(Sensitivity, primal-dual, duality)

Q1) ABC produces two models of an assembled product that use milling, drilling, and grinding facilities respectively. The table below shows the details:

Resource	Unit Resource	Requirements	Maximum Availability	
Resource	Model 1	Model 2	Maximum Availability	
Milling (m/c Hrs)	2	3	1200	
Drilling (m/c Hrs)	2	1	1000	
Grinding (m/c Hrs)	0	4	800	
Unit Profit (Rs.)	30	40		

- a) Find the optimum number of the models to be produced for the next plan period.
- b) If the available Milling machine-hours increase to 1300 units, find the new optimum solution
- c) If the available grinding machine-hours are reduced to 350 units, will you be able to determine the new optimum solution directly from the given information? Explain.
- d) Does the solution is profitable?

Solve using either Simplex or graphically.

Q2) Convert the following simplex problem into a dual problem.

Maximize
$$z = 5x_1 + 6x_2$$

Subject to $x_1 + 2x_2 = 5$
 $-x_1 + 5x_2 \ge 3$
 $4x_1 + 7x_2 \le 8$
 x_1 unrestricted, $x_2 \ge 0$

Q3) Consider the maximization problem in standard form.

Z	x_1	x_2	<i>x</i> ₃	χ_4	<i>x</i> ₅	RHS
1	0	4	0	0	-4	8
0	0	1	0	1	-1	5
0	1	2	0	0	-2	6
0	0	3	1	0	-3	7

- (a) What is the corresponding dual solution?
- (b) Is the dual solution feasible? If not, why not?
- (c) Show that, for any feasible dual solution the dual objective must be greater than or equal to 8. (Hint: Use weak duality).
- Q4) Consider the following LP:

Maximize
$$z = 5x_1 + 12 x_2 + 5x_3$$

Subject to $x_1 + 2 x_2 + 5x_3 \le 10$
 $2x_1 - x_2 + 3x_3 = 8$
 $x_1, x_2, x_3 \ge 0$

An optimal primal table is given below:

Basic	x_1	<i>x</i> ₂	<i>X</i> ₃	X4	R	Solution
Z	0	0	3/5	29/5	-2/5+M	274/5
<i>x</i> ₂	0	1	-1/5	2/5	-1/5	12/5
x_1	1	0	7/5	1/5	2/5	26/5

Find out the optimal dual solution by using complementary slackness condition

Q5) Solve the following LPP using primal-dual method.

Minimize
$$Z = 600 x_1 + 500 x_2$$
 Subject to
$$2 x_1 + x_2 \ge 80$$

$$x_1 + 2 x_2 \ge 60$$

$$x_1, x_2 \ge 0$$

Q6) Solve the following LPP using primal-dual method.

Minimize
$$Z = x_1 + 2x_2 + 3x_3$$
 Subject to
$$3x_1 + 4x_2 \le 5$$

$$5x_1 + x_2 + 6x_3 = 7$$

$$8x_1 + 9x_3 \ge 2$$

$$x_1, x_2, x_3 \ge 0$$