Krugman, AER, 1980

Utility → (1) $U = \sum_{i=1}^{n} c_i^{\theta}$; $0 < \theta < 1$

Cost → (2) $l_i = \alpha + \beta x_i$; $\alpha, \beta > 0$; $i = 1, 2, \dots, n$

eqm. → (3) $x_i = L c_i \quad \forall \; i = 1, 2, \dots, n$

full emplt → (4) $L = \sum_{i=1}^{n} l_i = \sum_{i=1}^{n} (\alpha + \beta x_i)$

[A] Utility maximization problem

$$\text{Max. } U = \sum_{i=1}^{n} c_i^{\theta}$$

$$\text{ST } I = \sum_{i=1}^{n} p_i c_i$$

$$L = \sum_{i=1}^{n} c_i^{\theta} + \lambda \left[I - \sum_{i=1}^{n} p_i c_i \right]$$

FOC : $\frac{\partial L}{\partial c_i} = 0 = n\theta c_i^{\theta - 1} - \lambda p_i n$

$$\Rightarrow \lambda p_i = \theta c_i^{\theta - 1}$$

$$\Rightarrow p_i = \lambda^{-1} \theta c_i^{\theta - 1} = \lambda^{-1} \theta \left(\frac{x_i}{L} \right)^{\theta - 1}$$

[B] Calculating ε (elasticity of dd. as faced by each producer)

$$\frac{dp}{dx} = \lambda^{-1} \theta L^{1-\theta} (\theta - 1) x_i^{\theta - 2}$$

$$\frac{p}{x} = \lambda^{-1} \theta L^{1-\theta} x_i^{\theta - 2}$$

$$\Rightarrow -\frac{dx}{dp} \cdot \frac{p}{x} = - \frac{\cancel{\lambda^{-1}} \cancel{\theta} \cancel{L^{1-\theta}} \cancel{x_i^{\theta - 2}}}{\cancel{\lambda^{-1}} \cancel{\theta} \cancel{L^{1-\theta}} (\theta - 1) \cancel{x_i^{\theta - 2}}} = -\frac{1}{\theta - 1} = \frac{1}{1 - \theta}$$

[C] MR = MC condition

$$p = \frac{\varepsilon}{\varepsilon - 1} \beta w = \theta^{-1} \beta w$$

$\boxed{D}$ Profit maximizn. (zero π condn.)

$$\pi_i = p_i x_i - (\alpha + \beta x_i) w \quad \forall \quad i = 1, 2, \ldots, n$$

zero π condn. $\Rightarrow$ $p_i x_i = (\alpha + \beta x_i) w$.

$$\text{or,} \quad (p_i - \beta w) x_i = \alpha w$$

$$\text{or,} \quad x_i = \frac{\alpha w}{p_i - \beta w} = \frac{\alpha}{p_i/w - \beta}$$

From $\boxed{C}$ we have $p = \theta^{-1} \beta w$

$$\text{or,} \quad p/w = \theta^{-1} \beta$$

$$\therefore \quad x_i = \frac{\alpha}{\theta^{-1}\beta - \beta} = \frac{\alpha}{\beta/\theta - \beta}$$

$$= \frac{\alpha\theta}{\beta - \beta\theta} = \frac{\theta}{1-\theta} \cdot \frac{\alpha}{\beta}$$

$\boxed{E}$ Full employment condn.

$$L = \sum_{i=1}^{n} (\alpha + \beta x_i) = n(\alpha + \beta x_i)$$

$$\text{or,} \quad n = \frac{L}{\alpha + \beta x_i}$$

$$= \frac{L}{\alpha + \cancel{\beta} \frac{\theta}{1-\theta} \cdot \frac{\alpha}{\cancel{\beta}}} = \frac{L}{\alpha\left(1 + \frac{\theta}{1-\theta}\right)}$$

$$= \frac{L(1-\theta)}{\alpha}$$

$\boxed{F}$ Trade

$$n = \frac{L(1-\theta)}{\alpha} \; ; \; n^* = \frac{L^*(1-\theta)}{\alpha}$$

$$U = \sum_{i=1}^{n+n^*} c_i^{\theta}$$