

General definition for any order PDE

1. A PDE of order m is called Quasi-linear if it is linear in the derivatives of order m with coefficients that depend on the independent variables and derivatives of the unknown function or order strictly less than m .

Example: $A\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) \frac{\partial^2 z}{\partial x^2} + B\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) \frac{\partial^2 z}{\partial y^2} + C\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) \frac{\partial^2 z}{\partial x \partial y} + E\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) \frac{\partial z}{\partial x} + F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) \frac{\partial z}{\partial y} + G(x, y, z) = 0$. Here $m = 2$.

2. A Quasi-linear PDE where the coefficients of derivatives of order m are functions of the independent variables alone is called a Semi-linear PDE.

Example: 1. $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x \partial y} + E\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) \frac{\partial z}{\partial x} + F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) \frac{\partial z}{\partial y} + G(x, y, z) = 0$. **2.** $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f(z)$. Here $m = 2$.

3. A PDE which is linear in the unknown function and all its derivatives with coefficients depending on the independent variables alone is called a Linear PDE.

Example: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = z$ or, $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f(x, y)$ or, $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. Here $m = 2$.

4. A PDE which is not Quasi-linear is called a Fully nonlinear PDE.

Example: $\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^2 = f(x, y, z)$