Product Differentiation

Address Model (Concepts)

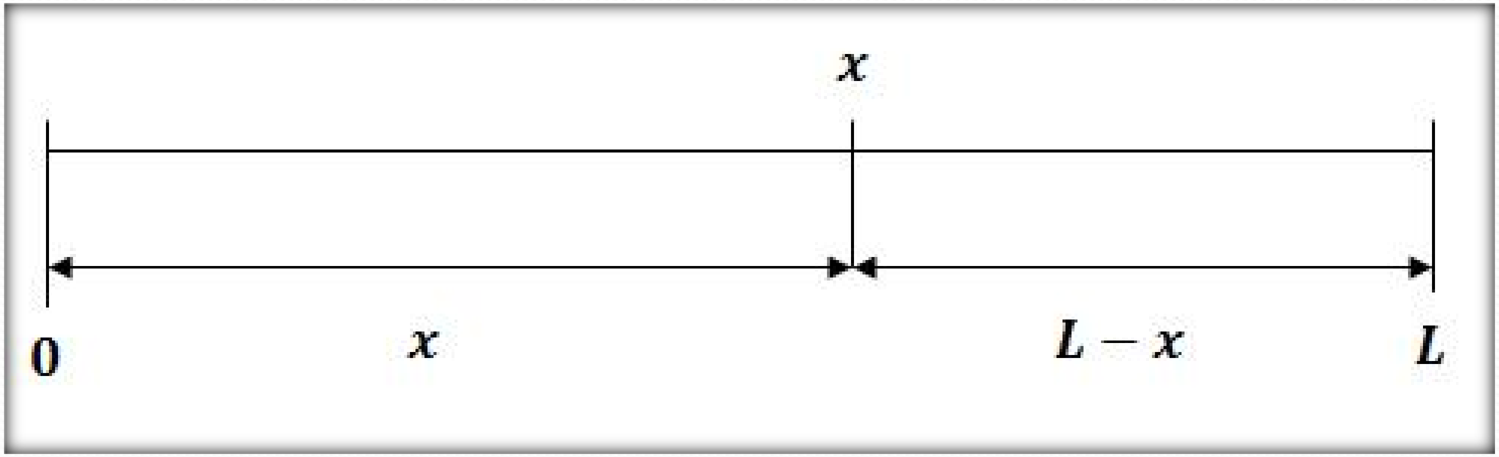
- Each person prefers a particular quality. Suppose you prefer a particular shade of blue emulsion paint. Anything else (even other shades of blue) you don't prefer. Now if such shade is not available in the market then you will have to deviate (may be slightly). In doing so you incur some cost.

- Location:

1. Physical location (consumer checks the price in all the stores and buys from that in which price plus transportation cost is minimum).
2. Distance between brand characteristics from that of the "ideal" brand. Consumer disutility arises from buying this less-than-ideal brand.
3. Brands are not uniformly ranked by all consumers.
4. Given same prices, consumers will buy their preferred brands.

Linear City Model (Hotelling)

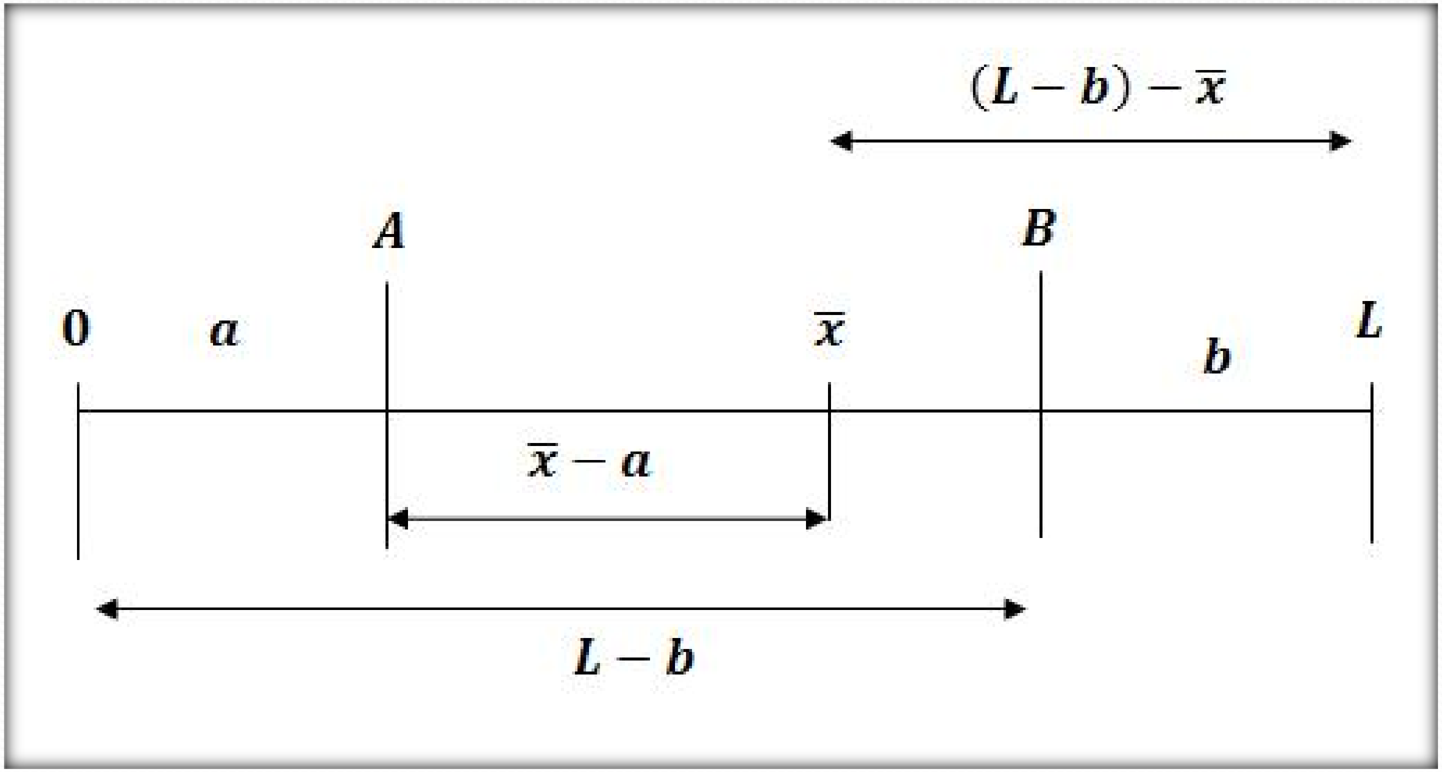
- Consider a linear street of length $L > 0$
- Consumers residing on that street are uniformly distributed $\Rightarrow n = L$
- Each consumer is indexed $x \in [0, L]$ $\Rightarrow$ consumer x is located at a distance x from the origin.



Price Game with Fixed Location

- Two firms, homogeneous product, location of product sold is different, zero production cost.

- Assume: Firm A located a distance *a* from the origin and firm B located at a distance *b* from *L*.



Price Game with Fixed Location

- Each consumer buys only one product.
- Unit transportation cost is t.
- Utility function of a consumer located at x is given by –

$$U_x = -p_A - t\,|\,x - a\,| \qquad \text{if he buys from A}$$
$$U_x = -p_B - t\,|\,x - (L - b)\,| \qquad \text{if he buys from B}$$

- Consumer at $\bar{x}$ is indifferent between buying from A and B.
- Formally if $a < \bar{x} < L - b$ then,

$$-p_A - t(\bar{x} - a) = -p_B - t(L - b - \bar{x})$$

Price Game with Fixed Location

- Or, $\bar{x} = \dfrac{p_B - p_A}{2t} + \dfrac{L - b + a}{2}$
- Anyone left of this point will buy from A
- On the right will buy from B
- Hence, $\bar{x} = \dfrac{p_B - p_A}{2t} + \dfrac{L - b + a}{2}$ gives the demand function faced by A
- Similarly for firm B we get $L - \bar{x} = \dfrac{p_A - p_B}{2t} + \dfrac{L + b - a}{2}$
- Assuming Bertrand competition find NE price strategies.

Price Game with Fixed Location

- Firm A: $\pi_A = [\frac{p_B - p_A}{2t} + \frac{L-b+a}{2}]p_A$
- FOC: $\frac{p_B - 2p_A}{2t} + \frac{L-b+a}{2} = 0$
- Similarly for firm B: $\frac{p_A - 2p_B}{2t} + \frac{L+b-a}{2} = 0$
- NE prices: $p_A^N = \frac{t(3L-b+a)}{3}$; $p_B^N = \frac{t(3L+b-a)}{3}$
- Equilibrium market share of A: $x_A^N = \frac{3L-b+a}{6}$
- Equilibrium profit of A: $\pi_A^N = p_A^N x_A^N = \frac{t(3L-b+a)^2}{18} > 0$

Price Game with Fixed Location

- If $a = b$ then equal market share.

- The profit of firm A: $\pi_A^N = p_A^N x_A^N = \dfrac{t(3L - b + a)^2}{18} > 0$

- This shows that the profit of each brand-producing firm increases with the distance between the firms.

- firms reach higher profit levels when the brands they produce are more differentiated

- What happens if $a + b = L$?

Price Game with Fixed Location

- If both firms are located at the same point $(a+b=L)$, meaning that the products are homogeneous), then the unique equilibrium is $p_A = p_B = 0$

- A unique equilibrium exists and is described by $p_A^N;\ p_B^N; x_A^N; x_B^N$ if and only if the two firms are not too close to each other

- Formally if and only if

$$\left(L+\frac{a-b}{3}\right)^2 \geq \left(\frac{4L(a+2b)}{3}\right)$$

and

$$\left(L+\frac{b-a}{3}\right)^2 \geq \left(\frac{4L(a+2b)}{3}\right)$$

Price Game with Fixed Location

- We consider the simple case where firms are located at equal distances along the edges.

- That is, assume that $(a = b; a < L/2)$

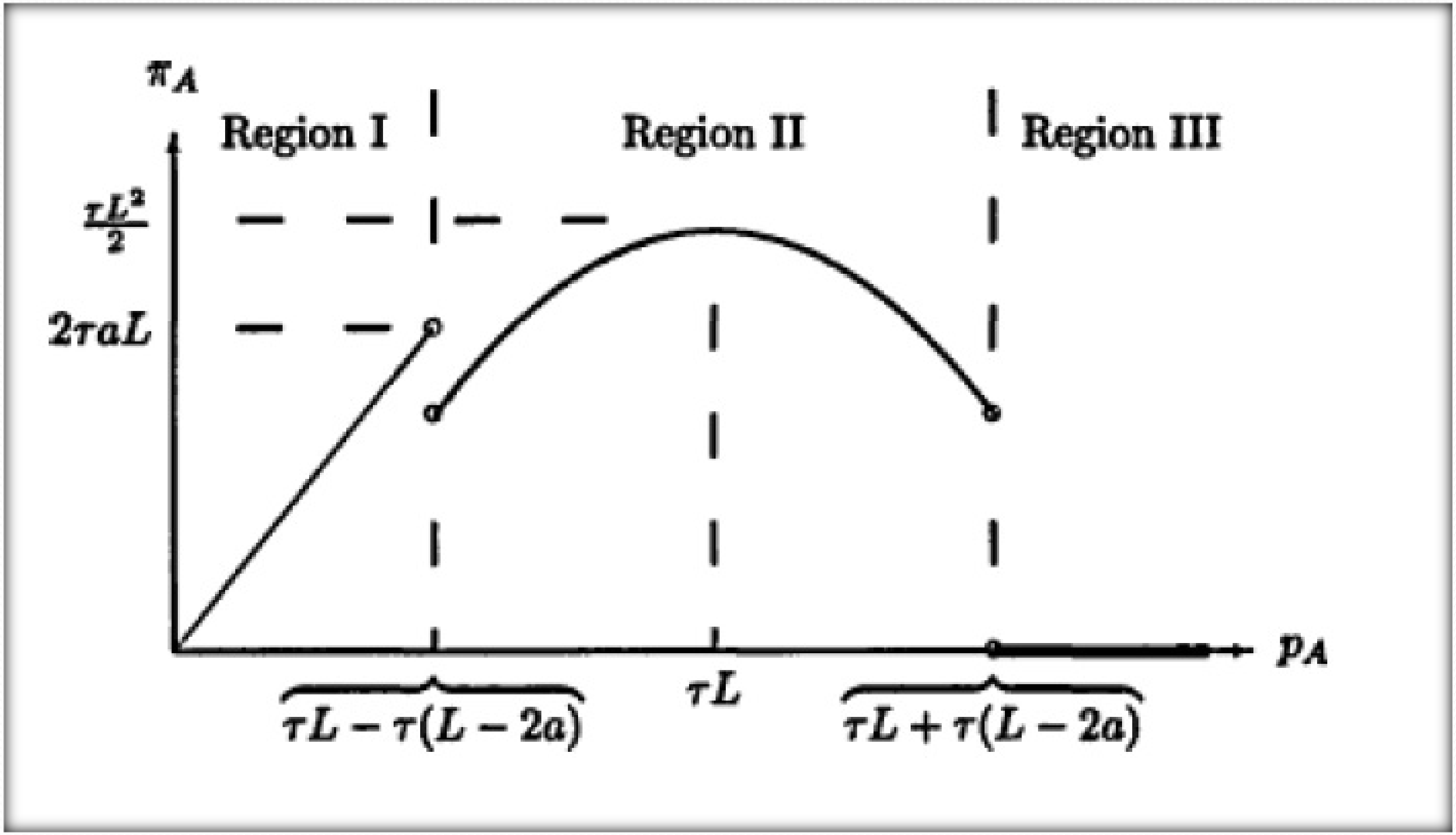
- we need to show that the equilibrium exists if and only if $L^2 \geq 4La$ or $a \leq L/4$

- When $a = b$, the distance between the two firms is $(L - 2a)$

- Hence, if equilibrium exists, we must have $p_A = p_B = tL$

Price Game with Fixed Location

- The profit level of firm A as a function of its own price p_A and a *given* B's price $\overline{p}_B = tL$ for the case of a = b



Price Game with Fixed Location

- Region I: $p_A < tL - t(L-2a)$

  In this zone p_A is very low, so that even the consumer located at the same point where firm B is located would purchase from firm A.

  Thus, firm A has the entire market, and its profit is given by $\pi_A = p_A L = [tL - t(L-2a) - \varepsilon]L$

- Region II: Here, both firms sell a strictly positive amount, so the profit of firm A as a function of p_A is given by

  $$\pi_A = [\frac{p_B - p_A}{2t} + \frac{L-b+a}{2}]p_A$$

  Substituting $\bar{p}_B = tL$ we have $\pi_A = p_A L - \frac{(p_A)^2}{2t}$

  Maximization wrt p_A yields $\pi_A = tL^2/2$

Price Game with Fixed Location

- Region III: Exact opposite of Region I.

  In this zone p_A is very high, all consumers purchase from firm *B*.

- Now, for a given $p_B = tL$; π_A has two local maxima.

- But in Region I, Firm A has the entire market.

- Thus to have an eqm defined by $p_A^N; p_B^N; x_A^N; x_B^N$; the globally profit-maximizing price for firm A would lie in Region II

Price Game with Fixed Location

- Hence, we must have profit in region II higher than profit in region I

- Formally, $[tL - t(L-2a)]L \le tL^2/2 \Rightarrow a \le L/4$

- When the two firms are located too closely, they start undercutting each other's prices, resulting in a process of price cuts that does not converge to an equilibrium.

- Thus, for an equilibrium to exist, the firms cannot be too closely located

Price Game with Fixed Location

- So far, we have assumed that the location of the firms is fixed
- What if firms can choose price and location both?
- Unfortunately, we now show that there is no solution for this two-dimensional strategy game.
- To show that, we ask what would firm A do if, given the price and location of its opponent, it would be allowed to relocate.

Price Game with Fixed Location

- Let's reconsider the profit function

$$\pi_A^N = p_A^N x_A^N = \frac{t(3L - b + a)^2}{18} > 0$$

- Then for any locations a and b, can firm A increase its profit by moving toward firm B (obviously, to gain a higher market share) or

$$\frac{\partial \pi_A}{\partial a} > 0$$

- This case, where firms tend to move toward the center, is called in the literature the ***principle of minimum differentiation*** since by moving toward the center the firms produce less-differentiated products.

Price Game with Fixed Location

- However, we have already seen that if firm A gets too close to firm B, an equilibrium will not exist.
- Also, if firm A locates at the same point where firm B locates, its profit will drop to zero
- implying that it is better off to move back to the left
- ***In the Hotelling linear-city game, there is no equilibrium for the game where firms use both prices and location as strategies***.

Price Game with Fixed Location

- Now, we have seen that even when the location is fixed, the linear-location model does not have an equilibrium in a price game when the firms are too close to each other.

- We also showed that there is no equilibrium in a game when firms choose both prices and location.

- However, it is important to observe that so far, we have assumed linear transportation costs.

- The existence problem can be solved if we assume quadratic transportation costs.

Price Game with Fixed Location

- Utility function of a consumer located at *x* is given by –

  $U_x = -p_A - t(x-a)^2$ if he buys from A

  $U_x = -p_B - t[x-(L-b)]^2$ if he buys from B

- we can formulate a two-period game in which firms decide where to locate in the first period, and set prices in the second period

- Solution – backward induction (SPNE)

Price Game with Fixed Location

- Second period:

1. For given location parameters a and b, find the Nash-Bertrand equilibrium prices, following the same steps we used in order to derive $(p_A^N;\ p_B^N)$

2. Substitute the equilibrium prices into the profit functions (in terms of prices, that to be maximized) to obtain the firms' profits as functions of the location parameters *a* and *b*.

Price Game with Fixed Location

- First period:

1. Maximize the firms' profit functions which you calculated for the second period with respect to a for firm A and with respect to b for firm B. Prove that for a given b, $\partial \pi_A / \partial a < 0$, meaning that firm A would choose a = 0. Similarly, show that firm B would locate at point L.

Circular City Location Model

- We have seen that an equilibrium in games in which firms jointly decide on prices and location does not exist in the Hotelling's linear city model.

- One way to approach this problem is to assume that the city be a circle with unit-circumference, where the consumers are uniformly distributed on the circumference.

- This type of location model can also be given an interpretation for describing differentiated products that differs from the physical-location interpretation.

- Consider for example airline, bus, and train firms which can provide a round-the-clock service.

- If we treat the circle as twenty-four hours, each brand can be interpreted as the time where an airline firm schedules a departure.

Circular City Location Model

- This model does not explicitly model how firms choose where to locate.

- It assumes a ***monopolistic-competition market structure***, in which the number of firms N is endogenously determined.

- All (infinitely many) potential firms have the same technology.

- Fixed cost $= F$
- marginal cost $= c$
- Output of the firm-producing brand $i = q_i$

Circular City Location Model

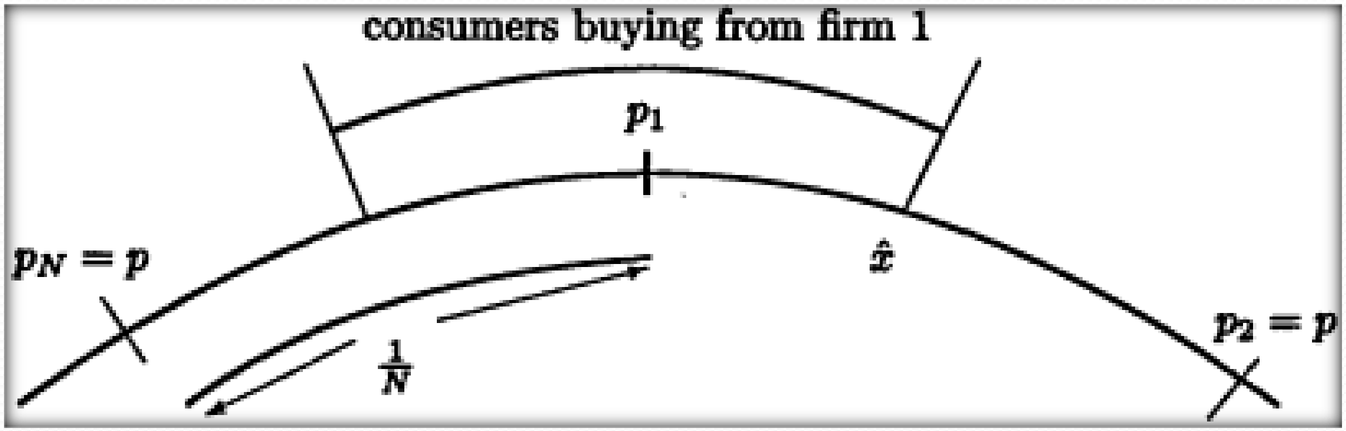
- Profit levels of the firm-producing brand $i = \pi_i(q_i)$ is given by –

$$\pi_i(q_i) = \begin{cases} (p_i - c)q_i - F & \forall \ q_i > 0 \\ 0 & if \ q_i = 0 \end{cases}$$

- Consumers are uniformly distributed on the unit circle.
- The consumers' transportation cost per unit of distance $= t$
- Each consumer buys one unit of the brand that minimizes the sum of the price and transportation cost.

Circular City Location Model

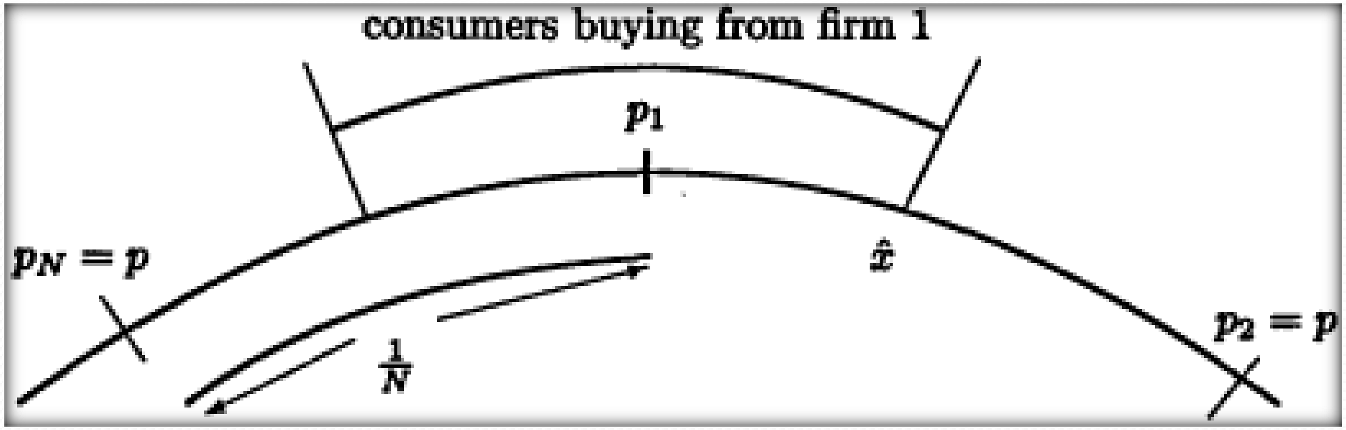
- Assuming that the N firms are located at an equal distance from one another yields that the distance between any two firms is $^1/_N$



- Lets' assume that firm 1 charges a price p_1 and firm 2 charges a price p.

- Then, the person located at a position $\hat{x}$ will be indifferent between buying from firm 1 or 2 iff -

Circular City Location Model



- Indifference condition: $p_1 + t\hat{x} = p + t(\frac{1}{N} - \hat{x})$

- Implying that the location of the consumer is: $\hat{x} = \frac{p - p_1}{2t} + \frac{1}{2N}$

- Similarly we can locate the location of the consumer at a distance $\hat{x}$ on the left of firm 1 (assuming firm N charges a price p)

Circular City Location Model

- Hence, the total demand (or, the number of consumers) that Firm 1 will face will be –

$$q_1(p, p_1) = 2\hat{x} = \frac{p - p_1}{t} + \frac{1}{N}$$

- Definition of equilibrium: The triplet $\{N^{\circ}, p^{\circ}, q^{\circ}\}$ is an equilibrium if

1. Each firm behaves as a monopoly on its brand; that is, given the demand for brand i and given that all other firms charge $p_j = p^{\circ}; i \neq j$, each firm i chooses p_i to

$$\max_{p_i} \pi_i\,(p_1, p^0) = p_i q_i(p_i) - (F - c q_i)$$

$$= (p_i - c)\left(\frac{p^0 - p_i}{t} + \frac{1}{N}\right) - F$$

Circular City Location Model

2. Free entry of firms (brands) will result in zero profits, that is,

$$\pi_i(p^0, p^0) = 0 \quad \forall \quad i = 1,2, \dots, N$$

- The FOC for firm i's profit maximization problem is –

$$\frac{\partial \pi_i(p_i, p^0)}{\partial p_i} = \frac{p^0 - 2p_i + c}{t} + \frac{1}{N} = 0$$

- Now, if all firms behave in the same fashion, then, in a symmetric equilibrium we will have –

$$p_i = p^0 = c + {}^{t}/_{N}$$

- Hence, to find the equilibrium number of brands (N^0), we set –

$$\pi_i(p^0, p^0) = 0$$

Circular City Location Model

- Or, $0 = (p^0 - c)\left(\frac{1}{N}\right) - F = (c + {}^{t}/_{N} - c)\left(\frac{1}{N}\right) - F = \frac{t}{N^2} - F$

- Hence, we have: $N^0 = \sqrt{{}^{t}/_{F}}$

- The equilibrium will thus be characterized by –

1. $N^0 = \sqrt{{}^{t}/_{F}}$
2. $p^0 = c + {}^{t}/_{N^0} = c + \sqrt{tF}$
3. $q^0 = {}^{1}/_{N^0}$

Circular City Location Model

- Welfare: We need to investigate whether the "free market" produces a larger or a smaller variety than the optimal variety level.

- Before defining the economy's welfare function, we calculate the economy's aggregate transportation costs, denoted by T.

- In equilibrium, all consumers purchasing from firm 1, say, are located between 0 and 1/(2N) units of distance from the firm (on each side).

- Remember, $\hat{x} = \frac{p-p_1}{2t} + \frac{1}{2N}$

- Hence, at equilibrium $\hat{x} = \frac{1}{2N}$ because, $p_i = p^0 \;\; \forall \;\; i$

Circular City Location Model

- Since there are $2N$ such intervals (if number of intervals is v, then $v\frac{1}{2N} = 1$, or, $v = 2n$), the economy's total transportation cost is given by –

$$T(n) = 2nt\left(\int_0^{\frac{1}{2n}} xdx\right) = 2nt\left[\frac{x^2}{2}\right]_0^{\frac{1}{2n}} = \frac{t}{4n}$$

- We define the economy's loss function, $L(F, t, N)$, as the sum of the fixed cost paid by the producing firms and the economy's aggregate transportation cost.

- The "Social Planner" chooses the optimal number of brands N* to minimize the economy's loss function, $L(F, t, N)$.

Circular City Location Model

- Formally, the "Social Pioneer" faces the following optimization –

$$\min_N L(F,t,N) \equiv NF + T(N) = NF + \frac{t}{4N}$$

- FOC: $\frac{\partial L}{\partial N} = 0 = F - \frac{t}{4N^2}$

- Hence, $N^* = \frac{1}{2}\sqrt{\frac{t}{F}} < \sqrt{t/_F} = N^0$

- Therefore, in a free-entry location model, too many brands are produced.

Circular City Location Model

- Notice, that there is a welfare tradeoff between the economies of scale and the aggregate transportation cost.

- That is, a small number of brands is associated with lower average production costs ($c + FN^0$) but higher aggregate transportation costs (because of fewer firms).

- A large number of brands means a lower scale of production (higher average cost) but with a lower aggregate transportation cost.

- Equation ($N^* < N^0$) shows that it is possible to raise the economy's welfare by reducing the number of brands.

Sequential entry to the linear city

- So far, we have not discussed any model in which firms strategically choose where to locate.
- Further, we have seen that the basic linear city model does not have an equilibrium where firms choose both prices and location.
- We now discuss the situation where prices are fixed at a uniform level set by the regulator (say prices are set by the government).
- The only choice variable left to firms is where to locate.

Sequential entry to the linear city

- Consider the unit interval (street) where there are three firms entering sequentially.

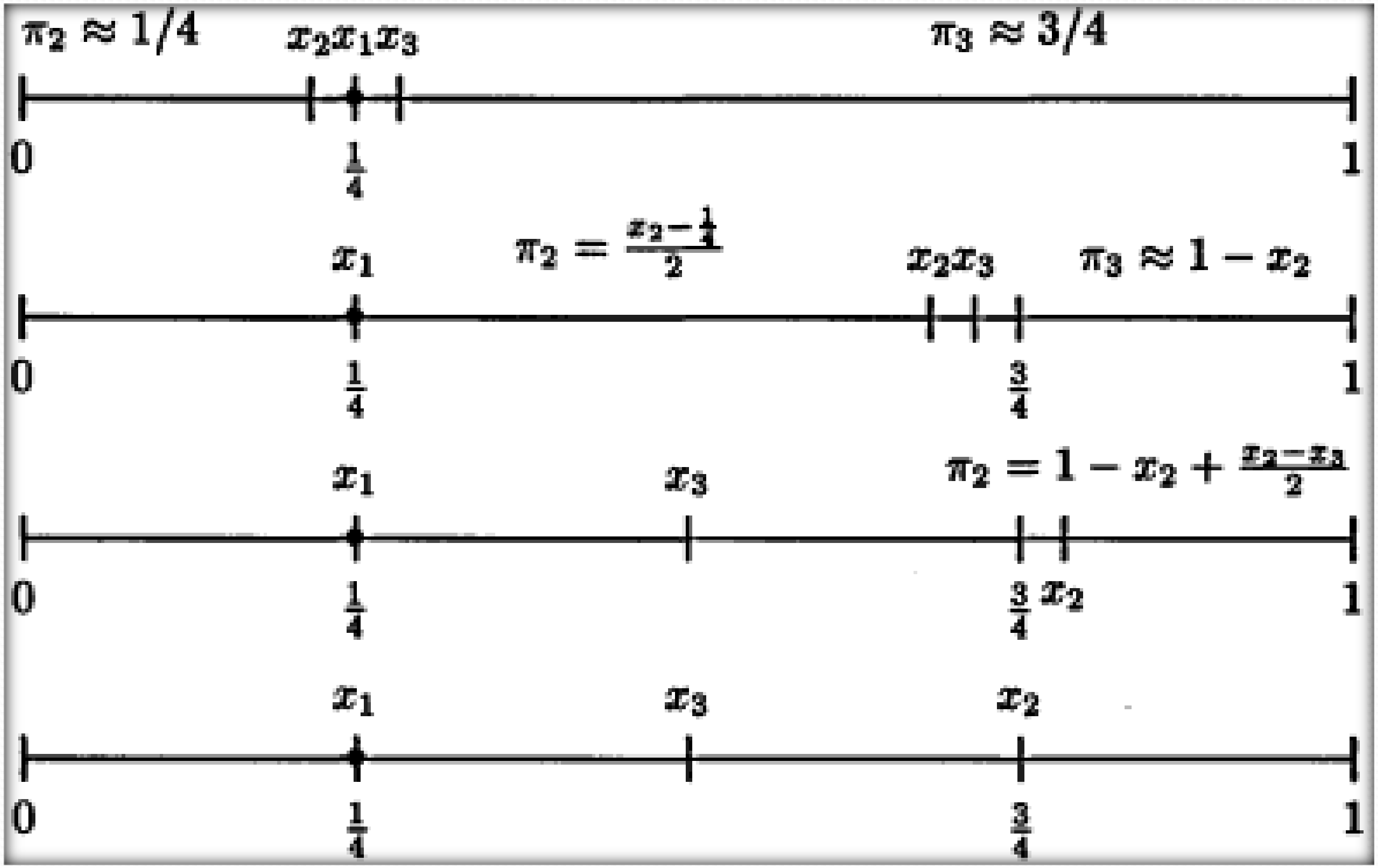
- In this three period model, firm 1 enters in period 1, firm 2 in period 2, and firm 3 in period 3.

- We look for a SPE in location strategies, where each firm maximizes its market share.

- We denote by $0 \leq x_i \leq 1$ the location strategy chosen by firm i (in period i), $i = 1, 2, 3$.
- Let ε denote a "very small" number, representing the smallest possible measurable unit of distance.

Sequential entry to the linear city

- We start by assuming that firm 1 has already moved and located itself at the point $x_1 = \frac{1}{4}$.



The third-period subgame

- Firm 3 decides on its location x_3 after firm 1 and firm 2 are already located.

- There are three possible locations of firm 2 corresponding to the three upper parts of the figure –

1. Next to Firm-1, towards 0 ($x_2 = \frac{1}{4} - \varepsilon$): In this case firm 3 would locate at ($x_3 = \frac{1}{4} + \varepsilon$).

- We can show that $\pi_2 = x_2 + \frac{1}{2}(\frac{1}{4} - x_2) < \frac{1}{4}$ and $\pi_3 \approx \frac{3}{4}$

The third-period subgame

2. Firm-2 locates in a position ($\frac{1}{4} < x_2 < \frac{3}{4}$): In this case firm 3 would locate to the right of firm 2, at $x_3 = x_2 + \varepsilon$.

- Here, we can show that, $\pi_3 \approx 1 - x_2$ while, $\pi_2 \approx \frac{x_2 - \frac{1}{4}}{2} < \frac{1}{4}$. That is, firm 2 shares the $[x_1, x_2]$ interval with firm 1.

3. Firm-2 locates in a position ($x_2 \geq \frac{3}{4}$): In this case firm 3 would locate between firm 1 and firm 2, at any point $x_1 < x_3 < x_2$. With no loss of generality, assume that $x_3 = \frac{x_2 + \frac{1}{4}}{2}$.

- Here, $\pi_3 = \frac{x_2 - \frac{1}{4}}{2}$ and $\pi_2 = 1 - x_2 + \frac{x_2 - x_3}{2} = \frac{15 - 12x_2}{16}$

The second-period subgame

- Firm 2 knows that in the third period, the location decision of firm 3 will be influenced by its own choice of location.

- Thus, firm 2 calculates the best-response function of firm 3 (which we calculated above).

- Hence, firm 2 takes the decision rule of firm 3 as given and chooses x_2 that would maximize π_2.

- Clearly, firm 2 will not locate at $x_2 = \frac{1}{4} - \varepsilon$ since this location yields a maximum profit of $\pi_2 \approx \frac{1}{4}$.

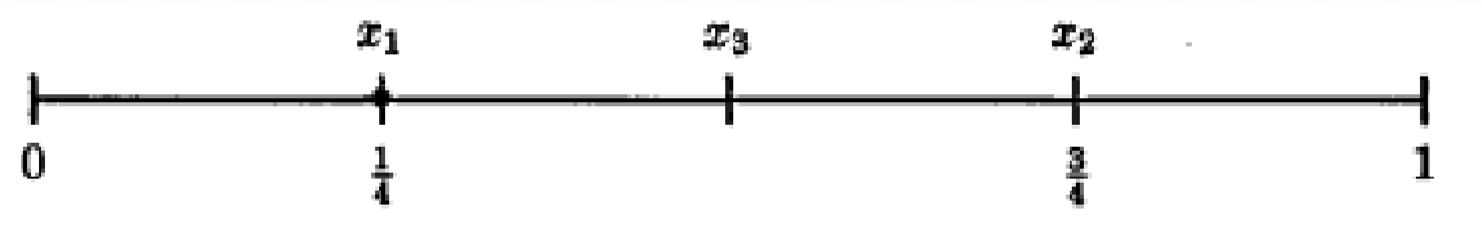
The second-period subgame

- If firm 2 locates at $\frac{1}{4} < x_2 < \frac{3}{4}$, we have seen that $\pi_2 < \frac{1}{4}$

- However, if firm 2 locates at $x_2 \geq \frac{3}{4}$, then, $\pi_2 = \frac{15-12x_2}{16}$ which can be maximized at $x_2 = \frac{3}{4}$, and, $\pi_2 = \frac{3}{8}$

- In summary, the SPE is reached where

$$x_2 = \frac{3}{4}, \pi_2 = \frac{3}{8}; x_3 = \frac{1}{2}, \pi_3 = \frac{1}{4}$$



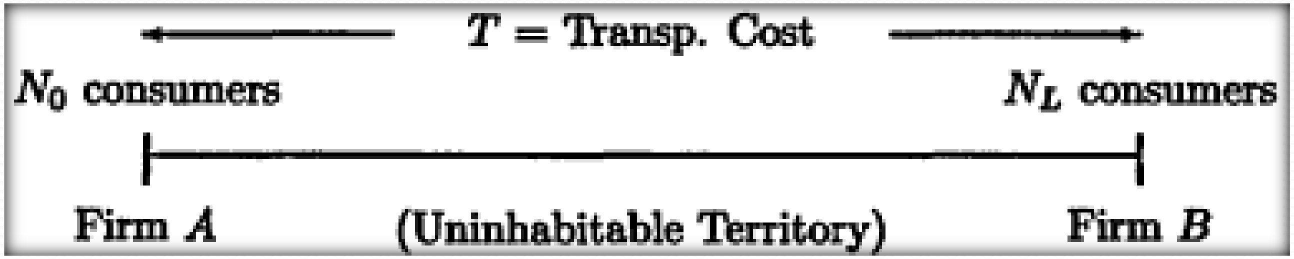
Location Model: Version

- Consider a city where consumers and producers are located only at the city's edges.

- Suppose that the city consists of N^0 consumers located at point $x = 0$ and N_L consumers located at the point $x = L$.

- There are two firms; firm A is located also at $x = 0$ and firm B is located at $x = L$.

- Assume that production is costless.



Location Model: Version

- Each consumer buys one unit either from the firm located where the consumer is, or from the firm located on the other side of town.

- Shopping nearby does not involve transportation cost, whereas shopping on the other side of town involves paying a fixed transportation cost of $t \geq 0$.

- Let p_A denote the price charged by firm A, and p_B the price charged by firm B.

- Thus, we assume that the utility of the consumer located at point $x = 0$ is given by –

$$U_0 \equiv \begin{cases} -p_A & \text{buying from A} \\ -p_B - t & \text{buying from B} \end{cases}$$

Location Model: Version

- Similarly, the utility of the consumer located at point $x = L$ is given by –

$$U_L \equiv \begin{cases} -p_A - t & \text{buying from A} \\ -p_B & \text{buying from B} \end{cases}$$

- Let n_A denote the number of consumers buying from firm A, and n_B denote the number of consumers buying from firm B.

- Then, from the above defined utility functions we must have –

$$n_A = \begin{cases} 0 & \text{if} \quad p_A > p_B + t \\ N^0 & \text{if} \quad p_B - t \le p_A \le p_B + t \\ N^0 + N_L & \text{if} \quad p_A < p_B - t \end{cases}$$

$$n_B = \begin{cases} 0 & \text{if} \quad p_B > p_A + t \\ N_L & \text{if} \quad p_A - t \le p_B \le p_A + t \\ N^0 + N_L & \text{if} \quad p_B < p_A - t \end{cases}$$

Non-existence of a Nash-Bertrand equilibrium

- A Nash-Bertrand equilibrium is the non-negative pair $\{p_A^N, p_B^N\}$, such that for a given p_B^N, firm A chooses p_A^N to maximize $\pi_A = p_A\, n_A$; and for a given p_A^N, firm B chooses p_B^N to maximize $\pi_B = p_B\, n_B$.

- Remember, n_A and n_B are given as mentioned

- ***Proposition***: There does not exist a Nash-Bertrand equilibrium in prices for the discrete version of Hotelling's location model.

Non-existence of a Nash-Bertrand equilibrium

- A Nash-Bertrand equilibrium is the non-negative pair $\{p_A^N, p_B^N\}$, such that for a given p_B^N, firm A chooses p_A^N to maximize $\pi_A = p_A\, n_A$; and for a given p_A^N, firm B chooses p_B^N to maximize $\pi_B = p_B\, n_B$.

- ***Proposition***: There does not exist a Nash-Bertrand equilibrium in prices for the discrete version of Hotelling's location model.

- Proof: By way of contradiction. Suppose that $\{p_A^N, p_B^N\}$ constitute a Nash equilibrium.

- Then, there can be three cases: $|p_A^N - p_B^N| > t$; $|p_A^N - p_B^N| < t$; $|p_A^N - p_B^N| = t$

Case 1: $|p_A^N - p_B^N| > t$

- With no loss of generality, suppose that $p_A^N - p_B^N > t$.
- Then, the set of equations (n_A, n_B) imply that $n_A^N = 0$ and, hence, $\pi_A^N = 0$
- However, firm A can deviate and increase its profit by reducing its price to $\bar{p}_A = p_B^N + t$ and by having $\bar{n}_A = N^0$, thereby earning a profit of
$$\bar{\pi}_A = N^0(p_B^N + t)$$
- Hence, by contradiction, $\{p_A^N, p_B^N\}$ can't be a Nash-Bertrand equilibrium

Case 2: $|p_A^N - p_B^N| < t$

- With no loss of generality, suppose that $p_A^N < p_B^N + t$.

- Then, firm A can deviate and increase its profit by slightly increasing its price to $\bar{p}_A$ such that $p_A^N < \bar{p}_A < p_B^N + t$

- Thus, obtaining a profit level of $\bar{\pi}_A = N^0 \bar{p}_A > \pi_A^N$

- Hence, by contradiction, $\{p_A^N, p_B^N\}$ can't be a Nash-Bertrand equilibrium

Case 3: $|p_A^N - p_B^N| = t$

- With no loss of generality, suppose that $p_A^N - p_B^N = t$.

- Then, $p_B^N = p_A^N - t < p_A^N + t$

- Hence, as firm A did in case (ii), firm B can increase its profit by slightly raising p_B^N

- Hence, by contradiction, $\{p_A^N, p_B^N\}$ can't be a Nash-Bertrand equilibrium

Undercutproof equilibrium

- In an undercutproof equilibrium, each firm chooses the highest possible price, subject to the constraint that the price is sufficiently low so that the rival firm would not find it profitable to set a sufficiently lower price in order to grab the entire market.

- That is, in an undercutproof equilibrium, firms set prices at the levels that ensure that competing firms would not find it profitable to completely undercut these prices.

- Thus, unlike behavior in a Nash-Bertrand environment, where each firm assumes that the rival firm does not alter its price, in an undercut-proof equilibrium environment, firms assume that rival firms are ready to reduce their prices whenever undercutting prices and grabbing their rival's market are profitable to them.

Undercutproof equilibrium

- This behavior is reasonable for firms competing in differentiated products.

- An undercutproof equilibrium for this economy is non-negative $\{p_A^U, p_B^U\}$ and $\{n_A^U, n_B^U\}$ such that –

1. For a given $\{p_B^U, n_B^U\}$, firm A chooses the highest price p_A^U subject to

$$\pi_B^U \equiv p_B^U n_B^U \geq (N_0 + N_L)(p_A^U - T).$$

2. For a given $\{p_A^U, n_A^U\}$, firm B chooses the highest price p_B^U subject to

$$\pi_A^U \equiv p_A^U n_A^U \geq (N_0 + N_L)(p_B^U - T).$$

3. The distribution of consumers between the firms is determined in the set of equations mentioned earlier

Undercutproof equilibrium

- Part 1 of the definition states that in an undercutproof equilibrium, firm A sets the highest price under the constraint that the price is sufficiently low to prevent firm B from undercutting p_A^U and grabbing the entire market.

- More precisely, firm A sets p_A^N sufficiently low so that B's equilibrium profit level exceeds B's profit level when it undercuts by setting $\bar{p}_B = p_A^U - \mathrm{T}$, and grabbing the entire market ($n_B = N_0 + N_L$).

- Part 2 is similar to part 1 but describes how firm B sets its price.

- We proceed with solving for the equilibrium prices.

Undercutproof equilibrium

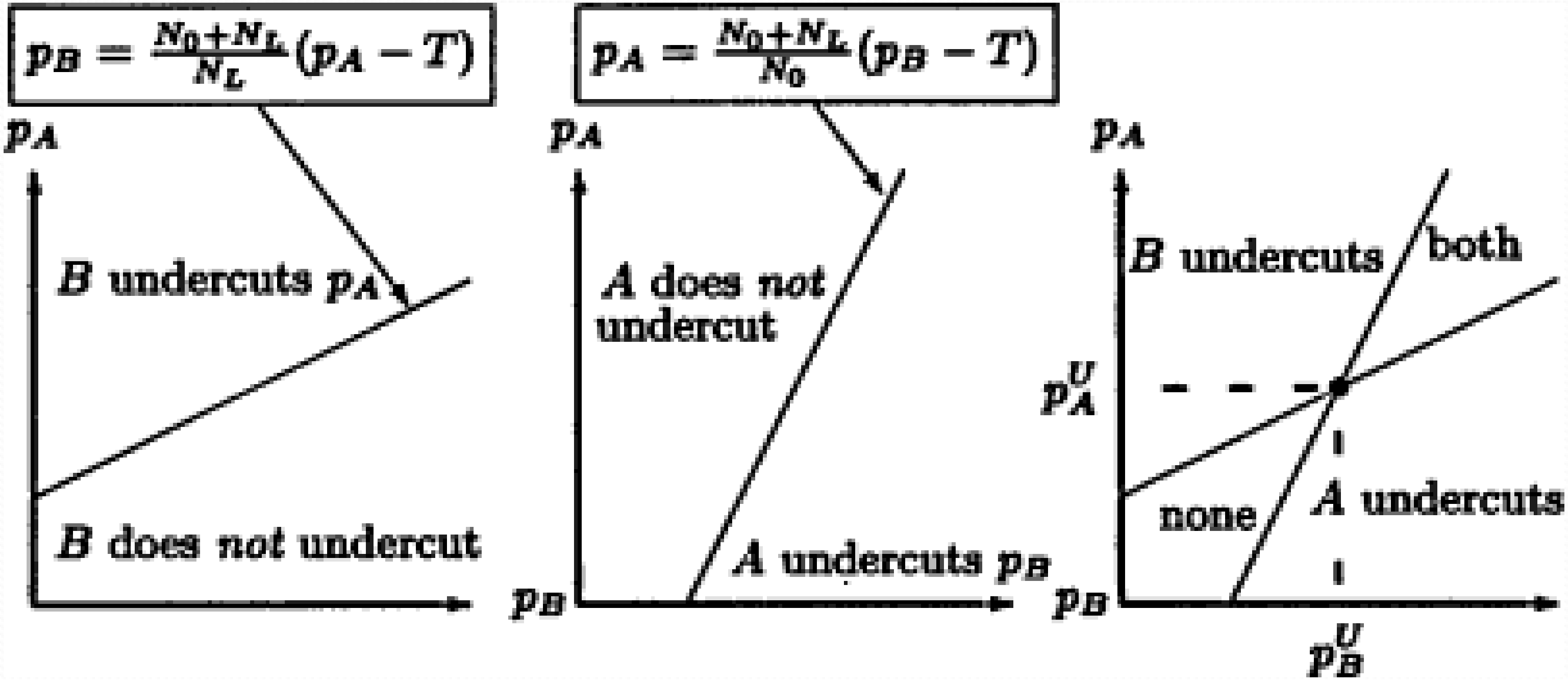
- ***Proposition***: There exists a unique undercutproof equilibrium for the discrete-location problem given by $n_A^U = N_0$, and $n_B^U = N_L$ and

$$p_A^U = \frac{(N_0 + N_L)(N_0 + 2N_L)T}{(N_0)^2 + N_0 N_L + (N_L)^2} \quad \text{and} \quad p_B^U = \frac{(N_0 + N_L)(2N_0 + N_L)T}{(N_0)^2 + N_0 N_L + (N_L)^2}$$

- *Proof*: First note that by setting $p_i \leq T$, each firm can secure a strictly positive market share without being undercut.
- Hence, in an undercut-proof equilibrium both firms maintain a strictly positive market share.
- From the set of equations mentioned earlier, we have $n_A^U = N_0$, and $n_B^U = N_L$.
- Substituting $n_A^U = N_0$, and $n_B^U = N_L$ into the two constraints in the definition and then verifying the previous equations yields the unique undercutproof equilibrium.

Undercutproof equilibrium

- *Undercutproof equilibrium for the discrete-location model*



Properties of undercutproof equilibrium

- Clearly, prices rise with transportation costs and monotonically decline to zero as transportation costs approach zero, reflecting a situation in which the products become homogeneous.

- More interestingly, $$\Delta p \equiv p_B - p_A = \frac{(N_0 + N_L)(N_0 - N_L)T}{(N_0)^2 + N_0 N_L + (N_L)^2}.$$

- Hence, $\Delta p \geq 0$ if and only if $N_0 > N_L$

- Thus, in an undercutproof equilibrium, the firm selling to the larger number of consumers charges a lower price.

- This lower price is needed to secure the firm from being totally undercut.

Properties of undercutproof equilibrium

- Finally, under symmetric distribution of consumers, $N_0 = N_L$, the equilibrium prices are given by $p_A^U = p_B^U = 2T$

- That is, each firm can mark up its price to twice the level of the transportation cost without being undercut.

Reference

- Oz Shy (1995). Industrial Organization. MIT Press. Chapter – 7.