Sol 1)

1 $\quad 1^{10} - 2^{8}$

2 $\quad 1^{3} - 4^{5} - 3^{6}$

3 $\quad 1^{4} - 4^{3}$

x_{ij}: time when the processing of job j started at machine i

s.t

$$x_{11} + 10 \le x_{12}$$
$$x_{21} + 3 \le x_{24}$$
$$x_{24} + 5 \le x_{23}$$
$$x_{31} + 4 \le x_{34}$$

Technological Constraints

$$x_{24} + 5 - x_{34} \le M\delta_1$$
$$x_{34} + 3 - x_{24} \le M(1-\delta_1)$$

Either or for machine 4

$$x_{11} + 10 \le x_{21} + 2My_1$$
$$x_{11} + 10 \le x_{31} + 2My_1$$
$$x_{21} + 3 \le x_{31} + 2My_1 - MY_1$$
$$x_{31} + 4 \le x_{21} + 2My_1 - M(1-Y_1)$$

$$x_{21} + 3 \le x_{11} + 2My_2$$
$$x_{21} + 3 \le x_{31} + 2My_2$$
$$x_{11} + 10 \le x_{31} + 2My_2 - MY_2$$
$$x_{31} + 4 \le x_{11} + 2My_2 - M(1-Y_2)$$

$$X_{21} + 4 \leq X_{11} + 2MY_3$$
$$X_{21} + 3 \leq X_{21} + 2MY_3$$
$$X_{11} + 10 \leq X_{21} + 2MY_3 - MY_3$$
$$X_{21} + 3 \leq X_{11} + 2MY_3 - M(1-Y_3)$$

$$Y_1 + Y_2 + Y_3 = 2$$

$$X_{12} + 8 \leq Z$$
$$X_{23} + 6 \leq Z$$
$$X_{34} + 3 \leq Z$$
$$Y \in \{0,1\}$$
$$Y_1 \in \{0,1\}$$

Sol 2)

Each project represents a city. The table below gives the number of *distinct* employees who enter/leave the manager's office when we switch from project *i* to project *j* (i.e., the number of mismatched "x" between column *i* and column *j*). The objective is to find a "tour" through all projects that will minimize the total traffic.

	1	2	3	4	5	6
1		4	4	6	6	5
2	4		6	4	6	3
3	4	6		4	8	7
4	6	4	4		6	5
5	6	6	8	6		5
6	5	3	7	5	5	

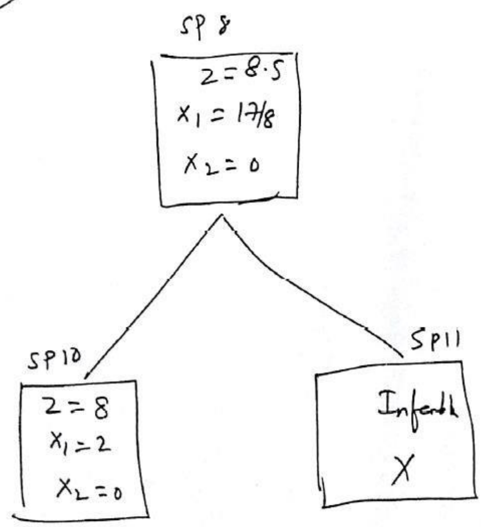
Sol 3)

(a) Maximization problem; $z = 7$

(b) SP4, SP9 → Infeasible } Fathomed
SP6 → Integer }

SP3, SP8 } → Not fathomed

(c)



Sol 4)

Write the problem in the matrix form, and get its reduced matrix by performing row operations followed by column operations. Write the amount of total reduction on the bottom of right corner of reduced matrix *S*. Both matrices are shown in the following table

Cost matrix

∞	5	9	∞
5	∞	3	4
∞	4	6	∞
9	3	∞	6

Reduced matrix

∞	0	4	∞
0	∞	0	0
∞	0	2	∞
4	0	∞	2

In the reduced matrix, city pairs with cost ‘0’ are candidates to be considered in the tour. From the reduced matrix (1,2), (2,1), (2,3), (2,4), (3,2), (4,2) are the candidate pairs.

The lower bound of total distance is

LB= 5+3+4+3=15

So the tour has to travel at least 15. Optimal solution will have to be more than or equal to the lower bound of 15.

Iteration 1. The pairs (1,2) and (2,1) have the largest LCE.

LB on $\overline{(1,2)}$ = 22

Now, the set of all four tours is portioned into two classes:

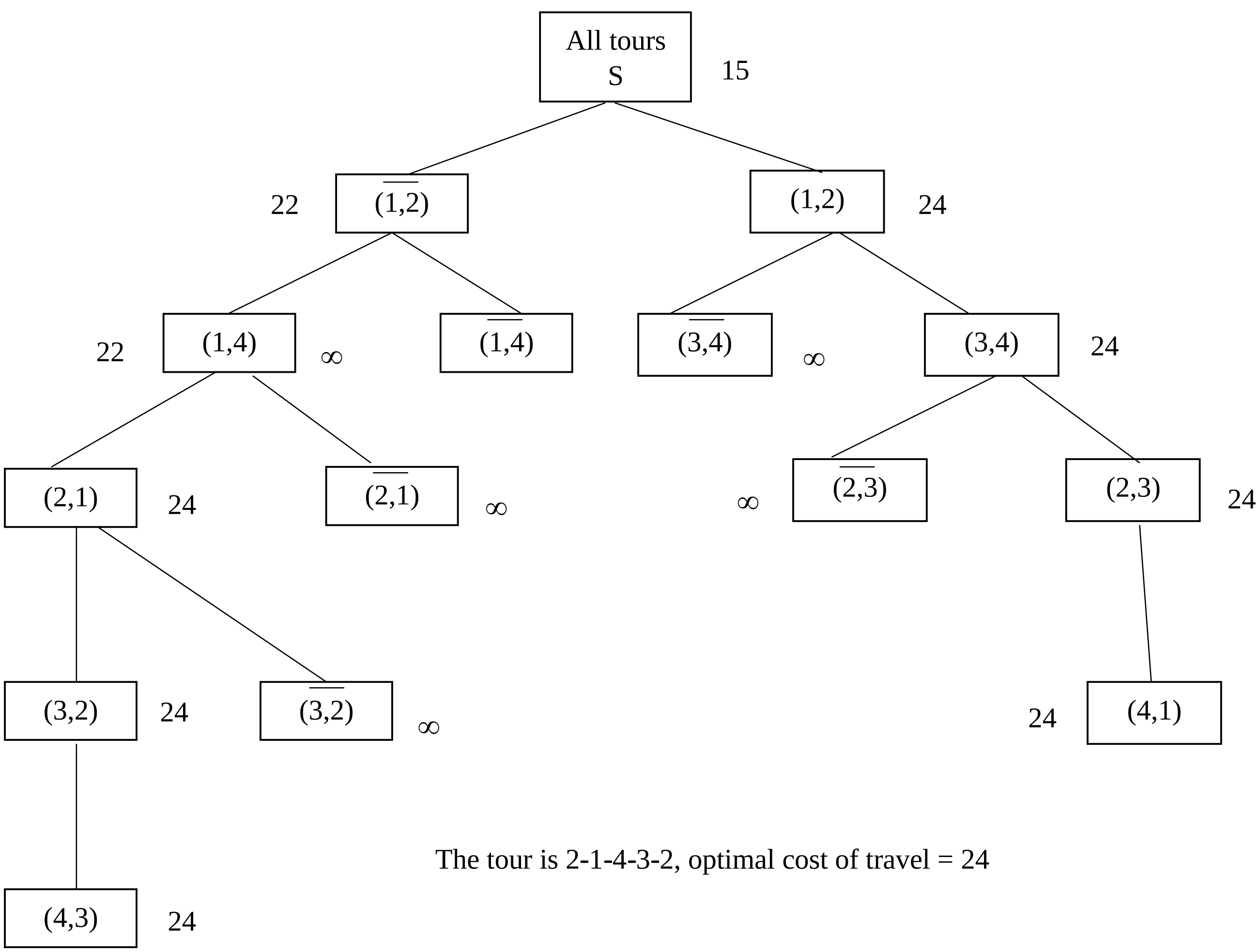
(a) Node (1,2) means all tours in which salesman travels from city 1 to 2.
(b) Node $\overline{(1,2)}$ means all tours in which salesman does not travel from city 1 to 2.

LB on (1,2): write the matrix of (1,2) by deleting the first row and second column of the present reduced matrix. Put ∞ in (2,1) cell. Reduce the matrix.

LB on (1,2) = 18+6 = 24

Now branching is done from <u>the</u> node which has the smallest LB, and hence in this case, branching is done from node (1,2).

Follow the same procedure and stops when there is a single entry. Below is the complete graph:



The tour is 2-1-4-3-2, optimal cost of travel = 24

Sol 5)

x_{ij} = Number of bottles of type i assigned to individual j, where $i = 1$ (full), 2 (half full), 3 (empty).

Constraints:

$$x_{11} + x_{12} + x_{13} = 7, x_{21} + x_{22} + x_{23} = 7, x_{31} + x_{32} + x_{33} = 7$$

$$x_{11} + .5x_{21} = 3.5, x_{12} + .5x_{22} = 3.5, x_{13} + .5x_{23} = 3.5$$

$$x_{11} + x_{21} + x_{31} = 7, x_{12} + x_{22} + x_{32} = 7, x_{13} + x_{23} + x_{33} = 7$$

All x_{ij} are nonnegative integers

Solution: Use a dummy objective function.

	Number of bottles assigned to individual		
Status	*1*	*2*	*3*
Full	1	3	3
Half full	5	1	1
Empty	1	3	3

6 Let,

$$x_j = \begin{cases} 1, & \text{if route `j' is selected} \\ 0, & \text{otherwise} \end{cases}$$

Total distance for route 1 = 10 + 32 + 14 + 15 + 9 = 80 miles.
Similarly for others, route 2 = 50 miles, route 3 = 70 miles,
route 4 = 52 miles. route 5 = 60 miles. route 6 = 44 miles.

Minimize, $Z = 80x_1 + 50x_2 + 70x_3 + 52x_4 + 60x_5 + 44x_6$

s.t.,

$$x_1 + x_3 + x_5 + x_6 \geq 1$$
$$x_1 + x_3 + x_4 + x_5 \geq 1$$
$$x_1 + x_2 + x_4 + x_6 \geq 1$$
$$x_1 + x_2 + x_5 \geq 1$$
$$x_2 + x_3 + x_4 + x_6 \geq 1$$

$x_j = (0,1)$ for all j

$\therefore$ Optimal solution, $Z = 210$, routes $\rightarrow$ (1, 4, 2) and (1, 3, 5)

8) Let, x_j represent the daily number of units produced of the product j.

~~Max.~~ ~~25~~ Maximize $25x_1 + 30x_2 + 22x_3$

s.t.,

Location 1 $\left| \begin{array}{l} 3x_1 + 4x_2 + 5x_3 \leq 100 \\ 4x_1 + 3x_2 + 6x_3 \leq 100 \end{array} \right.$

Or,

Location 2 $\left| \begin{array}{l} 3x_1 + 4x_2 + 5x_3 \leq 90 \\ 4x_1 + 3x_2 + 6x_3 \leq 120 \end{array} \right.$

Adding slack variables, after solving we got the solution:

Location 1: $x_1 = 4,\ x_2 = 12, x_3 = 8$.

Location 2: $x_1 = 30, x_2 = x_3 = 0$.

7) Let, x_e = No. of eastern (one-way) tickets.

x_u = No. of US Air tickets.

x_c = No. of continental tickets.

Maximize $1000(x_e + 1.5x_u + 1.8x_c + 5e_1 + 5e_2 + 10u + 7c)$

s.t.,

$$e_1 \leq x_e/2$$
$$e_2 \leq x_e/5$$
$$u \leq x_u/6$$
$$c \leq x_c/5$$

$$x_e + x_u + x_c = 12.$$

Solution: 10 tickets - continental, 2 tickets - eastern.

(9) Max $Z_A = 3x_1 + 2x_2$.

s.t.,

$2x_1 + 5x_2 \leq 9$

$4x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$

Max. $Z_A = 7.3125$

$(x_1 = 1.6875, x_2 = 1.125)$

$Z_L = 5 \; (x_1 = 1, x_2 = 1)$

Branch and bound diagram