Assignment 4

Topics: Linear 2nd order PDE with Const. coefficients→GS of homogeneous PDE, CF and PI of non-homogeneous PDE, Special forms of RHS function. 2^{nd} order PDE $R(x,y)r + S(x,y)s + T(x,y)t + f(x,y,z,p,q) = 0$ →Canonical forms, Classification & Reduction; Wave equation→Infinite & Semi-infinite string

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1. Find general solution of following 2^{nd} order linear PDEs
   a) $(4D^2 - 4D + 1)z = 0$
   b) $(2D - 1)(D + 3)Z = 0$

2. Find particular integral of following 2^{nd} order linear PDEs
   a) $25r - 40s + 16t = \exp(x - y)$
   b) $4\frac{\partial^2 z}{\partial x^2} - 12\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = x^2 y^3$
   c) $2s + t - 3q = \sin(2x - 3y)$

3. Find the general solution of following linear PDEs:
   a) $r + s - 4t = \ln(2x - 3y)$
   b) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = \mathrm{e}^{x+y}$

4. Determine the nature of following 2^{nd} order PDEs as hyperbolic, elliptic or parabolic:
   a) $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$
   b) $x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} = \exp x$
   c) $(1 + x^2)r + (1 + y^2)t + xp + yq = 0$
   d) $r - 2s\sin x - t\cos^2 x - q\cos x = 0$

5. Find the characteristic direction/s, if any, for PDEs a)-d) in Q 4.

6. Reduce PDEs a)-d) in Q 4 to canonical form by showing necessary transformation/s with steps.

7. Find d'Alembert's solution for the following Wave Equation with IC
$$\frac{\partial^2 \psi}{\partial t^2} = 9\frac{\partial^2 \psi}{\partial x^2}, \quad x \in (-\infty, +\infty), t \geq 0,$$
   IC: $\psi(x,0) = \sin(4x), \psi_t(x,0) = \cos(4x), \ x \in (-\infty, +\infty)$

8. Derive solution of following Wave Equation with IC and BC
   $\frac{\partial^2 \psi}{\partial t^2} = 4\frac{\partial^2 \psi}{\partial x^2}, \quad x \in [0, \infty), t \geq 0,$ BC: $\psi(0,t) = 0, t \geq 0$
   IC: $\psi(x,0) = \cos(9x), \psi_t(x,0) = \sin(9x), \ x \in [0, \infty)$

END