Solution of Sensitivity Assignment

Sol. 1) Let, x_1 = Number of model-1 is produced

x_2 = Number of model-2 is produced

Objective function:

$$\text{Maximize } Z = 30x_1 + 40x_2$$

Subject to:

$$2x_1 + 3x_2 \leq 1200$$

$$2x_1 + x_2 \leq 1000$$

$$4x_2 \leq 800$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Augmented problem:-

$$\text{Maximize } Z = 30x_1 + 40x_2 + 0S_1 + 0S_2 + 0S_3$$

S.T:

$$2x_1 + 3x_2 + S_1 = 1200$$

$$2x_1 + x_2 + S_2 = 1000$$

$$4x_2 + S_3 = 800$$

$$x_i \geq 0 \ \forall \ i = 1 \ldots 5.$$

Iteration 0

	Z	x_1	x_2	S_1	S_2	S_3	RHS
Z	1	-30	-40	0	0	0	0
S_1	0	2	3	1	0	0	1200
S_2	0	2	1	0	1	0	1000
S_3	0	0	4	0	0	1	800

Iteration 1

	Z	x_1	x_2	S_1	S_2	S_3	RHS
Z	1	-30	0	0	0	10	8000
S_1	0	2	0	1	0	-3/4	600
S_2	0	2	0	0	1	-1/4	800
x_2	0	0	1	0	0	1/4	200

Iteration 2

	Z	x_1	x_2	S_1	S_2	S_3	RHS
Z	1	0	0	15	0	-5/4	17000
x_1	0	1	0	1/2	0	-3/8	300
S_2	0	0	0	-1	1	1/2	200
x_2	0	0	1	0	0	1/4	200

Iteration 3

	Z	x_1	x_2	S_1	S_2	S_3	RHS
Z	1	0	0	25/2	5/2	0	17500
x_1	0	1	0	-1/4	¾	0	450
S_2	0	0	0	-2	2	1	400
x_2	0	0	1	1/2	-1/2	0	100

Optimal solution: x_1^*=450; x_2^* =100; Z=17,500

(b) Basis matrix (B) :- This is the matrix formed by the variables in the basis of the final table. Itsvalues, however, will be obtained from the initial table.

$$B=\begin{bmatrix} 2 & 0 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

Inverse of the basis matrix (B^{-1}): This matrix is formed by the variables in the basis of initial table and values from final table.

$$B^{-1}=\begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}$$

Resource (b_1) of milling machine changes from 1200 to 1300.

Changes in resource matrix affect the optimal solution. Therefore, new optimal solution will be:-

$$B^{-1}b=\begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}\begin{bmatrix} 1300 \\ 1000 \\ 800 \end{bmatrix}=\begin{bmatrix} 425 \\ 200 \\ 150 \end{bmatrix}$$

$x_1^* = 425$; $x_2^* = 150$; $Z = 18{,}750$

(C) Here, Resource (b_2) of Grinding machine changes from 800 to 350

$$B^{-1}b=\begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}\begin{bmatrix} 1200 \\ 1000 \\ 350 \end{bmatrix}=\begin{bmatrix} 450 \\ -50 \\ 100 \end{bmatrix}$$

No, we cannot determine the new optimal solution directly from the given information. Because $S_3 \leq 0$i.einfeasible.

Sol. 2) Formulation of the given problem will be,

$$\text{Max } Z = 250x_1+300x_2+400x_3+750x_4$$

$$\text{S.T} \quad 6x_1+9x_2+10x_3+10x_4 \leq 1600$$

$$x_1+2x_2+4x_3+5x_4 \leq 600$$

$$x_1+x_2+x_3+x_4 \leq 300$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Initial optimal Table:-

	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
Z	1	0	125	250	0	25	100	0	10^5
x_1	0	1	5/4	1/2	0	1/4	-1/2	0	100
x_4	0	0	3/20	7/10	1	-1/20	3/10	0	100
x_7	0	0	-2/5	-1/5	0	-1/5	1/5	1	100

Now, delux recorder is taking only 3 hours (instead of 5 hours) of testing, therefore change in coefficient is -2.

Which means $$\Delta Z = \begin{pmatrix} 25 & 100 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = -200$$

$$\Delta\, a_{14} = \begin{pmatrix} 1/4 & -1/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = 1$$

$$\Delta\, a_{24} = \begin{pmatrix} -1/20 & 3/10 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = -3/5$$

$$\Delta\, a_{34} = \begin{pmatrix} -1/5 & 1/5 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = -2/5$$

Due to this changes the initial table will change.

Updated Initial table

	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
Z	1	0	125	250	-200	25	100	0	10^5
x_1	0	1	5/4	1/2	1	1/4	-1/2	0	100
x_4	0	0	3/20	7/10	2/5	-1/20	3/10	0	100
x_7	0	0	-2/5	-1/5	-2/5	-1/5	1/5	1	100

Iteration 1:

	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
Z	1	0	200	600	0	0	250	0	10^5
x_1	0	1	7/8	-5/4	0	3/8	-5/4	0	-150
x_4	0	0	3/8	7/4	1	-1/8	3/4	0	250
x_7	0	0	-1/4	1/2	0	-1/4	1/2	1	200

Applying dual simplex method

Iteration 2:

	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
Z	1	200	375	350	0	75	0	0	120000
x_6	0	3/5	9/10	1	1	1/10	0	0	160
x_4	0	-4/5	-7/10	1	0	-3/10	1	0	120
x_7	0	2/5	-1/10	0	0	-1/10	0	1	140

New optimal solution is $x_6^*=160$; $x_4^*=120$; $x_7^*=140$; Z=120000

3 (a). augmented problem

$$Z^*=-z = x_1-2x_2-x_3+0x_4+0x_5$$

$$\text{s.t} \quad x_1+x_2+x_3+x_4=6$$

$$x_1-2x_2+x_5=4$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Iteration 0

	Z^*	x_1	x_2	x_3	x_4	x_5	RHS
Z^*	1	1	-2	-1	0	0	0
x_4	0	1	1	1	1	0	6
x_5	0	1	-2	0	0	1	4

Iteration 1

	Z^*	x_1	x_2	x_3	x_4	x_5	RHS
Z*	1	3	0	1	2	0	12
x_2	0	1	1	1	1	0	6
x_5	0	3	0	2	2	1	16

optimum solution is $x_1=0$, $x_2=6$ *and* $x_3=0$,

$z=-12$

(b) adding new constraint

$$-x_2+2x_3 \geq 4$$

Current optimal solution is $x_1=0$, $x_2=6$ *and* $x_3=0$, ***Z = -12***

Does it satisfy the new constaint ?

No, so the new constaint is not redundant and there be change in the optimal solution.

Now, we have to express all the basic variables in terms of non-basic variables x_1, x_3,and x_4,

$$x_2 = 6 - x_1 - x_3 - x_4$$

$$x_5 = 16 - 3x_1 - 2x_3 - 2x_4$$

the new constraint $-x_2+2x_3 \geq 4$ or $-x_2+2x_3-x_6+A_1=4$ should be expressed in the final table of x_2 *and* x_5. *This is possible by replacing* x_2 *with* $6-x_1-x_3-x_4$.

Hence , $x_1+x_3+x_4-6+2x_3-x_6+A_1=4$

$x_1+3x_3+x_4-x_6+A_1=10$ to add to the final table.

Iteration 0

	Z^*	x_1	x_2	x_3	x_4	x_5	x_6	A_1	RHS
Z*	1	3-M	0	1-3M	2-M	0	M	0	2+2M
x_2	0	1	1	1	1	0	0	0	6
x_5	0	3	0	2	2	1	0	0	16
A_1	0	1	0	3	1	0	-1	1	10

Iteration 1

	Z^*	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z*	1	8/3	0	0	5/3	0	1	26/3
x_2	0	2/3	1	0	2/3	0	1/3	8/3
x_5	0	7/3	0	0	4/3	1	2/3	28/3
x_3	0	11/3	0	1	1/3	0	-1/3	10/3

Optimal solution is $x_1=0$, $x_2=8/3$ *and* $x_3=10/3$,

$$z=-26/3$$

4.(a) Augmented problem

Max $z = 3x_1+x_2+0x_3+0x_4$

s.t $x_1+x_2+x_3=6$

$2x_1+3x_2+x_4=8$

$$x_1, x_2, x_3, x_4, \geq 0$$

	Z^*	x_1	x_2	x_3	x_4	*RHS*
Z*	1	-3	-1	0	0	0
x_3	0	1	1	1	0	6
x_4	0	2	3	0	1	8

	Z^*	x_1	x_2	x_3	x_4	*RHS*
Z*	1	0	7/2	0	3/2	12
x_3	0	0	-1/2	1	-1/2	2
x_1	0	1	3/2	0	1/2	4

Optimal solution z=12

(b) $\Delta a_{12} = 1$, $\Delta a_{22} = -3$

$$\Delta(z \quad c) = \begin{pmatrix}0 & \frac{3}{2}\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix} = -\frac{9}{2}$$

$$\Delta^* a_{12} \qquad = (1 \quad -1/2)\begin{pmatrix}1\\-3\end{pmatrix} = 5/2$$

$$\Delta^* a_{12} \qquad = (0 \quad 1/2)\begin{pmatrix}1\\-3\end{pmatrix} = -3/2$$

	Z^*	x_1	x_2	x_3	x_4	*RHS*
Z*	1	0	-1	0	3/2	12
x_3	0	0	2	1	-1/2	2
x_1	0	1	0	0	1/2	4

	Z^*	x_1	x_2	x_3	x_4	*RHS*
Z*	1	0	0	1/2	5/4	13
x_2	0	0	1	1/2	-1/4	1
x_1	0	1	0	0	1/2	4

Ne optimal solution : z=13, $x_1 = 4$ *and* $x_2 = 1$