Indian Institute of Technology Kharagpur
Mid-Semester Examination: Autumn 2022

Date of Examination: 30/09/2022 (FN) **Duration: 2 Hrs**
Subject. No: MA51109/MA60049 **Subject Name: Computational Statistics**
Department: Mathematics **TOTAL MARKS: 30**
Specific Chart, graph paper log book etc. required: None
Special Instruction: None

ANSWER ALL THE QUESTIONS

1. State whether the following statements are *TRUE* or *FALSE*. Justify your answer with a proof or a counter example. No marks will be awarded without justification. [8 marks]

   (a) Let $X_1, X_2, \ldots, X_n$ be iid with the pdf $f(x)$. Then the variance of $\bar{X} = \frac{1}{n}\sum_i^n X_i$ is smaller than the variance of X_1.

   (b) Let $X_1 \sim \exp(\lambda_1)$ and $X_2 \sim \exp(\lambda_2)$ be independent random variables. Then $\min\{X_1, X_2\} \sim \exp(\lambda_1 + \lambda_2)$.

   (c) Let $X \sim N(0,1)$. Then $E(X^4) = 3$.

   (d) Let $X_1, X_2, \ldots, X_n$ be iid exponential random variables with mean λ. Then $\sum_{i=1}^n X_i$ is a Gamma random variable with mean $n\lambda$.

2. Let $X \sim N(0,1)$ and define $Y = e^X$.

   (a) Determine the pdf of Y. [3 marks]

   (b) Determine $E(Y)$, if it exists. [2 marks]

3. Let $\{X_n, n \in \mathbb{N}\}$ be a Markov chain with the state space $\{0, 1, 2\}$ and the transition matrix given by

$$P = \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.4 & 0.4 & 0.2 \\ 0.1 & 0.7 & 0.2 \end{bmatrix},$$

   and the initial distribution $\pi = \begin{bmatrix} 0.2 & 0.5 & 0.3 \end{bmatrix}$. (Assume all the notations discussed in the class.) Calculate the following.

   (a) $P(X_1 = 2)$ [1 mark]

   (b) $P(X_3 = 2 \mid X_0 = 0)$ [2 marks]

   (c) $P(X_1 = 1, X_3 = 1)$ [2 marks]

4. Let $Y \sim \exp(\lambda)$ where λ is chosen such that $1 - p = e^{-\lambda}$ for some $p \in (0,1)$.

   (a) Prove that $Z = \lfloor Y \rfloor + 1 \sim$ geometric(p). Here the notation $\lfloor \alpha \rfloor$ denotes the integer part of α. [2 marks]

   (b) Given a random sample from $U(0,1)$, device an algorithm to obtain a random sample from geometric(p) using the result proved in the above question (4a). [4 marks]

5. Describe (with proving the necessary results) the inverse transform method to generate a random sample of the Laplace random variable with the following pdf. [4 marks]

$$f(x) = \frac{\lambda}{2} e^{-\lambda|x-\theta|}, \quad -\infty < x < \infty, \quad (\theta \in \mathbb{R},\ \lambda > 0)$$

6. Explain the accept/reject method of sampling from a density $f(x)$. [2 marks]

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