Demand for Money

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Quantity Theory of Money

- **Demand for money increases with number of transactions**

$$MV = PT \quad (1)$$

where,

- M : supply of money, P : price level, T : number of transactions
- V : velocity of money$\Rightarrow$ number of times money changes hand
- Number of transactions equals to income generated

$$T = Y \quad (2)$$

Quantity Theory of Money

- **Equation (1) and (2) gives,**

$$\begin{aligned} MV &= PY \\ V &= \frac{PY}{M} \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} M &= \frac{1}{V} PY \\ &= kPY \end{aligned}$$

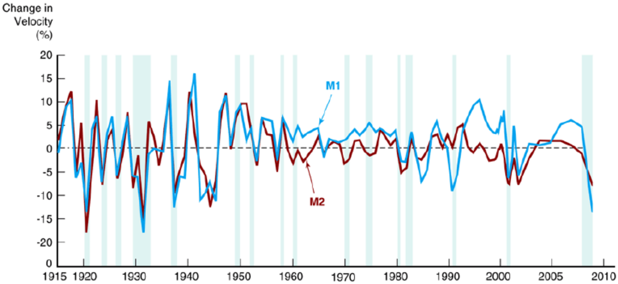
  - Smaller amount of money circulates faster

Quantity Theory of Money

- **Neutrality of Money:**
  - Velocity fairly constant in long-run
  - Aggregate output at full-employment level
  - Changes in money supply affect only the price level
  - Movement in the price level results solely from change in the quantity of money

- **Demand for money is determined by:**
  - The level of transactions generated by the level of nominal income PY
  - The institutions in the economy that affect the way people conduct transactions and thus determine velocity and hence k

- **Velocity of money:** varies in short run but fairly constant over long run



Sources: Economic Report of the President; Banking and Monetary Statistics; www.federalreserve.gov/releases/h6/hist/h6hist1.txt.

Keynes's Liquidity Preference Theory

- **Why do individuals hold money?**
  - **Transactions motive and Precautionary motive:** depends on income positively
  - **Speculative motive:** depends on interest rate negatively
    - interest rate is the opportunity cost of holding money. Individual earns interest rate i by foregoing one extra unit of money and investing in bond

Keynes's Liquidity Preference Theory

- **Demand for money:**

$$\begin{aligned} M &= PL(i, Y) \\ \frac{\partial L}{\partial i} &< 0, \frac{\partial L}{\partial Y} > 0 \end{aligned}$$

- Return for holding non-monetary assets is r, and return for holding money is, $-\pi^e$.
- Additional return for holding non-monetary assets over money is, $r - (-\pi^e) = r + \pi^e$
  - **Fischer equation**:

$$i = r + \pi^e$$

  - i: nominal interest rate, r : real interest rate, π^e: expected inflation

- **QTM:**

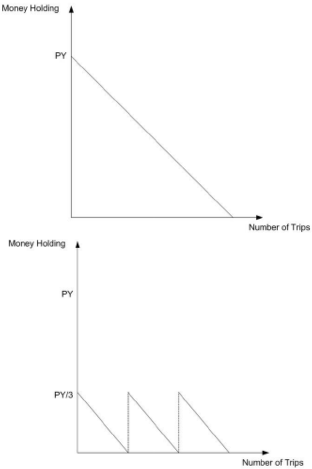
$$M = \frac{PY}{V(i)}, V^{'}(i) > 0$$

  - **velocity not constant**
    - rise in i reduces L and increases V
    - rise in Y increases both numerator and denominator. However, numerator rises more tha denominator causing V to rise

Baumol (1952) and Tobin (1956) Model of Transaction Demand

- **Suppose an individual holds money only for transaction purpose**
  - total amount of money holding: PY
  - total number of trip to bank: N
  - average money holding: $\frac{PY}{2N}$
  - interest rate: i
  - shoe leather cost: F per trip

Baumol (1952) and Tobin (1956) Model of Transaction Demand



Baumol (1952) and Tobin (1956) Model of Transaction Demand

- Average money holding when $N = 1$

$$\frac{PY + 0}{2} = \frac{PY}{2 \times 1} = \frac{PY}{2}$$

- Average money holding when $N = 2$

$$\frac{\frac{PY}{2} + 0}{2} = \frac{PY}{2 \times 2} = \frac{PY}{4}$$

- Average money holding when $N = 3$

$$\frac{\frac{PY}{3} + 0}{2} = \frac{PY}{2 \times 3} = \frac{PY}{6}$$

- Average money holding for N trips

$$\frac{PY}{2 \times N} = \frac{PY}{2N}$$

Baumol (1952) and Tobin (1956) Model of Transaction Demand

- **Total cost incurred by the individual:**

$$C = \frac{PY}{2N} i + PFN$$

- **Individual chooses N to minimize C:**

$$N^* = \sqrt{\frac{iY}{2F}}$$

- Higher interest rate induces people to withdraw less money in each trip$\Rightarrow$ number of trip to bank rises
- Higher shoe leather cost induces people to take less number of trips

Baumol (1952) and Tobin (1956) Model of Transaction Demand

- **Average money holding:**

$$M^* = \frac{PY}{2N^*} = P\sqrt{\frac{YF}{2i}}$$

- Even transaction demand for money depends both on income and interest rate
  - **positively on** Y : higher transaction needs more money
  - **negatively on** i : rise in i reduces reduces N and increases M
  - rise in F reduces N and increases M

Tobin (1958) Model of Speculative Demand

- **Total wealth or portfolio:** $\overline{W}$
  - Fraction $(1 - \alpha)$ held in cash
  - Rest in bond

$$\begin{aligned} W &= M + B \\ &= (1-\alpha)\, W + \alpha W, 0 < \alpha < 1 \end{aligned}$$

Tobin (1958) Model of Speculative Demand

- **Return on holding money:** 0
- **Expected return of bond:** r_b
- **Portfolio return:**

$$R = \alpha r_b \tag{3}$$

- **Portfolio risk:**

$$\begin{aligned} \sigma^2 &= \alpha^2 \sigma_b^2 \\ \alpha &= \frac{\sigma}{\sigma_b} \end{aligned} \tag{4}$$

- **Higher risk associated with bond** (σ_b) **reduces bond holding and increases money holding**

- **Equation (3) and (4) gives,**

$$R = \frac{r_b}{\sigma_b}\sigma \tag{5}$$

- **Slope:**

$$\frac{\partial R}{\partial \sigma} = \frac{r_b}{\sigma_b} > 0$$

Tobin (1958) Model of Speculative Demand

- **Individual earns utility from risk and return of the portfolio:**

$$
\begin{aligned}
u &= u(R, \sigma) \\
u_1 &> 0, u_2 < 0
\end{aligned}
$$

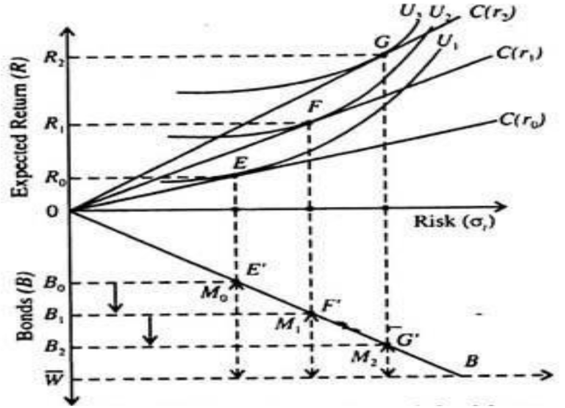
- **Slope of the indifference curve:**

$$
\frac{dR}{d\sigma} = -\frac{u_2}{u_1} > 0 \tag{6}
$$

- Higher risk should be compensated by higher return to keep a person indifferent for buying the bond

- **Maximizing (7) subject to (6) gives**

$$-\frac{u_2}{u_1} = \frac{r_b}{\sigma_b}$$



Tobin (1958) Model of Speculative Demand

- Higher expected return of bond (r_b) makes the budget constraint steeper and induces individual to hold more bond and less money
- Higher risk associated with bond (σ_b) reduces slope of the budget constraint and induces people to hold more money and less bond
- Higher wealth increases ddemand for both money and bond

- **Production Function in percapita form:**

$$Y(t) = F(K(t), N(t))$$

  - dimishing marginal productivity

$$\begin{aligned} F_K &> 0, F_{KK} < 0 \\ F_N &> 0, F_{NN} < 0 \\ F_{KK}F_{NN} - F_{KN}^2 &> 0 \end{aligned}$$

- **Constant Return to Scale (CRS):**

$$\begin{aligned} \frac{Y(t)}{N(t)} &= F\left(\frac{K(t)}{N(t)}, 1\right) \\ y(t) &= f(k(t)), f' > 0, f'' < 0 \end{aligned}$$

Sidrauski (1967) Dynamic General Equilibrium Model of Money Demand

Money in the Utility Function (MIU) Model

- **Lifetime utility function:**

$$U = \int_0^\infty e^{-\rho t} u(c(t), m(t))\, dt$$

- Individual derives utility from consumption and real money balance $\left(m(t) = \frac{M(t)}{P(t)}\right)$
  - $u_c > 0$, $u_m > 0$, $u_{cc} < 0$, $u_{mm} < 0$
  - discount factor: $0 < \rho < 1$

- **Resource constraint (without population growth):** Define total asset,

$$
\begin{aligned}
a(t) &= m(t) + b(t) + k(t) \\
\dot{a}(t) &= \dot{m}(t) + \dot{b}(t) + \dot{k}(t)
\end{aligned}
$$

$$
f(k(t)) + (i(t) - \pi(t))\, b(t) - \pi(t)\, m(t) + \tau(t) = \dot{a}(t)
$$

- $\tau(t)$: **lumpsum transfer**

- **Problem**

$$\max_{\{c(t),b(t),k(t),m(t)\}} U = \int_0^\infty e^{-\rho t} u\left(c\left(t\right), m\left(t\right)\right) dt$$

subject to,

$$\begin{aligned} \dot{m}\left(t\right) + \dot{b}\left(t\right) + \dot{k}\left(t\right) &= f\left(k\left(t\right)\right) - \delta k\left(t\right) - \pi\left(t\right) m\left(t\right) \\ &\quad + \left(i\left(t\right) - \pi\left(t\right)\right) b\left(t\right) + \tau\left(t\right) - c\left(t\right) \end{aligned}$$

- Given: k_0
- TVC: $\lim_{t\to\infty} e^{-\rho t} \lambda\left(t\right) x\left(t\right) = 0, x\left(t\right) = b\left(t\right), k\left(t\right), m\left(t\right)$

- **Current value Hamiltonian:**

$$H = u\left(c\left(t\right), m\left(t\right)\right) + \lambda\left(t\right)\left[\begin{array}{c} f\left(k\left(t\right)\right) - \delta k\left(t\right) - \pi\left(t\right) m\left(t\right) \\ +\left(i\left(t\right) - \pi\left(t\right)\right) b\left(t\right) + \tau\left(t\right) - c\left(t\right) \end{array}\right]$$

FOCs:

$$\frac{\partial H}{\partial \lambda\left(t\right)} = \dot{a}\left(t\right) \tag{7}$$

$$\frac{\partial H}{\partial c\left(t\right)} = u_c\left(c\left(t\right), m\left(t\right)\right) - \lambda\left(t\right) = 0 \tag{8}$$

- **Current value Hamiltonian:**

FOCs:

$$\begin{aligned}
\frac{\partial H}{\partial m(t)} &= -\dot{\lambda}(t) + \rho\lambda(t) = u_m(c(t), m(t)) - \lambda(t)\pi(t) \\
\frac{\partial H}{\partial k(t)} &= -\dot{\lambda}(t) + \rho\lambda(t) = \lambda(t)\left[f'(k(t)) - \delta\right] \\
\frac{\partial H}{\partial b(t)} &= -\dot{\lambda}(t) + \rho\lambda(t) = \lambda(t)\left[i(t) - \pi(t)\right]
\end{aligned} \tag{9}$$

- **Equation (9) gives Fischer equation**

$$i(t) - \pi(t) = f'(k(t)) - \delta$$

- **Taking log both sides of equation (8) and differenting and using (??) gives,**

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \delta - \rho}{\sigma_{u,c}} - \frac{u_{cm} m}{u_{cc} c} \frac{\dot{m}(t)}{m(t)} \tag{10}$$

$$\frac{u_m(c(t), m(t))}{u_c(c(t), m(t))} = i(t) \tag{11}$$

- **Equation (10):** Euler Equation
    - we get standard Ramsey back when utility is additively separable between $c(t)$ and $m(t) \Rightarrow u_{cm} = 0$
    - $\sigma_{u,c} = -\frac{u_{cc}(c(t),m(t))c(t)}{u_c(c(t),m(t))}$, elasticity of utility with respect to consumption. It is a measure of Relative Risk Aversion (RRA)
    - RRA rises as $\sigma_{u,c}$ rises.

- **Equation (11) gives,**
  - rise in $i(t)$ increases $\frac{u_m(c(t),m(t))}{u_c(c(t),m(t))}$
  - rise in $u_m(c(t), m(t))$ reduces $m(t)$ due to concavity of utility function
  - rise in $c(t)$ reduces $u_c(c(t), m(t))$ given $i(t)$. $u_m(c(t), m(t))$ should fall. Implies reduces $m(t)$ to rise

- **Steady State:** $\dot{m}(t) = 0 = \dot{b}(t) = \dot{k}(t)$
  - equation (10) gives,
    $$f'(k_{SS}) = \delta + \rho$$
  - $\tau_{SS} = \pi_{SS} m_{SS}$ $\Rightarrow$seigniorage (revenue generated by government by printing money)
  - budget constraint with $b_{SS} = 0$ gives,
    $$c_{SS} = f(k_{SS}) - \delta k_{SS}$$
  - $\dot{m}(t) = 0$ implies $\frac{\dot{M}(t)}{M(t)} = \pi_{SS}$. How to determine $\frac{\dot{M}(t)}{M(t)}$?
- **Real variables are independent of money and growth of money** $(\pi) \Rightarrow$ **money is superneutral and superneutral**

Seigniorage and Hyperinflation

- **Seigniorage:**

$$\frac{\dot{M}(t)}{P(t)} = \frac{\dot{M}(t)}{M(t)} \frac{M(t)}{P(t)}$$

- Printing money gives seigniorage revenue to the government but increase inflation too which reduces the revenue
- There exists an optimal inflation that maximizes seigniorage revenue
- Implies inflation has a Laffer curve

Seigniorage and Hyperinflation

- **Consider the following money demand function at**

$$\frac{M(t)}{P(t)} = Y(t)\, e^{-\alpha(r(t)+\pi(t))}, \alpha > 0$$

- **Seigniorage at steady state:**

$$\frac{\dot{M}(t)}{P(t)} \Big|_{SS} = \pi_{SS} Y_{SS} e^{-\alpha(r_{SS}+\pi_{SS})}$$

- Seigniorage revenue is maximum at $\pi_{SS} = \frac{1}{\alpha}$
- **Printing money gives revenue to government but may put government at wrong part of Laffer curve where $\pi_{SS} > \frac{1}{\alpha}$.**
  - Government earn less revenue but creates more inflation$\Rightarrow$ hyperinflation
  - Revenue can be increased by lowering inflation (nominal money growth, $\frac{\dot{M}(t)}{M(t)}$

Seigniorage and Hyperinflation

- **Printing money gives revenue to government but may put government at wrong part of Laffer curve where $\pi_{SS} > \frac{1}{\alpha}$.**
    - Laffer curve is hump-shaped. Government earns less revenue but creates more inflation$\Rightarrow$ hyperinflation
    - Revenue can be increased by lowering inflation (nominal money growth, $\frac{\dot{M}(t)}{M(t)}$

Cost of Inflation

- **Shoeleather cost:** Have to go to bank more frequently to withdraw money as value of money gets reduced
- **Menu Cost:** Firm has to change price frequently (restaurant has to print menu frequently)
- **Redistribution of Income:** Higher expected inflation reduces real interest rate. Creditors suffers but debtors gain
- **Liquidity effect:** increases nominal interest rate. Debt denominated in nominal terms (mortgage loan) rises

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