Assignment –1

Topics: Order & Degree of a PDE, Classification of PDEs as Linear/Semi-linear/Quasi-Linear/Non-linear PDE, Formation of PDE by eliminating arbitrary constants/Functions.

Symbols: $p \equiv \partial z/\partial x\ , q \equiv \partial z/\partial y$

1. Classify the following PDEs into linear / non-linear /semi-linear /quasi linear.
   a) $(x^2 - z^2)p + xyzq = z^2x^2 + y^2$
   b) $yp + xq = z - x^2$
   c) $pq = z$
   d) $xp - yxq = xz^2$
   e) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$
   f) $z\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} = 0$
   g) $\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} = z^2$
2. Determine order and degree of following PDEs:
   a) $p^2 + q^2 = 1$
   b) $z\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^2 z}{\partial y^2} = 0$
   c) $\frac{\partial z}{\partial x} + \left(\frac{\partial^2 z}{\partial y^2}\right)^3 = z^2$
   d) $py - qx = 0$
3. Check whether $z = (1 + \cos x)\cos y$ is a solution to the PDE $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$.
4. Eliminate the arbitrary constants a, b, c from
   a) $z = ax^2 + bxy + cy^2$ to form a 2nd order PDE.
   b) $-z + \frac{x^2}{y} = \ln\left(\frac{ax^2}{y^2} + \frac{b}{y}\right)$ to form a 1st order PDE.
5. Eliminate the arbitrary functions f, g from
   a) $z = f(\sqrt{x^2 + y^2})$ to form a 1st order PDE.
   b) $z = f(xy) + g(x/y)$ to form a 2nd order PDE.

END