Solution Approach to LP

Solution Approaches to LP models

Linear Programming problems can be solved efficiently and exact optimal solution can be found

- Graphical Approach
- Simplex Method
- Big-M and Two-Phase method
- Revised Simplex
- Dual Simplex

Each method has own limitations and requirements

Graphical solution Approach

Tech Edge problem: A product mix problem

- **Decision Variables**
  - x_1 : # of Deskpro manufactured/weak
  - x_2 : # of Portable manufactured/weak

- **LP Formulation**

Maximize, $Z = 50x_1 + 40x_2$

Subject to

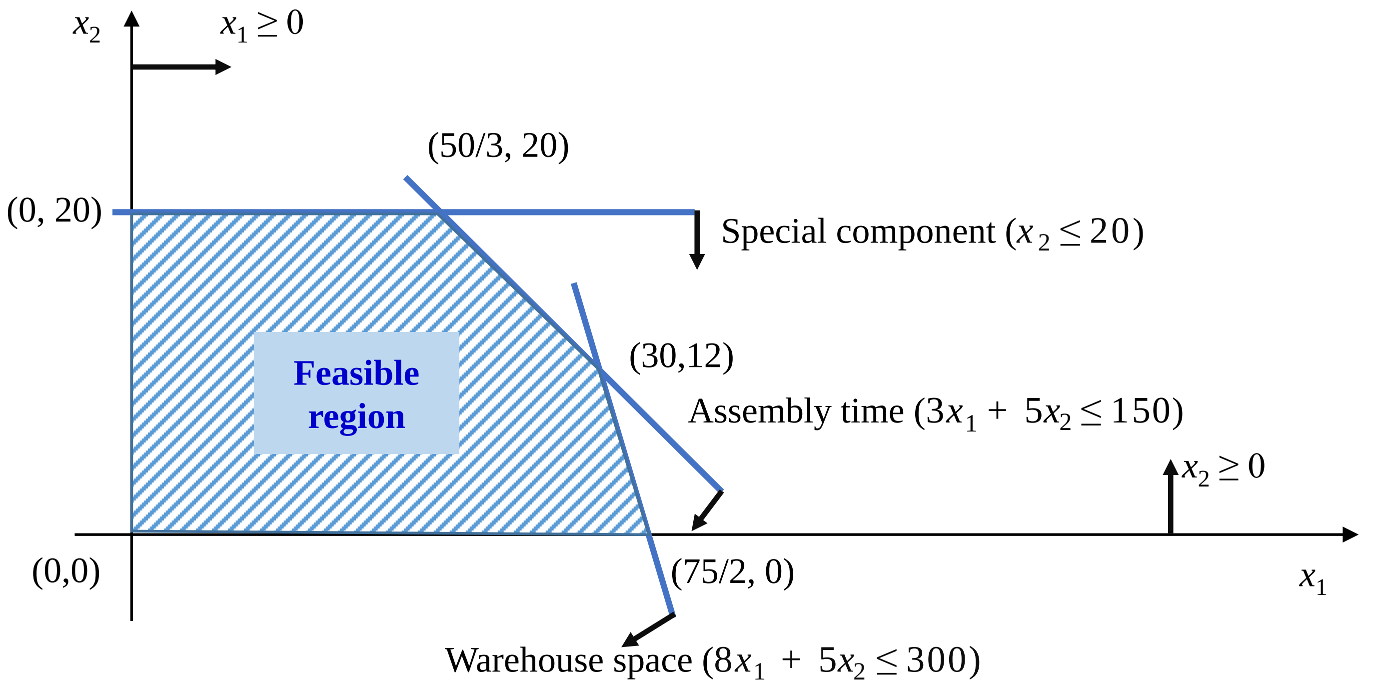
$3x_1 + 5x_2 \leq 150$ (Assembly time)

$x_2 \leq 20$ (special component)

$8x_1 + 5x_2 \leq 300$ (warehouse space)

$x_1, x_2 \geq 0$

Graphical solution Approach



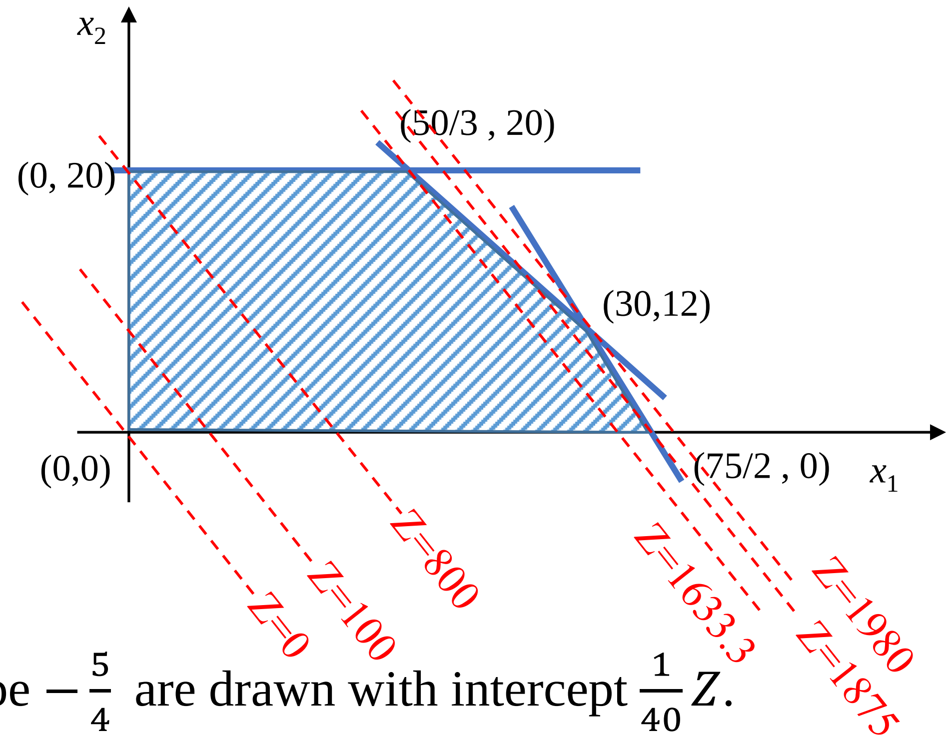
- Finding Optimal Solution
  - Pick that point which will give the maximum objective function value.
- A trivial solution
  - Check every point in the feasible region
  - Pick the best
- However, special properties of LP problems allow us to find it more efficiently.

- Consider corner points of feasible region.

Corner Point	(0, 0)	(75/2, 0)	(30, 12)	(50/3, 20)	(0, 20)
Z (= $50x_1 + 40x_2$)	0	1875	1980	1633.33	800

- Therefore, (30,12) is the optimal (x_1, x_2) and $Z = 1980$

Slope-Intercept Form



- First put Z=100 (say)

$\Rightarrow 50x_1 + 40x_2 \;=\; 100,$

- If any (x_1, x_2) combination on this straight line is feasible, increase Z.
- **Objective function:** $Z \;=\; 50x_1 + 40x_2$

$$\Rightarrow x_2 = -\frac{5}{4}x_1 + \frac{1}{40}Z$$

- A series of parallel lines with constant slope $-\frac{5}{4}$ are drawn with intercept $\frac{1}{40}Z$.

\- Start at (0, 0), $Z = 0$.

- Move the objective function line towards right

\- The first point encountered is (0, 20), therefore Z = 800

- Move towards right, the points encountered are

\- $\left(\frac{50}{3}, 200\right) \Rightarrow Z \;=\; 1633.3$

\- $\left(\frac{75}{2}, 0\right) \quad \Rightarrow Z \;=\; 1875$

\- $(30, 12) \quad \Rightarrow Z \;=\; 1980$

=> (30,12) optimal, $Z = 1980$

Change of Solution with Objective Function

Case (1): if $Z = 3x_1 - 5x_2$,

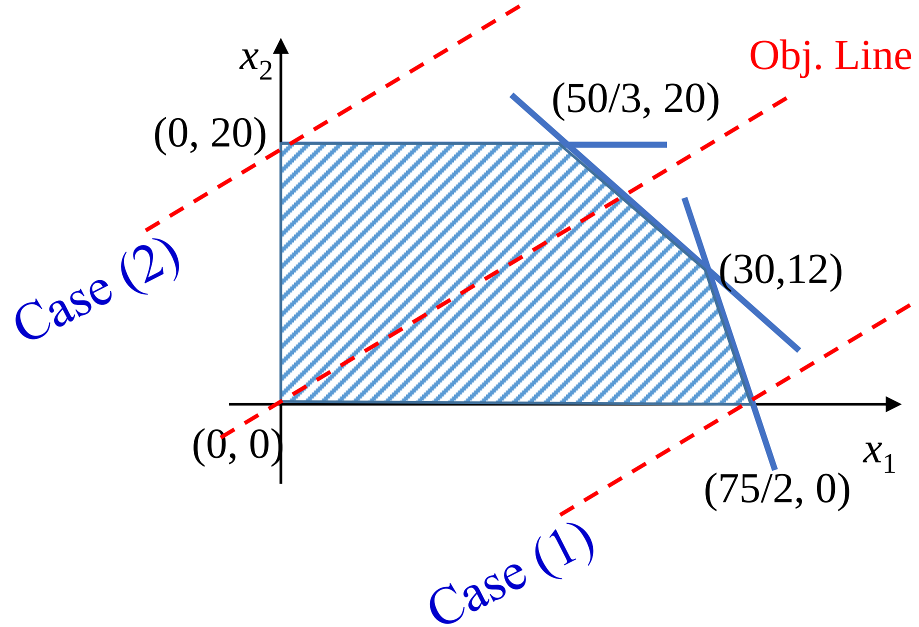
as $x_2 \uparrow$, $Z \downarrow$

=> the optimal solution is (75/2, 0)

Case (2): if $Z = -3x_1 + 5x_2$,

as $x_2 \uparrow$, $Z \uparrow$

=> and the optimal solution is (0, 20).



Case (3): if $Z = 64x_1 + 40x_2$

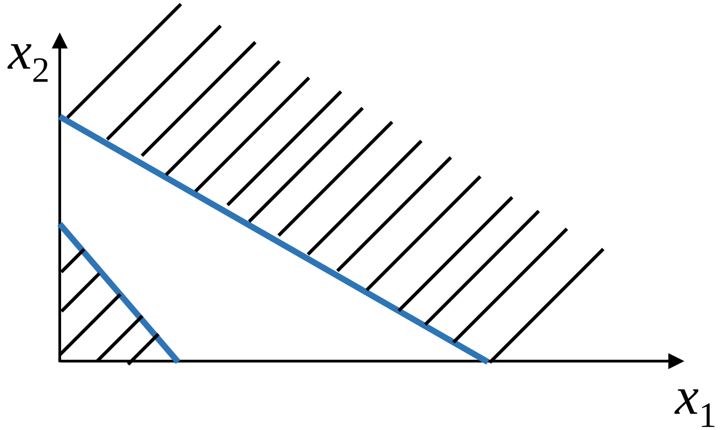
- Slope of Obj. line = Slope of warehouse space constarint
- Optimal solution

  - all the points on the line segment (75/2, 0) and (30, 12), $Z = 2400$

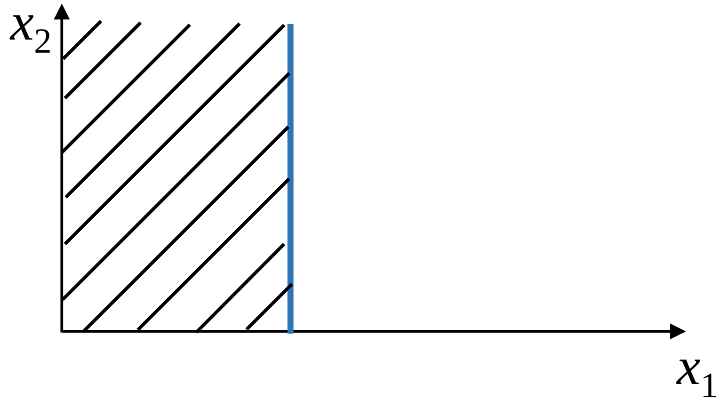
  - Multiple optimal solutions (also called alternative optima), each with the same value of the objective function

Infeasible Solution

- **No feasible solution: For any objective function**

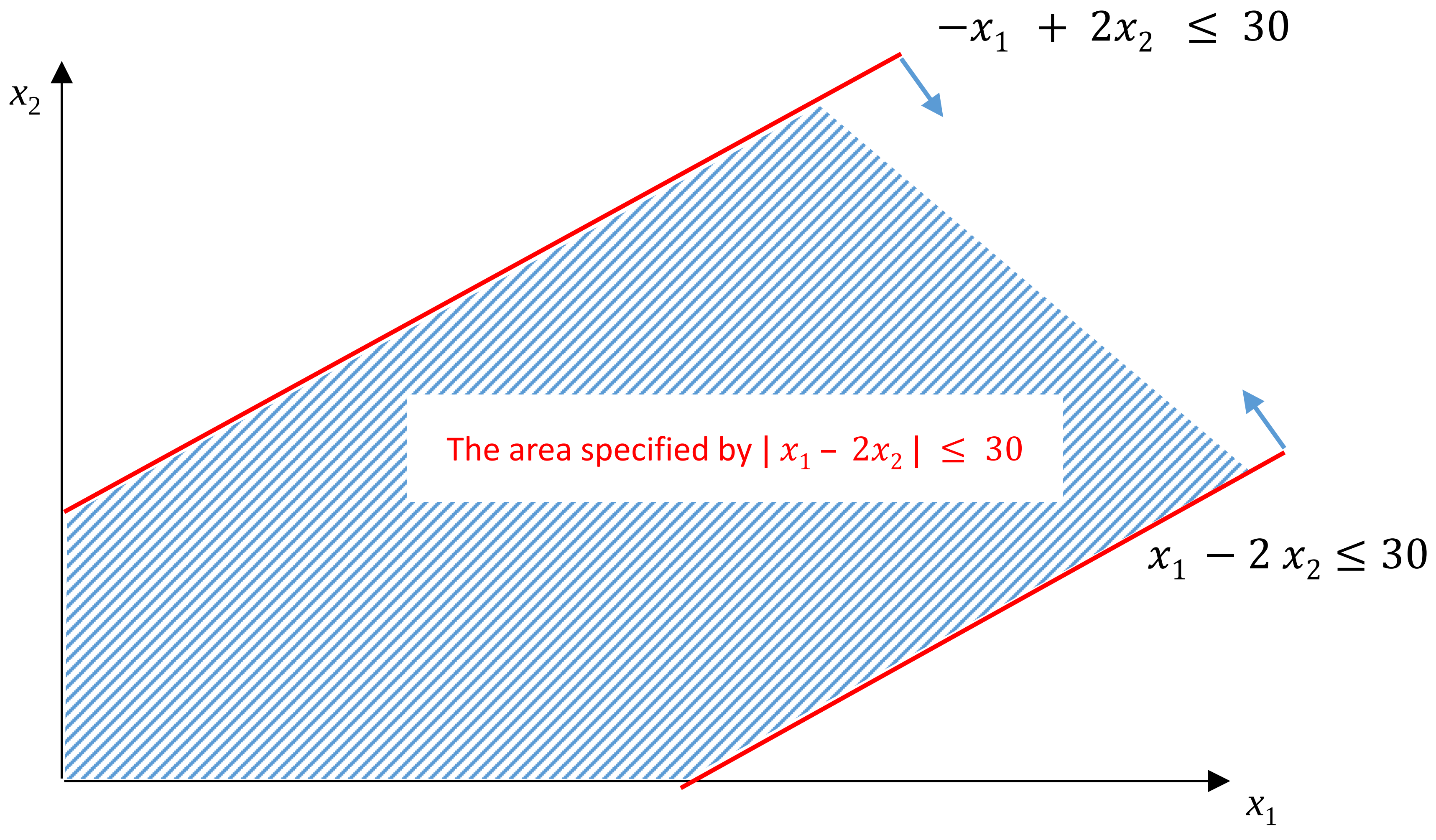


- **Unbounded solution** Max $Z = 50x_1 + 40x_2$



In both cases, No Optimal Solution

- How to interpret constraint $|x_1 - 2x_2| \le 300$?



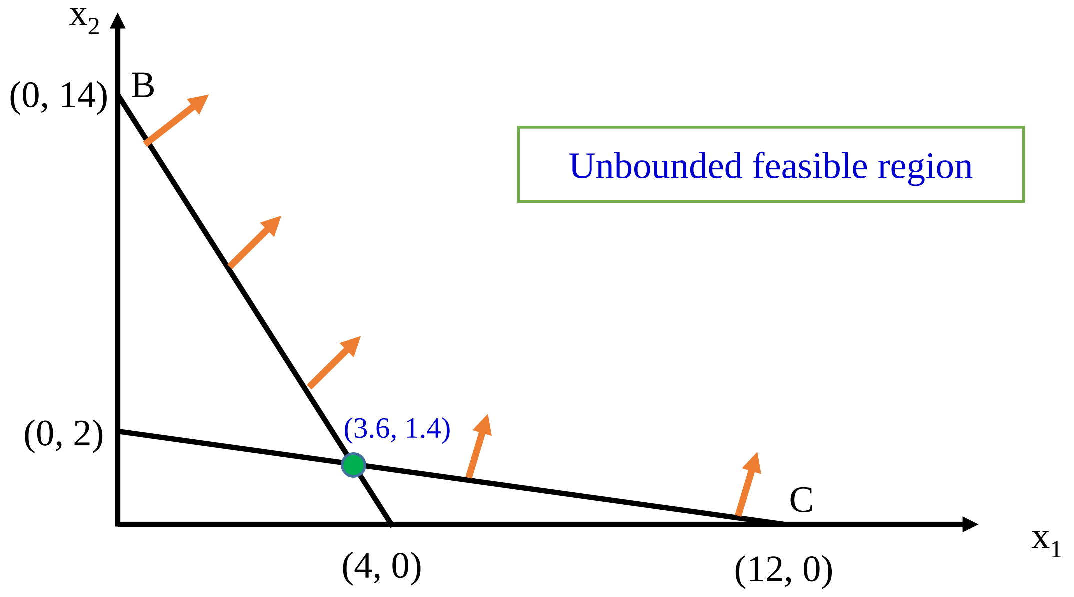
Minimization type problem: Graphical Solution Approach

Minimize, $Z = 50x_1 + 100x_2$

Subject to, $7x_1 + 2x_2 \geq 28$

$2x_1 + 12x_2 \geq 24$

$x_1, x_2 \geq 0$



Optimal Solution: $x_1 = 3.6, x_2 = 1.4, Z = 320$

Important terms in LP

- **Constraint boundary**

  Each constraint boundary is a line (plane) that forms the boundary of what is permitted by the corresponding constraint.

- **Corner points(or extreme points) solutions**

  Points of intersection of constraint boundaries.

- **Feasible solution**

  A solution for which all constraints are satisfied. e.g. any point within feasible region.

- **Infeasible solution**

  A solution for which at least one constraint is violated.

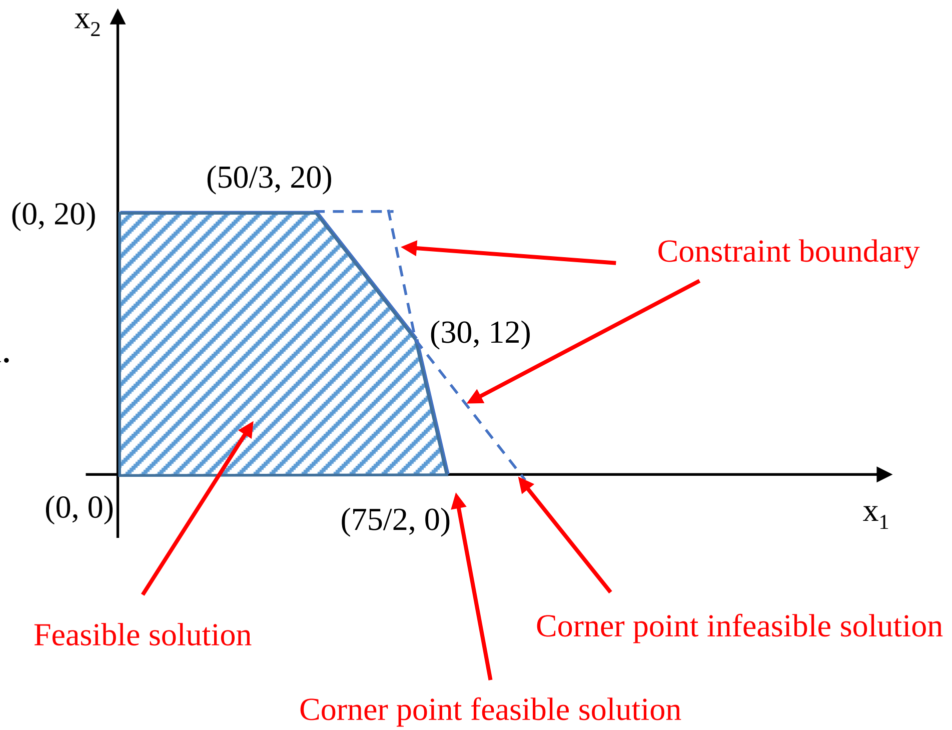
- **Feasible region**

  It is collection of all feasible solutions.

- **Corner-point feasible (CPF) solution**

A solution that lies at a corner of the feasible region.

- In any LPP with *n* decision variables, A CPF solution lies at the intersection of *n* constraint boundary



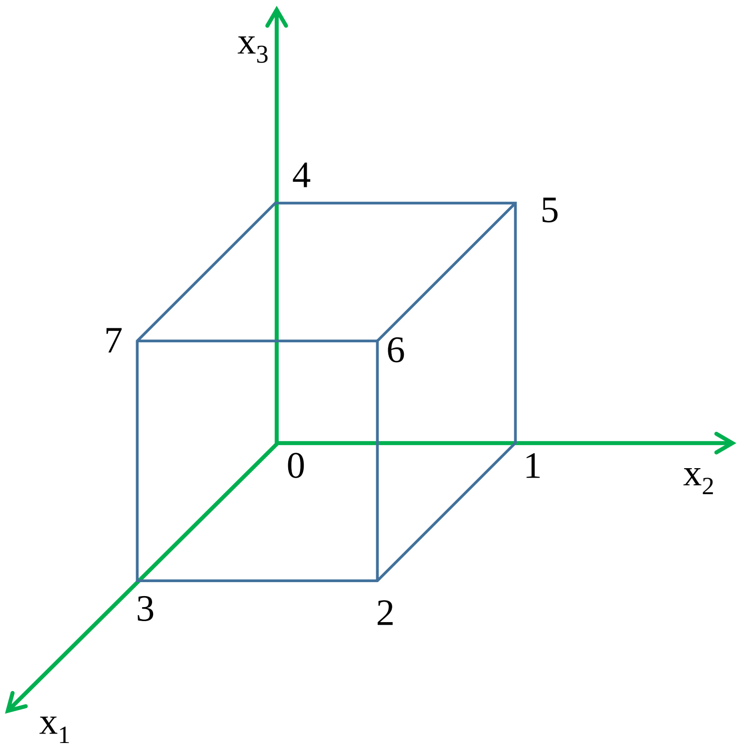
Important terms Contd…

Adjacent solutions

In any LPP, two corner point feasible solutions are adjust to each other if they share (*n*-1) constraint boundaries in *n* decision variables linear programming problem.

- **In Tech Edge problem, *n* = 2,** two of its CPF solutions are adjust if they share one constraint boundary. For example: (0, 0) & (0, 20) are adjust because they share, $x_1 = 0$ constraint boundary.

- **For LP with three decision variables, *n* = 3**



Example:

Max $Z = x_1 + x_2 + x_3$

$x_1 \leq 5$

$x_2 \leq 5$

$x_3 \leq 5$

$x_1 , x_2 , x_3 \geq 0$

e.g. Adjacent points 5 and 6 share constraint boundaries $x_2 = 5$ and $x_3 = 5$.

Points 0 and 5 share only $x_1 = 0$, so ***not adjacent***

Assumptions of LP Model

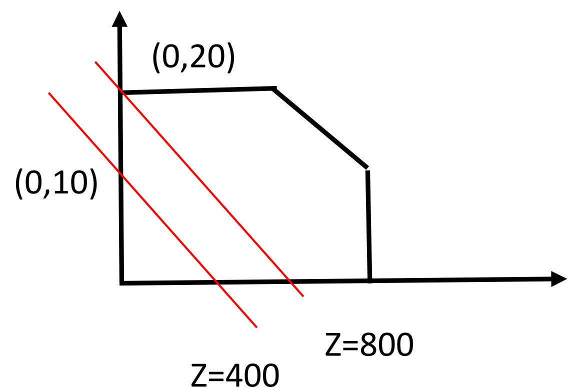
- Objective & constraints are linear functions

1. **Proportionality**

- The contribution of each decision variable in both the objective function & the constraints to be directly proportional to the value of the variable.
- In other word, double the amount, double the profit contribution or resource consumed.

i.e. linearity e.g. $c_1x_1 + c_2x_2\ (objective\ function)$

$$a_{11}x_1 + a_{12}x_2\ (constraint)$$



2. **Additivity**

The total contribution of all the variables in the objective function and in the constraints to be the direct sum of the individual contributions of each variable

Assumptions of LP Model

3. Divisibility

- Continuous (non- integer) value of decision variables possible.

4. Deterministic (certainty)

- All parameters (c_j , a_{ij} , bi) are known constant

Relaxation of Assumptions of LP Model

Violation of assumption	Model
1,2	NLP
3	IP
4	Stochastic programming

Problem:
Use graphical analysis to find optimal value of $\boldsymbol{x_1}$ & $\boldsymbol{x_2}$ for different values of $\boldsymbol{c_1}$ & $\boldsymbol{c_2}$

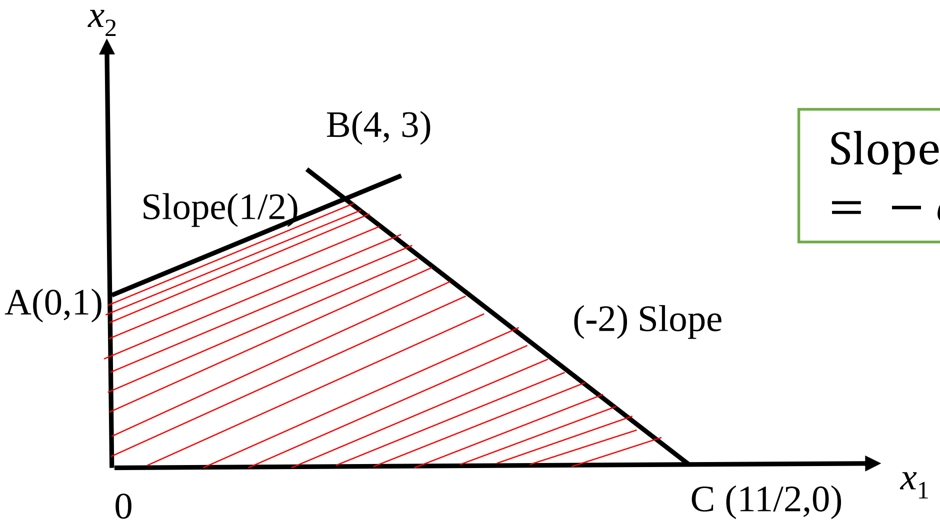
$$\text{Maximize } Z = c_1x_1 + c_2x_2$$

$$\textbf{\textit{Subject to }} 2x_1 + x_2 \leq 11$$

$$-x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

- **Solution**



Slope of objective line $= -c_1 / c_2$

Solution Contd..