Computational Statistics

$U[0,1] \longrightarrow \exp(\lambda)$ $\quad Y = -\lambda \log(1-U)$

$U[0,1) \longrightarrow N(0,1)$

$$X_1 = \sqrt{-2\log U_1}\ \cos 2\pi U_2$$

$$X_2 = \sqrt{-2\log U_2}\ \sin 2\pi U_2$$

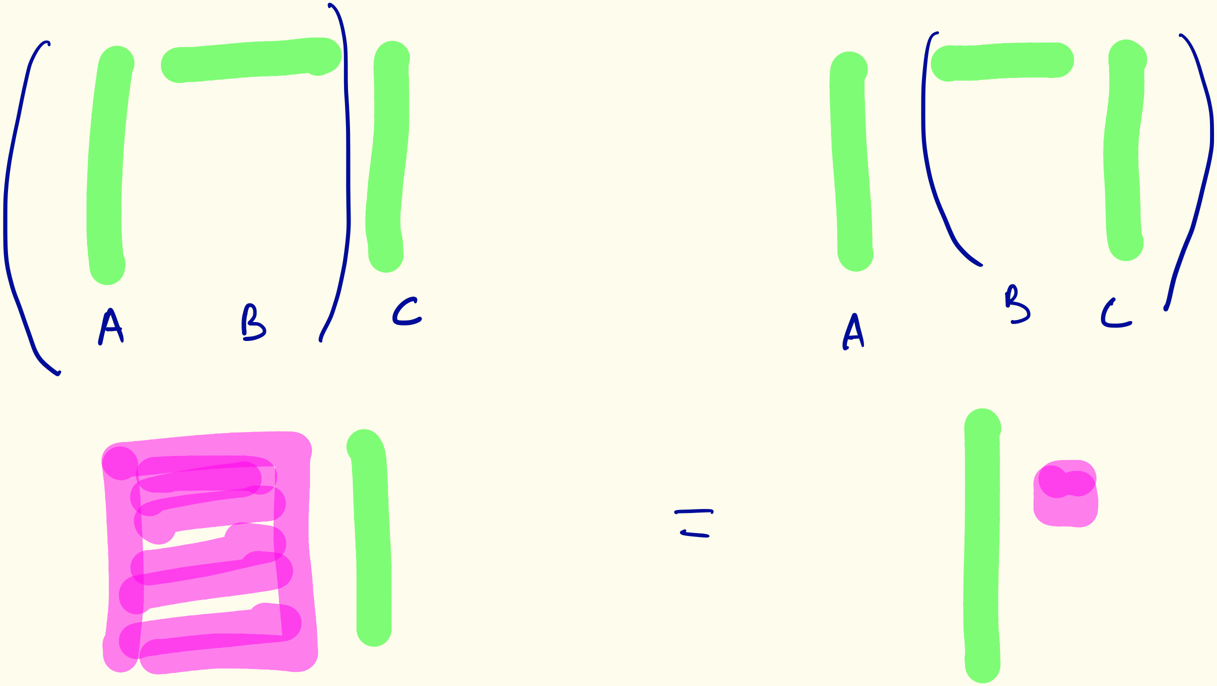
$$\mu + \sigma N(0,1) \sim N(\mu, \sigma^2)$$

Generate 100 samples of $\exp(\lambda)$

Assumption: $U[0,1]$ samples are "freely" available.

$\boxed{k}$ operations (flops) for one sample $U[0,1] \longrightarrow$ one sample of $\exp(\lambda)$

$100k$


A
B
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A
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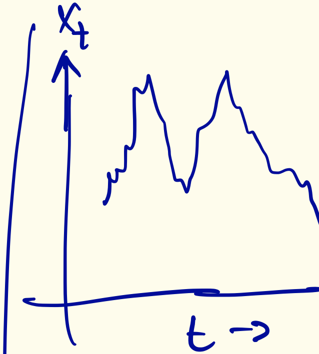
=

$X_1, X_2, \dots, X_n \overset{iid}{\sim} U[0,1]$



$Y = X_{(n)} : \max_n \{X_1, \dots, X_n\}$

$f_Y(y) = n y^{n-1}$ $\quad 0 \le y \le 1$

f_Y is $\beta(a,b)$ for some a & b.

$\{X_t$

Markov chains:

Consider a Markov Chain $X = \{X_t, t \in \mathbb{N}\}$

$X_1, X_2, X_3, X_4, \dots$

X_{t_0}

$\{X_t(w_0) : \forall t\}$

$\boxed{X_t}$ $\quad (\Omega, \mathcal{F}, P)$

$X_t : \Omega \longrightarrow \mathbb{R}^{\boxed{d}}$

$w_0 \in \Omega$

$X : \Omega \times T \to \mathbb{R}^d$

$\Omega = \mathbb{R}$, $\boxed{T = [0, \infty)}$

$t_0 \in T$, $\quad x(t_0)$

$X : \Omega \times T \longrightarrow \mathbb{R}^d$

Ω = countable set

$\boxed{T = [0, \infty)}$

$t_0 \quad x(t_0)$

$$P(X_{t+1} = x_{t+1} \mid X_0 = x_0, X_1 = x_1, \ldots, X_t = x_t)$$

$$= P(X_{t+1} = x_{t+1} \mid X_t = x_t)$$

Let $\mathcal{E} = \Omega$ be discrete/ countable

$\uparrow$

state space.

$$P(X_{t+1} = x_j \mid X_t = x_i)$$

$x_i, x_j \in \mathcal{E}$

$\{x_1, x_2, \ldots\} = \Omega$

is countable

$$P(X_0 = x_0, X_1 = x_1, \dots, X_t = x_t)$$

$$= P(X_0 = x_0)\, P(X_1 = x_1 \mid X_0 = x_0) \cdots P(X_t = x_t \mid X_{t-1} = x_{t-1}, \dots, X_0 = x_0)$$

$$= P(X_0 = x_0)\, P(X_1 = x_1 \mid X_0 = x_0) \cdots P(X_t = x_t \mid X_{t-1} = x_{t-1})$$

Transition probabilities.

$P(X_{t+1} = x_j \mid X_t = x_i)$ for fixed $x_i \in \Omega$ & $\forall x_j \in \Omega$

$$\Rightarrow \sum_{x_j \in \Omega} P(X_{t+1} = x_j \mid X_t = x_i) = 1$$

Consider $p_0 = (P(X_0 = x_0), P(X_0 = x_1), \dots, \dots)$

$p_1 = (P(X_1 = x_0), P(X_1 = x_1), P(X_1 = x_2), \dots)$

$P_1 = P_0 P$

P = transition matrix

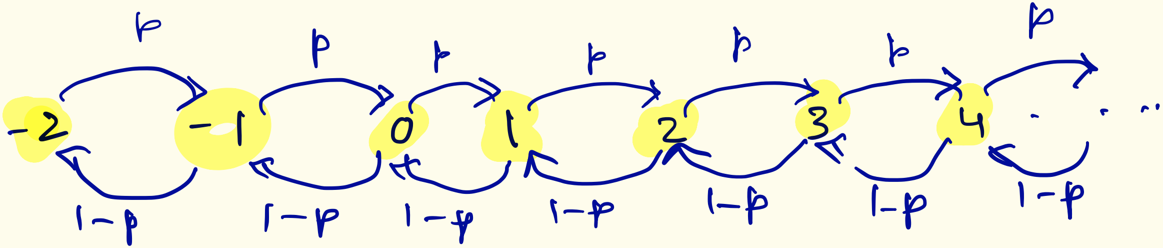
$$P_{ij} = P(X_{t+1} = x_j \mid X_t = x_i)$$

Random walk on the integers.

$p \in (0,1)$

state space / $\Omega = \mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, \dots\}$

$$P(X_0 = 0) = 1$$



$$
\begin{array}{c|cccccccccc}
 & \cdots & -3 & -2 & -1 & 0 & 1 & 2 & 3 & \cdots & \\
\hline
x_0 = 0 & \cdots & 0 & 0 & 1-p & 0 & p & 0 & 0 & \cdots & \\
x = 1 & & 0 & 0 & 0 & 1-p & 0 & p & 0 & \cdots &
\end{array}
$$

$p(X_0 = 0) = 1$

initial dist$^{\underline{n}}$

$$
\begin{array}{cccccccccc}
 & \cdots & -3 & -2 & -1 & 0 & 1 & 2 & 3 & \cdots \\
\pi_0 = [& \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots] \\
\pi_1 = [& \cdots & 0 & 0 & 1-p & 0 & p & 0 & 0 & \cdots]
\end{array}
$$

$$\pi_0 = [\; \cdots \; 0 \quad 0 \; \cdots \; 1 \quad 0 \quad 0 \; \cdots \;]$$

$$\pi_1 = [\; 0 \quad 0 \; \cdots \; 1-p \quad 0 \quad p \quad 0 \; \cdots \;]$$

$\mathbb{Z} \times \mathbb{Z}$