<u>Assignment – 2</u>

Topics: Classification of solutions (or integrals) as Complete Integral (CI), General Integral (GI), Particular Integral (PI), Singular Integral (SI) of 1st order PDE, Lagrange's method (or Method of Characteristics) to find general solution (or GI) of 1st order linear/semi-linear/quasi-linear/non-linear PDE, Integral Surface through a given curve (Cauchy Problem) for 1st order PDE, Surface orthogonal to given surface.

1. Find the general solution of each of the following PDE by Lagrange's method:-
   a) $yzp + zxq = xy$
   b) $(x - z)p + (z - y)q = 0$
   c) $x^2p - y^2q = (x - y)z$

2. Find the integral surface of each of the following PDE by method of characteristic:
   a) $(2z - y)p + (x + z)q + 2x + y = 0$
   b) $(p - q) = \log(x + y)$

3. For following 1st order PDEs, write down Lagrange's auxiliary equation and find two integral curves of that equation:
   a) $p\tan x + q\tan y = \tan z$
   b) $p + 5q = 9z + \tan(y - 5x)$

3. Find the general integral of each of the following equations in which $p_j = \frac{\partial z}{\partial x_j}$ $(j = 1,2,3)$:-
   a) $x_2p_1 + x_1p_2 + x_1x_2p_3 = 0$
   b) $x_1x_3p_1 - x_2x_3p_2 + p_3 = z$
   c) $z(p_1 + p_2 + p_3) = 1$
   d) $z(x_1p_1 + x_2p_2 + x_3p_3) = 1$
   e) $x_2x_3p_1 + x_1x_3p_2 + x_1x_2p_3 = 0$

4. Find the integral surface of the PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.

5. Find the integral surface of the equation $(y - z)\{2xyp + (x^2 - y^2)q\} + z(x^2 - y^2) = 0$ through the curve $x = t^2, y = 0, z = t^3$.

6. Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2) = 1$. By taking $b = 2a$, show that the envelope of the sub family is $(y - 2x)^2 + 5z^2 = 5$ which is a particular solution. Find the singular solution(s), if they exist.

7. Find the system of surfaces orthogonal to given system of surfaces given by $z = cxy(x^2 + y^2)$, c is a free parameter.

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