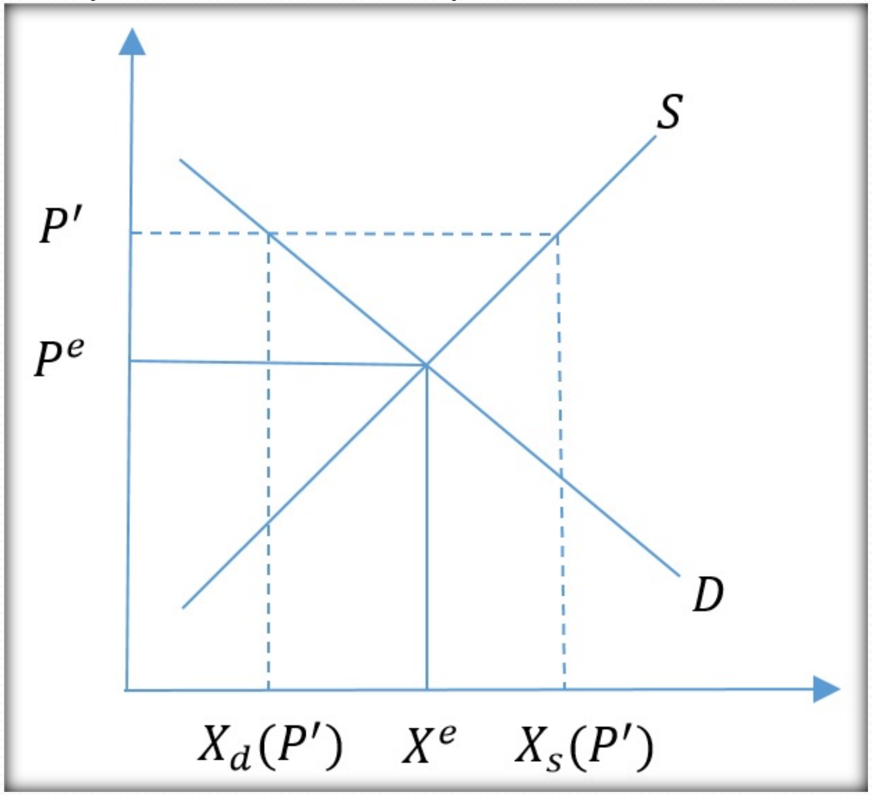
Partial Equilibrium

Equilibrium

- A price quantity configuration is said to be an equilibrium if plans of all the relevant economic agents (buyers and sellers) are realized simultaneously

- Single commodity demand supply framework



Equilibrium

- At any P'; such that $P' > P^e$; $X_d(P') < X_s(P')$

- Quantity transacted: $X_t = \min\{X_d(P'), X_s(P')\}$
$$\Longrightarrow X_t(P') = X_d(P')$$

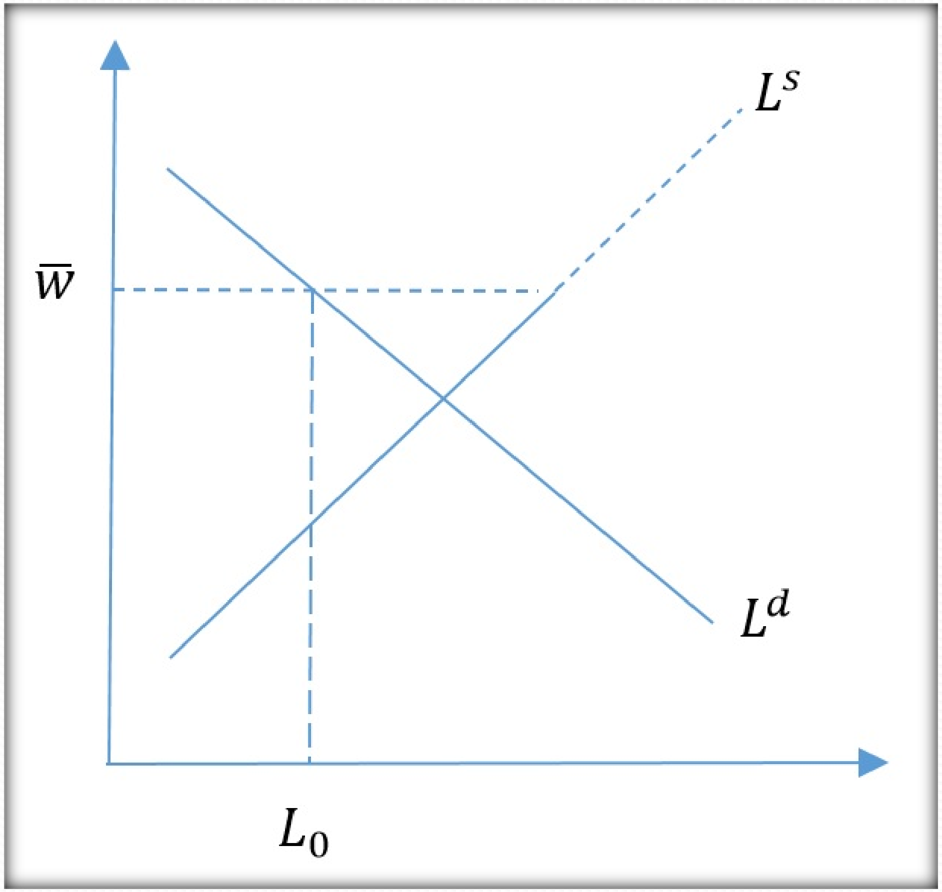
- Therefore at P' buyers' plans are realized, but not that of the sellers.

- Similarly, for any $P' < P^e$, sellers' plans would be realized.

- Only at P^e we have, $X_t(P^e) = X_d(P^e) = X_s(P^e)$

Equilibrium and Clearing of Market

- Let's consider the labour market with rigid wages (may be due to labour laws or presence of labour union); plans realised, but market does not clear.



Approaches to Equilibrium

1. Walrasian approach (price adjustment)

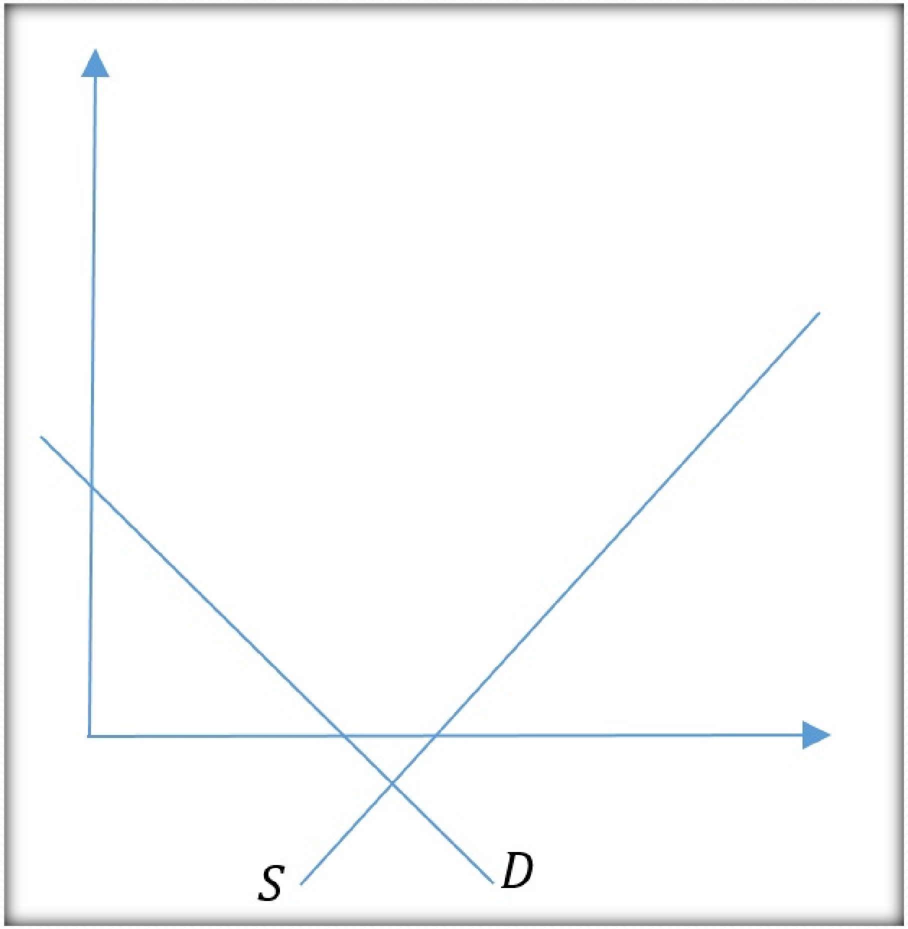
2. Marshallian approach (quantity adjustment)

- Walrashian approach – define excess demand function

- $E(P) = X_d(P) - X_s(P)$

- Any $P^e(\nless 0)$ is the equilibrium price *iff* $E(P^e) = 0$

Remember: $P^e \in [0, \infty)$



Marshallian Approach

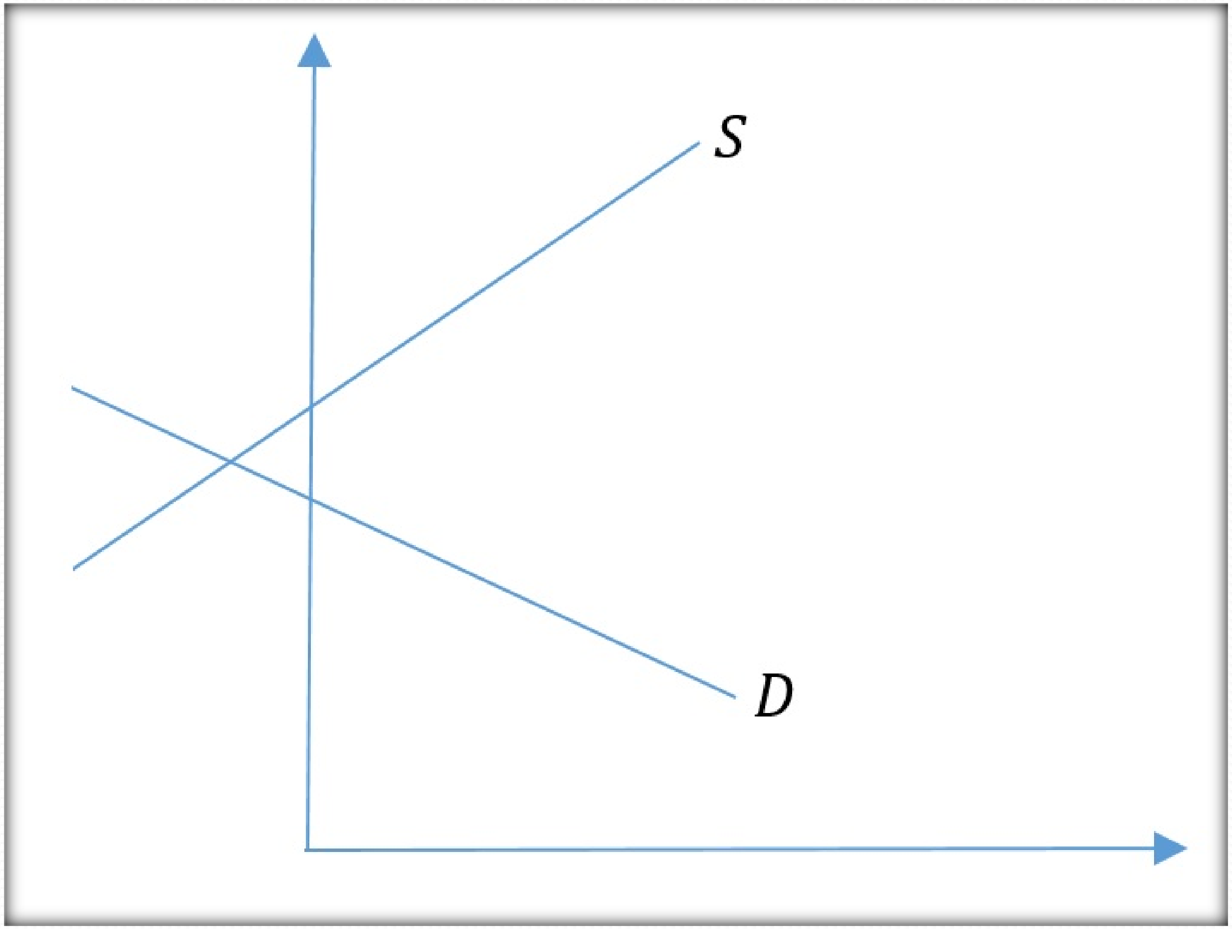
- The demand curve is the maximum willingness-to-pay for different quantities of the good

- Any X^e is an equilibrium transaction if the maximum price the buyers are willing to pay for X^e equals the minimum price the sellers are willing to charge for that quantity; that is
$$P^d(X^e) = P^s(X^e)$$

- We define the excess demand price function:
$$\mathrm{F(X)} = P^d(X) - P^s(X)$$

- $X^e(\nless 0)$ is an equilibrium quantity if $\mathrm{F}(X^e) = 0$

Remember: $X^e \in [0, \infty)$



Equilibrium Analysis

- We need to analyse three core characteristics –

1. **Existence**
2. **Uniqueness**
3. **Stability**

Existence (Walrasian Approach)

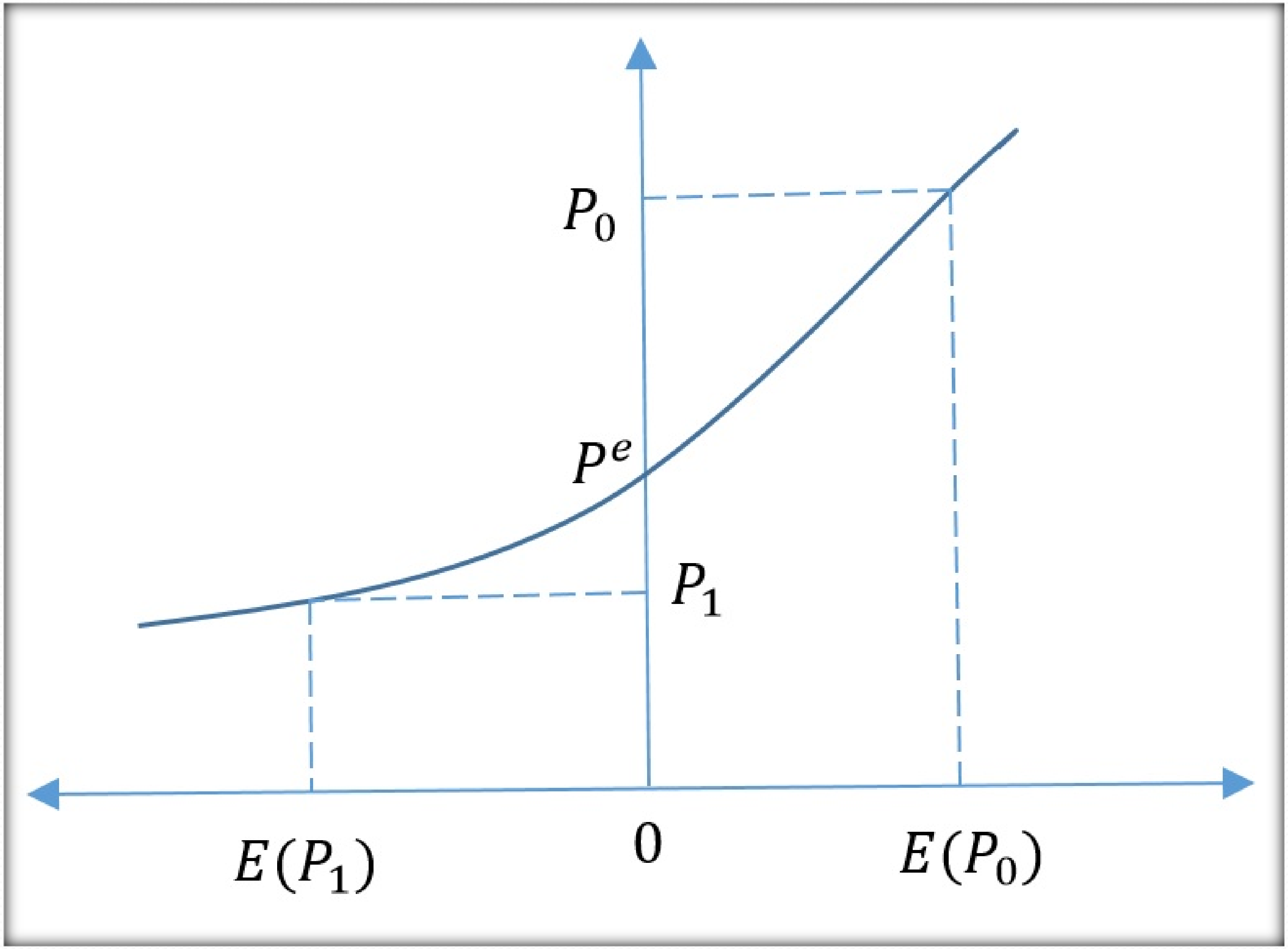
- *At least one* Walrasian equilibrium exists if –

1. $E(P) = X_d(P) - X_s(P)$ is continuous in $P \in [0, \infty)$
2. $\exists$ a price $P_0 > 0 \ni E(P_0) > 0$
3. $\exists$ a price $P_1 > 0 \ni E(P_1) < 0$

- Then $\exists$ a price $P^e > 0 \ni E(P^e) = 0$ and P^e will be the equilibrium price

- Condition 1 is sufficient but not necessary
- Condition 2 and 3 are necessary but not sufficient

Existence (Walrasian Approach)



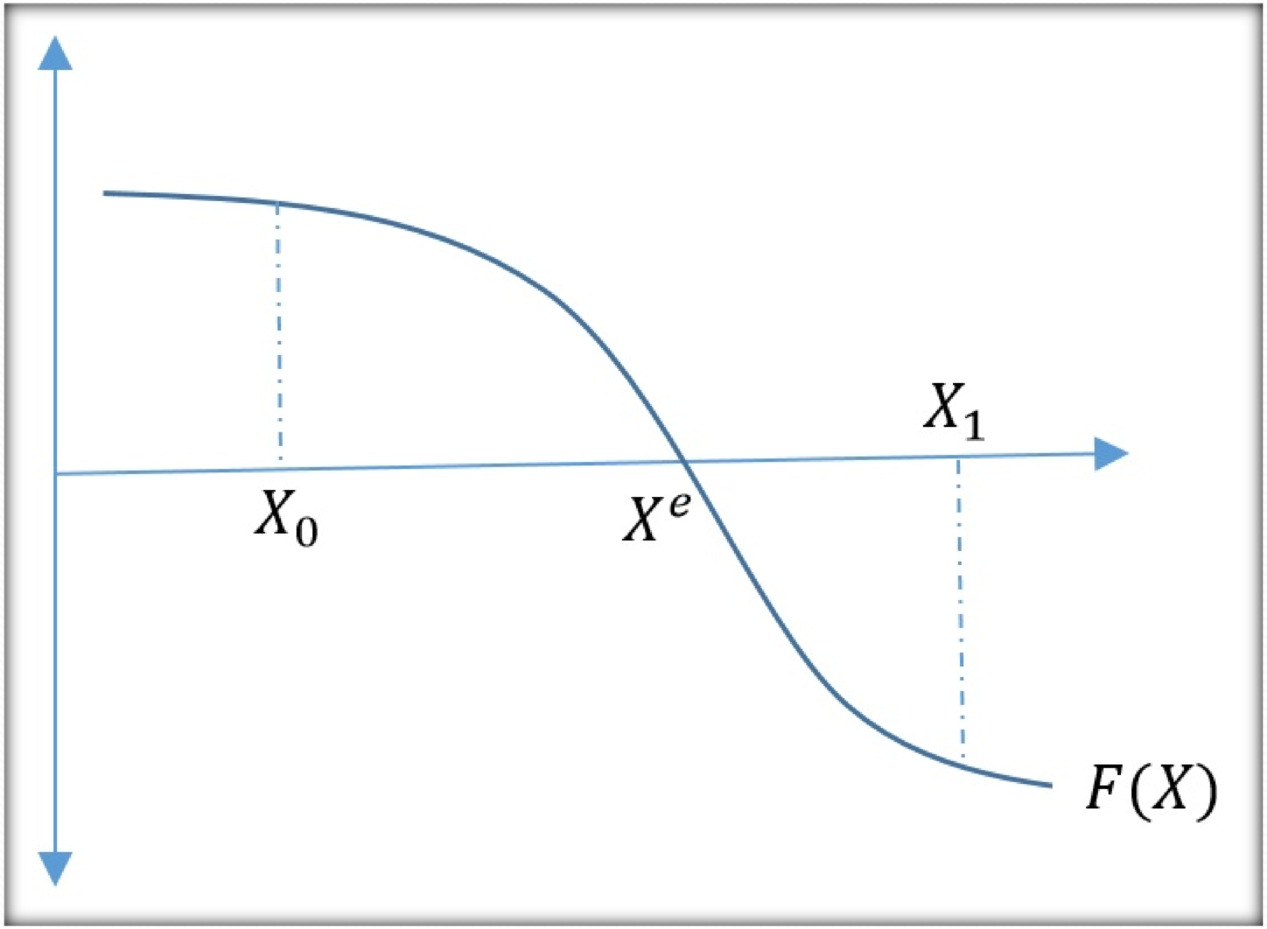
Existence (Marshallian Approach)

- At least one Marshallian equilibrium exists if –

1. $\mathrm{F(X)} = P^d(X) - P^s(X)$ is continuous in $X \in [0, \infty)$
2. $\exists$ a quantity $X_0 > 0 \ni F(X_0) > 0$
3. $\exists$ a quantity $X_1 > 0 \ni F(X_1) < 0$

- Then $\exists$ a quantity $X^e > 0 \ni F(X^e) = 0$ and X^e will be the equilibrium quantity

Existence (Marshallian Approach)



Cases

- Case 1: Supply schedule lies to the right of the demand schedule
- Implications: $E(P) < 0 \; \forall \; P$; Walrasian equilibrium does not exist
- Case 2: Supply schedule lies above the demand schedule
- Implications: $F(X) < 0 \; \forall \; X$; Marshallian equilibrium does not exist
- Case 3: Supply schedule lies to the left of the demand schedule
- Implications: $E(P) > 0 \; \forall \; P$; Walrasian equilibrium does not exist

Uniqueness

- Cases of multiple equilibria –

1. Backward bending supply curve
2. Supply curve cutting demand curve multiple times
3. Demand and supply curves have a common stretch

- D=S in linear models, D and S parallel – no equilibium
- In linear models, equilibrium if exists, is unique (except D=S)

- In non-linear models we might have multiple equilibrium

Uniqueness

- Let δ be the difference in slopes of dd and ss

$$\delta = D'(p) - S'(p)$$

$$\Rightarrow \delta \equiv \frac{\partial x^d}{\partial p} - \frac{\partial x^s}{\partial p} \equiv E'(p)$$

- If δ>0 [or δ<0] for all *p*, then, if equilibrium exists, then it is unique

- If $\delta < 0 \forall p$ and p^e is the equilibrium price, then

  dd< ss for $\forall p > p^e$

  dd> ss for $\forall p < p^e$

Uniqueness

- Hence, equilibrium is unique

- Similar argument for δ>0

- Consider the equilibrium:

$$p = p^e;\ x^d(p^e) = x^s(p^e)\ \&\ \delta < 0\ \forall\ p > p^e$$

- Case 1: Normal dd and ss
- Case 2:Both dd and ss are positively sloped with ss flatter
- Case 3: Both dd and ss are negatively sloped with dd flatter

Uniqueness

- Case $\delta=0$: Linear (D=S); non- linear (common stretch)
- Multiple eqm in backward bending ss
- At some section $\delta<0$ and $\delta>0$ in other

Stability

- Definition of eqm
- Eqm is stable if after a disturbance/shock, eqm restores
- That depends on ex. dd
- Suppose, price increases from $p^e \rightarrow p'$
- If $E(p') < 0$ then ex ss – competition among producers – price falls, eqm restored
- If $E(p') > 0$ then price increases further away from eqm

Stability

- Static (concerned with only direction of adjustment and not speed; adjustments are instantaneous and complete within the period of shock) – Walrashian, Marshallian

- Conditions : $E'(p) < 0 \; \forall \; p$ and $F'(x) < 0 \; \forall \; x$

- Dynamic (speed matters, study the time path)

- Continuous and lagged adjustments

- Condition: $\lim_{t \to \infty} p_t = p^e$

- Local and global stability (global is always unique)

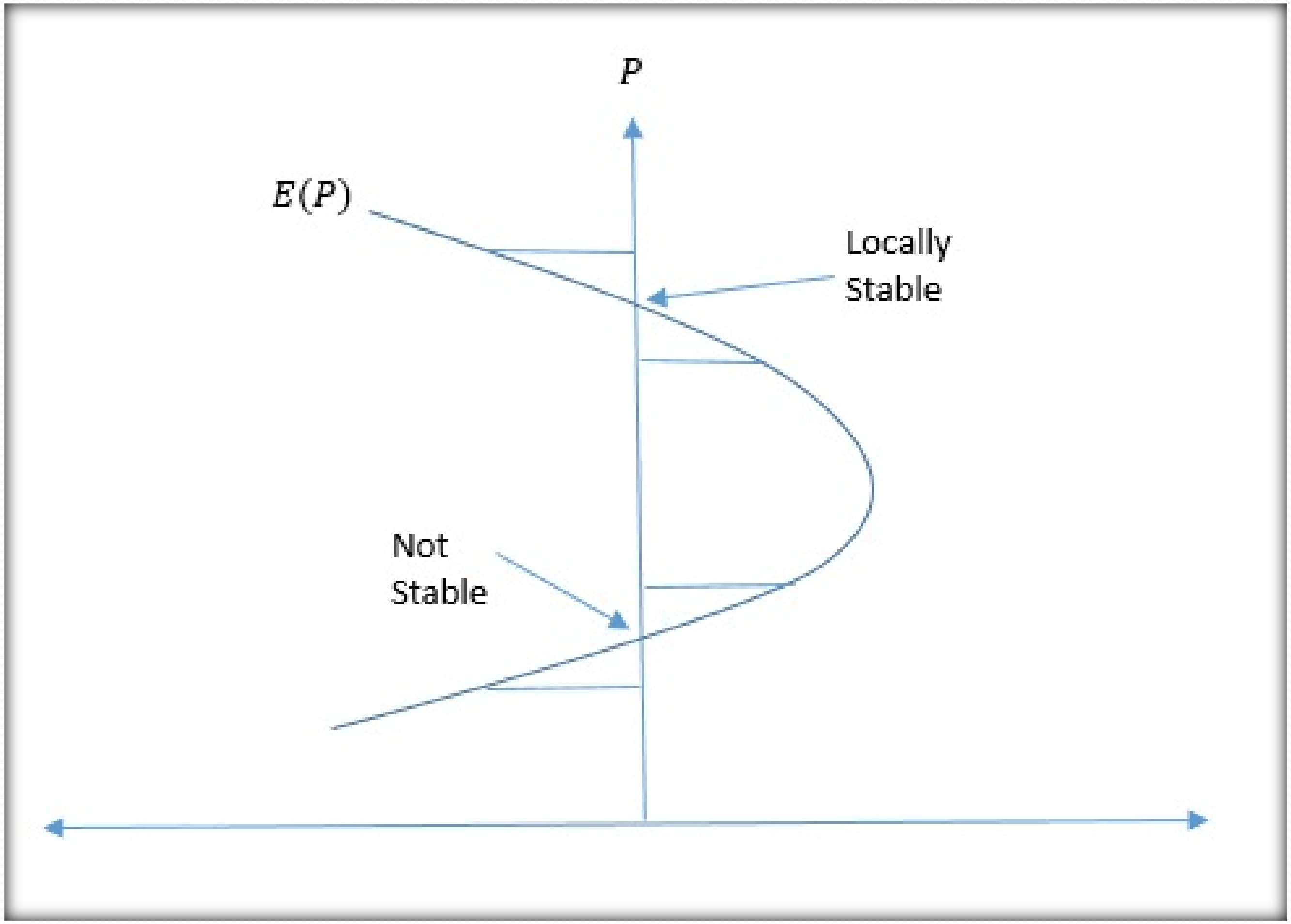
Condition for local (Walrasian) stability

$$\forall P \ \in [P^e, P^e + \varepsilon], E(P) < 0$$
$$\forall P \ \in [P^e - \varepsilon, P^e], E(P) > 0$$

- $\varepsilon > 0$

- Implying: $E'(P) < 0 \forall P \ \in [P^e - \varepsilon, P^e + \varepsilon]$

Locally Stable



Walrasian static stability

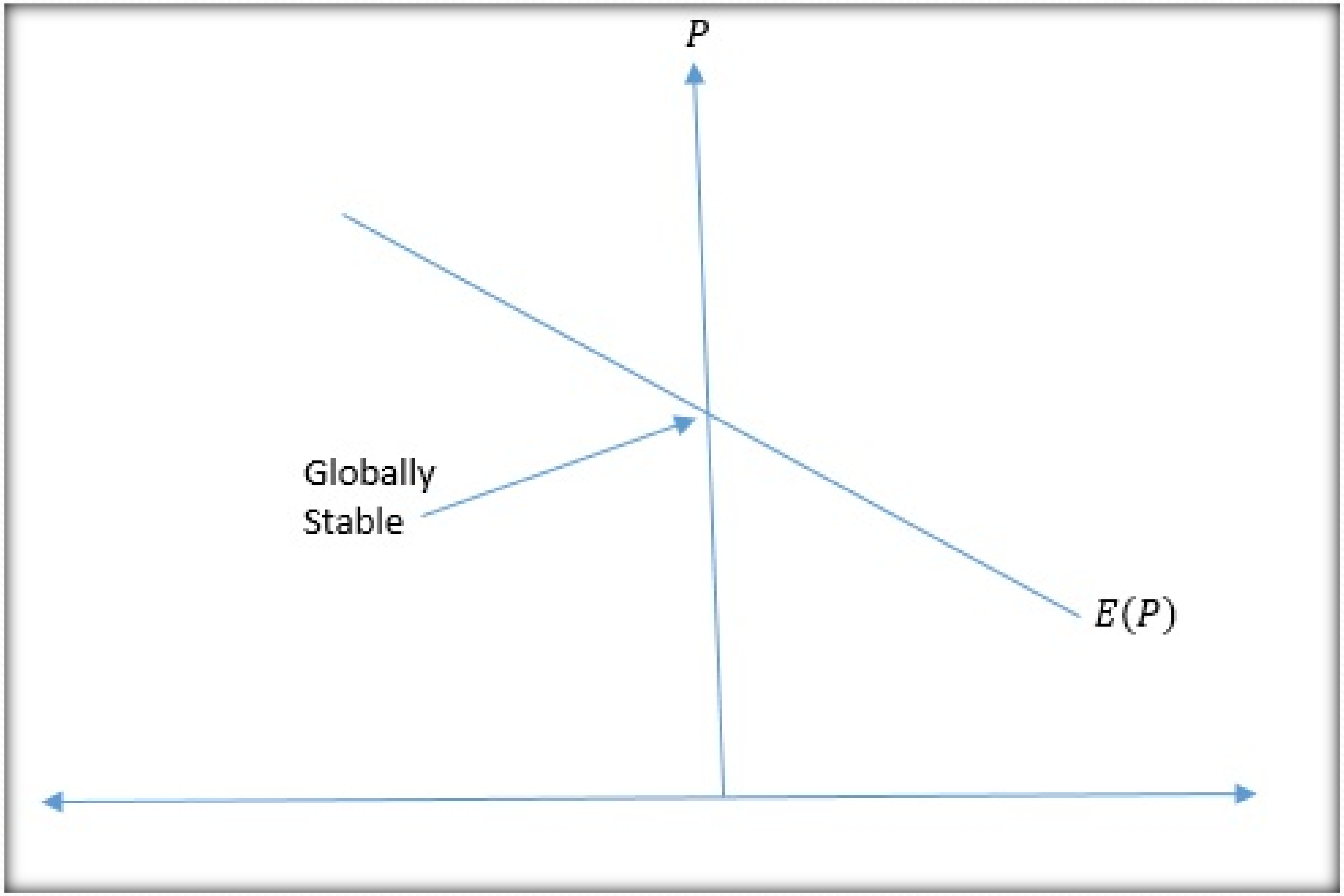
- The market is said to be in Walrasian static stability if –

$$\forall P > P^e, E(P) < 0$$
$$\forall P < P^e, E(P) > 0$$

- Implying: $E'(P) < 0 \; \forall \; P$

- The Walrasian stability condition is based on the premise that buyers tend to raise their bids if excess demand is positive and vice-versa

Globally Stable



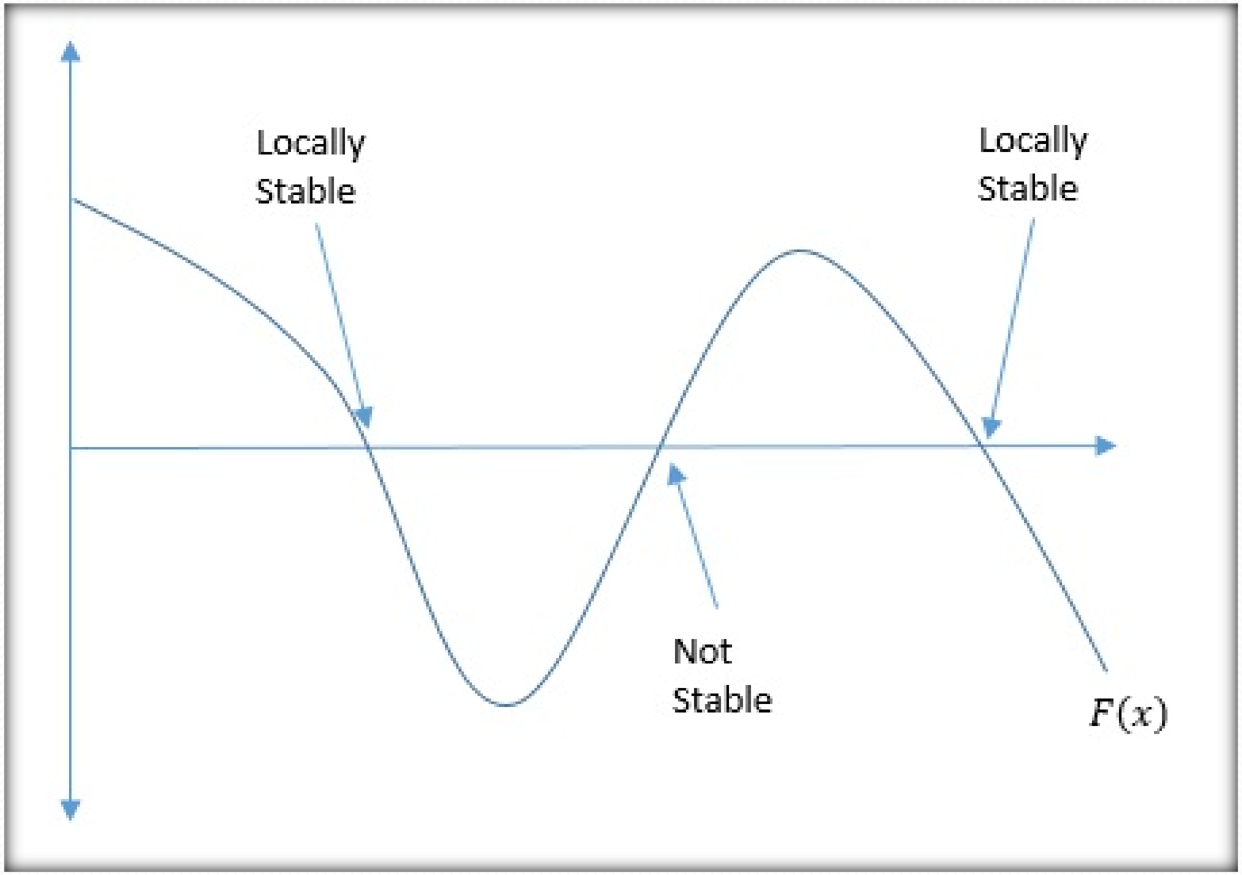
Condition for local (Marshallian) stability

$$\forall x \ \in [x^e, x^e + \varepsilon], F(x) < 0$$
$$\forall x \ \in [x^e - \varepsilon, x^e], F(x) > 0$$

- ε>0

- Implying: $F'(x) < 0 \forall x \ \in [x^e - \varepsilon, x^e + \varepsilon]$

Locally Stable



Marshallian static stability

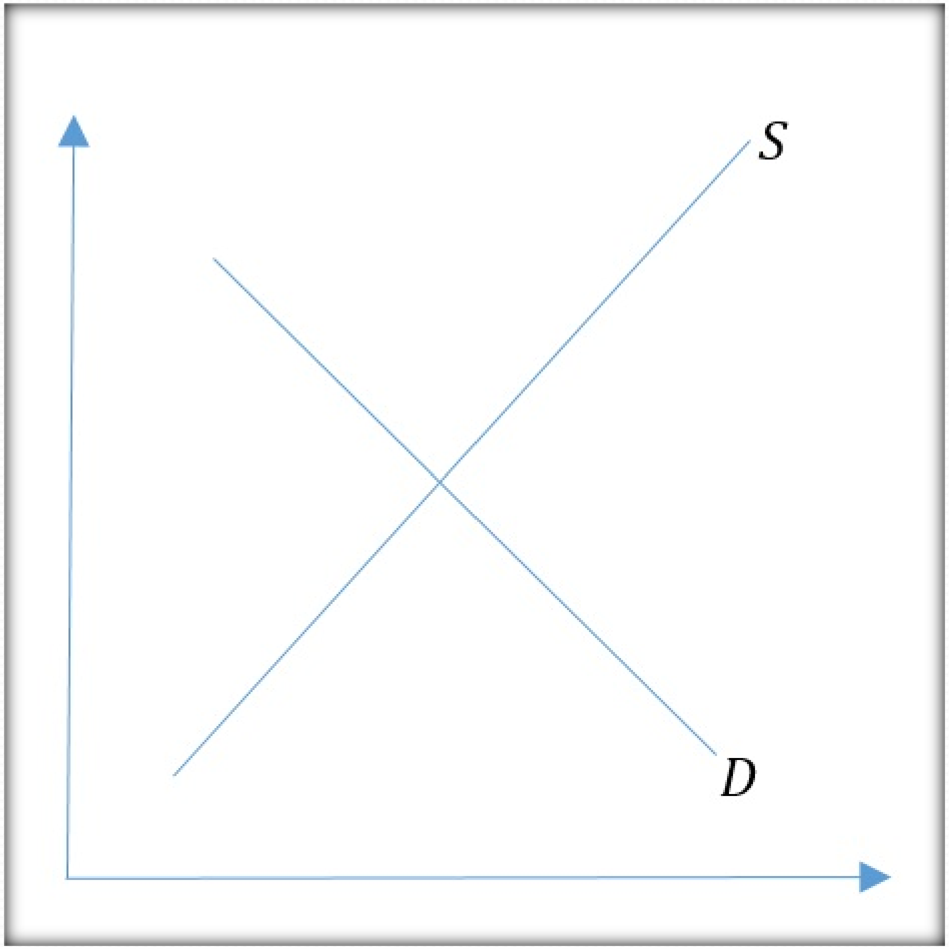
- The market is said to be in Marshallian static stability if –

$$\forall x > x^e, F(x) < 0$$
$$\forall x < x^e, F(x) > 0$$

- Implying: $F'(x) < 0 \; \forall \; x$

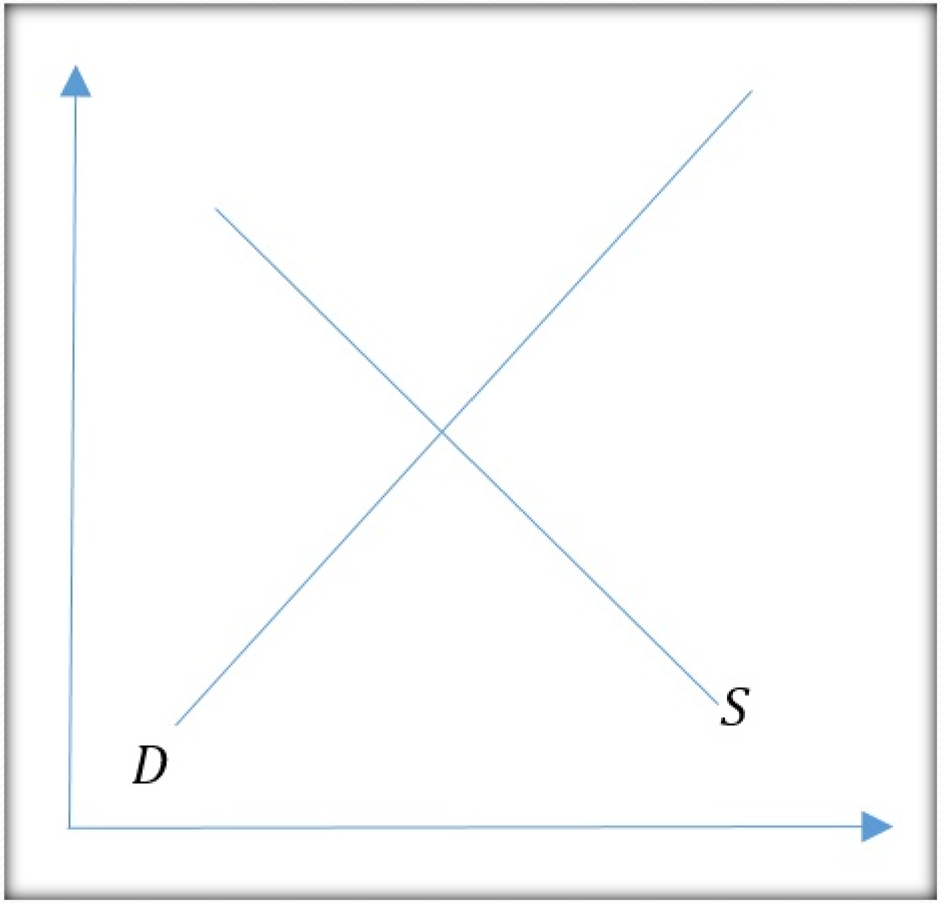
Case 1

- Stable – both Walrasian and Marshallian



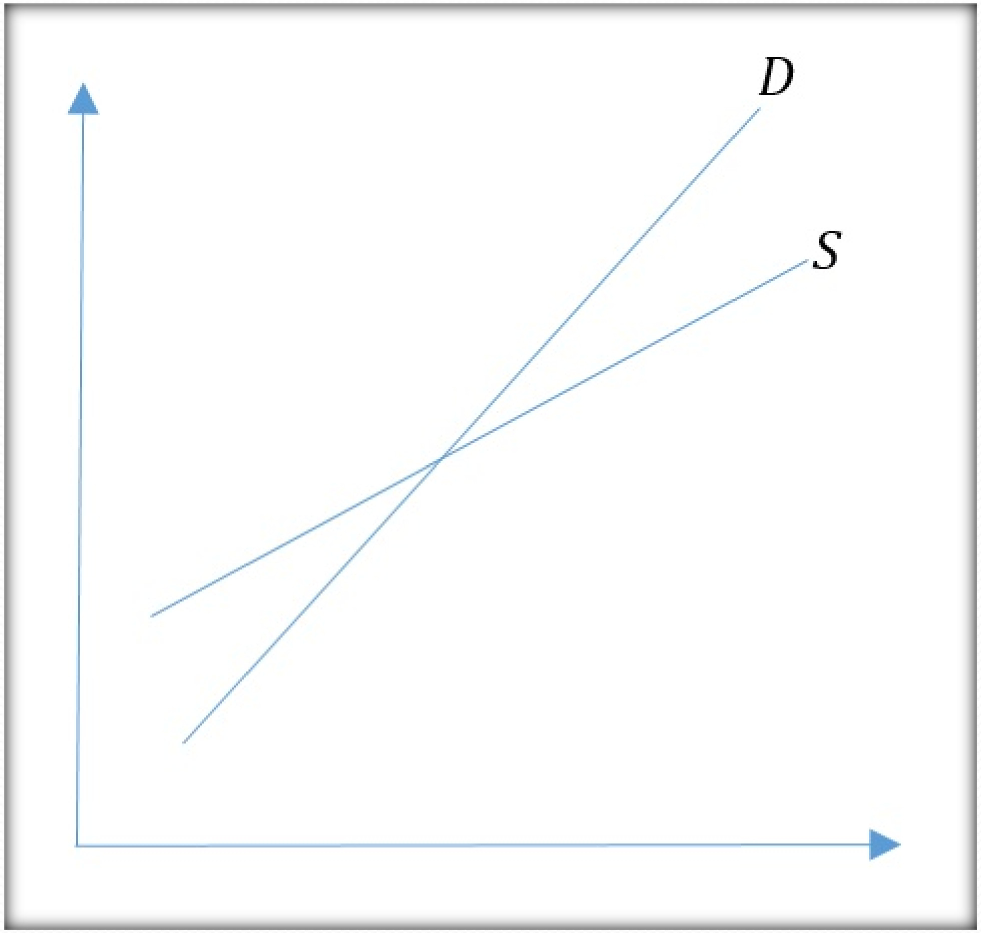
Case 2

- Not Stable – both Walrasian and Marshallian



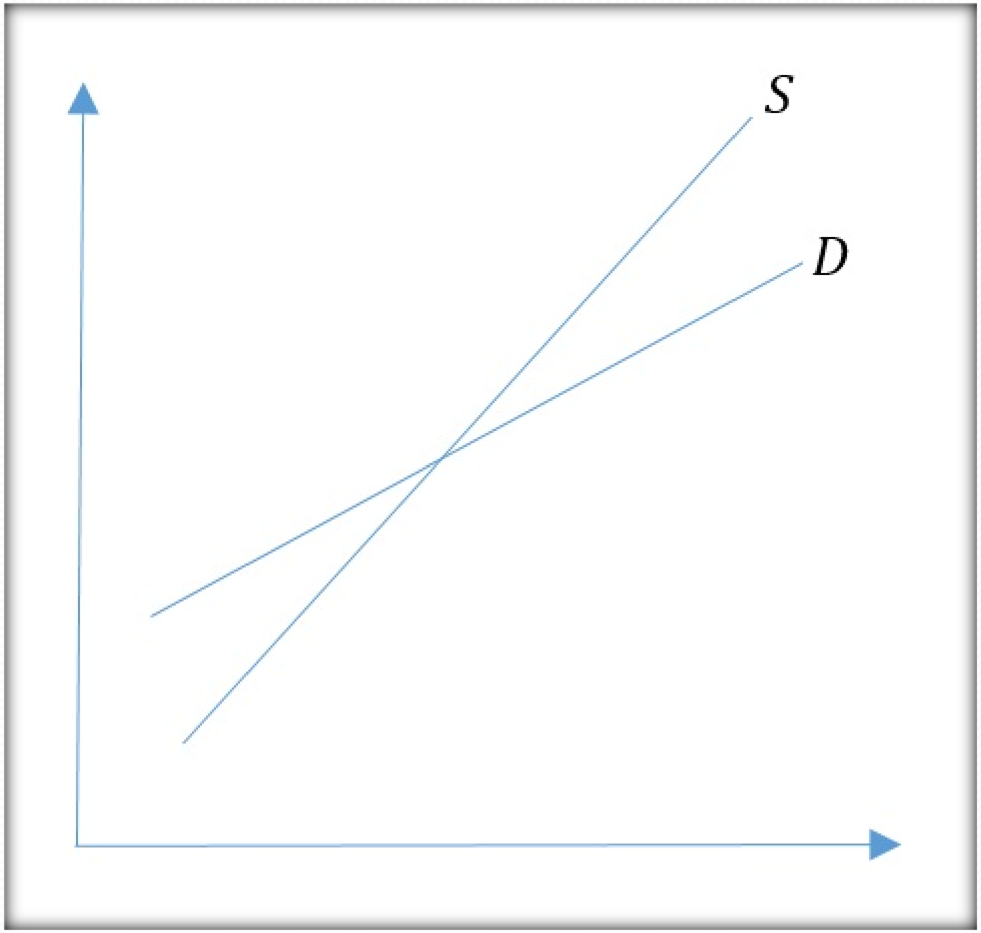
Case 3

- Walrasian stable but Marshallian not stable



Case 4

- Walrasian not stable but Marshallian stable



Relation between W and M Stability

- We can show that –

- $F'(x) < 0 \Rightarrow E'(P) > 0$ and
- $F'(x) > 0 \Rightarrow E'(P) < 0$

- Therefore, if both dd and ss are positively (or negatively) sloped, then, a market under W stability will be M not-stable and vice-versa

- This happens because $\frac{\partial P^d}{\partial x} . \frac{\partial P^s}{\partial x} > 0$

Walrasian Dynamic Stability

- Price can adjust in two ways –

1. Lagged adjustment
2. Continuous adjustment

- Lagged adjustment

- We assume that a positive excess demand tends to raise the price and the price in period-*t* is affected by the excess demand in period *t-1*

- $P_t - P_{t-1} = kE(P_{t-1});\ \ k > o$

Lagged Stability

- Let's consider the following demand and supply functions –

$$D_t = AP_t + B; S_t = aP_t + b$$

- Particular solution with $P_t = P^e \Rightarrow P^e = \dfrac{b-B}{A-a}$

- Excess dd fn for previous period –

$$E(P_{t-1}) = (A-a)P_{t-1} + (B-b)$$

- By characterization of lagged adjustment we had $P_t - P_{t-1} = kE(P_{t-1})$

- Combining: $P_t = [1+K(A-a)]P_{t-1} + k(B-b)$

Lagged Stability

- Complementary Solution
- Start with the homogenous part $P_t = [1 + K(A - a)]P_{t-1}$
- Trial solution: $P_t = \alpha z^t$

  $\therefore \quad \alpha z^t = [1 + K(A - a)]\alpha z^{t-1}$

  $\Rightarrow z = [1 + K(A - a)]$
- Complete solution: $P_t = \alpha z^t + P^e$

  $$= \alpha[1 + K(A - a)]^t + P^e$$
- At $t = 0$: $P_0 = \alpha + P^e \Rightarrow \alpha = (P_0 - P^e)$
- Solution: $P_t = (P_0 - P^e)[1 + K(A - a)]^t + P^e$

Lagged Stability

- B y definition, an equilibrium is dynamically stable if $\lim_{t\to\infty} p_t = p^e$

$$\therefore \; [1+K(A-a)]^t \to 0 \; as \; t \to \infty$$

- This is possible if $|1+K(A-a)| < 1$

$$or,\; 0 < 1+K(A-a) < 1$$

- RHS inequality requires: $a > A$ (fulfilled automatically if SS is positively sloped)

- LHS inequality requires: $k < \dfrac{1}{a-A}$

Lagged Stability

- If $-1 < 1 + K(A - a) < 0$ the amplitude of oscillation decreases over time and the time path approaches the eqm.

- If $1 + K(A - a) < -1$ the time path diverges

- Both static and dynamic stability depends upon the slopes of the dd and ss curves

- In addition, dynamic stability depends on the parameter k that indicates the extent to which the market adjusts

Continuous Stability

- Adjustment takes place continuously

- Price change is affected by the ex dd at every period

$$\frac{dp}{dt} = kE(p);\ k > 0$$

- Let's consider the following demand and supply functions –

$$D(p) = Ap + B; S(p) = ap + b$$

- Hence: $$\frac{dp}{dt} = k(A-a)p + k(B-b)$$

Continuous Stability

- Particular Solution: $D(P^e) = S(P^e) \Rightarrow P^e = \dfrac{b-B}{A-a}$

- Complementary function

- Homogenous equation: $\dfrac{dp}{dt} = k(A-a)p$

$$\Rightarrow \int \frac{dp}{p} = \int k(A-a)dt$$

$$\Rightarrow \log p = k(A-a) + c$$

- Hence, we have –

$$p(t) = \alpha e^{k(A-a)t} + P^e;\ \alpha = e^c$$

Continuous Stability

- Initial condition (at $t = t_0$): $\alpha = (P_0 - P^e)$

- Complete Solution: $p(t) = (P_0 - P^e)e^{k(A-a)t} + P^e$

- Stability requires: $\lim_{t\to\infty} p_t = p^e$

- This is possible if: $A < a \because k > 0$

- That is the slope of the supply schedule must be greater than that of the demand schedule

- Just like static stability

Continuous Stability

- Hence, if price adjustment is continuous, then stability condition is static an dynamic cases are the same

- Lagged adjustment different…

Case

- Consider the following case –

- Producer makes the decision based on previous period's price while the consumer makes the decision based on present period's price

$$D_t = AP_t + B$$
$$S_t = aP_{t-1} + b$$

- Further, at every period the market clears

- Hence we have: $P_t = \frac{a}{A} P_{t-1} + \frac{b-B}{A}$

- Market is dynamically stable if $P_t = P_{t-1} = P^e$

Case

- Complimentary Solution
- Homogenous equation: $P_t = \frac{a}{A} P_{t-1}$
- Trial Solution: $P_t = \alpha z^t$
- Implying: $z = \frac{a}{A}$
- At $t=0$; $\alpha = (P_0 - P^e)$
- Complete solution: $P_t = (P_0 - P^e)(\frac{a}{A})^t + P^e$

Case

- Stability requires: $\lim_{t \to \infty} p_t = p^e$
- This is possible if: $|a| < |A| \because k > 0$
- Implying absolute slope of dd > absolute slope of ss, in other words, dd must be flatter
- This is known as the famous cobweb model

References

- Microeconomic Theory, Henderson and Quandt, Chapter 6