DEPARTMENT OF MATHEMATICS, IIT - Kharagpur
Mid Semester Examination 2022
MA30207 / MA21007 Design & Analysis of Algorithms
Instructor: Prof. Sourav Mukhopadhyay
No. of students: 200 Total Points: 30 DURATION: 2 Hours

Answer **ALL QUESTIONS**. All the notations are standard and no query or doubts will be entertained. If any data/statement is missing, identify it in your answer script. Marks are indicated at the end of each question.

1. Consider the recurrence $T(n) = T(n/2) + T(n/4) + n$. Use the substitution method to give a tight upper bound on the solution to the recurrence using O-notation. [2]

2. For each of the following algorithms, (i) give a recurrence that describes its worst-case running time and (ii) its worst-case running time using Θ-notation: (a) Binary search, (b) Insertion Sort, (c) Merge Sort, (d) Randomized quicksort and (e) Strassen's algorithm. [5]

3. Consider the following sorting methods: Insertion Sort, Merge Sort, and Quick Sort. What is the running time using O-notation for each method

   (a) When all the the array values are equal?

   (b) When the values are in order?

   (c) When the values are in reverse order?

   Explain your answers. [3]

4. a) Describe a case where quicksort will result in quadratic behavior.

   b) Here is an array which has just been partitioned by the first step of Quicksort (sorting from smallest to largest):

   18 15 17 19 20 23 22 21 24

   Which of these elements could be a possible pivot?

   c) Consider the following array for sorting (sorting from smallest to largest)

   10 8 13 14 6 12 5 7 11 9

   (i) Draw the array after the FIRST iteration of the large loop in an Insersion sort.

   (ii) Draw the array after TWO recursive calls of Mergesort are completed, and before the final merge step has occured.

   (iii) Draw the array after applying Quicksort's partition function and using 10 for the pivot. Clearly describe the working of the partition function with this example.

   [4]

5.

a) Is the sequence $\langle 20, 15, 18, 7, 9, 5, 12, 3, 6, 2 \rangle$ is a max-heap? *Explain.*

b) Where in a max-heap can the smallest element reside, assuming all elements are distinct? Include both the location in the array and the location in the implicit tree structure.

c) Suppose that instead of using `Build Heap` to build a max-heap in place, the `Insert` operation is used n times. Starting with an empty heap, for each element, use `Insert` to insert it into the heap. After each insertion, the heap still has the max-heap property, so after n `Insert` operations, it is a max-heap on the n elements.

(i) Argue that this heap construction runs in $O(n \log n)$ time.

(ii) Argue that in the worst case, this heap construction runs in $\Omega(n \log n)$ time.

d) If bucket sort is implemented by using heapsort to sort the individual buckets, instead of by using insertion sort as in the normal algorithm, then what are the worst-case and Average case running time of bucket sort? Justify your answer.

[7]

6. **TRUE OR FALSE?** If the statement is correct, briefly state why. If the statement is wrong, explain why. [9]

i. The sum of the smallest $\sqrt{n}$ elements in an unsorted array of n distinct numbers can be found in $O(n)$ time.

ii. The information-theoretic (decision-tree) lower bound on comparison sorting can be used to prove that the number of comparisons needed to build a heap of n elements is $\Omega(n \lg n)$ in the worst case.

iii. If bucket sort is implemented by using heapsort to sort the individual buckets, instead of by using insertion sort as in the normal algorithm, then the worst-case running time of bucket sort is reduced to $\Theta(n \lg n)$.

iv. Radix sort works correctly even if insertion sort is used as its auxiliary sort instead of counting sort.

v. An adversary can present an input of n distinct numbers to RANDOMIZED-SELECT that will force it to run in $\Omega(n^2)$ time.

vi. Sorting 6 elements with a comparison sort requires at least 10 comparisons in the worst case.

——The End——