Coordination Games; Limitations of NE

Coordination Games

- Not all games have unique pure strategy NE
- Multiple NE can be explained using a class of games that have many applications.
- As a group, they may be labeled **coordination games.**
- The players in such games have some common interests.
- But, because they act independently (by virtue of the nature of non-cooperative games), the coordination of actions needed to achieve a jointly preferred outcome is problematic

Pure Coordination

- Two undergraduates, Harry and Sally, who meet in their college library.
- They are attracted to each other and would like to continue the conversation, but they have to go off to their separate classes.
- They arrange to meet for coffee after the classes are over at 4:30.
- Sitting separately in class, each realizes that in the excitement they forgot to fix the place to meet.
- There are two possible choices: Starbucks and Local Latte.
- Unfortunately, these locations are on opposite sides of the large campus, so it is not possible to try both.
- And Harry and Sally have not exchanged cell-phone numbers, so they can't send messages.
- What should each do?

Pure Coordination

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	1, 1	0, 0
	Local Latte	0, 0	1, 1

- Both (S,S) and (L,L) are NE
- Option: convergence, focal point [starbucks is expensive and hence, LL is more student friendly]

Pure Coordination

- Lets change the pay-offs a little
- Suppose both prefer LL

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	1, 1	0, 0
	Local Latte	0, 0	2, 2

- Again, Multiple NE but can get the preferred equilibrium outcome only if each has enough certainty or assurance that the other is choosing the appropriate action.
- For this reason, such games are called **assurance games**

Battle of Sexes

- Lets change the pay-offs again and let's assume Harry prefers S while Sally prefers LL.

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	2, 1	0, 0
	Local Latte	0, 0	1, 2

- Risk of coordination failure is higher

Chicken Game

- Two teenagers take their cars to opposite ends of Main Street, Middle-of-Nowhere, USA, at midnight and start to drive toward each other.
- The one who swerves to prevent a collision is the “chicken,” and the one who keeps going straight is the winner.
- If both maintain a straight course, there is a collision in which both cars are damaged and both players injured

		DEAN	
		Swerve (Chicken)	Straight (Tough)
JAMES	Swerve (Chicken)	0, 0	–1, 1
	Straight (Tough)	1, –1	–2, –2

Which equilibrium?

- As with the battle of the sexes, if the game is repeated, tacit coordination is a better route to a solution.

- They might logically choose to alternate between the two equilibria, taking turns being the winner every other week.

Development, Education, Coordination

- The complementarities phenomenon affects educational decisions, and as a result, coordination failures in education can lead to development traps.

- Some of the most striking features in any statistical comparison between developing and industrialized countries are differences in education and literacy

- There are also tremendous variations within countries.

- Poor regions within individual countries are nearly always associated with education levels that are below the average of even a poor country

Education by world regions

Table 3.1. Education and literacy by world region

Region	% School enrollment[a]	Literacy
Sub-Saharan Africa	44	62.4
South Asia	54	56.3
East Asia/Pacific	65	87.1
Latin America/Caribbean	81	89.2
Arab States	60	60.8
C&E Europe/fmr. USSR	79	99.3
OECD Countries[b]	87	99.0

Notes: [a]Combined primary, secondary, and tertiary enrollment ratios.
[b]United States, Canada, Western Europe, Japan, Australia, and New Zealand.
Source: UNDP, *Human Development Report*, 2003.

Development, Education, Coordination

- Is there a reason that families in impoverished regions might *rationally choose* a low level of education for their children?

- In a world of complementarities, the unfortunate answer might very well be yes.

- In education, we have an area in which complementarities abound: My optimal level of education is positively related to what I expect yours to be, and vice versa.

- Where there are complementarities, there is strategic interdependence in which multiple Nash equilibria and development traps are possible, and where coordination and confidence matter.

Development, Education, Coordination

- Coordination failures in education arise because the returns to education depend on how many other people in the same economy are educated.
- Individual choices about education are not made in a vacuum; they are made in the context of the relevant society and its corresponding division of labor.

Wydick's Experience

- Despite the embarrassingly large number of years I had logged in school, many of my Dominican neighbors, though lacking a high school education and living in relative poverty, were better at doing more things than I was.
- Doubtless, I held a considerable advantage in solving economic and statistical problems.
- Nevertheless, from the perspective of the Dominicans, I lacked a number of essential skills.
- Foremost among these were the ability to fix a broken carburetor, repair an old shoe, slay a chicken, and, most importantly, connect a hidden wire to a special place on the nearest telephone pole to receive free ESPN.
- I was a specialist; the average Dominican was a proud generalist.
- In the Dominican Republic, my specialized skill set was a mismatch for the market.

Coordination Game in Education

		Familia Postigo		
		Primary School	Secondary School	College
Familia Lora	Primary School	1 1	-3 1	-5 1
	Secondary School	1 -3	3 3	-1 3
	College	1 -5	3 -1	5 5

Trap and Solution

- Parents, who might ordinarily like to keep their children in school, are forced to send them into the labor market simply because their own reduced wages make the family unable to reach subsistence consumption solely on the adult family labor.

- **The trap**: Low adult wages, caused significantly by child labor, create even more child labor.

- **The solution**: Every family could remove its children from the labor market all at once, driving up wages and removing the impetus for child labor

Criticism of NE – Risk

- Some critics argue that the Nash equilibrium concept does not pay due attention to risk. In some games, people might find strategies different from their Nash equilibrium strategies to be safer and might therefore choose those strategies.

		COLUMN		
		A	B	C
ROW	A	2, 2	3, 1	0, 2
	B	1, 3	2, 2	3, 2
	C	2, 0	2, 3	2, 2

- NE: (A, A)

Criticism of NE – Risk

		COLUMN		
		A	B	C
ROW	A	2, 2	3, 1	0, 2
	B	1, 3	2, 2	3, 2
	C	2, 0	2, 3	2, 2

- But one may argue that playing C *guarantees you the same payoff* as you would get in the Nash equilibrium—namely, 2; whereas if you play your NE strategy A, you will get a 2 only if the other player also plays A.
- Why take that chance? What is more, if you think the other player might use this rationale for playing C, then you would be making a serious mistake by playing A; you would get only a 0 when you could have gotten a 2 by playing C

Criticism of NE – Risk: Rebuttal

		COLUMN		
		A	B	C
ROW	A	2, 2	3, 1	0, 2
	B	1, 3	2, 2	3, 2
	C	2, 0	2, 3	2, 2

- If you really believe that the other player would think this way and play C, then you should play B to get the payoff 3. And if you think the other person would think this way and play B, then your best response to B should be A. And if you think the other person would figure this out, too, you should be playing your best response to A—namely, A. Back to the Nash equilibrium!

- Criticising NE and its rebuttal is a game in itself!!!

Krep's Example

		B	
		Left	Right
A	Up	9, 10	8, 9.9
	Down	10, 10	–1000, 9.9

- As player A, which one would you play?
- Note that player B has a dominant strategy. But marginal.
- What if B was a player with a substantially different value system or was not a truly rational player and might choose the "wrong" action just for fun?
- What if A doesn't know B's payoffs?

Krep's Example

- With probability $(1 - p)$, B's dominant strategy is Left (the case shown in the figure), and with probability p, it is Right.

- Therefore, if A chooses Up, then with probability $(1 - p)$ he will meet B playing Left and so get a payoff of 9 and with probability p, he will meet B playing Right and so get a payoff of 8.

- Thus, A's statistical or probability weighted average payoff from playing Up is $9(1 - p) + 8p$ and A's statistical average payoff from playing Down is $10(1 - p) - 1000p$.

- Hence, A will play Up if

  $9(1 - p) + 8p > 10(1 - p) - 1000p$, or $p > 1/1{,}009$

Criticism of NE: Multiple Equilibria

- Another criticism of the Nash equilibrium concept is based on the observation that many games have multiple Nash equilibria.
- Thus, the argument goes, ***the concept fails to pin down outcomes*** of games sufficiently precisely to give unique predictions.
- This argument does not automatically require us to abandon the Nash equilibrium concept.
- Rather, it suggests that if we want a unique prediction from our theory, we must ***add some criterion*** for deciding which one of the multiple Nash equilibria we want to select.
- Solutions –

1. Focal point – convergence of expectations
2. Credibility
3. Refinements (like, perfect Bayesian equilibrium)

Criticism of NE: Rationality

- Remember that Nash equilibrium can be regarded as a system of the strategy choices of each player and the belief that each player holds about the other players' choices.

- In equilibrium, (1) the choice of each should give her the best payoff given her belief about the others' choices, and (2) the belief of each player should be correct—that is, her actual choices should be the same as what this player believes them to be.

- These seem to be natural expressions of the requirements of the ***mutual consistency of individual rationality***.

- If all players have common knowledge that they are all rational, how can any one of them rationally believe something about others' choices that would be inconsistent with a rational response to her own actions?

Criticism of NE: Rationality

		COLUMN		
		C1	C2	C3
ROW	R1	0, 7	2, 5	7, 0
	R2	5, 2	3, 3	5, 2
	R3	7, 0	2, 5	0, 7

- Best-response analysis quickly reveals that it has only one Nash equilibrium—namely, (R2, C2), leading to payoffs (3, 3).
- In this equilibrium, Row plays R2 because she believes that Column is playing C2.
- Why does she believe this? Because she knows Column to be rational, Row must simultaneously believe that Column believes that Row is choosing R2, because C2 would not be Column's best choice if she believed Row would be playing either R1 or R3.
- Thus, the claim goes, in any rational process of formation of beliefs and responses, ***beliefs would have to be correct***.

Rationalizability

- What strategy choices in games can be justified on the basis of rationality alone?

- In the last example, we can justify any one of the nine logically conceivable combinations depending on beliefs

- Rationality alone does not give us any power to narrow down or predict outcomes at all

- When players recognize that other players, being rational, will not play dominated strategies, iterated elimination of dominated strategies can be performed on the basis of common knowledge of rationality.

Rationalizability

- Some more ruling out of strategies can be done, by using a property slightly stronger than being dominated in pure strategies.
- This property identifies strategies that are **never a best response.**
- The set of strategies that survive elimination on this ground are called **rationalizable**
- The concept itself is known as **rationalizability**
- It is useful to know how far we can narrow down the possible outcomes of a game based on the players' rationality alone, without invoking correctness of expectations about the other player's actual choice

Rationalizability

- It is sometimes possible to figure out that the other player *will not choose some available action or actions,* even when it is not possible to pin down the single action that she *will choose*.

- In some cases rationalizability may not narrow down the outcomes at all

- In some cases it narrows down the possibilities to some extent, but not all the way down to the Nash equilibrium if the game has a unique one, or to the set of Nash equilibria if there are several.

- In some other cases, the narrowing down may go all the way to the Nash equilibrium; in these cases, we have a more powerful justification for the Nash equilibrium that relies on rationality alone, without assuming correctness of expectations.

Never a Best Response

		COLUMN			
		C1	C2	C3	C4
ROW	R1	0, 7	2, 5	7, 0	0, 1
	R2	5, 2	3, 3	5, 2	0, 1
	R3	7, 0	2, 5	0, 7	0, 1
	R4	0, 0	0, −2	0, 0	10, −1

- Note that, although C4 is never a best response, it is not dominated by any of C1, C2, and C3.

- "never a best response" is a more general concept than "dominated." Eliminating strategies that are never a best response may be possible even when eliminating dominated strategies is not. So ***eliminating strategies that are never a best response can narrow down*** the set of possible outcomes more than can elimination of dominated strategies

References

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