Assignment - 6 (no need to submit)

Note: Unless otherwise stated, notation used is as defined in the class.

1. The "Second order" Fibonacci sequence is defined by the rule:

$$U_0 = 0, U_1 = 1, U_{n+2} = U_{n+1} + U_n + F_n$$

   where F_n is the n-th Fibonacci number. Express U_n in terms of F_n and F_{n+1} (Hints: Use generating functions)

2. Suppose the worker a is suitable for jobs $3, 4, 5$, worker b is suitable for jobs $2, 3$, and worker c is suitable for jobs $1, 5$. Also, each worker can be assigned to at most one job, no more than one worker per job, and a worker only gets a job to which he or she is suited. Set up a generating function and use it to answer the following questions:
   (a) In how many ways can we assign one worker to a job?
   (b) In how many ways can we assign two workers to jobs?
   (c) In how many ways can we assign three workers to jobs?

3. Find the number of codewords of length k from an alphabet $\{a, b, c, d, e\}$ if b occurs an odd number of times.

4. Find the number of derangements of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ in which the first four elements are mapped into:
   (a) $1, 2, 3, 4$ in some order
   (b) $5, 6, 7, 8$ in some order

5. Use the method of characteristic roots to solve the following recurrences
   (a) $a_n = -2a_{n-1} - a_{n-2}, a_0 = 2, a_1 = 2$
   (b) $a_n = 9a_{n-2}, a_0 = 4, a_1 = 2$

6. Use generating function to solve each of the recurrences in Q.5

7. Let D_n be the number of derangements of $1, 2, \ldots, n$. Derive a formula for D_n as follows:
   (a) Let $C_n = \frac{D_n}{n!} - \frac{D_{n-1}}{(n-1)!}$ Find a recurrence relation for C_{n+1} in terms of C_n
   (b) Solve the recurrence for C_n by iteration.
   (c) Use the formula for C_n to solve for D_n.

8. A coding system encodes messages using strings of octal (base 8) digits. A codewors is considered valid if and only if it contains an even number of 7s.
   (a) Find a linear nonhomogeneous recurrence relation for the number of valid codewords of length n. What are the initial conditions?
   (b) Solve this recurrence relation using generating functions.

9. Solve the recurrence relation

$$a_n = 10a_{n-1} - 25a_{n-2} + 5^{n+1}, n \geq 2$$

   subject to the initial values $a_0 = 5, a_1 = 15$.

10. Draw the tree whose Prüfer code is $(2, 2, 2, 2)$.

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