

Monopoly

Prelude

- Monopoly: theory of a single seller facing competitive (price-taking) consumers in one or several markets, over one or several periods.
- The monopolist faces a downward sloping demand curve
- The monopolist needs to devote resources to the careful study of the demand curve facing its product – then, the monopolist can determine either the price for the product or the quantity supplied
- After estimating the demand curve, the monopoly has to study the market demand to determine its profit-maximizing output.

Monopolist's profit maximization

- Let $TC(Q)$ denote the total cost function of the monopoly.
- Denoting by $\pi(Q)$ the monopoly's profit level when producing Q units of output, the monopoly chooses Q^m to maximize –

$$\max_Q \pi(Q) = TR(Q) - TC(Q)$$

- Profit maximization (FOC) yields $Q^m > 0$ such that –

$$\frac{dTR(Q^m)}{dQ} = \frac{dTC(Q^m)}{dQ} \Rightarrow MR(Q^m) = MC(Q^m)$$

Monopolist's profit maximization

- Now, $MR(Q^m) = MC(Q^m)$ is a necessary condition, not sufficient!
- $MR(Q^m) = MC(Q^m)$ is only a necessary condition, meaning that if the profit maximizing output is strictly positive, then the condition has to be satisfied
- If the monopoly has to pay high fixed costs, it is possible that the monopoly's profit-maximizing output level is $Q^m = 0$.
- How to ensure?

Monopolist's profit maximization

- Solve for Q^m from, $MR(Q^m) = MC(Q^m)$
- Plug the Q^m thus obtained into the total profit function to check whether $\pi(Q^m) \geq 0$.
- If $\pi(Q^m) \not\geq 0$ for the Q^m obtained then monopolist chooses $Q^m = 0$
- If $\pi(Q^m) \geq 0$ then the output level solved from $MR(Q^m) = MC(Q^m)$ is the profit-maximizing output level.
- After finding the monopoly's profit-maximizing output, the price charged by the monopoly can be found by substituting Q^m into the demand function

Case

- Total cost: $TC(Q) = F + cQ^2$
- Demand curve: $p(Q) = a - bQ$
- FOC: $MR(Q^m) = MC(Q^m) \Rightarrow a - 2bQ^m = 2cQ^m$
- Hence, $Q^m = \frac{a}{2(b+c)}$
- Implying, $p(Q^m) = a - b \left[\frac{a}{2(b+c)} \right] = \frac{a(b+2c)}{2(b+c)}$

Case

- Therefore we have the profit function as –

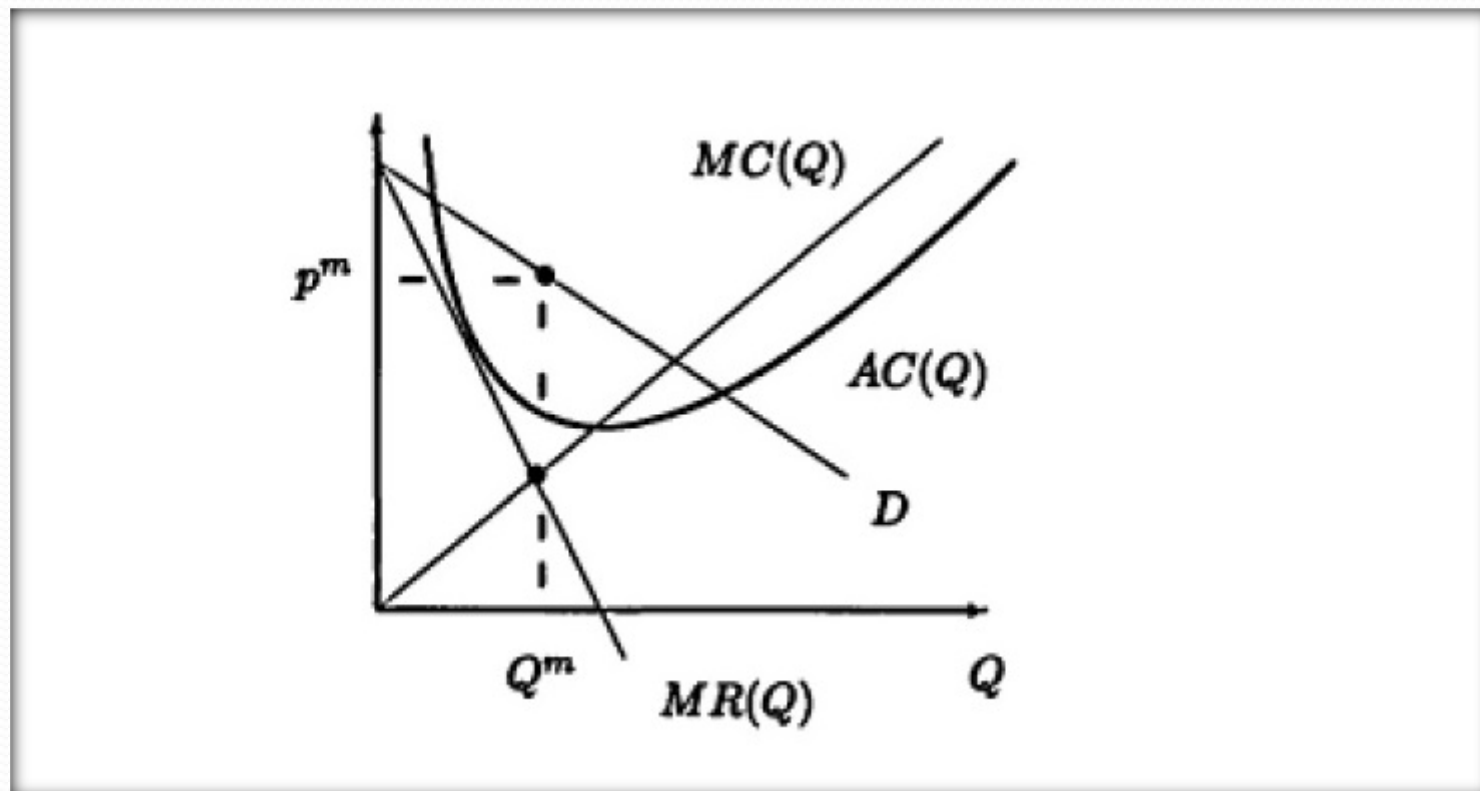
$$\pi(Q^m) \equiv TR(Q^m) - TC(Q^m) = p^m Q^m - F - c(Q^m)^2 = \frac{a^2}{4(b+c)} - F$$

- So, we can formally write down the monopolist's profit maximization output as –

$$(Q^m) = \begin{cases} \frac{a}{2(b+c)} & \text{if } F \leq \frac{a^2}{4(b+c)} \\ 0 & \text{otherwise} \end{cases}$$

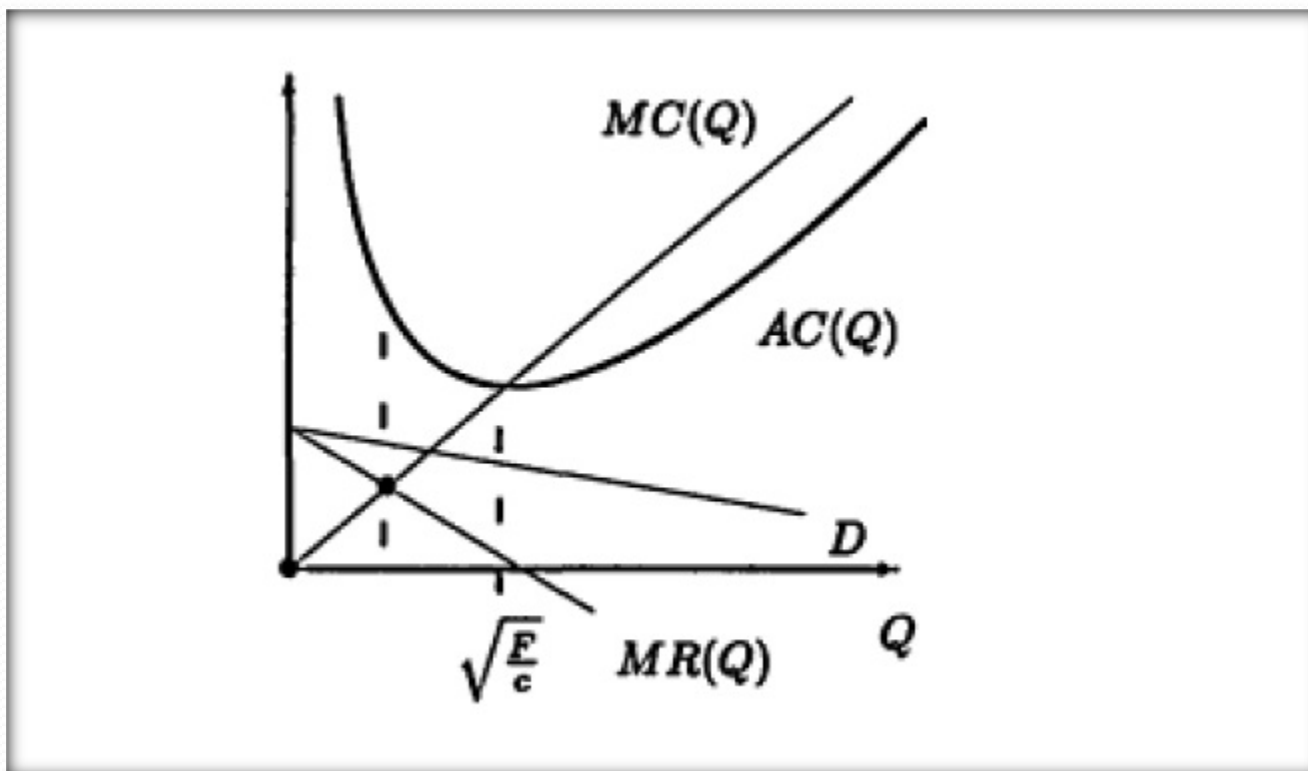
Graphical Illustration ($Q^m > 0$)

- Case where the demand is high enough (or the fixed cost is low enough) so that the monopolist would produce $(Q^m) > 0$



Graphical Illustration ($(Q^m) = 0$)

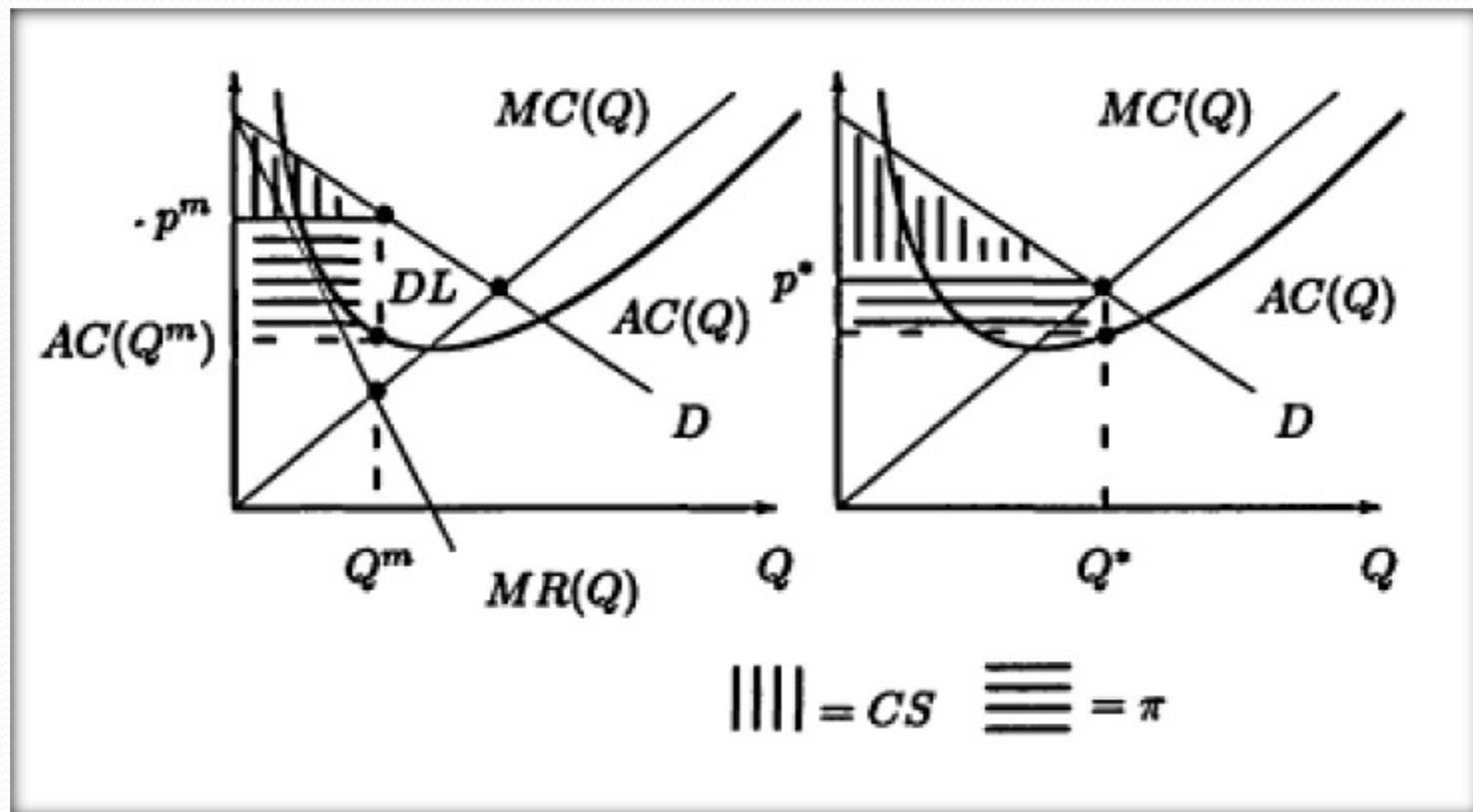
- Case where the demand is so low (or the fixed cost is high enough) so that the monopolist would produce $(Q^m) = 0$



Monopoly and Social Welfare

- Monopolies are discouraged in a number of economies across the world.
- Reason 1: Social welfare — remember, that social welfare (as calculated by the sum of CS and PS) is maximum under PC
- Social welfare under monopoly reduces by the amount of the dead-weight loss

Monopoly and Social Welfare



Monopoly and Social Welfare

- Reason 2: Social cost of a monopoly
- The cost to the society associated with the existence of a monopoly is much higher than the deadweight-loss area.
- The pursuit of monopoly rents is itself a competitive activity, and one that consumes resources (rent-seeking). The social cost of having a monopoly should also include the costs of deterring competition.
- The point is that firms, wishing to obtain a monopoly status or wishing to maintaining a monopoly position, must allocate resources for that goal.
- These resources may or may not be counted as a waste to the economy.

Monopoly and Social Welfare

- Resources allocated to establishing or maintaining monopoly power that *should not be considered* as welfare reducing include –
 1. R&D leading to a patent monopoly right since the R&D improves technologies and results in new products.
 2. Bribes to politicians or civil servants for the purpose of getting exclusive business rights (since this constitutes only a transfer of wealth)

Monopoly and Social Welfare

- Resources allocated to establishing or maintaining monopoly power that *may count* as social waste include –
 1. Persuasive advertising, needed to convince consumers that alternative brands are inferior.
 2. Resources needed to preempt potential entrants from entering the industry. Also, excessive production or investment in capital for the purpose of making entry unprofitable for potential competitors.
 3. Lobbying costs, needed to convince the legislators that a particular monopoly is not harmful (provided that these costs divert resources from productive activities).
 4. Excessive R&D resulting from a patent race.

Discriminating Monopoly

- The analysis so far has focused, on monopolies charging a single, uniform price to all customers.
- A monopolist can, however, increase its profit by charging different prices to consumers with different characteristics. That is, a firm may be able to differentiate among consumers according to tastes, income, age, and location in order to charge consumers with different characteristics different prices.
- In order to be able to charge consumers different prices, a monopolist must possess the means for making arbitrage (buying low for the purpose of reselling at a high price) impossible. Price discrimination is impossible when those consumers who are able to purchase at a low price can make a profit by reselling the product to the consumers who buy at high prices.

Discriminating Monopoly

- Monopolies resort to various marketing techniques to prevent arbitrage from taking place (market segmentation) –
 1. Charge different prices at different locations (in order for price discrimination to be sustained, the markets should be isolated by geography, by prohibitive taxes (such as tariffs), or by prohibitive transportation costs).
 2. Monopolies that provide services (such as transportation companies, restaurants, and places of entertainment) charge senior citizens lower prices than they charge younger consumers – senior citizen ID or say, student discounts
 3. Book publishers manage to charge institutions higher prices than they charge individuals by selling hardcovers to institutions and softcovers to individuals

Discriminating Monopoly

- Let's assume that arbitrage cannot take place and a monopolist is selling in two different markets
- Question: How does a monopolist determine the output level (and, hence, the price) in each market?
- The monopolist chooses the output levels sold in each market, q_1^m and q_2^m , that solve –

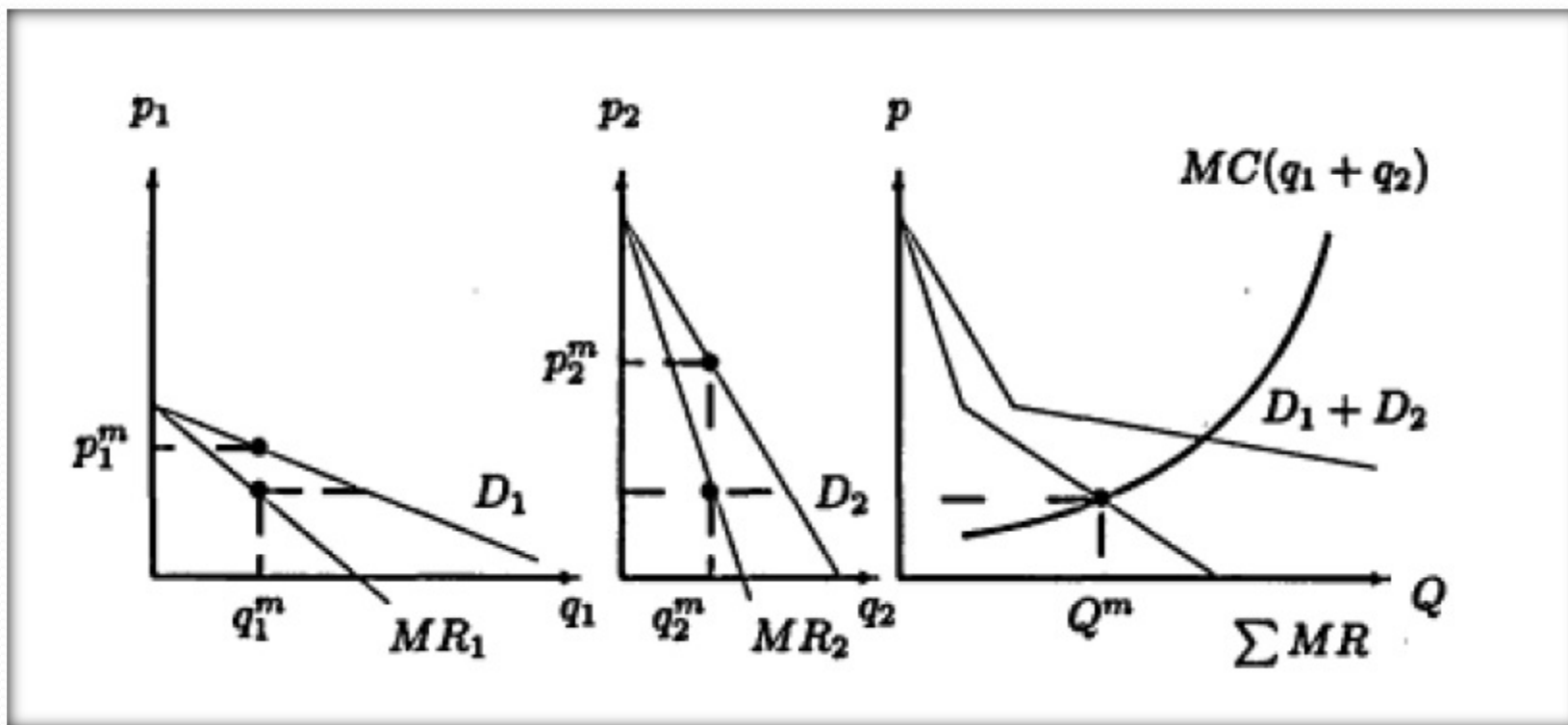
$$\max_{q_1, q_2} \pi(q_1, q_2) = TR_1(q_1) + TR_2(q_2) - TC(q_1 + q_2)$$

- FOCs: $MR_i(q_i^m) = MC(q_1^m + q_2^m) \forall i = 1, 2$

Discriminating Monopoly

- If the monopolist chooses q_1^m and q_2^m such that $MR_1(q_1^m) > MR_2(q_2^m)$, then, it is clear that the monopoly should transfer one unit from market 2 to market 1.
- In this case the reduction in revenue in market 2 is smaller than the increase in revenue in market 1.
- To solve for the profit-maximizing output levels q_1^m and q_2^m , we need to solve two equations (FOCs) with the two variables.
- Graphically – 3 steps
- Prices charged p_1^m and p_2^m such that $p_1^m \left(1 + \frac{1}{\varepsilon_1}\right) = p_2^m \left(1 + \frac{1}{\varepsilon_2}\right)$

Discriminating Monopoly



Discriminating Monopoly

- Implication: *A discriminating monopoly selling a strictly positive amount in each market will charge a higher price at the market with the less elastic demand*
- If $\varepsilon_2 > \varepsilon_1$ (or $|\varepsilon_2| < |\varepsilon_1|$) then $p_2^m > p_1^m$

Cartel and Multi-plant Monopoly

- The cartel and the multi-plant monopoly are forms of organizations and contractual agreements among plants, firms, or countries
- Cartel: cartel is an organization that contracts with the plants on how much each would produce and hence on what would be the price
- Examples: OPEC (assuming each oil producing country is a plant), IATA, Bar Associations
- The multiplant monopoly is very similar to the cartel, except that all the plants are put under a single ownership.
- Multiplant monopoly occurs when several firms in the industry merge together into a single firm (horizontal merger), or when a monopoly firm opens several plants producing the same product.

Cartel and Multi-plant Monopoly

- Thus, unlike the cartel, the multiplant monopoly has the power to decide whether to shut down some of its plants (or whether to open several more).
- A cartel generally does not shut down plants or countries for the simple legal reason that the cartel does not own the plants, and no plant would join the cartel knowing that it could be shut down.
- We assume a linear aggregate demand given by $p = a - bQ$.
- We assume that there are N plants, indexed by i , ($i = 1, 2, \dots, N$).
- Each plant produces q_i

Cartel and Multi-plant Monopoly

- Let's assume that each plant has the technology summarized by the total cost function given by –

$$TC_i(q_i) = F + c(q_i)^2; \quad F, c > 0$$

- Thus, we assume that all plants have identical cost functions, and that each plant has a fixed (output independent) cost of F .
- The plant's average and marginal-cost functions are given by –

$$ATC_i(q_i) = \frac{F}{q_i} + cq_i$$
$$MC_i(q_i) = 2cq_i$$

Cartel

- The cartel organizes all the N plants by directing each plant to produce a certain amount.
- The objective is to maximize the Sum of the profits of all the N plants.
- Let's denote $\pi_i(q_i)$ be the profit of the i^{th} plant and $Q = \sum_{i=1}^N q_i$
- Objective of the cartel: Choose q_1, q_2, \dots, q_N such that –

$$\begin{aligned} \max_{q_1, q_2, \dots, q_N} \pi(q_1, q_2, \dots, q_N) &\equiv \sum_{i=1}^N \pi_i(q_i) \\ &= \left[a - b \sum_{i=1}^N q_i \right] \left(\sum_{i=1}^N q_i \right) - \sum_{i=1}^N TC_i(q_i) \end{aligned}$$

Cartel

- N FOCs: $\frac{\partial \pi}{\partial q_j} = 0 = a - 2b \sum_{i=1}^N q_i - MC_j(q_j)$
 $= MR(Q) - MC_j(q_j); j = 1, 2, \dots, N$
- Implication: *The cartel's profit-maximizing output produced by each plant is found by equating the marginal revenue function (derived from the market demand curve, evaluated at the aggregate cartel-output level) to the marginal-cost function of each plant.*
- Since all plants have identical cost functions, we search for a symmetric equilibrium where the cartel directs each plant to produce the same output level. Hence, $q_j = q \forall j$, and we have $q = \frac{a}{2(bN+c)}$ such that
$$Q = \frac{Na}{2(bN+c)} \text{ and } p = a - bQ = \frac{a(bN+2c)}{2(bN+c)}$$

Cartel

- Note that for $N = 1$ the cartel's output and price coincide with the pure monopoly levels.
- It can be easily verified that as the number of firms in the cartel increases (N increases), both the output level of each firm and the market price fall (q and p decrease).
- Hence, the total revenue and profit of each firm must fall with an increase in the number of cartel members.
- For this reason, many professional organizations, such as those of lawyers and accountants, impose restrictions on new candidates who wish to practice in their profession.

Multi-plant Monopoly

- The multiplant monopoly is very similar to the cartel, except that it has the authority (ownership) to shut down some plants, thereby "saving" variable and fixed costs associated with maintaining the plant.
- Thus, if we suppose that the multiplant monopoly can choose the number of plants, that is, N is a choice variable by the multiplant monopoly owner, then the question is: *What is the profit maximizing number of plants operated by the multiplant monopoly?*
- Given that the multiplant monopoly can add or discard plants, the monopoly would seek to adjust the number of plants to minimize the cost per unit of production

Multi-plant Monopoly

- In other words, the multiplant monopoly will adjust the number of plants to minimize $ATC_i(q_i)$ for every plant in operation.
- Let N be continuous (quite an assumption!!)
- Like the cartel, the multiplant monopoly would equate $MR(Q) = MC_i(q_i)$ for every operating plant, yielding, $q_i = \frac{a}{2(bN+c)}$
- In addition, the monopoly will adjust N so that each operating plant would operate at minimum $ATC_i(q_i)$ where $\min ATC_i(q_i) = \sqrt{F/c}$
- Hence, $\sqrt{F/c} = \frac{a}{2(bN+c)}$ yields $N^m = \frac{a\sqrt{c}}{2b\sqrt{F}} - \frac{c}{b}$

Multi-plant Monopoly

- Thus, the multiplant monopoly's profit-maximizing number of plants increases with an increase in the demand parameter a , and decreases with the fixed cost parameter of each plant F .

Durable-good Monopoly

- Flow goods: goods that are purchased repeatedly and that perish after usage
- Durable goods are bought only once in a long time and can be used for long time
- Land
- Coase (1972) first pointed out that a monopoly selling a durable good will behave differently from the monopoly selling a perishable good analyzed so far

Durable-good Monopoly

- Coase's example: one person who owns all the land in the world, and wants to sell it at the largest discounted profit.
- Had land been perishable, then given our analyses, the monopoly would restrict output (land) and raise the price high enough so that not all the land would be sold
- Now, suppose that the monopoly charges the monopoly price and sells half of its land by the end of this year.
- What will happen next year?

Durable-good Monopoly

- The monopoly still owns the remainder of the world's land, and there is no reason why the monopoly will not offer that land for sale next year.
- However, it is clear that next year (if population is not growing very fast) the demand for land will be lower than the demand for land this year.
- Thus, the monopoly land price next year will be lower than the monopoly price this year.

Durable-good Monopoly

- Given that the monopoly's next-year price will be (substantially) lower than the monopoly land price this year, it is clear that those consumers who do not discount time too heavily would postpone buying land until next year.
- Hence, the current demand facing the monopoly falls, implying that the monopoly will charge a lower price than what a monopoly selling a perishable would charge.

Durable-good Monopoly

- *Durable-good monopoly facing a downward sloping demand*
- there is a continuum of consumers having different valuations for the annual services of a car
- Suppose that consumers live for two periods denoted by t , $t = 1, 2$,
- a monopoly sells a durable product that lasts for two periods
- Thus, if a consumer purchases the product, she will have it for her entire life, and she will not have to replace it ever again.

Durable-good Monopoly

- The consumers have different valuations for the product summarized by the aggregate period $t = 1$ inverse demand function for one period of service given by $p = 100 - Q$
- In particular, we assume that in period 1 there is a continuum of consumers, each having a different valuation for purchasing one unit of the product.
- Altogether, they form a downward sloping demand

Durable-good Monopoly

- **Definitions**
- By *selling* a product to a consumer, for a price of p^S , the firm transfers all rights of ownership for using the product and getting the product back from the consumer, from the time of purchase extended indefinitely
- By *renting* a product to a consumer, for a price of p^R , the renter maintains ownership of the product, but contracts with the consumer to allow the consumer to derive services from the product for a given period specified in the renting contract.

Durable-good Monopoly

- It should be emphasized that the definition of selling does not imply that by selling, the manufacturer always transfers all rights on the product sold.
- For example, even when a product is sold (rather than rented) the new owner does not have the rights to produce identical or similar products if the product is under patent protection.

Renting Monopoly

- Assume that each period the monopoly rents a durable product for one period only.
- For example, a common practice of firms in several industries, in particular in the car industry, is to lease a car for a given time period rather than sell the car.
- Suppose that in each of the two periods the monopoly faces the demand $p = 100 - Q$
- If we assume 0 production cost, the monopoly would rent an amount determined by the condition $MR=MC$

Renting Monopoly

- Therefore, we will have –

$$MR = 100 - 2Q_t = 0 = MC$$

$$\Rightarrow Q_t = 50$$

- Hence, $P_t = 50$ and $\pi_t = 2500$
- Total (lifetime) profit $\pi^R = 5000$

Selling Monopoly

- A seller monopoly knows that those consumers who purchase the durable good in $t = 1$ will not repurchase in period $t = 2$.
- That is, in $t = 2$ the monopoly will face a demand for its product that is lower than the period 1 demand by exactly the amount it sold in $t = 1$.
- Therefore, in period 2 the monopoly will have to sell at a lower price resulting from a lower demand, caused by its own earlier sales.

Two-period Game

- The payoff to the monopoly is the total revenue generated by period 1 and period 2 sales.
- The strategies of the seller are the prices set in period 1, p_1 , and the price set in period 2 as a function of the amount purchased in period 1, $p_2(q_1)$
- The strategies of the buyers are to buy or not to buy as a function of first period price, and to buy or not to buy as a function of second period price.
- We look for a SPE for this simple game. The methodology for solving this finite horizon game is to solve it backwards.

The 2nd Period

- Second period demand: $q_2 = 100 - \bar{q}_1 - p_2$
 $\Rightarrow p_2 = 100 - \bar{q}_1 - q_2$
- MR: $MR_2(q_2) = 100 - \bar{q}_1 - 2q_2 = 0$
 $\Rightarrow q_2 = 50 - \frac{\bar{q}_1}{2}$
- 2nd period price (as a function of 1st period quantity): $p_2 = 50 - \frac{\bar{q}_1}{2}$
- 2nd period profit: $p_2 q_2 = \left(50 - \frac{\bar{q}_1}{2}\right)^2$

The 1st Period

- Suppose that the monopolist sells in the first period to \bar{q}_1 buyers with the highest reservation prices.
- Then, the marginal buyer, with a reservation price $(100 - \bar{q}_1)$, will be indifferent between purchasing in the first period and buying in the second period.
- Implying : $2(100 - \bar{q}_1) - p_1 = (100 - \bar{q}_1) - p_2$
$$\Rightarrow p_1 = 150 - \frac{3\bar{q}_1}{2}$$

- The selling monopoly chooses a first-period output level \bar{q}_1 that solves

$$\max_{q_1} (\pi_1 + \pi_2) = (150 - \frac{3q_1}{2})q_1 + (50 - \frac{q_1}{2})^2$$

- FOC: $0 = \frac{\partial(\pi_1 + \pi_2)}{\partial q_1} = 100 - \frac{5q_1}{2} \Rightarrow q_1^S = 40$
- Therefore we will have: $q_2^S = 30; p_2^S = 30; p_1^S = 90$
- Total profit under selling: $\pi^S = 4500 < 5000 = \pi^R$

Outcome

- *A monopoly selling a durable goods earns a lower profit than a renting monopoly.*
- **Intuition:** rational consumers are able to calculate that a selling durable- good monopoly would lower future prices due to future fall in the demand resulting from having some consumers purchasing the durable product in earlier periods. This calculation reduces the willingness of consumers to pay high prices in the first period the monopoly offers the product for sale.

Durable-good monopoly

- *Durable-good monopoly facing a discrete demand.*
- No more continuum of buyers – finite number of buyers
- Let us consider an economy with two consumers living only for two periods.
- Two consumers differ substantially in MWP – high type (H) has a MWP V_H while the low type (L) has MWP V_L with type H consumers are willing to pay more than twice as much for a period of car service as type L consumers.
- Formally, $V_H > 2V_L > 0$

Durable-good monopoly

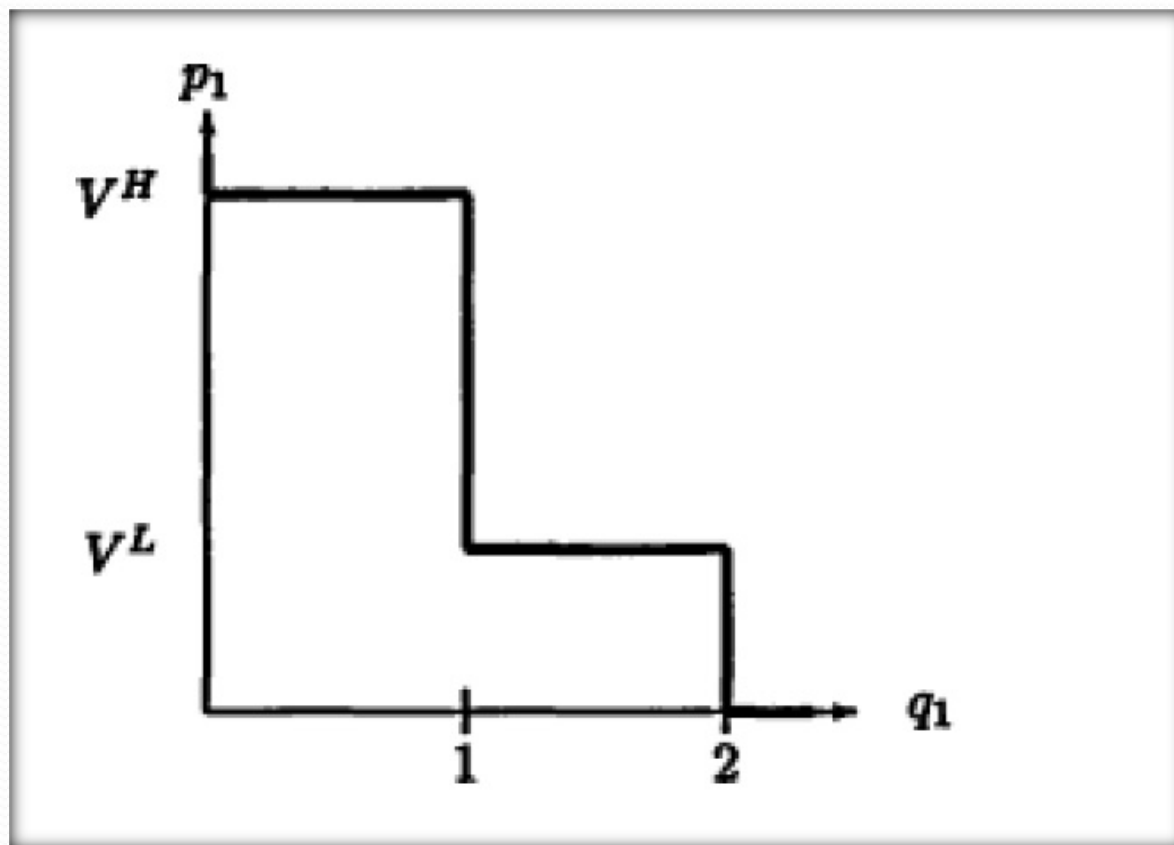
- Because the product is durable, consumers buy it once in their life either at $t = 1$ or $t = 2$.
- The utility functions for consumers type $i = H, L$ are given by –

$$U_i \equiv \begin{cases} 2V_i - P_1 & \text{if buyes at } t = 1 \\ V_i - P_2 & \text{if buyes at } t = 2 \\ 0 & \text{if does not buy at all} \end{cases}$$

- Thus, if consumer i , $i = H, L$, buys a car in the first period, he gains a benefit of $2V_i$ since the car provides services for two periods, and he pays whatever the monopoly charges in $t = 1$.
- In contrast, if the consumer waits and purchases the car in $t = 2$, he gains only one period of utility of V_i minus the price charged in period 2.

Durable-good monopoly

- The aggregate inverse demand function for one period of service facing the monopoly each period looks like –



Durable-good monopoly

- On the production side, we assume that there is only one firm producing cars, at zero cost.
- Like the consumers, the monopoly firm lives for two periods and maximizes the sum of profits from the sales during the two periods.
- We denote by q_t the amount produced and sold by the monopoly, and by p_t the period t price of a car set by the monopoly in period t , $t = 1, 2$.
- The monopoly chooses p_1 and p_2 to maximize the sum of revenue from two periods worth of sales given by $\pi = p_1 q_1 + p_2 q_2$

Renting monopoly

- Suppose now that the monopoly firm does not sell cars, but instead rents cars for one period only.
- Thus, each consumer who rents a car in $t = 1$, has to return the car at the end of the first period and rent it again in the second period.
- Rental price for one period of renting in period t : P_t^R
- Since car rentals last for one period only, it is sufficient to calculate the price for each period separately.

Renting monopoly

- The monopolist has two options –
- Option 1: Set $P_t^R = V_H$
- Implication: Only H-type will rent, L-type won't. Monopolist extracts entire surplus from the H-type.
- Profit: $\pi_R = 2V_H$
- Option 2: Set $P_t^R = V_L$
- Implication: Both types rent. Monopolist extracts entire surplus from the L-type. H-type will be left with some positive surplus.
- Profit: $\pi_R = 4V_L$

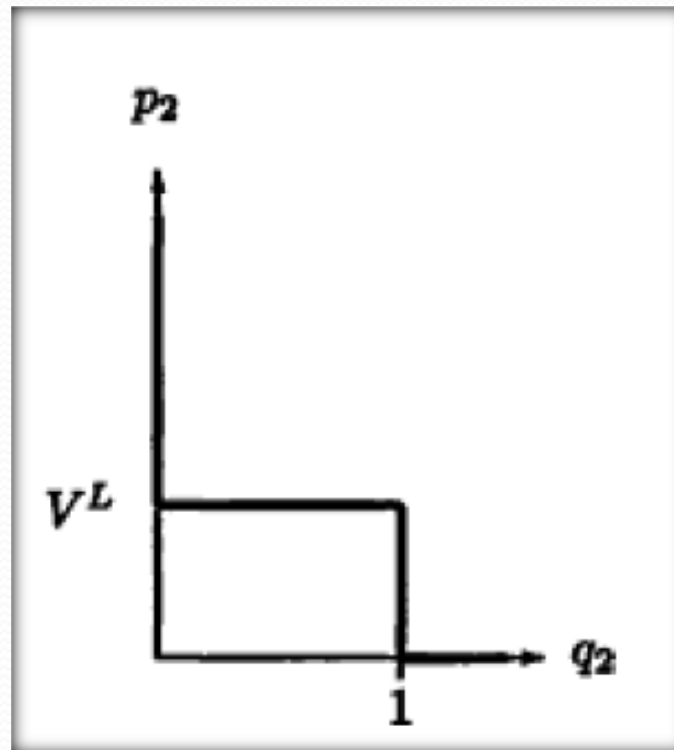
Renting monopoly

- However, we had assumed $V_H > 2V_L > 0$
- Hence, $2V_H > 4V_L$
- A renting monopoly would rent cars only to the high-valuation consumer by setting a rental price $P_t^R = V_H$ ($t = 1, 2$) and it will earn a two-period profit of $\pi_R = 2V_H$.

Selling monopoly

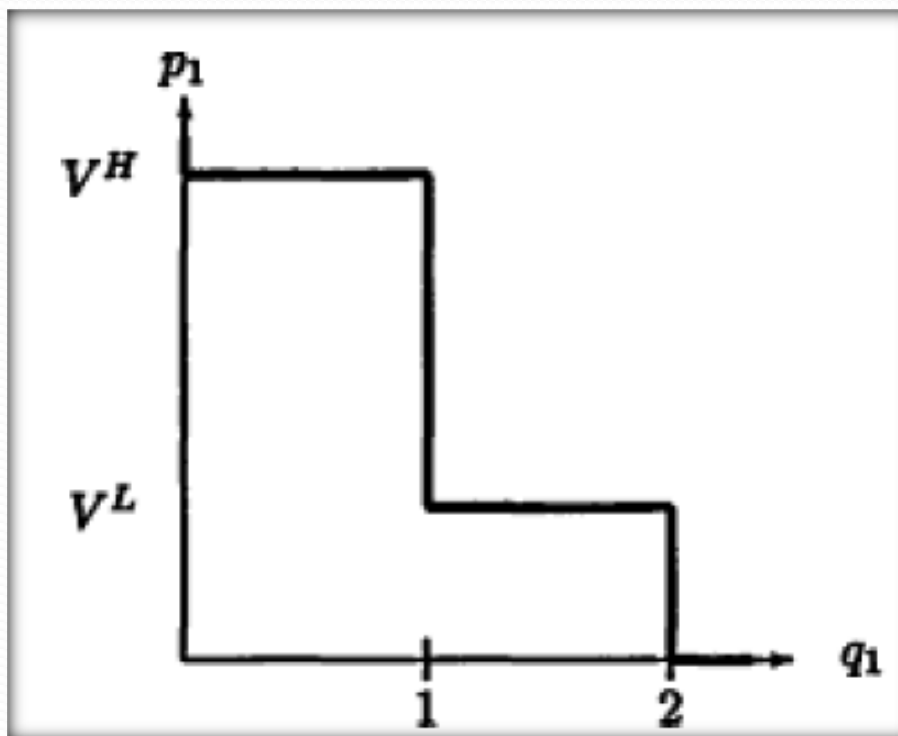
- Now, suppose that the monopoly sells the cars to consumers.
- We denote the selling prices by P_t^S ; $t = 1, 2$.
- The period 1 selling price, P_1^S , means that the consumer pays for two periods of using the car (compared with the renting price P_1^R that entitles the consumer to use the car for period 1 only).
- If consumer H purchases in period 1, only consumer L demands a car in the second period.
- This effect of selling in the first period on the second period demand can be shown as —

Selling monopoly

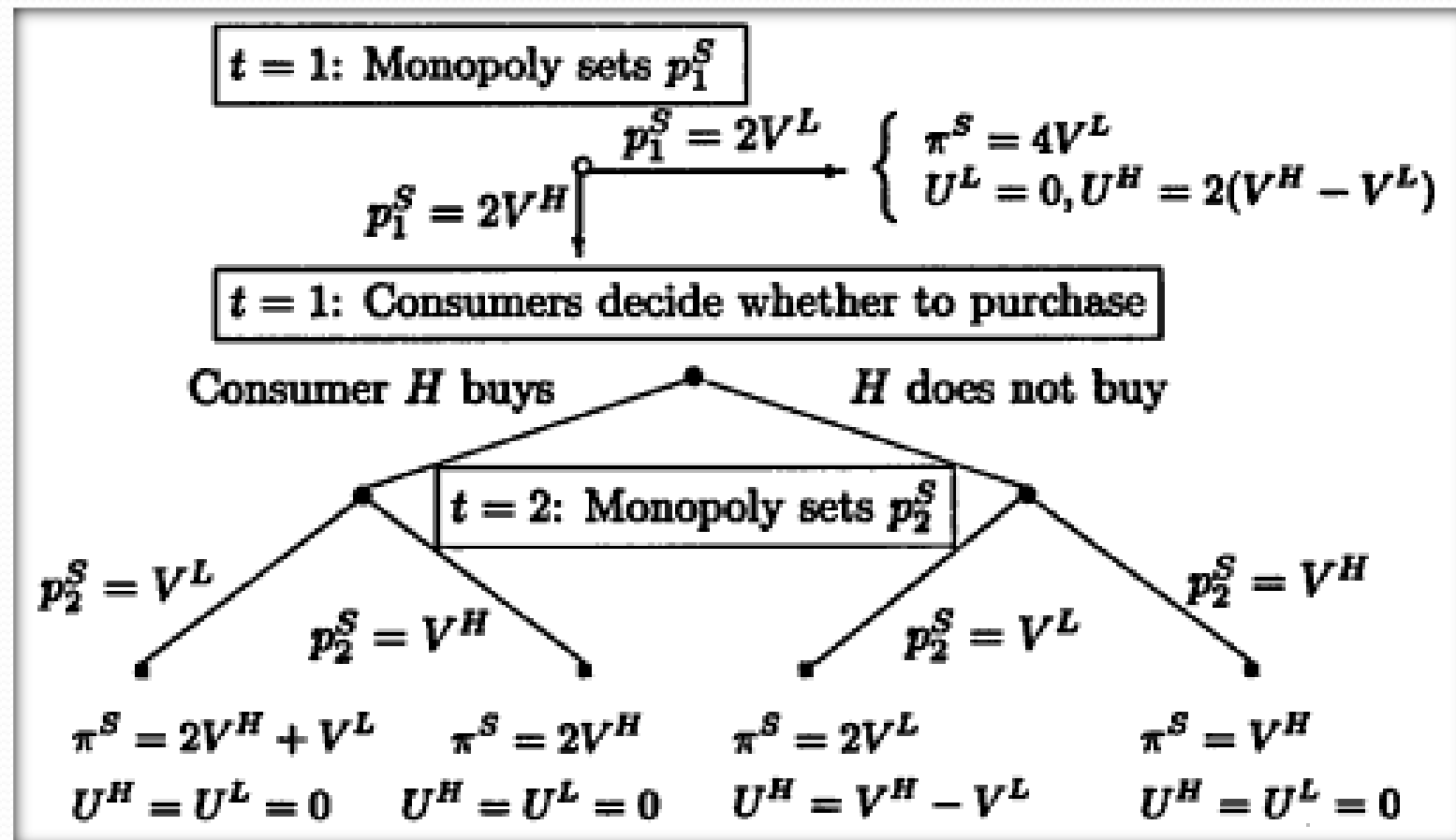


Selling monopoly

- If consumer H does not purchase in the first period, then the second period demand is the given rental demand curve –



Game Tree



Selling monopoly

- When consumer H buys in the first period, the monopoly will maximize second period profit by setting $P_2^S = V_L$
- The monopolist will earn a second period profit of $\pi_2 = V_L$ (the monopoly will extract all surplus from consumer L).
- If consumer H does not buy in period 1, then in the second period the monopoly faces the entire demand, hence, the monopoly charges $P_2^S = V_H$ (selling only to consumer H) yielding a second period profit of $\pi_2 = V_H$

Selling monopoly

- *First Period*
- In the first period, the monopoly sets P_1^S , and consumers decide whether to purchase or not.
- Since consumer L knows that the price in the last period will never fall below V_L , consumer L will buy in the first period at any price below $2V_L$.
- Hence, if the seller sets $P_1^S = 2V_L$ both consumers would purchase initially.

Selling monopoly

- The monopoly will not set $P_1^S > 2V_H$ because this price exceeds the two period sum of consumer H's valuation.
- Therefore, we now check whether $P_1^S = 2V_H$ is the profit maximizing first period price for the seller monopoly.
- From the second period analysis we conclude that consumer H earns a utility of zero ($U_H = 0$) whether or not he buys the product in the first period.
- Hence, buying the product is an optimal response for consumer H to the first period price $P_1^S = 2V_H$

Selling monopoly

- Thus, $P_1^S = 2V_H$, consumer H buys in period 1, $P_2^S = V_L$, and consumer L buys in period 2, constitute a SPE equilibrium path for this game.
- A durable-good selling monopoly facing a discrete demand will –
 1. charge a first period selling price that is equal to the sum of the per-period rental prices, $P_1^S = 2V_H = 2P_t^R$
 2. earn a higher profit than the renting monopoly;
$$\pi_S = 2V_H + V_L > 2V_H = \pi_R$$
- Thus, in the case of discrete demand, a selling monopoly can extract a higher surplus from consumers than the renting monopoly

Selling monopoly

- Coase conjectured that the ability of a durable-good monopoly to extract consumer surplus is reduced when the monopoly is forced to sell rather than rent.
- This case demonstrates the opposite, where selling enables the monopoly to price discriminate among different consumers by setting prices which would induce different consumers to purchase at different time periods

Reference

- Industrial Organization: Theory and Applications (1995). Oz Shy. MIT Press. Chapter – 5.