Class Test – I, Operations Research – I, Total Marks: 60, Duration: 1 Hour

Question (1) **[10 + 10]**

(a) Consider the following LPP

Minimize $Z = c_1 x_1 + x_2$

Subject to: $-x_1 + x_2 \leq 1$, x_1 unrestricted, $x_2 \geq 0$.

For all possible values of $c_1 \in (-\infty, \infty)$, use graphical analysis to find the optimal solution of the problem.

(b) Consider the LP feasible region defined by the constraints

$x_1 + x_2 \leq 6;\ x_2 \leq 3;\ x_1 \geq 0\ x_2 \geq 0.$

Find all basic solutions and the basic feasible solutions.

Question (2) **[3 + 3 + 2 + 2]**

Suppose the following system of equations was obtained in the course of applying Simplex method to solve a linear program with nonnegative variables x_1, x_2, x_3 and two inequalities. The objective function (Z) is to maximize and slack variables s_1 and s_2 were added.

$Z + ax_2 + bx_3 + 4s_2 = 82$

$-2x_2 + 2x_3 + s_1 + 3s_2 = c$

$x_1 - x_2 + 3x_3 - 5s_2 = 3$

Give conditions on a, b and c that are required for the following statements to be true

(i) The current basic solution is a feasible basic solution.

Assume that the condition found in (i) holds for the following:

(ii) The current basic solution is optimal.

(iii) The linear program is unbounded (for this question, assume that $b > 0$).

Question (3) Consider the following LP: **[10+5+5]**

Max. $Z = 3x_1 + 2x_2 + 3x_3$

s.t. $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

Find a basic feasible solution at the end of phase I of the 2-phase method. Show that all variable columns except x_2 can be dropped from the corresponding table before we start phase II. Solve the reduced problem and write down the optimal solution in terms of x_1, x_2 and x_3.

Question (4) Consider the following LP: **[10]**

Max. $Z = x_1 + 2x_2 + 3x_3 + 4x_4$

s.t. $x_1 + 2x_2 + 2x_3 + 3x_4 \leq 20$

$2x_1 + x_2 + 3x_3 + 2x_4 \leq 20$

$x_1, x_2, x_3, x_4 \geq 0$

Using the principle of the revised simplex method, prove that an optimal solution exists with x_3 and x_4 as basic variables.