chapter 6

Return and Risk: The Foundation of Investing Worldwide

As you continue to prepare yourself to put together and manage a $1 million portfolio, you realize you need to have a very clear understanding of risk and return. After all, as you recall from your introductory finance class, these are the basic parameters of all investing decisions. While you agree that the past is not a sure predictor of the future, it seems reasonable that knowing the history of the returns and risks on the major financial assets will be useful. After all, if stocks in general have never returned more than about 10 percent on average, does it make sense for you to think of earning 15 or 20 percent annually on a regular basis? And what about compounding, supposedly an important part of long-term investing? How much, realistically, can you expect your portfolio to grow over time? Finally, exactly what does it mean to talk about the risk of stocks? How can you put stock risk into perspective? If stocks are really as risky as people say, maybe they should be only a small part of your portfolio.

Although math is not your long suit, you realize that it is not unreasonable that a $1 million gift should impose a little burden on you. Therefore, you resolve to get out your financial calculator and go to work on return and risk concepts, knowing that with some basic understanding of the concepts you can take the easy way out and let your computer spreadsheet program do the hard work.

Chapter 6 analyzes the returns and risks from investing. We learn how well investors have done in the past investing in the major financial assets. Investors need a good understanding of the returns and risk that have been experienced to date before attempting to estimate returns and risk, which they must do as they build and hold portfolios for the future.

AFTER READING THIS CHAPTER YOU WILL BE ABLE TO:

- Calculate important return and risk measures for financial assets, using the formulation appropriate for the task.
- Use key terms involved with return and risk, including geometric mean, cumulative wealth index, inflation-adjusted returns, and currency-adjusted returns.
- Understand clearly the returns and risk investors have experienced in the past, an important step in estimating future returns and risk.

An Overview

How do investors go about calculating the returns on their securities over time? What about the risk of these securities? Assume you invested an equal amount in each of three stocks over a five-year period, which has now ended. The five annual returns for stocks 1, 2, and 3 are as follows:

1	2	3
−0.1	0.04	0.4
−0.2	0.05	−0.02
0.29	0.07	−0.1
0.19	0.06	−0.15
0.12	0.09	0.17

Stock 1 started off with two negative returns but then had three good years. Stock 2's returns are all positive, but quite low. Stock 3 had a 40 percent return in one year and a 17 percent return in another year, but it also suffered three negative returns. Which stock would have produced the largest final wealth for you, and which stock had the lowest risk over this five-year period? Which stock had the lowest compound annual average return over this five-year period? How would you proceed to determine your answers?

How would investors have fared, on average, over the past by investing in each of the major asset classes such as stocks and bonds? What are the returns and risk from investing, based on the historical record? What about nominal returns versus inflation-adjusted returns? We answer important questions such as these in this chapter.

Although there is no guarantee that the future will be exactly like the past, a knowledge of historical risk-return relationships is a necessary first step for investors in making investment decisions for the future. Furthermore, there is no reason to assume that *relative* relationships will differ significantly in the future. If stocks have returned more than bonds, and Treasury bonds more than Treasury bills, over the entire financial history available, there is every reason to assume that such relationships will continue over the *long-run* future. Therefore, it is very important for investors to understand what has occurred in the past.

Return

In Chapter 1, we learned that the objective of investors is to maximize expected returns subject to constraints, primarily risk. Return is the motivating force in the investment process. It is the reward for undertaking the investment.

Returns from investing are crucial to investors; they are what the game of investments is all about. The measurement of realized (historical) returns is necessary for investors to assess how well they have done or how well investment managers have done on their behalf. Furthermore, the historical return plays a large part in estimating future, unknown returns.

THE TWO COMPONENTS OF ASSET RETURNS

Return on a typical investment consists of two components:

- **Yield**: The basic component many investors think of when discussing investing returns is the periodic cash flows (or income) on the investment, either interest (from bonds) or

Yield The income component of a security's return

Capital Gain (Loss) The change in price on a security over some period

dividends (from stocks). The distinguishing feature of these payments is that the issuer makes the payments in cash to the holder of the asset. **Yield** measures a security's cash flows relative to some price, such as the purchase price or the current market price.

- **Capital gain (loss)**: The second component is the appreciation (or depreciation) in the price of the asset, commonly called the **capital gain (loss)**. We will refer to it simply as the price change. In the case of an asset purchased (long position), it is the difference between the purchase price and the price at which the asset can be, or is, sold; for an asset sold first and then bought back (short position), it is the difference between the sale price and the subsequent price at which the short position is closed out. In either case, a gain or a loss can occur.[1]

Putting the Two Components Together Add these two components together to form the total return:

$$\text{Total return} = \text{Yield} + \text{Price change} \tag{6-1}$$

where the yield component can be 0 or +
the price change component can be 0, +, or −

Example 6-1

A bond purchased at par ($1,000) and held to maturity provides a yield in the form of a stream of cash flows or interest payments, but no price change. A bond purchased for $800 and held to maturity provides both a yield (the interest payments) and a price change, in this case a gain. The purchase of a nondividend-paying stock, such as Apple, that is sold six months later produces either a capital gain or a capital loss but no income. A dividend-paying stock, such as Microsoft, produces both a yield component and a price change component (a realized or unrealized capital gain or loss).

Equation 6-1 is a conceptual statement for the total return *for any security*. Investors' returns from financial assets come only from these two components—an income component (the yield) and/or a price change component, regardless of the asset. Investors sometimes mistakenly focus only on the yield component of their investments, rather than the total return, and mistakenly assume they are achieving acceptable performance when they are not.

Example 6-2

In one recent year a $500,000 portfolio was invested half in stocks and half in bonds. At the end of the year this portfolio had yielded about $19,000 in dividends and interest. However, because of the declining stock market, the value of the portfolio at the end of the year was about $475,000. Therefore, the capital loss exceeded the yield, resulting in a negative total return for that one-year period.

[1] This component involves only the difference between the beginning price and the ending price in the transaction. An investor can purchase or short an asset and close out the position one day, one hour, or one minute later for a capital gain or loss. Furthermore, gains can be realized or unrealized. See online Appendix 2-A for more discussion on capital gains and losses and their taxation.

Measuring Returns

TOTAL RETURN

Total Return (TR) Percentage measure relating all cash flows on a security for a given time period to its purchase price

We now know that a correct returns measure must incorporate the two components of return, yield and price change, keeping in mind that either component could be zero. The **Total Return (TR)** for a given holding period is a decimal or percentage number relating all the cash flows received by an investor during any designated time period to the purchase price of the asset calculated as

$$TR = \frac{CF_t + (P_E - P_B)}{P_B} = \frac{CF_t + PC}{P_B} \quad \textbf{(6-2)}$$

where

CF_t = cash flows during the measurement period t

P_E = price at the end of period t or sale price

P_B = purchase price of the asset or price at the beginning of the period

PC = change in price during the period, or P_E minus P_B

The periodic cash flows from a bond consists of the interest payments received, and for a stock, the dividends received. For some assets, such as a warrant or a stock that pays no dividends, there is only a price change. Part A of Exhibit 6-1 illustrates the calculation of TR for a bond, a common stock, and a warrant. Although one year is often used for convenience, the TR calculation can be applied to periods of any length.

EXHIBIT 6-1

Examples of Total Return and Price Relative Calculations

A. Total Return (TR) Calculations

I. Bond TR

$$\text{Bond TR} = \frac{I_t + (P_E - P_B)}{P_B} = \frac{I_t + PC}{P_B}$$

I_t = the interest payment(s) received during the period:

P_B and P_E = the beginning and ending prices, respectively

PC = the change in price during the period

Example: Assume the purchase of a 10-percent-coupon Treasury bond at a price of $960, held one year, and sold for $1,020. The TR is

$$\text{Bond TR} = \frac{100 + (1{,}020 - 960)}{960} = \frac{100 + 60}{960} = 0.1667 \text{ or } 16.67\%$$

II. Stock TR

$$\text{Stock TR} = \frac{D_t + (P_E - P_B)}{P_B} = \frac{D_t + PC}{P_B}$$

D_t = the dividend(s) paid during the period

Example: 100 shares of DataShield are purchased at $30 per share and sold one year later at $26 per share. A dividend of $2 per share is paid.

$$\text{Stock TR} = \frac{2 + (26 - 30)}{30} = \frac{2 + (-4)}{30} = -.0667 \text{ or } -6.67\%$$

III. Warrant TR

$$\text{Warrant TR} = \frac{C_t + (P_E - P_B)}{P_B} = \frac{C_t + PC}{P_B} = \frac{PC}{P_B}$$

where, C_t = any cash payment received by the warrant holder during the period. Because warrants pay no dividends, the only return to an investor from owning a warrant is the change in price during the period.

Example: Assume the purchase of warrants of DataShield at $3 per share, a holding period of six months, and the sale at $3.75 per share.

$$\text{Warrant TR} = \frac{0 + (3.75 - 3.00)}{3.00} = \frac{0.75}{3.00} = 0.25 \text{ or } 25\%$$

B. Return Relative Calculations

The return relative for the preceding bond example shown is

$$\text{Bond return relative} = \frac{100 + 1020}{960} = 1.1667$$

The return relative for the stock example is

$$\text{Stock return relative} = \frac{2 + 26}{30} = 0.9333$$

The return relative for the warrant example is

$$\text{Warrant return relative} = \frac{3.75}{3.00} = 1.25$$

To convert from a return relative to a total return, subtract 1.0 from the return relative.

Calculating Total Returns for the S&P 500 Index Table 6-1 shows the Standard & Poor's (S&P) 500 Stock Composite Index for the years 1926 through 2011 (a total of 86 years because the data start on January 1, 1926). Included in the table are *end-of-year* values for the index, from which capital gains and losses can be computed, and dividends on the index, which constitute the income component.

Table 6-1 Historical Composite Stock Price Index, Based on Standard & Poor's 500 Index, Dividends in Index Form, and Total Returns (TRs), 1926–2011. Values are End-of-Year. (No monthly compounding.)

Year	Index Val	Div	TR%	Year	Index Val	Div	TR%
1925	10.34			1969	92.06	3.16	−8.32
1926	15.03	0.75	8.20	1970	92.15	3.14	3.51
1927	19.15	0.81	32.76	1971	102.09	3.07	14.12
1928	25.61	0.84	38.14	1972	118.05	3.15	18.72
1929	22.05	0.95	−10.18	1973	97.55	3.38	−14.50
1930	15.31	0.90	−26.48	1974	68.56	3.60	−26.03
1931	7.89	0.76	−43.49	1975	90.19	3.68	36.92
1932	6.80	0.46	−8.01	1976	107.46	4.05	23.64
1933	10.19	0.36	55.34	1977	95.10	4.67	−7.16
1934	10.10	0.40	3.00	1978	96.11	5.07	6.39
1935	13.91	0.41	41.79	1979	107.94	5.65	18.19
1936	17.60	0.68	31.38	1980	135.76	6.16	31.48
1937	11.14	0.78	32.29	1981	122.55	6.63	−4.85
1938	13.60	0.52	26.70	1982	140.64	6.87	20.37
1939	13.19	0.59	1.31	1983	164.93	7.09	22.31
1940	11.51	0.67	−7.63	1984	167.24	7.53	5.97
1941	9.59	0.75	−10.24	1985	211.28	7.90	31.06
1942	10.45	0.64	15.67	1986	242.17	8.28	18.54
1943	12.59	0.64	26.71	1987	247.08	8.81	5.67
1944	14.33	0.68	19.18	1988	277.72	9.73	16.34
1945	18.87	0.67	36.43	1989	353.40	11.05	31.23
1946	17.08	0.77	−5.44	1990	330.22	12.10	−3.14
1947	16.74	0.92	3.43	1991	417.09	12.20	30.00
1948	16.11	1.05	2.45	1992	435.71	12.38	7.43
1949	18.11	1.14	19.48	1993	466.45	12.58	9.94
1950	21.94	1.40	28.92	1994	459.27	13.18	1.29
1951	24.98	1.35	19.99	1995	615.93	13.79	37.11
1952	26.94	1.37	13.34	1996	740.74	14.90	22.68
1953	25.85	1.41	1.17	1997	970.43	15.50	33.10
1954	36.73	1.49	47.87	1998	1229.23	16.38	28.36
1955	43.89	1.68	24.06	1999	1469.25	16.48	20.87
1956	46.30	1.82	9.65	2000	1,320.28	15.97	−9.05
1957	39.99	1.87	−9.59	2001	1,148.08	15.71	−11.85
1958	55.21	1.75	42.44	2002	879.82	16.07	−22.10
1959	59.89	1.83	11.79	2003	1111.92	17.49	28.37
1960	58.11	1.95	0.28	2004	1,211.92	19.54	10.75
1961	71.55	2.02	26.60	2005	1248.29	22.22	4.83
1962	63.10	2.13	−8.83	2006	1,418.30	24.88	15.61
1963	75.02	2.28	22.50	2007	1468.36	27.73	5.48
1964	84.75	2.50	16.3	2008	903.25	28.39	−36.55
1965	92.43	2.72	12.27	2009	1115.10	22.31	25.92
1966	80.33	2.87	−9.99	2010	1,257.64	23.12	14.86
1967	96.47	2.92	23.73	2011	1257.64	26.41	2.1
1968	103.86	3.07	10.84				

Example 6-3 The TRs for each year as shown in Table 6-1 can be calculated as shown in Equation 6-2. As a demonstration of these calculations, the TR for 2010 for the S&P 500 Index was 14.86 percent, calculated as:[2]

$$TR_{2010} = [1257.64 - 1115.10 + 23.12]/1115.10 = .1486 \text{ or } 14.86\%$$

In contrast, in 2000 the TR was −9.07 percent, calculated as:

$$TR_{2000} = [1320.28 - 1469.25 + 15.69]/1469.25 = -.0907 = -9.07\%$$

Conclusions About Total Return In summary, the TR concept is valuable as a measure of return because it is all-inclusive, measuring the total return per dollar of original investment.

✓ TR is *the* basic measure of the return earned by investors on any financial asset for any specified period of time. It can be stated on a decimal or percentage basis.

TR facilitates the comparison of asset returns over a specified period, whether the comparison is of different assets, such as stocks versus bonds, or different securities within the same type, such as several common stocks. Remember that using this concept does not mean that the securities have to be sold and the gains or losses actually realized—that is, the calculation applies to realized or unrealized gains (see Appendix 2-A).

Some Practical Advice

As you analyze and consider common stocks, never forget the important role that dividends have played historically in the TRs shown for large common stocks. For example, for the 85-year period 1926–2010, for the S&P 500 Index, the compound annual average return was 9.6 percent (rounded). Dividends averaged 4 percent, and obviously were an important component of the TR. However, in the 1990s the dividend yield on the major stock indexes continued to decline, and reached levels of about 1.5 percent in 2001 and 2002. Clearly, if all other things remained equal, TRs on the S&P 500 Index would decline relative to the past because of the significant decreases in the dividend yield. Not surprisingly, given the turmoil in the economy, more large companies cut dividends in 2008 than in any year since 2001. At the beginning of 2012, the dividend yield on the S&P 500 Index was approximately 2.1 percent.

What about the importance of dividends for individual stocks? Consider a company with an ordinary product consumed daily around the world, Coca-Cola.

What if a member of your family bought one share in 1919 for $40 when Coca-Cola had its IPO? One share would be worth $322,421 at the end of 2011.[3] Coca-Cola also paid dividends. How much impact do you think the reinvested dividends would have on the

[2] Note carefully that these calculations do not account for the reinvestment of dividends during the year and will differ from the total returns calculated as part of the official series of returns later in the chapter.

[3] This example is based on "*Never Underestimate the Winning Role Dividends Play," AAII Dividend Investing*, Internet mailing, February 25, 2012.

terminal wealth of this one share at the end of 2011? According to one calculation, that one share would have been worth \$9.3 million at the end of 2011! Such is the impact of compounding reinvested dividends over a very long period of time.

RETURN RELATIVE

It is often necessary to measure returns on a slightly different basis than total returns. This is particularly true when calculating either a cumulative wealth index or a geometric mean, both of which are explained below, because negative returns cannot be used in the calculation.

Return Relative (RR) The total return for an investment for a given period stated on the basis of 1.0

✓ The **Return Relative (RR)** eliminates negative numbers by adding 1.0 to the TR. It provides the same information as the TR, but in a different form.

- RR = TR in decimal form + 1.0
- TR in decimal form = RR − 1.0

Example 6-4 A TR of 0.10 for some holding period is equivalent to a return relative of 1.10, and a TR of −9.07, as calculated in Example 6-3 for the year 2000, is equivalent to a return relative of 0.9093.

Equation 6-2 can be modified to calculate return relatives directly by using the price at the end of the holding period in the numerator, rather than the change in price, as in Equation 6-3.

$$\text{Return relative} = \text{RR} = \frac{\text{CF}_t + \text{P}_\text{E}}{\text{P}_\text{B}} \tag{6-3}$$

Examples of RR calculations for the same three assets as the preceding are shown in Part B of Exhibit 6-1.

Example 6-5 The RR for 2010 for the S&P 500 calculated using Equation 6-3 is

$$(1257.64 + 23.12)/1115.10 = 1.1486$$

CUMULATIVE WEALTH INDEX

Return measures such as TRs measure the rate of change in an asset's price or return, and percentage rates of return have multiple uses. Nevertheless, we all understand dollar amounts! Therefore, it is often desirable to measure how one's wealth in dollars changes over time. In other words, we measure the cumulative effect of returns compounding over time given *some stated initial investment*, which typically is shown as \$1 for convenience (\$1 is the default value). Note that having calculated ending wealth (cumulative wealth) over some time period on the basis of a \$1 initial investment, it is simple enough to multiply by an investor's actual beginning amount invested, such as \$10,000 or \$22,536 or any other beginning amount.

Cumulative Wealth Index Cumulative wealth over time, given an initial wealth and a series of returns on some asset

The **Cumulative Wealth Index**, CWI_n, is computed as

$$\text{CWI}_n = \text{WI}_0(1 + \text{TR}_1)(1 + \text{TR}_2)\ldots(1 + \text{TR}_n) \tag{6-4}$$

where

CWI_n = the cumulative wealth index as of the end of period n

WI_0 = the beginning index value; typically \$1 is used but any amount can be used

$TR_{1,n}$ = the periodic TRs in decimal form (when added to 1.0 in Equation 6-4, they become return relatives)

Example 6-6

Let's calculate cumulative wealth per \$1 invested for the 1990s, one of the two greatest decades in the 20th century in which to own common stocks. This will provide you with a perspective on common stock returns at their best. Using the S&P Total Returns in Table 6-1, and converting them to return relatives, the CWI for the decade of the 1990s (the 10-year period 1990–1999) would be

$$CWI_{90-99} = 1.00(0.969)(1.30)(1.0743)(1.0994)(1.0129)(1.3711)(1.2268)(1.331)(1.2834)(1.2088)$$
$$= 5.23$$

Thus, \$1 (the beginning value arbitrarily chosen) invested at the beginning of 1990 would have been worth \$5.23 by the end of 1999. Obviously, any beginning wealth value can be used to calculate cumulative wealth. For example, \$10,000 invested under the same conditions would have been worth \$52,300 at the end of 1999, and \$37,500 invested under the same conditions would have been worth \$196,125.

✓ Cumulative wealth is always stated in dollars, and represents the effects of compounding returns over some period of time, given any initial investment. Typically, \$1 is used as the initial investment.

Our three returns measures are shown in Figure 6-1.

A Global Perspective

As noted in Chapter 1, international investing offers potential return opportunities and potential reduction in risk through diversification. Based on the historical record, investments in certain foreign markets would have increased investor returns during certain periods of time. For example, in the first decade of the 21st century European stocks performed much better than U.S. stocks. Dividend yields abroad were about 1 percentage point higher than U.S. dividend yields during that period.

U.S. investors need to understand how the returns on their investment in foreign securities are calculated, and the additional risk they are taking relative to domestic securities. This additional risk may pay off, or penalize them.

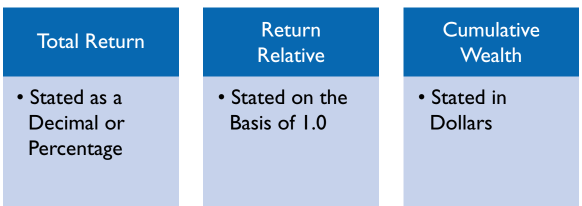


Figure 6-1

Three Measures of Return from a Financial Asset.

INTERNATIONAL RETURNS AND CURRENCY RISK

When investors buy and sell assets in other countries, they must consider exchange rate risk or currency risk. This risk can convert a gain from the asset itself into a loss on the investment or a loss from the asset itself into a gain on the investment. We need to remember that international stocks are priced in local currencies—for example, a French stock is priced in Euros, and a Japanese stock is priced in yen. For a U.S. investor who buys a foreign security, the ultimate return to him or her in spendable dollars depends on the rate of exchange between the foreign currency and the dollar, and this rate typically changes daily.

Currency Risk (Exchange Rate Risk) The risk of an adverse impact on the return from a foreign investment as a result of movements in currencies

Currency risk (exchange rate risk) is the risk that any change in the value of the investor's home currency relative to the foreign currency involved will be unfavorable; however, like risk in general, currency risk can work in the investor's favor, enhancing the return that would otherwise be received.

How Currency Changes Affect Investors An investment denominated in an appreciating currency relative to the investor's domestic currency will experience a gain from the currency movement, while an investment denominated in a depreciating currency relative to the investor's domestic currency will experience a decrease in the return because of the currency movement. Said differently, when you buy a foreign asset, you sell the home currency to do so, and when you sell the foreign asset, you buy back the home currency. For a U.S. investor,

- ✓ if the foreign currency strengthens while you hold the foreign asset, when you sell the asset you will be able to buy back more dollars using the now stronger foreign currency. Your dollar-denominated return will increase.
- ✓ if the dollar strengthens while you hold the foreign asset, when you sell your asset and convert back to dollars, you will be able to buy back fewer of the now more-expensive dollars, thereby decreasing your dollar-denominated return.

Example 6-7 In one recent year, the Brazilian market was up about 150 percent, but the currency adjustment for U.S. investors was negative (83 percent), leaving a U.S. dollar return for the year of approximately 67 percent instead of 150 percent. On the other hand, the Japanese market enjoyed a 47 percent return, and the currency adjustment was positive, 15 percent, resulting in a U.S. dollar return of approximately 62 percent for the year.

Calculating Currency-Adjusted Returns To understand the logic of currency adjustments on investor returns, consider Table 6-2. It shows the actual change in the dollar relative to the Euro over the period 2002–2004, from 1 Euro = \$1.05 to 1 Euro = \$1.35 (in other words, the value of the dollar dropped sharply during this period). Assume one share of EurTel at the end of 2002 was €75 (75 Euros). The dollar cost at this time was

Table 6-2 Impact of Currency Changes on an Investment in EurTel Stock Denominated in Euros

December 30, 2000	December year-end, 2004	Return to Investor
Exchange Rate	1.00 = \$1.05	1.00 = \$1.35
Cost in Euros of 1 share of EurTel 40% (in euros)	€75	€105
Cost in Dollars of 1 share of EurTel 80% (in dollars)	\$78.75	\$141.75

$1.05 (75) = $78.75. At the end of 2004 the value of one share had risen to €105, an increase of 40 percent for an investor transacting only in Euros. However, the value of the dollar fell sharply during this period, and when the shares of EurTel were sold, the euro proceeds bought back more dollars. The dollar value of one share is now $1.35 (105) = $141.75. The return on this investment, in dollar terms, is (141.75/78.75) − 1.0 = 80 percent.

To calculate directly the return to a U.S. investor from an investment in a foreign country, we can use Equation 6-5. The foreign currency is stated in domestic terms; that is, the amount of domestic currency necessary to purchase one unit of the foreign currency.

$$\text{TR in domestic terms} = \left[\text{RR} \times \frac{\text{Ending value of foreign currency}}{\text{Beginning value of foreign currency}}\right] - 1.0 \quad \textbf{(6-5)}$$

Example 6-8 Consider a U.S. investor who invests in WalMex at 40.25 pesos when the value of the peso stated in dollars is $0.10. One year later WalMex is at 52.35 pesos, and the stock did not pay a dividend. The peso is now at $0.093, which means that the dollar appreciated against the peso.

$$\text{RR for WalMex} = 52.35/40.25 = 1.3006$$

TR to the U.S. investor *after currency adjustment* is

$$\begin{aligned}\text{TR denominated in \$} &= \left[1.3006 \times \frac{\$0.093}{\$0.10}\right] - 1.0\\ &= [1.3006 \times 0.93] - 1.0\\ &= 1.2096 - 1.0\\ &= .2096 \text{ or } 20.96\%\end{aligned}$$

In this example, using round numbers, the U.S. investor earned a 30 percent TR denominated in Mexican currency, but only 21 percent denominated in dollars because the peso declined in value against the U.S. dollar. With the strengthening of the dollar, the pesos received when the investor sells WalMex buy fewer U.S. dollars, decreasing the 30 percent return a Mexican investor would earn to only 21 percent for a U.S. investor.

The Dollar and Investors How much difference can currency adjustments make to investors? They can make a substantial difference for selected periods of time.

Example 6-9 Let's consider the impact of the falling dollar on U.S. investors for one year, 2007. Canadian stocks earned Canadian investors 10.5 percent, but the gain for U.S. investors was 28.4 percent because of the strengthening of the Canadian dollar against the U.S. dollar. Meanwhile, French investors in a French stock index fund earned only 1.2 percent TR in 2007, while U.S. investors in the same index earned 12.1 percent (as a benchmark, the S&P 500 had a TR of 5.5 percent for 2007).

In 2000 a Euro was worth about \$.82. By mid-2008 it was worth roughly \$1.56, which means that the value of the dollar declined sharply over this entire period. As the dollar fell, foreign investors owning U.S. stocks suffered from the declining stock market and an unfavorable currency movement. On the other hand, U.S. investors in foreign securities benefited from currency movements. In 2011, despite Greece's sovereign debt crises and other issues about some European countries sovereign debt, as well as concerns about the viability of some financial institutions, the Euro continued to hover around \$1.36. This was at least partly a reflection of problems in the United States regarding the national debt and the weak economy. However, in early 2012 the Euro was at \$1.27, reflecting the ongoing European crisis.

Investments Intuition

As we now know, a declining dollar benefits U.S. investors in foreign securities, but also benefits investors holding large multinational companies like McDonald's and Apple. First, U.S. exports increase because they become more competitive around the world. Second, the foreign sales and earnings become more valuable in dollar terms.

Checking Your Understanding

1. The Cumulative Wealth Index can be calculated for nominal stock returns, but it cannot show the impact of inflation. Agree or disagree, and explain your reasoning.
2. What does it mean to say that when you buy a foreign asset, you are selling the dollar?
3. Is it correct to say that in recent years anti-dollar bets by U.S. investors paid off?

Summary Statistics for Returns

The total return, return relative, and cumulative wealth index are useful measures of return for a specified period of time. Also needed for investment analysis are statistics to describe a series of returns. For example, investing in a particular stock for 10 years or a different stock in each of 10 years could result in 10 TRs, which need to be described by summary statistics. Two such measures used with returns data are described below.

ARITHMETIC MEAN

The best-known statistic to most people is the arithmetic mean. Therefore, when someone refers to the *mean return* they usually are referring to the arithmetic mean unless otherwise specified. The arithmetic mean, customarily designated by the symbol X-bar, of a set of values is calculated as

$$\overline{X} = \frac{\sum X}{n} \tag{6-6}$$

or the sum of each of the values being considered divided by the total number of values n.

Example 6-10 Based on data from Table 6-1 for the 10 years of the 1990s ending in 1999, the arithmetic mean is calculated in Table 6-3.

$$\begin{aligned}\overline{X} &= [-3.14 + 30.00 + \cdots + 20.88]/10\\ &= 187.63/10\\ &= .1876 \text{ or } 18.76\%\end{aligned}$$

Table 6-3 Calculation of the Arithmetic and Geometric Mean for the Years 1990–1999 for the S&P 500 Stock Composite Index

Year	S&P 500 TRs (%)	S&P 500 RR
1990	−3.14	0.9687
1991	30.00	1.3000
1992	7.43	1.0743
1993	9.94	1.0994
1994	1.29	1.0129
1995	37.11	1.3711
1996	22.68	1.2268
1997	33.10	1.3310
1998	28.34	1.2834
1999	20.88	1.2088

Arithmetic Mean $= [-3.14 + 30.00 + \cdots + 20.88]/10$
$= 18.76\%$

Geometric Mean $= [(0.9687)(1.30001)(1.07432)(1.09942)(1.01286)(1.37113)(1.22683)(1.33101)(1.28338)(1.2088)]^{1/10} - 1$
$= 1.18 - 1$
$= 0.18, \text{ or } 18\%$

GEOMETRIC MEAN

The arithmetic mean return is an appropriate measure of the central tendency of a distribution consisting of returns calculated for a particular time period, such as 10 years. However, when an ending value is the result of compounding over time, the geometric mean, is needed to describe accurately the "true" average rate of return over multiple periods.

Geometric Mean The compound rate of return over time

The **geometric mean** is defined as the *nth* root of the product resulting from multiplying a series of return relatives together, as in Equation 6-7.

$$G = [(1 + TR_1)\ (1 + TR_2)\ \ldots\ (1 + TR_n)]^{1/n} - 1 \tag{6-7}$$

where TR is a series of total returns in decimal form. Note that adding 1.0 to each total return produces a return relative. RRs are used in calculating geometric mean returns, because TRs, which can be negative or zero, cannot be used in the calculation.[4]

[4] An alternative method of calculating the geometric mean is to find the log of each return relative, sum them, divide by n, and take the antilog.

The geometric mean return measures the compound rate of growth over time. It is important to note that the geometric mean assumes that all cash flows are reinvested in the asset and that those reinvested funds earn the subsequent rates of return available on that asset for some specified period. It reflects the steady *growth rate* of invested funds over some past period; that is, the uniform rate at which money actually grew over time per period, taking into account all gains and losses.

Example 6-11 Continuing the example from Table 6-3, consisting of the 10 years of data ending in 1999 for the S&P 500, the geometric mean is

$$G = [(0.969)(1.30)(1.0743)(1.0994)(1.0129)(1.3711)(1.2268)(1.331)(1.2834)(1.2088)]^{1/10} - 1$$
$$= 1.1800 - 1 = .18, \text{ or } 18\%$$

Using the Calculator

In Example 6-6, we calculated the CWI for 1990–1999 as 5.23. Knowing this number, we can calculate the geometric mean return for these years by raising 5.23 to y^x, taking the 10th root, and subtracting 1.0:

5.23y^x; 10; 1/x; =; answer is 1.1799; 1.1799 − 1.0 = .1799 or .18 or 18%

✓ Think of the annual geometric mean as the equal annual return that makes a beginning amount of money grow to a particular ending amount of money.

For example, we saw in Example 6-6 that $1 invested in the S&P 500 Composite Index on January 1, 1990 would have grown to $5.23 by December 31, 1999 (10 years). This is a result of the money compounding at the annual rate of 18 percent. At the end of year 1, the $1 would grow to $1.18; at the end of year 2, the $1.18 would grow to $1.39; at the end of year 3, the $1.39 would grow to $1.64, and so on, until at the end of year 10 the original $1 is worth $5.23. Notice that this geometric average rate of return is lower than the arithmetic average rate of return of 18.76 percent, because it reflects the variability of the returns.

✓ The geometric mean will always be less than the arithmetic mean unless the values being considered are identical, an unlikely event. The spread between the two depends on the dispersion of the distribution: the greater the dispersion, the greater the spread between the two means.

ARITHMETIC MEAN VERSUS GEOMETRIC MEAN

When should we use the arithmetic mean and when should we use the geometric mean to describe the returns from financial assets? The answer depends on the investor's objective:

- The arithmetic mean is a better measure of average (typical) performance over single periods. It is the best estimate of the expected return for next period.
- The geometric mean is a better measure of the change in wealth over the past (multiple periods). It is typically used by investors to measure the realized compound rate of return at which money grew over a specified period of time.

Example 6-12 As an illustration of how the arithmetic mean can be misleading in describing returns over multiple periods, consider the data in Table 6-4, which show the movements in price for two stocks over two successive holding periods. Both stocks have a beginning price of \$10. Stock A rises to \$20 in period 1 and then declines to \$10 in period 2. Stock B falls to \$8 in period 1 and then rises 50 percent to \$12 in period 2. For stock A, the indicated annual average arithmetic rate of change in price is 25 percent [(100% − 50%)/2]. This is clearly not sensible, because the price of stock A at the end of period 2 is \$10, the same as the beginning price. The geometric mean calculation gives the correct annual average rate of change in price of 0 percent per year.

For stock B, the arithmetic average of the annual percentage changes in price is 15 percent. However, if the price actually increased 15 percent each period, the ending price in period 2 would be \$10 (1.15) (1.15) = \$13.23. We know that this is not correct, because the price at the end of period 2 is \$12. The annual geometric rate of return, 9.54 percent, produces the correct price at the end of period 2: \$10 (1.0954)(1.0954) = \$12.

Table 6-4 Contrasting the Arithmetic and Geometric Means

Stock	Period 1	Period 2	Annual Arithmetic Rate of Return	Annual Geometric Rate of Return
A	\$20	\$10	$[100\% + (-50\%)]/2 = 25\%$	$[2.0(0.5)]^{1/2} - 1 = 0\%$
B	\$8	\$12	$[-20\% + (50\%)]/2 = 15\%$	$[0.8(1.5)]^{1/2} - 1 = 9.54\%$

✓ Over multiple periods, such as years, the geometric mean shows the true average compound rate of growth that actually occurred—that is, the *annual average rate* at which an invested dollar grew, taking into account the gains and losses over time.

On the other hand, we should use the arithmetic mean to represent the likely or typical performance for a single period. Consider the TR data for the S&P Index for the years 1990–1999 as described earlier. Our best representation of any one year's performance would be the arithmetic mean of 18.76 percent because it was necessary to average this rate of return, given the variability in the yearly numbers, in order to realize an annual compound growth rate of 18 percent after the fact.

Concepts in Action

Using the Geometric Mean to Measure Market Performance

The geometric mean for the S&P 500 Index for the 20th century was 10.35 percent. Thus, \$1 invested in this index compounded at an average rate of 10.35 percent every year during the period 1900–1999. What about the first decade of the 21st century (defined as 2000–2009)?

The S&P 500 Index suffered losses for the first three years of the decade, followed by positive returns during 2003–2007. 2008 was a disaster, but 2009 showed a very large return. The geometric mean for the first decade is calculated as

Return Relatives for the S&P 500 Index:

2000	.909	2004	1.107	2008	.634
2001	.881	2005	1.049	2009	1.265
2002	.779	2006	1.157		
2003	1.287	2007	1.055		

$$[(.909)(.881)(.779)(1.287)(1.107)(1.049)(1.157)(1.055)(.634)(1.265)]^{1/10} - 1.0 = [.9127]^{1/10} - 1.0 = .9909 - 1.0 = -.0091$$

This indicates that $1 invested in the S&P 500 Index at the beginning of 2000 compounded at an average annual rate of approximately minus one percent a year for the first 10 years of the 21st century. The market, as measured by the S&P 500 Index, got a very bad start for the first three years of the decade, and it is quite difficult to overcome such a bad start. The severely negative performance in 2008 sealed the fate for this decade. This poor performance has led some to name this period the "Lost Decade" for common stocks. In fact, this decade was the worst-performing decade for stocks in the history of reliable stock market data.

INFLATION-ADJUSTED RETURNS

Nominal Return Return in current dollars, with no adjustment for inflation

Real Returns Nominal (dollar) returns adjusted for inflation

All of the returns discussed above are **nominal returns**, based on dollar amounts that do not take inflation into account. Typically, the percentage rates of return we see daily on the news, being paid by financial institutions, or quoted to us by lenders, are nominal rates of return.

We need to consider the purchasing power of the dollars involved in investing. To capture this dimension, we analyze **real returns**, or inflation-adjusted returns. When nominal returns are adjusted for inflation, the result is in constant purchasing-power terms.

Why is this important to you? What really matters is the purchasing power that your dollars have. It is not simply a case of how many dollars you have, but what those dollars will buy.[5]

Since 1871, the starting point for reliable data on a broad cross-section of stocks, the United States has had a few periods of deflation, but on average it has experienced mild inflation over a long period of time. Therefore, on average, the purchasing power of the dollar has declined over the long run. We define the rate of inflation or deflation as the percentage change in the CPI.[6]

Example 6-13 Suppose one of your parents or relatives earned a salary of $35,000 in 1975, and by 2010 his or her salary had increased to $135,000. How much better off is this individual in terms of purchasing power? We can convert the $35,000 in 1975 dollars to 2010 dollars. When we do this, we find that the 1975 salary is worth $141,822 dollars in 2010 dollars. So in terms of purchasing power, this individual has suffered a lost over this long time period.

The Consumer Price Index The Consumer Price Index (CPI) typically is used as the measure of inflation. The compound annual rate of inflation over the period 1926–2010 and over 1926–2011 was 3.00 percent. This means that a basket of consumer goods purchased at the beginning of for $1 would cost approximately $12.34 at year-end 2010. This is calculated as $(1.03)^{85}$, because there are 85 years from the beginning of 1926 through the end of 2010.[7] For 1926–2011, the calculation is $(1.03)^{86}$, which is approximately $12.71.

[5] A handy calculator for making the conversions illustrated in Example 6-13 can be found at http://buyupside.com/calculators/purchasepowerjan08.htm.

[6] Detailed information on the CPI can be found at the Bureau of Labor Statistics (BLS) website.

[7] To determine the number of years in a series such as this, subtract the beginning year from the ending year and add 1.0. For example, 1926 − 2010 = 84, and we add 1.0 to account for the fact that we start at the beginning of 1926.

Relation Between Nominal Return and Real Return As an approximation, the nominal return (nr) is equal to the real return (rr) plus the expected rate of inflation (expinf), or

$$\text{nr} \approx \text{rr} + \text{expinf}$$

Reversing this equation, we can approximate the real return as

$$\text{rr} \approx \text{nr} - \text{expinf}$$

To drop the approximation, we can calculate inflation-adjusted returns by dividing 1 + (nominal) total return by 1 + the inflation rate as shown in Equation 6-8.

$$TR_{IA} = \frac{(1 + TR)}{(1 + IF)} - 1 \tag{6-8}$$

where TR_{IA} = the inflation—adjusted total return
IF = the rate of inflation

This equation can be applied to both individual years and average TRs.

Example 6-14 The TR for the S&P 500 Composite in 2004 was 10.87 percent (assuming monthly reinvestment of dividends). The rate of inflation was 3.26 percent Therefore, the real (inflation-adjusted) total return for large common stocks in 2004, as measured by the S&P 500, was

$$1.1087/1.0326 = 1.0737$$
$$1.0737 - 1.0 = .0737 \text{ or } 7.37\%$$

Example 6-15 Consider the period 1926—2011. The geometric mean for the S&P 500 Composite for the entire period was 9.5 percent, and for the CPI, 3.00 percent. Therefore, the real (inflation-adjusted) geometric mean rate of return for large common stocks for the period 1926—2011 was

$$1.095/1.03 = 1.063$$
$$1.063 - 1.0 = .063 \text{ or } 6.3\%$$

Checking Your Understanding

4. Assume that you invest $1,000 in a stock at the beginning of year 1. The rate of return is 15 percent for year 1, followed by a loss of 15 percent for year two. Did you break even by the end of year 2? Would it matter if the sequence of returns were reversed?

5. It is well known that the inflation rate was very low in recent years. Why, then, should investors be concerned with inflation-adjusted returns?

Risk

It is not sensible to talk about investment returns without talking about risk, because investment decisions involve a tradeoff between the two. Investors must constantly be aware of the risk they are assuming, understand how their investment decisions can be impacted, and be prepared for the consequences.

✓ Return and risk are opposite sides of the same coin.

Risk was defined in Chapter 1 as the chance that the actual outcome from an investment will differ from the expected outcome. Specifically, most investors are concerned that the actual outcome will be less than the expected outcome. The more variable the possible outcomes that can occur (i.e., the broader the range of possible outcomes), the greater the risk.

Investors should be willing to purchase a particular asset if the expected return is adequate to compensate for the risk, but they must understand that their expectation about the asset's return may not materialize. If not, the realized return will differ from the expected return. In fact, realized returns on securities show considerable variability—sometimes they are larger than expected, and other times they are smaller than expected, or even negative. Although investors may receive their expected returns on risky securities on a long-run average basis, they often fail to do so on a short-run basis. It is a fact of investing life that realized returns often differ from expected returns.

Investments Intuition

It is important to remember how risk and return go together when investing. An investor cannot reasonably *expect* larger returns without being willing to assume larger risks. Consider the investor who wishes to avoid any practical risk on a nominal basis. Such an investor can deposit money in an insured savings account, thereby earning a guaranteed return of a known amount. However, this return will be fixed, and the investor cannot earn more than this rate. Although risk is effectively eliminated, the chance of earning a larger return is also removed. To have the opportunity to earn a return larger than the savings account provides, investors must be willing to assume risks—and when they do so, they may gain a larger return, but they may also lose money.

SOURCES OF RISK

What makes a financial asset risky? In this text we equate risk with variability of returns. One-period rates of return fluctuate over time. Traditionally, investors have talked about several sources of total risk, such as interest rate risk and market risk, which are explained below because these terms are used so widely.

Interest Rate Risk The variability in a security's returns resulting from changes in interest rates

Market Risk The variability in a security's returns resulting from fluctuations in the aggregate market

Interest Rate Risk

The variability in a security's return resulting from changes in the level of interest rates is referred to as **interest rate risk**. Such changes generally affect securities inversely; that is, other things being equal, security prices move inversely to interest rates.[8] Interest rate risk affects bonds more directly than common stocks, but it affects both and is a very important consideration for most investors.

Market Risk

The variability in returns resulting from fluctuations in the overall market—that is, the aggregate stock market—is referred to as **market risk**. All securities are exposed to market risk, although it affects primarily common stocks.

Market risk includes a wide range of factors exogenous to securities themselves, including recessions, wars, structural changes in the economy, and changes in consumer preferences.

Inflation Risk

A factor affecting all securities is purchasing power risk, or the chance that the purchasing power of invested dollars will decline. With uncertain inflation, the real (inflation-adjusted) return involves risk even if the nominal return is safe (e.g., a Treasury bond). This risk is related to interest rate risk, since interest rates generally rise as inflation increases, because lenders demand additional inflation premiums to compensate for the loss of purchasing power.

Business Risk

The risk of doing business in a particular industry or environment is called business risk. For example, AT&T, the traditional telephone powerhouse, faces major changes today in the rapidly changing telecommunications industry.

Financial Risk

Financial risk is associated with the use of debt financing by companies. The larger the proportion of assets financed by debt (as opposed to equity), the larger the variability in the returns, other things being equal. Financial risk involves the concept of financial leverage, explained in managerial finance courses.

Liquidity Risk

Liquidity risk is the risk associated with the particular secondary market in which a security trades. An investment that can be bought or sold quickly and without significant price concession is considered liquid. The more uncertainty about the time element and the price concession, the greater the liquidity risk. A Treasury bill has little or no liquidity risk, whereas a small OTC stock may have substantial liquidity risk.

Currency Risk (Exchange Rate Risk)

All investors who invest internationally in today's increasingly global investment arena face the prospect of uncertainty in the returns after they convert the foreign gains back to their own currency. Investors today must recognize and understand exchange rate risk, which was illustrated earlier in the chapter.

As an example, a U.S. investor who buys a German stock denominated in Euros must ultimately convert the returns from this stock back to dollars. If the exchange rate has moved against the investor, losses from these exchange rate movements can partially or totally negate the original return earned.

Obviously, U.S. investors who invest only in U.S. stocks on U.S. markets do not face this risk, but in today's global environment where investors increasingly consider alternatives from other countries, currency fluctuations have become important. U.S. investors who invest in

[8] The reason for this movement is tied up with the valuation of securities and will be explained in later chapters.

such financial assets as international mutual funds, global mutual funds, closed-end single-country funds, foreign stocks, and foreign bonds can be affected by currency risk.

Country Risk Country risk, also referred to as political risk, is an important risk for investors today—probably more important now than in the past. With more investors investing internationally, both directly and indirectly, the political, and therefore economic, stability and viability of a country's economy need to be considered. The United States has one of the lowest country risks, and other countries can be judged on a relative basis using the United States as a benchmark. In today's world, countries that may require careful attention include Russia, Pakistan, Greece, Portugal, and Mexico.

Measuring Risk

We can easily calculate the average return on stocks over a period of time. Why, then, do we need to know anything else? The answer is that while the average return, however measured, is probably the most important piece of information to an investor, it tells us only the center of the data. It does not tell us anything about the spread of the data.

Risk is often associated with the dispersion in the likely outcomes. Dispersion refers to variability. Risk is assumed to arise out of variability, which is consistent with our definition of risk as the chance that the actual outcome of an investment will differ from the expected outcome. If an asset's return has no variability, in effect it has no risk. Thus, a one-year Treasury bill purchased to yield 10 percent and held to maturity will, in fact, yield (a nominal) 10 percent. No other outcome is possible, barring default by the U.S. government, which is typically not considered a likely possibility.

Consider an investor analyzing a series of returns (TRs) for the major types of financial assets over some period of years. Knowing the mean of this series is not enough; the investor also needs to know something about the variability, or dispersion, in the returns. Relative to the other assets, common stocks show the largest variability (dispersion) in returns, with small common stocks showing even greater variability. Corporate bonds have a much smaller variability and therefore a more compact distribution of returns. Of course, Treasury bills are the least risky. The dispersion of annual returns for bills is compact.

In order to appreciate the range of outcomes for major financial asset classes, consider Figure 6-2. It shows the range of outcomes, and the mean (given by the circle) for each of the following asset classes for the period 1926 through 2011, in order from left to right: inflation, Treasury bills, Treasury bonds, corporate bonds, large common stocks (S&P 500 Composite Index), and smaller common stocks. (Don't worry about the numbers and the means here—instead, concentrate on the visual range of outcomes for the different asset classes.)

As we can see from Figure 6-2, stocks have a considerably wider range of outcomes than do bonds and bills. Smaller common stocks have a much wider range of outcomes than do large common stocks. Given this variability, investors must be able to measure it as a proxy for risk. They often do so using the standard deviation.

Variance A statistical term measuring dispersion—the standard deviation squared

Standard Deviation A measure of the dispersion in outcomes around the expected value

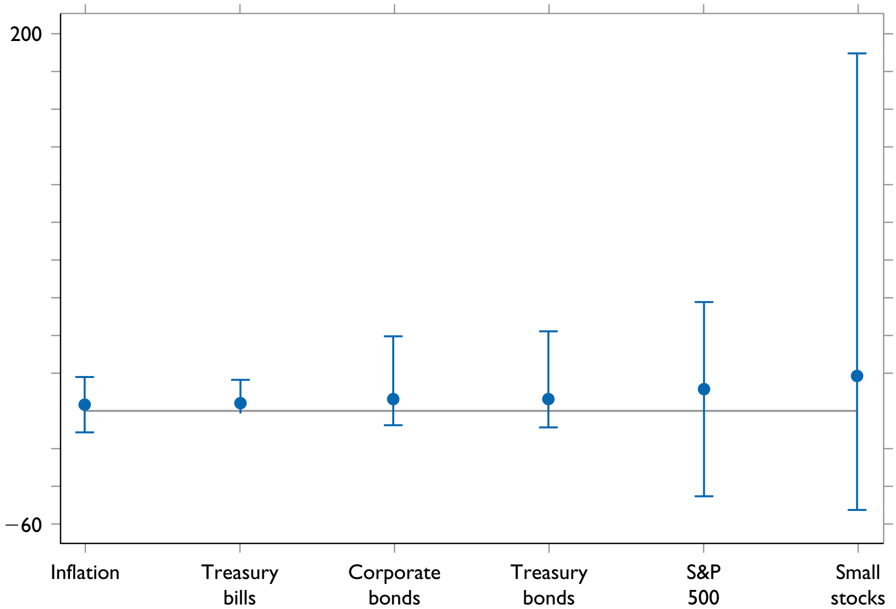
VARIANCE AND STANDARD DEVIATION

In this text we equate risk with the variability of returns—how do one-period rates of return vary over time? The risk of financial assets can be measured with an absolute measure of dispersion, or variability of returns, called the **variance**. An equivalent measure of total risk is the square root of the variance, the **standard deviation**, which measures the deviation of each observation from the arithmetic mean of the observations and is a reliable measure of

Figure 6-2

Graph of Spread in Returns for Major Asset Classes for the Period 1926–2011.

Source: Jack W. Wilson and Charles P. Jones.



variability because all the information in a sample is used.[9] The symbol σ^2 is used to denote the variance, and σ to denote the standard deviation.

The standard deviation is a measure of the total risk of an asset or a portfolio. It captures the total variability in the asset's or portfolio's return, whatever the source(s) of that variability. The standard deviation can be calculated from the variance, which is calculated as

$$\sigma^2 = \frac{\sum_{i=1}^{n} (X - \overline{X})^2}{n - 1} \tag{6-9}$$

where

σ^2 = the variance of a set of values
X = each value in the set
$\overline{X}$ = the mean of the observations
n = the number of returns in the sample
σ = $(\sigma^2)^{1/2}$ = standard deviation

Knowing the returns from the sample, we can calculate the standard deviation quite easily.

Example 6-16 The standard deviation of the 10 TRs for the decade of the 1970s, 1970–1979, for the Standard & Poor's 500 Index can be calculated as shown in Table 6-5.

[9] The variance is the standard deviation squared. The variance and the standard deviation are similar and can be used for the same purposes; specifically, in investment analysis, both are used as measures of risk. The standard deviation, however, is used more often.

Table 6-5 Calculating the Historical Standard Deviation for the Period 1970–1979

Year	TR (%), X	$X-\bar{X}$		$(X-\bar{X})^2$
1970	3.51	−3.87		14.98
1971	14.12	6.74		45.43
1972	18.72	11.34		128.6
1973	−14.50	−21.88		478.73
1974	−26.03	33.41		1116.23
1975	36.92	29.54		872.61
1976	23.64	16.26		264.39
1977	−7.16	−14.54		211.41
1978	6.39	−0.99		0.98
1979	18.19	10.81		116.86
	$\bar{X} = 7.38$		$\sum (X-\bar{X})^2$	$= 3250.22$
			σ^2	$= \frac{3250.22}{9} = 361.14$
			σ	$= (361.14)^{1/2} = 19.00\%$

In summary, the standard deviation of return measures the total risk of one security or the total risk of a portfolio of securities. The historical standard deviation can be calculated for individual securities or portfolios of securities using TRs for some specified period of time. This *ex post* value is useful in evaluating the total risk for a particular historical period and in estimating the total risk that is expected to prevail over some future period.

The standard deviation, combined with the normal distribution, can provide some useful information about the dispersion or variation in returns. For a *normal distribution*, the probability that a particular outcome will be above (or below) a specified value can be determined. With one standard deviation on either side of the arithmetic mean of the distribution, 68.3 percent of the outcomes will be encompassed; that is, there is a 68.3 percent probability that the actual outcome will be within one (plus or minus) standard deviation of the arithmetic mean. The probabilities are 95 and 99 percent that the actual outcome will be within two or three standard deviations, respectively, of the arithmetic mean.

RISK PREMIUMS

Risk Premium That part of a security's return above the risk-free rate of return

A **risk premium** is the additional return investors expect to receive, or did receive, by taking on increasing amounts of risk. It measures the payoff for taking various types of risk. Such premiums can be calculated between any two classes of securities. For example, a time premium measures the additional compensation for investing in long-term Treasuries versus Treasury bills, and a default premium measures the additional compensation for investing in risky corporate bonds versus riskless Treasury securities.

Equity Risk Premium The difference between stock returns and the risk-free rate

Defining the Equity Risk Premium

An often-discussed risk premium is the **equity risk premium**, defined as the difference between the return on stocks and a risk-free rate using Treasury securities. The equity risk premium measures the additional compensation for assuming risk, since Treasury securities have little risk of default. The equity risk premium is an important concept in finance. Note that the *historical equity risk premium* measures the difference between stock returns and Treasuries over some past period of time. When we talk about the future, however, we must consider the *expected equity risk premium* which is, of course, an unknown quantity, since it involves the future.

The equity risk premium affects several important issues and has become an often-discussed topic in Investments. The size of the risk premium is controversial, with varying estimates as to the actual risk premium in the past as well as the prospective risk premium in the future.

Calculating the Equity Risk Premium There are alternative ways to calculate the equity risk premium, involving arithmetic means, geometric means, Treasury bonds, and so forth.

It can be calculated as

1. Equities minus Treasury bills, using the arithmetic mean or the geometric mean
2. Equities minus long-term Treasury bonds, using the arithmetic mean or the geometric mean

Historically the equity risk premium (based on the S&P 500 Index) had a wide range depending upon time period and methodology, but 5–6 percent would be a reasonable average range.

The Expected Equity Risk Premium Obviously, common stock investors care whether the expected risk premium is 5 percent, or 6 percent, because that affects what they will earn on their investment in stocks. Holding interest rates constant, a narrowing of the equity risk premium implies a decline in the rate of return on stocks because the amount earned beyond the risk-free rate is reduced. A number of prominent observers have argued that the equity risk premium in the future is likely to be very different from that of the past, specifically considerably lower.

Checking Your Understanding

6. What do we mean when we say that we need to know something about the spread of the data?
7. Why do some market observers expect the equity risk premium in the future to be much lower than it has been in the past?

Realized Returns and Risks From Investing

We can now examine the returns and risks from investing in major financial assets that have occurred in the United States. We also will see how the preceding return and risk measures are typically used in presenting realized return and risk data of interest to virtually all financial market participants.

TOTAL RETURNS AND STANDARD DEVIATIONS FOR THE MAJOR FINANCIAL ASSETS

Table 6-6 shows the average annual geometric and arithmetic returns, as well as standard deviations, for major financial assets for the period 1926–2010 (85 years). Included are both nominal returns and real returns. These data are comparable to those produced and distributed by Ibbotson Associates on a commercial basis. This is an alternative series reconstructed

Table 6-6 Summary Statistics of Annual Total Returns for Major Financial Assets for 85 Years, January 1, 1926–December 31, 2010, Nominal and Inflation-Adjusted

	Arithmetic		Geometric
	Mean	Std.Dev.	Mean
Nominal Total Returns Summary			
S&P 500 Composite	11.5%	19.9%	9.6%
Aaa Corporate Bond	6.3	8.5	5.9
US Treasury Bond	5.8	9.2	5.4
Treasury bill	3.7	3.0	3.6
Inflation	3.1	4.2	3.0
Inflation-Adjusted Total Returns Summary			
S&P 500 Composite	8.3%	19.9%	6.3%
Aaa Corporate Bond	3.3	9.7	2.8
US Treasury Bond	2.7	10.3	2.3
Treasury bill	0.7	3.9	0.6

Source: Jack W. Wilson and Charles P. Jones.

by Jack Wilson and Charles Jones that provides basically the same information (but with a more comprehensive set of S&P 500 companies for the period 1926–1957).[10]

Table 6-6 indicates that large common stocks, as measured by the well-known Standard & Poor's 500 Composite Index, had a geometric mean annual return over this 85-year period of 9.6 percent (rounded). Hence, $1 invested in this market index at the *beginning* of 1926 would have grown at an average annual compound rate of 9.6 percent over this very long period. In contrast, the arithmetic mean annual return for large stocks was 11.4 percent. The best estimate of the "average" return for stocks in any one year, using only this information, would be 11.4 percent, based on the arithmetic mean, and not the 9.6 percent based on the geometric mean return. The standard deviation for large stocks for 1926–2010 was 19.9 percent.

The difference between these two means is related to the variability of the stock return series. The linkage between the geometric mean and the arithmetic mean is approximated by Equation 6-10:

$$(1 + \text{G})^2 \approx (1 + \text{A.M.})^2 - (\text{S.D.})^2 \qquad \textbf{(6-10)}$$

where

G = the geometric mean of a series of asset returns
A.M. = the arithmetic mean of a series of asset returns
S.D. = the standard deviation of the arithmetic series of returns

[10] A primary difference in the return series shown here and that of Ibbotson Associates is the return for large common stocks (S&P 500 Index). Wilson and Jones used a larger set of stocks between 1926 and 1957, whereas Ibbotson Associates used 90 large stocks. Large stocks did better during this period than did stocks in general, resulting in a larger geometric mean for the Ibbotson Associates data. Wilson and Jones believe that the S&P series used here, which was laboriously reconstructed after the Ibbotson Associates series was put together and used, is a more complete representation of the S&P 500 Index because of this difference.

Example 6-17 Using data with more decimal places than Table 6-6 for 1926–2010 for the S&P 500 Index:

$$(1.0957)^2 \approx (1.1152)^2 - (0.1992)^2$$
$$1.201 \approx 1.2437 - 0.0397$$
$$1.201 \approx 1.204$$

✓ If we know the arithmetic mean of a series of asset returns and the standard deviation of the series, we can approximate the geometric mean for this series. As the standard deviation of the series increases, holding the arithmetic mean constant, the geometric mean decreases.

Although not shown in Table 6-6, smaller common stocks have greater returns and greater risk relative to large common stocks. "Smaller" here means the smallest stocks on the NYSE and not the really small stocks traded on the over-the-counter market. The arithmetic mean for this series is much higher than for the S&P 500 Index, typically 5 or 6 percentage points.[11] However, because of the much larger standard deviation for smaller common stocks, roughly 30 percent, the geometric mean is considerably less than that, typically around 2 percentage points more than the geometric mean for large common stocks (the framework of Equation 6-10 explains why there is a large difference between the two means). Small common stocks have by far the largest variability of any of the returns series considered in Table 6-6.

Corporate and Treasury bonds had geometric means that were roughly 50 to 60 percent of the S&P 500 Composite Index, at 5.9 and 5.4 percent, respectively, but the risk was considerably smaller. Standard deviations for the bond series were less than half as large as that for the S&P 500 Composite.[12]

Finally, as we would expect, Treasury bills had the smallest returns of any of the major assets shown in Table 6-6, 3.6 percent, as well as the smallest standard deviation by far.

The deviations for each of the major financial assets in Table 6-6 reflect the dispersion of the returns over the 85-year period covered in the data. The standard deviations clearly show the wide dispersion in the returns from common stocks compared with bonds and Treasury bills. Furthermore, smaller common stocks can logically be expected to be riskier than the S&P 500 stocks, and the standard deviation indicates a much wider dispersion.

CUMULATIVE WEALTH INDEXES

Figure 6-3 shows the CWIs for the major financial assets and the corresponding index number for inflation from the data in Table 6-6. The series starts at the beginning of 1926 and shows the cumulative results of starting with $1 in each of these series and going through the end of 2010. Note that the vertical axis of Figure 6-3 is a log scale.[13]

[11] The data for small common stocks is omitted because of the difficulty in getting one series that spans the entire time period, determining the yield for the series, and because of some unusual values in the 1930s exceeding 100 percent.

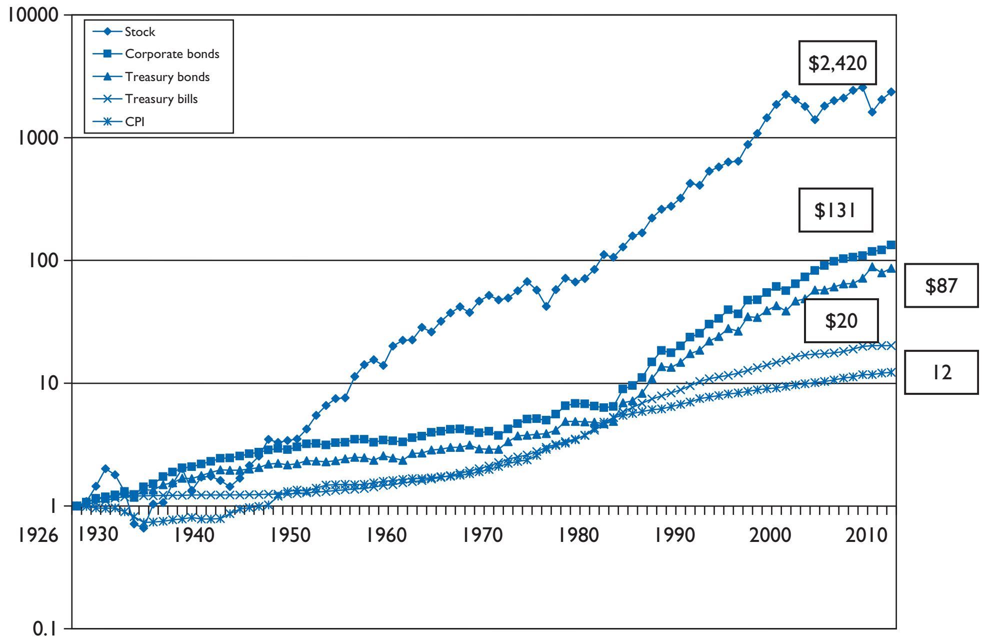
[12] The reason for the distribution of Treasury bonds and Treasury bills, which have no practical risk of default, is that this is a distribution of annual returns, where negative numbers are possible. Thus, a new 10-year Treasury bond purchased at $1,000 on January 1 could decline to, say, $900 by December 31, resulting in a negative TR for that year. For the full 10-year period, its return will, of course, be positive since it will be redeemed for $1,000.

[13] A logarithmic scale greatly facilitates comparisons of different series across time because the same vertical distance represents the same percentage change in a particular series return. The logarithmic scale allows the user to concentrate on rates of return and ignore the dollar amounts involved.

Figure 6-3

Cumulative Wealth for Major Asset Classes and Cumulative Inflation (Amounts are Rounded).

SOURCE: Jack W. Wilson and Charles P. Jones.



As Figure 6-3 shows, the cumulative wealth for stocks, as measured by the S&P 500 Composite Index, completely dominated the returns on corporate bonds over this period—\$2,420.46 versus \$130.66. Note that we use the geometric mean from Table 6-6 to calculate cumulative ending wealth for each of the series shown in Figure 6-3 by raising (1 + the geometric mean stated as a decimal) to the power represented by the number of periods, which in this case is 85.

Example 6-18 The ending wealth value of \$2,420.46 for common stocks in Figure 6-3 is the result of compounding at 9.6 percent for 85 years, or

$$CWI = WI_0(1.096)^{85} = \$1.00(2{,}420.46) = \$2{,}420.46$$

The large CWI value for stocks shown in Figure 6-3 speaks for itself. Remember, however, that the variability of this series is considerably larger than that for bonds or Treasury bills, as shown by the standard deviations in Table 6-6.[14]

[14] On an inflation-adjusted basis, the cumulative ending wealth for any of the series can be calculated as

$$CWI_{IA} = \frac{CWI}{CI_{INF}}$$

where

CWI_{IA} = the cumulative wealth index value for any asset on inflation – adjusted basis
CWI = the cumulative wealth index value for any asset on a nominal basis
CI_{INF} = the ending index value for inflation, calculated as $(1 + \text{geometric rate of inflation})^n$, where n is the number of periods considered

UNDERSTANDING CUMULATIVE WEALTH AS INVESTORS

All of us appreciate dollar totals, and the larger the final wealth accumulated over some period of time, the better. If we fully understand how cumulative wealth comes about from financial assets, in particular stocks, we have a better chance of enhancing that wealth. The CWI can be decomposed into the two components of Total Return: the dividend component and the price change component. To obtain TR, we add these two components, but for Cumulative Wealth, we multiply these two components together. Thus,

$$\text{CWI} = \text{CDY} \times \text{CPC} \tag{6-11}$$

where

CWI = the cumulative wealth index or total return index for a series
CDY = the cumulative dividend yield component of total return
CPC = the cumulative price change component of total return

Conversely, to solve for either of the two components, we divide the CWI by the other component, as in Equation 6-12.

$$\text{CPC} = \frac{\text{CWI}}{\text{CDY}} \tag{6-12}$$

$$\text{CDY} = \frac{\text{CWI}}{\text{CPC}} \tag{6-13}$$

Example 6-19 The CWI for common stocks (S&P 500) for 1926–2010 (85 years) was $2,364.78, based on a geometric mean of 9.57 percent for that period (rounded to 9.6 percent in Table 6-6). The average dividend yield for those 85 years was 3.99 percent. Raising 1.0399 to the 85th power, the cumulative dividend yield, CDY, was $27.82. Therefore, the cumulative price change index for the S&P 500 for that period was $2,364.78/$27.82, or $85.00. This is a compound annual average return of

$$(\$85.00)^{1/85} - 1.0 = .0537 \text{ or } 5.37\%$$

Note that the annual average geometric mean return relative for common stocks is the product of the corresponding geometric mean return relatives for the two components:

For 1926–2010, an 85-year period,

$$G_{DY} \times G_{PC} = G_{TR} \tag{6-14}$$

$$1.0399 \times 1.0537 = 1.0957$$
$$1.0957 - 1.0 = .0957 \text{ or } 9.57\% \text{ (rounded to 9.6\% in Table 6-6)}$$

It is important to understand that for the S&P 500 stocks historically, which many investors hold as an index fund or EFT, dividends played an important role in the overall compound rate of return that was achieved. As we can see, almost 42 percent (3.99%/9.57%) of the TR from large stocks over that long period of time was attributable to dividends.

Dividend yields on the S&P 500 have been low for the past several years, roughly half their historical level. This means that either investors will have to earn more from the price change component of cumulative wealth, or their cumulative wealth will be lower than it was in the past because of the lower dividend yield component.

Compounding and Discounting

Of course, the single most striking feature of Figure 6-3 is the tremendous difference in ending wealth between stocks and bonds. This difference reflects the impact of compounding substantially different mean returns over long periods, which produces almost unbelievable results. The use of compounding points out the importance of this concept and of its complement, discounting. Both are important in investment analysis and are used often. *Compounding* involves future value resulting from compound interest—earning interest on interest. As we saw, the calculation of wealth indexes involves compounding at the geometric mean return over some historical period.

Present value (discounting) is the value today of a dollar to be received in the future. Such dollars are not comparable, because of the time value of money. In order to be comparable, they must be discounted back to the present. Present value concepts are used extensively in Chapters 10 and 17, and in other chapters as needed.

Summary

- Return and risk go together in investments; indeed, these two parameters are the underlying basis of the subject. Everything an investor does, or is concerned with, is tied directly or indirectly to return and risk.
- The term *return* can be used in different ways. It is important to distinguish between realized (ex post, or historical) return and expected (ex ante, or anticipated) return.
- The two components of return are yield and price change (capital gain or loss).
- The total return is a decimal or percentage return concept that can be used to correctly measure the return for any security.
- The return relative, which adds 1.0 to the total return, is used when calculating the geometric mean of a series of returns.
- The cumulative wealth index (total return index) is used to measure the cumulative wealth over time given some initial starting wealth—typically, $1—and a series of returns for some asset.
- Return relatives, along with the beginning and ending values of the foreign currency, can be used to convert the return on a foreign investment into a domestic return.
- The geometric mean measures the compound rate of return over time. The arithmetic mean, on the other hand, is simply the average return for a series and is used to measure the typical performance for a single period.
- Inflation-adjusted returns can be calculated by dividing 1 + the nominal return by 1 + the inflation rate as measured by the CPI.
- Risk is the other side of the coin: risk and expected return should always be considered together. An investor cannot reasonably expect to earn large returns without assuming greater risks.
- The primary components of risk have traditionally been categorized into interest rate, market, inflation, business, financial, and liquidity risks. Investors today must also consider exchange rate risk and country

risk. Each security has its own sources of risk, which we will discuss when we discuss the security itself.

- Historical returns can be described in terms of a frequency distribution and their variability measured by use of the standard deviation.
- The standard deviation provides useful information about the distribution of returns and aids investors in assessing the possible outcomes of an investment.
- Common stocks over the period 1926–2010 had an annualized geometric mean total return of 9.6 percent, compared to 5.4 percent for long-term Treasury bonds.
- Over the period 1920–2010, common stocks had a standard deviation of returns of approximately 19.9 percent, about two and one-half times that of long-term government and corporate bonds and about six times that of Treasury bills.

Questions

6-1 Distinguish between historical return and expected return.

6-2 How long must an asset be held to calculate a TR?

6-3 Define the components of TR. Can any of these components be negative?

6-4 Distinguish between TR and holding period return.

6-5 When should the geometric mean return be used to measure returns? Why will it always be less than the arithmetic mean (unless the numbers are identical)?

6-6 When should the arithmetic mean be used in describing stock returns?

6-7 What is the mathematical linkage between the arithmetic mean and the geometric mean for a set of security returns?

6-8 What is an equity risk premium?

6-9 According to Table 6-6, common stocks have generally returned more than bonds. How, then, can they be considered more risky?

6-10 Distinguish between market risk and business risk. How is interest rate risk related to inflation risk?

6-11 Classify the traditional sources of risk as to whether they are general sources of risk or specific sources of risk.

6-12 Explain what is meant by country risk. How would you evaluate the country risk of Canada and Mexico?

6-13 Assume that you purchase a stock on a Japanese market, denominated in yen. During the period you hold the stock, the yen weakens relative to the dollar. Assume you sell at a profit on the Japanese market. How will your return, when converted to dollars, be affected?

6-14 Define risk. How does use of the standard deviation as a measure of risk relate to this definition of risk?

6-15 Explain verbally the relationship between the geometric mean and a CWI.

6-16 As Table 6-6 shows, the geometric mean return for stocks over a long period has been 9.6 percent. Thus, on average, on a compound basis, stocks have averaged a 9.6 percent TR per year over a long time period. Should investors be surprised if they hold stocks for a 10-year period, or a 15-year period, and earn an average return of only 1 or 2 percent?

6-17 Explain how the geometric mean annual average inflation rate can be used to calculate inflation-adjusted stock returns over the period 1926–2010.

6-18 Explain the two components of the CWI for common stocks. Assume we know one of these two components. How can the other be calculated?

6-19 Common stocks have returned slightly less than twice the compound annual rate of return for corporate bonds. Does this mean that common stocks are about twice as risky as corporates?

6-20 Can cumulative wealth be stated on an inflation-adjusted basis?

6-21 Over the long run, stocks have returned a lot more than bonds, given the compounding effect? Why, then, do investors buy bonds?

6-22 Given the strong performance of stocks over the last 85 years, do you think it is possible for stocks to show an average negative return over a 10-year period?

6-23 Is there a case to be made for an investor to hold only Treasury bills over a long period of time?

6-24 How can we calculate the returns from holding gold?

6-25 Don't worry too much if your retirement funds earn 5.5 percent over the next 40 years instead of 6 percent. It won't affect final wealth very much. Evaluate this claim.

6-26 Suppose someone promises to double your money in 10 years. What rate of return are they implicitly promising you?

6-27 A technical analyst claimed in the popular press to have earned 25 percent a month for 10 years using his technical analysis technique. Is this claim feasible?

6-28 Which alternative would you prefer: (a) 1 percent a month, compounded monthly, or (2) ½ percent a month, compounded semimonthly (24 periods)?

Demonstration Problems

6-1 Calculation of Arithmetic Mean and Geometric Mean: Data for Extell Corp.

Year (t)	(1) End-of-Year Price (P_t)	(2) Calendar-Year Dividends (D_t)	TR%
2007	$ 74.60	$2.88	--
2008	64.30	3.44	−9.2%
2009	67.70	3.44	10.6%
2010	56.70	3.44	−11.2%
2011	96.25	3.44	75.8%
2012	122.00	3.71	30.6%

The arithmetic mean of the TR for Extell, 2008–2012:

$$\frac{\sum(\text{TR\%})}{n} = \frac{96.6}{5} = 19.32\%$$

To calculate the *geometric* mean in this example, convert the TRs to decimals, and add 1.0, producing return relatives, RR. The GM is the fifth root of the product of the RRs.

Year	TR%	TR decimal	RR
2008	−9.2%	−0.092	0.908
2009	10.6%	0.106	1.106
2010	−11.2%	−0.112	0.888
2011	75.8%	0.758	1.758
2012	−30.6%	0.306	1.306

The geometric mean is $GM = [(1 + RR_1)(1 + RR_2)\ldots(1 + RR_n)]^{1/n} - 1$. Therefore, take the fifth root of the product

$$(0.908)(1.106)(0.888)(1.758)(1.306) = 2.0474, \text{ and } (2.0474)^{1/5} = 1.154; 1.154 - 1.0 = .154 \text{ or } 15.4\% \text{ TR}$$

6-2 **The Effects of Reinvesting Returns:** The difference in meaning of the arithmetic and geometric mean, holding Extell stock over the period January 1, 2008 through December 31, 2012 for two different investment strategies, is as follows:

Strategy A—keep a fixed amount (say, $1,000) invested and do not reinvest returns.
Strategy B—reinvest returns and allow compounding.

First, take Extell's TRs and convert them to decimal form (r) for Strategy A, and then to $(1 + r)$ form for Strategy B.

Strategy A				Strategy B			
Jan. 1 Year	Amt. Inv. ×	r_t =	Return	Jan. 1 Year	Amt. Inv. ×	$(1 + r_t)$ =	Terminal Amt.
2008	$1000	−0.092	−$92.00	2008	$1000	0.908	$908.00
2009	1000	0.106	106.00	2009	908.0	1.106	1004.25
2010	1000	−0.112	−112.00	2010	1004.25	0.888	891.77
2011	1000	0.758	758.00	2011	891.77	1.758	1567.74
2012	1000	0.306	306.00	2012	1567.74	1.306	2047.46

Using Strategy A, keeping $1,000 invested at the beginning of the year, TRs for the years 2008–2012 were $966, or $193.20 per year average ($966/5), which on a $1,000 investment is $193.20/1000 = 0.1932, or 19.32 percent per year—the same value as the arithmetic mean in Demonstration Problem 6-1 earlier.

Using Strategy B, compounding gains and losses, TR was $1,047.46 (the terminal amount $2,047.46 minus the initial $1,000). The average annual rate of return in this situation can be found by taking the nth root of the terminal/initial amount:

$$[2047.46/1000]^{1/5} = (2.0474)^{1/5} = 1.1541 = (1 + r), r\% = 15.41\%$$

which is exactly the set of values we ended up with in Demonstration Problem 6-1 when calculating the geometric mean.

6-3 **Calculating the Standard Deviation:** Using the TR values for Extell for the years 2008–2012, we can illustrate the deviation of the values from the mean.

The numerator for the formula for the variance of these Y_t values is $\sum (Y_t\text{-}Y)^2$, which we will call SS_y, the sum of the squared deviations of the Y_t around the mean. Algebraically, there is a simpler alternative formula.

$$SS_y = \sum (Y_t - \overline{Y})^2 = \sum Y^2{}_t - \frac{(\sum Y_t)^2}{n}$$

Using Extell's annual total returns, we will calculate the SS_y both ways.

Year	Y_t=TR	$(Y_t - \overline{Y})$	$(Y_t - \overline{Y})^2$	Y_t^2
2008	−9.2%	28.52	813.3904	84.64
2009	10.6%	−8.72	76.0384	112.36
2010	−11.2%	−30.52	931.4704	125.44
2011	75.8%	56.48	3189.9904	5745.64
2012	30.6%	11.28	127.2384	936.36
Sum	96.6%	−0−	5138.1280	7004.44

$$\overline{Y} = 19.32\%$$

$$SS_y = \sum (Y_t - \overline{Y})^2 = 5138.128, \text{ and also}$$

$$SS_y = \sum Y^2 - \frac{(\sum Y_t)^2}{n} = 7004.44 - \frac{(96.6)^2}{5} = 5138.128$$

The variance is the "average" squared deviation from the mean:

$$\sigma^2 = \frac{SS_y}{(n-1)} = \frac{5138.128}{4} = 1284.532 \text{ "squared percent"}$$

The standard deviation is the square root of the variance:

$$\sigma = \left(\sigma^2\right)^{1/2} = (1284.532)^{1/2} = 35.84\%$$

The standard deviation is in the same units of measurement as the original observations, as is the arithmetic mean.

6-4 **Calculation of Cumulative Wealth Index and Geometric Mean:** By using the geometric mean annual average rate of return for a particular financial asset, the CWI can be found by converting the TR on a geometric mean basis to a return relative, and raising this return relative to the power representing the number of years involved. Consider the geometric mean of 12.47 percent for small common stocks for the period 1926–2007. The CWI, using a starting index value of \$1, is (note the 82 periods):

$$\$1(1.1247)^{82} = \$15{,}311.19$$

Conversely, if we know the CWI value, we can solve for the geometric mean by taking the *n*th root and subtracting out 1.0.

$$(\$15{,}311.19)^{1/82} - 1.0 = 1.1247 - 1.0 = .1247 \text{ or } 12.47\%$$

note : number of years to use = [ending year-beginning year] + 1

6-5 **Calculation of Inflation-Adjusted Returns:** Knowing the geometric mean for inflation for some time period, we can add 1.0 and raise it to the *n*th power. We then divide the CWI on a nominal basis by the ending value for inflation to obtain inflation-adjusted returns. For example, given a CWI of \$2,364.78 for common stocks for 1926–2010, and a geometric mean inflation rate of 3.00 percent, the inflation-adjusted cumulative wealth index for this 85-year period is calculated as

$$\$2364.78/(1.03)^{85} = \$2364.78/12.336 = \$191.70$$

6-6 **Analyzing the Components of a Cumulative Wealth Index:** Assume that we know that for the period 1926–2010 the yield component for common stocks was 3.99 percent, and that the CWI was $2,364.78. The CWI value for the yield component was

$$(1.0399)^{85} = 27.82$$

The CWI value for the price change component was

$$\$2{,}364.78/27.82 = 85.00$$

The geometric mean annual average rate of return for the price change component for common stocks was

$$(85.00)^{1/85} = 1.0537$$

The geometric mean for common stocks is linked to its components by the following:

$$1.0399(1.0537) = 1.0957; 1.0957 - 1.0 = .0957 \text{ or } 9.57\%$$

The CWI can be found by multiplying together the individual component CWIs:

$$\$85.00(\$27.82) = \$2{,}364.70^{*}$$

(*rounding accounts for any differences)

Problems

6-1 Calculate the TR and the RR for the following assets:

a. A preferred stock bought for $70 per share, held one year during which $5 per share dividends are collected, and sold for $62

b. A warrant bought for $10 and sold three months later for $13

c. A 12 percent bond bought for $830, held two years during which interest is collected, and sold for $920

6-2 Calculate, using a calculator, the arithmetic and geometric mean rate of return for the Standard & Poor 500 Composite Index (Table 6-1) for the years 2001–2004. How does this change when 2005 is included?

6-3 Calculate the index value for the S&P 500 (Table 6-1) assuming a $1 investment at the beginning of 1990 and extending through the end of 1999. Using only these index values, calculate the geometric mean for these years.

6-4 Assume that one of your relatives, on your behalf, invested $50,000 in a trust holding S&P 500 stocks at the beginning of 1926. Using the data in Table 6-6, determine the value of this trust at the end of 2010.

6-5 Now assume that your relative had invested $50,000 in a trust holding "small stocks" at the beginning of 1926. Determine the value of this trust at the end of 2010.

6-6 What if your relative had invested $50,000 in a trust holding long-term Treasury bonds at the beginning of 1926. Determine the value of this trust at the end of 2010.

6-7 Finally, what if this relative had invested $50,000 in a trust holding Treasury bills at the beginning of 1926. Determine the value of this trust by the end of 2010.

6-8 Calculate cumulative wealth for corporate bonds for the period 1926–2010, using a geometric mean of 5.9 percent (85-year period).

6-9 Given a CWI for Treasury bills of $20.21 for the period 1926–2010, calculate the geometric mean.

6-10 Given an inflation rate of 3 percent over the period 1926–2010 (geometric mean annual average), calculate the inflation-adjusted CWI for corporate bonds as of year-end 2010.

6-11 Given a geometric mean inflation rate of 4 percent, determine how long it would take to cut the purchasing power of money in half using the rule of 72.

6-12 If a basket of consumer goods cost $1 at the beginning of 1926 and $12.34 at the end of 2010, calculate the geometric mean rate of inflation over this period.

6-13 Assume that over the period 1926–2010 the yield index component of common stocks had a geometric mean annual average of 3.99 percent. Calculate the CWI for this component as of year-end 2010. Using this value, calculate the CWI for the price change component of common stocks using information in Figure 6-3.

6-14 Assume that Treasury bonds continued to have a geometric mean as shown in Table 6-6 until 100 years have elapsed. Calculate the cumulative ending wealth per $1 invested for this 100-year period.

6-15 Assume that over the period 1926–2010 the geometric mean rate of return for Treasury bonds was 5.4 percent. The corresponding number for the rate of inflation was 3 percent. Calculate, two different ways, the CWI for government bonds for the period, on an inflation-adjusted basis.

6-16 Using the TRs for the years 1926–1931 from Table 6-1, determine the geometric mean for this period. Show how the same result can be obtained from the ending wealth index value for 1931 of 0.7405.

6-17 Using data for three periods, construct a set of TRs that will produce a geometric mean equal to the arithmetic mean.

6-18 According to Table 6-6, the standard deviation for all common stocks for the period 1926–2010 was 19.9 percent. Using data from Table 6-1, calculate the standard deviation for the years 1981–1991 and compare your results.

6-19 Someone offers you a choice between $50,000 to be received 10 years from now, or a $20,000 portfolio of stocks guaranteed to earn a compound annual average rate of return of 8 percent per year for the next 10 years. Determine the better alternative based solely on this information

Computational Problems

6-1 Assume that we know the performance of the S&P 500 Index for the first five years of the second decade of the 21st century, defined as 2010–2019. What annual geometric mean must the market average for the last five years (2015–2019) to produce an overall geometric mean for the decade of 10 percent? The TRs for the first 5 years are 2010, 15.06 percent; 2011, −9.9 percent; 2012, 16.1 percent; 2013, −19.7 percent; and 2014, 10.7 percent.

6-2 Using the five years of TRs in 6−1, assume that one of the five years during the second half of the decade, 2015−2019, shows a loss of 15 percent. What would the geometric mean of the remaining four years have to be for the decade as a whole to average the 10 percent return for the S&P 500 Index?

6-3 The geometric mean for the TR for the S&P 500 Index for the period 1926−2010 was 9.57 percent. Assume that the geometric mean for the yield component of the TR on the S&P 500 for the period 1926−2010 was 3.99 percent. What is the cumulative wealth for the other component of the CWI for the S&P 500 for the period 1926−2010?

6-4 Suppose you know that cumulative inflation for the period 1926−2010 was 12.34. You also know that the geometric mean for Treasury bills for this period was 3.6 percent. What was the real return for Treasury bills for the period 1926−2010?

6-5 Based on some calculations you have done, you know that the cumulative wealth for corporate bonds for the period 2008−2012 was $1.234. However, you have misplaced the return for 2010. The other four returns are: 9.3 percent, −6.2 percent, 12.1 percent, and 7.4 percent. What is the return for 2010, based on this information? (Use 3 decimal places.)

Spreadsheet Exercises

6-1 Warren Buffett, arguably the most famous investor in the United States, is the CEO of Berkshire Hathaway (BRK), a company that has enjoyed great success in terms of its stock price. Below are the actual year-end stock prices for BRK-A from 1965 through 2011. (Yes, these are the actual stock prices, believe it or not.)

a. Calculate the RRs for each year starting in 1966. Use two decimal places.

b. Calculate the arithmetic mean and geometric mean for these price relatives, using three decimal places.

c. Calculate the CWI for 1966−2011, assuming an initial investment of $1,000, and $10,000. State the answers without decimal places.

Year-End Price	
1965	$16.25
1966	17.50
1967	20.25
1968	37.00
1969	42.00
1970	39.00
1971	70.00
1972	80.00
1973	71.00
1974	40.00
1975	38.00

Year-End Price	
1976	94.00
1977	138.00
1978	157.00
1979	320.00
1980	425.00
1981	560.00
1982	775.00
1983	1,310.00
1984	1,275.00
1985	2,470.00
1986	2,820.00

Year-End Price		Year-End Price	
1987	2,950.00	2000	71,000.00
1988	4,700.00	2001	75,600.00
1989	8,675.00	2002	72,750.00
1990	6,675.00	2003	84,250.00
1991	9,050.00	2004	87,900.00
1992	11,750.00	2005	88,620.00
1993	16,325.00	2006	109,990.00
1994	20,400.00	2007	141,600.00
1995	32,100.00	2008	96,600.00
1996	34,100.00	2009	99,200.00
1997	46,000.00	2010	120,450.00
1998	70,000.00	2011	114,755.00
1999	56,100.00		

(*Continued*)

6-2 **a.** Using the spreadsheet data in 6–1, and spreadsheet formulas, determine the compound annual average rate of return for Berkshire Hathaway for the last 10 years and the last five years.

b. What was the percentage rate of return on this stock in 2008?

6-3 The following data for Coca-Cola (ticker symbol = KO) are the December ending prices (adjusted for stock splits and dividends) and the annual dividend. This information can be obtained from a source such as *Yahoo! Finance*. Place these data in a spreadsheet for columns A-C. Use 3 decimal places and calculate results in decimal form (not percentages). You will need the 2000 price to calculate the 2001 return.
For each year 2001–2010:

a. Calculate as column D the RR for the price change only.

b. Calculate as column E the TR based on price change only.

c. Calculate as column F the RR based on price change and dividends.

d. Calculate as column G the TR based on price change and dividends.

e. Calculate the arithmetic and geometric means for 2001–2010 for price change only and for TR.

f. Calculate the ending wealth as of December 31, 2010, based on TRs, for $1 invested in Coca-Cola stock at the beginning of 2001.

g. Calculate the standard deviation of the TRs for the years 2001–2010. (Note: use the total returns and not the return relatives.)

Coca-Cola ending prices and dividends

2010 $64.39, $1.76; 2009 $54.10, $1.64; 2008 $41.52, $1.52; 2007 $54.70, $1.36; 2006 $41.93, $1.24
2005 $34.06, $1.12; 2004 $34.29, $1,00; 2003 $40.88, $0.88; 2002 $34.61, $0.80; 2001 $36.62, $0.72, 2000 $46.80

Checking Your Understanding

6-1 Disagree. The Cumulative Wealth Index can be calculated for nominal stock returns or inflation-adjusted (real) returns, just as Total Returns and Return Relatives can be used on either a nominal or real basis.

6-2 When an investor buys a foreign asset, he or she is, in effect, selling dollars to obtain the foreign currency needed to buy the security. When this security is sold in the foreign market by a U.S. investor, the proceeds will need to be converted back to dollars.

6-3 Yes. The dollar declined against many foreign currencies in the first few years of the 21st century. This increased the returns to U.S. investors from investing in foreign countries. Investors often viewed this approach as a bet against the dollar.

6-4 This investment showed a loss on a geometric mean basis even though the arithmetic mean is zero. It is calculated as $1000*1.15*.85 = $977.50. The sequence of returns does not matter.

6-5 The long-term financial history of the United States shows that inflation is an issue over a period of many years. While it was very low in recent years, it is expected to be higher in the future. Even at an average inflation rate of 3 percent a year, the purchasing power of money will be cut in half in approximately 24 years. Many retirees will live this long after retiring.

6-6 The spread of the data tells us something about the risk involved. How likely is the average, or mean, to be realized?

6-7 A number of market observers expect equity returns to be lower in the future because dividend yields are currently about half what they were for many years, on average. Other things equal, equity returns will be reduced (unless price appreciation makes up the difference) and, unless interest rates decline, equity risk premiums will be lower.

NOTE: Answers to the three-stock example at the beginning of the chapter using 2 decimal places:

Stock 2 has the largest geometric mean and would produce the greatest ending wealth. It also has the lowest risk by far based on calculating standard deviations. Stocks 1 and 3 have identical geometric means and would produce identical ending wealths.

−0.1	0.04	0.4	0.9	1.04	1.4
−0.2	0.05	−0.02	0.8	1.05	0.98
0.29	0.07	−0.1	1.29	1.07	0.9
0.19	0.06	−0.15	1.19	1.06	0.85
0.12	0.09	0.17	1.12	1.09	1.17
0.204	0.019	0.226 Std Dev	1.04	1.06	1.04 G.M.
0.060	0.062	0.060 A.M.			

chapter 7

Portfolio Theory is Universal

Everyone keeps telling you that with $1 million to invest, you can have a nice portfolio of securities. And then the thought occurs: what is a portfolio exactly? You begin to wonder if a portfolio has characteristics you need to consider, because you can recall someone talking about portfolio theory, and what it means to investors. You have heard people say *don't put all of your eggs in one basket*, so how does that apply to investing? If you decide to put the entire $1 million in two stocks, what will the trustee say? It is not difficult to pick up a popular press article on investing and read about the importance of diversification, or hear about how some stocks seem to react negatively to threats of rising inflation while others seem to respond positively. Therefore, it seems like a good idea to expose yourself to at least the basics of portfolio theory. Then you will not be intimidated when someone starts talking about Markowitz portfolio theory, a universal concept in today's global investing world that is widely known and discussed.

In this chapter, we outline the nature of risk and return as it applies to making investment decisions. Unlike Chapter 6, we are talking about the future—which involves *expected returns*, and not the past—which involves *realized returns*. Investors must estimate and manage the returns and risk from their investments. They reduce risk to the extent possible without affecting returns by building diversified portfolios. Therefore, we must be concerned with the investor's total portfolio and analyze investment risk accordingly. As we shall see, *diversification is the key to effective risk management*. We will consider the critically important principle of Markowitz diversification, focusing primarily on the concepts of the correlation coefficient and covariance as applied to security returns.

AFTER READING THIS CHAPTER YOU WILL BE ABLE TO:

- Understand the meaning and calculation of expected return and risk measures for an individual security.
- Recognize what it means to talk about modern portfolio theory.
- Calculate portfolio return and risk measures as formulated by Markowitz.
- Understand how diversification works.

Dealing With Uncertainty

In Chapter 6 we discussed the average returns, both arithmetic and geometric, that investors have experienced over the years from investing in the major financial assets available to them. We also considered the risk of these asset returns as measured by the standard deviation. Analysts often refer to the realized returns for a security, or class of securities, over time using these measures as well as other measures such as the cumulative wealth index.

Realized returns are important for several reasons. For example, investors need to know how their portfolios have performed relative to relevant market indexes. Realized returns are also important in helping investors to form expectations about future returns by providing a foundation upon which to make estimates of expected returns. For example, if over a long period Treasury bills have averaged less than 4 percent on a geometric mean basis, it would be unrealistic to expect long-run average compound returns of 6 or 7 percent in the future from Treasury bills unless the investing environment has changed significantly.

How do we go about estimating returns, which is what investors must actually do in managing their portfolios? First of all, note that we will use the return and risk measures developed in Chapter 6. The total return measure, TR, is applicable whether one is measuring realized returns or estimating future (expected) returns. Because it includes everything the investor can expect to receive over any specified future period, the TR is useful in conceptualizing the estimated returns from securities.

Similarly, the variance, or its square root, the standard deviation, is an accepted measure of variability for both realized returns and expected returns. We will calculate both the variance and the standard deviation below and use them interchangeably as the situation dictates. Sometimes it is preferable to use one, and sometimes the other.

To estimate the returns from various securities, investors must estimate the cash flows these securities are likely to provide. The basis for doing so for bonds and stocks will be covered in their respective chapters. For now it is sufficient to remind ourselves of the uncertainty of estimates of the future, a problem emphasized at the outset of Chapter 1.

Box 7-1 is an interesting discussion of risk, and how best to understand it. In this essay Peter Bernstein, one of the most prominent observers of the investing environment over many years, argues that risky decisions are all about three elements. His second point, “expect the unexpected,” turned out to be particularly relevant given what happened to the financial system in 2008. The unexpected did occur, and very few were prepared to deal with the situation. The resulting damage has been enormous.

USING PROBABILITIES

The return an investor will earn from investing is not known; it must be estimated. Future return is an *expected* return and may or may not actually be realized. An investor may expect the TR on a particular security to be 0.10 for the coming year, but in truth this is only a “point estimate.” Risk, or the chance that some unfavorable event will occur, is involved when investment decisions are made. Investors are often overly optimistic about expected returns. We can use the term *random variable* to describe the one-period rate of return from a stock (or bond)—it has an uncertain value which fluctuates randomly.

To deal with the uncertainty of returns, investors need to think explicitly about a security’s distribution of probable TRs. In other words, while investors may expect a security to return 10 percent, for example, this is only a one-point estimate of the entire range of possibilities. Given that investors must deal with the uncertain future, a number of possible returns can, and will, occur.

BOX 7-1

Risk: The Whole Versus the Parts

Many years ago, in the middle of a staff meeting, a colleague passed me a scrap of paper on which he had written, "When all is said and done, more things are said than done." When I consider the plethora of books, articles, consultants, and conferences on risk in today's world, my friend's aphorism has never seemed more appropriate. Are we never going to nail risk down and bring it under control? How much more can anyone reveal to us beyond what we have already been told?

In a very real sense, this flood of material about risk is inherently risky. Sorting out the pieces and searching for main themes has become an escalating challenge. The root of the matter gets lost in the shuffle while we are analyzing all the elegant advances in risk measurement and the impressive broadening of the kinds of risks we seek to manage. More is said than is done, or what is done loses touch with what has been said.

If we go back to first principles for a moment, perhaps we can put the multifarious individual pieces into some kind of a larger framework and optimize the choices among the masses of information we are attempting to master.

Professor Elroy Dimson of the London Business School once said risk means more things can happen than will happen. Dimson's formulation is only a fancy way of saying that we do not know what is going to happen—good or bad. Even the range of possible outcomes remains indeterminate, much as we would like to nail it down. Remember always: Risk is not about uncertainty but about the unknown, the inescapable darkness of the future.

If more things can happen than will happen, and if we are denied precise knowledge of the range of possible outcomes, *some decisions we make are going to be wrong*. How many, how often, how seriously? We have no way of knowing even that. Even the most elegant model, as Leibniz reminded Jacob Bernouilli in 1703, is going to work "only for the most part." What lurks in the smaller part is hidden from us, but it could turn into a load of dynamite.

The beginning of wisdom in life is in accepting the inevitability of being wrong on occasion. Or, to turn that phrase around, the greatest risks we take are those where we are certain of the outcome—as masses of people are at classic market bottoms and tops. My investment philosophy has always been that victory in the long run accrues to the humble rather than to the bold.

This emphasis on ignorance is the necessary first step toward the larger framework we need if we hope to sort out the flood of information about risk that assails us. Now we can break down the problem of risk into what appear to me to be its three primary constituent parts.

First, what is the balance between the consequences of being wrong and the probability of being wrong? Many mistakes do not matter. Other mistakes can be fatal. No matter how small the probability you will be hit by a car when you cross against the lights, the consequences of being hit deserve the greater weight in the decision. This line of questioning is the beginning, and in some ways the end, of risk management. All decisions must pass through this sieve. It is the end if you decide not to take the risk, but it is also the end in the sense that distinguishing between consequences and probabilities is what risk management is all about.

Second, expect the unexpected. That sounds like an empty cliché, but it has profound meaning for risk management. It is easy to prepare for the risks you know—earnings fail to meet expectations, clients depart, bonds go sour, a valued associate goes to a competitor. Insurance and hedging strategies cover other kinds of risks lying in wait out there, from price volatility to premature death.

But preparation for the unexpected is a matter of the decision-making structure, *and nothing else*. Who is in charge here? That is the critical question in any organization. And if it is just you there when the unexpected strikes, then you should prepare in advance for where you will turn for help when matters seem to be running out of control.

Finally, note that word "control." With an exit strategy—when decisions are easily reversible—control over outcomes can be a secondary matter. But with decisions such as launching a new product or getting married, the costs of reversibility are so high that you should not enter into them unless you have some control over the outcome if things turn out differently from what you expect. Gambling is fun because your bet is irreversible and you have no control over the outcome. But real life is not a gambling casino.

These three elements are what risky decisions are all about—consequences versus probabilities, preparation for dealing with unexpected outcomes, and the distinction between reversibility and control. These are where things get done, not said.

Source: Peter Bernstein, "Risk:The Whole Versus the Parts," *CFA Magazine*, March/ April 2004, p. 5. Reprinted by permission.

In the case of a Treasury bond paying a fixed rate of interest, the interest payment will be made with 100 percent certainty barring a financial collapse of the economy. The probability of occurrence is 1.0, because no other outcome is possible.

With the possibility of two or more outcomes, which is the norm for common stocks, each possible likely outcome must be considered and a probability of its occurrence assessed. The probability for a particular outcome is simply the chance that the specified outcome will occur and is typically expressed as a decimal or fraction.

PROBABILITY DISTRIBUTIONS

A *probability distribution* for a security brings together the likely outcomes that may occur for that security for a specified time period along with the probabilities associated with these likely outcomes. The set of probabilities in a probability distribution must sum to 1.0, or 100 percent, because they must completely describe all the (perceived) likely occurrences.

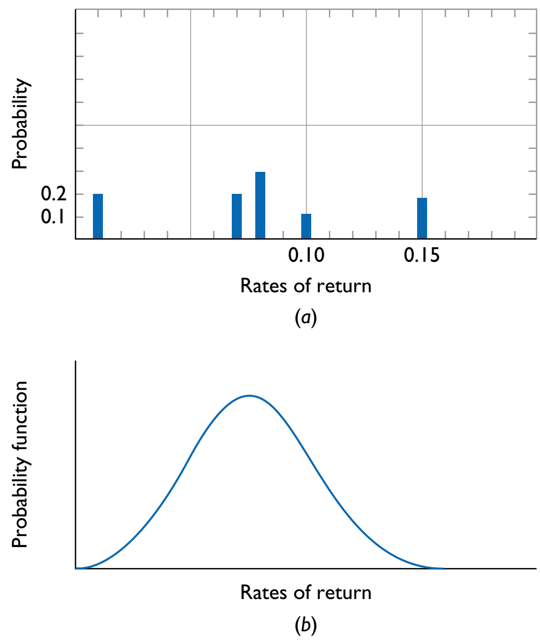
How are these probabilities and associated outcomes obtained? In the final analysis, investing for some future period involves uncertainty, and therefore subjective estimates. Although past occurrences (frequencies) may be relied on heavily to estimate the probabilities, the past must be modified for any changes expected in the future.

Probability distributions can be either discrete or continuous. With a discrete probability distribution, a probability is assigned to each possible outcome. In Figure 7-1*a*, five possible TRs are assumed for General Foods for next year. Each of these five possible outcomes—.01, .07, .08, .10, and .15—has an associated probability; these probabilities sum to 1.0, indicating that the possible outcomes that an investor foresees for General Foods for next year have been accounted for.

With a continuous probability distribution, as shown in Figure 7-1*b*, an infinite number of possible outcomes exist. Because probability is now measured as the area under the curve in Figure 7-1*b*, the emphasis is on the probability that a particular outcome is within some range of values.

Figure 7-1

(a) A Discrete Probability Distribution.
(b) A Continuous Probability Distribution.



The most familiar continuous distribution is the normal distribution depicted in Figure 7-1*b*. This is the well-known bell-shaped curve often used in statistics. It is a two-parameter distribution in that the mean and the variance fully describe it.

CALCULATING EXPECTED RETURN FOR A SECURITY

To describe the single most likely outcome from a particular probability distribution, it is necessary to calculate its *expected value*. The expected value of a probability distribution is the weighted average of all possible outcomes, where each outcome is weighted by its respective probability of occurrence. Since investors are interested in returns, we will call this expected value the *expected rate of return*, or simply **expected return**, and for any security, it is calculated as

Expected Return The ex ante return expected by investors over some future holding period

$$E(R) = \sum_{i=1}^{m} R_i pr_i \tag{7-1}$$

where

$E(R)$ = the expected rate of return on a security
R_i = the ith possible return
pr_i = the probability of the ith return R_i
m = the number of possible returns

Example 7-1 Based on your analysis, you think that General Foods will have a positive return for next period, ranging from 1 percent to 5 percent as described above. The expected value of the probability distribution for General Foods is calculated in the first three columns of Table 7-1. We will call this expected value the expected rate of return, or simply expected return, for General Foods.

CALCULATING RISK FOR A SECURITY

Investors must be able to quantify and measure risk. To calculate the total (standalone) risk associated with the expected return, the variance or standard deviation is used. As we know from Chapter 6, the variance and its square root, standard deviation, are measures of the spread or dispersion in the probability distribution; that is, they measure the dispersion of a random variable around its mean. The larger this dispersion, the larger the variance or standard deviation.

✓ The tighter the probability distribution of expected returns, the smaller the standard deviation, and the smaller the risk.

To calculate the variance or standard deviation from the probability distribution, first calculate the expected return of the distribution using Equation 7-1. Essentially, the same procedure used in Chapter 6 to measure risk applies here, but now the probabilities associated with the outcomes must be included, as in Equation 7-2.

$$\text{the variance of returns} = \sigma^2 = \sum_{i=1}^{m} [R_i - E(R)]^2 pr_i \tag{7-2}$$

and

$$\text{the standard deviation of returns} = \sigma = (\sigma^2)^{1/2} \quad \textbf{(7-3)}$$

where all terms are as defined previously.

Note that the standard deviation is simply a weighted average of the deviations from the expected value. As such, it provides some measure of how far the actual value may be from the expected value, either above or below. With a normal probability distribution, the actual return on a security will be within ± 1 standard deviation of the expected return approximately 68 percent of the time, and within ± 2 standard deviations approximately 95 percent of the time.

Example 7-2 The variance and standard deviation for General Foods, using the information above, is calculated in Table 7-1.

Calculating a standard deviation using probability distributions involves making subjective estimates of the probabilities and the likely returns. However, we cannot avoid such estimates because future returns are uncertain. The prices of securities are based on investors' expectations about the future. The relevant standard deviation in this situation is the *ex ante* standard deviation and not the *ex post* based on realized returns.

Although standard deviations based on realized returns are often used as proxies for *ex ante* standard deviations, investors should be careful to remember that the past cannot always be extrapolated into the future without modifications.

✓ Standard deviations calculated using historical data may be convenient, but they are subject to errors when used as estimates of the future.

Checking Your Understanding

1. The expected return for a security is typically different from any of the possible outcomes (returns) used to calculate it. How, then, can we say that it is the security's expected return?
2. Having calculated a security's standard deviation using a probability distribution, how confident can we be in this number?

Table 7-1 Calculating the Standard Deviation Using Expected Data

(1) Possible Return	(2) Probability	(3) (1) × (2)	(4) $R_i - E(R)$	(5) $(R_i - E(R))^2$	(6) $(R_i - E(R))^2 pr_i$
0.01	0.2	0.002	−0.070	0.0049	0.00098
0.07	0.2	0.014	−0.010	0.0001	0.00002
0.08	0.3	0.024	0.000	0.0000	0.00000
0.10	0.1	0.010	0.020	0.0004	0.00004
0.15	0.2	0.030	0.070	0.0049	0.00098
	1.0	0.080 = E(R)			0.00202

$\sigma = (0.00202)^{1/2} = 0.0449 = 4.49\%$

Introduction to Modern Portfolio Theory

In the 1950s, Harry Markowitz, considered the father of Modern Portfolio Theory (MPT), developed the basic portfolio principles that underlie modern portfolio theory. His original contribution was published in 1952, making portfolio theory about 60 years old. Over time, these principles have been widely adopted by the financial community in a variety of ways, with the result that his legacy of MPT is very broad today.[1]

The primary impact of MPT is on portfolio management, because it provides a framework for the systematic selection of portfolios based on expected return and risk principles. Most portfolio managers today are aware of, and to varying degrees use, the basic principles of MPT. Major mutual fund families employ the implications of MPT in managing their funds, financial advisors use the principles of MPT in advising their individual investor clients, many financial commentators use MPT terms in discussing the current investing environment, and so forth.

Before Markowitz, investors dealt loosely with the concepts of return and risk. Investors have known intuitively for many years that it is smart to diversify, that is, not to "put all of your eggs in one basket." Markowitz, however, was the first to develop the concept of portfolio diversification in a formal way—he quantified the concept of diversification. He showed quantitatively why, and how, portfolio diversification works to reduce the risk of a portfolio to an investor when individual risks are correlated.

Markowitz sought to organize the existing thoughts and practices into a more formal framework and to answer a basic question: Is the risk of a portfolio equal to the sum of the risks of the individual securities comprising it? The answer is no! Markowitz was the first to show that we must account for the interrelationships among security returns in order to calculate portfolio risk, and in order to reduce portfolio risk to its minimum level for any given level of return.

Investments Intuition

Clearly, investors thought about diversifying a portfolio before Markowitz's landmark work. But they did so in general terms. And it is true that not everyone uses his analysis today. However, the tenets of portfolio theory are widely used today, by themselves or in conjunction with other techniques, and by both institutional investors and individual investors.

Portfolio Return and Risk

When we analyze investment returns and risks, we must be concerned with the total portfolio held by an investor. Individual security returns and risks are important, but it is the return and risk to the investor's total portfolio that ultimately matters. Optimal portfolios can be constructed if portfolios are diversified correctly. As we learned in Chapter 1, an investor's portfolio is his or her combination of assets.

As we will see, portfolio risk is a unique characteristic and not simply the sum of individual security risks. A security may have a large risk if it is held by itself but much less risk when held in a portfolio of securities. Since the investor is concerned primarily with the risk to

[1] See Frank J. Fabozzi, Francis Gupta, and Harry M. Markowitz, "The Legacy of Modern Portfolio Theory," *The Journal of Investing*, Fall 2002, pp. 7–22.

his or her total wealth, as represented by his or her portfolio, individual stocks are risky only to the extent that they add risk to the total portfolio.

✓ Investors should always diversify to reduce their risk. Because they should not hold only one security, that security's risk, taken by itself, is not the relevant issue for investors.

PORTFOLIO EXPECTED RETURN

Portfolio Weights Percentages of portfolio funds invested in each security, summing to 1.0.

Portfolio Weights The percentages of a portfolio's total value that are invested in each portfolio asset are referred to as **portfolio weights**, which we will denote by *w*. The combined portfolio weights are assumed to sum to 100 percent of total investable funds, or 1.0, indicating that all portfolio funds are invested. That is,

$$w_1 + w_2 + \cdots + w_n = \sum_{i=1}^{n} w_i = 1.0 \tag{7-4}$$

Example 7-3

With equal dollar amounts in three securities, the portfolio weights are 0.333, 0.333, and 0.333. Under the same conditions with a portfolio of five securities, each security would have a portfolio weight of 0.20. Of course, dollar amounts do not have to be equal. A five-stock portfolio could have weights of .40, .10, .15, .25, and .10, or .18, .33, .11, .22, and .16.

Calculating the Expected Return on a Portfolio The expected return on any portfolio *p* can be calculated as a weighted average of the individual securities expected returns.

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{7-5}$$

where

$E(R_p)$ = the expected return on the portfolio
w_i = the portfolio weight for the *i*th security
$\sum w_i$ = 1.0
$E(R_i)$ = the expected return on the *i*th security
n = the number of different securities in the portfolio

Example 7-4

Consider a three-stock portfolio consisting of stocks G, H, and I with expected returns of 12 percent, 20 percent, and 17 percent, respectively. Assume that 50 percent of investable funds is invested in security G, 30 percent in H, and 20 percent in I. The expected return on this portfolio is:

$$E(R_p) = 0.5(12\%) + 0.3(20\%) + 0.2(17\%) = 15.4\%$$

- The Expected Returns for Individual Securities
- Multiplied by the Weights Assigned to Each Security
- When Added Together Form the Expected Return on a Portfolio

✓ Regardless of the number of assets held in a portfolio, or the proportion of total investable funds placed in each asset, *the expected return on the portfolio is always a weighted average of the expected returns for individual assets in the portfolio.*

Investments Intuition

The expected return for a portfolio must fall between the highest and lowest expected returns for the individual securities making up the portfolio. Exactly where it falls is determined by the percentages of investable funds placed in each of the individual securities in the portfolio.

PORTFOLIO RISK

We know that return and risk are the basis of all investing decisions. Therefore, in addition to calculating the expected return for a portfolio, we must also measure the risk of the portfolio. Risk is measured by the variance (or standard deviation) of the portfolio's return, exactly as in the case of each individual security. Typically, portfolio risk is stated in terms of standard deviation.

It is at this point that the basis of modern portfolio theory emerges, which can be stated as follows: Although the expected return of a portfolio is a weighted average of its expected returns, portfolio risk (as measured by the variance or standard deviation) is typically *not* a weighted average of the risk of the individual securities in the portfolio. Symbolically,

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{7-6}$$

But

$$\sigma_p^2 \neq \sum_{i=1}^{n} w_i \sigma_i^2 \tag{7-7}$$

Precisely because Equation 7-7 is an inequality, investors can reduce the risk of a portfolio beyond what it would be if risk were, in fact, simply a weighted average of the individual securities' risk. In order to see how this risk reduction can be accomplished, we will analyze portfolio risk in detail.

✓ Portfolio risk is always less than a weighted average of the risks of the securities in the portfolio unless the securities have outcomes that vary together exactly, an almost impossible occurrence. Thus, diversification almost always lowers risk, and should be taken advantage of.

Analyzing Portfolio Risk

RISK REDUCTION—THE INSURANCE PRINCIPLE

To begin our analysis of how a portfolio of assets can reduce risk, assume that all risk sources in a portfolio of securities are independent. As we add securities to this portfolio, the exposure to any particular source of risk becomes small. According to the *Law of Large Numbers*, the larger the sample size, the more likely it is that the sample mean will be close to the population expected value. Risk reduction in the case of independent risk sources can be thought of as the *insurance principle*, named for the idea that an insurance company reduces its risk by writing many policies against many independent sources of risk.

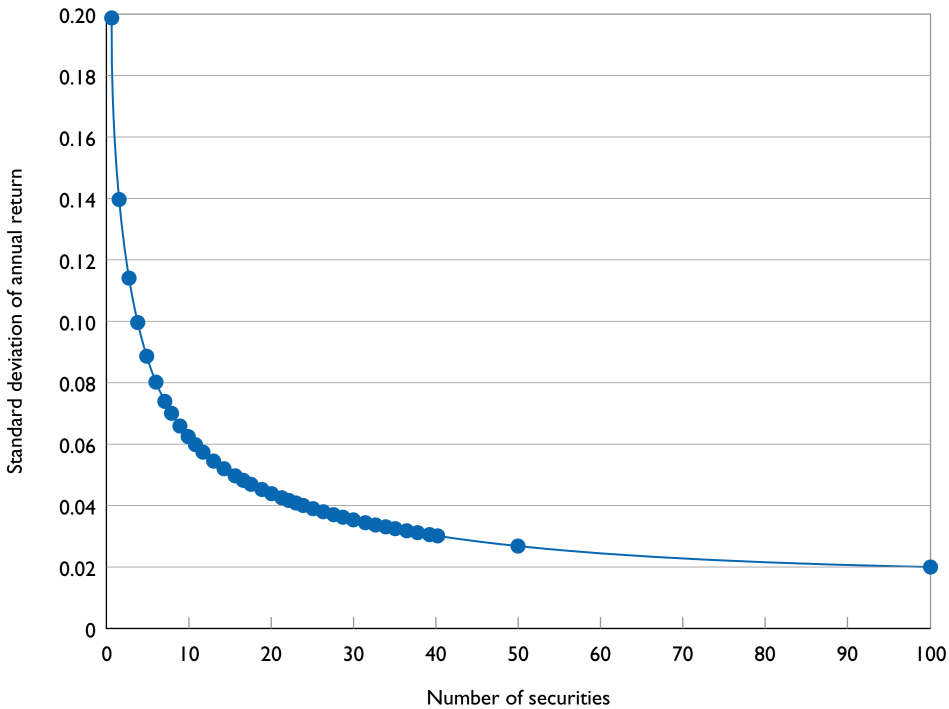
Note that in the case of the insurance principle, we are assuming that rates of return on individual securities are *statistically independent* such that any one security's rate of return is unaffected by another's rate of return. In this situation, and only in this situation, the standard deviation of the portfolio is given by

$$\sigma_p = \frac{\sigma_i}{n^{1/2}} \qquad \textbf{(7-8)}$$

As Figure 7-2 shows, the risk of the portfolio will quickly decline as more securities are added. Notice that no decision is to be made about which security to add because all have identical properties. The only issue is how many securities are added.

Figure 7-2

Risk Reduction When Returns are Independent.



Example 7-5 Figure 7-2 shows how risk declines given that the risk of each security is 0.20. The risk of the portfolio will quickly decline as more and more of these securities are added. Equation 7-8 indicates that for the case of 100 securities the risk of the portfolio is reduced to 0.02:

$$\sigma_p = \frac{0.20}{100^{1/2}}$$
$$= 0.02$$

Figure 7-2 illustrates the case of independent risk sources. As applied to investing, all risk in this situation is firm-specific. Therefore, the total risk *in this situation* can continue to be reduced. Unfortunately, when it comes to investing in financial assets the assumption of statistically independent returns is unrealistic.

Going back to the definition of market risk in Chapter 6, we find that most stocks are positively correlated with each other; that is, the movements in their returns are related. We can call this market risk. While total risk can be reduced, it cannot be eliminated because market risk cannot be eliminated. Unlike firm-specific risk, common sources of risk affect all firms and cannot be diversified away. For example, a rise in interest rates will affect most firms adversely, because most firms borrow funds to finance part of their operations.

DIVERSIFICATION

The insurance principle illustrates the concept of risk reduction when the sources of risk are independent. This situation does not apply to stocks because of market risk. Therefore, we must consider how to reduce risk when the sources of risk are not independent. We do this by diversifying our portfolio of stocks, taking into account how the stocks interact with each other.

✓ Diversification is the key to the management of portfolio risk because it allows investors to significantly lower portfolio risk without adversely affecting return.

Therefore, we focus on portfolio diversification, beginning with random diversification and moving to efficient portfolio diversification based on modern portfolio theory principles.

Random Diversification *Random or naive diversification* refers to the act of randomly diversifying without regard to how security returns are related to each other. An investor simply selects a relatively large number of securities randomly—the proverbial "throwing a dart at *The Wall Street Journal* page showing stock quotes." For simplicity, we assume equal dollar amounts are invested in each stock. As we add securities to a portfolio, the total risk associated with the portfolio of stocks declines rapidly. The first few stocks cause a large decrease in portfolio risk.

The benefits of diversification kick in immediately—two stocks are better than one, three stocks are better than two, and so on. However, diversification cannot eliminate the risk in a portfolio. As additional stocks are added, risk is reduced but the marginal risk reduction is small. Furthermore, and very important to note, recent studies suggest that it takes far more securities to diversify properly than has traditionally been believed to be the case (this point is discussed in detail in Chapter 8).

Although random diversification is clearly beneficial, it is generally not optimal. To take full advantage of the benefits of diversification, we need to understand efficient diversification; that is, we need to understand portfolio risk within a modern portfolio theory context.

Checking Your Understanding

3. What does it mean to an investor that the benefits of diversification kick in immediately, but are limited?

The Components of Portfolio Risk

In order to remove the inequality sign from Equation 7-7 and develop an equation that will calculate the risk of a portfolio as measured by the variance, we must account for two factors:

1. Weighted individual security risks (i.e., the variance of each individual security, weighted by the percentage of investable funds placed in each individual security).
2. Weighted co-movements between securities' returns (i.e., the covariance between the securities' returns, again weighted by the percentage of investable funds placed in each security).

As explained below, covariance is an absolute measure of the co-movements between security returns used in the calculation of portfolio risk. We need the actual covariance between securities in a portfolio in order to calculate portfolio variance or standard deviation. Before considering covariance, however, we can easily illustrate how security returns move together by considering the correlation coefficient, a relative measure of association learned in statistics.

THE CORRELATION COEFFICIENT

Correlation Coefficient A statistical measure of the extent to which two variables are associated

As used in portfolio theory, the **correlation coefficient** ρ_{ij} (pronounced "rho") is a statistical measure of the *relative* co-movements between security returns. It measures the extent to which the returns on any two securities move together; however, it denotes only association, not causation. It is a relative measure of association that is bounded by $+1.0$ and -1.0, with

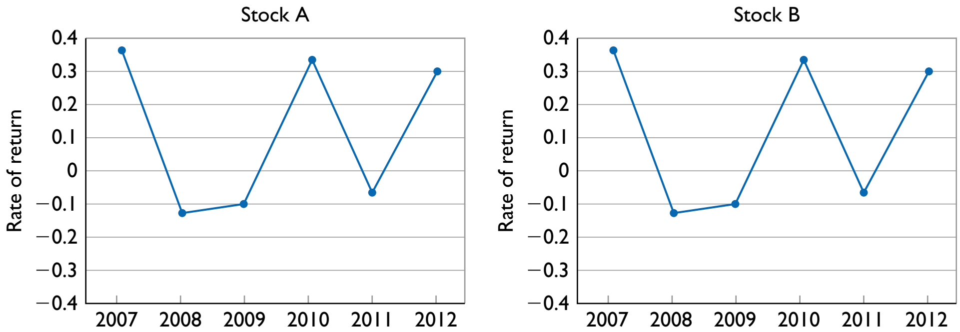
$$\begin{aligned} \rho_{ij} &= +1.0 \\ &= \text{perfect positive correlation} \\ \rho_{ij} &= -1.0 \\ &= \text{perfect negative (inverse) correlation} \\ \rho_{ij} &= 0.0 \\ &= \text{zero correlation} \end{aligned}$$

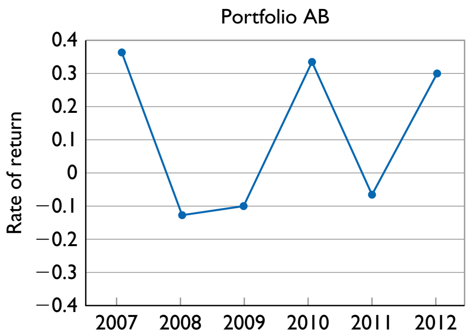
Perfect Positive Correlation With perfect positive correlation, the returns have a perfect direct linear relationship. Knowing what the return on one security will do allows an investor to forecast perfectly what the other will do. In Figure 7-3, stocks A and B have identical return patterns over the six-year period 2007–2012. When stock A's return goes up, stock B's does also. When stock A's return goes down, stock B's does also.

Consider the return and standard deviation information in Figure 7-3. Notice that a portfolio combining stocks A and B, with 50 percent invested in each, has exactly the same return

Figure 7-3

Returns for the Years 2007-2012 on Two Stocks, A and B, and a Portfolio Consisting of 50 Percent A and 50 Percent of B, When the Correction Coefficient is +1.0.





Year	Stock A	Stock B	Portfolio AB
2007	0.36	0.36	0.36
2008	−0.12	−0.12	−0.12
2009	−0.1	−0.1	−0.1
2010	0.34	0.34	0.34
2011	−0.06	−0.06	−0.06
2012	0.3	0.3	0.3
Av. Return	0.12	0.12	0.12
Std. Dev.	0.215	0.215	0.215

as does either stock by itself, since the returns are identical. The risk of the portfolio, as measured by the standard deviation, is identical to the standard deviation of either stock by itself.

✓ When returns are perfectly positively correlated, the risk of a portfolio is simply a weighted average of the individual risks of the securities. This is the one case where diversification does not lead to a reduction in risk.

Perfect Negative Correlation On the other hand, with perfect negative correlation, the securities' returns have a perfect inverse linear relationship to each other. Therefore, knowing the return on one security provides full knowledge about the return on the second security. When one security's return is high, the other is low.

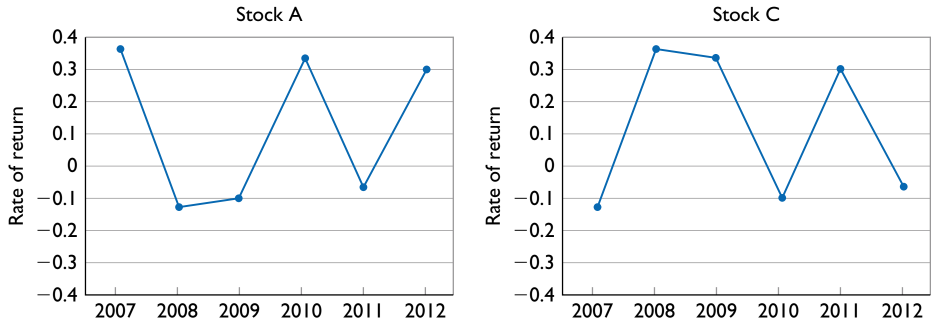
In Figure 7-4, stocks A and C are perfectly negatively correlated with each other. Notice that the information given for these two stocks states that each stock has exactly the same return and standard deviation. When combined, however, the deviations in the returns on these stocks around their average return of 12 percent cancel out, resulting in a portfolio return of 12 percent. This portfolio has no risk. It will earn 12 percent each year over the period measured, and the average return will be 12 percent.

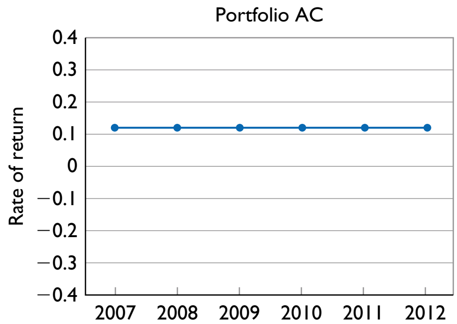
Notice carefully what perfect negative correlation does for an investor. By offsetting all the variations around the expected return for the portfolio, the investor is assured of earning the expected return. At first glance it might appear that offsetting a negative return with an exactly equal positive return produces a zero return, but that is not the case. The expected return for the portfolio is a positive number (otherwise, we would not invest). What is being offset in this case are any variations around that expected return.

Zero Correlation With zero correlation, there is no *linear* relationship between the returns on the two securities. Combining two securities with zero correlation with each other

Figure 7-4

Returns for the Years 2007-2012 on Two Stocks, A and C, and a Portfolio Consisting of 50 Percent A and 50 Percent of C, When the Correction Coefficient is −1.0.





Year	Stock A	Stock C	Portfolio AC
2007	0.36	−0.12	0.12
2008	−0.12	0.36	0.12
2009	−0.1	0.34	0.12
2010	0.34	−0.1	0.12
2011	−0.06	0.3	0.12
2012	0.3	−0.06	0.12
Av. Return	0.12	0.12	0.12
Std. Dev.	0.215	0.215	0.000

reduces the risk of the portfolio. If more securities with uncorrelated returns are added to the portfolio, significant risk reduction can be achieved. However, portfolio risk cannot be eliminated in the case of zero correlation. While a zero correlation between two security returns is better than a positive correlation, it does not produce the risk reduction benefits of a negative correlation coefficient.

Less Than Perfect Positive Correlation Figure 7-5 illustrates a case where stocks A and D are positively correlated with each other at a level of $\rho = +0.55$. Investors may encounter situations such as this and feel there is not much benefit to be gained from diversifying. Note that the standard deviation of each security is still .215, with an average return of .12, but when combined with equal weights of .50 into the portfolio the risk is somewhat reduced, to a level of .18. Any reduction in risk that does not adversely affect return has to be considered beneficial.

With positive correlation risk can be reduced but it cannot be eliminated. Other things being equal, investors wish to find securities with the least positive correlation possible.

✓ Ideally, investors would like securities with negative correlation or low positive correlation, but they generally will be faced with positively correlated security returns.

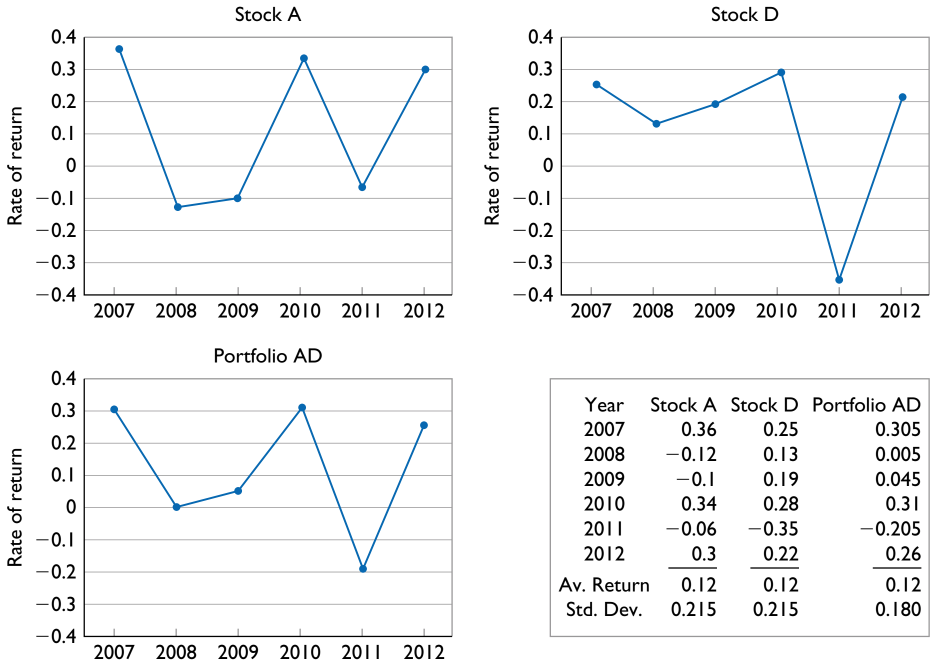
Over the decade ending in 2011, the average correlation between the stocks in the S&P 500 and the Index itself was about .55, so our example above reflects the actual situation, on average, that has existed. Of course, the correlation fluctuates. For example, in 2011 it rose as high as .86 before dropping back some.

Checking Your Understanding

4. Why is negative correlation between two securities in a portfolio better than no (zero) correlation?

Figure 7-5

Returns for the Years 2007-2012 on Two Stocks, A and D, and a Portfolio Consisting of 50 Percent A and 50 Percent of D, When the Correction Coefficient is +0.55.



Year	Stock A	Stock D	Portfolio AD
2007	0.36	0.25	0.305
2008	−0.12	0.13	0.005
2009	−0.1	0.19	0.045
2010	0.34	0.28	0.31
2011	−0.06	−0.35	−0.205
2012	0.3	0.22	0.26
Av. Return	0.12	0.12	0.12
Std. Dev.	0.215	0.215	0.180

COVARIANCE

The previous discussion of the correlation coefficient shows us that the variability in security returns may offset each other to some extent. This is why, to calculate a portfolio's risk (variance), we must take account not only of each security's own risk but also the interactions among the returns of the securities in the portfolio based on the correlation coefficients. However, we must measure the actual amount of co-movement among security returns and incorporate it into the calculation of portfolio risk because the size of the co-movements affect the portfolio's variance (or standard deviation). The covariance measure does this.

✓ Whereas the correlation coefficient measures the relative association between the returns for a pair of securities, the covariance is an absolute measure of the degree of association between the returns for a pair of securities.

Covariance An absolute measure of the extent to which two variables tend to covary, or move together

Covariance is defined as the extent to which two random variables covary (move together) over time. As is true throughout our discussion, the variables in question are the returns (TRs) on two securities. Similar to the correlation coefficient, the covariance can be

1. Positive, indicating that the returns on the two securities tend to move in the same direction at the same time; when one increases (decreases), the other tends to do the same.[2] When the covariance is positive, the correlation coefficient will also be positive.
2. Negative, indicating that the returns on the two securities tend to move inversely; when one increases (decreases), the other tends to decrease (increase). When the covariance is negative, the correlation coefficient will also be negative.

[2] Another way to say this is that *higher* than average values of one random variable tend to be paired with *higher*-than-average values of the other random variable.

3. Zero, indicating that the returns on two securities are independent and have no tendency to move in the same or opposite directions together.

The formula for calculating covariance on an expected basis is

$$\sigma_{AB} = \sum_{i=1}^{m} [R_{A,i} - E(R_A)][R_{B,i} - E(R_B)]pr_i \qquad \textbf{(7-9)}$$

where

σ_{AB} = the covariance between securities A and B[3]
$R_{A,i}$ = one possible return on security A
$E(R_A)$ = the expected value of the return on security A
m = the number of likely outcomes for a security for the period

Equation 7-9 indicates that covariance is the expected value of the product of deviations from the mean. In Equation 7-9, if the stock returns for both A and B are above their mean or below their mean at the same time, the product will be positive, leading to a positive covariance. If, on the other hand, A is above its mean when B is below its mean, the product will be negative, and with enough similar occurrences the covariance will be negative.

The size of the covariance measure depends on the units of the variables involved and usually changes when these units are changed. Therefore, the covariance primarily provides information to investors about whether the association between asset returns is positive, negative, or zero because simply observing the number itself, without any context with which to assess the number, is not very useful.

RELATING THE CORRELATION COEFFICIENT AND THE COVARIANCE

The covariance and the correlation coefficient can be related in the following manner:

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B} \qquad \textbf{(7-10)}$$

This equation shows that the correlation coefficient is simply the covariance standardized by dividing by the product of the two standard deviations of returns.

Given this definition of the correlation coefficient, the covariance can be written as

$$\sigma_{AB} = \rho_{AB}\sigma_A\sigma_B \qquad \textbf{(7-11)}$$

Therefore, knowing the correlation coefficient, we can calculate the covariance because the standard deviations of the assets' rates of return will already be available. Knowing the covariance, we can easily calculate the correlation coefficient.

When analyzing how security returns move together, it is always convenient to talk about the correlation coefficients because we can immediately assess the degree of association (the boundaries are +1 and −1). However, our final objective is to calculate portfolio risk, and to do that we must understand and calculate the covariances.

[3] The order does not matter, because $\sigma_{AB} = \sigma_{BA}$.

Calculating Portfolio Risk

Now that we understand that covariances quantitatively account for the co-movements in security returns, we are ready to calculate portfolio risk. First, we will consider the simplest possible case, two securities, in order to see what is happening in the portfolio risk equation. We will then consider the case of many securities, where the calculations soon become too large and complex to analyze with any means other than a computer.

THE TWO-SECURITY CASE

The risk of a portfolio, as measured by the standard deviation of returns, for the case of two securities, 1 and 2, is

$$\sigma_P = [w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2(w_1)(w_2)(\rho_{1,2})\sigma_1\sigma_2]^{1/2} \tag{7-12}$$

Equation 7-12 shows us that the risk for a portfolio encompasses not only the individual security risks but also the covariance between these two securities and that *three factors, not two, determine portfolio risk*:

- The variance of each security, as shown by σ_1^2 and σ_2^2 in Equation 7-12
- The covariance between securities, as shown by $\rho_{1,2}\sigma_1\sigma_2$ in Equation 7-12
- The portfolio weights for each security, as shown by the w_i's in Equation 7-12

Note the following about Equation 7-12:

- The covariance term contains two covariances—the (weighted) covariance between stock 1 and stock 2, and between stock 2 and stock 1. Since each covariance is identical, we simply multiply the first covariance by two. Otherwise, there would be four terms in Equation 7-12, rather than three.
- We first solve for the variance of the portfolio, and then take the square root to obtain the standard deviation of the portfolio.

Example 7-6 Consider the Total Returns between Southeast Utilities and Precision Instruments for the period 2003–2012. The summary statistics for these two stocks are as follows:

	Southeast		Precision
Return (%)	10.1		15.4
Standard Deviation (%)	16.8		27.5
Correlation Coeff.		.29	

Assume, for expositional purposes, we place equal amounts in each stock; therefore, the weights are 0.5 and 0.5.

$$\begin{aligned}
\sigma_P &= [w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2(w_1)(w_2)(\rho_{1,2})\sigma_1\sigma_2]^{1/2} \\
&= [(.5)^2(16.8)^2 + (.5)^2(27.5)^2 + 2(.5)(.5)(.29)(16.8)(27.5)]^{1/2} \\
&= [70.56 + 189.06 + 66.99]^{1/2} \\
&= 18.1\%
\end{aligned}$$

Alternatively,

$$\begin{aligned}\sigma_P &= [w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + (w_1)(w_2)(\rho_{1,2})\sigma_1\sigma_2 + (w_2)(w_1)(\rho_{2,1})\sigma_2\sigma_1]^{1/2}\\ &= [(.5)^2(16.8)^2 + (.5)^2(27.5)^2 + (.5)(.5)(.29)(16.8)(27.5)\\ &\quad + (.5)(.5)(.29)(27.3)(16.8)]^{1/2}\\ &= [70.56 + 189.06 + 33.5 + 33.5]^{1/2}\\ &= 18.1\%\end{aligned}$$

The Impact of the Correlation Coefficient The standard deviation of the portfolio is directly affected by the correlation between the two stocks. Portfolio risk will be reduced as the correlation coefficient moves from +1.0 downward, everything else constant.

Example 7-7 Let's continue with the data in Example 7-6. The correlation coefficient between Southeast Utilities and Precision Instruments returns is +0.29. In order to focus on the effects of a changing correlation coefficient, we continue to assume weights of 0.5 each—50 percent of investable funds is to be placed in each security. Summarizing the data in this example,

$$\begin{aligned}\sigma_{SU} &= 16.8\\ \sigma_{PI} &= 27.5\\ w_{SU} &= 0.5\\ w_{PI} &= 0.5\end{aligned}$$

With these data, the standard deviation, or risk, for this portfolio, σ_ρ, is

$$\begin{aligned}\sigma_P &= [(0.5)^2(16.8)^2 + (0.5)^2(27.5)^2 + 2(0.5)(0.5)(16.8)(27.5)\rho]^{1/2}\\ &= [70.56 + 189.06 + 229.32\rho]^{1/2}\end{aligned}$$

since $2(0.5)(0.5)(16.8)(27.5) = 229.32$.

The risk of this portfolio clearly depends heavily on the value of the third term, which in turn depends on the correlation coefficient between the returns for SEUT and PI. To assess the potential impact of the correlation, consider the following cases: a ρ of +1, +0.5, +0.29, 0, −0.5, and −1.0. Calculating portfolio risk under each of these scenarios produces the following portfolio risks:

$$\begin{aligned}&\text{If } \rho = +1.0: && \sigma_p = 22.2\%\\ &\text{If } \rho = +0.5: && \sigma_p = 19.4\%\\ &\text{If } \rho = +0.29: && \sigma_p = 18.1\%\\ &\text{If } \rho = 0.0: && \sigma_p = 16.1\%\\ &\text{If } \rho = -0.5: && \sigma_p = 12.0\%\\ &\text{If } \rho = -1.0: && \sigma_p = 5.4\%\end{aligned}$$

These calculations clearly show the impact that combining securities with less than perfect positive correlation will have on portfolio risk. The risk of the portfolio steadily decreases from 22.2 percent to 5.4 percent as the correlation coefficient declines from +1.0 to −1.0. Note, however, that the risk has declined from 22.2 percent to only 16.1 percent as the correlation coefficient drops from +1 to 0, and it has only been cut in half (approximately) by the time ρ drops to −0.5.

Investments Intuition

Correlations are a key variable when considering how diversification can reduce risk. However, a little reflection indicates they are not the complete story. As Equation 7-12 shows, the benefits also depend on the standard deviations of the asset returns and the portfolio weights.

The Impact of Portfolio Weights We saw earlier (Figure 7-4) that with a two-stock portfolio and perfect negative correlation, the risk can be reduced to zero. Notice that this did not happen in Example 7-8 (the risk when $\rho = -1.0$ was 5.4 percent). The reason for this is that the weights for each stock were selected to be 0.50 for illustration purposes. To reduce the risk to zero in the two-security case, and to minimize risk in general, it is necessary to select optimal weights, which can be calculated.

Let's consider the impact of the portfolio weights in the calculation of portfolio risk. The size of the portfolio weights assigned to each security has an effect on portfolio risk, holding the correlation coefficient constant.

Example 7-8

Using the same data as Example 7-7, let's consider the portfolio risk for these two securities. Recall that the correlation coefficient between Southeast Utilities and Precision Instruments is +0.29. For illustration purposes, we will examine five different sets of weights, each of which must sum to 1.0.

Southeast	Precision	σ_p
0.1	0.9	25.3%
0.3	0.7	21.3%
0.5	0.5	18.1%
0.7	0.3	16.2%
0.9	0.1	16.1%

As we can see, in this two-stock portfolio example, holding the correlation coefficient constant at +0.29, the risk of the portfolio varies as the weights for each of the assets changes. Because Southeast has a substantially lower standard deviation than does Precision, portfolio risk decreases as the weight assigned to Southeast increases. However, with a positive correlation coefficient, portfolio risk can decrease only so much.

✓ Portfolio risk is affected both by the correlation between assets and by the percentages of funds invested in each asset.

THE *n*-SECURITY CASE

The two-security case can be generalized to the *n*-security case. Portfolio risk can be reduced by combining assets with less than perfect positive correlation. Furthermore, the smaller the positive correlation, the better.

Portfolio risk is a function of each individual security's risk and the covariances between the returns on the individual securities. Stated in terms of variance, portfolio risk is

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} w_i w_j \sigma_{ij} \tag{7-13}$$

where

σ_p^2 = the variance of the return on the portfolio

σ_i^2 = the variance of return for security i

σ_{ij} = the covariance between the returns for securities i and j

w_i = the portfolio weights or percentage of investable funds invested in security i

$\sum_{i=1}^{n}\sum_{j=1}^{n}$ = a double summation sign indicating that n^2 numbers are to be added together (*i*.e., all possible pairs of values for i and j)

Although Equation 7-13 appears formidable, it states exactly the same message as Equation 7-12 for the two-stock portfolio:

Portfolio risk is a function of

- The weighted risk of each individual security (as measured by its variance)
- The weighted covariances among all pairs of securities

We can rewrite Equation 7-13 into a shorter format:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{7-14}$$

or

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j$$

These equations account for both the variance and the covariances, because when $i = j$, the variance is calculated; when $i \neq j$, the covariance is calculated.

✓ As noted previously, three variables determine portfolio risk: variances, covariances, and weights.

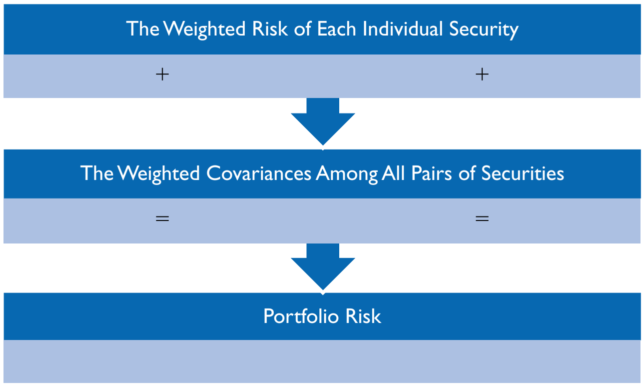
Because of its importance, we emphasize the components of portfolio risk in Figure 7-6.

Checking Your Understanding

5. Given the use of the correlation coefficient, which is clear and easy to understand, why do we need to consider covariances?
6. Suppose we add a very risky stock to a well-diversified portfolio. Could such an action lower the portfolio's risk?

Figure 7-6

The Components of Portfolio Risk.



The Importance of Covariance One of Markowitz's important contributions to portfolio theory is his insight about the relative importance of the variances and covariances. When we add a new security to a large portfolio of securities, there are two impacts.

1. The asset's own risk, as measured by its variance, is added to the portfolio's risk.
2. A covariance between the new security and every other security already in the portfolio is also added.

✓ As the number of securities held in a portfolio increases, the importance of each individual security's risk (variance) decreases, while the importance of the covariance relationships increases.

For example, in a portfolio of 150 securities, the contribution of each security's own risk to the total portfolio risk will be extremely small. When a new security is added to a large portfolio of securities, what matters most is its average covariance with the other securities in the portfolio.

Portfolio risk will consist almost entirely of the covariance risk between securities. Thus, individual security risks can be diversified away in a large portfolio, but the covariance terms cannot be diversified away and therefore contribute to the risk of the portfolio.

Obtaining the Data

To calculate portfolio risk using Equation 7-13, we need estimates of the variance for each security and estimates of the correlation coefficients or covariances. Both variances and correlation coefficients can be (and are) calculated using either *ex post* or *ex ante* data. If an analyst uses *ex post* data to calculate the correlation coefficient or the covariance and then uses these estimates in the Markowitz model, the implicit assumption is that the relationship that existed in the past will continue into the future. The same is true of the variances for individual securities. If the historical variance is thought to be the best estimate of the expected variance, it should be used. However, it must be remembered that an individual security's variance and the correlation coefficient between securities can change over time (and does), as can the expected return.

Table 7-2 The Variance-Covariance Matrix Involved in Calculating the Standard Deviation of a Portfolio

Two securities:				
	$\sigma_{1,1}$	$\sigma_{1,2}$		
	$\sigma_{2,1}$	$\sigma_{2,2}$		
Four securities:				
	$\sigma_{1,1}$	$\sigma_{1,2}$	$\sigma_{1,3}$	$\sigma_{1,4}$
	$\sigma_{2,1}$	$\sigma_{2,2}$	$\sigma_{2,3}$	$\sigma_{2,4}$
	$\sigma_{3,1}$	$\sigma_{3,2}$	$\sigma_{3,3}$	$\sigma_{3,4}$
	$\sigma_{4,1}$	$\sigma_{4,2}$	$\sigma_{4,3}$	$\sigma_{4,4}$

SIMPLIFYING THE MARKOWITZ CALCULATIONS

Equation 7-13 illustrates the problem associated with the calculation of portfolio risk using the Markowitz mean-variance analysis. In the case of two securities, there are two covariances, and we multiply the weighted covariance term in Equation 7-12 by two since the covariance of A with B is the same as the covariance of B with A. In the case of three securities, there are six covariances; with four securities, 12 covariances; and so forth, based on the fact that the total number of covariances in the Markowitz model is calculated as $n\ (n - 1)$, where n is the number of securities.

Table 7-2 shows the variance-covariance matrix associated with these calculations. For the case of two securities, there are n^2, or four, total terms in the matrix—two variances and two covariances. For the case of four securities, there are n^2, or 16 total terms in the matrix—four variances and 12 covariances. The variance terms are on the diagonal of the matrix and, in effect, represent the covariance of a security with itself. Note that the covariance terms above the diagonal are a mirror image of the covariance terms below the diagonal—that is, each covariance is repeated twice since COV_{AB} is the same as $COV_{BA.}$

✓ The number of covariances in the Markowitz model is based on the calculation $n(n - 1)$, where n is the number of securities involved. Because the covariance of A with B is the same as the covariance of B with A, there are $[n(n - 1)]/2$ unique covariances.

Example 7-9 An analyst considering 100 securities must estimate $[100(99)]/2 = 4{,}950$ unique covariances. For 250 securities, the number is $[250(249)]/2 = 31{,}125$ unique covariances.

Obviously, estimating large numbers of covariances quickly becomes a major problem for model users. Since many institutional investors follow as many as 250 or 300 securities, the number of inputs required may become an impossibility. In fact, until the basic Markowitz model was simplified in terms of the covariance inputs, it remained primarily of academic interest.

On a practical basis, analysts are unlikely to be able to directly estimate the large number of correlations necessary for a complete Markowitz analysis. In his original work, Markowitz suggested using an index to which securities are related as a means of generating covariances.

Summary

- The expected return from a security must be estimated. Since this is done under conditions of uncertainty, it may not be realized. Risk (or uncertainty) is always present in the estimation of expected returns for risky assets.
- Probability distributions are involved in the calculation of a security's expected return.
- The standard deviation or variance of expected return for a security is a measure of the risk involved in the expected return; therefore, it also incorporates the probabilities used in calculating the expected return.
- The expected return for a portfolio is always a weighted average of the individual security expected returns.
- Portfolio weights, designated w_i, are the percentages of a portfolio's total funds that are invested in each security, where the weights sum to 1.0.
- Portfolio risk is not a weighted average of the individual security risks. To calculate portfolio risk, we must take account of the relationships between the securities' returns.
- The correlation coefficient is a relative measure of the association between security returns. It is bounded by +1.0 and −1.0, with 0 representing no association.
- The covariance is an absolute measure of association between security returns and is used in the calculation of portfolio risk.
- Portfolio risk is a function of security variances, covariances, and portfolio weights.
- The covariance term captures the correlations between security returns and determines how much portfolio risk can be reduced through diversification.
- The risk of a well-diversified portfolio is largely attributable to the impact of the covariances. When a new security is added to a large portfolio of securities, what matters most is its average covariance with the other securities in the portfolio.
- As the number of securities held in a portfolio increases, the importance of each individual security's risk (variance) decreases, while the importance of the covariance relationships increases.
- The major problem with the Markowitz model is that it requires a full set of covariances between the returns of all securities being considered in order to calculate portfolio variance.
- The number of covariances in the Markowitz model is $n(n-1)$; the number of unique covariances is $[n(n-1)]/2$.

Questions

7-1 Distinguish between historical return and expected return.

7-2 How is expected return for one security determined? For a portfolio?

7-3 The Markowitz approach is often referred to as a mean-variance approach. Why?

7-4 How would the expected return for a portfolio of 500 securities be calculated?

7-5 What does it mean to say that portfolio weights sum to 1.0 or 100 percent?

7-6 What are the boundaries for the expected return of a portfolio?

7-7 Many investors have known for years that they should not "put all of their eggs in one basket." How does the Markowitz analysis shed light on this old principle?

7-8 Evaluate this statement: With regard to portfolio risk, the whole is not equal to the sum of the parts.

7-9 How many, and which, factors determine portfolio risk?

7-10 What is the relationship between the correlation coefficient and the covariance, both qualitatively and quantitatively?

7-11 How many covariance terms would exist for a portfolio of 10 securities using the Markowitz analysis? How many unique covariances?

7-12 How many total terms (variances and covariances) would exist in the variance-covariance matrix for a portfolio of 30 securities using the Markowitz analysis? How many of these are variances, and how many covariances?

7-13 When, if ever, would a stock with a large risk (standard deviation) be desirable in building a portfolio?

7-14 Evaluate the following statement: As the number of securities held in a portfolio increases, the importance of each individual security's risk decreases.

7-15 Should investors generally expect positive correlations between stocks and bonds? Bonds and bills? Stocks and real estate? Stocks and gold?

7-16 What are the inputs for a set of securities using the Markowitz model?

7-17 Evaluate this statement: For any two-stock portfolio, a correlation coefficient of -1.0 guarantees a portfolio risk of zero.

7-18 Agree or disagree with this statement: The variance of a portfolio is the expected value of the squared deviations of the returns for the portfolio from its mean return.

7-19 Evaluate this statement: Portfolio risk is the key issue in portfolio theory. It is not a weighted average of individual security risks.

7-20 Agree or disagree with these statements: There are n^2 terms in the variance covariance matrix, where n is the number of securities.

There are $n(n-1)$ total covariances for any set of n securities. Divide by two to obtain the number of unique covariances.

7-21 Holding a large number of stocks ensures an optimal portfolio. Agree or disagree and explain your reasoning.

CFA

7-22 The variance of a stock portfolio depends on the variances of each individual stock in the portfolio and also the covariances among the stocks in the portfolio. If you have five stocks, how many unique covariances (excluding variance) must you use in order to compute the variance of return on your portfolio? (Recall that the covariance of a stock with itself is the stock's variance.)

CFA

7-23 Given the large-cap stock index and the government bond index data in the following table, calculate the expected mean return and standard deviation of return for a portfolio 75 percent invested in the stock index and 25 percent invested in the bond index.

Assumed Returns, Variances, and Correlations

	Large-Cap Stock Index	Government Bond Index
Expected return	15%	5%
Variance	225	100
Standard Deviation	15%	10%
Correlation	0.5	

CFA

7-24 Suppose a risk-free asset has a 5 percent return and a second asset has an expected return of 13 percent with a standard deviation of 23 percent. Calculate the expected portfolio return and standard deviation of a portfolio consisting of 10 percent of the risk-free asset and 90 percent of the second asset.

7-25 Consider the following information for Exxon and Merck:

- Expected return for each stock is 15 percent.
- Standard deviation for each stock is 22 percent.
- Covariances with other securities vary.

Everything else being equal, would the prices of these two stocks be expected to be the same? Why or why not?

7-26 Select the **CORRECT** statement from among the following:

a. The risk for a portfolio is a weighted average of individual security risks.

b. Two factors determine portfolio risk.

c. Having established the portfolio weights, the calculation of the expected return on the portfolio is independent of the calculation of portfolio risk.

d. When adding a security to a portfolio, the average covariance between it and the other securities in the portfolio is less important than the security's own risk.

7-27 Select the **CORRECT** statement from among the following:

a. The risk of a portfolio of two securities, as measured by the standard deviation, would consist of two terms.

b. The expected return on a portfolio is usually a weighted average of the expected returns of the individual assets in the portfolio.

c. The risk of a portfolio of four securities, as measured by the standard deviation, would consist of 16 covariances and four variances.

d. Combining two securities with perfect negative correlation could eliminate risk altogether.

7-28 Select the **INCORRECT** statement from among the following:

a. Under the Markowitz formulation, a portfolio of 30 securities would have 870 covariances.

b. Under the Markowitz formulation, a portfolio of 30 securities would have 30 variances in the variance-covariance matrix.

c. Under the Markowitz formulation, a portfolio of 30 securities would have 870 terms in the variance-covariance matrix.

d. Under the Markowitz formulation, a portfolio of 30 securities would require 435 unique covariances to calculate portfolio risk.

7-29 Concerning the riskiness of a portfolio of two securities using the Markowitz model, select the **CORRECT** statements from among the following set:

a. The riskiness depends on the variability of the securities in the portfolio.

b. The riskiness depends on the percentage of portfolio assets invested in each security.

c. The riskiness depends on the expected return of each security.

d. The riskiness depends on the amount of correlation among the security returns.

e. The riskiness depends on the beta of each security.

7-30 Select the **CORRECT** statement from the following statements regarding the Markowitz model:

a. As the number of securities held in a portfolio increases, the importance of each individual security's risk also increases.

b. As the number of securities held in a portfolio increases, the importance of the covariance relationships increases.

c. In a large portfolio, portfolio risk will consist almost entirely of each security's own risk contribution to the total portfolio risk.

d. In a large portfolio, the covariance term can be driven almost to zero.

Problems

7-1 Calculate the expected return and risk (standard deviation) for General Foods for 2012, given the following information:

Probabilities:	0.10	0.20	0.40	0.15	0.15
Expected returns:	0.20	0.16	0.12	0.05	−0.05

7-2 Four securities have the following expected returns:

$$A = 12\%, B = 15\%, C = 22\%, \text{and } D = 30\%$$

Calculate the expected returns for a portfolio consisting of all four securities under the following conditions:

a. The portfolio weights are 25 percent each.

b. The portfolio weights are 10 percent in A, with the remainder equally divided among the other three stocks.

c. The portfolio weights are 20 percent each in A and B, and 30 percent each in C and D.

7-3 Assume the additional information provided below for the four stocks in Problem 7-2.

		Correlations With			
	σ(%)	A	B	C	D
A	10	1.0			
B	8	0.6	1.0		
C	20	0.2	−1.0	1.0	
D	16	0.5	0.3	0.8	1.0

a. Assuming equal weights for each stock, what are the standard deviations for the following portfolios?
A, B, and C
B and C
B and D
C and D

b. Calculate the standard deviation for a portfolio consisting of stocks B and C, assuming the following weights: (1) 30 percent in B and 70 percent in C; (2) 70 percent in C and 30 percent in B.

c. In part a, which portfolio(s) would an investor prefer?

Computational Problems

The following data apply to Problems 7-1 through 7—4.
Assume expected returns and standard deviations as follows:

	EG&G	GF
Return (%)	25	23
Standard deviation (%)	30	25
Covariance (%)	112.5	

The correlation coefficient, ρ, is +.15.

Proportion In EG&G w_i	Proportion In GF $w_j = (1 - w_i)$	(1) Portfolio Expected Returns (%)	(2) Variance (%)	(3) Standard Deviation (%)
1.0	0.0	25.0	900	30.0
0.8	0.2	24.6	637	25.2
0.6	0.4	24.2	478	21.9
0.2	0.8	23.4	472	21.7
0.0	1.0	23.0	625	25.0

7-1 Confirm the expected portfolio returns in column 1.

7-2 Confirm the expected portfolio variances in column 2.

7-3 Confirm the expected standard deviations in column 3.

7-4 On the basis of these data, determine the lowest risk portfolio.

7-5 Assume that RF is 4 percent, the estimated return on the market is 15 percent, and the standard deviation of the market's expected return is 20 percent. Calculate the expected return and risk (standard deviation) for the following portfolios:

a. 60 percent of investable wealth in riskless assets, 40 percent in the market portfolio

b. 150 percent of investable wealth in the market portfolio

c. 100 percent of investable wealth in the market portfolio

Spreadsheet Exercises

7-1 Given two stocks and returns for five or six periods, construct combinations of returns in Excel for these two stocks that will produce the following four different correlation coefficients: −1, 0, +.20, −.20. Use the CORREL function to show that your returns achieve the indicated correlation coefficient. The following example shows returns for two stocks, A and B, that produce a correlation coefficient of 1.0. You can use either five periods or six periods. Note that numerous combinations are possible in each case, so there is no one correct answer.

A	B
−2	−2
9	9
6	6
8	8
3	3
20	20

CORREL = +1.0

7-2 The data below are annual total returns for General Foods (GF) and Sigma Technology (ST) for the period 1997–2011. Sigma Technology is highly regarded by many investors for its innovative products. It had returns more than twice as large as that of General Foods. What would have been the results if an investor had placed half her funds in General Foods and half in Sigma Technology during this 15-year period in order to try to earn a larger return than that available in General Foods alone? Would the risk have been too large?

a. Calculate the arithmetic mean returns for each stock.

b. Calculate the standard deviation for each stock using the STDEV function in the spreadsheet.

c. Calculate the correlation coefficient using the CORREL function in the spreadsheet.

d. Calculate the covariance using the COVAR function in the spreadsheet.

e. Calculate the portfolio return assuming equal weights for each stock.

f. Set up a calculation for the standard deviation of the portfolio that will allow you to substitute different values for the correlation coefficient or the standard deviations of the stocks. Using equal weights for the two stocks, calculate the standard deviation of the portfolio consisting of equal parts of the two stocks.

g. How does the portfolio return compare to the return on General Foods alone? How does the risk of the portfolio compare to the risk of having held General Foods alone?

h. Assume that the correlation between the two stocks had been −0.20. How much would portfolio risk have changed relative to the result calculated in f?

	GF	ST
2011	−0.141	0.222
2010	0.203	0.079
2009	−0.036	−0.220
2008	−0.204	0.527
2007	0.073	−0.628
2006	−0.111	0.684
2005	0.023	1.146
2004	0.291	0.564
2003	0.448	0.885
2002	0.482	0.433
2001	0.196	0.516
2000	0.103	−0.056
1999	0.075	0.153
1998	0.780	1.207
1997	0.254	0.736

7-3 Fill in the spreadsheet below to calculate the portfolio return and risk between Zenon and Dynamics, given the 10 years of annual returns for each stock, and portfolio weights of 50/50.

a. How would your answer change if the weights were 40 percent for Zenon and 60 percent for Dynamics?

b. How would your answer change if the weights were 30 percent for Zenon and 70 percent for Dynamics?

	Zenon	Dynamics
Expected Return		
Variance		
Standard Deviation		
Covariance		
Weight for Zenon	50%	
Weight for Dynamics	50%	
Expected Portfolio Return		
Portfolio Variance		
Portfolio Standard Deviation		

	Zenon	Dynamics	
	Zenon Ret	Dynam Ret	
	9.89%	−47.67%	2011
	−12.34%	30.79%	2010
	13.56%	24.78%	2009
	34.56%	7.89%	2008
	−15.23%	24.42%	2007
	20.09%	34.56%	2006
	7.56%	67.56%	2005
	16.47%	44.67%	2004
	18.34%	78.56%	2003
	15.56%	51.00%	2002

Checking Your Understanding

7-1 The expected return for a security is a weighted average of the possible outcomes that could occur. It is the best one-point estimate of the return. If this opportunity were to be repeated for a large number of trials, the average return realized would be the expected return.

7-2 Assuming a normal probability distribution, we can be quite confident.

7-3 The benefits of diversification do kick in immediately. Therefore, two securities provide better risk reduction than one, three are better than two, and so forth. However, at some point there is very little benefit to be gained by adding securities (the gains are so small as to be insignificant), and, therefore, the benefits of diversification are limited.

7-4 Negative correlation means that security returns move inversely to each other. This provides better risk reduction because the negative movement of one security can be offset by the positive movement of another security.

7-5 Covariances are needed to calculate portfolio risk since it consists of weighted variances and weighted covariances. The correlation coefficient is a component of the covariance, given by $COV_{AB} = \rho_{AB}\sigma_A\sigma_B$.

7-6 When adding a security to a well-diversified portfolio, what matters is its relationship to the other securities and not its own individual risk. If this security is negatively correlated with the other securities in the portfolio, having a large risk will work to reduce the overall risk of the portfolio.

chapter 8

Portfolio Selection for All Investors

Having learned about the importance of diversification, it seems logical that there are limits to its use. How many stocks are enough? How can you know if you have chosen the right portfolio?

We know that return and risk are the key parameters to consider, but how do we balance them against each other? It seems prudent at this point to learn about optimal portfolios, and in fact the basic principles about optimal portfolios can now be readily understood, given what we have learned so far. Going further, what about an overall plan to ensure that you have evaluated all of your investing opportunities? It is time to consider asset allocation, one of the most important decisions when it comes to investing. After all, many of the websites devoted to investing refer to asset allocation when discussing what investors should be doing. With a good asset allocation plan in place for your $1,000,000, you can sleep better at night.

Suppose someone whose opinions you respect suggest that you invest a sizeable portion of your $1 million in gold bullion, given the rise in gold prices. How would you respond?

Calculation of portfolio risk is a key issue in portfolio management. Risk reduction through diversification is a very important concept. Closely related to the principle of diversification is the concept of asset allocation. This involves the choices the investor makes among asset classes, such as stocks, bonds, and cash equivalents. The asset allocation decision is the most important single decision made by investors in terms of the impact on the performance of their portfolios.

AFTER READING THIS CHAPTER YOU WILL BE ABLE TO:

- Appreciate the significance of the efficient frontier and understand how an optimal portfolio of risky assets is determined.
- Understand the importance of the asset allocation decision.
- Apply the Markowitz optimization procedure to asset classes and understand the practical implications of doing so.
- Recognize how the total risk of a portfolio can be broken into two components.

Building a Portfolio using Markowitz Principles

To select an optimal portfolio of financial assets using the Markowitz analysis, investors should:

1. Identify optimal risk-return combinations (the efficient set) available from the set of risky assets being considered by using the Markowitz efficient frontier analysis. This step uses the inputs from Chapter 7, the expected returns, variances and covariances for a set of securities.
2. Select the optimal portfolio from among those in the efficient set based on an investor's preferences.

In Chapter 9 we examine how investors can invest in both risky assets and riskless assets, and buy assets on margin or with borrowed funds. As we shall see, the use of a risk-free asset changes the investor's ultimate portfolio position from that derived under the Markowitz analysis.

IDENTIFY OPTIMAL RISK-RETURN COMBINATIONS

As we saw in Chapter 7, even if portfolios are selected arbitrarily, some diversification benefits are gained. This results in a reduction of portfolio risk. However, to take the full information set into account, we use portfolio theory as developed by Markowitz. Portfolio theory is normative, meaning that it tells investors how they should act to diversify optimally. It is based on a small set of assumptions, including

1. A single investment period; for example, one year.
2. Liquidity of positions; for example, there are no transaction costs.
3. Investor preferences based only on a portfolio's expected return and risk, as measured by variance or standard deviation.

THE ATTAINABLE SET OF PORTFOLIOS

Markowitz's approach to portfolio selection is that an investor should evaluate portfolios on the basis of their expected returns and risk as measured by the standard deviation. Therefore, we must first determine the risk-return opportunities available to an investor from a given set of securities. Figure 8-1 illustrates the opportunities available from a given set of securities. A large number of possible portfolios exist when we realize that varying percentages of an investor's wealth can be invested in each of the assets under consideration.

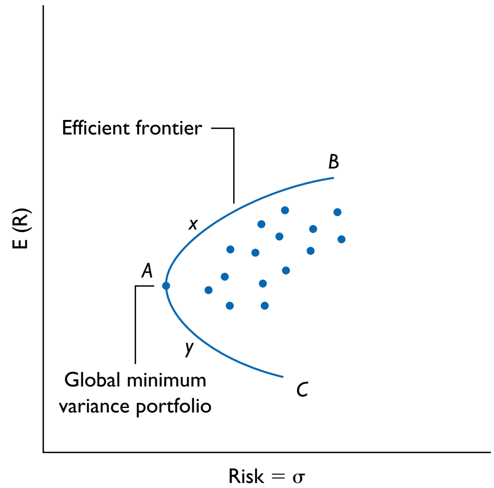
The assets in Figure 8-1 constitute the *attainable set* of portfolios, or the opportunity set. The attainable set is the entire set of all portfolios that could be found from a group of *n* securities. However, risk-averse investors should be interested only in those portfolios with the lowest possible risk for any given level of return. All other portfolios in the attainable set are *dominated*.

Efficient Portfolio A portfolio with the highest level of expected return for a given level of risk or a portfolio with the lowest risk for a given level of expected return

Efficient Portfolios Markowitz was the first to derive the concept of an **efficient portfolio**, defined as one that has the smallest portfolio risk for a given level of expected return or the largest expected return for a given level of risk. Investors can identify efficient portfolios by specifying an expected portfolio return and minimizing the portfolio risk at this level of return. Alternatively, they can specify a portfolio risk level they are willing to assume and maximize the expected return on the portfolio for this level of risk. Rational investors will seek efficient portfolios, because these portfolios are optimized on the basis of the two dimensions of most importance to investors, expected return and risk.

Figure 8-1

The Attainable Set and the Efficient Set of Portfolios.



Using the inputs described earlier—expected returns, variances, and covariances—we can calculate the portfolio with the smallest variance, or risk, for a given level of expected return based on these inputs. Given the minimum-variance portfolios, we can plot the *minimum-variance frontier* as shown in Figure 8-1. Point A represents the *global minimum-variance portfolio* because no other minimum-variance portfolio has a smaller risk. The bottom segment of the minimum-variance frontier, AC, is dominated by portfolios on the upper segment, AB. For example, since portfolio X has a larger return than portfolio Y for the same level of risk, investors would not want to own portfolio Y.

Efficient Set The set of portfolios generated by the Markowitz portfolio model

Efficient Frontier The Markowitz tradeoff between expected portfolio return and portfolio risk (standard deviation) showing all efficient portfolios given some set of securities

The Efficient Set (Frontier) The segment of the minimum-variance frontier above the global minimum-variance portfolio, AB, offers the best risk-return combinations available to investors from this particular set of inputs. This segment is referred to as the **efficient set** or **efficient frontier** of portfolios. The efficient set is determined by the principle of dominance—portfolio X dominates portfolio Y if it has the same level of risk but a larger expected return, or the same expected return but a lower risk.

✓ An efficient portfolio has the smallest portfolio risk for a given level of expected return or the largest expected return for a given level of risk. All efficient portfolios for a specified group of securities are referred to as the efficient set of portfolios.

The arc AB in Figure 8-1 is the Markowitz efficient frontier. Note again that expected return is on the vertical axis while risk, as measured by the standard deviation, is on the horizontal axis. There are many efficient portfolios on the arc AB in Figure 8-1.

Understanding the Markowitz Solution The solution to the Markowitz model revolves around the portfolio weights, or percentages of investable funds to be invested in each security. Because the expected returns, standard deviations, and correlation coefficients for the securities being considered are inputs in the Markowitz analysis, the portfolio weights are the only variable that can be manipulated to solve the portfolio problem of determining efficient portfolios.

✓ A computer program varies the portfolio weights to determine the set of efficient portfolios.

Think of efficient portfolios as being derived in the following manner. The inputs are obtained and a level of desired expected return for a portfolio is specified, for example, 10 percent. Then all combinations of securities that can be combined to form a portfolio with an expected return of 10 percent are determined, and the one with the smallest variance of return is selected as the efficient portfolio. Next, a new level of portfolio expected return is specified—for example, 11 percent—and the process is repeated. This continues until the feasible range of expected returns is processed. Of course, the problem could be solved by specifying levels of portfolio risk and choosing that portfolio with the largest expected return for the specified level of risk.

SELECTING AN OPTIMAL PORTFOLIO OF RISKY ASSETS

Once the efficient set of portfolios is determined using the Markowitz model, investors must select from this set the portfolio most appropriate for them.

✓ The Markowitz model does not specify one optimum portfolio.

Rather, it generates the efficient set of portfolios, all of which, by definition, are optimal portfolios (for a given level of expected return or risk). From this efficient set an investor chooses the portfolio that is optimal for him or her.

Indifference Curves Curves describing investor preferences for risk and return

Indifference Curves We assume investors are risk-averse.[1] To illustrate the expected return-risk combination that will satisfy such an investor's personal preferences, Markowitz used **indifference curves** (which are assumed to be known for an investor). These curves, shown in Figure 8-2 for a risk-averse investor, describe investor preferences for risk and return.[2] Each indifference curve represents the combinations of risk and expected return that are equally desirable to a particular investor (that is, they provide the same level of utility).[3]

Selecting the Optimal Portfolio The optimal portfolio for a risk-averse investor is the one on the efficient frontier tangent to the investor's highest indifference curve. In Figure 8-3 this occurs at point 0. This portfolio maximizes investor utility because the indifference curves reflect *investor preferences,* while the efficient set represents *portfolio possibilities*.

✓ In selecting one portfolio from the efficient frontier, we are matching investor preferences (as given by his or her indifference curves) with portfolio possibilities (as given by the efficient frontier).

Notice that curves U2 and U1 are unattainable, and that U3 is the highest indifference curve for this investor that is tangent to the efficient frontier. On the other hand, U4, though

[1] This means that investors, if given a choice, will not take a "fair gamble," defined as one with an expected payoff of zero and equal probabilities of a gain or a loss. In effect, with a fair gamble, the disutility from the potential loss is greater than the utility from the potential gain. The greater the risk-aversion, the greater the disutility from the potential loss.

[2] Although not shown, investors could also be risk-neutral (the risk is unimportant in evaluating portfolios) or risk-seekers. A risk-seeking investor, given a fair gamble, will want to take the fair gamble, and larger gambles are preferable to smaller gambles.

[3] A few important points about indifference curves should be noted. Indifference curves cannot intersect since they represent different levels of desirability. Investors have an infinite number of indifference curves. The curves for all risk-averse investors will be upward-sloping, but the shapes of the curves can vary depending on risk preferences. Higher indifference curves are more desirable than lower indifference curves. The greater the slope of the indifference curves, the greater the risk-aversion of investors. Finally, the farther an indifference curve is from the horizontal axis, the greater the utility.

Figure 8-2

Indifference Curves.

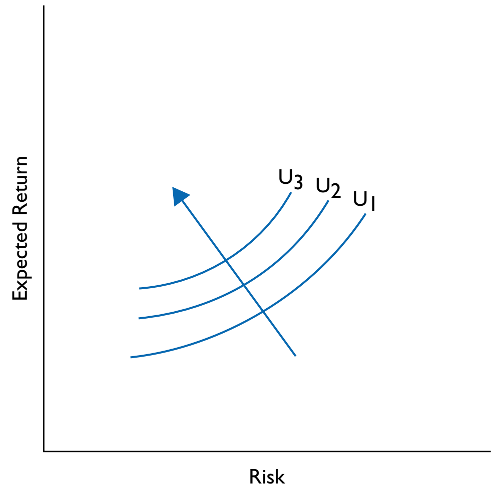
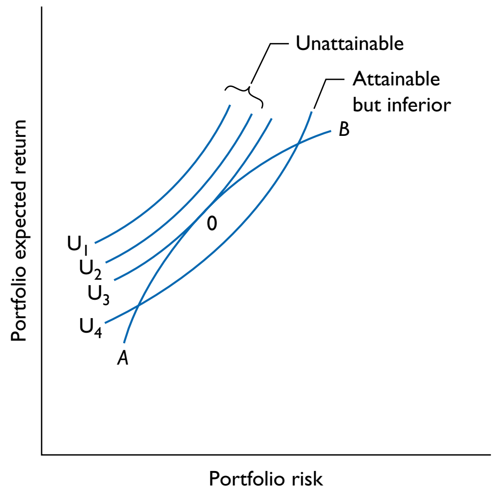


Figure 8-3

Selecting a Portfolio on the Efficient Frontier.



attainable, is inferior to U3, which offers a higher expected return for the same risk (and therefore more utility). If an investor had a different preference for expected return and risk, he or she would have different indifference curves, and another portfolio on the efficient frontier would be optimal.

Investments Intuition

Stated on a practical basis, conservative investors would select portfolios on the left end of the efficient set AB in Figure 8-3 because these portfolios have less risk (and, of course, less expected return). Conversely, aggressive investors would choose portfolios toward point B because these portfolios offer higher expected returns (along with higher levels of risk). Investors typically select their optimal efficient portfolio based on their risk tolerance, which can change depending on conditions. For example, given two really bad stock markets in the first decade of the 21st century (2000–2002 and 2008), along with the natural aging of the population, we would expect risk tolerance to decrease. And surveys of U.S. households show that the willingness to take risk when investing has dropped sharply since 2008.

The Global Perspective—International Diversification

Our discussion has implicitly assumed diversification in domestic securities such as stocks traded on the NYSE and on NASDAQ. However, we now know the importance of taking a global approach to investing. The United States may be the world's largest financial market, but it still accounts for less than half of the total market value of the world's stocks.

What effect would the addition of international stocks have on our diversification analysis? Considering only the potential for risk reduction and ignoring the additional risks of foreign investing, such as currency risk, we could reasonably conclude that if domestic diversification is good, international diversification must be better. And empirical studies have confirmed that at least historically, adding foreign stocks to a well-diversified portfolio reduced the overall volatility.

Bruno Solnik, a leading authority on international investing, has noted that *in the past* country factors dominated stock prices and the correlation of country factors was weak.[4] This means equity markets around the world were in fact different, and because of the low correlations investors could reduce the total variance of their portfolio by diversifying across countries. However, conditions changed dramatically in recent years as financial markets became more and more integrated. There is enormous growth in what is called cross-border mergers and acquisitions, which means, for example, that a British company wishing to grow will buy the same type of business in another country rather than buying another type of British company.

When correlations among country returns increased significantly starting around 1995, the immediate benefits of risk reduction through combining assets with low correlations was reduced. By 2008 global equity correlations were historically high and the MSCI-EAFE index, an international equity index, moved in unison with the S&P 500 index about 90 percent of the time. Even the MSCI Emerging Markets index was correlated with the S&P 500 index at the 80 percent level.[5]

Some Important Conclusions about the Markowitz Model

Five important points must be noted about the Markowitz portfolio selection model:

1. Markowitz portfolio theory is referred to as a two-parameter model because investors are assumed to make decisions on the basis of two parameters, expected return and risk. Thus, it is sometimes referred to as the mean-variance model.
2. The Markowitz analysis generates an entire set, or frontier, of efficient portfolios, all of which are equally "good." No portfolio on the efficient frontier, as generated, dominates any other portfolio on the efficient frontier.
3. The Markowitz model does not address the issue of investors using borrowed money along with their own portfolio funds to purchase a portfolio of risky assets; that is, investors are not allowed to use leverage. As we shall see in Chapter 9, allowing investors to purchase a risk-free asset increases investor utility and leads to a different efficient set on what is called the capital market line.

[4] These comments are based on Bruno Solnik, "Global Considerations for Portfolio Construction," in *AIMR Conference Proceedings: Equity Portfolio Construction,* Association for Investment Management and Research, Charlottesville, VA, 2002, pp. 29–35.

[5] Based on Alec Young, "Dwindling Diversification," Standard & Poor's *The Outlook*, Vol. 80, No. 43, November 12, 2008, p. 5.

4. In practice, different investors, or portfolio managers, will estimate the inputs to the Markowitz model differently. This will produce different efficient frontiers. This results from the uncertainty inherent in the security analysis part of investments as described in Chapter 1.
5. The Markowitz model remains cumbersome to work with because of the large variance-covariance matrix needed for a set of stocks. For example, using only 100 stocks the variance-covariance matrix has 10,000 terms in it (although each covariance is repeated twice).

   This raises two issues, which we will deal with in turn below:

   (a) Are there simpler methods for computing the efficient frontier?

   (b) Can the Markowitz analysis be used to optimize asset classes rather than individual assets?

Checking Your Understanding

1. Given the large number of portfolios in the attainable set, why are there so relatively few portfolios in the efficient set?
2. On an intuitive level, what is the value of talking about indifference curves when discussing the efficient frontier?
3. How should evidence of high correlations between domestic and foreign stock indexes influence investor behavior with regard to international investing?

Alternative Methods of Obtaining the Efficient Frontier

Single-Index Model A model that relates returns on each security to the returns on a market index

The **single-index model** provides an alternative expression for portfolio variance, which is easier to calculate than in the case of the Markowitz analysis. This alternative approach can be used to solve the portfolio problem as formulated by Markowitz—determining the efficient set of portfolios. It requires considerably fewer calculations. Multi-index models have also been examined and evaluated. Both are discussed in Appendix 8-A.

Selecting Optimal Asset Classes—The Asset Allocation Decision

The Markowitz model is typically thought of in terms of selecting portfolios of individual stocks; indeed, that is how Markowitz expected his model to be used. As we know, however, it is a cumbersome model to employ because of the number of covariance estimates needed when dealing with a large number of individual securities.

An alternative way to use the Markowitz model as a selection technique is to think in terms of asset classes, such as domestic stocks, foreign stocks of industrialized countries, the stocks of emerging markets, bonds, and so forth. Using the model in this manner, investors decide what asset classes to own and what proportions of the asset classes to hold.

Asset Allocation Decision The allocation of a portfolio's funds to classes of assets, such as cash equivalents, bonds, and equities

✓ The **asset allocation decision** refers to the allocation of portfolio assets to broad asset markets; in other words, how much of the portfolio's funds is to be invested in stocks, how much in bonds, money market assets, and so forth. Each weight can range from 0 to 100 percent.

Not only is asset allocation one of the most widely used applications of MPT, it is likely the most important single decision an investor makes when holding a portfolio of securities. Examining the asset allocation decision globally leads us to ask the following questions:

1. What percentage of portfolio funds is to be invested in each of the countries for which financial markets are available to investors?
2. Within each country, what percentage of portfolio funds is to be invested in stocks, bonds, bills, and other assets?
3. Within each of the major asset classes, what percentage of portfolio funds is to be invested in various individual securities?

Some Practical Advice

Investors making asset allocation decisions may wish to separate short-term accounts from long-term accounts. For example, we know from Chapter 6 that stocks historically have outperformed other asset classes over very long periods of time. Therefore, a young investor should seriously consider a heavy allocation of funds to stocks in an account that is considered as a long-term holding. On the other hand, when saving for a short-term objective, such as the down payment on a house purchase within a few years, investors need to seriously weigh the risk of common stocks. Consider what happened in 2000–2002 when the S&P 500 Index declined a cumulative 20 percent in two years, and a cumulative 38 percent in three years—measured exactly, the S&P 500 Index declined almost 50 percent from March 24, 2000 (the peak) to October 9, 2002 (the trough), a period of 929 days.

Many knowledgeable market observers agree that the asset allocation decision is the most important decision made by an investor. For example, a widely circulated study found that the asset allocation decision accounts for more than 90 percent of the variance in quarterly returns for a typical large pension fund.[6] A followup study by Ibbotson and Kaplan confirmed these results, finding that approximately 90 percent of the variability in a fund's return across time is explained by the variability in the asset allocation decision.[7] Furthermore, this study concluded that "On average, the pension funds and balanced mutual funds are not adding value above their policy benchmarks because of a combination of timing, security selection, management fees, and expenses."[8]

✓ Asset allocation largely determines an investor's success or lack thereof.

Example 8-1 Consider the 25-month bear market that occurred during 2000–2002. A 100 percent stock portfolio (Wilshire 5000 Index) would have lost about 44 percent of its value, while an investor who chose a 60 percent stock/40 percent bond combination would have lost only about 17 percent. On the other hand, a 100 percent bond portfolio (Lehman Bond Index) would have gained about 23 percent in value.

[6] Gary P. Brinson, L. Randolph Hood, and Gilbert L. Beebower, "Determinants of Portfolio Performance," *Financial Analysts Review* (July/August 1986).

[7] Roger D. Ibbotson and Paul D. Kaplan, "Does Asset Allocation Policy Explain 40, 90, or 100 Percent of Performance?" *Financial Analysts Journal,* January/February 2000, Vol. 56, No. 1, pp. 26–33.

[8] Ibbotson and Kaplan, op. cit., p. 33.

Of course, if we knew stocks were going to go up strongly during some period of time, such as this year, or the next two years, we would like to be 100 percent invested in stocks to take full advantage of this. We know in such a market stocks are very likely to outperform bonds. But the point is no one can be sure what financial markets are going to do over some future period of time. And if, in fact, stocks decline sharply, as they invariably will, asset allocation becomes critical to wealth preservation.

Example 8-2 Consider a shorter period of recent history. The stock market hit a record high on October 9, 2007, and officially entered a declining phase by June 30, 2008. The typical U.S. stock index fund lost about 16 percent over this roughly nine-month period. On the other hand, a 60 percent stock/40 percent bond portfolio lost only half that amount.

Different asset classes offer various potential returns and various levels of risk, and the correlation coefficients between some of these asset classes may be quite low, thereby providing beneficial diversification effects. As with the Markowitz analysis applied to individual securities, inputs remain a problem because they must be estimated. However, this will always be a problem in investing because we are selecting assets to be held over the uncertain future.

ASSET ALLOCATION AND DIVERSIFICATION

The emphasis in Chapter 7 was on diversification of a stock portfolio. Here we have been discussing asset allocation. How do these two concepts connect?

Choosing an asset allocation model does not assure you of a diversified portfolio. For example, choosing to put 90 percent of your funds in an equity mutual fund concentrating on technology stocks and 10 percent in cash is not a diversified portfolio. And if you hold only a diversified stock portfolio, you are making a one-dimensional bet on asset classes.

✓ For many investors a diversified portfolio consists of two elements: diversifying between asset categories and diversifying within asset categories. Such an action can provide a truly diversified portfolio.

SOME MAJOR ASSET CLASSES

Let's consider some of the major asset classes, in addition to U.S. stocks, that investors can use in building a portfolio. It should be noted that investors have more money in Treasury bills, bonds, accounts at banks and real estate than they do in stocks. This is a non-exhaustive list, although it does encompass asset classes that many investors either consider or use.

1. International Investing Investment counselors have regularly recommended that investors diversify internationally by holding foreign securities. The rationale for this has been that such investing reduces the risk of the portfolio because domestic and foreign markets may not move together and potential opportunities in other markets may be greater than those available in the United States.

U.S. investors have taken this rationale to heart. Whereas the average allocation for international equities was about 15 percent in 2001, it was twice that by 2011. Despite the amount of U.S. investor money flowing into international funds, there are limits. A study by the Vanguard Group found that investing more than 40 percent of one's equity allocation in foreign equities did not provide any additional diversification benefits.

Historically, international diversification clearly provided some risk-reducing benefits because of some low positive correlations between asset returns in various countries. Numerous studies confirmed these lower correlations and led many in the investing business to recommend foreign holdings as an asset class. Regardless of the previous studies showing how international diversification can lower portfolio risk, it appears that the benefits of international diversification have decreased recently as the correlation between U.S. stocks and international stocks increased.

The world is changing rapidly. It is clear that many economies have become more integrated as a result of global mergers, rapid money flows around the world, a more-integrated European community, and so forth. Therefore, we might reasonably expect that the correlation between U.S. stocks and some index of foreign stocks has increased over time, and this is exactly the case as noted earlier. Whereas the correlation between the S&P 500 Index and MSCI-EAFE Index was only 58 percent in 1992, by 2008 it was about 90 percent. Furthermore, the new, higher correlation between markets can stay high when markets decline, thereby failing investors when they most need it.

Should investors give up on international diversification? In short, NO! Good opportunities are going to exist in different countries and regions at different times, and a diversified portfolio can capture some of these opportunities. For example, for the decade ending in 2011, European and Pacific index funds produced average returns that were higher than those available on broad U.S. indexes. Other strong economies are emerging, and will emerge in the future. Also, as we know from Chapter 6, a weakening dollar increases dollar-denominated foreign returns to U.S. investors, and if a weakening dollar is anticipated it might be a good time to invest internationally.

How easy is it to choose foreign markets to add to a domestic portfolio? History teaches us that the best-performing markets differ from year to year. Emerging markets may produce good returns for certain periods, and very bad returns during other periods. The same is true of developed countries. Japan had great equity returns in the 1980s and disastrous returns in the 1990s and into the 21st century. History also teaches us that past returns are not necessarily accurate predictors of future returns. For the 10 years ending in 1994, the EAFE Index showed higher returns than did the broadest measure of U.S. stock returns. However, the five years starting in 1995 and ending in 1999 were the greatest consecutive five years in U.S. market history, and clearly where U.S. investors would have liked to be invested during that time period.

2. Bonds are an obvious choice as one of the asset classes to hold in a diversified portfolio. Traditionally, asset allocation was described as dividing one's funds between stocks, bonds, and Treasury bills. The average correlation between the returns on the S&P 500 Index and 15-year Treasury bonds over a very long period was about 0.20. In some time periods, such as 1989–2010, the correlation between these two asset classes was negative.[9]

3. Treasury Inflation-Indexed Securities (TIPS) Inflation-indexed bonds are a relatively new asset class of growing importance because they are the only asset class to provide systematic protection against inflation risk. They are now regarded as a major asset class because these securities often do not follow the movements of other types of securities, including conventional bonds.

TIPS pay a base interest rate that is fixed at the time the bonds are auctioned.[10] However, the principal value of the bonds is adjusted for inflation. Therefore, the fixed rate of interest is applied semiannually to the inflation-adjusted principal of the bonds rather than their par value.

[9] See Burton Malkiel, "How Much Diversification Is Enough?" in *AIMR Conference Proceedings: Equity Portfolio Construction*, Association for Investment Management and Research, Charlottesville, VA, 2002, p. 23.

[10] Details as well as buying instructions for TIPS can be found at http://www.treasurydirect.gov/indiv/products/prod_tips_glance.htm.

Malkiel estimated that the correlation between the S&P 500 Index and TIPS has fluctuated around zero but would actually have often been negative during the 1980s and 1990s.[11] During the period 1999–2004 TIPS had a negative correlation with both stocks (S&P 500) and bonds (U.S. Aggregate Bond Index). As we know, negative correlations provide significant risk-reducing possibilities. Furthermore, while TIPS prices will fluctuate as inflation expectations change, they are about one third less volatile than regular Treasury bonds of similar maturity.

TIPS performed well in 2010, with an average 6.1 percent return. For 2011, the average return was 13.5 percent.

4. Real Estate Real estate is another obvious choice for portfolio diversification. Investors can easily hold real estate by buying Real Estate Investment Trusts (REITs). In recent years REITs have been positively correlated with U.S. stocks, but over a wide range. Based on the FTSE EPRA/NAREIT Global Real Estate Index Series, U.S. equity REITS had an average annual compound rate of return of 11.9 percent for the 30 years ending in 2010.

5. Gold In 2011 gold reached prices of $1,900+ an ounce, although it typically traded for somewhat less. Given that investors can own gold through mutual funds, ETFs, coins, gold mining stocks, and the commodity itself, gold may appeal to a range of portfolio builders. The correlation between gold and the S&P 500 index varies, although it has generally been positive for 12-month periods since 2003. In 2011, however, it was negative.

6. Commodities As most of us know, commodities enjoyed a great rise in some recent years, with steel, cooper, oil, cement, agricultural products, and so forth showing large increases in price. Many investors rushed to cash in on this booming alternative investment. By 2012, there were roughly 200 commodity funds available to investors, a very large change from a few years prior. Commodities often have had low positive correlation with U.S. stocks. Many commodity prices declined sharply in 2012.

COMBINING ASSET CLASSES

As an indication of what can be accomplished using asset classes for an investment program, consider a simple analysis whereby investors diversify across mutual funds representing different asset classes. For example, portfolio funds are spread across asset classes such as blue-chip stocks, small-cap stocks, international equities, domestic bonds, international bonds, gold, and money markets. Tests of such portfolios indicate that they have outperformed the S&P 500 Index over long periods, and with less risk. And this analysis does not employ the Markowitz efficient frontier technique, because it simply uses equal portfolio weights for each of the asset classes. Presumably, the Markowitz optimization procedure could improve the results obtained from this simple strategy.

Programs exist to calculate efficient frontiers using asset classes. These programs allow for a variety of constraints, such as minimum yield and no short selling.

Table 8-1 shows an example of calculating efficient portfolios using the Markowitz optimization technique. It contains return and risk data for "traditional" asset allocation portfolios consisting of stocks (S&P 500 Index), Treasury bonds, and Treasury bills, as well as "nontraditional" portfolios which could also include real estate and TIPS. Notice that three different portfolios are shown: (1) a low-risk portfolio, with a standard deviation of 5 percent, (2) a moderate-risk portfolio, with a standard deviation of 10 percent, (3) and a high-risk portfolio with a standard deviation of 15 percent.

The nontraditional portfolios can include all five assets, as opposed to three for the traditional. As we can see in Table 8-1, the standard deviations for both portfolios are the same

[11] Ibid., p. 22.

Table 8-1 Comparison of Traditional Portfolio and Nontraditional Portfolio, March 1991–September 2001

Characteristic	Low Risk	Moderate Risk	High Risk
Traditional			
Expected return (%)	9.13	12.98	14.51
Standard deviation (%)	5.00	10.00	15.00
Sharpe ratio	0.88	0.83	0.65
Efficient asset allocation			
S&P 500 Index (%)	22.80	56.54	92.34
U.S. long-term government bonds (%)	36.28	43.46	7.66
U.S. T-bills (%)	40.92	0.00	0.00
Nontraditional			
Expected return (%)	10.11	13.57	14.80
Standard deviation (%)	5.00	10.00	15.00
Sharpe ratio	1.08	0.89	0.67
Efficient asset allocation			
S&P 500 Index (%)	18.65	39.23	88.20
U.S. long-term government bonds (%)	26.47	26.93	0.00
U.S. T-bills (%)	0.00	0.00	0.00
TIPS (%)	41.53	0.00	0.00
NAREIT Equity Index (%)	13.08	33.85	11.80

Note: The average risk-free rate during the period was 4.71percent.

SOURCE: From "How Much Diversification Is Enough?" by Burton Malkiel from the CFA Institute Conference and Proceedings EQUITY PORTFOLIO CONSTRUCTION.

for each of three risk levels: 5, 10, and 15 percent. But note that the expected returns are higher in each case for the nontraditional portfolios as compared to the traditional portfolios.

For the traditional portfolios, an investor seeking low risk (5 percent standard deviation) would place funds in each of the three major asset classes, ranging from 22.8 percent in stocks to 40.92 percent in Treasury bills. With a nontraditional portfolio, four of the five asset classes would be held for a low-risk position, with no funds in Treasury bills. In contrast, for the high-risk portfolio, funds are allocated only to stocks and bonds with the traditional portfolio and only to stocks and real estate for the nontraditional.

Figure 8-4 shows a plot of the efficient frontiers for the traditional and nontraditional portfolios. Note that the boundaries are Treasury bills on the low end and stocks on the high end. As we would expect, the nontraditional efficient frontier plots above the traditional efficient frontier. Thus, using the Markowitz analysis investors can determine efficient portfolios by calculating the optimal allocations to each asset class being considered.

Whether we use the Markowitz analysis for asset classes or individual securities, the end result is an efficient frontier of risky portfolios and the choice of an optimal risky portfolio based on investor preferences.

ASSET CLASSES AND CORRELATION COEFFICIENTS

The correlation between asset classes is obviously a key factor in building an optimal portfolio. Investors would like to have asset classes that are negatively correlated with each other, or at least not highly positively correlated with each other.

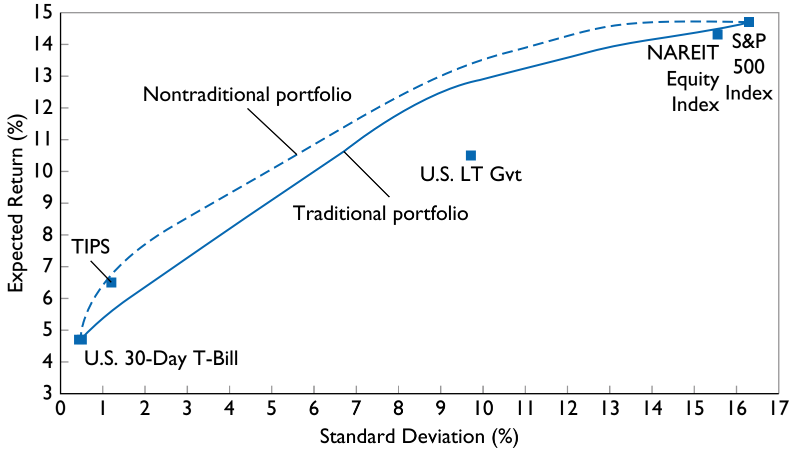


Figure 8-4

Efficient Frontiers of a Traditional and a Nontraditional Portfolio, March 1991–September 2001.

SOURCE: From "How Much Diversification is Enough!" by Burton Malkiel from the CFA Institute Conference and Proceedings EQUITY PORTFOLIO CONSTRUCTION. Copyright © 2002, CFA Institute. Reproduced and Republished from Equity Portfolio Construction with Permission from CFA Institute. All Rights Reserved.

For many years, up until about 2000, the correlation between stocks and bonds ranged from +.3 to +.6. More recently, this relationship seems to have broken down, and been negative at times. Nor is the change surprising. We should expect changes in the correlations among asset classes over time. Correlations between U.S. stocks and REITS have been rising quite sharply since 2002. While the correlation between U.S. stocks and international stocks has risen in recent years from about .60 to .70, the correlation between U.S. stocks and foreign government bonds in developed markets has fallen sharply in recent years. Gold is often considered to have low or negative correlation with U.S. stocks, but there are yearly periods recently when this was not true.

It is obvious that correlation coefficients change over time. It is also clear that the historical correlation between two asset classes will vary somewhat depending on the time period chosen to do the calculation; the frequency of the data (monthly, yearly, etc.); and the exact asset class used (S&P 500, DJIA, Wilshire 5000, etc.). Of course, what really matters when building a portfolio are the future correlation coefficients, which may well be different from the historical.

Asset Allocation and the Individual Investor

Individual investors must confront the asset allocation issue if they are to be successful over time. Having a diversified portfolio of stocks is often not enough. Of course, owning only a portfolio of stocks and not properly diversifying is a prescription for poor, if not disastrous, investment performance. All investors should diversify, simply because we live in an uncertain world, and proper diversification does eliminate some of the risk of owning stocks.

Chapter 7 should convince you that Markowitz diversification pays; that is, portfolio risk can be reduced depending on the covariance relationships. Figure 8-5 makes a strong case for asset allocation by demonstrating that the traditional efficient frontier using stocks, bonds, and risk-free assets can be improved with the addition of other asset classes which have low or negative correlation with the traditional asset classes. For additional evidence on the importance of asset allocation for individual investors, see Box 8-1.

For individual investors, the asset allocation decision depends heavily upon their time horizon and their risk tolerance. Investors tend to be more comfortable with equities when they have long time horizons, given the year to year volatility of stocks. Investors with a low tolerance for risk may only be able to tolerate a relatively modest allocation to stocks.

Asset Allocation Using Stocks and Bonds Let's first consider owning the two major asset classes that a majority of investors are familiar with, and own, in addition to cash assets such as a money market fund. Most investors own portfolios of stocks or bonds or a

BOX 8-1

Spread It Around

Diversifying May Help Reduce Risk in Your Portfolio

Over the last few years, investors have learned a hard lesson in market volatility. One small example: The S&P 500, which ended 2002 with a total return of -23.37 percent, finished 2003 with a flourish, up 26.38 percent. This dramatic one-year change in performance demonstrates how much the financial markets can fluctuate. Alas, performance ups and downs—whether over the short or long term—are a given in the world of investing. And in a sharp market downturn, this volatility can significantly shrink your holdings.

Certainly, last year's rise in equity values came as a great relief to investors after three years of steep stock market declines. But with stock returns flat so far this year and interest rates beginning to go back up, you may wonder whether you want (or need) to adjust your investment strategy.

When reviewing your portfolio, first realize that you cannot predict how the markets will perform. As a result, trying to "time" the market—attempting to guess which way the markets will move, and basing your investment decisions on these predictions—is bound to fail, at least most of the time.

Since market timing is not the answer, you need a better approach for building your portfolio. A tried and true method, based on substantial research, is to diversify your money across different types of investments. "Given the uncertainty of the markets, *asset allocation*, or dividing holdings among different asset classes like stocks, bonds and real estate, provides a good way to manage risk and to build a portfolio for the long term," says Leonard Govia, participant advice manager, TIAA-CREF. (However, diversification doesn't guarantee against loss.)

The Birth of a Theory

The concept of asset allocation is based on modern portfolio theory, which was developed in the 1950s by the economist Harry M. Markowitz, who later shared a Nobel Prize for his work. Markowitz measured the risk inherent in various types of securities and developed methods for combining investments to maximize the tradeoff between risk and return.

Basically, the theory says that investors shouldn't view the prospects of a particular security in isolation, but instead look at each investment and how it fits into an overall portfolio. By combining securities that have a low (or, better yet, negative) *correlation* with each other—that is, securities that don't perform in the same way under similar market conditions—investors will create a less risky portfolio than if they invested only in securities that perform similarly (i.e., have a high correlation).

"The advantage of diversifying investments is that each type of security won't react to the ups and downs of the market in the same way," says Govia. "So by diversifying, you spread the risk in your portfolio around. The result is a more balanced portfolio that can help you withstand drops in the market."

Other studies demonstrate the impact asset allocation has on volatility. For example, in a notable 10-year study of large pension funds, Gary P. Brinson, L. Randolph Hood, and Gilbert Beebower found that, over time, more than 90 percent of the variability of a portfolio's performance is due to allocation among specific asset classes, while less than 5 percent of the variability of performance results from investment selection.

Create a Portfolio for You

If diversification works, your next question may be "How do I ensure that my portfolio is right for my needs?" Many investment companies give you a simple way to develop an appropriate strategy: model portfolios, diversified among asset classes like stocks, bonds and money markets, that are based on different risk tolerances, investment preferences and "time horizons" (the number of years you have to invest before needing to use the money, and how many years you'll need that money to last).

At TIAA-CREF, we've developed model portfolios diversified among five asset classes—stocks, fixed income, real estate, guaranteed and money market—for a variety of investor types. To ensure appropriate diversification for retirement, our portfolios are diversified among at least three asset classes, with one being stocks; virtually all our after-tax mutual fund portfolios are diversified among at least two asset classes, including stocks.

Source: "Spread It Around," *Balance*, Quarterly News and Tools From TIAA-CREF, Summer 2004, pp. 10–11. Reprinted by permission.

Table 8-2 Annual Average Compound Returns and Risks for Portfolio Combinations of Stocks and Bonds for Two 20-Year Periods

		1963–1982		1983–2002	
Stocks	**Bonds**	**Return**	**SD**	**Return**	**SD**
1.00	.00	8.3154	17.6383	12.6902	16.9656
.95	.05	8.1115	16.7606	12.6115	16.1947
.90	.10	7.9080	15.9057	12.5327	15.4504
.85	.15	7.7048	15.0760	12.4541	14.7353
.80	.20	7.5021	14.2739	12.3754	14.0525
.75	.25	7.2997	13.5025	12.2969	13.4052
.70	.30	7.0977	12.7655	12.2184	12.7975
.65	.35	6.8961	12.0673	12.1399	12.2336
.60	.40	6.6949	11.4130	12.0615	11.7184
.55	.45	6.4941	10.8086	11.9832	11.2572
.50	.50	6.2936	10.2611	11.9049	10.8556
.45	.55	6.0935	9.7781	11.8266	10.5192
.40	.60	5.8938	9.3679	11.7484	10.2532
.35	.65	5.6941	9.0389	11.6703	10.0626
.30	.70	5.4955	8.7987	11.5922	9.9508
.25	.75	5.2969	8.6540	11.5142	9.9203
.20	.80	5.0987	8.6087	11.4363	9.9716
.15	.85	4.9009	8.6644	11.3584	10.1038
.10	.90	4.7034	8.8193	11.2805	10.3140
.05	.95	4.5063	9.0690	11.2027	10.5984
00	1.00	4.3666	9.5131	11.0475	11.1496

combination of the two. It stands to reason that bonds are the safer of the two assets, and this in fact is why many investors allocate at least part of their portfolio to bonds. Bonds historically have provided a lower return than stocks, but with a considerably lower risk. We saw in Table 6-6 that the standard deviation for bonds has been roughly 40 percent of the standard deviation for stocks. A severe stock market decline such as that of 2000–2002 convinced a number of investors that they should be holding bonds, thereby lessening or avoiding the really sharp losses in stocks that occurred during that period. An important question remains: What is the best approach for an investor given what we can learn from asset allocation strategies and the history of asset returns?

Table 8-2 shows the geometric mean return and risk combinations for bonds and stocks for two recent 20-year periods.[12] Shown are the returns and standard deviations of portfolio combinations of stocks and bonds in 5 percent increments. Clearly, in general, return and risk go together. A portfolio consisting only of stocks (the first row) has a higher return than does a portfolio consisting only of bonds, or a portfolio consisting of 50 percent stocks and 50 percent bonds. However, the risk of such a portfolio is also higher than the alternatives.

[12] These data come from Charles P. Jones and Jack W. Wilson, "The Changing Nature of Stock and Bond Volatility," unpublished manuscript.

Now consider the situation for an investor who because of his or her risk tolerance really wishes to own a portfolio of bonds. This investor understands that the return on such a portfolio is expected to be lower than that of a stock portfolio, but also knows that the risk will be lower, and the investor's risk tolerance drives the decision. Let us assume the investor owned a 100 percent bond portfolio over the second 20-year period.

This investor earned an annual compound rate of return of 11.05 percent with a risk level of 11.15 percent. However, Table 8-2 shows us that a portfolio of 50 percent stocks and 50 percent bonds had a lower standard deviation, 10.85 percent, and a higher annual return of 11.90 percent. The same is true for the prior period. A portfolio of 65 percent bonds and 35 percent stocks had a slightly lower risk than a portfolio of 100 percent bonds, but a return that was almost 1.2 percentage points higher on a compound annual basis.

Clearly, for the 40-year period shown here, asset allocation between stocks and bonds paid off for investors. Unless one expects the future to be quite different from the past, it is difficult to justify holding a portfolio consisting only of bonds.

Some Limitations on Asset Allocation Individual investors, in choosing asset classes, should be aware that the benefits of asset allocation are not always present. We noted above that gold was typically negatively correlated with U.S. stocks in 2011. This would seem to make gold a good candidate for at least some percentage of an investor's portfolio. A recent study over the period 1975–2005 found, not unexpectedly, that gold as a standalone investment performed poorly relative to stocks over the long run. What about a 5 percent buy-and-hold gold investment added to an optimized global portfolio? This study found that over this long period, there was no real benefit to investing in gold. Over certain time periods, of course, gold performed well, but on balance it did not.[13] Given the hype about gold in 2011 when it reached very high price levels, this is good information to keep in mind as a long-run investor.

Investors should also note that asset allocation does not guarantee that an investor will not lose money during some time period. During the financial crisis of 2008 almost all asset classes declined. This has led some observers to argue that asset allocation/diversification is overrated and can be ignored. Such an argument is wrong. 2008 was a true financial crisis in the same sense that the Great Depression was a true financial crisis. Such an event happens only rarely, fortunately. For all other bad times in the U.S. economy, asset allocation/diversification pays off, reducing the investor's risk and losses. Investors would be ill advised to go against the entire history of diversification because of one catastrophic event in recent history.

Some Practical Advice

Despite investor interest in, and use of, several different asset classes such as gold or commodities, it is still true today that the three major categories for asset allocation for many investors are stocks, bonds and cash equivalents. U.S. investment-grade bonds tend to have low correlations with stocks over long time periods, and provide investors the opportunity to have a balanced portfolio that cushions against market shocks.

For investors who feel that three asset classes are not sufficient, consider the following. Considerable research suggests that an investor needs no more than seven asset classes to achieve optimal asset allocation: blue-chip U.S. stocks, blue-chip foreign stocks, small company stocks, value stocks, high quality bonds, inflation-protected bonds, and cash equivalents. Furthermore, almost all of these asset classes can be built using index funds, which as we know from Chapter 3 have minimal costs and good diversification.

[13] See Mitchell Ratner and Steven Klein, "The Portfolio Implications of Gold Investment," *The Journal of Investing,* 17, Spring 2008, pp. 77–87.

ASSET ALLOCATION AND INDEX MUTUAL FUNDS

Investors can build a sound portfolio using index mutual funds or ETFs. Starting with equities, funds would be needed to cover domestic large cap stocks and small cap stocks. An intermediate cap stock fund might be added. Because we live in a global economy, we would add an international large stock fund and an emerging markets fund. An intermediate U.S. government bond fund could add stability and income. Individuals in high tax brackets might opt for a municipal bond fund.

This asset allocation plan, using only stock and bond funds, both domestic and international, should be sufficient for many investors. Adding additional asset classes may add value, but they also increase the overall portfolio risk. For example, gold, with no cash flow, could easily decline in price following a big runup.

LIFE CYCLE ANALYSIS

Traditionally, recommended asset allocations for investors have focused on the stage of the life cycle they are in. For example, young investors with a 30-year working horizon ahead of them were assumed to be able to invest in risky common stocks, while investors nearing retirement were assumed to favor mostly bonds in their portfolio. However, as investors have become more familiar with what inflation can do to the value of a fixed portfolio over a long period, and as they realize that a retiree may well have a life expectancy of 25 years or more, some have changed their views of asset allocation.

Life-Cycle Fund Funds that automatically become more conservative as your retirement date approaches

A simple approach for some individual investors in managing their retirement funds is to buy a **life-cycle fund** (also called a **target-date fund**). Life-cycle funds are balanced funds (holding both equity and fixed income investments) with an asset allocation that automatically adjusts to a more conservative posture as your retirement date approaches. They are available in many 401(k) plans.

Fidelity and Vanguard are the two largest providers of life-cycle funds. Fidelity uses 18 underlying mutual funds in an active management approach while Vanguard uses only a few funds in a passive management approach. For example, the Vanguard Target Retirement 2050 Fund is for people in their 20s with approximately 40 years to retirement. The fund starts out mostly invested in stocks, but by approximately 2026 the fund starts to annually reduce stocks and increase bonds.

OTHER APPROACHES

There is no “one” answer to the question, what is an ideal asset allocation for a particular investor? There are a number of suggested allocations readily available, with differences that might well be justified depending on the circumstances.

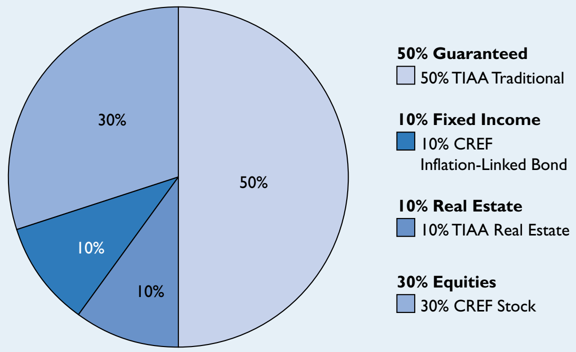
Concepts in Action

Making Asset Allocation Recommendations for Investors

As noted in the chapter, studies suggest that the asset allocation decision can account for more than 90 percent of the variance in returns for large pension fund portfolios. Many investors now regard the asset allocation decision as the most important one to be made in determining the success of their portfolio over time.

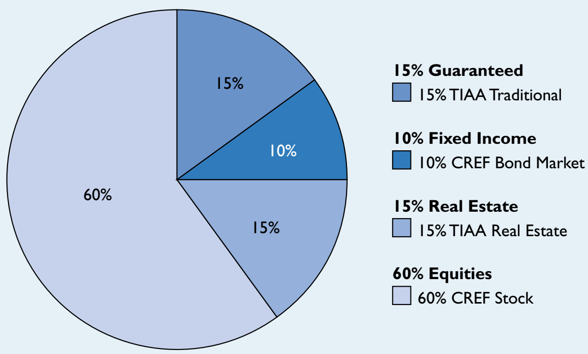
How does asset allocation get implemented in practice? Consider the model portfolios of TIAA-CREF, one of the largest financial service providers in the world. This organization provides retirement planning and investment services for a very large clientele.

TIAA-CREF illustrates several model portfolios that accommodate a range of investor risk tolerances. For example, for a conservative investor who emphasizes safety and stability, the following asset allocation is recommended.

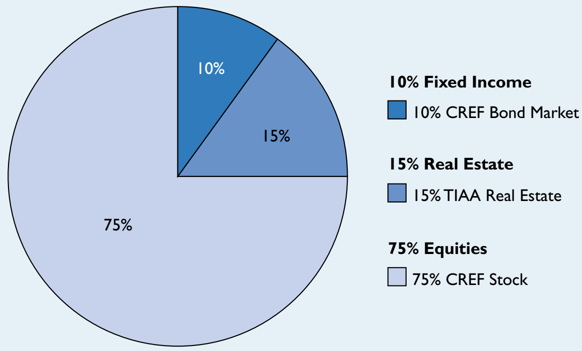


Notice that 50 percent of the portfolio is allocated to a guaranteed fund. Even so, 30 percent is allocated to equities to provide some growth opportunities over time.

What about a moderately aggressive investor who seeks more growth possibilities while still emphasizing stability? TIAA-CREF recommends the following asset allocation:



Finally, what about the "aggressive" investor building a retirement portfolio (as opposed to a speculator). This approach offers investors both growth and income stocks, and both domestic and international opportunities. Note that even at this stage only 75 percent of funds are invested in equities, because the investor still needs diversification and some stability.



Source: TIAA-CREF website.

Other asset allocation examples are readily available. AAII, the American Association of Individual Investors, has a website with a section on asset allocation models. Suggested allocation breakdowns are shown for conservative, moderate, and aggressive investors using seven asset classes.

Asset allocation calculators are also readily available. By supplying your data, a suggested asset allocation model tailored to you can be generated. As one example, see http://www.ipers.org/calcs/AssetAllocator.html.

Checking Your Understanding

4. Relative to Figure 8-5, what does it mean to say that an efficient frontier is pushed out?
5. Explain why, using the bear markets of 2000–2002 or 2008, one can argue that the Asset Allocation decision is the most important decision made by an investor.

The Impact of Diversification on Risk

The Markowitz analysis demonstrates that the standard deviation of a portfolio is typically less than the weighted average of the standard deviations of the securities in the portfolio. Thus, diversification typically reduces the risk of a portfolio—as the number of portfolio holdings increases, portfolio risk declines. In fact, almost half of an average stock's risk can be eliminated if the stock is held in a well-diversified portfolio.

SYSTEMATIC AND NONSYSTEMATIC RISK

Nonsystematic Risk Risk attributable to factors unique to a security

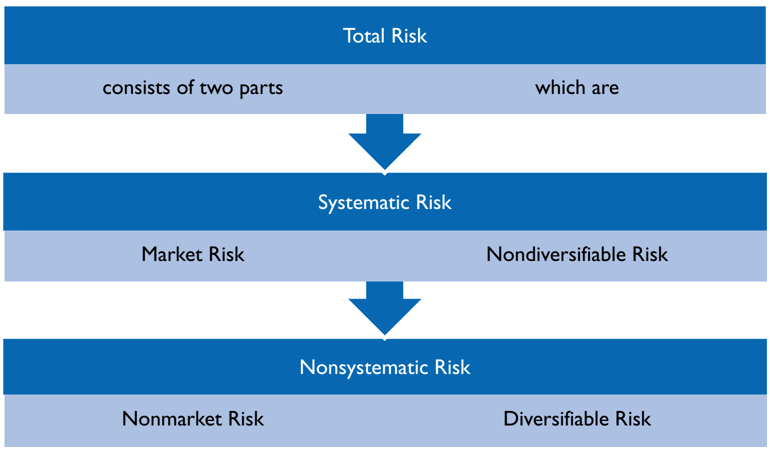
Diversifiable (Nonsystematic) Risk The riskiness of the portfolio generally declines as more stocks are added because we are eliminating the **nonsystematic risk**, or company-specific risk. This is unique risk related to a particular company. However, the extent of the risk reduction depends on the degree of correlation among the stocks. As a general rule, correlations among stocks, at least domestic stocks and particularly large domestic stocks, are positive, although less than 1.0. Adding more stocks will reduce risk at first, but no matter how many partially correlated stocks we add to the portfolio, we cannot eliminate all of the risk.

Systematic Risk Risk attributable to broad macro factors affecting all securities

Nondiversifiable (Systematic) Risk Variability in a security's total returns that is directly associated with overall movements in the general market or economy is called **systematic risk**, or market risk, or nondiversifiable risk. Virtually all securities have some systematic risk, whether bonds or stocks, because systematic risk directly encompasses interest rate risk, recession, inflation, and so on. Most stocks are negatively impacted by such factors; therefore, diversification cannot eliminate market risk.

After the nonsystematic risk is eliminated, what is left is the nondiversifiable portion, or the market risk (systematic part). This part of the risk is inescapable, because no matter how well an investor diversifies, the risk of the overall market cannot be avoided. If the stock market rises strongly, as it did in 1998 and 1999, most stocks will appreciate in value; if it declines sharply, as in 2000, 2001, and 2002, most stocks will be adversely affected. These movements occur regardless of what any single investor does.

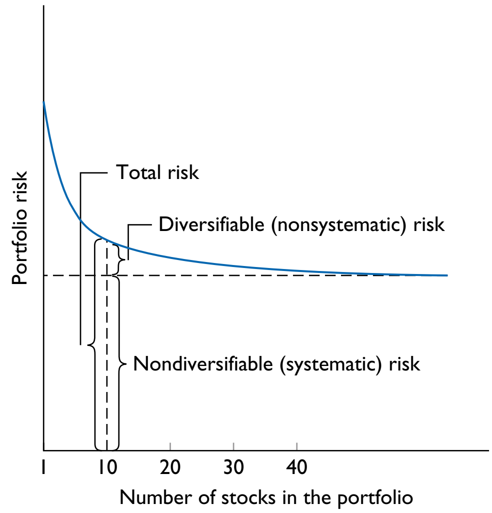
Remember:



Risk and the Number of Securities Investors can construct a diversified portfolio and eliminate part of the total risk, the diversifiable or nonmarket part. Figure 8-5

Figure 8-5

Systematic and Nonsystematic Risk.



illustrates this concept of declining nonsystematic risk in a portfolio of securities. As more securities are added, the nonsystematic risk becomes smaller and smaller, and the total risk for the portfolio approaches its systematic risk. Since diversification cannot reduce systematic risk, total portfolio risk can be reduced no lower than the total risk of the market portfolio.

Diversification can substantially reduce the unique risk of a portfolio. However, Figure 8-5 indicates that no matter how much we diversify, we cannot eliminate systematic risk. The declining total risk curve in that figure levels off and at most becomes asymptotic to the systematic risk. Clearly, market risk is critical to all investors. It plays a central role in asset pricing because it is the risk that investors can expect to be rewarded for taking.

HOW MANY SECURITIES ARE NEEDED TO FULLY DIVERSIFY?

A study done by Evans and Archer in 1968 is often cited in answering the question of how many securities are needed to have a well-diversified portfolio.[14] Their analysis suggested as few as 15 stocks could be adequate. Thus, based on studies done in the 1960s, 1970s, and 1980s it had become commonplace for investors to believe that 15 or so stocks provides adequate diversification, and investors will often find reference to this belief today. This belief is now being revised.

According to a recent study by Campbell, Lettau, Malkiel, and Xu, between 1962 and 1997 the market's overall volatility did not change while the volatility of individual stocks increased sharply.[15] Market volatility was found to be essentially trendless, while the volatility of individual stocks has risen. This study suggests that investors need more stocks in today's environment to adequately diversify.

In a separate article, Malkiel illustrates how today's situation differs from the past in terms of idiosyncratic (nonsystematic) risk and risk reduction.[16] Figure 8-6 shows how total risk declines based on the 1960s and the 1990s. Using the 1960s, which is the typical diagram traditionally shown to illustrate this, total risk declines rapidly as the idiosyncratic (labeled

[14] See John Evans and Stephen Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," *Journal of Finance,* 23, 1968, pp. 761–767.

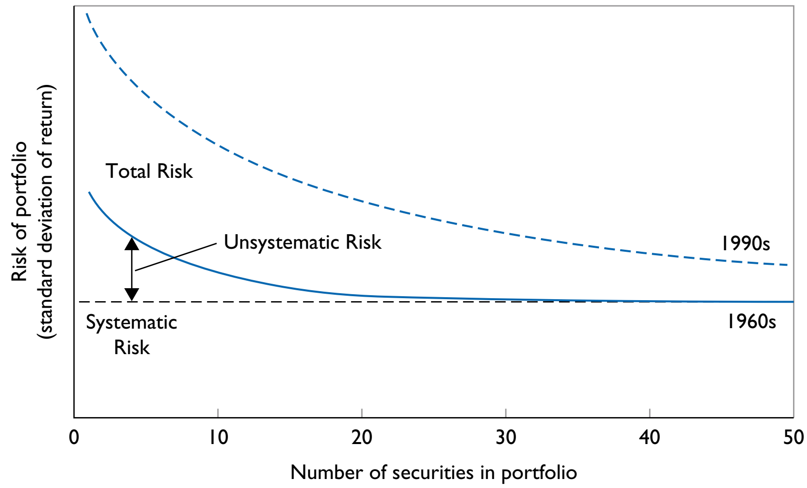
[15] See John Campbell, Martin Lettau, Burton Malkiel, and Yexiao Xu, "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance*, 56, (February 2001), pp. 1–43.

[16] Malkiel, op. cit., p. 19.

Figure 8-6

Diversification and the Number of Securities Past and Present.

SOURCE: From "How Much Diversification is Enough!" by Burton Malkiel from the CFA Institute Conference and Proceedings EQUITY PORTFOLIO CONSTRUCTION. Copyright © 2002, CFA Institute. Reproduced and Republished from *Equity Portfolio Construction* with Permission from CFA Institute. All Rights Reserved.



unsystematic in Figure 8-6) risk is eliminated. Twenty stocks diversified by sector could effectively eliminate the company-specific risk. In contrast, for the 1990s even a 50-stock portfolio contains a significant amount of idiosyncratic risk. Malkiel goes on to say that in "today's market, a portfolio must hold many more stocks than the 20 stocks that in the 1960s achieved sufficient diversification."[17] Malkiel has suggested that it could take as many as 200 stocks to provide the level of diversification in the earlier studies.

So how many securities are needed to adequately diversify a portfolio? Some research suggests 40 stocks are needed. Campbell et al. suggest that for recent periods at least 50 randomly selected stocks are needed. Another study by Boscaljon et al. focused on portfolios of about 60 stocks chosen from different industries.[18] Based on the recent research done on diversification, it seems reasonable to state that at least 40 securities, and perhaps 50 or 60, are needed to ensure adequate diversification.

The Implications of Reducing Risk by Holding Portfolios

The construction of optimal portfolios and the selection of the best portfolio for an investor have implications for the pricing of financial assets. As we saw in the previous discussion, part of the riskiness of the average stock can be eliminated by holding a well-diversified portfolio. This means that part of the risk of the average stock can be eliminated and part cannot. Investors need to focus on that part of the risk that cannot be eliminated by diversification because this is the risk that should be priced in the financial markets.

The relevant risk of an individual stock is its contribution to the riskiness of a well-diversified portfolio. The return that should be expected on the basis of this contribution can be estimated by the capital asset pricing model. We consider these topics in Chapter 9.

[17] Malkiel, op. cit., p. 19.

[18] See Brian Boscaljon, Greg Filback, and Chia-Cheng Ho, "How Many Stocks Are Required for a Well-Diversified Portfolio?" *Advances in Financial Education*, 3, Fall 2005, pp. 60–71.

Summary

- Markowitz portfolio theory provides the way to select optimal portfolios based on using the full information set about securities.
- Having calculated the expected returns and standard deviations for a set of portfolios, the efficient set (or efficient frontier) can be determined.
- The expected returns, standard deviations, and correlation coefficients for the securities being considered are inputs in the Markowitz analysis. Therefore, the portfolio weights are the variable manipulated to determine efficient portfolios.
- An efficient portfolio has the highest expected return for a given level of risk, or the lowest level of risk for a given level of expected return.
- The Markowitz analysis determines the efficient set of portfolios, all of which are equally desirable. The efficient set is an arc in expected return—standard deviation space.
- The efficient frontier captures the possibilities that exist from a given set of securities. Indifference curves express investor preferences.
- The optimal portfolio for a risk-averse investor occurs at the point of tangency between the investor's highest indifference curve and the efficient set of portfolios.
- The single-index model provides an alternative expression for portfolio variance, which is easier to calculate than in the case of the Markowitz analysis. This alternative approach can be used to solve the portfolio problem as formulated by Markowitz.
- The asset allocation decision refers to the allocation of portfolio assets to broad asset markets; in other words, how much of the portfolio's funds is to be invested in stocks, how much in bonds, money market assets, and so forth. Each weight can range from 0 to 100 percent. Asset allocation is one of the most widely used applications of MPT.
- The Markowitz analysis can be applied to asset classes to determine optimal portfolios to hold. Efficient frontiers involving asset classes can be generated.
- Diversification can substantially reduce the unique risk of a portfolio. However, no matter how much we diversify, we cannot eliminate systematic risk. Therefore, systematic (market) risk is critical to all investors.
- New research indicates that it takes substantially more stocks to diversify adequately than has previously been thought. This number appears to be at least 40, and could be more.
- The relevant risk of an individual stock is its contribution to the riskiness of a well-diversified portfolio.

Questions

8-1 Consider a diagram of the efficient frontier. The vertical axis is ___________. The horizontal axis is ___________, as measured by the ___________.

8-2 How many portfolios are on an efficient frontier? What is the Markowitz efficient set?

8-3 Why do rational investors seek efficient portfolios?

8-4 Using the Markowitz analysis, how does an investor select an optimal portfolio?

8-5 How is an investor's risk-aversion indicated in an indifference curve? Are all indifference curves upward-sloping?

8-6 What does it mean to say that the efficient frontier with indifference curves matches possibilities with preferences?

8-7 With regard to international investing, how has the situation changed in recent years with regard to correlations among the stocks of different countries?

8-8 If the correlations among country returns have increased in recent years, should U.S. investors give up, or decrease significantly, their positions in foreign securities?

8-9 What is meant by the asset allocation decision? How important is this decision?

8-10 When efficient frontiers are calculated using asset classes, what types of results are generally found?

8-11 As we add securities to a portfolio, what happens to the total risk of the portfolio?

8-12 How well does diversification work in reducing the risk of a portfolio? Are there limits to diversification? Do the effects kick in immediately?

8-13 Assume that you have an investment portfolio worth $100,000 invested in bonds because you are a conservative investor. Based on the discussion in this chapter, is this a sound decision?

8-14 Now assume that you inherit $25,000 and decide to invest this amount in bonds also, adding the new bonds to your existing bond portfolio. Is such a decision consistent with the lessons of modern portfolio theory?

8-15 What is the difference between traditional beliefs (starting in the 1960s) as to the number of securities needed to properly diversify, and the very recent evidence that has been presented by Malkiel and others?

8-16 Can gold be used as part of an asset allocation plan? If so, how can this be accomplished?

8-17 Suppose you are considering a stock fund and a bond fund and determine that the covariance between the two is −179. Does this indicate a strong negative relationship?

8-18 Can a single asset portfolio be efficient?

8-19 Can the original Markowitz efficient frontier ever be a straight line?

Problems

8-1 Given the following information:

Standard deviation for stock X = 12%
Standard deviation for stock Y = 20%
Expected return for stock X = 18%
Expected return for stock Y = 25%
Correlation coefficient between X and Y = 0.60

The covariance between stock X and Y is

a. .048
b. 144.00
c. 3.60
d. 105.6

8-2 Given the information in Problem 8-1 regarding risk, the expected return for a portfolio consisting of 50 percent invested in X and 50 percent invested in Y can be seen to be

a. 21.5%
b. 18%
c. Less than 18%
d. More than 25%

8-3 Given the information in Problem 8-1, assume now that the correlation coefficient between stocks X and Y is −1.0. Choose the investment below that represents the minimum-risk portfolio.

a. 100% investment in stock X
b. 100% investment in stock Y

c. 50% investment in stock X and 50% investment in stock Y

d. 80% investment in stock X and 20% investment in stock Y

8-4 Assume a family member is approaching retirement. Her retirement assets include her house and Social Security payments. She also has a 401(k) plan representing one-third of her assets. If she wants to own some foreign securities and decides to invest 15 percent of her 401(k) assets accordingly, what percentage of her total assets will this constitute?

Spreadsheet Exercises

8-1 Closing prices for SilTech and New Mines for the years 1997–2012 are shown below.

a. Calculate the total returns for each stock for the years 2012–1998 to 3 decimal places. Note that the price for 1997 is used to calculate the total return for 1998.

b. Assume that similar returns will continue in the future (i.e., average returns = expected returns). Calculate the expected return, variance and standard deviation for both stocks and insert these values in the spreadsheet. Use Average, Var, and Stdev functions.

c. Calculate the covariance between these two stocks based on the 15 years of returns.

d. Using the 11 different proportions that SilTech could constitute of the portfolio ranging from 0 percent to 100 percent in 10 percent increments, calculate the portfolio variance, standard deviation, and expected return.

e. Plot the tradeoff between return and risk for these two stocks based on the calculation in (d). Use the XY scatter diagram in Excel.

	SILTECH	NEWMINES
2012	198.08	21.634
2011	84.84	34.867
2010	71.89	44.67
2009	32.2	49.8
2008	10.69	49.55
2007	7.16	46.86
2006	10.95	53.11
2005	7.44	48.75
2004	25.7	63.12
2003	10.23	37.04
2002	3.28	31.67
2001	5.22	21.78
2000	7.97	14.45
1999	9.64	9.39
1998	7.13	14.99
1997	14.39	10.72

8-2 You are trying to decide whether to buy Banguard's Large Stock Equity Fund and/or its Treasury Bond Fund. You believe that next year involves several possible scenarios to which you have assigned probabilities. You have also estimated expected returns for each of the two funds for each scenario. Your spreadsheet looks like the following.

Next Year's Possibilities	Probability	Stock Fund Rate of Return	Column D	Bond Fund Rate of Return	Column F
Recession	0.2	−13		15	
Weak Econ	0.15	5		3	
Average Econ	0.6	10		7	
Strong Econ	0.05	24		−9	
		Exp Value =		Exp Value =	

a. Fill in columns D and F and calculate the expected return for each fund, given the probabilities for the four possible economic conditions and their associated rates of return.

b. Given the expected value for each fund for next year, fill out the following spreadsheet to calculate the standard deviation of each fund. Note that you need to fill in columns D, E, and F for the stock fund, and columns H, I, and J for the bond fund. The first two columns in each set are labeled; you need to determine what goes in columns F and J, respectively, which will lead to the variance, and then the standard deviation.

c.

		Stock Fund				Bond Fund			
Scenario	**Probability**	**Forecast Return**	**Column D Deviation from Exp.**	**Column E Squared Deviation**	**Column F**	**Forecast Return**	**Column H Deviation from Exp.**	**Column I Squared Deviation**	**Column J**
Recession	0.2	−13				15			
Weak Econ	0.15	5				3			
Moderate Econ	0.6	10				7			
Strong Econ	0.05	24				−9			
	Exp. Ret.	9.1		**Variance =**				**Variance =**	
				Std Dev =				**Std Dev =**	

d. Now calculate the covariance between the two funds, and the correlation coefficient, using the following format.

Scenario	Column B Probability	Deviation from Exp. Return for Stock fund	Deviation from Exp. Return for Bond fund	Product of Deviations	Col B × Col E
Recession	0.2				
Weak Econ	0.15				
Moderate Econ	0.6				
Strong Econ	0.05				
				Covariance =	**Corr Coeff =**

e. Using the formulas for the expected return and risk of a portfolio, calculate these values for each of the following portfolio weights.

w1 = stock fd % of funds in	w2 = bond fd % of funds in	Portfolio Expected Ret	Std Dev
0.1	0.9		
0.2	0.8		
0.3	0.7		
0.4	0.6		
0.5	0.5		
0.6	0.4		
0.7	0.3		
0.8	0.2		
0.9	0.1		

f. Which of the portfolios in (d) is the minimum variance portfolio?

g. Based on your analysis, should investors hold a portfolio of 100 percent bonds?

Answer to Question at the Beginning of the Chapter The price of gold can be very volatile, going down as well as up. Gold bullion has no current return (no income component) and has a carrying cost (interest cost if borrowed money is used to buy it, storage costs, and opportunity costs—an income-producing asset could have been bought instead).

Checking Your Understanding

8-1 Most portfolios are dominated by another portfolio that has either a higher return for the same level of risk or a lower risk for the same level of return.

8-2 Indifference curves allow us to talk about preferences with regard to the return-risk tradeoff.

8-3 International investing may not be as beneficial today as it was several years ago, but it is still beneficial, and investors should diversify internationally.

8-4 An efficient frontier that is pushed out has a higher level of return for a given level of risk than does the frontier below it.

8-5 Having made an asset allocation decision in 2001 to own common stocks, and stick with them, an investor's performance was essentially determined. Since the market performed badly during that period, this investor's portfolio would almost assuredly perform badly also. Conversely, another investor who decided to invest in Treasury bills in 2001 would have a positive performance.

chapter 9

Asset Pricing Principles

Although you have postponed dealing with the issue, in the back of your mind you remember from your finance course a well-known model called the CAPM. It occurs to you that this model, which was said to be so important in finance, probably has a role to play in your investing decisions. And in fact it does because it captures the concept of a required rate of return for a stock, which is important to consider when you are trying to decide which stocks to buy. So the time has come to bite the bullet and review some theory regarding asset prices and markets, and consider the CAPM once again. Knowing about the required rate of return will be important when you start trying to value common stocks, a topic that you are almost ready to tackle. Furthermore, understanding how risk is priced in financial markets can be very valuable to an investor.

In the last chapter we discussed portfolio theory, which is normative, describing how investors should act in selecting an optimal portfolio of risky securities. In this chapter we consider theories about asset pricing. What happens if all investors seek portfolios of risky securities using the Markowitz framework under idealized conditions? How will this affect equilibrium security prices and returns? In other words, how does optimal diversification affect the market prices of securities? Under these idealized conditions, what is the risk-return tradeoff that investors face? In general, we wish to examine models that explain security prices under conditions of market equilibrium. These are asset pricing models, or models for the valuation of risky assets.

We devote most of our attention to capital market theory (CMT), which begins where portfolio theory ends. CMT provides a model for pricing risky assets.[1] While CMT has its shortcomings, and arbitrage pricing theory provides an alternative, it remains the case that most investors are much more likely to encounter, and use, CMT in the form of the CAPM.

[1] Much of this analysis is attributable to the work of Sharpe. See W. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *The Journal of Finance*, 19 (September 1964): pp. 425–442. Lintner and Mossin developed a similar analysis.

AFTER READING THIS CHAPTER YOU WILL BE ABLE TO:

- Understand capital market theory as an extension of portfolio theory.
- Recognize the capital market line, which applies to efficient portfolios, and the security market line, which applies to all portfolios as well as individual securities.
- Understand and use the capital asset pricing model (CAPM) equation to calculate the required rate of return for a security.
- Recognize an alternative theory of how assets are priced, arbitrage pricing theory.

Capital Market Theory

Capital market theory is a positive theory in that it hypothesizes how investors do behave rather than how investors should behave, as in the case of modern portfolio theory (MPT). It is reasonable to view capital market theory as an extension of portfolio theory, but it is important to understand that MPT is not based on the validity, or lack thereof, of capital market theory.

The specific equilibrium model of interest to many investors is known as the capital asset pricing model, typically referred to as the CAPM. It allows us to assess the relevant risk of an individual security as well as to assess the relationship between risk and the returns expected from investing. The CAPM is attractive as an equilibrium model because of its simplicity and its implications. As a result of serious challenges to the model over time, however, alternatives have been developed. The primary alternative to the CAPM is arbitrage pricing theory, or APT, which allows for multiple sources of risk.

CAPITAL MARKET THEORY ASSUMPTIONS

Capital market theory involves a set of predictions concerning equilibrium expected returns on risky assets. It typically is derived by making some simplifying assumptions in order to facilitate the analysis and help us to more easily understand the arguments without fundamentally changing the predictions of asset pricing theory.

Capital market theory builds on Markowitz portfolio theory. Each investor is assumed to diversify his or her portfolio according to the Markowitz model, choosing a location on the efficient frontier that matches his or her return-risk references. Because of the complexity of the real world, additional assumptions are made to make individuals more alike:

1. All investors can borrow or lend money at the risk-free rate of return (designated RF in this text).
2. All investors have identical probability distributions for future rates of return; they have **homogeneous expectations** with respect to the three inputs of the portfolio model explained in Chapter 7: expected returns, the variance of returns, and the correlation matrix. Therefore, given a set of security prices and a risk-free rate, all investors use the same information to generate an efficient frontier.
3. All investors have the same one-period time horizon.
4. There are no transaction costs.
5. There are no personal income taxes—investors are indifferent between capital gains and dividends.
6. There is no inflation.

Homogeneous Expectations Investors have the same expectations regarding the expected return and risk of securities

7. There are many investors, and no single investor can affect the price of a stock through his or her buying and selling decisions. Investors are price-takers and act as if prices are unaffected by their own trades.
8. Capital markets are in equilibrium.

Realism of the Assumptions These assumptions appear unrealistic and often disturb investors encountering capital market theory for the first time. However, the important issue is how well the theory predicts or describes reality, and not the realism of its assumptions. If capital market theory does a good job of explaining the returns on risky assets, it is very useful and the assumptions made in deriving the theory are of less importance.

Most of these assumptions can be relaxed without significant effects on the capital asset pricing model (CAPM) or its implications; in other words, the CAPM is robust.[2] Although the results from such a relaxation of the assumptions may be less clear-cut and precise, no significant damage is done. Many conclusions of the basic model still hold.

Finally, most investors recognize that all of the assumptions of capital market theory are not unrealistic. For example, some institutional investors such as pension funds are tax-exempt, and brokerage costs today, as a percentage of the transaction, are very, very small. Nor is it too unreasonable to assume that for the one-period horizon of the model, inflation may be fully (or mostly) anticipated and, therefore, not a major factor.

INTRODUCTION OF THE RISK-FREE ASSET

The first assumption of capital market theory listed above is that investors can borrow and lend at the risk-free rate. Although the introduction of a risk-free asset appears to be a simple step to take in the evolution of portfolio and capital market theory, it is a very significant step. In fact, it is the introduction of a risk-free asset that allows us to develop capital market theory from portfolio theory.

With the introduction of a risk-free asset, investors can now invest part of their wealth in this asset and the remainder in any of the risky portfolios in the Markowitz efficient set. This allows Markowitz portfolio theory to be extended in such a way that the efficient frontier is completely changed, which in turn leads to a general theory for pricing assets under uncertainty.

Defining a Risk-Free Asset A risk-free asset can be defined as one with a certain-to-be-earned expected return and a variance of return of zero. (Note, however, that this is a nominal return and not a real return, which is uncertain because inflation is uncertain.) Since variance $= 0$, the nominal risk-free rate in each period will be equal to its expected value. Furthermore, the covariance between the risk-free asset and any risky asset i will be zero.

The true risk-free asset is best thought of as a Treasury security, which has little or no practical risk of default, with a maturity matching the holding period of the investor. In this case, the amount of money to be received at the end of the holding period is known with certainty at the beginning of the period. The Treasury bill (discussed in Chapter 2) typically is taken to be the risk-free asset, and its rate of return is referred to here as RF.

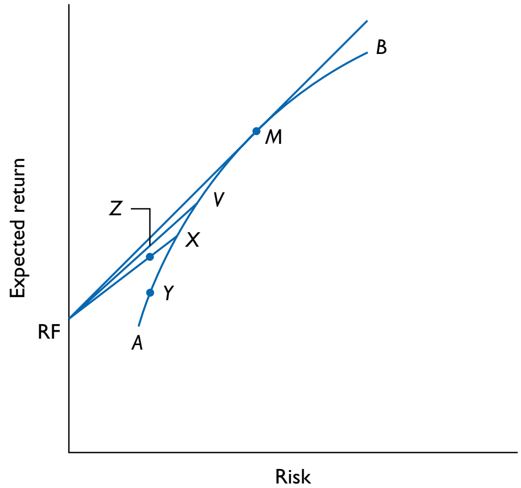
RISK-FREE BORROWING AND LENDING

Assume that the efficient frontier, as shown by the arc AB in Figure 9-1, has been derived by an investor. The arc AB delineates the efficient set of portfolios of *risky* assets as explained in

[2] For a discussion of changing these assumptions, see E. Elton, M. Gruber, S. Brown, and W. Goetzmann, *Modern Portfolio Theory and Investment Analysis*, 8th edition (New York: John Wiley & Sons, 2010), Chapter 14.

Figure 9-1

The Markowitz Efficient Frontier and the Possibilities Resulting from Introducing a Risk-Free Asset.



Chapter 8. (For simplicity, assume these are portfolios of common stocks.) We now introduce a risk-free asset with return RF and $\sigma = 0$.

As shown in Figure 9-1, the return on the risk-free asset (RF) will plot on the vertical axis because the risk is zero. Investors can combine this riskless asset with the efficient set of portfolios on the efficient frontier. By drawing a line between RF and various risky portfolios on the efficient frontier, we can examine combinations of risk-return possibilities that did not exist previously.

Lending Possibilities In Figure 9-1 a new line could be drawn between RF and the Markowitz efficient frontier above point X, for example, connecting RF to point V. Each successively higher line will dominate the preceding set of portfolios. This process ends when a line is drawn tangent to the efficient set of risky portfolios, given a vertical intercept of RF. In Figure 9-1, we call this tangency point M. The set of portfolio opportunities on this line (RF to M) dominates all portfolios below it.

The straight line from RF to the efficient frontier at point M, RF-M, dominates all straight lines below it and contains the superior *lending portfolios* given the Markowitz efficient set depicted in Figure 9-1. Lending refers to the purchase of a riskless asset such as Treasury bills, because by making such a purchase, the investor is lending money to the issuer of the securities, the U.S. government. We can think of this risk-free lending simply as *risk-free investing*.

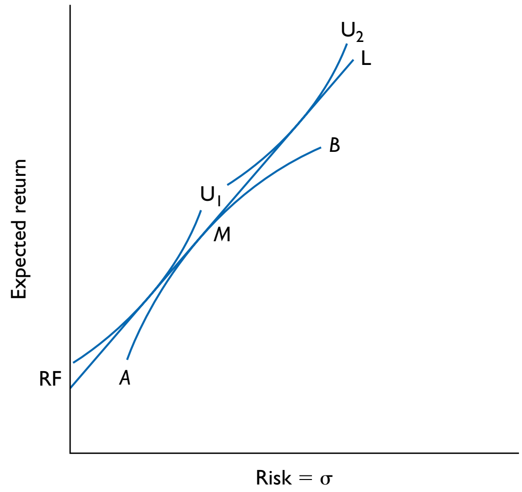
Borrowing Possibilities What if we extend this analysis to allow investors to borrow money? The investor is no longer restricted to his or her wealth when investing in risky assets. Technically, we are short-selling the riskless asset. One way to accomplish this borrowing is to buy stocks on margin, which has a current initial margin requirement of 50 percent. We will assume that investors can also borrow at the risk-free rate RF.[3] This assumption can be removed without changing the basic arguments.

Borrowing additional investable funds and investing them together with the investor's own wealth allows investors to seek higher expected returns while assuming greater risk. These

[3] Keep in mind that with lending the investor earns a rate RF, whereas with borrowing the investor pays the rate RF on the borrowed funds.

Figure 9-2

The Efficient Frontier when Lending and Borrowing Possibilities are Allowed.



borrowed funds can be used to leverage the portfolio position beyond point M, the point of tangency between the straight line emanating from RF and the efficient frontier AB. As in the lending discussion, point M represents 100 percent of an investor's wealth in the risky asset portfolio M. The straight line RF-M is now extended upward, as shown in Figure 9-2, and can be designated RF-M-L.

Checking Your Understanding

1. Why is the introduction of risk-free borrowing and lending such an important change relative to where the Markowitz analysis left off?

The Equilibrium Return-Risk Tradeoff

Given the previous analysis, we can now derive some predictions concerning equilibrium expected returns and risk. On an overall basis, we need an equilibrium model that encompasses two important relationships.

- The capital market line specifies the equilibrium relationship between expected return and risk for efficient portfolios.
- The security market line specifies the equilibrium relationship between expected return and systematic risk. It applies to individual securities as well as portfolios.

THE CAPITAL MARKET LINE

The straight line shown in Figure 9-2, which traces out the risk-return tradeoff for efficient portfolios, is tangent to the Markowitz efficient frontier at point M and has a vertical intercept RF. We now know that portfolio M is the tangency point to a straight line drawn from RF to the efficient frontier, and that this straight line is the best obtainable efficient-set line. All investors will hold portfolio M as their optimal risky portfolio, and all investors will be somewhere on this steepest tradeoff line between expected return and risk, because it represents those combinations of risk-free investing/borrowing and portfolio M that yield the highest return obtainable for a given level of risk.

Let's summarize what we have learned so far:

1. A risk-averse investor makes investment decisions based on Markowitz principles. Such investors select efficient portfolios of risky assets.
2. Investors can borrow and lend freely at the risk-free rate.
3. Each investor should construct an optimal portfolio that matches his or her preferred risk-return combination. Conservative investors lend, and aggressive investors borrow.
4. All investors can construct an optimal portfolio by combining an efficient portfolio, M, with a risk-free asset (fund). The result is a straight line in expected return, standard deviation of return space.

Capital Market Line (CML) The tradeoff between expected return and risk for efficient portfolios

Defining the Capital Market Line This straight line, usually referred to as the **Capital Market Line (CML)**, depicts the equilibrium conditions that prevail in the market for *efficient portfolios* consisting of the optimal portfolio of risky assets and the risk-free asset. All combinations of the risk-free asset and the risky portfolio M are on the CML, and, in equilibrium, all investors will end up with a portfolio somewhere on the CML based on their risk tolerance.

Understanding the CML The CML is shown as a straight line in Figure 9-3 without the now-dominated Markowitz frontier. We know that this line has an intercept of RF. If investors are to invest in risky assets, they must be compensated for this additional risk with a risk premium. The vertical distance between the risk-free rate and the CML at point M in Figure 9-3 is the amount of return expected for bearing the risk of owning a portfolio of stocks, that is, the excess return above the risk-free rate. At that point, the amount of risk for the risky portfolio of stocks is given by the horizontal dotted line between RF and σ_M.

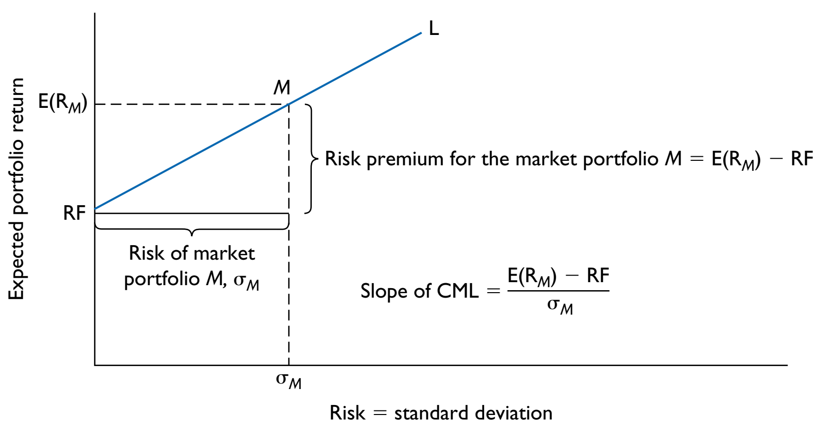
Therefore,

$$\frac{E(R_M) - RF}{\sigma_M} = \text{Slope of the CML}$$

$$= \text{Expected return-risk tradeoff for efficient portfolios}$$

Figure 9-3

The Capital Market Line and the Components of its Slope.



The slope of the CML is the *market price of risk* for efficient portfolios. It is also called the equilibrium market price of risk.[4] It indicates the additional return that the market demands for each percentage increase in a portfolio's risk, that is, in its standard deviation of return.

Example 9-1

Assume that the expected return on portfolio M is 13 percent, with a standard deviation of 20 percent, and that RF is 5 percent. The slope of the CML is

$$(0.13 - 0.05)/0.20 = 0.40$$

In our example a risk premium of 0.40 indicates that the market demands this amount of return for each percentage increase in a portfolio's risk.

The Equation for the CML We now know the intercept and slope of the CML. Since the CML is the tradeoff between expected return and risk for efficient portfolios, and risk is being measured by the standard deviation, the equation for the CML is shown as Equation 9-1:

$$E(R_p) = RF + \frac{E(R_M) - RF}{\sigma_M}\sigma p \tag{9-1}$$

where

$E(R_p)$ = the expected return on any efficient portfolio on the CML
RF = the rate of return on the risk-free asset
$E(R_M)$ = the expected return on the market portfolio M
σ_M = the standard deviation of the returns on the market portfolio
σ_p = the standard deviation of the efficient portfolio being considered

In words, the expected return for any portfolio on the CML is equal to the risk-free rate plus a risk premium. The risk premium is the product of the market price of risk and the amount of risk for the portfolio under consideration.

Important Points About the CML The following points should be noted about the CML:

1. Only efficient portfolios consisting of the risk-free asset and portfolio M lie on the CML. Portfolio M, the market portfolio of risky securities, contains all securities weighted by their respective market values—it is the optimum combination of risky securities and is, by definition, an efficient portfolio. The risk-free asset has no risk. Therefore, all combinations of these two assets on the CML are efficient portfolios.
2. As a statement of equilibrium, the CML must always be upward-sloping, because the price of risk must always be positive. Remember that the CML is formulated in a world

[4] The assumption throughout this discussion is that $E(R_M)$ is greater than RF. This is the only reasonable assumption to make, because the CAPM is concerned with expected returns (i.e., ex ante returns). After the fact, this assumption may not hold for particular periods—that is, over historical periods such as a year, RF has exceeded the return on the market, which is sometimes negative.

of expected return, and risk-averse investors will not invest unless they expect to be compensated for the risk. The greater the risk, the greater the expected return.

3. On a historical basis, for some particular period of time such as a year or two, or four consecutive quarters, the CML can be downward-sloping; that is, the return on RF exceeds the return on the market portfolio. This does not negate the validity of the CML; it merely indicates that returns actually realized differ from those that were expected. Obviously, investor expectations are not always realized. (If they were, there would be no risk.) Thus, although the CML must be upward-sloping ex ante (before the fact), it can be, and sometimes is, downward-sloping ex post (after the fact).
4. The CML can be used to determine the optimal expected returns associated with different portfolio risk levels. Therefore, the CML indicates the required return for each portfolio risk level.

Market Portfolio The portfolio of all risky assets, with each asset weighted by the ratio of its market value to the market value of all risky assets

The Market Portfolio

Portfolio M in Figure 9-2 is called the **market portfolio** of risky securities. It is the highest point of tangency between RF and the efficient frontier and is *the* optimal risky portfolio. All investors would want to be on the optimal line RF-M-L, and, unless they invested 100 percent of their wealth in the risk-free asset, they would own portfolio M with some portion of their investable wealth, or they would invest their own wealth plus borrowed funds in portfolio M.

✓ Portfolio M is the optimal portfolio of risky assets.[5]

Why do all investors hold identical risky portfolios? Based on our assumptions above, all investors use the same Markowitz analysis on the same set of securities, have the same expected returns and covariances, and have an identical time horizon. Therefore, they will arrive at the same optimal risky portfolio, and it will be the market portfolio, designated M.

It is critical to note that although investors take different positions on the straight-line efficient set in Figure 9-2, all investors are investing in portfolio M, the same portfolio of risky assets. This portfolio will always consist of all risky assets in existence. The emergence of the market portfolio as the optimal efficient portfolio is the most important implication of the CAPM.

In equilibrium, all risky assets must be in portfolio M because all investors are assumed to arrive at, and hold, the same risky portfolio. If the optimal portfolio did not include a particular asset, the price of this asset would decline dramatically until it became an attractive investment opportunity. At some point investors will purchase it, and it will be included in the market portfolio. Because the market portfolio includes all risky assets, *portfolio M is completely diversified*. Portfolio M contains only market (systematic) risk, which, even with perfect diversification, cannot be eliminated because it is the result of macroeconomic factors that affect the value of all securities.

[5] All assets are included in portfolio M in proportion to their market value. For example, if the market value of IBM constitutes 2 percent of the market value of all risky assets, IBM will constitute 2 percent of the market value of portfolio M, and, therefore, 2 percent of the market value of each investor's portfolio of risky assets. Therefore, we can state that security i's percentage in the risky portfolio M is equal to the total market value of security i relative to the total market value of all securities.

In theory, the market portfolio should include all risky assets worldwide, both financial (bonds, options, futures, etc.) and real (gold, real estate, etc.), in their proper proportions. The global aspects of such a portfolio are important to note. By one estimate, the value of non-U.S. assets exceeds 60 percent of the world total. U.S. equities make up only about 10 percent of total world assets. Therefore, international diversification is clearly important. A worldwide portfolio, if it could be constructed, would be completely diversified. Of course, the market portfolio is unobservable.

The market portfolio is often proxied by the portfolio of all common stocks, which, in turn, is proxied by a market index such as the Standard & Poor's 500 Composite Index, which has been used throughout the text. Therefore, to facilitate this discussion, think of portfolio M as a broad market index such as the S&P 500 Index. The market portfolio is, of course, a risky portfolio, because it consists of risky common stocks, and its risk will be designated σ_M.

The Separation Theorem We have established that each investor will hold combinations of the risk-free asset (either lending or borrowing) and the tangency portfolio from the efficient frontier, which we now know is the market portfolio M. Because we are assuming homogeneous expectations, in equilibrium all investors will determine the same tangency portfolio. Further, under the assumptions of CMT all investors agree on the risk-free rate. Therefore, the linear efficient set shown in Figure 9-2 now applies to all investors.

Borrowing and lending possibilities, combined with one portfolio of risky assets, M, offer an investor whatever risk-expected return combination he or she seeks; that is, investors can be anywhere they choose on this line, depending on their risk-return preferences. An investor could

(a) Invest 100 percent of investable funds in the risk-free asset, providing an expected return of RF and zero risk
(b) Invest 100 percent of investable funds in risky-asset portfolio M, offering $E(R_M)$, with its risk σ_M
(c) Invest in any combination of return and risk between these two points, obtained by varying the proportion w_{RF} invested in the risk-free asset
(d) Invest more than 100 percent of investable funds in the risky-asset portfolio M by borrowing money at the rate RF, thereby increasing both the expected return and the risk beyond that offered by portfolio M

Different investors will choose different portfolios because of their risk preferences (they have different indifference curves), but they will choose the same combination of risky securities as denoted by the tangency point in Figure 9-2, M. Investors will then borrow or lend to achieve various positions on the linear tradeoff between expected return and risk.

Unlike the Markowitz analysis, it is not necessary to match each client's indifference curves with a particular efficient portfolio, because only one efficient portfolio is held by all investors. Rather, each client will use his or her indifference curves to determine where along the new efficient frontier RF-M-L he or she should be. In effect, each client must determine how much of investable funds should be lent or borrowed at RF and how much should be invested in portfolio M. This result is referred to as a separation property.

Separation Theorem The idea that the decision of which portfolio of risky assets to hold is separate from the decision of how to allocate investable funds between the risk-free asset and the risky asset

The **separation theorem** states that the investment decision (which portfolio of risky assets to hold) is separate from the financing decision (how to allocate investable funds between the risk-free asset and the risky asset).

- The investment decision is a technical decision not involving the investor. The risky portfolio M is optimal for every investor regardless of that investor's utility function.
- The financing decision depends on an investor's preferences and is the decision of the investor.

All investors, by investing in the same portfolio of risky assets (M) and either borrowing or lending at the rate RF, can achieve any point on the straight line RF-M-L in Figure 9-2. Each point on that line represents a different expected return-risk tradeoff. An investor with utility curve U1 will be at the lower end of the line, representing a combination of lending and investment in M. On the other hand, utility curve U2 represents an investor borrowing at the rate RF to invest in risky assets—specifically, portfolio M.

The capital market line depicts the risk-return tradeoff in the financial markets in equilibrium. However, it applies only to efficient portfolios and cannot be used to assess the equilibrium expected return on a single security.

What about individual securities or inefficient portfolios? To relate expected return and risk for any asset or portfolio, efficient or inefficient, we need the expected return–beta form of the capital asset pricing model.

Checking Your Understanding

2. Explain why the CML applies only to efficient portfolios.
3. Why, under capital market theory, do investors not have to make the investment decision?

THE SECURITY MARKET LINE

The capital market line depicts the risk-return tradeoff in the financial markets in equilibrium. However, it applies only to efficient portfolios and cannot be used to assess the equilibrium expected return for a single security. What about individual securities or inefficient portfolios?

Under the CAPM all investors will hold the market portfolio, which is the benchmark portfolio against which other portfolios are measured. How does an individual security contribute to the risk of the market portfolio?

Investors should expect a risk premium for buying a risky asset such as a stock. The greater the riskiness of that stock, the higher the risk premium should be. If investors hold well-diversified portfolios, they should be interested in portfolio risk rather than individual security risk. Different stocks will affect a well-diversified portfolio differently. The relevant risk for an individual stock is its contribution to the riskiness of a well-diversified portfolio. And the risk of a well-diversified portfolio is market risk, or systematic risk, which is nondiversifiable (see Chapter 8).

We now know that investors should hold diversified portfolios to reduce the portfolio risk. When an investor adds a security to a large portfolio, what matters is the security's average covariance with the other securities in the portfolio. We also now know that under CMT all investors will hold the same portfolio of risky assets, the market portfolio. Therefore, the risk that matters when we consider any security is its covariance with the market portfolio.

✓ The major conclusion of the CAPM is: The relevant risk of any security is the amount of risk that security contributes to a well-diversified portfolio.

We could relate the expected return on a stock to its covariance with the market portfolio. This would result in an equation similar to Equation 9-1, except now it applies to any single asset *i*.

$$E(R_i) = RF + \frac{E(R_M) - RF}{\sigma_M^2} Cov_{i,M} \qquad \textbf{(9-2)}$$

where

$E(R_i)$ = the expected return on any individual security *i*
RF = the rate of return on the risk-free asset
$E(R_M)$ = the expected return on the market portfolio M
σ_M^2 = the variance of the returns on the market portfolio
$Cov_{i,M}$ = the covariance of the stock with the market

Equation 9-2 states that the expected return for any security is the sum of the risk-free rate and a risk premium. This risk premium reflects the asset's covariance with the market portfolio.

Beta A measure of volatility, or relative systematic risk, for a stock or a portfolio

Beta We know that the relevant risk measure for any asset *i* is its covariance with the market portfolio. However, it is more convenient to use a standardized measure of the systematic risk that cannot be avoided through diversification. **Beta** relates the covariance of an asset with the market portfolio to the variance of the market portfolio, and is defined as

$$\beta_i = \mathrm{COV}_{i,M}/\sigma_M^2$$

✓ Beta is a *relative measure* of risk—the risk of an individual stock relative to the market portfolio of all stocks.

If the security's returns move more (less) than the market's returns as the latter changes, the security's returns have more (less) volatility (fluctuations in price) than those of the market. For example, a security whose returns rise or fall on average 15 percent when the market return rises or falls 10 percent is said to be an aggressive, or volatile, security.

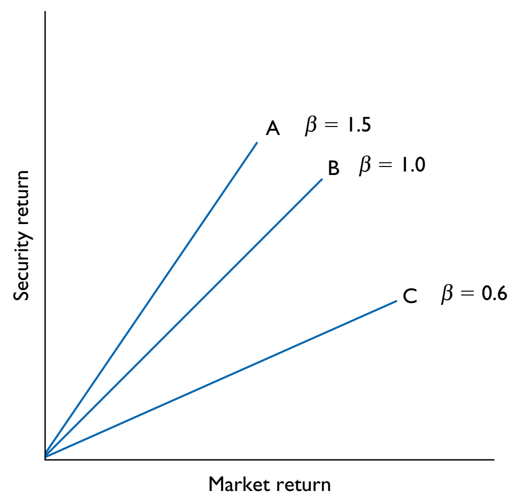
Securities with different slopes have different sensitivities to the returns of the market index. If the slope of this relationship for a particular security is a 45-degree angle, as shown for security B in Figure 9-4, the beta is 1.0. This means that for every 1 percent change in the market's return, *on average* this security's returns change 1 percent. The market portfolio has a beta of 1.0.

Example 9-2

In Figure 9-4 Security A's beta of 1.5 indicates that, *on average*, security returns are 1.5 times as volatile as market returns, both up and down. A security whose returns rise or fall on average 15 percent when the market return rises or falls 10 percent is said to be an aggressive, or volatile, security. If the line is less steep than the 45-degree line, beta is less than 1.0; this indicates that on average, a stock's returns have less volatility than the market as a whole. For example, security C's beta of 0.6 indicates that stock returns move up or down, on average, only 60 percent as much as the market as a whole.

Figure 9-4

Illustrative Betas of 1.5 (A), 1.0 (B), and 0.6 (C).



In summary, the aggregate market has a beta of 1.0. More volatile (risky) stocks have betas larger than 1.0, and less volatile (risky) stocks have betas smaller than 1.0. As a relative measure of risk, beta is very convenient. Beta is useful for comparing the relative systematic risk of different stocks and, in practice, is used by investors to judge a stock's riskiness. Stocks can be ranked by their betas. Because the variance of the market is a constant across all securities for a particular period, ranking stocks by beta is the same as ranking them by their absolute systematic risk. Stocks with high (low) betas are said to be high- (low-) risk securities.

Investments Intuition

Investors first exposed to the concepts of beta and CAPM may hear someone say that a high-beta stock or group of stocks held last year produced a lower return than did low-beta stocks, and therefore something is wrong with this concept. This is a fallacy, however, because the CAPM relationship is an equilibrium relationship expected to prevail. High-beta stocks are more risky than low-beta stocks and are expected to produce higher returns. However, they will not typically produce higher returns every period and over all intervals of time. If they did, they would be less risky than low-beta stocks, not more risky. The correct statement is that over long periods of time, high-beta stocks should produce higher average returns.

Security Market Line (SML) The graphical depiction of the CAPM

The CAPM's Expected Return–Beta Relationship The **Security Market Line (SML)** is the CAPM specification of how risk and expected rate of return for any asset, security or portfolio, are related. This theory posits a linear relationship between an asset's risk and its expected rate of return. This linear relationship, called the security market line (SML), is shown in Figure 9-5. Required rate of return is on the vertical axis and beta, the measure of risk, is on the horizontal return.[6] The slope of the line is the difference between the required rate of return on the market index and RF, the risk-free rate.

Investments Intuition

As we could (and should) expect, Figure 9-5 again demonstrates that if investors are to seek higher expected returns, they must assume a larger risk as measured by beta, the relative measure of systematic risk. The tradeoff between expected return and risk must always be positive. In Figure 9-5 the vertical axis can be thought of as the expected return for an asset. In equilibrium, investors require a minimum expected return before they will invest in a particular security. That is, given its risk, a security must offer some minimum expected return before a given investor can be persuaded to purchase it. Thus, in discussing the SML concept, we are simultaneously talking about the required and expected rate of return.

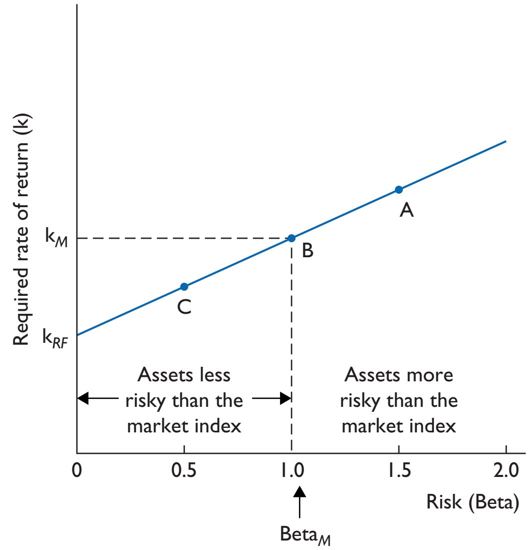
Capital Asset Pricing Model (CAPM) Relates the required rate of return for any security with the risk for that security as measured by beta

The **Capital Asset Pricing Model (CAPM)** formally relates the expected rate of return for any security or portfolio with the relevant risk measure. The CAPM's expected return–beta relationship is the most-often cited form of the relationship. Beta is the relevant measure of risk that cannot be diversified away in a portfolio of securities and, as such, is the measure that investors should consider in their portfolio management decision process.

[6] We use required rate of return in the figure to point out that in equilibrium the expected and required rates of return are the same for a security.

Figure 9-5

The Security Market Line (SML).



The CAPM in its expected return–beta relationship form is a simple but elegant statement. It says that the expected rate of return on an asset is a function of the two components of the required rate of return—the risk-free rate and the risk premium. Thus,

$$\begin{aligned} k_i &= \text{Risk-free rate} + \text{Risk premium} \\ &= \text{RF} + \beta_i[E(R_M) - \text{RF}] \end{aligned} \tag{9-3}$$

where

k_i = the required rate of return on asset i
$E(R_M)$ = the expected rate of return on the market portfolio
β_i = the beta coefficient for asset i

The risk premium should reflect all the uncertainty involved in the asset. Thinking of risk in terms of its traditional sources, such components as the business risk and the financial risk of a corporation would certainly contribute to the risk premium demanded by investors for purchasing the common stock of the corporation. After all, the risk to the investor is that the expected return will not be realized because of unforeseen events.

The particular business that a company is in will significantly affect the risk to the investor. One has only to look at the textile and steel industries in the last few years to appreciate business risk [which leads to an understanding of why industry analysis (Chapter 14) is important]. And the financial decisions that a firm makes (or fails to make) will also affect the riskiness of the stock.

Market Risk Premium The difference between the expected return for the equities market and the risk-free rate of return

The CAPM relationship described in Equation 9-3 provides an explicit measure of the risk premium. It is the product of the beta for a particular security i and the **market risk premium**, $E(R_M) - \text{RF}$. Thus,

$$\begin{aligned} \text{Risk premium for security } i &= \beta_i(\text{market risk premium}) \\ &= \beta_i[E(R_M) - \text{RF}] \end{aligned}$$

Investments Intuition

Equation 9-3 indicates that securities with betas greater than the market beta of 1.0 should have larger risk premiums than that of the average stock and therefore, when added to RF, larger required rates of return. This is exactly what investors should expect, since beta is a measure of risk, and greater risk should be accompanied by greater return. Conversely, securities with betas less than that of the market are less risky and should have required rates of return lower than that for the market as a whole. This will be the indicated result from the CAPM, because the risk premium for the security will be less than the market risk premium and, when added to RF, will produce a lower required rate of return for the security.

The CAPM's expected return–beta relationship is a simple but elegant statement about expected (required) return and risk for any security or portfolio. It formalizes the basis of investments, which is that the greater the risk assumed, the greater the expected (required) return should be. This relationship states that an investor requires (expects) a return on a risky asset equal to the return on a risk-free asset plus a risk premium, and the greater the risk assumed, the greater the risk premium.

Example 9-3

Assume that the beta for IBM is 1.15. Also assume that RF is 0.05 and that the expected return on the market is 0.12. The required return for IBM can be calculated as

$$\begin{aligned} k_{IBM} &= 0.05 + 1.15(0.12 - 0.05) \\ &= 13.05\% \end{aligned}$$

The required (or expected) return for IBM is, as it should be, larger than that of the market because IBM's beta is larger—once again, the greater the risk assumed, the larger the required return.

Over-and-Undervalued Securities The SML has important implications for security prices. In equilibrium, each security should lie on the SML because the expected return on the security should be that needed to compensate investors for the systematic risk.

What happens if investors determine that a security does not lie on the SML? To make this determination, they must employ a separate methodology to estimate the expected returns for securities. In other words, a SML can be fitted to a sample of securities to determine the required return-risk tradeoff that exists. Knowing the beta for any stock, we can determine the required return from the SML. Then, independently estimating the expected return from, say, fundamental analysis, an investor can assess a security in relation to the SML and determine whether it is under- or overvalued.

Example 9-4

In Figure 9-6, two securities are plotted around the SML. Security X has a high expected return derived from fundamental analysis and plots above the SML; security Y has a low expected return and plots below the SML. Which is undervalued?

Security X, plotting above the SML, is undervalued because it offers more expected return than investors require, given its level of systematic risk. Investors require a minimum expected return of $E(R_X)$, but security X, according to fundamental analysis, is offering $E(R_X{}')$. If investors recognize this, they will do the following:

> Purchase security X, because it offers more return than required. This demand will drive up the price of X, as more of it is purchased. The return will be driven down, until it is at the level indicated by the SML.

Now consider security Y. This security, according to investors' fundamental analysis, does not offer enough expected return given its level of systematic risk. Investors require $E(R_Y)$ for security Y, based on the SML, but Y offers only $E(R_Y{}')$. As investors recognize this, they will do the following:

> Sell security Y (or perhaps sell Y short), because it offers less than the required return. This increase in the supply of Y will drive down its price. The return will be driven up for new buyers because any dividends paid are now relative to a lower price, as is any expected price appreciation. The price will fall until the expected return rises enough to reach the SML and the security is once again in equilibrium.

Alpha The difference between an independently determined expected rate of return on a stock and the required rate of return on that stock.

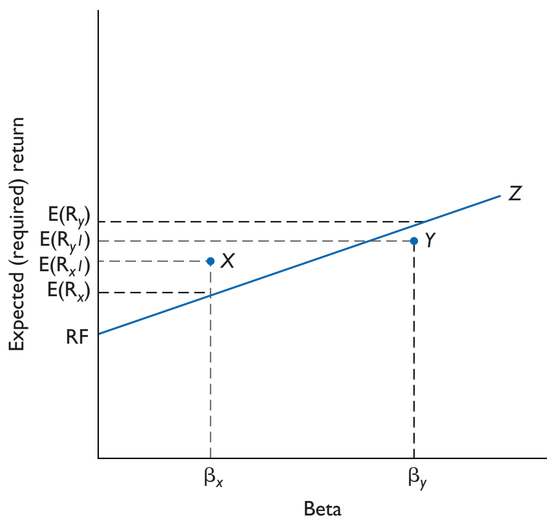
Alpha is the expected return for a stock above or below the expected return predicted for the stock by the CAPM. We can see this clearly in Figure 9-6. Alpha will be zero for stocks that are fairly priced; that is, on the SML. According to the CAPM, the expected value for alpha is zero for all securities.

Checking Your Understanding

4. We can relate the expected return on a security to its covariance with the market portfolio. Why, then, is the CAPM equation written using beta instead of covariance?
5. Why do overvalued securities plot below the SML?

Figure 9-6

Overvalued and Undervalued Securities Using the SML.



Estimating the SML

To implement the SML approach described here, an investor needs estimates of the return on the risk-free asset, the expected return on the market index, and the beta for an individual security. How difficult are these to obtain?

The return on a risk-free asset, RF, should be the easiest of the three variables to obtain. In estimating RF, the investor can use the return on Treasury bills for the coming period (e.g., a year).

Estimating the market return is more difficult, because the expected return for the market index is not observable. Furthermore, several different market indexes could be used. Estimates of the market return could be derived from a study of previous market returns (such as the Standard & Poor's data in Table 6-1). Alternatively, probability estimates of market returns could be made, and the expected value calculated. This would provide an estimate of both the expected return and the standard deviation for the market.

Finally, it is necessary to estimate the betas for individual securities. This is a crucial part of the CAPM estimation process. The estimates of RF and the expected return on the market are the same for each security being evaluated. Only beta is unique, bringing together the investor's expectations of returns for the stock with those for the market. Beta is the only company-specific factor in the CAPM; therefore, risk is the only asset-specific forecast that must be made in the CAPM.

ESTIMATING BETA

Market Model Relates the return on each stock to the return on the market, using a linear relationship with intercept and slope

A less restrictive form of the single index model referred to in Chapter 8 is known as the **market model**. This model is identical to the single index model except that the assumption of the error terms for different securities being uncorrelated is not made.

The market model equation can be expressed as

$$R_i = \alpha_i + \beta_i R_M + e_i \tag{9-4}$$

where

R_i = the return (TR) on security *i*
R_M = the return (TR) on the market index
α_i = the intercept term
β_i = the slope term
e_i = the random residual error

The market model produces an estimate of return for any stock.

To estimate the market model, the TRs for stock *i* can be regressed on the corresponding TRs for the market index. Estimates will be obtained of α_i (the constant return on security *i* that is earned regardless of the level of market returns) and β_i (the slope coefficient that indicates the expected increase in a security's return for a 1 percent increase in market return). This is how the estimate of a stock's beta is often derived.

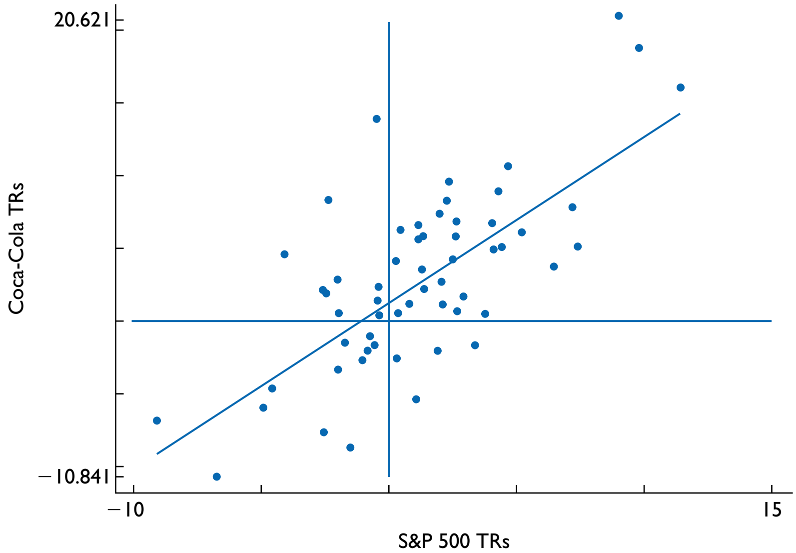
Example 9-5

To illustrate the calculation of the Market Model, we use Total Return (TR) data for the Coca-Cola company (ticker symbol "KO"). Fitting a regression equation to 60 months of return data along with corresponding TRs for the S&P 500, the estimated equation is

$$R_{KO} = 1.06 + 1.149 R_{S\&P500}$$

Figure 9-7

The Characteristic Line for Coca-Cola, Monthly Data.



Characteristic Line A regression equation used to estimate beta by regressing stock returns on market returns

When the TRs for a stock are plotted against the market index TRs, the regression line fitted to these points is referred to as the **characteristic line**. Coca-Cola's characteristic line is shown in Figure 9-7.

The characteristic line is often fitted using *excess returns*. The excess return is calculated by subtracting out the risk-free rate, RF, from both the return on the stock and the return on the market.

In excess return form, the same analysis as before applies. The alpha is the intercept of the characteristic line on the vertical axis and, in theory, should be zero for any stock. It measures the excess return for a stock when the excess return for the market portfolio is zero. In excess return form, the beta coefficient remains the slope of the characteristic line. It measures the sensitivity of a stock's excess return to that of the market portfolio.

The variance of the error term measures the variability of a stock's excess return not associated with movements in the market's excess return. Diversification can reduce this variability.

Many brokerage houses and investment advisory services report betas as part of the total information given for individual stocks. For example, *The Value Line Investment Survey* reports the beta for each stock covered, as do such brokerage firms as Merrill Lynch. Both measures of risk discussed above, standard deviation and beta, are widely known and discussed by investors.

Whether we use the single index model or the market model, beta can be estimated using regression analysis. However, the values of α_i and β_i obtained in this manner are estimates of the true parameters and are subject to error. Furthermore, beta can shift over time as a company's situation changes. A legitimate question, therefore, is how accurate are the estimates of beta?

As noted, beta is usually estimated by fitting a characteristic line to the data. However, this is an estimate of the beta called for in the CAPM. The market proxy used in the equations for estimating beta may not fully reflect the market portfolio specified in the CAPM. Furthermore, several points should be kept in mind:

1. We are trying to estimate the future beta for a security, which may differ from the historical beta.
2. In theory, the independent variable R_M represents the total of all marketable assets in the economy. This is typically approximated with a stock market index, which, in turn, is an approximation of the return on all common stocks.

3. The characteristic line can be fitted over varying numbers of observations and time periods. There is no one correct period or number of observations for calculating beta. As a result, estimates of beta will vary. For example, *The Value Line Investment Survey* calculates betas from weekly rates of return for five years, whereas other analysts often use monthly rates of return over a comparable period.
4. The regression estimates of α and β from the characteristic line are only estimates of the true α and β, and are subject to error. Thus, these estimates may not be equal to the true α and β.
5. As the fundamental variables (e.g., earnings, cash flow) of a company change, beta should change; that is, the beta is not perfectly stationary over time. This issue is important enough to be considered separately.

Blume found that in comparing nonoverlapping seven-year periods for 1, 2, 4, 7, 10, 21, etc., stocks in a portfolio, the following observations could be made:[7]

1. Betas estimated for individual securities are unstable; that is, they contain relatively little information about future betas.
2. Betas estimated for large portfolios are stable; that is, they contain much information about future betas.

In effect, a large portfolio (e.g., 50 stocks) provides stability because of the averaging effect. Although the betas of some stocks in the portfolio go up from period to period, others go down, and these two movements tend to cancel each other. Furthermore, the errors involved in estimating betas tend to cancel out in a portfolio. Therefore, estimates of portfolio betas show less change from period to period and are much more reliable than are the estimates for individual securities.

Researchers have found that betas in the forecast period are, on average, closer to 1.0 than the estimate obtained using historical data. This would imply that we can improve the estimates of beta by measuring the adjustment in one period and using it as an estimate of the adjustment in the next period. For example, we could adjust each beta toward the average beta by taking half the historical beta and adding it to half of the average beta. Merrill Lynch, the largest brokerage firm, reports adjusted betas based on a technique such as this. Other methods have also been proposed, including a Bayesian estimation technique.

Concepts in Action

Beta Management

Beta has emerged as a key concept in Investments. A measure of relative systematic risk, it relates stock or portfolio returns to the overall market. A portfolio with a beta of 1.0 should perform like the overall market, which by definition has a beta of 1.0. A portfolio with a beta greater than 1.0 would be expected to be more volatile than the market, while a portfolio with a beta less than one would be expected to be less volatile.

Beta management involves adjusting the beta of the portfolio to take account of expected market conditions. If the market is expected to go up (down), the portfolio manager would increase (decrease) the beta of the portfolio. During the 1980s and 1990s the market performed, on average, very strongly, and investors in stocks typically enjoyed good performance across the board. However, a beta management approach could show even better results by

[7] See M. Blume, "Betas and Their Regression Tendencies," *The Journal of Finance*, 10 (June 1975): pp. 785–795; and R. Levy, "On the Short-Term Stationarity of Beta Coefficients," *Financial Analysts Journal*, 27 (December 1971): pp. 55–62.

increasing the beta of the portfolio, leading to returns even better than the market.

With the market declines in 2000–2002, good returns were difficult to come by. Those who had to maintain stock portfolios regardless during these years (such as equity fund managers) but lowered the beta of the portfolio significantly were able to avoid the steep losses suffered by many. Of course, owning Treasury securities and avoiding stocks altogether in 2000–2002 was an even better strategy, in hindsight.

Simple beta management strategies continue to be in play. With low costs, a disciplined approach to managing the beta of the portfolio, along with well-defined goals, should continue to be rewarding. Nevertheless, in today's institutional investing world, merely providing some beta exposure while charging active-management fees is no longer considered appropriate by some overseers, such as pension fund executives. In today's world of lower stock returns and flattened yield curves, the pressures are intense for managers to produce some results beyond what could be attained by a passive approach such as owning a market index.

How has beta management changed over time? In the latter part of the twentieth century, based on the development of the CAPM and beta, investing centered on being in cash or in the market (risky assets), depending upon one's expectations. In today's global environment, the emphasis may be more on examining and shifting assets from one market to another. Some argue that global markets weaken the case for beta management, because the emphasis is more on getting out of one market and into another at the right time.

Tests of the CAPM

The conclusions of the CAPM are entirely sensible:

1. Return and risk are positively related—greater risk should carry greater return.
2. The relevant risk for a security is a measure of its effect on portfolio risk.

The question, therefore, is how well the theory works. After all, the assumptions on which capital market theory rest are, for the most part, unrealistic. To assess the validity of this or any other theory, empirical tests must be performed. If the CAPM is valid, and the market tends to balance out so that realized security returns average out to equal expected returns, equations of the following type can be estimated:

$$R_i = a_1 + a_2\beta i \qquad \textbf{(9-5)}$$

where

R_i = the average return on security i over some number of periods
β_i = the estimated beta for security i

When Equation 9-5 is estimated, a_1 should approximate the average risk-free rate during the periods studied, and a_2 should approximate the average market risk premium during the periods studied.

An extensive literature exists involving tests of capital market theory, in particular, the CAPM. Although it is not possible to summarize the scope of this literature entirely and to reconcile findings from different studies that seem to be in disagreement, the following points represent a reasonable consensus of the empirical results:[8]

1. The SML appears to be linear; that is, the tradeoff between expected (required) return and risk is an upward-sloping straight line.

[8] For a discussion of empirical tests of the CAPM, see Elton, Gruber, Brown and Goetzmann, *Modern Portfolio Theory and Investment Analysis*, 8th edition, John Wiley & Sons, Inc,. 2010.

2. The intercept term, a_1, is generally found to be higher than RF.
3. The slope of the CAPM, a_2, is generally found to be less steep than posited by the theory.
4. Although the evidence is mixed, no persuasive case has been made that unsystematic risk commands a risk premium. In other words, investors are rewarded only for assuming systematic risk.

The major problem in testing capital market theory is that it is formulated on an ex ante basis but can be tested only on an ex post basis. We can never know investor expectations with certainty. Therefore, it should come as no surprise that tests of the model have produced conflicting results in some cases, and that the empirical results diverge from the predictions of the model. In fact, it is amazing that the empirical results support the basic CAPM as well as they do. Based on studies of many years of data, it appears that the stock market prices securities on the basis of a linear relationship between systematic risk and return, with diversifiable (unsystematic) risk playing little or no part in the pricing mechanism.

The CAPM has not been proved empirically, nor will it be. In fact, Roll has argued that the CAPM is untestable because the market portfolio, which consists of all risky assets, is unobservable.[9] In effect, Roll argues that tests of the CAPM are actually tests of the mean-variance efficiency of the market portfolio. Nevertheless, the CAPM remains a logical way to view the expected return-risk tradeoff as well as a frequently used model in finance.

Arbitrage Pricing Theory

Arbitrage Pricing Theory (APT) An equilibrium theory of expected returns for securities involving few assumptions about investor preferences

The CAPM is not the only model of security pricing. Another model that has received attention is based on **Arbitrage Pricing Theory (APT)** as developed by Ross and enhanced by others. In recent years APT has emerged as an alternative theory of asset pricing to the CAPM. Its appeal is that it is more general than the CAPM, with less restrictive assumptions. However, like the CAPM, it has limitations, and like the CAPM, it is not the final word in asset pricing.

Similar to the CAPM, or any other asset pricing model, APT posits a relationship between expected return and risk. It does so, however, using different assumptions and procedures. Very importantly, APT is not critically dependent on an underlying market portfolio as is the CAPM, which predicts that only market risk influences expected returns. Instead, APT recognizes that several types of risk may affect security returns.

THE LAW OF ONE PRICE

APT is based on the *law of one price*, which states that two otherwise identical assets cannot sell at different prices. APT assumes that asset returns are linearly related to a set of indexes, where each index represents a factor that influences the return on an asset. Market participants develop expectations about the sensitivities of assets to the factors. They buy and sell securities so that, given the law of one price, securities affected equally by the same factors will have equal expected returns. This buying and selling is the arbitrage process, which determines the prices of securities.

APT states that equilibrium market prices will adjust to eliminate any arbitrage opportunities, which refer to situations where a *zero investment portfolio* can be constructed that will yield a risk-free profit. If arbitrage opportunities arise, a relatively few investors can act to restore equilibrium.

[9] R. Roll, "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory," *Journal of Financial Economics*, 4 (March 1977): pp. 129–176.

ASSUMPTIONS OF APT

Unlike the CAPM, APT does not assume

1. A single-period investment horizon
2. The absence of taxes
3. Borrowing and lending at the rate RF
4. Investors select portfolios on the basis of expected return and variance

APT, like the CAPM, does assume that

1. Investors have homogeneous beliefs
2. Investors are risk-averse utility maximizers
3. Markets are perfect
4. Returns are generated by a factor model

FACTOR MODELS

Factor Model Used to depict the behavior of security prices by identifying major factors in the economy that affect large numbers of securities

A **factor model** is based on the view that there are underlying *risk factors* that affect realized and expected security returns. These risk factors represent broad economic forces and not company-specific characteristics and, by definition, they represent the element of surprise in the risk factor—the difference between the actual value for the factor and its expected value.

The factors must possess three characteristics:[10]

1. Each risk factor must have a pervasive influence on stock returns. Firm-specific events are not APT risk factors.
2. These risk factors must influence expected return, which means they must have non-zero prices. This issue must be determined empirically, by statistically analyzing stock returns to see which factors pervasively affect returns.
3. At the beginning of each period, the risk factors must be unpredictable to the market as a whole. This raises an important point. In our example above, we used inflation and the economy's output as the two factors affecting portfolio returns. The rate of inflation is **not** an APT risk factor, because it is at least partially predictable. In an economy with reasonable growth where the quarterly rate of inflation has averaged 3 percent on an annual basis, we can reasonably assume that next quarter's inflation rate is not going to be 10 percent. On the other hand, unexpected inflation—the difference between actual inflation and expected inflation—is an APT risk factor. By definition, it cannot be predicted since it is unexpected.

What really matters are the *deviations* of the factors from their expected values. For example, if the expected value of inflation is 5 percent and the actual rate of inflation for a period is only 4 percent, this 1 percent deviation will affect the actual return for the period.

[10] See Michael A. Berry, Edwin Burmeister, and Marjorie B. McElroy, "Sorting Out Risks Using Known APT Factors," *Financial Analysts Journal* (March–April 1988): pp. 29–42.

Example 9-6 An investor holds a portfolio of stocks that she thinks is influenced by only two basic economic factors, inflation and the economy's output. Diversification once again plays a role, because the portfolio's sensitivity to all other factors can be eliminated by diversification.

Portfolio return varies directly with output, and inversely with inflation. Each of these factors has an expected value, and the portfolio has an expected return when the factors are at their expected values. If either or both of the factors deviates from expected value, the portfolio return will be affected.

We must measure the sensitivity of each stock in our investor's portfolio to changes in each of the two factors. Each stock will have its own sensitivity to each of the factors. For example, stock #1 (a mortgage company) may be particularly sensitive to inflation and have a sensitivity of 2.0, while stock #2 (a food manufacturer) may have a sensitivity to inflation of only 1.0.

UNDERSTANDING THE APT MODEL

Based on this analysis, we can now understand the APT model. It assumes that investors believe that asset returns are randomly generated according to a *n*-factor model, which, for security *i*, can be formally stated as

$$\mathrm{R}_i = E(R_i) + \beta_{i1}\mathrm{f}_1 + \beta_{i2}\mathrm{f}_2 + \cdots + \beta_{in}\mathrm{f}_n + \mathrm{e}_i \tag{9-6}$$

where

R_i = the actual (random) rate of return on security i in any given period t
$E(R_i)$ = the expected return on security i
f = the deviation of a systematic factor F from its expected value
β_i = sensitivity of security i to a factor
e_i = random error term, unique to security i[11]

It is important to note that the expected value of each factor, F, is zero. Therefore, the *fs* in Equation 9-6 are measuring the deviation of each factor from its expected value. Notice in Equation 9-6 that the actual return for a security in a given period will be at the expected or required rate of return if the factors are at expected levels [e.g., $\mathrm{F}_1 - \mathrm{E}(\mathrm{F}_1) = 0$, $\mathrm{F}_2 - \mathrm{E}(\mathrm{F}_2) = 0$, and so forth] and if the chance element represented by the error term is at zero.

A factor model makes no statement about equilibrium. If we transform Equation 9-10 into an equilibrium model, we are saying something about *expected* returns across securities. APT is an equilibrium theory of expected returns that requires a factor model such as Equation 9-6. The equation for expected return on a security is given by Equation 9-7:

$$E(R_i) = \mathrm{a}_0 + \mathrm{b}_{i1}\overline{\mathrm{F}}_1 + \mathrm{b}_{i2}\overline{\mathrm{F}}_2 + \cdots + \mathrm{b}_{in}\overline{\mathrm{F}}_n \tag{9-7}$$

where

$E(R_i)$ = the expected return on security i
a_0 = the expected return on a security with zero systematic risk
$\overline{\mathrm{F}}$ = the risk premium for a factor (for example, the risk premium for F_1 is equal to $\mathrm{E}(F_1) - \mathrm{a}_0$)

[11] It is assumed that all covariances between returns on securities are attributable to the effects of the factors; therefore, the error terms are uncorrelated.

With APT, risk is defined in terms of a stock's sensitivity to basic economic factors, while expected return is directly related to sensitivity. As always, expected return increases with risk.

The expected return-risk relationship for the CAPM is

$$E(R_i) = \mathrm{RF} + \beta_i[\text{market risk premium}]$$

The CAPM assumes that the only required measure of risk is the sensitivity to the market. The risk premium for a stock depends on this sensitivity and the market risk premium (the difference between the expected return on the market and the risk-free rate).

The expected return-risk relationship for the APT can be described as

$$\begin{aligned} \mathrm{E}(\mathrm{R}_i) = \mathrm{RF} &+ \mathrm{b}_{i1}(\text{risk premium for factor 1}) \\ &+ \mathrm{b}_{i2}(\text{risk premium for factor 2}) + \cdots \\ &+ \mathrm{b}_{in}(\text{risk premium for factor } n) \end{aligned}$$

Note that the sensitivity measures (β_i and b_i) have similar interpretations. They are measures of the relative sensitivity of a security's return to a particular risk premium. Also notice that we are dealing with risk premiums in both cases. Finally, notice that the CAPM relationship is the same as would be provided by APT if there were only one pervasive factor influencing returns. APT is more general than CAPM.

IDENTIFYING THE FACTORS

The problem with APT is that the factors are not well specified, at least ex ante. To implement the APT model, we need to know the factors that account for the differences among security returns. The APT makes no statements about the size or the sign of the F_i's. Both the factor model and these values must be identified empirically. In contrast, with the CAPM the factor that matters is the market portfolio, a concept that is well understood conceptually; however, as noted earlier, Roll has argued that the market portfolio is unobservable.

Early empirical work by Roll and Ross suggested that three to five factors influence security returns and are priced in the market.[12] Typically, systematic factors such as the following have been identified:

1. Changes in expected inflation
2. Unanticipated changes in inflation
3. Unanticipated changes in industrial production
4. Unanticipated changes in the default-risk premium
5. Unanticipated changes in the term structure of interest rates

These factors are related to the components of a valuation model. The first three affect the cash flows of a company while the last two affect the discount rate.

According to APT models, different securities have different sensitivities to these systematic factors, and investor risk preferences are characterized by these dimensions. Each

[12] R. Roll and S. Ross, "An Empirical Investigation of the Arbitrage Pricing Theory," *The Journal of Finance*, 35 (December 1980), pp. 1073–1103.

investor has different risk attitudes. Investors could construct a portfolio depending upon desired risk exposure to each of these factors. Knowing the market prices of these risk factors and the sensitivities of securities to changes in the factors, the expected returns for various stocks could be estimated.

Another study has suggested that an APT model that incorporates unanticipated changes in five macroeconomic variables is superior to the CAPM. These five variables are[13]

1. Default risk
2. The term structure of interest rates
3. Inflation or deflation
4. The long-run expected growth rate of profits for the economy
5. Residual market risk

USING APT IN INVESTMENT DECISIONS

Roll and Ross have argued that APT offers an approach to strategic portfolio planning. The idea is to recognize that a few systematic factors affect long-term average returns. Investors should seek to identify the few factors affecting most assets in order to appreciate their influence on portfolio returns. Based on this knowledge, they should seek to structure the portfolio in such a way as to improve its design and performance.

Some researchers have identified and measured, for both economic sectors and industries, the risk exposures associated with APT risk factors such as the five identified above as changes in five macroeconomic variables. These "risk exposure profiles" vary widely. For example, the financial, growth, and transportation sectors were found to be particularly sensitive to default risk, while the utility sector was relatively insensitive to both unexpected inflation and the unexpected change in the growth rate of profits.

An analysis of 82 different industry classifications showed the same result—exposure to different types of risk varies widely. For example, some industries were particularly sensitive to unexpected inflation risk, such as the mobile home building industry, retailers, hotels and motels, toys, and eating places. The industries least sensitive to this risk factor included foods, tire and rubber goods, shoes, and breweries. Several industries showed no significant sensitivity to unexpected inflation risk, such as corn and soybean refiners and sugar refiners.

A portfolio manager could design strategies that would expose them to one or more types of these risk factors, or "sterilize" a portfolio such that its exposure to the unexpected change in the growth rate of profits matched that of the market as a whole. Taking an active approach, a portfolio manager who believes that he or she can forecast a factor realization can build a portfolio that emphasizes or deemphasizes that factor. In doing this, the manager would select stocks that have exposures to the remaining risk factors that are exactly proportional to the market. If the manager is accurate with the forecast—and remember that such a manager must forecast the unexpected component of the risk factor—he or she can outperform the market for that period.

Checking Your Understanding

6. Can the CAPM be considered simply a special case of APT?

[13] These factors are based on Berry et al.

Some Conclusions about Asset Pricing

The question of how security prices and equilibrium returns are established—whether as described by the CAPM or APT or some other model—remains open. Some researchers are convinced that the APT model is superior to the CAPM. For example, based on their research using the five factors discussed above, the authors concluded that "[t]he APT model with these five risk factors is vastly superior to both the market model and the CAPM for explaining stock returns." The CAPM relies on the observation of the market portfolio, which, in actuality, cannot be observed. On the other hand, APT offers no clues as to the identity of the factors that are priced in the factor structure.

In the final analysis, neither model has been proven superior. Both rely on expectations which are not directly observable. Additional testing is needed.

Summary

- Capital market theory, based on the concept of efficient diversification, describes the pricing of capital assets in the marketplace.
- Capital market theory is derived from several assumptions that appear unrealistic; however, the important issue is the ability of the theory to predict. Relaxation of most of the assumptions does not change the major implications of capital market theory.
- Risk-free borrowing and lending changes the efficient set to a straight line.
- Borrowing and lending possibilities, combined with one portfolio of risky assets, offer an investor whatever risk-expected return combination he or she seeks; that is, investors can be anywhere they choose on this line, depending on their risk-return preferences.
- Given risk-free borrowing and lending, the new efficient frontier has a vertical intercept of RF and is tangent to the old efficient frontier at point M, the market portfolio. The new efficient set is no longer a curve, or arc, as in the Markowitz analysis. It is now linear.
- All investors can achieve an optimal point on the new efficient frontier by investing in portfolio M and either borrowing or lending at the risk-free rate RF.
- The new efficient frontier is called the capital market line, and its slope indicates the equilibrium price of risk in the market. In effect, it is the expected return-risk tradeoff for efficient portfolios.
- Ex ante, the CML must always be upward-sloping, although ex post it may be downward-sloping for certain periods.
- In theory, the market-value-weighted market portfolio, M, should include all risky assets, although in practice it is typically proxied by a stock market index such as the Standard & Poor's 500.
- The separation theorem states that the investment decision (what portfolio of risky assets to buy) can be separated from the financing decision (how much of investable funds should be put in risky assets and how much in the risk-free asset).
- Under the separation theorem, all investors should hold the same portfolio of risky assets and achieve their own position on the return-risk tradeoff through borrowing and lending.
- Investors need to focus on that part of portfolio risk that cannot be eliminated by diversification because this is the risk that should be priced in financial markets.
- Total risk can be divided into systematic risk and nonsystematic risk. Nonsystematic risk, also called diversifiable risk, can be eliminated by diversification.
- Market risk cannot be eliminated by diversification and is the relevant risk for the pricing of financial assets in the market.
- Based on the separation of risk into its systematic and nonsystematic components, the security market line can be constructed for individual securities (and portfolios). What is important is each security's contribution to the total risk of the portfolio, as measured by beta.
- Using beta as the measure of risk, the SML depicts the tradeoff between required return and risk for all securities and all portfolios.