Two-Phase Method

Maximize $Z = 4x_1 + x_2$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

- **Modified problem**

Maximize $Z = 4x_1 + x_2$

subject to $3x_1 + x_2 + \bar{x}_5 = 3$

$4x_1 + 3x_2 - x_3 + \bar{x}_6 = 6$

$x_1 + 2x_2 + x_4 = 4$

$x_1, x_2, \ldots, x_4, \bar{x}_5, \bar{x}_6 \geq 0$

- **Phase I: Problem**

Minimize $Z' = \bar{x}_5 + \bar{x}_6 \quad \Rightarrow \quad Maximize, -Z' = -\bar{x}_5 - \bar{x}_6$

$subject\ to \qquad 3x_1 + x_2 + \bar{x}_5 = 3$

$$4x_1 + 3x_2 - x_3 + \bar{x}_6 = 6$$

$$x_1 + 2x_2 + x_4 = 4$$

$$x_1, x_2, .., x_4, \bar{x}_5, \bar{x}_6 \geq 0$$

Apply simplex procedure for Phase I

	Basis	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	RHS	Ratio
Iteration 0	$\bar{x}_5$	**3**	1	0	0	1	0	3	**1**
	$\bar{x}_6$	4	3	-1	0	0	1	6	3/2
	x_4	1	2	0	1	0	0	4	4
	-Z'	0	0	0	0	**1**	**1**	0	
	-Z'	–7	– 4	1	0	0	0	-9	
Iteration 1	x_1	1	1/3	0	0	1/3	0	1	3
	$\bar{x}_6$	0	**5/3**	-1	0	-4/3	1	2	**6/5**
	x_4	0	5/3	0	1	-1/3	0	3	9/5
	-Z'	0	**-5/3**	1	0	7/3	0	-2	
Iteration 2	x_1	1	0	1/5	0	3/5	-1/5	3/5	
	x_2	0	1	-3/5	0	-4/5	3/5	6/5	
	x_4	0	0	1	1	1	-1	1	
	-Z'	0	0	0	0	1	1	0	

Z row is not in proper form (coefficient of basic variable must be zero)

$R_0 \rightarrow R_0 - R_1 - R_2$

Indicates that artificial variables left the basis.

- If minimum value of sum of artificial variables is positive, then LP has no feasible solution. Otherwise, proceed for phase II

- Now, Phase II could be started from the final constraints row manipulation at phase I and original objective function.

- **Phase II**

	Basis	x_1	x_2	x_3	x_4	RHS
Iteration 0	x_1	1	0	1/5	0	**3/5**
	x_2	0	1	-3/5	0	**6/5**
	x_4	0	0	1	1	**1**
	z	**-4**	**-1**	0	0	**0**
	z	0	0	1/5	0	**18/5**

Z row in not in proper form (coefficient of basic variable must be zero)

$R_o \rightarrow R_o + 4R_1 + R_2$

- Optimal solution : $(x_1 = 3/5, x_2 = 6/5, z = 18/5)$

Dealing with unrestricted variables

- Convert such variable as difference of two non-negative variables.
- **Example**:
    - x_k – unrestricted in sign
    - $x_k = x_k^+ - x_k^-$

- **Assignment**

$$\begin{aligned} &Minimize, && z = x_1 + 2x_2 + x_3 \\ &subject\ to, && 2x_1 + 3x_2 + 4x_3 \geq -4 \\ & && 3x_1 + 5x_2 + 2x_3 \geq 7 \\ & && x_1, x_2 \geq 0\ and\ x_3\ is\ unrestricted \end{aligned}$$