Big M and Two-Phase Methods

for

Non-Standard form of LPP

Non-Standard form of LPP

- **Simplex method is suitable for LPP in standard form**

  i.e. Maximization type objective

  functional constraints $\leq$ type

  non-negativity constraints

  and RHS non-negative

- **How to deal with the LPP in other (Non-standard) forms**

  - LPP with "$\geq$" and "$=$" type constraints.

  - Negative RHS

  Difficulty: In identifying initial BFS

- **Example:**

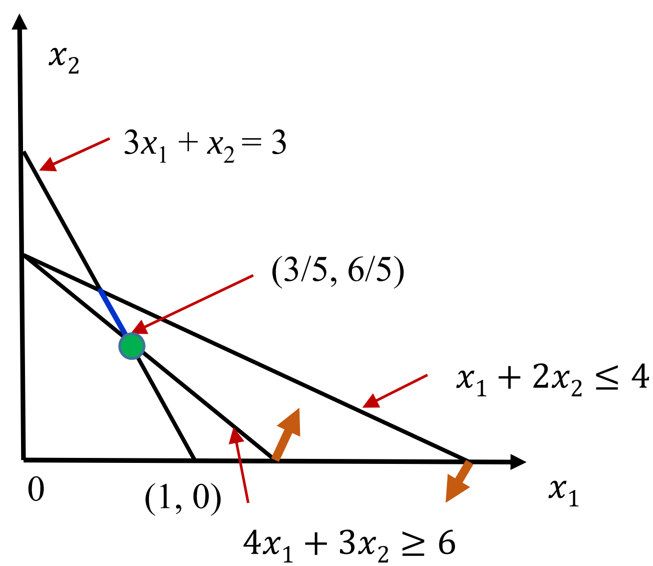
  Maximize $Z = 4x_1 + x_2$

  Subject to $3x_1 + x_2 = 3$

  $4x_1 + 3x_2 \geq 6$

  $x_1 + 2x_2 \leq 4$

  $x_1, x_2 \geq 0$



Identify feasible region?

Approach for Non-Standard form of LPP

- **Augmented form**

$$\text{maximize } Z = 4x_1 + x_2$$

Subject to $\quad 3x_1 + x_2 = 3$

$$4x_1 + 3x_2 - x_3 = 6$$

$$x_1 + 2x_2 + x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

x_3 : surplus variable, and x_4 : slack variable

But, the initial BFS is not available.

- **Artificial-Variable Techniques**

  1. **Big-M method**
  2. **Two-phase method**

Big M-method

- Introduce artificial variable to get initial BFS.
- **Modified problem**

$$\text{Maximize } Z = 4x_1 + x_2 - M\bar{x}_5 - M\bar{x}_6$$

$$\text{Subject to} \quad 3x_1 + x_2 + \bar{x}_5 = 3$$

$$4x_1 + 3x_2 - x_3 + \bar{x}_6 = 6$$

$$x_1 + 2x_2 + x_4 = 4$$

$$x_1, \dots, x_4 \geq 0 \text{ and } \bar{x}_5, \bar{x}_6 \geq 0$$

- For each artificial variable $\bar{x}_j$ subtract $M\bar{x}_j$ from objective function (add for minimum type problem), where M is a Large positive number
- In order to get rid of the artificial variables in the final optimal solution, we assign a very large penalty in the objective function

Apply the procedure of Simplex method

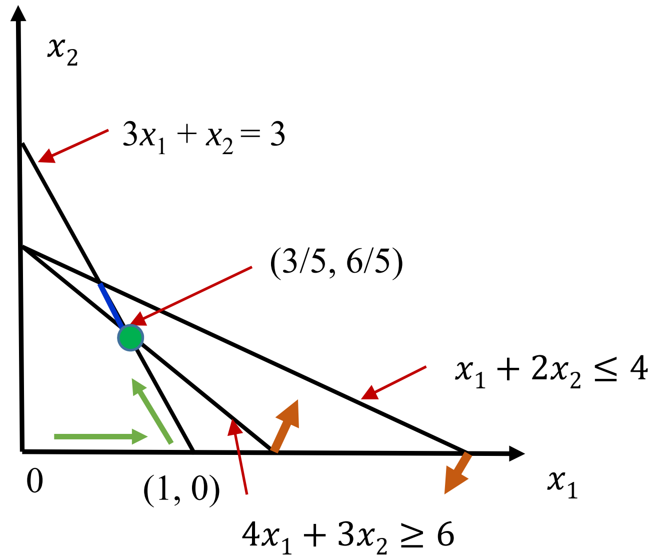
	Basis	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	RHS	Ratio
Iteration 0	$\bar{x}_5$	**3**	1	0	0	1	0	3	**1**
	$\bar{x}_6$	4	3	-1	0	0	1	6	3/2
	x_4	1	2	0	1	0	0	4	4
	Z	-4	-1	0	0	M	M	0	
	Z	**-4-7M**	-1-4M	M	0	0	0	-9M	
Iteration 1	x_1	1	1/3	0	0	1/3	0	1	3
	$\bar{x}_6$	0	**5/3**	-1	0	-4/3	1	2	**6/5**
	x_4	0	5/3	0	1	-1/3	0	3	9/5
	Z	0	**(1-5M)/3**	M	0	(4+7M)/3	0	4-2M	
Iteration 2	x_1	1	0	1/5	0	3/5	-1/5	3/5	
	x_2	0	1	-3/5	0	-4/5	3/5	6/5	
	x_4	0	0	1	1	1	-1	1	
	Z	0	0	1/5	0	(5M+8)/5	(5M-1)/5	18/5	

Z row in not in proper form (coefficient of basic variable must be zero)

$\mathbf{R_0 \rightarrow R_0 - MR_1 - MR_2}$

Optimal solution : $\left(x_1 = \dfrac{3}{5}, x_2 = \dfrac{6}{5}\right)$, $Z = 18/5$

Path traced by Big M



Note:

If the optimality condition is satisfied and

- No artificial variable remains in the basis $\Longrightarrow$ current solution is optimal
- At least one artificial variable appears in basis at zero level $\Longrightarrow$ current solution is optimal but degenerate solution
- At least one artificial variable in basis at non-zero level $\Longrightarrow$ the problem has no feasible solution