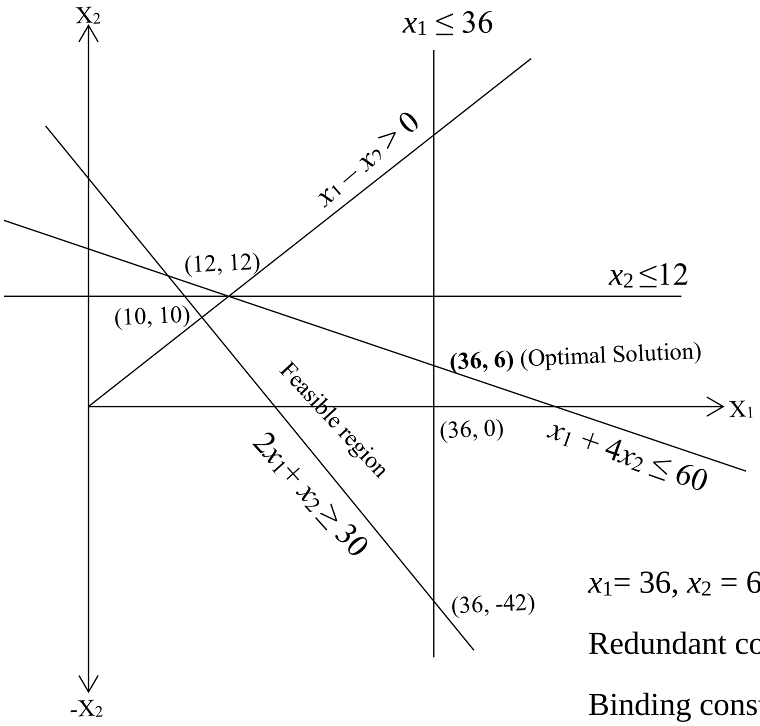
Solutions of Assignment 1

Sol 1)



$x_1 = 36$, $x_2 = 6$ and Z=312

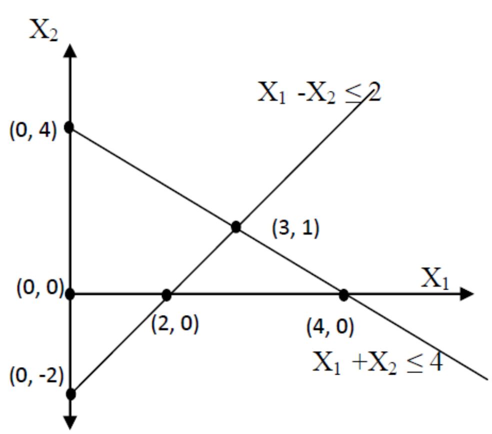
Redundant constraint $x_2 \leq 12$

Binding constraint $x_1 \leq 36$

$x_1 + 4x_2 \leq 60$

Sol 2)

(b) (i)



Maximize $Z = x_1 + 3x_2$

Subject to: $X_1 + X_2 + X_3 = 4$
$X_1 - X_2 + X_4 = 2$
$X_1, X_3, X_4 \geq 0$,
X_2 => Unrestricted

SN	NBV	BV and Solution	Feasibility
1	X_1, X_2	$X_3 = 4, X_4 = 2$	Yes
2	X_1, X_3	$X_2 = 4, X_4 = 6$	Yes
3	X_1, X_4	$X_2 = -2, X_3 = 6$	Yes
4	X_2, X_3	$X_1 = 4, X_4 = -2$	No
5	X_2, X_4	$X_1 = 2, X_3 = 2$	Yes
6	X_3, X_4	$X_1 = 3, X_2 = 1$	Yes

(ii) Optimal solution : $X_1 = 0$, $X_2 = 4$; $Z = 12$

Sol 3) (A) Formulation of LP problem after introducing slack variables:

Max. $Z = 3x_1 + 4x_2 + 0S_1 + 0S_2$

Subject to: $3x_1 + 2x_2 + S_1 = 30$

$x_1 + 2x_2 + S_2 = 22$

$x_1, x_2, S_1, S_2 \geq 0$

Iteration 1

C_i			3	4	0	0	
	Variables	RHS	x_1	x_2	S_1	S_2	Ratio
0	S_1	30	3	2	1	0	15
0	S_2	22	1	2	0	1	11
	Z_j		0	0	0	0	
	C_i-Z_j		3	4	0	0	

Iteration 2

C_i			3	4	0	0	
	Variables	RHS	x_1	x_2	S_1	S_2	Ratio
0	S_1	8	2	0	1	-1	4
4	x_2	11	1/2	1	0	1/2	22
	Z_j		2	4	0	2	
	C_i-Z_j		1	0	0	-2	

Iteration 3

C_i			3	4	0	0
	Variables	RHS	x_1	x_2	S_1	S_2
3	x_1	4	1	0	1/2	-1/10
4	x_2	9	0	1	-1/4	3/20
	Z_j		3	4	1/2	3/5
	C_i-Z_j		0	0	-1/2	-3/2

Since there is no entering variable which means this is a final table giving optimal solution, *i.e.* x_1=4 and x_2=9. The optimal value of Z is 48.

(B) Formulation of LP problem after introducing slack variables:

Max. $Z = x_1 + 2x_2 + 0S_1 + 0S_2$

Subject to: $3x_1 + 2x_2 + S_1 = 30$

$x_1 + 5x_2 + S_2 = 22$

$x_1, x_2, S_1, S_2 \geq 0$

Iteration 1

C_i			1	2	0	0	
	Variables	RHS	x_1	x_2	S_1	S_2	Ratio
0	S_1	30	3	2	1	0	15
0	S_2	22	1	2	0	1	11
	Z_j		0	0	0	0	
	C_i-Z_j		1	2	0	0	

Iteration 2

C_i			1	2	0	0
	Variables	RHS	x_1	x_2	S_1	S_2
0	S_1	8	2	0	1	-1
2	x_2	11	1/2	1	0	1/2
	Z_j		1	2	0	1
	C_i-Z_j		0	0	0	-1

Since there is no entering variable which means this is a final table giving optimal solution, i.e. x_1=0 and x_2=11. The optimal value of Z is 22.

Sol 4) Formulation of LP problem after introducing slack variables:

Max. $Z = 3x_1 + 4x_2 + x_3 + 0S_1 + 0S_2 + 0S_3$

Subject to: $x_1 + 2x_2 + 3x_3 + S_1 = 90$

$2x_1 + x_2 + x_3 + S_2 = 60$

$3x_1 + x_2 + 2x_3 + S_3 = 80$

x_1, x_2, x_3, S_1, S_2 and $S_3 \geq 0$

Iteration 1

C_i			3	4	1	0	0	0	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
0	S_1	90	1	2	3	1	0	0	45
0	S_2	60	2	1	1	0	1	0	60
0	S_3	80	3	1	2	0	0	1	80
	Z_j		0	0	0	0	0	0	
	C_i-Z_j		2	4	1	0	0	0	

Iteration 2

C_i			3	4	1	0	0	0	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
4	x_2	45	1/2	1	3/2	1/2	0	0	90
0	S_2	15	3/2	0	-1/2	-1/2	1	0	10
0	S_3	35	5/2	0	1/2	-1/2	0	1	14
	Z_j		2	4	6	2	0	0	
	C_i-Z_j		1	0	-5	-2	0	0	

Iteration 3

C_i			3	4	1	0	0	0
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3
4	x_2	40	0	1	5/3	2/3	-1/3	0
3	x_1	10	1	0	-1/3	-1/3	2/3	0
0	S_3	10	0	0	4/3	1/3	-5/3	1
	Z_j		3	4	17/3	5/3	2/3	0
	C_i-Z_j		0	0	-17/3	-5/3	-2/3	0

This is a final table with optimum solution (Z=190) at x_1= 10, x_2 = 40 and x_3=0.

Sol 5). Formulation of LP problem after introducing slack variables:

Minimize $Z = 3x_1 + 2.5x_2 - 0S_1 - 0S_2 + MA_1 + MA_2$

Subject to: $2x_1 + 4x_2 - S_1 + A_1 = 40$

$5x_1 + 2x_2 - S_2 + A_2 = 50$

Iteration 1

C_i			3	2.5	0	0	M	M	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
M	A_1	40	2	4	-1	0	1	0	20
M	A_2	50	5	2	0	-1	0	1	10
	Z_j		7M	6M	-M	-M	M	M	
	C_i-Z_j		3-7M	2.5-6M	M	M	0	0	

Iteration 2

C_i			3	2.5	0	0	M	M	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
M	A_1	20	0	16/5	-1	2/5	1	-2/5	6.25
3	x_1	10	1	2/5	0	-1/5	0	1/5	25
	Z_j		3	(16M+6)/5	-M	(2M-3)/5	M	(3-2M)/5	

	C_i-Z_j		0	2.5-[(16M+6)/5]	M	(3-2M)/5	0	(7M-3)/5	

Iteration 3

C_i			3	2.5	0	0	M	M
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2
2.5	x_2	6.25	0	1	-5/16	1/8	5/16	-1/8
3	x_1	7.5	1	0	1/8	-1/4	-1/8	1/4
	Z_j		3	2.5	-13/32	-7/16	13/32	7/16
	C_i-Z_j		0	0	13/32	7/16	M-(13/32)	M-(7/16)

Optimal solution: Since all C_i-Z_j are ≥ 0, the table provides the optimal solution, i.e. x_1= 7.5, x_2=6.25. The optimal value of Z is 38.125

Sol 6). Formulation of LP problem after introducing slack variables:

Max. $Z = 30x_1 + 20x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

Subject to: $x_1 + x_2 + S_1 = 8$

$6x_1 + 4x_2 - S_2 + A_1 = 12$

$5x_1 + 8x_2 + A_2 = 20$

Iteration 1

C_i			30	20	0	0	-M	-M	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
0	S_1	8	1	1	1	0	0	0	8
-M	A_1	12	6	4	0	-1	1	0	3
-M	A_2	20	5	8	0	0	0	1	5/2
	Z_j		-11M	-12M	0	M	-M	-M	
	C_i-Z_j		30+11M	20+12M	0	0	0	0	

Iteration 2

C_i			30	20	0	0	-M	-M	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
0	S_1	11/2	3/8	0	1	0	0	-1/8	44/3
-M	A_1	2	7/2	0	0	-1	-1	-1/2	4/7
20	x_2	5/2	5/8	1	0	0	0	1/8	4
	Z_j		(25-7M)/2	20	0	M	M	(2M+5)/2	
	C_i-Z_j		(35+7M)/2	0	0	-M	0	-5/2	

Iteration 3

C_i			30	20	0	0	-M	-M	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio

0	S_1	37/7	0	0	1	2/28	3/28	-1/14	148/3
30	x_1	4/7	1	0	0	-2/7	-2/7	-1/7	-2
20	x_2	15/7	0	1	0	5/28	5/28	3/14	12
	Z_j		30	20	0	-20/28	-20/28	0	
	C_i-Z_j		0	0	0	20/28	(20-28M)/28	-M	

Iteration 4

C_i			30	20	0	0	-M	-M
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2
0	S_1	4	0	0	2/5	0	0	-4/35
30	x_1	4	1	8/5	0	0	0	17/35
0	S_2	12	0	28/5	0	1	1	6/5
	Z_j		30	48	0	0	0	102/7
	C_i-Z_j		0	-28	0	0	-M	-(7M+102)/7

Optimal solution: Since all $C_i - Z_j$ are ≤ 0 the solution is optimum at $x_1 = 4$ and $x_2 = 0$. Thus, the value of Z is 120.

Sol 7. Minimization $0.4x_1 + 0.5x_2 + MA_1 + MA_2$

Subject to $0.3x_1 + 0.1x_2 + S_1 = 1.8$

$$x_1 + x_2 + A_1 = 12$$

$$0.6x_1 + 0.4x_2 - S_2 + A_2 = 6$$

$$x_1 , x_2 , S_1, S_2 \geq 0$$

Where S_1 is slack variable A_1 and A_2 are artificial variable and S_2 is surplus variable.

Phase I:

Minimize Z : $A_1 + A_2$

Subject to $0.3x_1 + 0.1x_2 + S_1 = 1.8$

$$x_1 + x_2 + A_1 = 12$$

$$0.6x_1 + 0.4x_2 - S_2 + A_2 = 6$$

$$x_1 , x_2 , S_1, S_2 \geq 0$$

Drop the artificial variable A_1 and A_2 , we get the objective function for phase I:

Iteration 1

C_i			1.6	1.4	0	-1	0	0	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
0	S_1	1.8	0.3	0.1	1	0	0	0	6

0	A_1	12	1	1	0	0	1	0	12
0	A_2	6	0.6	0.4	0	-1	0	1	10
	Z_j		0	0	0	0	0	0	
	C_i-Z_j		1.6	1.4	0	-1	0	0	

Iteration 2

C_i			1.6	1.4	0	-1	0	0	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
1.6	x_1	6	1	1/3	10/3	0	0	0	18
0	A_1	6	0	2/3	-10/3	0	1	0	9
0	A_2	2.4	0	1/5	-2	-1	0	1	12
	Z_j		1.6	0.53	5.3	0	0	0	
	C_i-Z_j		0	0.87	-5.3	-1	0	0	

Iteration 3

C_i			1.6	1.4	0	-1	0	0	
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
1.6	x_1	3	1	0	5/3	0	-1/2	0	9/5
1.4	x_2	9	0	1	-5	0	3/2	0	-9/5
0	A_2	0.6	0	0	-1	-1	-0.2	1	
	Z_j		1.6	1.4	-4.33	0	1.3	0	
	C_i-Z_j		0	0	4.33	-1	0	0	

Iteration 4

C_i			1.6	1.4	0	-1	0	0
	Variables	RHS	x_1	x_2	S_1	S_2	A_1	A_2
0	S_1	9/5	3/5	0	1	0	-3/10	0
1.4	x_2	18	3	1	0	0	0	0
0	A_2	0.6	3/5	0	0	-1	-0.5	1
	Z_j		4.2	1.4	0	0	0	0
	C_i-Z_j		-2.6	0	0	-1	0	0

This is the final table and A_2 is still present in the basis which makes this L P infeasible.

Sol 8. $\max Z = 2x_1 + 3x_2 + x_3 - 0S_1 - 0S_2 - 0S_3 + MA_1 + MA_2$

Subject to $x1 + x2 + x3 + S_1 = 40$

$2x1 + x2 - x3 - S_2 + A_1 = 10$

$-x_2 + x_3 - S_3 + A_2 = 10$

Phase I:

Iteration 1

C_i			2	0	0	0	-1	-1	0	0	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	A_1	A_2	Ratio
0	S_1	40	1	1	1	1	0	0	0	0	40
0	A_1	10	2	1	-1	0	-1	0	1	0	5
0	A_2	10	0	-1	1	0	0	-1	0	1	∞
	Z_j		0	0	0	0	0	0	0	0	
	C_i-Z_j		2	0	0	0	-1	-1	0	0	

Iteration 2

C_i			2	0	0	0	-1	-1	0	0	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	A_1	A_2	Ratio
0	S_1	35	0	1/2	3/2	1	1/2	0	-1/2	0	23.3
2	x_1	5	1	1/2	-1/2	0	-1/2	0	1/2	0	-10
0	A_2	10	0	-1	1	0	0	-1	0	1	10
	Z_j		2	1	-1	0	1	0	1	0	
	C_i-Z_j		0	-1	1	0	-2	-1	-1	0	

Iteration 3

C_i			2	0	0	0	-1	-1	0	0	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	A_1	A_2	Ratio
0	S_1	20	0	2	0	1	1/2	3/2	-1/2	-3/2	
2	x_1	10	1	0	0	0	-1/2	-1/2	1/2	1/2	
0	x_3	10	0	-1	1	0	0	-1	0	1	
	Z_j		2	0	0	0	-1	-1	1	1	
	C_i-Z_j		0	0	0	0	0	0	-1	-1	

The optimal value of the Phase I problem is $w = 0$. So the original problem is feasible, and a basic feasible solution is $x_1 = 10;\ x3 = 10;\ S_1 = 20;\ x_2 = S_2 = S_3 = 0$. Now we can start Phase II. Again the objective value z should be represented by the non-basic variables:

$z = 2x_1 + 3x_2 + x_3 = 30 + 4x_2 + S_2 + 2S_3$

The initial tableau is (the last Phase I tableau with A_1 and A_2 taken away):

C_i			0	4	0	0	1	2	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
0	S_1	20	0	2	0	1	1/2	3/2	10

0	x_1	10	1	0	0	0	-1/2	-1/2	∞
0	x_3	10	0	-1	1	0	0	-1	-10
	Z_j		0	0	0	0	0	0	
	C_i-Z_j		0	4	0	0	1	2	

Iteration 4

C_i			0	4	0	0	1	2
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3
4	x_2	10	0	1	0	1/2	1/4	3/4
0	x_1	10	1	0	0	0	-1/2	-1/2
0	x_3	20	0	0	1	1/2	1/4	-1/4
	Z_j		0	4	0	2	1	3
	C_i-Z_j		0	0	0	-2	0	-1

Optimal solution: $x_1 = 10$; $x_2 = 10$; $x_3 = 20$; Z=70

Sol 9. Formulation of LP problem after introducing slack variables

Max. $Z = 3x_1 + x_2 + 0x_3 + 0S_1 + 0S_2 + 0S_3$

Subject to: $x_1 + 2x_2 + S_1 = 5$

$x_1 + x_2 - x_3 + S_2 = 2$

$7x_1 + 3x_2 - 5x_3 + S_3 = 20$

Iteration 1

C_i			3	1	0	0	0	0	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
0	S_1	5	1	2	0	1	0	0	5
0	S_2	2	1	1	-1	0	1	0	2
0	S_3	20	7	3	-5	0	0	1	20/7
	Z_j		0	0	0	0	0	0	
	C_i-Z_j		3	1	0	0	0	0	

Iteration 2

C_i			3	1	0	0	0	0	
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
0	S_1	3	0	1	1	1	-1	0	3
3	x_1	2	1	1	-1	0	1	0	-2
0	S_3	6	0	-4	2	0	-1	1	3
	Z_j		3	3	-3	0	3	0	
	C_i-Z_j		0	-2	3	0	-3	0	

Iteration 3

C_i			3	1	0	0	0	0
	Variables	RHS	x_1	x_2	x_3	S_1	S_2	S_3
0	x_3	3	0	1	1	1	-1	0
3	x_1	5	1	2	0	1	0	0
0	S_3	0	0	-6	0	-2	-5	1
	Z_j		3	6	0	3	0	0
	C_i-Z_j		0	-5	0	-3	0	0

Optimal solution: $x_1 = 5$; $x_2 = 0$; $x_3 = 3$; Z=15

Sol 10. Minimize $Z = -x_1 + x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2$

Subject to: $x_1 - 4x_2 - S_1 + A_1 = 5$

$x_1 - 3x_2 + S_2 = 1$

$2x_1 - 5x_2 - S_3 + A_2 = 1$

Iteration 1

C_i			-1	1	0	0	0	M	M	Ratio
	Variables	RHS	x_1	x_2	S_1	S_2	S_3	A_1	A_2	
M	A_1	5	1	-4	-1	0	0	1	0	5
0	S_2	1	1	-3	0	1	0	0	0	1
M	A_2	1	2	-5	0	0	-1	0	1	1/2
	Z_j		3M	-9M	-M	0	-M	M	M	
	C_i-Z_j		-1-3M	1+9M	M	0	M	0	0	

Iteration 2

C_i			-1	1	0	0	0	M	M	
	Variables	RHS	x_1	x_2	S_1	S_2	S_3	A_1	A_2	Ratio
M	A_1	9/2	0	-3/2	-1	0	1/2	1	-1/2	9
0	S_2	1/2	0	-1/2	0	1	1/2	0	-1/2	1
-1	x_1	1/2	1	-5/2	0	0	-1/2	0	1/2	-1
	Z_j		-1	(5-3M)/2	-M	0	(M-1)/2	M	(1-M)/2	
	C_i-Z_j		0	(3M-3)/2	M	0	-(M+1)/2	0	(3M+1)/2	

Iteration 3

C_i			-1	1	0	0	0	M	M
	Variables	RHS	x_1	x_2	S_1	S_2	S_3	A_1	A_2
M	A_1	4	0	-1	-1	-1	0	1	0
0	S_3	1	0	-1	0	2	1	0	-1
-1	x_1	1	1	-3	0	1	0	0	0

	Z_j		-1	3-M	-M	-(1+M)	0	M	0
	C_i-Z_j		0	M-2	M	M+1	0	0	M

Optimality conditions are satisfied but presence of artificial variable in the final table indicates infeasibility.