Test 2: MA20104 Probability and Statistics

Time: 45 minutes   Marks: 40

Instructions: There are ten questions. You have to answer all. They carry equal marks.

Q.1: A particle can move randomly form origin (0,0,0) to any of the four points (1,0,1), (1,1,0), (0,1,1) and (0,1,0) with equal probability.   Find the expected displacement of the particle. (answer should be correct up to three decimal places)

ANS 1: 1.31 (error range: 0.005)

Q.2: A fair coin is tossed independently 6 times by Player A and 10 times by Player B. Find the probability that both the players get the same number of heads. (answer should be correct up to three decimal places)

ANS 2: 0.122 (error range: 0.005)

Q.3: Let a random variable X have c.d.f. $F(x) = 2\pi^{-1} sin^{-1}\sqrt{x}$, if $0 < x < 1$,

$F(x) = 0$, $if\, x \le 0$ and $F(x) = 1$, $if\, x \ge 1$. Find the (mean + median) of the distribution.

ANS 3: 1 (error range : 0)

Q.4: Let X be a random variable satisfying the property for the moment generating function $M_X(t) = M_X(-t)$ for all real values of t. Find the value of

$$\sum_{m=0}^{10} E(X^{2m+1})$$

ANS 4: 0 (error range: 0)

Q.5: If the variance of a Bernoulli random variable is maximum with respect to the probability of success p, find the probability that six independent realizations of this Bernoulli random variable will produce at most two successes.

ANS 5: 0.344 (error range: 0.005)

Q.6: Suppose that the moment generating function of a continuous random variable X has the property $M_X(t) = e^t M_X(-t)$. Find its expectation.

ANS 6: 0.5 (error range: 0)

Q. 7: Let a random variable X have mean 5 and standard deviation 0.1. Using Chebyshev's inequality, find the minimum value of k such that the random variable X belongs to the interval $\left(5 - \dfrac{k}{10}, \quad 5 + \dfrac{k}{10}\right)$ with a probability of at least 0.99.

ANS 7: 10 (error range: 0)

Q. 8: Let X be a random variable with the probability density function given by

$$f(x) = \begin{cases} cx^2, & if \ |x| \le 2 \\ 0, & elsewhere. \end{cases}$$

Find fourth central moment. (answer should be correct up to three decimal places)

ANS 8: 6.857 (error range: 0.005)

Q.9: The probability density function of a random variable X is

$$f(x) = \begin{cases} a + bx^2 & if \ 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

If $E(X) = \dfrac{3}{5}$, find the value of $\dfrac{b}{a}$.

ANS 9: 2 (error range: 0)

Q. 10: A discrete random variable X can take values 1, 2, ..., 10 and $P(X = i)$ is proportional to $\dfrac{1}{i(i+1)}$ for $i = 1, 2, ..., 10$. Find $P(X \le 2)$. (answer should be correct up to three decimal places)

ANS: 0.733 (error range 0.005)

Q. 11: Five balls are drawn without replacement from an urn containing 3 white and 7 red balls. Find the probability that at least two white balls are drawn. (answer should be correct up to three decimal places)

ANS: 0.5 (error range: 0.005)

Q. 12: Let the c.d.f. of a random variable X be

$$F(x) = \begin{cases} 1 - \frac{1}{x^2} & if \ x \geq 1 \\ 0 & if \ x < 1 \end{cases}$$

Let Q_1, Q_2, Q_3 be quartiles of the distribution. Find the value of $Q_1^2 + Q_2^2 + Q_3^3$. (answer should be correct up to three decimal places)

ANS: ~~7.333 (error range: 0.005)~~   correct answer 11.333 (error range 0.005)   done

Q. 13: The probability that an experiment has a successful outcome is 0.8. The experiment is to be repeated independently until three successful outcomes have occurred. Let X be the number of repetitions required in order to have 3 successful outcomes. What is the probability that at least 5 repetitions will be required? (answer should be correct up to three decimal places)

ANS: 0.181 (error range: 0.005)

Q. 14: Let X be a geometric random variable with $p = \frac{2}{5}$. Find $P(X > 7 | X > 3)$. (answer should be correct up to three decimal places)

ANS: 0.130 (error range: 0.005)

Q. 15: Let the random variable X have c.d.f. as

$$F(x) = \begin{cases} 0 & if\, x \leq 0 \\ \frac{x^2}{2} & if\, 0 < x < 1 \\ \frac{13}{12} - \frac{1}{3x^2} & if\, 1 \leq x < 2 \\ 1 & if\, x \geq 2 \end{cases}$$

Find E(X). (answer should be correct up to three decimal places)

ANS: 0.917 (error range: 0.005)

Q. 16: A coin with probability of head 0.3 is tossed independently in a sequence. The first head is observed on the fifth toss. The coin is tossed further five times. Find the probability that three heads will be observed in these total 10 trials. (answer should be correct up to three decimal places)

Correct answer
0.022 (error range: 0.005) or
0.309 (error range: 0.005)

ANS: ~~0.022 (error range: 0.005)~~

Q. 17: If $P(X = 1) = P(X = -1) = \frac{1}{8}$ and $P(X = 0) = \frac{3}{4}$, then find the value of

$$P\left(|X - E(X)| \geq 2\sqrt{Var(X)} \right).$$ (answer should be correct up to three decimal places)

ANS: 0.25 (error range: 0)

Q. 18: The life of a machine (in years) is a continuous random variable with the p.d.f. given by

$$f_X(x) = \begin{cases} \frac{3}{x^4} & if\, x > 1 \\ 0 & if\, x \leq 1 \end{cases}$$

Find the probability that out of five such machines operating independently, at least three machines will be working for at least two years. (answer should be correct up to three decimal places)

ANS: 0.013 (error range: 0.005)

Q. 19: Abhinav and Sunil appear in job interviews independently of one another until each of them get placed. The probabilities of each of them getting placed at each interview are $\frac{3}{5}$ and $\frac{5}{7}$ respectively. Find the probability that Sunil will require to appear in more interviews than Abhinav. (answer should be correct up to three decimal places)

ANS: 0.194 (error range: 0.005)

Q. 20: Items produced in an assembly line are defective with a probability 0.02 independently of each other. They are packed in sets of ten each. If a set has more than one defective, then the company agrees to take return of the set. If a customer buys 10 sets, what is the probability that no set will be required to be returned to the company. (answer should be correct up to three decimal places)

ANS: 0.850 (error range: 0.005)