Oligopoly

Ref. 1. Pindyck and Rubinfeld

2. Gravelle & Rees

Characteristics of Oligopoly

- In an oligopoly market there are small number of large firms with significant market power.
- The important element of an oligopoly is the strategic interactions between firms and interdependence of firms.
- Whether firms produce homogeneous or differentiated good and / or whether entry is limited or prohibited are not that important.
- Examples: Automobiles, Boeing and Airbus, Home appliances.

Equilibrium in Oligopoly market: A Topology

- The study of equilibrium in an Oligopoly market is complicated because of wide range of strategic interactions depending on specification of the strategy apace (or variable), sequence of move and non-cooperative or collusive behavior.

- No single model and unique equilibrium can describe an oligopolistic market.

Application of Game Theory in understanding oligopoly

- The development of Game Theory, in particular the concept of Nash Equilibrium in the year 1995 and its subsequent refinements, have made it easier to understand and interpret not only the classic oligopoly model like Cournot (1838), Bertrand and Stackelberg, but also many other complicated oligopoly behaviors.

- Due to the interplay of strategies and counter strategies, oligopoly behavior is essentially like a game of Chess.

- Equilibrium in an Oligopolistic Market

When a market is in equilibrium, firms are doing the best they can and have no reason to change their price or output.

Nash Equilibrium Equilibrium in oligopoly markets means that each firm will want to do the best it can *given what its competitors are doing,* and *these competitors will do the best they can given what that firm is doing.*

- **Nash equilibrium** Set of strategies or actions in which each firm does the best it can given its competitors' actions.

- **duopoly** Market in which two firms compete with each other.

The Cournot Model

- **Cournot model** developed by French economist Augustin Cournot 1838

  Oligopoly model in which firms produce a homogeneous good, each firm treats the output of its competitors as fixed, and all firms decide simultaneously how much to produce.

(q_i^*, q_j^*) is a NE pair of output , if ,

$$\pi_i\left(q_i^*, q_j^*\right) \geq \pi_i\left(q_i, q_j^*\right) \forall q, \& i \neq j$$

- Derivation of reaction function

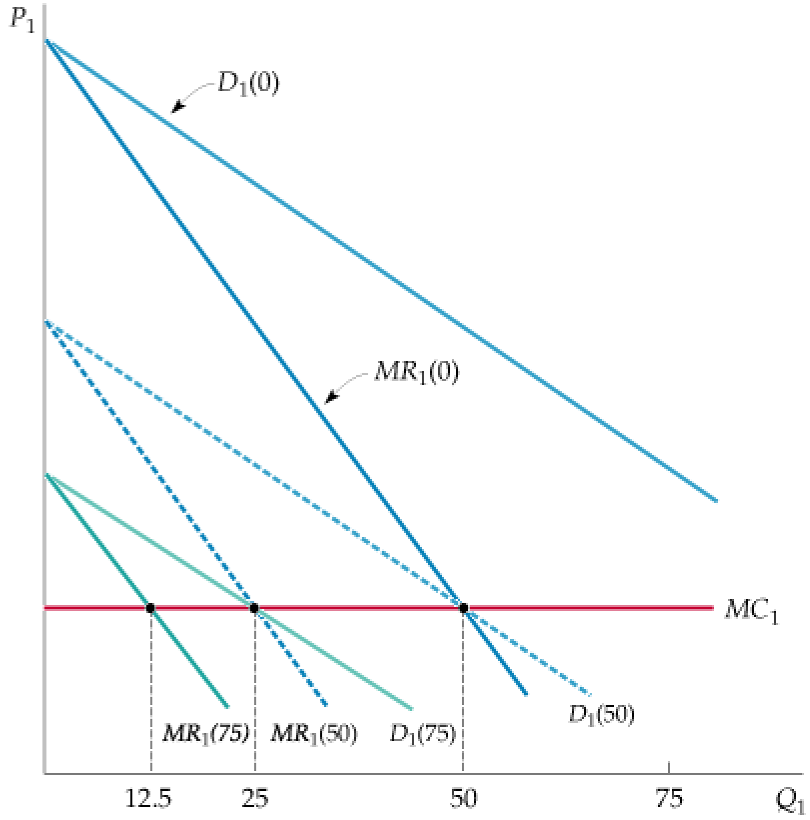
Firm 1's Output Decision

Firm 1's profit-maximizing output depends on how much it thinks that Firm 2 will produce.

If it thinks Firm 2 will produce nothing, its demand curve, labeled $D_1(0)$, is the market demand curve. The corresponding marginal revenue curve, labeled $MR_1(0)$, intersects Firm 1's marginal cost curve MC_1 at an output of 50 units.

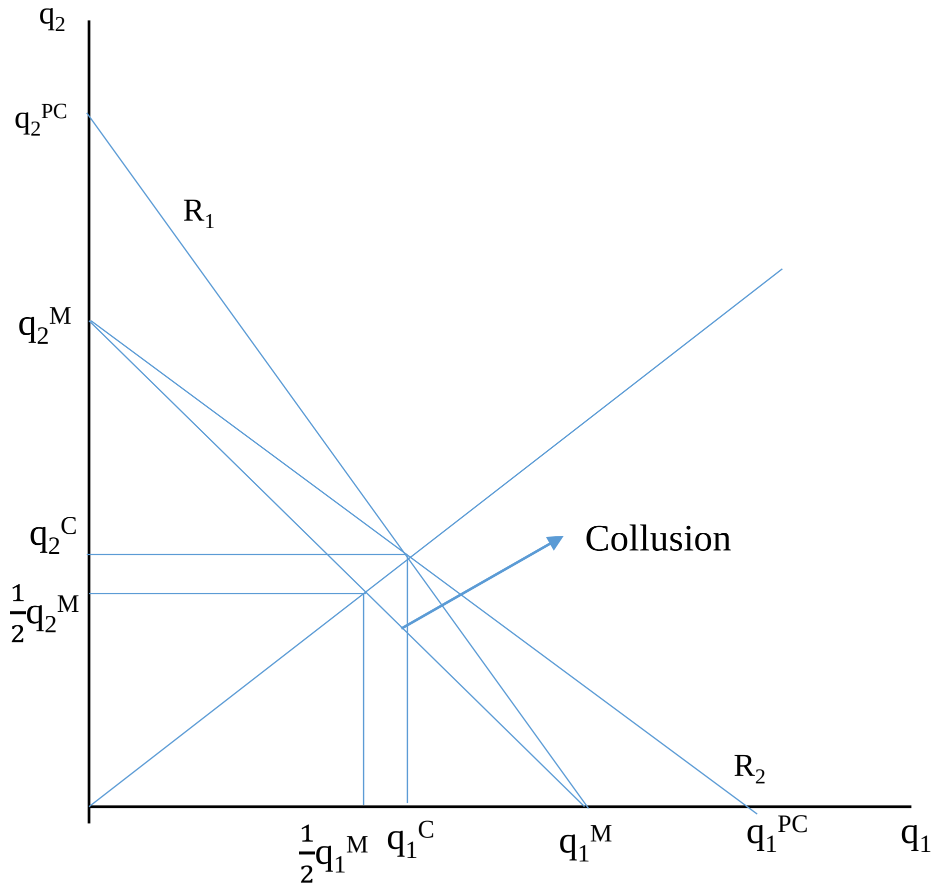
If Firm 1 thinks that Firm 2 will produce 50 units, its demand curve, $D_1(50)$, is shifted to the left by this amount. Profit maximization now implies an output of 25 units.

Finally, if Firm 1 thinks that Firm 2 will produce 75 units, Firm 1 will produce only 12.5 units.



Cournot Model

R_1 is steeper than R_2 as competitive output > monopoly output



Reaction functions

- Vertical intercept of R1: Firm 1 thinks Firm 2 produces the highest possible level of output which is perfectly competitive output. In that case Firm 1 produces nothing. $q_2=q_{pc}$, $q_1=0$.

- Horizontal intercept of R1: Firm 1 becomes monopoly when it think Firm 2 produces nothing. produces nothing. $q_2=0$, $q_1=q_m$.

- Horizontal intercept of R2: Firm 2 thinks Firm 1 produces the highest possible level of output which is perfectly competitive output. In that case Firm 1 produces nothing. $q_1=q_{pc}$, $q_2=0$.

- Vertical intercept of R1: Firm 2 becomes monopoly when it think Firm 1 produces nothing. produces nothing. $q_1=0$, $q_2=q_m$.

Explanation of reaction function and its slpe

- When Firm 2 produces nothing, the entire market is left to Firm 1 who behaves as a monopolist. As x2 increases, x1 falls. As Firm 2 produces the largest possible output which is perfectly competitive output, Firm 1 produces nothing. $q_2=q_{pc}$, $q_1=0$.
- Since competitive output is better than monopoly output Reaction function of Firm 1 is steeper than Reaction function of Firm 2.

Determination of best response functions

$$P = a - q_1 - q_2, C_1 = cq_1, C_2 = cq_2$$
$$\pi_1 = pq_1 - cq_1, \pi_2 = pq_2 - cq_2$$

$$TR_1 = pq_1 = (a - q_1 - q_2)q_1$$
$$= aq_1 - q_1^2 - q_2q_1$$
$$\pi_1 = aq_1 - q_1^2 - q_2q_1 - cq_1$$
$$\frac{\partial \pi_1}{\partial q_1} = a - 2q_1 - q_2 - c = 0$$
$$\therefore q_1 = \frac{1}{2}(a - q_2 - c)\ldots\ldots\ldots(1)$$
$$\rightarrow R_1: q_2 \rightarrow q_1$$
$$Sim, R_2 = pq_2 = aq_2 - q_2^2 - q_2q_1$$
$$\pi_2 = aq_2 - q_2^2 - q_2q_1 - cq_2$$
$$\frac{\partial \pi_2}{\partial q_2} = a - 2q_2 - q_1 - c = 0$$
$$\therefore q_2 = \frac{1}{2}(a - q_1 - c)\ldots\ldots\ldots(2)$$
$$\rightarrow R_2: q_1 \rightarrow q_2$$

$$2q_1 = a - c - \frac{1}{2}(a - q_1 - c)$$

$$\Rightarrow 4q_1 = 2a - 2c - a + q_1 + c$$

$$\Rightarrow 3q_1 = a - c \Rightarrow q_1 = \frac{a-c}{3}$$

$$2q_2 = a - c - \frac{a-c}{3} = \frac{2(a-c)}{3}$$

$$\Rightarrow q_2 = \frac{a-c}{3}$$

$$So, Q = q_1 + q_2 = \frac{2(a-c)}{3}$$

$$So, P = a - q_1 - q_2 = a - \frac{a-c}{3} - \frac{a-c}{3}$$

$$= \frac{3a - a + c - a + c}{3} = \frac{a+2c}{3}$$

$$Cournot\_outcome : q_1^c = q_2^c = \frac{a-c}{3}$$

$$P = \frac{a+2c}{3}$$

N firms

N = 2 ,

$$\Rightarrow P = \frac{a+2c}{3}$$

With N firms :

$$P = \frac{a+Nc}{N+1} = \frac{\frac{a}{N}+c}{1+\frac{1}{N}}$$

$$\lim_{N\to\infty} P = \frac{\frac{a}{N}+c}{1+\frac{1}{N}} = c$$

As N ↑ , P tends to MC (P = c) as in perfect competition

$$\pi_1 = TR_1 - C_1 = pq_1 - cq_1 = \left(\frac{a+2c}{3} - c\right)\left(\frac{a-c}{3}\right)$$

$$= \left(\frac{a-c}{3}\right)\left(\frac{a-c}{3}\right) = \left(\frac{a-c}{3}\right)^2 > 0$$

$$\pi_2 = TR_2 - C_2 = pq_2 - cq_2 = \left(\frac{a+2c}{3} - c\right)\left(\frac{a-c}{3}\right)$$

$$= \left(\frac{a-c}{3}\right)^2 > 0$$

Symmetric outcome: outputs and prices are same as we have assumed identical firms with same cost function. Please refer to your home work with different MCs, taxes.

Collusive outcome

Monopoly output : In (1) and (2) , putting $q_2 = 0$ & $q_1 = 0$ respectively , we have

$$q_1 = \frac{a-c}{2}, q_2 = 0$$
$$q_1 = 0, q_2 = \frac{a-c}{2}$$
$$\& \; P = a - q_1 - q_2$$
$$= a - \frac{a-c}{2} - 0$$
$$= \frac{a+c}{2}$$

Prisoner’s dilemma Problem

Cournot NE is not Pareto Efficient.

Monopoly profit is unique and highest possible. Total Cournot output is > monopoly output. Hence Cournot profits are less.

Hence NE output levels are not efficient.

Collusion is not the NE

- Each one has unilateral incentive of deviation from collusive output.
- Graphically we see that the collusive eqm is “off” the best responses of both the firms.
- In one-shot game Cournot-NE is the only prediction of the game.
- Collusion can be sustained if the game is repeated (that too infinite times). In finitely repeated games collusion cannot be sustained.

Example

Duopolists face the following market demand curve

$P = 30 - Q$

Also, $MC_1 = MC_2 = 0$

Total revenue for firm 1: $R_1 = PQ_1 = (30 - Q)Q_1$

then $MR_1 = \Delta R_1/\Delta Q_1 = 30 - 2Q_1 - Q_2$

Setting $MR_1 = 0$ (the firm's marginal cost) and solving for Q_1, we find

Firm 1's reaction curve: $Q_1 = 15 - \frac{1}{2}Q_2$ **(1)**

By the same calculation, *Firm 2's reaction curve:* $Q_2 = 15 - \frac{1}{2}Q_1$ **(2)**

Cournot equilibrium: $Q_1 = Q_2 = 10$

Total quantity produced: $Q = Q_1 + Q_2 = 20$

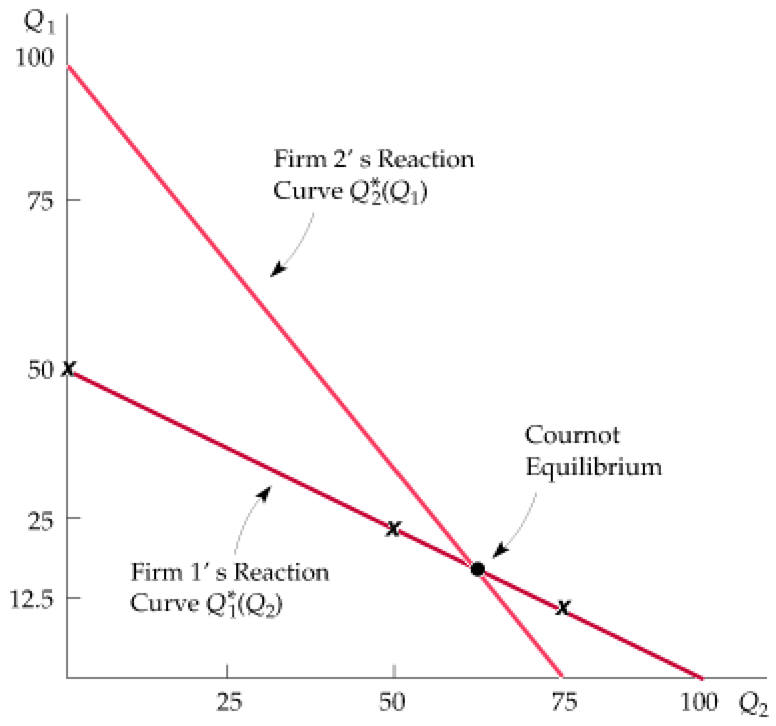
- **reaction curve** Relationship between a firm's profit-maximizing output and the amount it thinks its competitor will produce.

Reaction Curves and Cournot Equilibrium

Firm 1's reaction curve shows how much it will produce as a function of how much it thinks Firm 2 will produce.

Firm 2's reaction curve shows its output as a function of how much it thinks Firm 1 will produce.

In Cournot equilibrium, each firm correctly assumes the amount that its competitor will produce and thereby maximizes its own profits. Therefore, neither firm will move from this equilibrium.



- **Cournot equilibrium** Equilibrium in the Cournot model in which each firm correctly assumes how much its competitor will produce and sets its own production level accordingly.

If the two firms collude, then the total profit-maximizing quantity can be obtained as follows:

Total revenue for the two firms: $R = PQ = (30 - Q)Q = 30Q - Q^2$, then $MR = \Delta R/\Delta Q = 30 - 2Q$

Setting MR = 0 (the firms' marginal cost) we find that total profit is maximized at $Q = 15$.

Then, $Q_1 + Q_2 = 15$ is the *collusion curve*.

If the firms agree to share profits equally, each will produce half of the total output:
$Q_1 = Q_2 = 7.5$

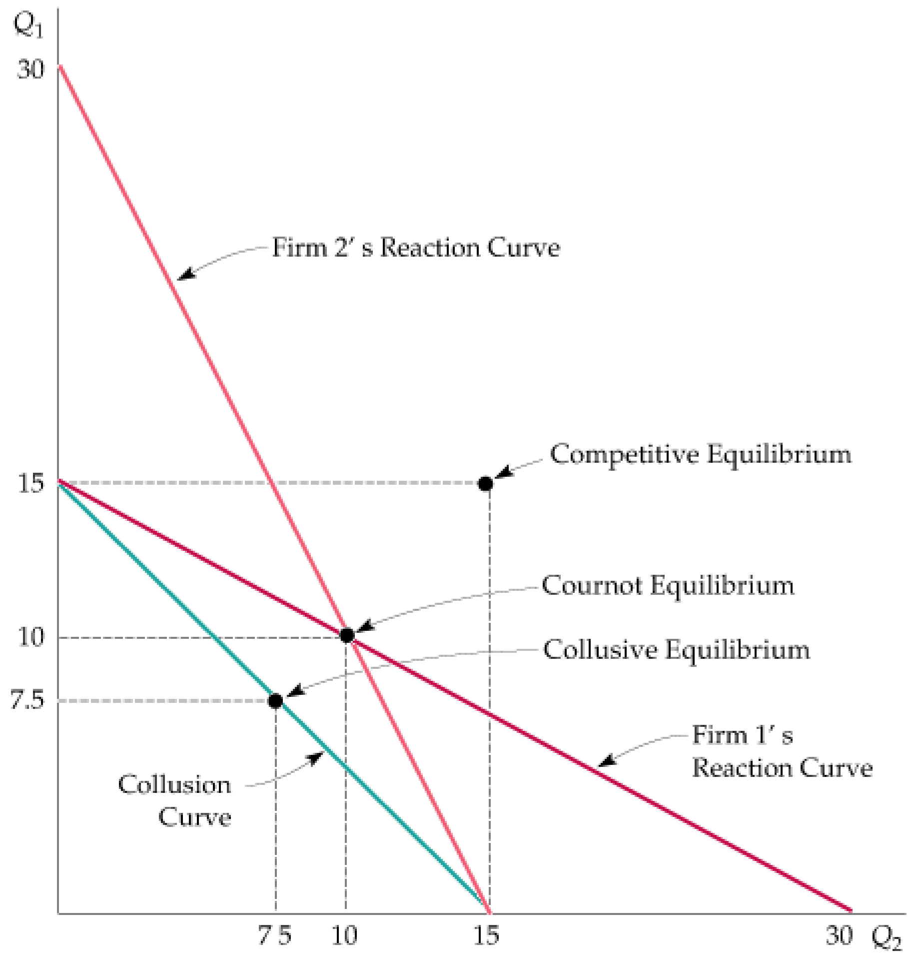
Duopoly Example

The demand curve is $P = 30 - Q$, and both firms have zero marginal cost. In Cournot equilibrium, each firm produces 10.

The collusion curve shows combinations of Q_1 and Q_2 that maximize *total* profits.

If the firms collude and share profits equally, each will produce 7.5.

Also shown is the competitive equilibrium, in which price equals marginal cost and profit is zero.



H.W.

1. When $C_1 \neq C_2$
2. When tariff is imposed only on Firm 1
3. When tariff / tax is imposed on both the firms

Bertrand Model

The Bertrand Model was developed in 1883 by French Economist Joseph Bertrand .

Characteristics:

- Firms choose prices (P_1 , P_2) and
- Firms makes their decisions at same time (simultaneously)

Case I : Firms produce identical goods (homogenous)

Proposition if
- MCs are constant and identical
- Firms have no capacity constraint

Then Unique Nash Equilibrium is

$$p_1 = p_2 = c$$
$$\Rightarrow \pi_1 = \pi_2 = 0$$

Bertrand Paradox

In Perfect Competition , firms also earn 0 profit . But in Perfect Competition, there are many firms in the market and the firms has no market power . But in Bertrand model , there are only two firms in the market and both have market power . Despite that they both earn zero profit , this is the paradox .

NE Test →

How should we show a NE ?

To show that

$$p_1 = p_2 = c$$

is a NE note that , if

$P_1 > P_2 = c$ then $x_1 = 0$

$P_2 > P_1 = c$ then $x_2 = 0$

That means there is no deviation incentive , as both the firms are producing identical goods .
If
$P_1 < P_2 = c \Rightarrow \pi_1 < 0$ (incurring lose)

$P_2 < P_1 = c \Rightarrow \pi_2 < 0$

Uniqueness

$$p_1 = p_2 = c$$

Consider the following possibilities

$$p_1 = p_2 > c$$

$$p_1 = p_2 < c$$

- $P_1 > P_2 = c$
- $P_1 < P_2 = c$

Example

$P = 30 - Q$

$MC_1 = MC_2 = \$3$

$Q_1 = Q_2 = 9$, and in Cournot equilibrium, the market price is \$12, so that each firm makes a profit of \$81.

Nash equilibrium in the Bertrand model results in both firms setting price equal to marginal cost: $P_1 = P_2 = \$3$. Then industry output is 27 units, of which each firm produces 13.5 units, and both firms earn zero profit.

In the Cournot model, because each firm produces only 9 units, the market price is \$12. Now the market price is \$3. In the Cournot model, each firm made a profit; in the Bertrand model, the firms price at marginal cost and make no profit.

Case II : Firms produce differentiated goods

Price Competition with Differentiated Products

Suppose each of two duopolists has fixed costs of $20 but zero variable costs, and that they face the same demand curves:

Firm 1's demand: $Q_1 = 12 - 2P_1 + P_2$ **(a)**

Firm 2's demand: $Q_2 = 12 - 2P_2 + P_1$ **(b)**

Choosing Prices

Firm 1's profit: $\pi_1 = P_1 Q_1 - 20 = 12P_1 - 2P_1^2 + P_1 P_2 - 20$

Firm 1's profit maximizing price: $\Delta\pi_1 / \Delta P_1 = 12 - 4P_1 + P_2 = 0$

Firm 1's reaction curve: $P_1 = 3 + \frac{1}{4} P_2$

Firm 2's reaction curve: $P_2 = 3 + \frac{1}{4} P_1$

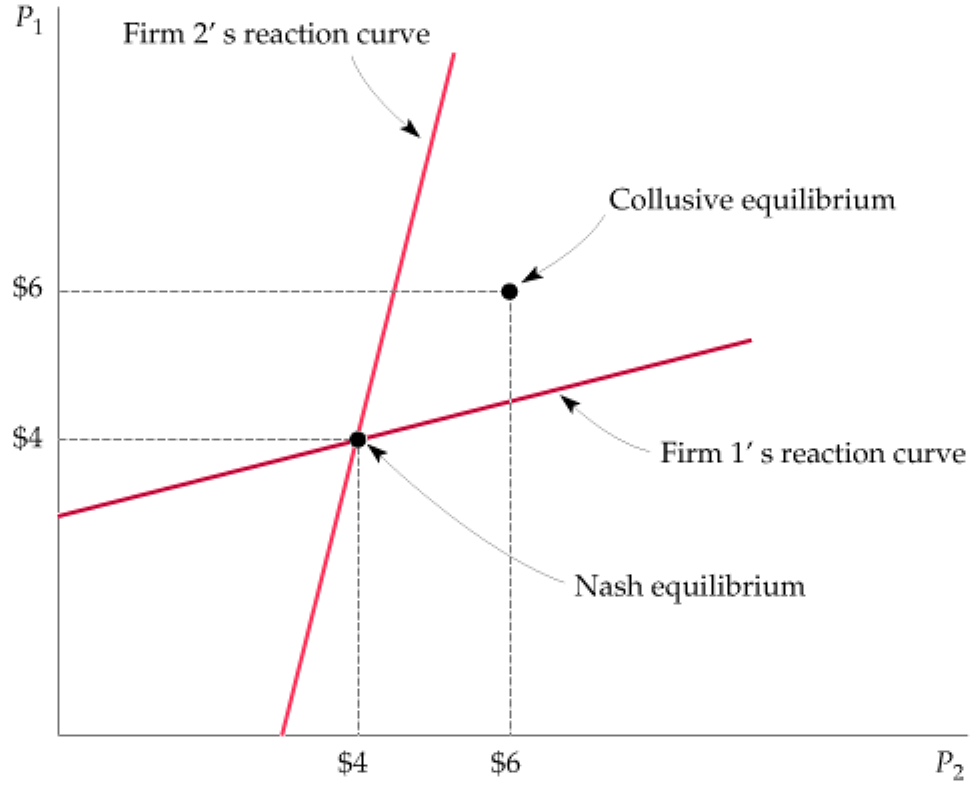
Nash Equilibrium in Prices

Here two firms sell a differentiated product, and each firm's demand depends both on its own price and on its competitor's price. The two firms choose their prices at the same time, each taking its competitor's price as given.

Firm 1's reaction curve gives its profit-maximizing price as a function of the price that Firm 2 sets, and similarly for Firm 2.

The Nash equilibrium is at the intersection of the two reaction curves: When each firm charges a price of $4, it is doing the best it can given its competitor's price and has no incentive to change price.

Also shown is the collusive equilibrium: If the firms cooperatively set price, they will choose $6.



Application of BI : Sequential move output competition

- **The Stackelberg Model (developed by German Economist : H Von Stackelberg)**

Firm 1 and 2 choose q_1 & q_2

Let Nature selected F_1 as the leader - it chooses $q_{1\,,}$ then F_2 chooses q_2 .

$Q_1 \in [\ 0\ ,\ D\ (\ c\ )\ ]$

$s_2\ :\ q_1 \rightarrow q_2$

Apply BI method:

Start with **stage 2 :** F2 will choose q_2 to maximize $\pi_2(\ q_1\ ,\ q_2\ )$ for any given q

i.e. $q_2^{\ *} = \arg\max \pi_2(\ q_1\ ,\ q_2\ ) \rightarrow$ Cournot best response function ,

$q_2^{\ *} = \ q_2\,B(q_1)$

Stage 1 : choose q_1 to max $\pi_1(\ q_1\ ,\ q_2\ )$ s.t $q_2^{\ *} = \ q_2\,B(q_1)$

i.e. F1 will choose q_1 to max $q_1(\ q_1\ ,\ q_2\,B(q_1))$

$$P = a - q_1 - q_{2,} C_1 = cq_1, C_2 = cq_2$$
$$\pi_2 = TR_2 - C_2 = pq_2 - cq_2$$
$$= aq_2 - q_1 q_2 - q_2^2 - cq_2$$
$$\frac{\partial \pi_2}{\partial q_2} = a - q_1 - 2q_2 - c = 0$$
$$\therefore q_2 = \frac{1}{2}(a - q_1 - c) \ldots\ldots\ldots\ldots (1)$$
$$\pi_1 = aq_1 - q_1 q_2 - q_1^2 - cq_1$$
$$= aq_1 - cq_1 - q_1^2 - \frac{aq_1}{2} + \frac{cq_1}{2} + \frac{{q_1}^2}{2}$$
$$= \frac{aq_1}{2} - \frac{cq_1}{2} - \frac{{q_1}^2}{2}$$

Application of Backward Induction: Sequential move output competition:

- The Stackelberg Model (German Economist : H Von Stackelberg)

Firm 1 and 2 choose q_1 & q_2

Let Nature selected F_1 as the leader - it chooses $q_{1,}$ then F_2 chooses q_2.

$Q_1 \in [\, 0, D\,(c)]$

$s_{2:}\ \ q_1 \rightarrow q_2$

Apply BI method:

Start with **stage 2:** F2 will choose q_2 to maximize $\pi_2(q_1, q_2\,)$ for any given q_1

i.e. $q_2^* = \arg\max \pi_2(q_1, q_{2)}$ $\rightarrow$ Cournot best response function,

$q_2^* = q_2\, B(q_1)$

Stage 1: When Firm 1 chooses its output level it anticipates that Firm 2 will choose output according to its best response function.

Hence Firm 1 chooses q_1 to max $\pi_1(q_1, q_2)$ subject to $q_2^* = q_2^B(q_1)$

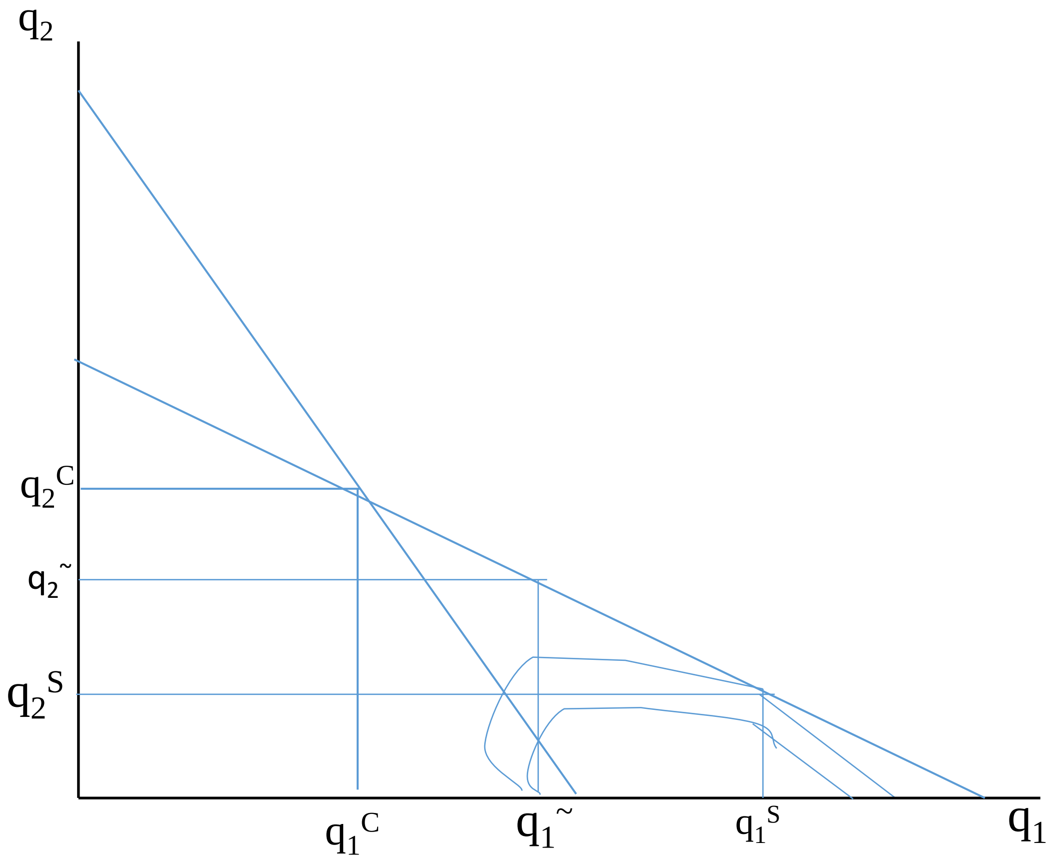
i.e., Firm 1 will choose q_1 to max $q_1(q_1, q_2^B(q_1))$

Iso profit curves: The curves are the contours of Firm 1's profit function. Lower iso profit curve corresponds to higher profit of Firm 1.

The **Stackelberg eqm** occurs at the tangency between the lowest possible iso-profit and best response function of Firm 2. It is also the **Nash eqm** of the game due to the assumption that Firm 1 makes a credible commitment to an output level which it selects before the follower firm selects its output. The follower firm takes the large output of the leader firm as given and chooses output.

First movers' advantage: Output and profit of the Stackelberg leader firm is greater than Cournot output and profit.

Stackelberg Model



Solution

$$P = a - q_1 - q_{2,} C_1 = cq_1, C_2 = cq_2$$

$$\pi_2 = TR_2 - C_2 \ = pq_2 - cq_2 = aq_2 - q_1 q_2 - q_2^2 - cq_2 \frac{\partial \pi_2}{\partial q_2}$$

$$= a - q_1 - 2q_2 - c = 0 \therefore q_2 = \frac{1}{2}(a - q_1 - c) \ldots\ldots\ldots\ldots (1)$$

$$\pi_1 = aq_1 - q_1 q_2 - q_1^2 - cq_1 = aq_1 - cq_1 - q_1^2 - \frac{aq_1}{2} + \frac{cq_1}{2} + \frac{{q_1}^2}{2}$$

$$= \frac{aq_1}{2} - \frac{cq_1}{2} - \frac{{q_1}^2}{2}$$

$$q_1 = \frac{a-c}{2}, q_2 = \frac{a-c}{4}$$

$$\pi_1 = \frac{(a-c)^2}{8}, \pi_2 = \frac{(a-c)^2}{16}$$

Example

Suppose Firm 1 sets its output first and then Firm 2, after observing Firm 1's output, makes its output decision. In setting output, *Firm 1 must therefore consider how Firm 2 will react*.

$P = 30 - Q$

Also, $MC_1 = MC_2 = 0$

Firm 2's reaction curve: $Q_2 = 15 - \frac{1}{2}Q_1$

Firm 1's revenue: $R_1 = PQ_1 = 30Q_1 - Q_1^2 - Q_2Q_1$

And $MR_1 = \Delta R_1/\Delta Q_1 = 15 - Q_1$

Setting $MR_1 = 0$ gives $Q_1 = 15$, and $Q_2 = 7.5$

We conclude that Firm 1 produces twice as much as Firm 2 and makes twice as much profit. *Going first gives Firm 1 an advantage.*