NR(2)-11

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Spring Semester Examination 2023

Date of Examination: **Session: (FN/AN) :** **Duration: 2 hrs.** **Full Marks: 30**

Subject No.: EM20204 **Subject: Statistics for Economics**

Department/Center/School: Humanities and Social Sciences

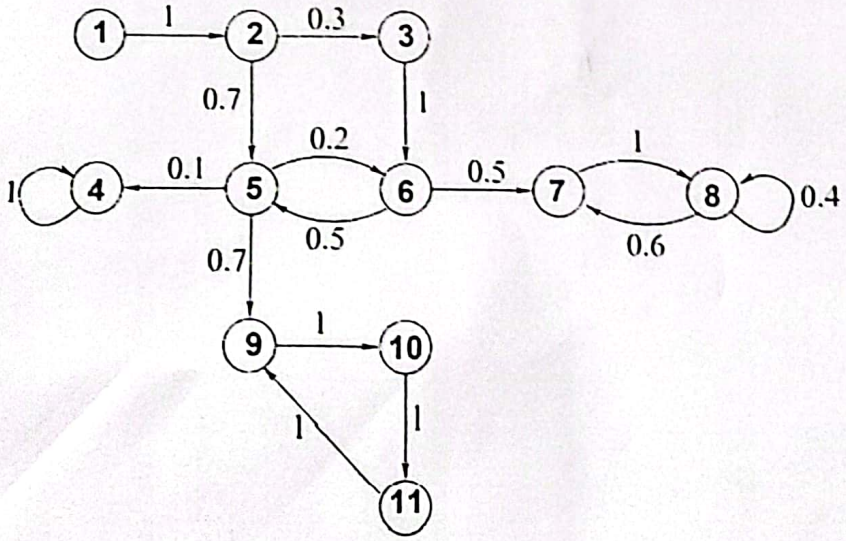
Special Instructions (if any): Answer all Questions

Question 1:

Assume that a man's profession can be classified as professional, skilled labourer, or unskilled labourer. Assume that, of the sons of professional men, 80 percent are professional, 10 percent are skilled labourers, and 10 percent are unskilled labourers. In the case of sons of skilled labourers, 60 percent are skilled labourers, 20 percent are professional, and 20 percent are unskilled. Finally, in the case of unskilled labourers, 50 percent of the sons are unskilled labourers, and 25 percent each are in the other two categories. Assume that every man has at least one son, and form a Markov chain by following the profession of a randomly chosen son of a given family through several generations. Set up the matrix of transition probabilities. Find the probability that a randomly chosen grandson of an unskilled labourer is a professional man.

[8 marks]

Question 2:



i. Which states are transient?

ii. What are the recurrent classes?

iii. Find the probability of absorption to the different recurrent classes, given that the system starts at state - 1?

[3+3+4 = 10 marks]

Question 3:

Let $X_1,\ldots,X_n$ be i.i.d. random variables with the probability density function

$$f(x)=\begin{cases}e^{-x}, & x>0,\\ 0, & \text{otherwise.}\end{cases}$$

If $X_{(n)}=\max\{X_1,\ldots,X_n\}$, then $\lim_{n\to\infty} P\left(X_{(n)}-\log_e n\le 2\right)$ equals

(A) $1-e^{-2}$

(B) $e^{-e^{-0.5}}$

(C) $e^{-e^{-2}}$

(D) $e^{-e^{2}}$

[6 marks]

Question 4:

Let (X,Y) have the joint probability density function

$$f(x,y)=\begin{cases}\frac{3}{4}(y-x), & 0\le x<y<2\\ 0, & \text{otherwise}\end{cases}$$

Then the conditional expectation $E(X|Y=1)$ equals _____ *(round off to two decimal places)*

[6 marks]