Revised Simplex Method

- Streamlined versions of the simplex method for computer implementations do not follow the original simplex algorithm (algebraic or tabular form)

- *The Revised Simplex Algorithm* explicitly uses matrix manipulations

  it computes and stores only the following information required for each iteration,

  – the coefficients of the non-basic variables in Equation 0,
  – the coefficients of the entering variable in the other equations,
  – the RHS of all the equations.

Standard form of LP

$$\text{Max } Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$\text{S.t.} \quad a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}\,x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}\,x_n \leq b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}\,x_n \leq b_m$$

$$\text{and} \quad x_j \geq 0, \text{ for } j = 1, 2, \ldots, n$$

Augmented form of LP

$$\text{Max } Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$\text{S.t.} \quad a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}\,x_n + x_{n+1} = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}\,x_n + x_{n+2} = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}\,x_n + x_{n+m} = b_m$$

$$\text{and} \quad x_j \geq 0, \text{ for } j = 1, 2, \ldots, n, n+1, \ldots. n+m$$

Matrix form of Standard form of LP

$$\text{Max } Z = \mathbf{cx}$$

$$\text{S.t. } \mathbf{Ax} \leq \mathbf{b}$$

$$\mathbf{x} \geq \mathbf{0}$$

Matrix form of Augmented form of LP

$$\text{Max } Z = \mathbf{cx}$$

$$\text{S.t. } \mathbf{Ax} + \mathbf{Ix}_s = \mathbf{b}$$

$$\mathbf{x}, \mathbf{x}_s \geq \mathbf{0}$$

$$\Rightarrow$$

$$\text{Max } Z = \mathbf{cx} + \mathbf{0x}_s$$

$$\text{S.t } \begin{bmatrix} \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} = \mathbf{b}$$

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} \geq \mathbf{0}$$

Where,

$$\mathrm{c}_{1\times n} = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \qquad \mathrm{A}_{m\times n} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad \mathrm{b}_{m\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$\mathrm{x}_{n\times 1} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathrm{I}_{m\times m} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & 1 \end{bmatrix} \qquad \mathrm{x}_{s(1\times m)} = \begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \vdots \\ x_{n+m} \end{bmatrix}$$

LP problem in matrix Form

Tech edge Comp Problem

Max $Z = 50x_1+40x_2$
S.t.
$3x_1 + 5x_2 \leq 150$
$x_2 \leq 20$
$8x_1 + 5x_2 \leq 300$
$x_1, x_2 \geq 0$

In Augmented Form

Max $Z = 50x_1+40x_2$
S.t.
$3x_1 + 5x_2 + x_3 = 150$
$x_2 + x_4 = 20$
$8x_1 + 5x_2 + x_5 = 300$
$x_1, x_2, x_3, x_4 , x_5 \geq 0$

Matrix Form

$$\text{Max } Z = \mathbf{cx} + \mathbf{0x}_s$$

$$[\mathbf{A} \quad \mathbf{I}]\begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} = \mathbf{b} \qquad \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} \geq \mathbf{0}$$

$$\mathbf{c} = [50 \quad 40]$$

$$[\mathbf{A} \quad \mathbf{I}] = \begin{bmatrix} 3 & 5 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 8 & 5 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 150 \\ 20 \\ 300 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \mathbf{x_s} = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

Solving for a basic feasible solution (for the given basic and nonbasic variables)

$$\text{Max } Z = \mathbf{c}\mathbf{x} + \mathbf{0}\mathbf{x}_s$$

$$\text{S.t} \quad [\text{A} \quad \text{I}]\begin{bmatrix} \text{x} \\ \text{x}_s \end{bmatrix} = \text{b}$$

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} \geq \mathbf{0}$$

$\Rightarrow$

$$\textbf{Max}\, Z = [\mathbf{c}_B \quad \mathbf{c}_N]\begin{bmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{bmatrix}$$

$$\text{S.t} \; [\mathbf{B} \quad \mathbf{N}]\begin{bmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{bmatrix} = \mathbf{b}$$

$$\begin{bmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{bmatrix} \geq \mathbf{0}$$

Where

$\text{X}_{\text{B}(m\times 1)}$: vector of basic variables obtained by eliminating the nonbasic variables from $\begin{bmatrix} \text{x} \\ \text{x}_s \end{bmatrix}$, where the first row in X_B is the basic variable corresponding to the 1st functional constraint, and so on

$\text{B}_{m\times m}$: the basis matrix obtained by eliminating the columns corresponding to coefficients of nonbasic variables from $[\text{A I}]$, where the 1st column in B is the column corresponding to the 1st basic variable in X_B, and so on

Solution key

- **Total number of variables =** $m + n$
- **Number of basic variables =** m **number of nonbasic variables =** n
- **Set all nonbasic variables to zero**
- **results in** m **equations in** m **unknowns (basic variables)**

$$\mathbf{B}\mathbf{x}_B + \mathbf{N}\mathbf{x}_N = \mathbf{b} \;\Rightarrow\; \mathbf{B}^{-1}\mathbf{B}\mathbf{x}_B + \mathbf{B}^{-1}\mathbf{N}\mathbf{x}_N = \mathbf{B}^{-1}\mathbf{b} \text{ (where, } \mathbf{B} \text{ is nonsingular so that } \mathbf{B}^{-1} \text{exits)}$$

$$\Rightarrow \boxed{\mathbf{x}_B = \mathbf{B}^{-1}\mathbf{b}} \quad (\text{Since, } \mathbf{x}_N = \mathbf{0})$$

$$\boxed{\textbf{The value of objective function, } Z = \mathbf{c}_B\mathbf{x}_B = \mathbf{c}_B\mathbf{B}^{-1}\mathbf{b}}$$