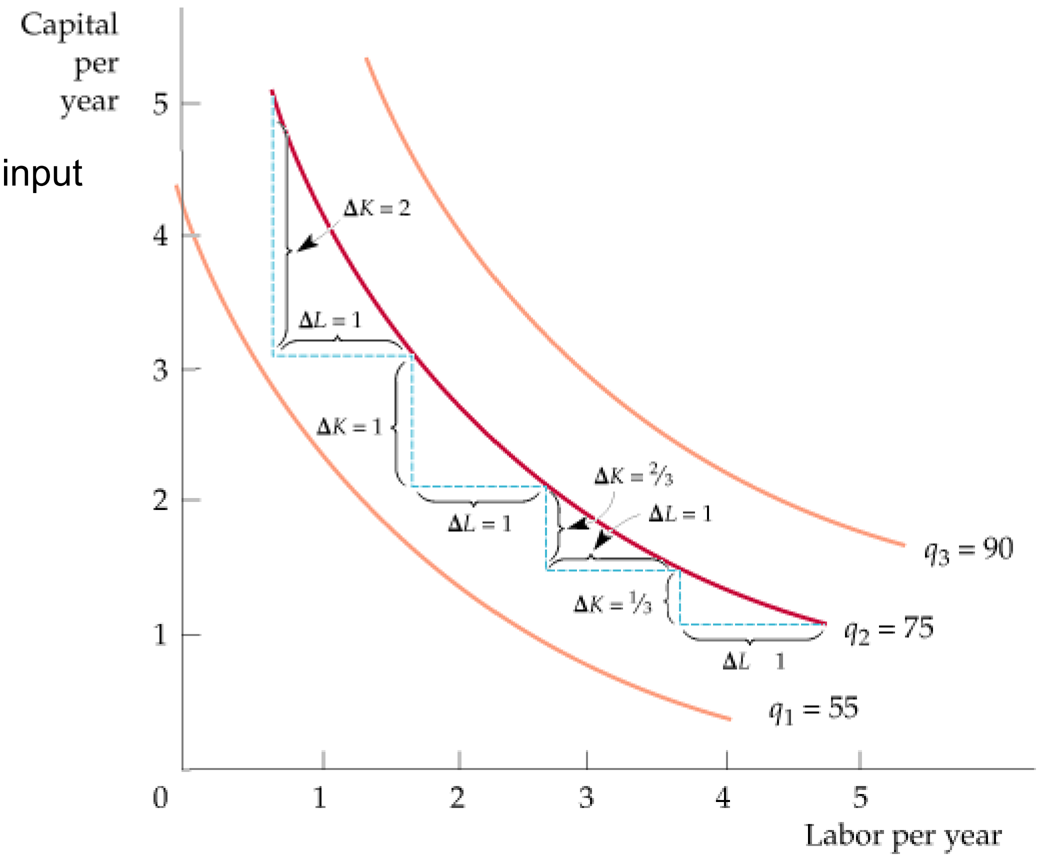


PRODUCTION WITH TWO VARIABLE INPUTS

- Substitution Among Inputs
  - **marginal rate of technical substitution (MRTS)** Amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant.

MRTS = − Change in capital input/change in labor input
= − $\Delta K/\Delta L$ (for a fixed level of q)

$$(\mathrm{MP}_L)/(\mathrm{MP}_K) = -(\Delta K/\Delta L) = \mathrm{MRTS}$$



Difference between production technology and technique of production:

Technique of production:

Technologically feasible proportions in which inputs are combined to produce a good.

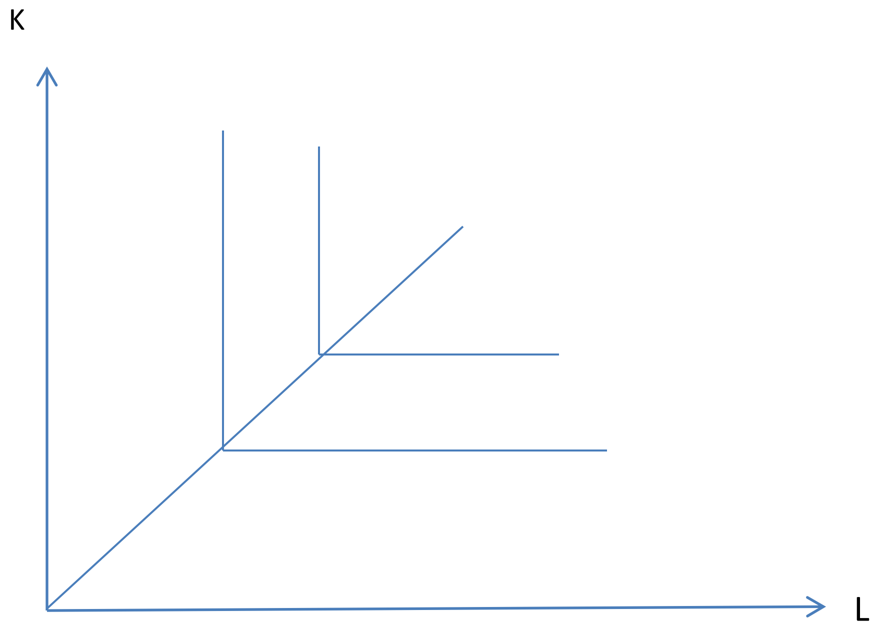
Technology of production:

Set of all feasible and techniques of production

Change in technology versus change in technique of production:

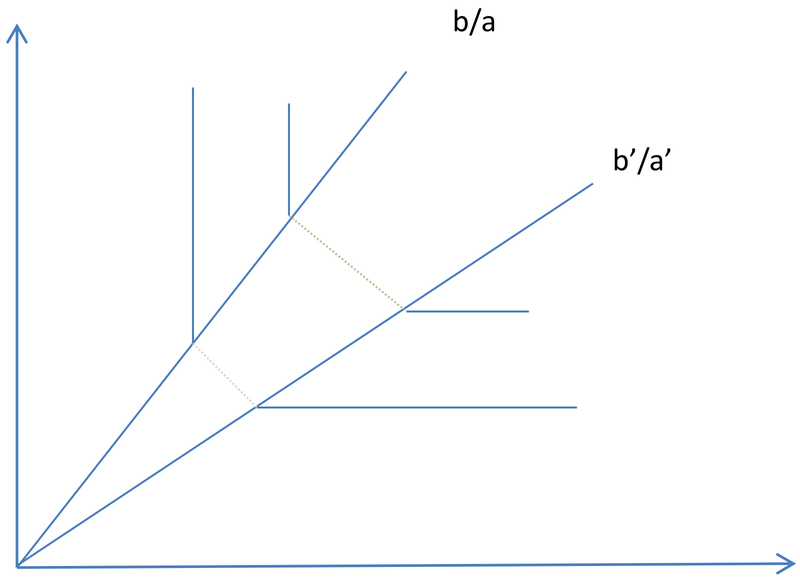
Shift of IQ versus movement along the same IQ.

Technology and shape of IQ



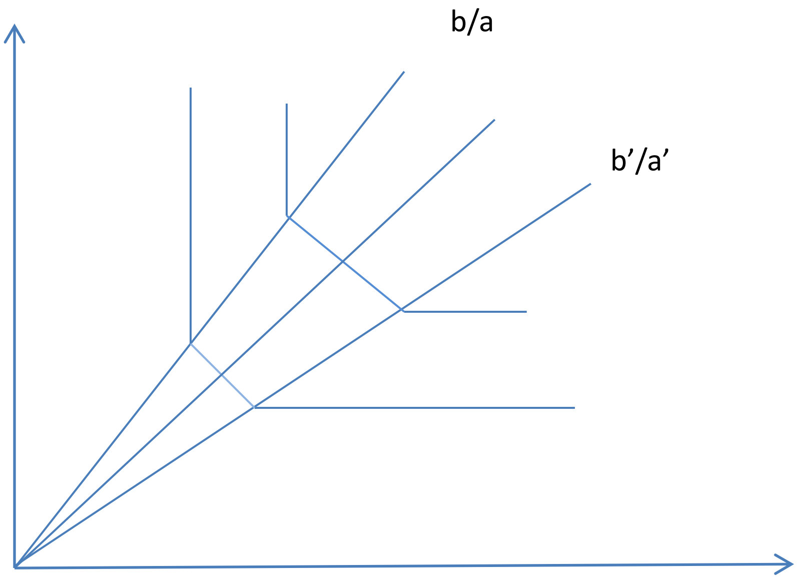
Case I. Fixed coefficient technology: Suppose technology allows only one way of combining the inputs.

Leontief production function: $Q = \min\{K/a, L/b\}$



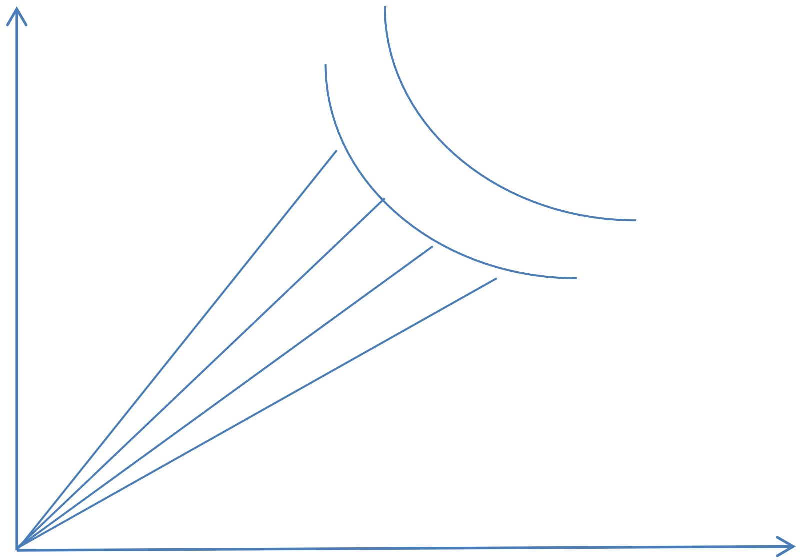
Case II. Rigid technology:
Technology permits two ways of combining L & K: b/a and b’/a’

Discontinuity in the IQ.



Case III. Suppose two ways of combining L & K and any convex combination of them

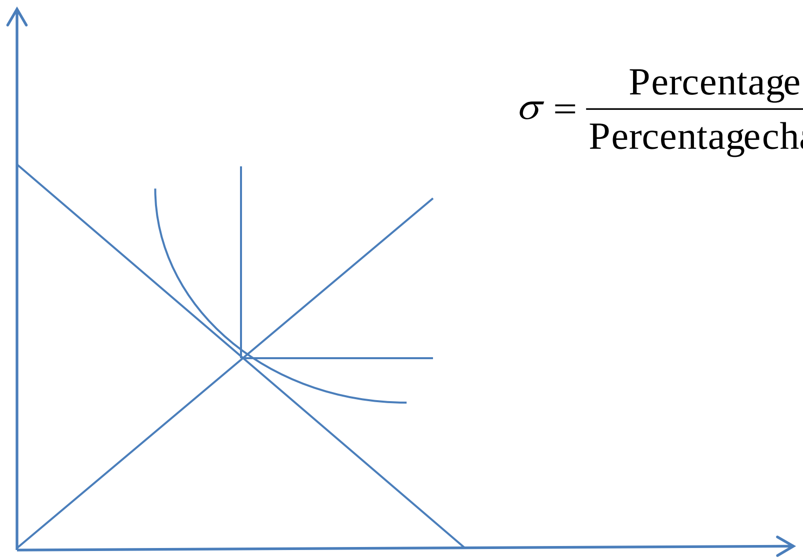
As possibility of substitution increases, IQ gets smoother.



Case IV. Suppose production technology allows infinite ways of combining L & K: flexible coefficient production function. IQ is smooth.

Degree of input substitutability is related to the curvature of IQ.

Degree of input substitution



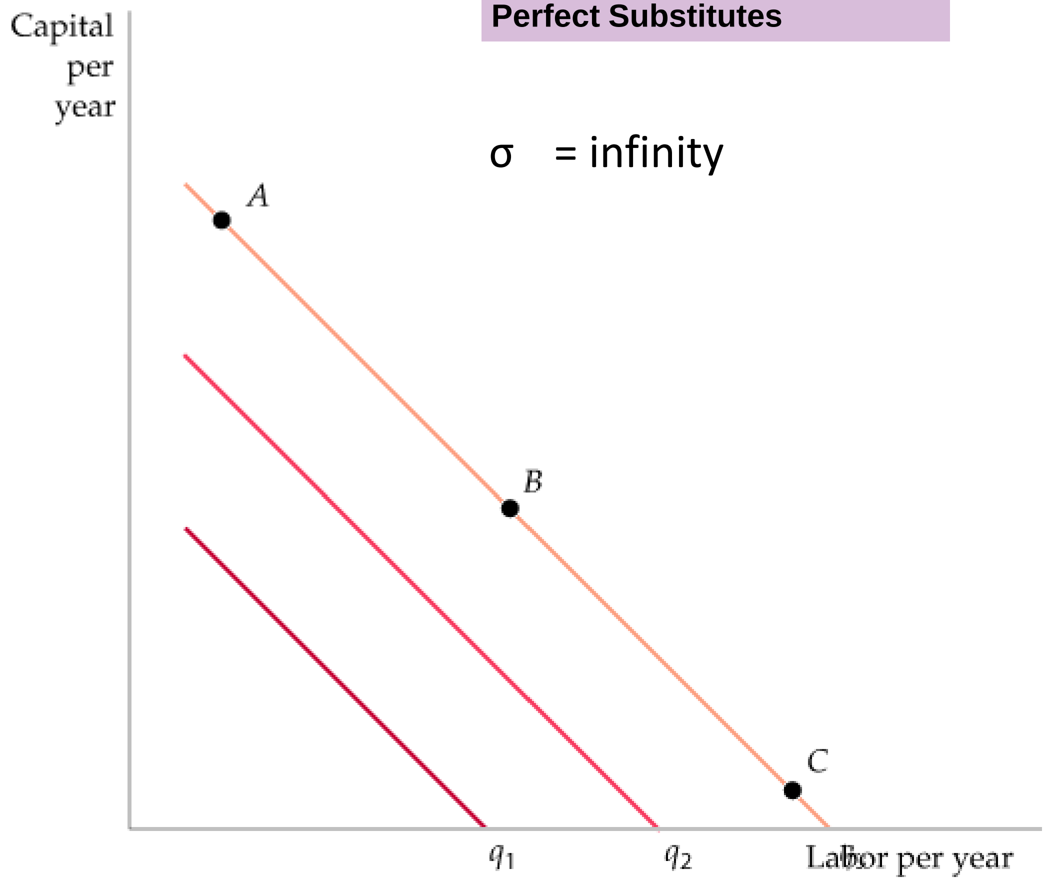
$$\sigma = \frac{\text{Percentage change in K/L}}{\text{Percentage change in MRTS}_{\text{LK}}} = \frac{d(K/L)}{d\text{MRTS}_{\text{LK}}} \cdot \frac{\text{MRTS}_{\text{LK}}}{(K/L)}$$

Fixed coefficient Technology: σ =0

PRODUCTION WITH TWO VARIABLE INPUTS

Isoquants When Inputs Are Perfect Substitutes

σ = infinity



THE TECHNOLOGY OF PRODUCTION

- The Short Run versus the Long Run

  - **short run** Period of time in which quantities of one or more production factors cannot be changed.
  - **fixed input** Production factor that cannot be varied.
  - **long run** Amount of time needed to make all production inputs variable.

Average product of labor = Output/labor input = q/L

Marginal product of labor = Change in output/change in labor input = $\Delta q/\Delta L$

Two laws of production

To achieve higher output, producer can

Either alter K/L (Short run)), that is, change the technique of production;

Or can proportionately increase use of K & L (long run), that is, change the scale of production.

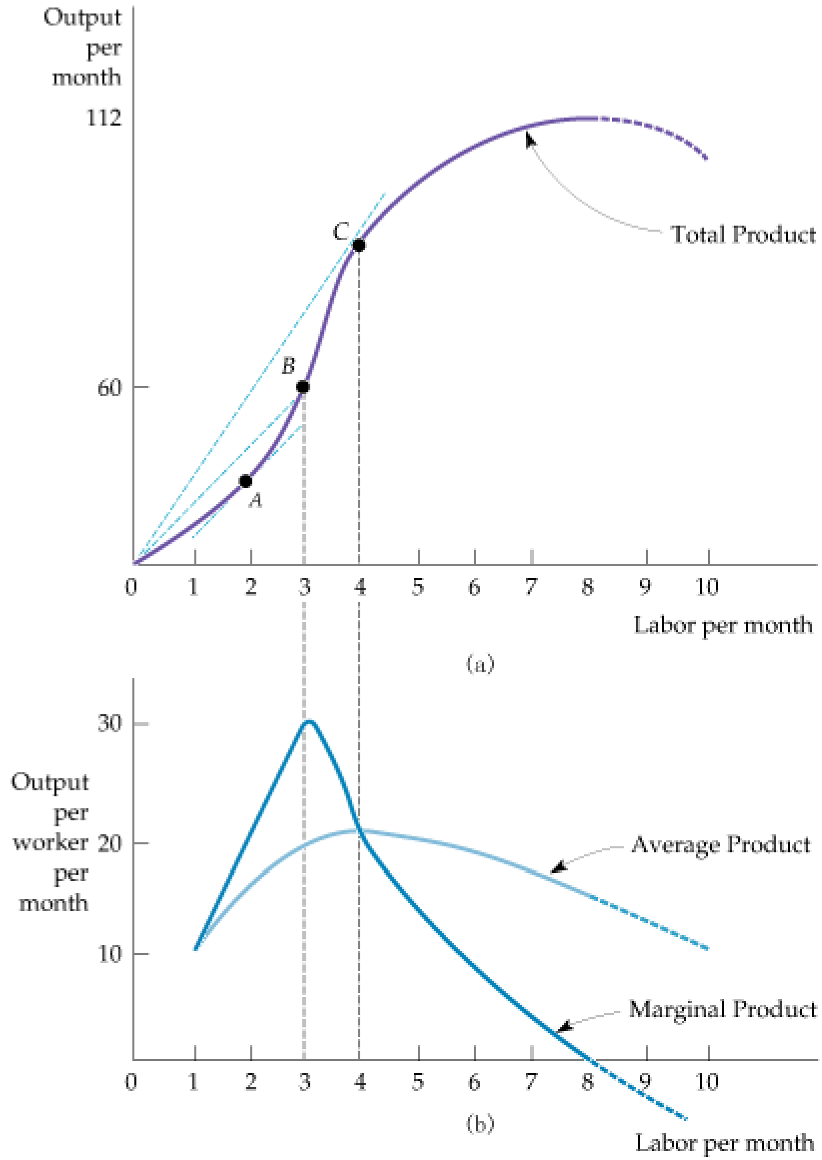
- **law of diminishing returns to the variable factor or Law of variable proportion:**

  Statement: as we employ more and more of the variable factor, output will first increase at an increasing rate and then at a decreasing rate.

PRODUCTION WITH ONE VARIABLE INPUT (LABOR)

Production with One Variable Input				
Amount of Labor (*L*)	**Amount of Capital (*K*)**	**Total Output (*q*)**	**Average Product (*q*/*L*)**	**Marginal Product ($\Delta q/\Delta L$)**
0	10	0	—	
1	10	10	10	10
2	10	30	15	20
3	10	60	20	30
4	10	80	20	20
5	10	95	19	15
6	10	108	18	13
7	10	112	16	4
8	10	112	14	0
9	10	108	12	–4
10	10	100	10	–8

PRODUCTION WITH ONE VARIABLE INPUT (LABOR)



Relationship between AP and MP

$$\frac{\partial(q/xi\,)}{\partial x_i} = \frac{\frac{\partial q}{\partial x_i}xi - q}{x_i^2} = \frac{1}{x_i}\left(\frac{\partial q}{\partial x_i} - \frac{q}{x_i}\right)$$

$$= \frac{1}{x_i}\,(MP_i - AP_i\,)$$

Therefore, $\frac{\partial(q/xi\,)}{\partial x_i} <_{>} 0$, according as $MP_i <_{>} AP_i$

And $\frac{\partial(q/xi\,)}{\partial x_i} = 0$ when MP_i =AP_i

When MP cuts AP, MP is diminishing (at the maximum of AP)

The second order of maximum requires that $\frac{\partial^2(q/xi)}{\partial x_i^2} < 0$

Now $\frac{\partial^2(q/xi)}{\partial x_i^2}$ = $\frac{(f_{ii}.x_i + f_i.1 - f_i)\,x_i^2 - (f_i\,x_i - q)2x_i}{x_i^4}$

where f_{ii} = $\frac{\partial^2 q}{\partial x_i^2}$ and f_i = $\frac{\partial q}{\partial x_i}$

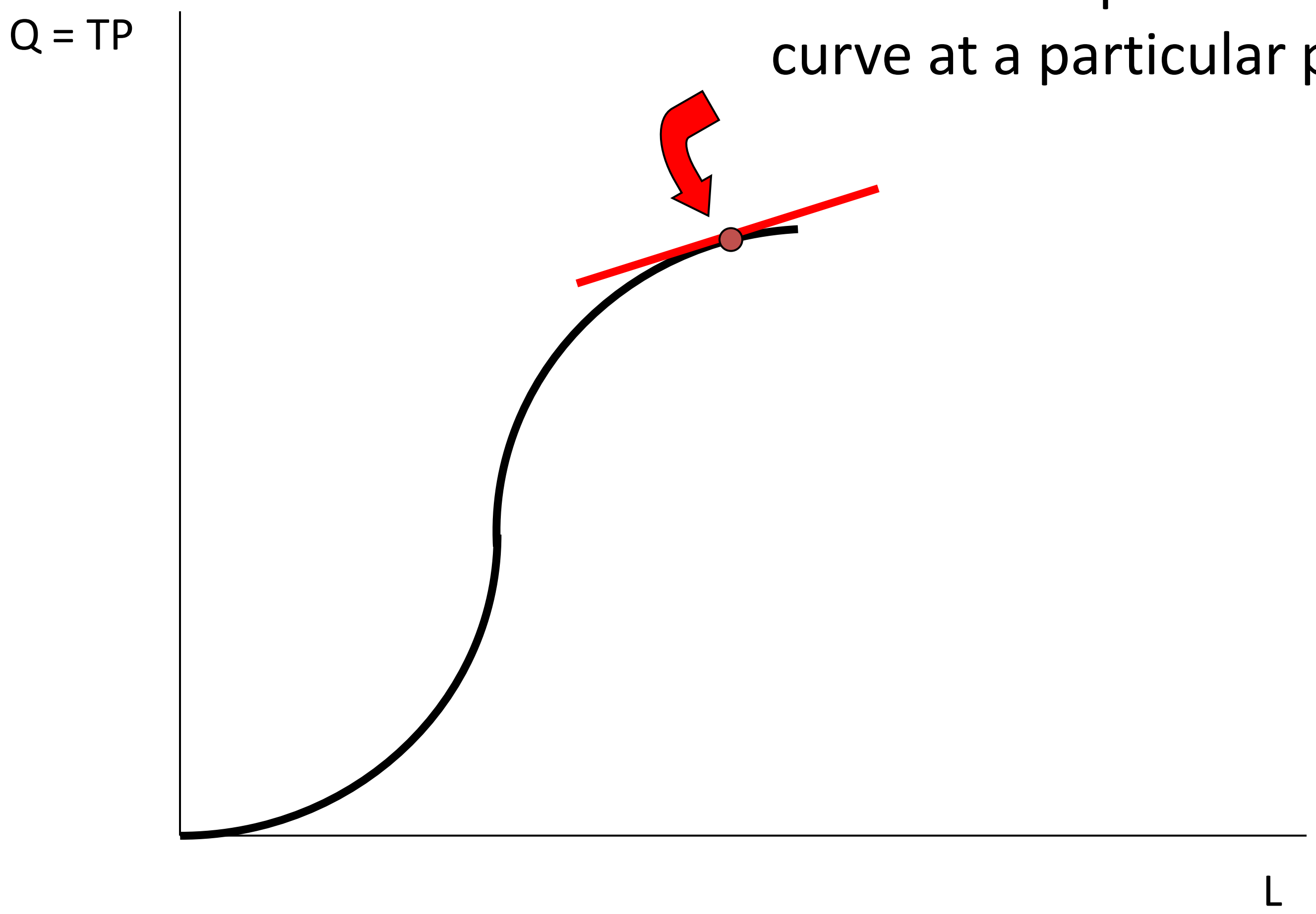
$$\frac{f_{ii}x_i^3 - 2f_i x_i^2 + 2\partial x_i}{x_i^4} < 0 = \frac{f_{ii}}{x_i} - \frac{2x_i^2\left(f_i - \frac{q}{xi}\right)}{x_i^4} < 0$$

But at the maximum point f_i = $\frac{q}{x_i}$ from the first order condition.

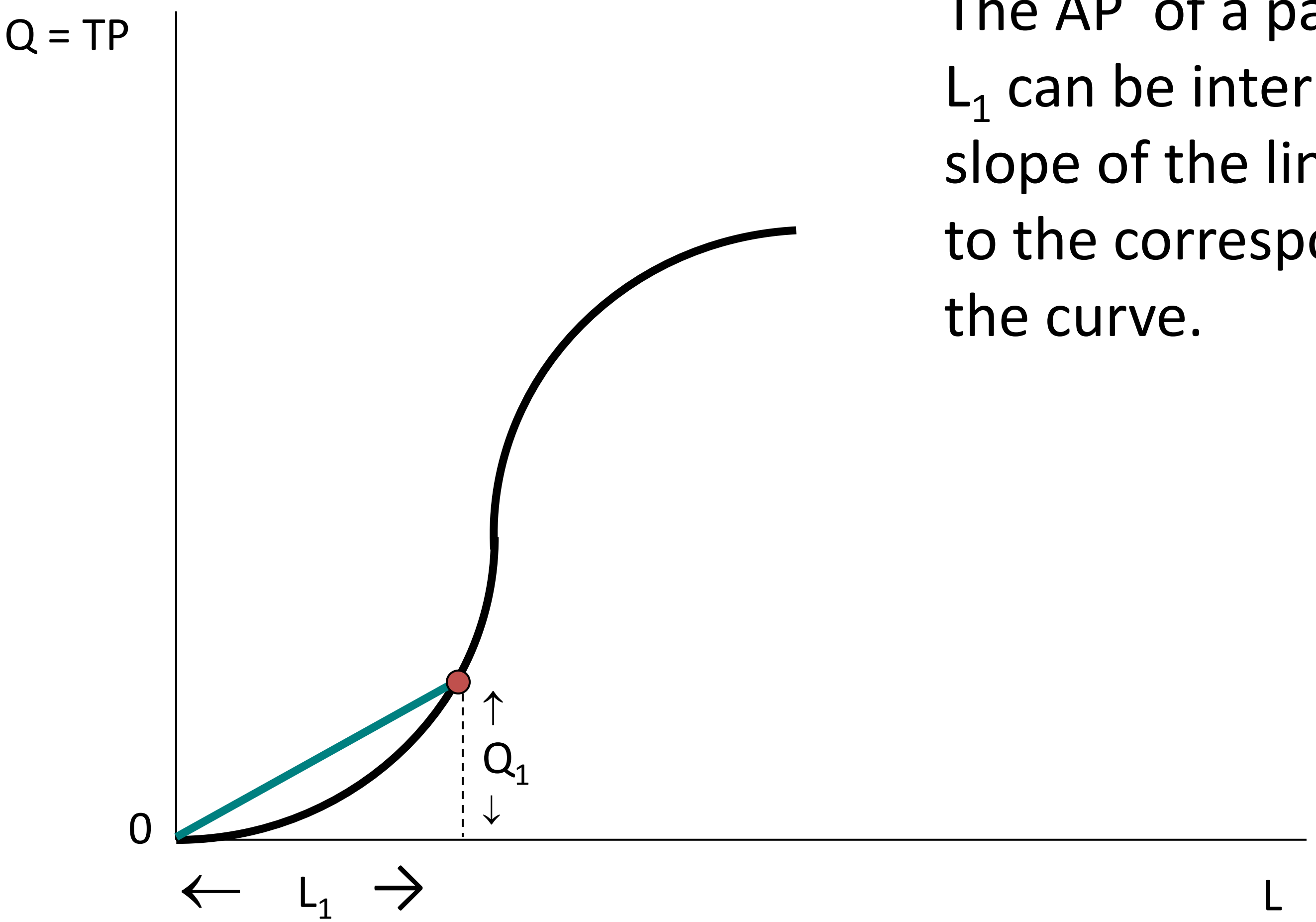
Therefore, $\frac{\partial^2(q/xi)}{\partial x_i^2} = \frac{f_{ii}}{x_i} < 0$, i.e., $f_{ii} < 0$ i.e., $\frac{\partial^2 q}{\partial x_i^2} < 0$

Graphical Interpretation of MP

MP is the slope of the total product curve at a particular point.

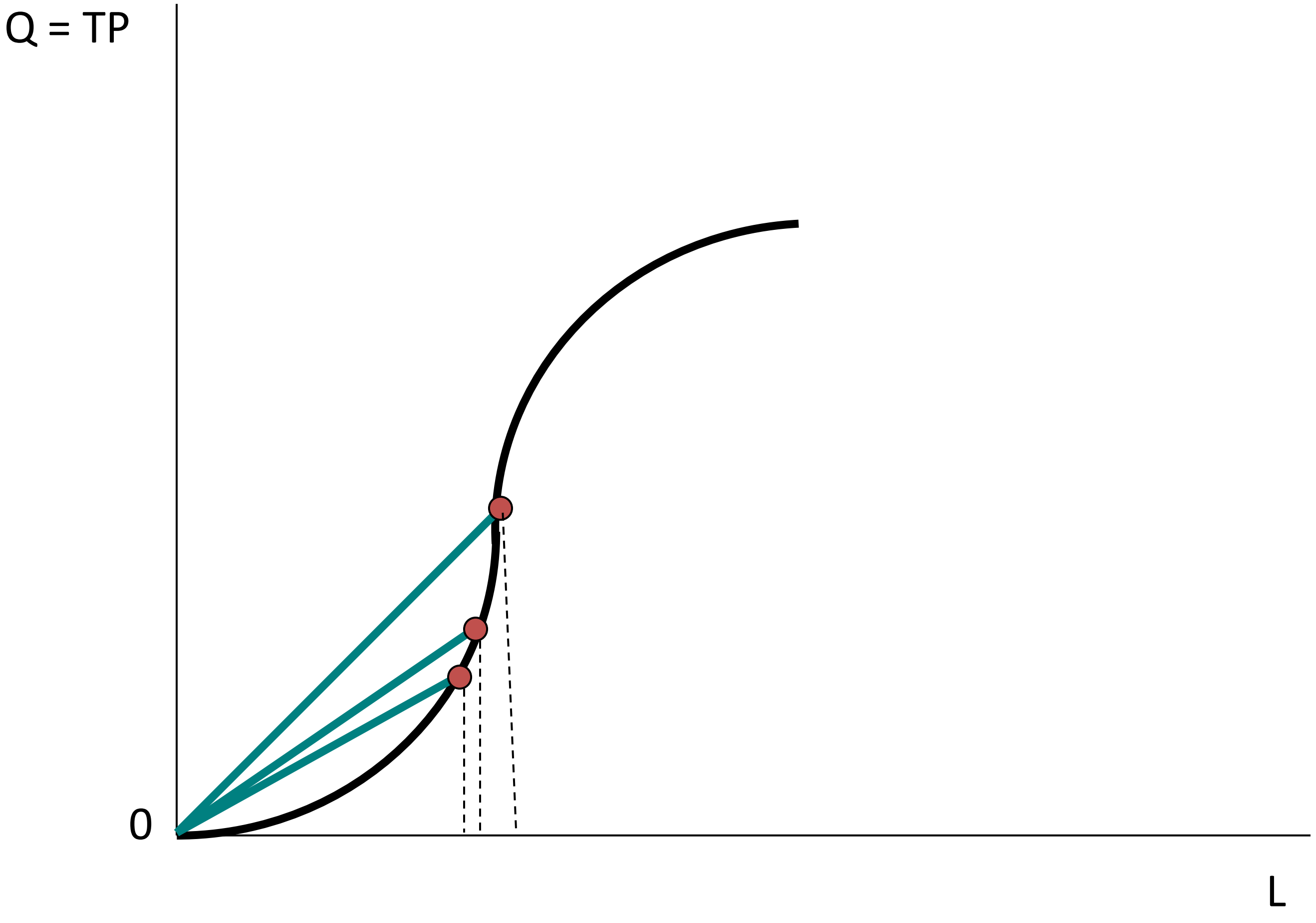


Graphical Interpretation of $AP = Q / L$

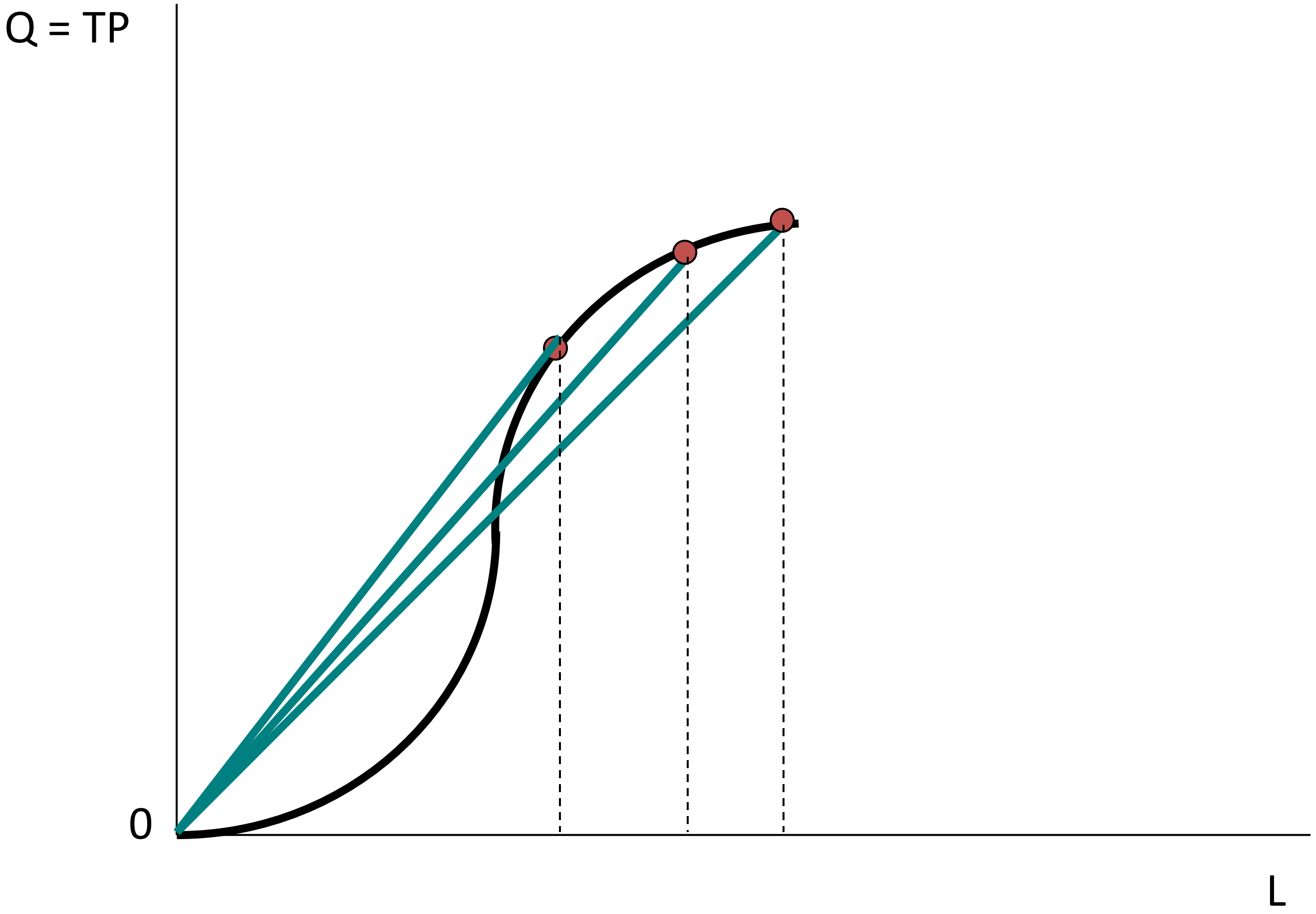


The AP of a particular value of L_1 can be interpreted as the slope of the line from the origin to the corresponding point on the curve.

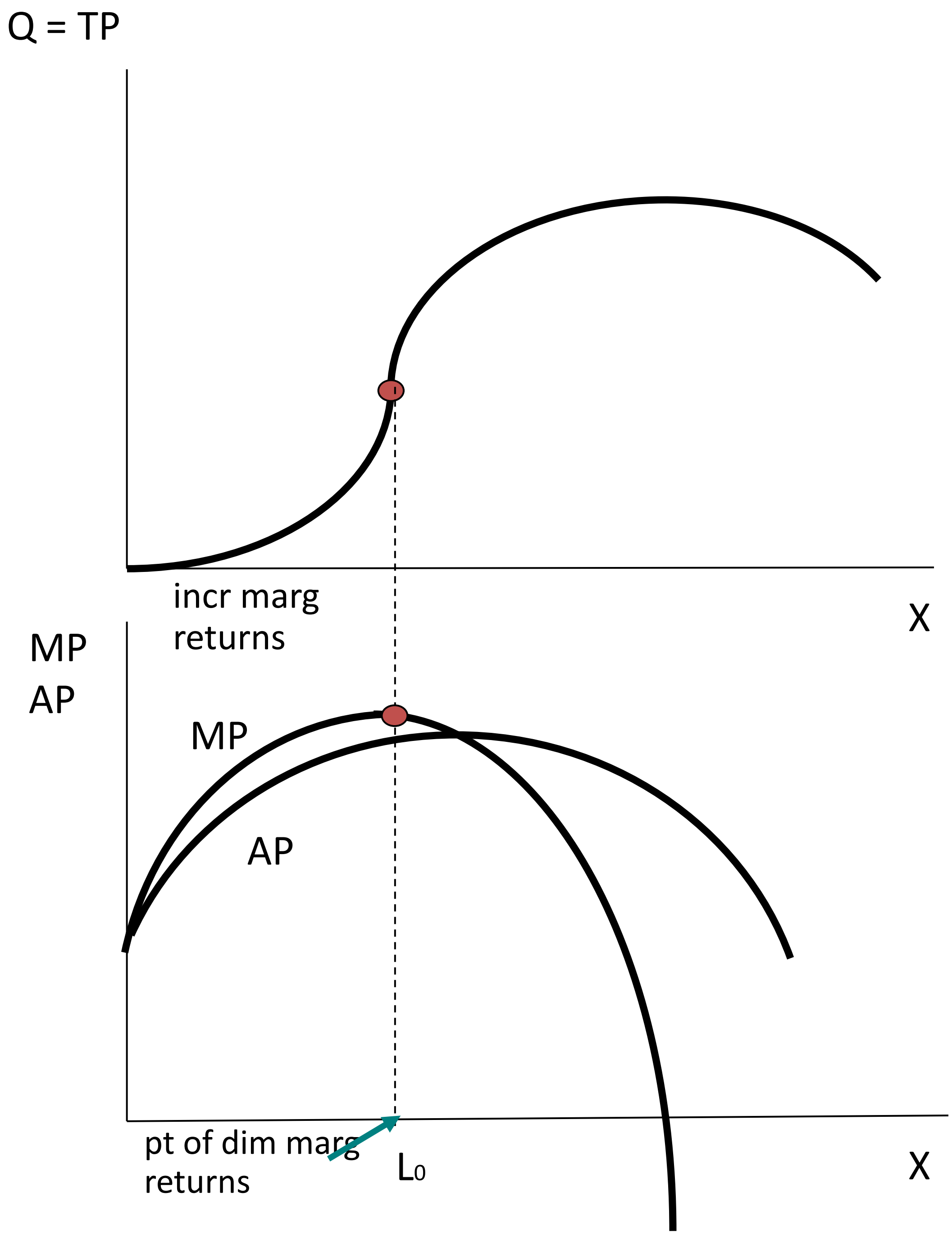
In this graph, we see that initially, AP is increasing



and then decreasing



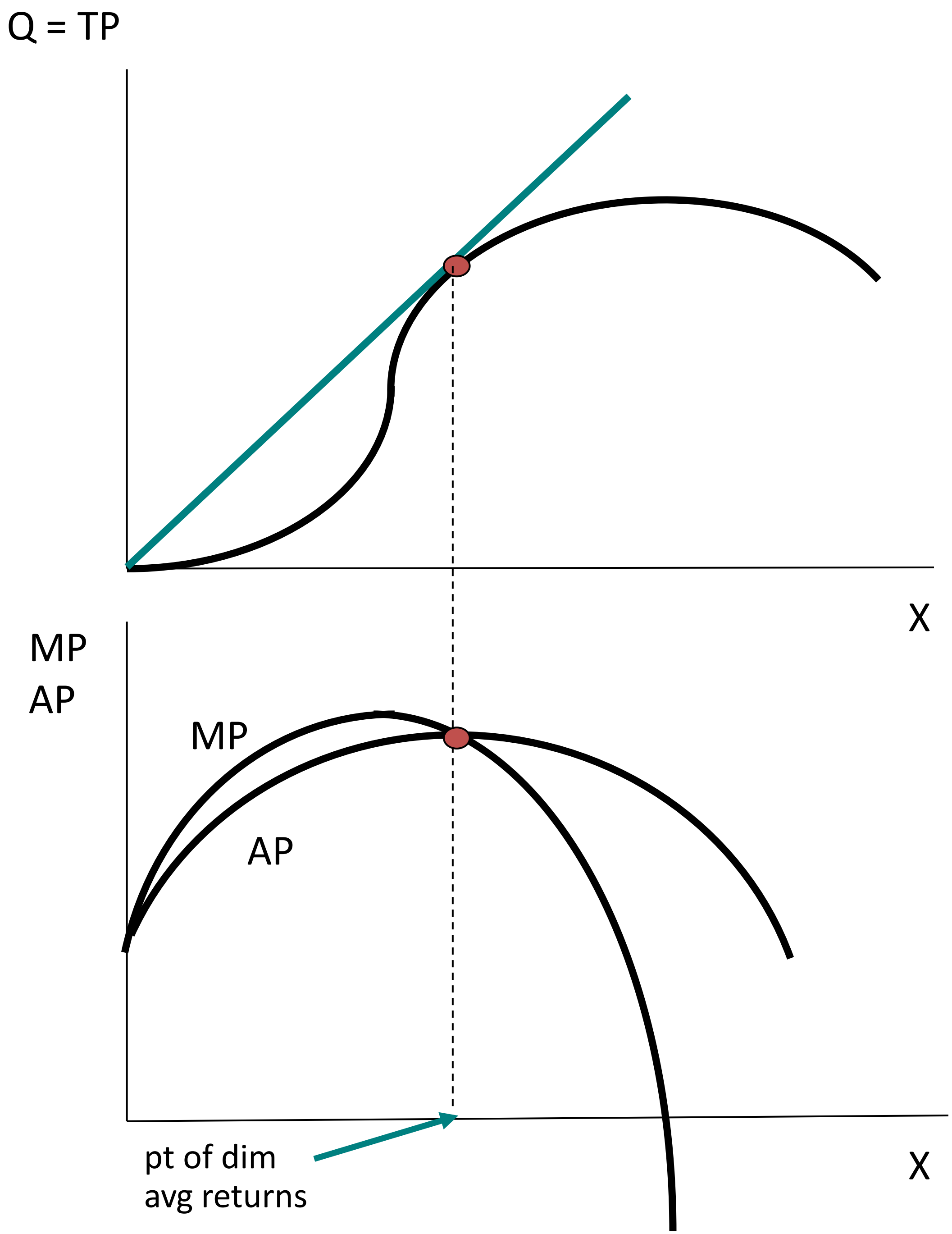
Total Product, Marginal Product, & Average Product Curves



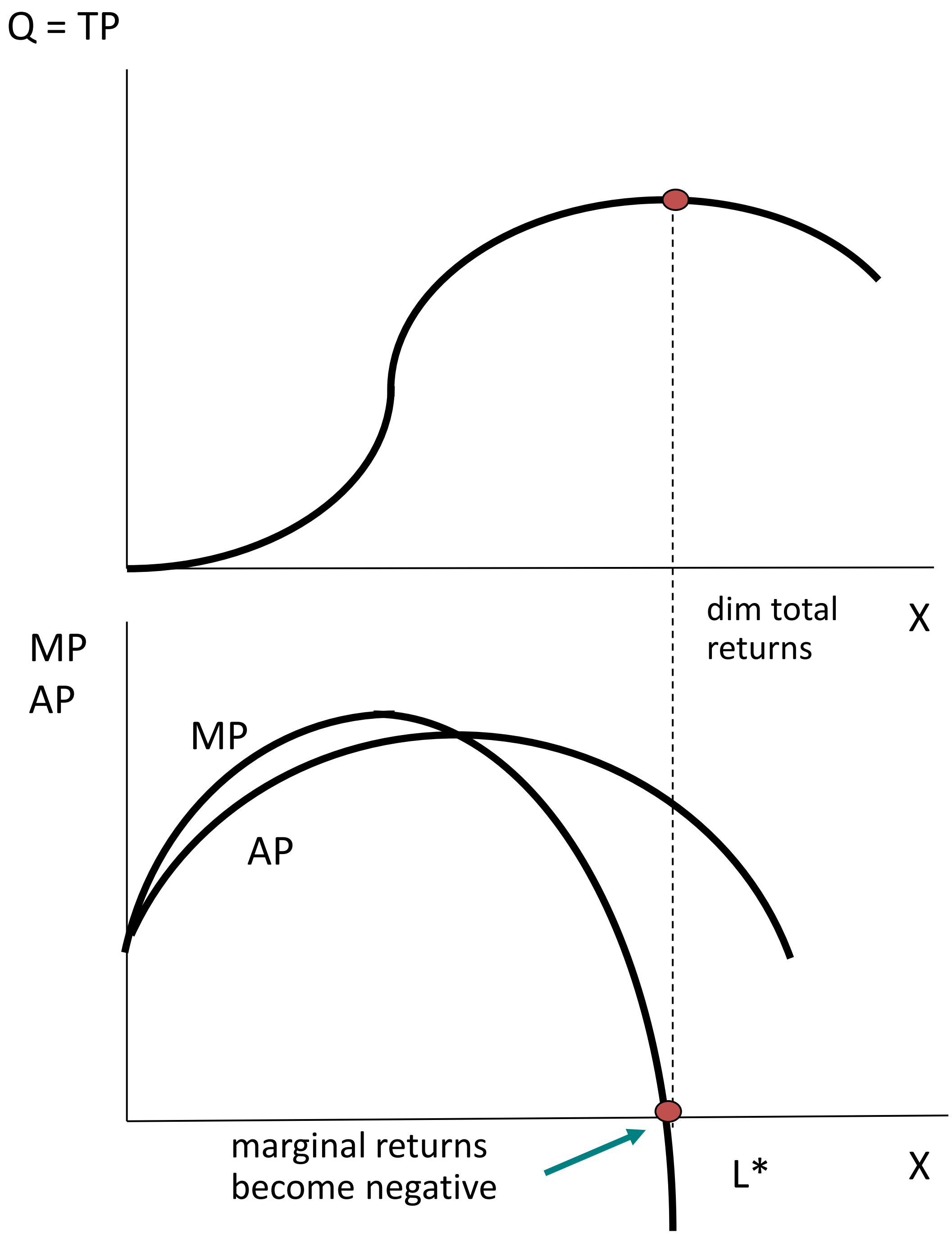
Diminishing marginal returns set in when MP starts to fall (but is still positive).

The TP curve gets flatter as the slope of TP falls.

Region I: $(0, L_0)$



Diminishing <u>average</u> returns set in when AP starts to fall.



Diminishing <u>total</u> returns set in when the TP curve turns downward and MP becomes negative.

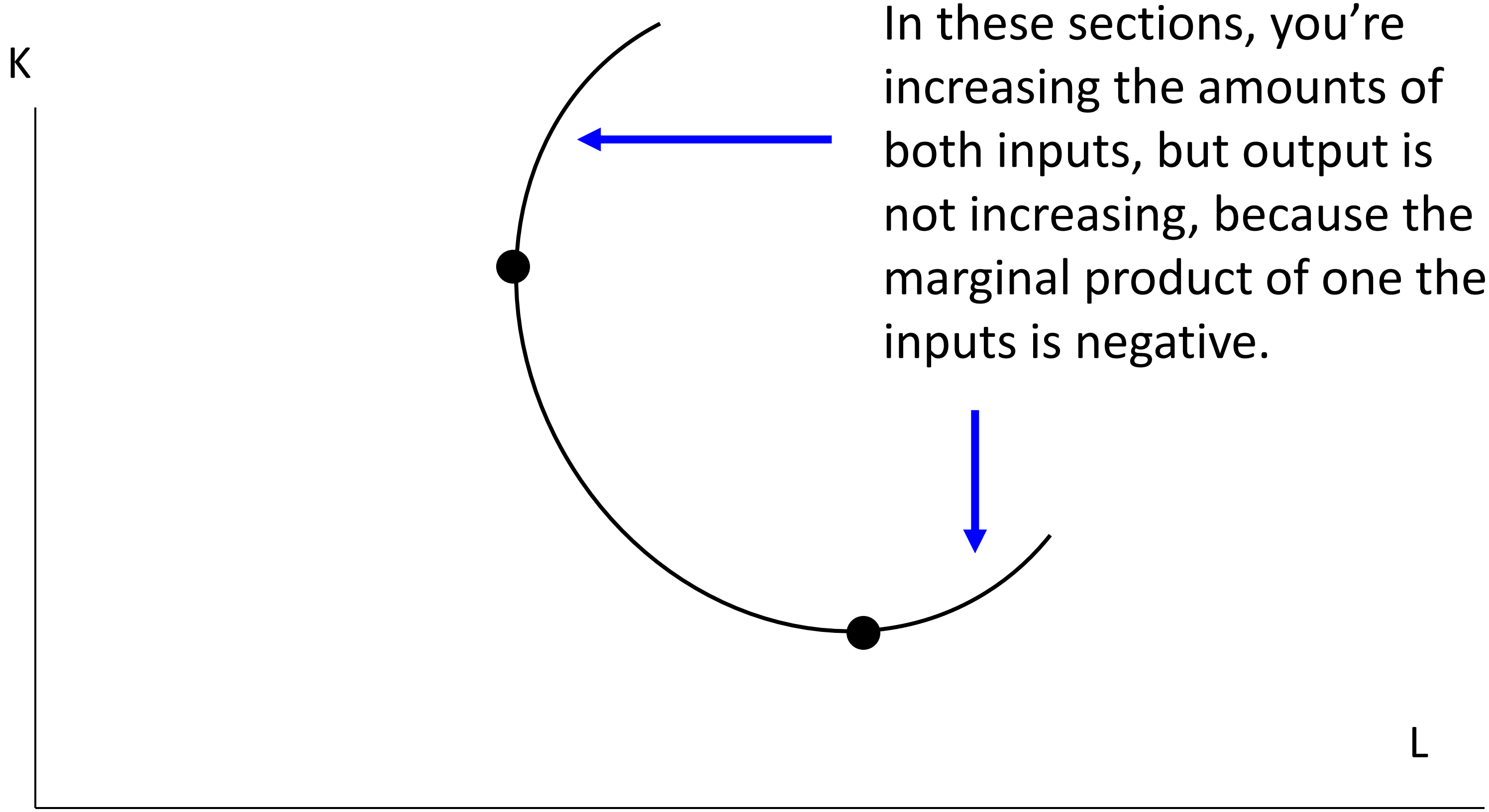
Region II: (L_0, L*)

Is called the region of economic operation.

Region III: (L*,….)

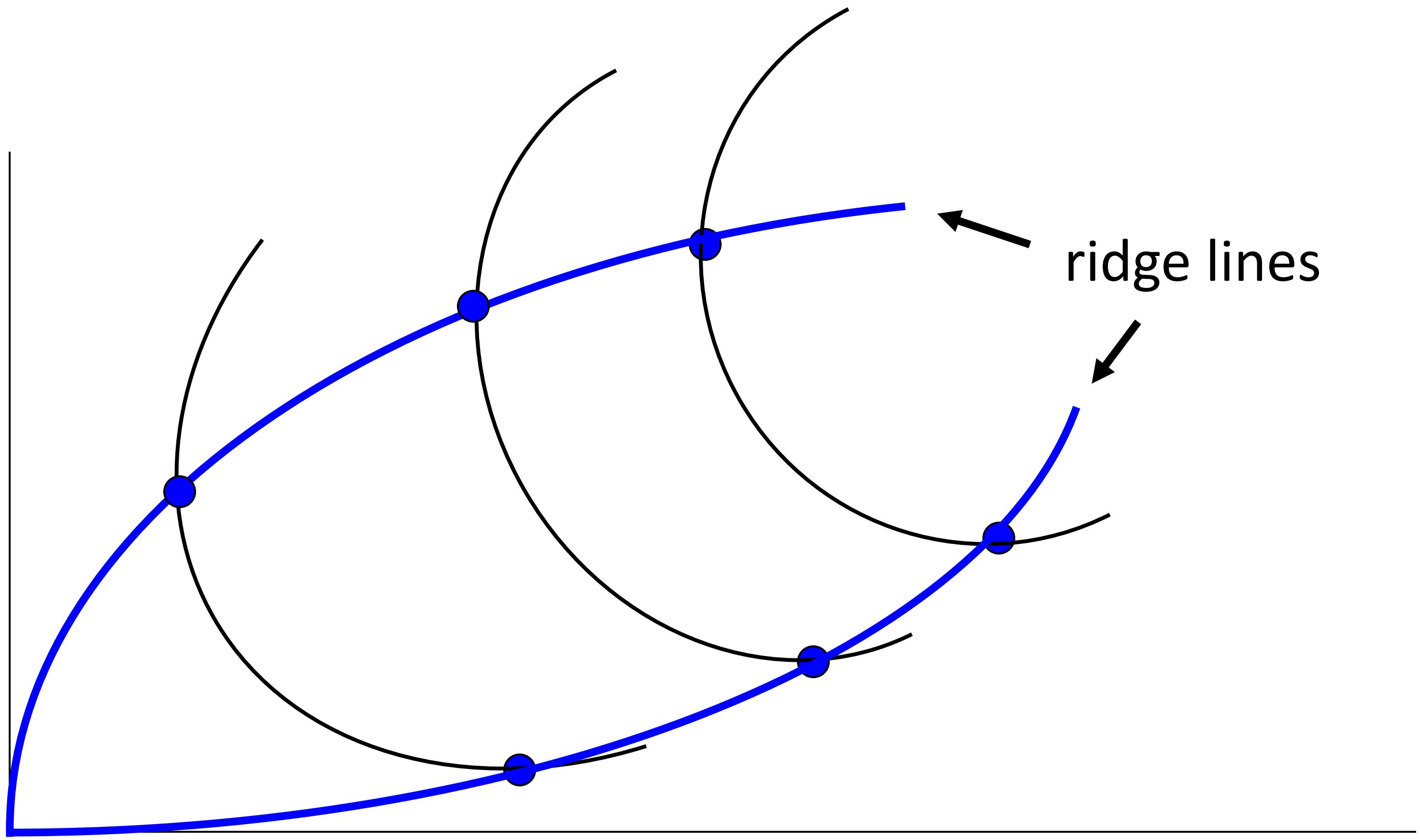
MP_L falls as we employ more L.

Is it possible for an isoquant to have positively sloped segment?

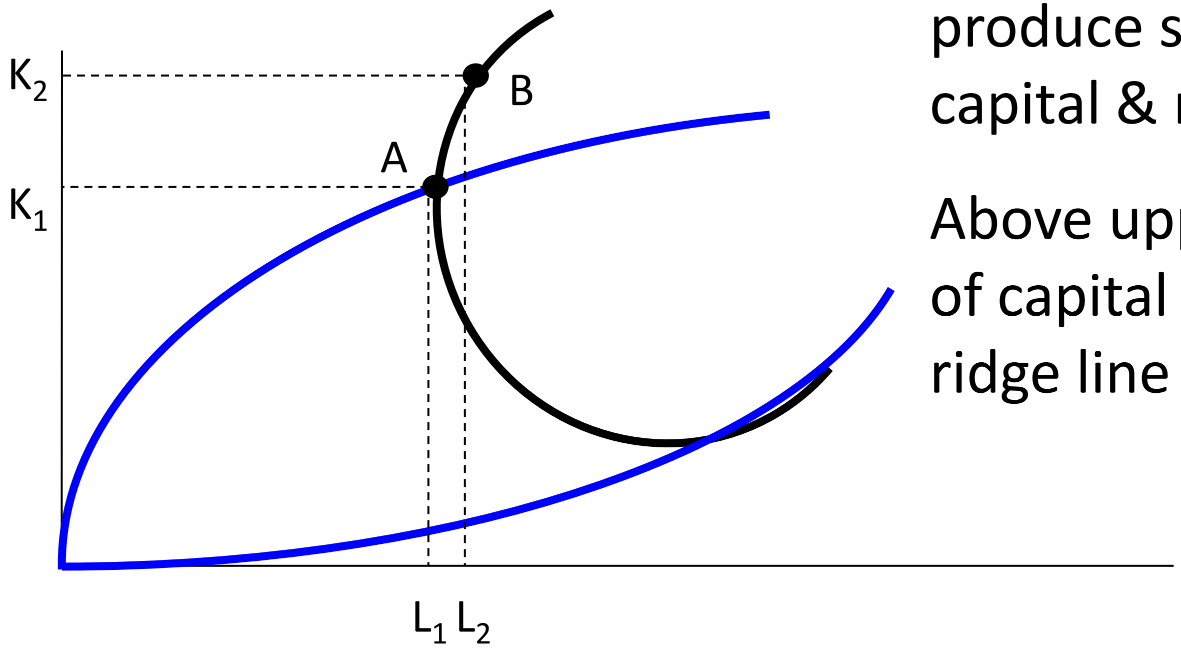


In these sections, you're increasing the amounts of both inputs, but output is not increasing, because the marginal product of one the inputs is negative.

The lines connecting the points where the isoquants begin to slope upward are called ridge lines.



No profit-maximizing firm will operate at a point outside the ridge lines, since it can produce the same output with less of both outputs.



Though A & B produce the same amount of output, B is a more expensive way to produce since it uses more capital & more labor.

Above upper ridge line MP of capital <0, below lower ridge line MP of labour <0

RETURNS TO SCALE

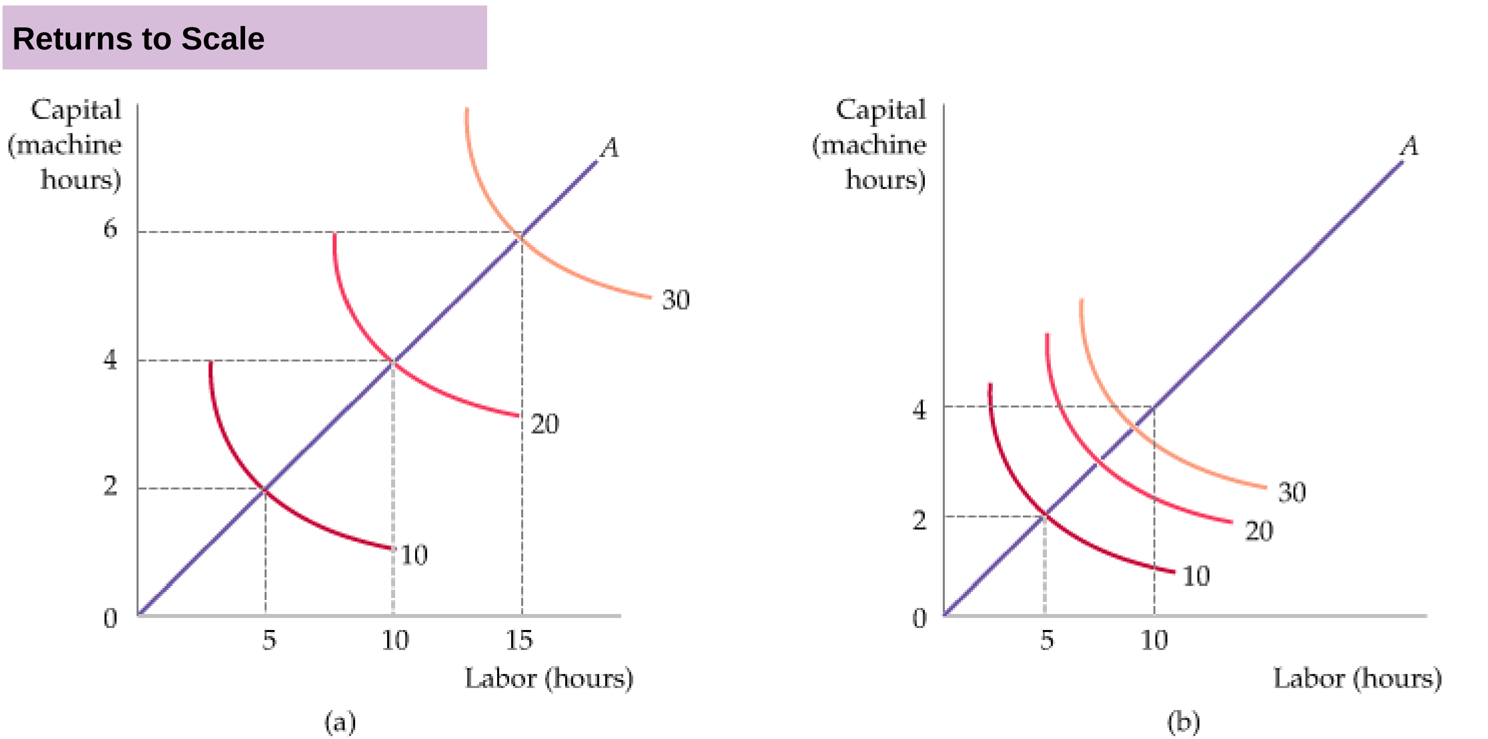
- **Law of returns to scale** Rate at which output changes when scale of production changes (that is, inputs are changed proportionately).

- **increasing returns to scale** Situation in which output more than doubles when all inputs are doubled.

- **constant returns to scale** Situation in which output doubles when all inputs are doubled.

- **decreasing returns to scale** Situation in which output less than doubles when all inputs are doubled.

RETURNS TO SCALE



With CRS the isoquants are equally spaced as output increases proportionally.

With IRS the isoquants move closer together as inputs are increased along the line.

Relationship between

law of variable proportion & law of returns to scale

For the Cobb-Douglas production function:

Law of variable proportion: $\alpha < 1$
$\beta < 1$

CRS: $\alpha + \beta = 1$
DRS: $\alpha + \beta < 1$
IRS: $\alpha + \beta > 1$

IRS can occur with **anything** (that is, diminishing / constant/) but CRS & DRS require diminishing MP, that is, law of variable proportion.

Homogeneous production function

A production function Q =f (K , L) is said to be homogeneous of degree r if

$F(\lambda K, \lambda L) = \lambda^r f(K, L) = \lambda^r Q$

The degree of homogeneity of a production function indicates the returns to scale that the production function exhibits.

If $r>1$=> IRS

$r=1$=> CRS (also called linearly homogeneous production function)

$r<1$=> DRS

Properties of homogeneous production function

If a production function is homogeneous of degree r: Redefine $Q = L^r . F(K/L)$

1. Average and marginal productivities are homogeneous of degree r-1;

2. $MRTS_{LK}$ is a function of K/L;

3. $MRTS_{LK}$ is homogeneous of degree 0;

4. Expansion path is straight line through the origin.

5. Euler's Theorem: $r\,Q = (\delta f/\delta L)L + (\delta f/\delta K)K$

   Implication: If r=1, $PQ = w\,L + r\,K$

   Product exhaustion theorem: value of production will be exhausted by factor payments.

- Cobb-Douglas production function $Q = AL^{\alpha}K^{\beta}$

σ = 1

is a special case of a CES production function.

Homothetic production function: It is a monotonic transformation of a linearly homogeneous production function.

The homothetic production function may not necessarily be CRS.

$$Q = A[\alpha L^{\rho} + (1-\alpha)K^{\rho}]^{1/\rho}$$

A is the technological parameter, α is the distributional parameter, ρ is the substitution parameter.