CHAPTER 5

# DISCOUNTED DIVIDEND VALUATION

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## LEARNING OUTCOMES

*After completing this chapter, you will be able to do the following:*

- compare dividends, free cash flow, and residual income as inputs to discounted cash flow models and identify investment situations for which each measure is suitable;
- calculate and interpret the value of a common stock using the dividend discount model (DDM) for single and multiple holding periods;
- calculate the value of a common stock using the Gordon growth model and explain the model's underlying assumptions;
- calculate and interpret the implied growth rate of dividends using the Gordon growth model and current stock price;
- calculate and interpret the present value of growth opportunities (PVGO) and the component of the leading price-to-earnings ratio (P/E) related to PVGO;
- calculate and interpret the justified leading and trailing P/Es using the Gordon growth model;
- calculate the value of noncallable fixed-rate perpetual preferred stock;
- describe strengths and limitations of the Gordon growth model and justify its selection to value a company's common shares;
- explain the assumptions and justify the selection of the two-stage DDM, the H-model, the three-stage DDM, or spreadsheet modeling to value a company's common shares;

The data and examples for this reading were updated in 2014 by Professor Stephen Wilcox, CFA.

- explain the growth phase, transitional phase, and maturity phase of a business;
- describe terminal value and explain alternative approaches to determining the terminal value in a DDM;
- calculate and interpret the value of common shares using the two-stage DDM, the H-model, and the three-stage DDM;
- estimate a required return based on any DDM, including the Gordon growth model and the H-model;
- explain the use of spreadsheet modeling to forecast dividends and to value common shares;
- calculate and interpret the sustainable growth rate of a company and demonstrate the use of DuPont analysis to estimate a company's sustainable growth rate;
- evaluate whether a stock is overvalued, fairly valued, or undervalued by the market based on a DDM estimate of value.

## 1. INTRODUCTION

Common stock represents an ownership interest in a business. A business in its operations generates a stream of cash flows, and, as owners of the business, common stockholders have an equity ownership claim on those future cash flows. Beginning with John Burr Williams (1938), analysts have developed this insight into a group of valuation models known as discounted cash flow (DCF) valuation models. DCF models—which view the intrinsic value of common stock as the present value of its expected future cash flows—are a fundamental tool in both investment management and investment research. This reading is the first of several that describe DCF models and address how to apply those models in practice.

Although the principles behind discounted cash flow valuation are simple, applying the theory to equity valuation can be challenging. Four broad steps in applying DCF analysis to equity valuation are:

- choosing the class of DCF model—equivalently, selecting a specific definition of cash flow;
- forecasting the cash flows;
- choosing a discount rate methodology; and
- estimating the discount rate.

In this reading, we take the perspective that dividends—distributions to shareholders authorized by a company's board of directors—are an appropriate definition of cash flows. The class of models based on this idea is called dividend discount models, or DDMs. The basic objective of any DDM is to value a stock. The variety of implementations corresponds to different ways to model a company's future stream of dividend payments. The steps of choosing a discount rate methodology and estimating the discount rate involve the same considerations for all DCF models, so they have been presented separately in a reading on return concepts.

The reading is organized as follows: Section 2 provides an overview of present value models. A general statement of the dividend discount model follows in Section 3. Forecasting dividends, individually and in detail, into the indefinite future is not generally practicable, so the dividend-forecasting problem is usually simplified. One approach is to assign dividends to a stylized growth pattern. The simplest pattern—dividends growing at a constant rate forever—is the constant growth (or Gordon growth) model, discussed in Section 4. For some companies, it is more appropriate to view earnings and dividends as having multiple stages of growth; multistage dividend discount models are presented in Section 5 along with spreadsheet modeling. Section 6 lays out the determinants of dividend growth rates, and the final section summarizes the reading.

## 2. PRESENT VALUE MODELS

Present value models as a group constitute a demanding and rigorous approach for valuing assets. In this section, we discuss the economic rationale for valuing an asset as the present value of its expected future cash flows. We also discuss alternative definitions of cash flows and present the major alternative methods for estimating the discount rate.

### 2.1. Valuation Based on the Present Value of Future Cash Flows

The value of an asset must be related to the benefits or returns we expect to receive from holding it. Those returns are called the asset's future cash flows (we will define *cash flow* more concretely and technically later). We also need to recognize that a given amount of money received in the future is worth less than the same amount of money received today. Money received today gives us the option of immediately spending and consuming it, so money has a time value. Therefore, when valuing an asset, before adding up the estimated future cash flows, we must **discount** each cash flow back to the present: the cash flow's value is reduced with respect to how far away it is in time. The two elements of discounted cash flow valuation—estimating the cash flows and discounting the cash flows to account for the time value of money—provide the economic rationale for discounted cash flow valuation. In the simplest case, in which the timing and amounts of future cash flows are known with certainty, if we invest an amount equal to the present value of future cash flows at the given discount rate, that investment will replicate all of the asset's cash flows (with no money left over).

For some assets, such as government debt, cash flows may be essentially known with certainty—that is, they are default risk free. The appropriate discount rate for such a risk-free cash flow is a risk-free rate of interest. For example, if an asset has a single, certain cash flow of \$100 to be received in two years, and the risk-free interest rate is 5 percent a year, the value of the asset is the present value of \$100 discounted at the risk-free rate, $\$100/(1.05)^2 = \$90.70$.

In contrast to risk-free debt, future cash flows for equity investments are not known with certainty—they are risky. Introducing risk makes applying the present value approach much more challenging. The most common approach to dealing with risky cash flows involves two adjustments relative to the risk-free case. First, discount the *expected* value of the cash flows, viewing the cash flows as random variables.[1] Second, adjust the discount rate to reflect the risk of the cash flows.

The following equation expresses the concept that an asset's value is the present value of its (expected) future cash flows:

$$V_0 = \sum_{t=1}^{n} \frac{\mathrm{CF}_t}{(1+r)^t} \tag{1}$$

[1] The expected value of a random quantity is the mean, or average, value of its possible outcomes, in which each outcome's weight in the average is its probability of occurrence. See DeFusco, McLeavey, Pinto, and Runkle (2004) for all statistical concepts used in this reading.

where

$V_0$ = the value of the asset at time $t = 0$ (today)
$n$ = number of cash flows in the life of the asset ($n$ is set equal to ∞ for equities)
$CF_t$ = the cash flow (or the expected cash flow, for risky cash flows) at time $t$
$r$ = the discount rate or required rate of return

For simplicity, the discount rate in Equation 1 is represented as the same for all time periods (i.e., a flat term structure of discount rates is assumed). The analyst has the latitude in this model, however, to apply different discount rates to different cash flows.[2]

Equation 1 gives an asset's value from the perspective of today ($t = 0$). Likewise, an asset's value at some point in the future equals the value of all subsequent cash flows discounted back to that point in time. Example 1 illustrates these points.

## EXAMPLE 1 Value as the Present Value of Future Cash Flows

An asset is expected to generate cash flows of \$100 in one year, \$150 in two years, and \$200 in three years. The value of this asset today, using a 10 percent discount rate, is

$$\begin{aligned} V_0 &= \frac{100}{(1.10)^1} + \frac{150}{(1.10)^2} + \frac{200}{(1.10)^3} \\ &= 90.909 + 123.967 + 150.263 = \$365.14 \end{aligned}$$

The value at $t = 0$ is \$365.14. The same logic is used to value an asset at a future date. The value of the asset at $t = 1$ is the present value, discounted back to $t = 1$, of all cash flows after this point. This value, $V_1$, is

$$\begin{aligned} V_1 &= \frac{150}{(1.10)^1} + \frac{200}{(1.10)^2} \\ &= 136.364 + 165.289 = \$301.65 \end{aligned}$$

At any point in time, the asset's value is the value of future cash flows (CF) discounted back to that point. Because $V_1$ represents the value of $CF_2$ and $CF_3$ at $t = 1$, the value of the asset at $t = 0$ is also the present value of $CF_1$ and $V_1$:

$$\begin{aligned} V_0 &= \frac{100}{(1.10)^1} + \frac{301.653}{(1.10)^1} \\ &= 90.909 + 274.23 = \$365.14 \end{aligned}$$

Finding $V_0$ as the present value of $CF_1$, $CF_2$, and $CF_3$ is logically equivalent to finding $V_0$ as the present value of $CF_1$ and $V_1$.

[2] Different discount rates could reflect different degrees of cash flow riskiness or different risk-free rates at different time horizons. Differences in cash flow riskiness may be caused by differences in business risk, operating risk (use of fixed assets in production), or financial risk or leverage (use of debt in the capital structure). The simple expression given, however, is adequate for this discussion.

In the next section, we present an overview of three alternative definitions of cash flow. The selected cash flow concept defines the type of DCF model we can use: the dividend discount model, the free cash flow model, or the residual income model. We also broadly characterize the types of valuation problems for which analysts often choose a particular model. (Further details are supplied when each model is discussed individually.)

### 2.2. Streams of Expected Cash Flows

In present value models of stock valuation, the three most widely used definitions of returns are dividends, free cash flow, and residual income. We discuss each definition in turn.

The dividend discount model defines cash flows as dividends. The basic argument for using this definition of cash flow is that an investor who buys and holds a share of stock generally receives cash returns only in the form of dividends.[3] In practice, analysts usually view investment value as driven by earnings. Does the definition of cash flow as dividends ignore earnings not distributed to shareholders as dividends? Reinvested earnings should provide the basis for increased future dividends. Therefore, the DDM accounts for reinvested earnings when it takes all future dividends into account. Because dividends are less volatile than earnings and other return concepts, the relative stability of dividends may make DDM values less sensitive to short-run fluctuations in underlying value than alternative DCF models. Analysts often view DDM values as reflecting long-run intrinsic value.

A stock either pays dividends or does not pay dividends. A company might not pay dividends on its stock because the company is not profitable and has no cash to distribute. Also, a company might not pay dividends for the opposite reason: because it is very profitable. For example, a company may reinvest all earnings—paying no dividends—to take advantage of profitable growth opportunities. As the company matures and faces fewer attractive investment opportunities, it may initiate dividends. Generally, mature, profitable companies tend to pay dividends and are reluctant to reduce the level of dividends.[4]

Dividend policy practices have international differences and change through time, even in one market. Typically, a lower percentage of companies in a given US stock market index have paid dividends than have companies in a comparable European stock market index. Wanger (2007) noted a much higher propensity for European and Asian small-cap companies to pay dividends compared with US companies. In addition, the following broad trends in dividend policy have been observed:

- The fraction of companies paying cash dividends has been in long-term decline in most developed markets (e.g., the United States, Canada, the European Union, the United Kingdom, and Japan).[5] For example, Fama and French (2001) found that although 66.5 percent of US stocks paid dividends in 1978, only 20.8 percent did in 1999, with later research documenting a small rebound since 2001.[6] In the United States, the decline was caused by a reduced propensity to pay dividends (controlling for differences in profitability and growth opportunities) and by growth in the number of smaller, publicly traded companies with low profitability and large growth opportunities.[7]

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[3] Corporations can also effectively distribute cash to stockholders through stock repurchases (also called buybacks). This fact, however, does not affect the argument.
[4] See Lintner (1956) and Grullon, Paye, Underwood, and Weston (2007).
[5] See von Eije and Megginson (2008) and references therein.
[6] Julio and Ikenberry (2004).
[7] Fama and French (2001).

- Since the early 1980s in the United States[8] and the early 1990s in the United Kingdom and continental Europe,[9] the fraction of companies engaging in share repurchases (an alternative way to distribute cash to shareholders) has trended upwards.

Analysts will frequently need to value non-dividend-paying shares. Can the DDM be applied to non-dividend-paying shares? In theory it can, as is illustrated later, but in practice it generally is not.

Predicting the timing of dividend initiation and the magnitude of future dividends without any prior dividend data or specifics about dividend policy to guide the analysis is generally not practical. For a non-dividend-paying company, analysts usually prefer a model that defines returns at the company level (as free cash flow or residual income—these concepts are defined shortly) rather than at the stockholder level (as dividends). Another consideration in the choice of models relates to ownership perspective. An investor purchasing a small ownership share does not have the ability to meaningfully influence the timing or magnitude of the distribution of the company's cash to shareholders. That perspective is the one taken in applying a dividend discount model. The only access to the company's value is through the receipt of dividends, and dividend policy is taken as a given. If dividends do not bear an understandable relation to value creation in the company, applying the DDM to value the stock is prone to error.

Generally, the definition of returns as dividends, and the DDM, is most suitable when:

- the company is dividend-paying (i.e., the analyst has a dividend record to analyze);
- the board of directors has established a dividend policy that bears an understandable and consistent relationship to the company's profitability; and
- the investor takes a noncontrol perspective.

Often, companies with established dividends are seasoned companies, profitable but operating outside the economy's fastest-growing subsectors. Professional analysts often apply a dividend discount model to value the common stock of such companies.

### EXAMPLE 2 Coca-Cola Bottling Company and Hormel Foods: Is the DDM an Appropriate Choice?

As director of equity research at a brokerage, you have final responsibility in the choice of valuation models. An analyst covering consumer/noncyclicals has approached you about the use of a dividend discount model for valuing the equity of two companies: Coca-Cola Bottling Company Consolidated (NASDAQ: COKE) and Hormel Foods (NYSE: HRL). Exhibit 1 gives the most recent 15 years of data. (In the table, EPS is earnings per share, DPS is dividends per share, and payout ratio is DPS divided by EPS.)

[8] Important in the United States was the adoption of Securities and Exchange Commission Rule 10b-18 in 1982, which relieved companies from concerns of stock manipulation in repurchasing shares so long as companies follow certain guidelines.

[9] See von Eije and Megginson (2008).

EXHIBIT 1 COKE and HRL: The Earnings and Dividends Record

| | COKE | | | HRL | | |
|---|---|---|---|---|---|---|
| Year | EPS ($) | DPS ($) | Payout Ratio (%) | EPS ($) | DPS ($) | Payout Ratio (%) |
| 2012 | 3.08 | 1.00 | 32 | 1.86 | 0.60 | 32 |
| 2011 | 3.08 | 1.00 | 32 | 1.74 | 0.51 | 29 |
| 2010 | 3.94 | 1.00 | 25 | 1.51 | 0.42 | 28 |
| 2009 | 3.56 | 1.00 | 28 | 1.27 | 0.38 | 30 |
| 2008 | 1.77 | 1.00 | 56 | 1.04 | 0.37 | 36 |
| 2007 | 2.17 | 1.00 | 46 | 1.07 | 0.30 | 28 |
| 2006 | 2.55 | 1.00 | 39 | 1.03 | 0.28 | 27 |
| 2005 | 2.53 | 1.00 | 40 | 0.91 | 0.26 | 29 |
| 2004 | 2.41 | 1.00 | 41 | 0.78 | 0.23 | 29 |
| 2003 | 3.40 | 1.00 | 29 | 0.67 | 0.21 | 31 |
| 2002 | 2.56 | 1.00 | 39 | 0.68 | 0.20 | 29 |
| 2001 | 1.07 | 1.00 | 93 | 0.65 | 0.19 | 29 |
| 2000 | 0.71 | 1.00 | 141 | 0.61 | 0.18 | 30 |
| 1999 | 0.37 | 1.00 | 270 | 0.54 | 0.17 | 31 |
| 1998 | 1.75 | 1.00 | 57 | 0.41 | 0.16 | 39 |

*Source:* The Value Line Investment Survey, sec.edgar-online.com.

Answer the following questions based on the information in Exhibit 1:

1. State whether a dividend discount model is an appropriate choice for valuing COKE. Explain your answer.
2. State whether a dividend discount model is an appropriate choice for valuing HRL. Explain your answer.

*Solution to 1:* Based only on the data given in Exhibit 1, a DDM does not appear to be an appropriate choice for valuing COKE. COKE's dividends have been $1.00 per share since 1998. In 1998, COKE's EPS was $1.75 but fell sharply to $0.37 in 1999. EPS recovered to $2.56 in 2002 but has varied from $1.77 to $3.94 since with a value of $3.08 in 2012. In short, during the 10-year period of 2002–2012, COKE achieved compound annual growth of just 1.9 percent with considerable variability while DPS were flat. Based on the record presented, it is hard to discern an understandable and consistent relationship of dividends to earnings. Because dividends do not appear to adjust to reflect changes in profitability, applying a DDM to COKE is probably inappropriate. Valuing COKE on another basis, such as a company-level definition of cash flows, appears to be more appropriate.

*Solution to 2:* The historical earnings of HRL show a long-term upward trend, with the exception of 2003 and 2008. Although you might want to research those divergent payout ratios, HRL's dividends have generally followed its growth in earnings. Earnings per share and dividends per share grew at comparable compound annual growth rates of 11.4 percent and 9.9 percent during the entire period. During the most recent four-year period, EPS and DPS also grew at comparable rates, reflecting a dividend payout ratio varying only between 28 percent and 32 percent. In summary, because HRL is dividend-paying and dividends bear an understandable and consistent relationship to earnings, using a DDM to value HRL is appropriate.

Valuation is a forward-looking exercise. In practice, the analyst would check for public disclosures concerning changes in dividend policy going forward.

A second definition of returns is free cash flow. The term *cash flow* has been given many meanings in different contexts. Earlier the term was used informally, referring to returns to ownership (equity). We now want to give it a more technical meaning, related to accounting usage. Over a given period of time, a company can add to cash (or use up cash) by selling goods and services. This money is cash flow from operations (for that time period). Cash flow from operations is the critical cash flow concept addressing a business's underlying economics. Companies can also generate (or use up) cash in two other ways. First, a company affects cash through buying and selling assets, including investment and disinvestment in plant and equipment. Second, a company can add to or reduce cash through its financing activities. Financing includes debt and equity. For example, issuing bonds increases cash, and buying back stock decreases cash (all else equal).[10]

Assets supporting current sales may need replacement because of obsolescence or wear and tear, and the company may need new assets to take advantage of profitable growth opportunities. The concept of free cash flow responds to the reality that, for a going concern, some of the cash flow from operations is not "free" but rather needs to be committed to reinvestment and new investment in assets. **Free cash flow to the firm** (FCFF) is cash flow from operations minus capital expenditures. Capital expenditures—reinvestment in new assets, including working capital—are needed to maintain the company as a going concern, so only that part of cash flow from operations remaining after such reinvestment is "free." (This definition is conceptual; a later reading defines free cash flow concepts in detail.) FCFF is the part of the cash flow generated by the company's operations that can be withdrawn by bondholders and stockholders without economically impairing the company. Conceptually, the value of common equity is the present value of expected future FCFF—the total value of the company—minus the market value of outstanding debt.

Another approach to valuing equity works with free cash flow to equity. **Free cash flow to equity** (FCFE) is cash flow from operations minus capital expenditures, or FCFF, from which we net all payments to debtholders (interest and principal repayments net of new debt issues). Debt has a claim on the cash of the company that must be satisfied before any money

[10] Internationally, accounting definitions may not be fully consistent with the presented concepts in distinguishing between types of sources and uses of cash. Although the implementation details are not the focus here, an example can be given. US generally accepted accounting principles include a financing item, net interest payments, in *cash flow from operating activities*. So, careful analysts working with US accounting data often add back after-tax net interest payments to cash flow from operating activities when calculating cash flow from operations. Under International Accounting Standards, companies may or may not include interest expense as an operating cash flow.

can be paid to stockholders, so money paid on debt is not available to common stockholders. Conceptually, common equity can be valued as the present value of expected FCFE. FCFF is a predebt free cash flow concept; FCFE is a postdebt free cash flow concept. The FCFE model is the baseline free cash flow valuation model for equity, but the FCFF model may be easier to apply in several cases, such as when the company's leverage (debt in its capital structure) is expected to change significantly over time.

Valuation using a free cash flow concept is popular in current investment practice. Free cash flow (FCFF or FCFE) can be calculated for any company. The record of free cash flows can also be examined even for a non-dividend-paying company. FCFE can be viewed as measuring what a company can afford to pay out in dividends. Even for dividend-paying companies, a free cash flow model valuation may be preferred when dividends exceed or fall short of FCFE by significant amounts.[11] FCFE also represents cash flow that can be redeployed outside the company without affecting the company's capital investments. A controlling equity interest can effect such redeployment. As a result, free cash flow valuation is appropriate for investors who want to take a control perspective. (Even a small shareholder may want to take such a perspective when potential exists for the company to be acquired, because stock price should reflect the price an acquirer would pay.)

Just as there are cases in which an analyst would find it impractical to apply the DDM, applying the free cash flow approach is a problem in some cases. Some companies have intense capital demands and, as a result, have negative expected free cash flows far into the future. As one example, a retailer may be constantly constructing new outlets and be far from saturating even its domestic market. Even if the retailer is currently very profitable, free cash flow may be negative indefinitely because of the level of capital expenditures. The present value of a series of negative free cash flows is a negative number. The use of a free cash flow model may entail a long forecast horizon to capture the point at which expected free cash flow turns positive. The uncertainty associated with distant forecasts may be considerable. In such cases, the analyst may have more confidence using another approach, such as residual income valuation.

Generally, defining returns as free cash flow and using the FCFE (and FCFF) models are most suitable when:

- the company is not dividend-paying;
- the company is dividend-paying but dividends significantly exceed or fall short of free cash flow to equity;
- the company's free cash flows align with the company's profitability within a forecast horizon with which the analyst is comfortable; and
- the investor takes a control perspective.

The third and final definition of returns that we will discuss in this overview is residual income. Conceptually, **residual income** for a given time period is the earnings for that period in excess of the investors' required return on beginning-of-period investment (common stockholders' equity). Suppose shareholders' initial investment is $200 million, and the required rate of return on the stock is 8 percent. The required rate of return is investors' **opportunity cost** for investing in the stock: the highest expected return available from other equally risky investments, which is the return that investors forgo when investing in the stock. The company

[11] In theory, when period-by-period dividends equal FCFE, the DDM and FCFE models should value stock identically, if all other assumptions are consistent. See Miller and Modigliani (1961), a classic reference for the mathematics and theory of present value models of stock value.

earns \$18 million in the course of a year. How much value has the company added for shareholders? A return of $0.08 \times \$200$ million = \$16 million just meets the amount investors could have earned in an equivalent-risk investment (by the definition of opportunity cost). Only the residual or excess amount of \$18 million − \$16 million = \$2 million represents value added, or an economic gain, to shareholders. So, \$2 million is the company's residual income for the period. The residual income approach attempts to match profits to the time period in which they are earned (but not necessarily realized as cash). In contrast to accounting net income (which has the same matching objective in principle), however, residual income attempts to measure the value added in excess of opportunity costs.

The residual income model states that a stock's value is book value per share plus the present value of expected future residual earnings. (**Book value per share** is common stockholders' equity divided by the number of common shares outstanding.) In contrast to the dividend and free cash flow models, the residual income model introduces a stock concept, book value per share, into the present value expression. Nevertheless, the residual income model can be viewed as a restatement of the dividend discount model, using a company-level return concept. Dividends are paid out of earnings and are related to earnings and book value through a simple expression.[12] The residual income model is a useful addition to an analyst's toolbox. Because the record of residual income can always be calculated, a residual income model can be used for both dividend-paying and non-dividend-paying stocks. Analysts may choose a residual income approach for companies with negative expected free cash flows within their comfortable forecast horizon. In such cases, a residual income valuation often brings the recognition of value closer to the present as compared with a free cash flow valuation, producing higher value estimates.

The residual income model has an attractive focus on profitability in relation to opportunity costs.[13] Knowledgeable application of the residual income model requires a detailed knowledge of accrual accounting; consequently, in cases for which the dividend discount model is suitable, analysts may prefer it as the simpler choice. Management sometimes exercises its discretion within allowable accounting practices to distort the accuracy of its financials as a reflection of economic performance. If the quality of accounting disclosure is good, the analyst may be able to calculate residual income by making appropriate adjustments (to reported net income and book value, in particular). In some cases, the degree of distortion and the quality of accounting disclosure can be such that the application of the residual income model is error-prone.

Generally, the definition of returns as residual income, and the residual income model, is most suitable when:

- the company is not paying dividends, as an alternative to a free cash flow model, or
- the company's expected free cash flows are negative within the analyst's comfortable forecast horizon.

[12] Book value of equity at $t$ = (Book value of equity at $t - 1$) + (Earnings over $t - 1$ to $t$) − (Dividends paid at $t$), as long as anything that goes through the balance sheet (affecting book value) first goes through the income statement (reflected in earnings), apart from ownership transactions. The condition that all changes in the book value of equity other than transactions with owners are reflected in income is known as **clean surplus accounting**. US and international accounting standards do not always follow clean surplus accounting; the analyst, therefore, in using this expression, must critically evaluate whether accounting-based results conform to clean surplus accounting and, if they do not, adjust them appropriately.

[13] Executive compensation schemes are sometimes based on a residual income concept, including branded variations such as Economic Value Added (EVA®) from Stern Stewart & Co.

In summary, the three most widely used definitions of returns to investors are dividends, free cash flow, and residual income. Although claims are often made that one cash flow definition is inherently superior to the rest—often following changing fashions in investment practice—a more flexible viewpoint is practical. The analyst may find that one model is more suitable to a particular valuation problem. The analyst may also develop more expertise in applying one type of model. In practice, skill in application—in particular, the quality of forecasts—is frequently decisive for the usefulness of the analyst's work.

In the next section, we present the general form of the dividend discount model as a prelude to discussing the particular implementations of the model that are suitable for different sets of attributes of the company being valued.

## 3. THE DIVIDEND DISCOUNT MODEL

Investment analysts use a wide range of models and techniques to estimate the value of common stock, including present value models. In Section 2.2, we discussed three common definitions of returns for use in present value analysis: dividends, free cash flow, and residual income. In this section, we develop the most general form of the dividend discount model.

The DDM is the simplest and oldest present value approach to valuing stock. In a survey of CFA Institute[14] members by Block (1999), 42 percent of respondents viewed the DDM as "very important" or "moderately important" for determining the value of individual stocks. Beginning in 1989, the *Merrill Lynch Institutional Factor Survey* has assessed the popularity of 23 valuation factors and methods among a group of institutional investors. The highest recorded usage level of the DDM was in the first survey in 1989, when more than 50 percent of respondents reported using the DDM. Since 1993, reported usage has been in the 25 to 40 percent range with usage increasing to over 35 percent in 2012. Besides its continuing significant position in practice, the DDM has an important place in both academic and practitioner equity research. The DDM is, for all these reasons, a basic tool in equity valuation.

### 3.1. The Expression for a Single Holding Period

From the perspective of a shareholder who buys and holds a share of stock, the cash flows he or she will obtain are the dividends paid on it and the market price of the share when he or she sells it. The future selling price should in turn reflect expectations about dividends subsequent to the sale. In this section, we will show how this argument leads to the most general form of the dividend discount model. In addition, the general expression developed for a finite holding period corresponds to one practical approach to DDM valuation; in that approach, the analyst forecasts dividends over a finite horizon, as well as the terminal sales price.

If an investor wishes to buy a share of stock and hold it for one year, the value of that share of stock today is the present value of the expected dividend to be received on the stock plus the present value of the expected selling price in one year:

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{P_1}{(1+r)^1} = \frac{D_1 + P_1}{(1+r)^1} \tag{2}$$

[14] Then called and referred to in the Block (1999) paper as the Association for Investment Management and Research. The name was changed to CFA Institute in 2004.

where

$V_0$ = the value of a share of stock today, at $t = 0$
$P_1$ = the expected price per share at $t = 1$
$D_1$ = the expected dividend per share for Year 1, assumed to be paid at the end of the year at $t = 1$
$r$ = the required rate of return on the stock

Equation 2 applies, to a single holding period, the principle that an asset's value is the present value of its future cash flows. In this case, the expected cash flows are the dividend in one year (for simplicity, assumed to be received as one payment at the end of the year)[15] and the price of the stock in one year.

## EXAMPLE 3 DDM Value with a Single Holding Period

Suppose that you expect Carrefour SA (NYSE Euronext Paris: CA) to pay a €0.58 dividend next year. You expect the price of CA stock to be €27.00 in one year. The required rate of return for CA stock is 9 percent. What is your estimate of the value of CA stock?

Discounting the expected dividend of €0.58 and the expected sales price of €27.00 at the required return on equity of 9 percent, we obtain

$$V_0 = \frac{D_1 + P_1}{(1+r)^1} = \frac{0.58 + 27.00}{(1+0.09)^1} = \frac{27.58}{1.09} = 25.30$$

### 3.2. The Expression for Multiple Holding Periods

If an investor plans to hold a stock for two years, the value of the stock is the present value of the expected dividend in Year 1, plus the present value of the expected dividend in Year 2, plus the present value of the expected selling price at the end of Year 2.

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{P_2}{(1+r)^2} = \frac{D_1}{(1+r)^1} + \frac{D_2 + P_2}{(1+r)^2} \tag{3}$$

The expression for the DDM value of a share of stock for any finite holding period is a straightforward extension of the expressions for one-year and two-year holding periods. For an $n$-period model, the value of a stock is the present value of the expected dividends for the $n$ periods plus the present value of the expected price in $n$ periods (at $t = n$).

$$V_0 = \frac{D_1}{(1+r)^1} + \cdots + \frac{D_n}{(1+r)^n} + \frac{P_n}{(1+r)^n} \tag{4}$$

[15] Throughout the discussion of the DDM, we assume that dividends for a period are paid in one sum at the end of the period.

If we use summation notation to represent the present value of the first $n$ expected dividends, the general expression for an $n$-period holding period or investment horizon can be written as

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \tag{5}$$

Equation 5 is significant in DDM application because analysts may make individual forecasts of dividends over some finite horizon (often two to five years) and then estimate the terminal price, $P_n$, based on one of a number of approaches. (We will discuss valuation using a finite forecasting horizon in Section 5.) Example 4 reviews the mechanics of this calculation.

## EXAMPLE 4 Finding the Stock Price for a Five-Year Forecast Horizon

For the next five years, the annual dividends of a stock are expected to be \$2.00, \$2.10, \$2.20, \$3.50, and \$3.75. In addition, the stock price is expected to be \$40.00 in five years. If the required return on equity is 10 percent, what is the value of this stock?

The present values of the expected future cash flows can be written out as

$$V_0 = \frac{2.00}{(1.10)^1} + \frac{2.10}{(1.10)^2} + \frac{2.20}{(1.10)^3} + \frac{3.50}{(1.10)^4} + \frac{3.75}{(1.10)^5} + \frac{40.00}{(1.10)^5}$$

Calculating and summing these present values gives a stock value of $V_0$ = 1.818 + 1.736 + 1.653 + 2.391 + 2.328 + 24.837 = \$34.76.

The five dividends have a total present value of \$9.926, and the terminal stock value has a present value of \$24.837, for a total stock value of \$34.76.

With a finite holding period, whether one, two, five, or some other number of years, the dividend discount model finds the value of stock as the sum of 1) the present values of the expected dividends during the holding period, and 2) the present value of the expected stock price at the end of the holding period. As the holding period is increased by one year, we have an extra expected dividend term. In the limit (i.e., if the holding period extends into the indefinite future), the stock's value is the present value of all expected future dividends.

$$V_0 = \frac{D_1}{(1+r)^1} + \ldots + \frac{D_n}{(1+r)^n} + \ldots \tag{6}$$

This value can be expressed with summation notation as

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \tag{7}$$

Equation 7 is the general form of the dividend discount model, first presented by John Burr Williams (1938). Even from the perspective of an investor with a finite investment horizon, the value of stock depends on all future dividends. For that investor, stock value today depends *directly* on the dividends the investor expects to receive before the stock is sold and *indirectly* on the expected dividends after the stock is sold, because those future dividends determine the expected selling price.

Equation 7, by expressing the value of stock as the present value of expected dividends into the indefinite future, presents a daunting forecasting challenge. In practice, of course, analysts cannot make detailed, individual forecasts of an infinite number of dividends. To use the DDM, the forecasting problem must be simplified. Two broad approaches exist, each of which has several variations:

1. Future dividends can be forecast by assigning the stream of future dividends to one of several stylized growth patterns. The most commonly used patterns are:
   - constant growth forever (the Gordon growth model);
   - two distinct stages of growth (the two-stage growth model and the H-model); and
   - three distinct stages of growth (the three-stage growth model).

   The DDM value of the stock is then found by discounting the dividend streams back to the present. We present the Gordon growth model in Section 4, and the two-stage, H-model, and three-stage growth models are presented in Section 5.
2. A finite number of dividends can be forecast individually up to a terminal point, by using pro forma financial statement analysis, for example. Typically, such forecasts extend from 3 to 10 years into the future. Although some analysts apply the same horizon to all companies under analysis, the horizon selected often depends on the perceived predictability (sometimes called the **visibility**) of the company's earnings. We can then forecast either:
   - the remaining dividends from the terminal point forward by assigning those dividends to a stylized growth pattern, or
   - the share price at the terminal point of our dividend forecasts (**terminal share price**), by using some method (such as taking a multiple of forecasted book value or earnings per share as of that point, based on one of several methods for estimating such multiples).

The stock's DDM value is then found by discounting the dividends (and forecasted price, if any) back to the present.

Spreadsheets are particularly convenient tools for implementing a DDM with individual dividend forecasts, but are useful in all cases. We address spreadsheet modeling in Section 5.

Whether analysts are using dividends or some other definition of cash flow, they generally use one of the above forecasting approaches when valuing stock. The challenge in practice is to choose an appropriate model for a stock's future dividends and to develop quality inputs to that model.

## 4. THE GORDON GROWTH MODEL

The Gordon growth model, developed by Gordon and Shapiro (1956) and Gordon (1962), assumes that dividends grow indefinitely at a constant rate. This assumption, applied to the general dividend discount model (Equation 7), leads to a simple and elegant valuation formula that has been influential in investment practice. This section explores the development of the Gordon growth model and illustrates its uses.

## 4.1. The Gordon Growth Model Equation

The simplest pattern that can be assumed in forecasting future dividends is growth at a constant rate. In mathematical terms, this assumption can be stated as

$$D_t = D_{t-1}(1+g)$$

where $g$ is the expected constant growth rate in dividends and $D_t$ is the expected dividend payable at time $t$. Suppose, for example, that the most recent dividend, $D_0$, was €10. Then, if a 5 percent dividend growth rate is forecast, the expected dividend at $t = 1$ is $D_1 = D_0(1 + g) =$ €10 × 1.05 = €10.5. For any time $t$, $D_t$ also equals the $t = 0$ dividend, compounded at $g$ for $t$ periods:

$$D_t = D_0(1+g)^t \tag{8}$$

To continue the example, at the end of five years the expected dividend is $D_5 = D_0(1 + g)^5 =$ €10 × $(1.05)^5$ = €10 × 1.276282 = €12.76. If $D_0(1 + g)^t$ is substituted into Equation 7 for $D_t$, it gives the Gordon growth model. If all of the terms are written out, they are

$$V_0 = \frac{D_0(1+g)}{(1+r)} + \frac{D_0(1+g)^2}{(1+r)^2} + \ldots + \frac{D_0(1+g)^n}{(1+r)^n} + \ldots \tag{9}$$

Equation 9 is a geometric series; that is, each term in the expression is equal to the previous term times a constant, which in this case is $(1 + g)/(1 + r)$. This equation can be simplified algebraically into a much more compact equation:[16]

$$V_0 = \frac{D_0(1+g)}{r-g}, \text{ or } V_0 = \frac{D_1}{r-g} \tag{10}$$

Both equations are equivalent because $D_1 = D_0(1 + g)$. In Equation 10, it must be specified that the required return on equity must be greater than the expected growth rate: $r > g$. If $r = g$ or $r < g$, Equation 10 as a compact formula for value assuming constant growth is not valid. If $r = g$, dividends grow at the same rate at which they are discounted, so the value of the stock (as the undiscounted sum of all expected future dividends) is infinite. If $r < g$, dividends grow faster than they are discounted, so the value of the stock is infinite. Of course, infinite values do not make economic sense; so constant growth with $r = g$ or $r < g$ does not make sense.

To illustrate the calculation, suppose that an annual dividend of €5 has just been paid ($D_0$ = €5). The expected long-term growth rate is 5 percent, and the required return on equity is 8 percent. The Gordon growth model value per share is $D_0(1 + g)/(r - g)$ = (€5 × 1.05)/(0.08 − 0.05) = €5.25/0.03 = €175. When calculating the model value, be careful to use $D_1$ and not $D_0$ in the numerator.

The Gordon growth model (Equation 10) is one of the most widely recognized equations in the field of security analysis. Because the model is based on indefinitely extending future

[16]The simplification involves the expression for the sum of an infinite geometric progression with the first term equal to $a$ and the growth factor equal to $m$ with $|m| < 1$ [i.e., the sum of $a + am + am^2 + \ldots$ is $a/(1 - m)$]. Setting $a = D_1/(1 + r)$ and $m = (1 + g)/(1 + r)$, gives the Gordon growth model.

dividends, the model's required rate of return and growth rate should reflect long-term expectations. Further, model values are very sensitive to both the required rate of return, *r*, and the expected dividend growth rate, *g*. In this model and other valuation models, it is helpful to perform a sensitivity analysis on the inputs, particularly when an analyst is not confident about the proper values.

Earlier we stated that analysts typically apply DDMs to dividend-paying stocks when dividends bear an understandable and consistent relation to the company's profitability. The same qualifications hold for the Gordon growth model. In addition, the Gordon growth model form of the DDM is most appropriate for companies with earnings expected to grow at a rate comparable to or lower than the economy's nominal growth rate. Businesses growing at much higher rates than the economy often grow at lower rates in maturity, and the horizon in using the Gordon growth model is the entire future stream of dividends.

To determine whether the company's growth rate qualifies it as a candidate for the Gordon growth model, an estimate of the economy's nominal growth rate is needed. This growth rate is usually measured by the growth in **gross domestic product** (GDP). (GDP is a money measure of the goods and services produced within a country's borders.) National government agencies as well as the World Bank (www.worldbank.org) publish GDP data, which are also available from several secondary sources. Exhibit 2 shows the recent real GDP growth record for a number of major developed markets.

EXHIBIT 2 Average Annual Real GDP Growth Rates: 1983–2012 (in Percent)

| | Time Period | | |
|---|---|---|---|
| Country | 1983–1992 | 1993–2002 | 2003–12 |
| Australia | 3.4% | 3.8% | 2.4% |
| Canada | 2.7 | 3.5 | 1.9 |
| Denmark | 2.1 | 2.4 | 0.6 |
| France | 2.3 | 2.0 | 1.1 |
| Germany | 3.0 | 1.4 | 1.2 |
| Italy | 2.5 | 1.6 | 0.0 |
| Japan | 4.3 | 0.8 | 0.9 |
| Netherlands | 2.9 | 3.0 | 1.1 |
| Sweden | 1.9 | 2.7 | 2.3 |
| Switzerland | 2.1 | 1.3 | 1.9 |
| United Kingdom | 2.6 | 3.4 | 1.4 |
| United States | 3.5 | 3.4 | 1.7 |

*Source:* OECD.

Based on historical and/or forward-looking information, nominal GDP growth can be estimated as the sum of the estimated real growth rate in GDP plus the expected long-run inflation rate. For example, an estimate of the underlying real growth rate of the Canadian economy is 1.2 percent as of early 2013. By using the Bank of Canada's inflation target of 2 percent as the expected inflation rate, an estimate of the Canadian economy's nominal annual growth rate is 1.2 percent + 2 percent = 3.2 percent. Publicly traded companies constitute varying amounts of the total corporate sector, but always less than 100 percent. As a result, the overall growth rate of the public corporate sector can diverge from the nominal GDP growth rate during a long horizon; furthermore, within the public corporate sector, some subsectors may experience persistent growth rate differentials. Nevertheless, an earnings growth rate far above the nominal GDP growth rate is not sustainable in perpetuity.

When forecasting an earnings growth rate far above the economy's nominal growth rate, analysts should use a multistage DDM in which the final-stage growth rate reflects a growth rate that is more plausible relative to the economy's nominal growth rate, rather than using the Gordon growth model.

## EXAMPLE 5 Valuation Using the Gordon Growth Model (1)

Joel Williams follows Sonoco Products Company (NYSE: SON), a manufacturer of paper and plastic packaging for both consumer and industrial use. SON appears to have a dividend policy of recognizing sustainable increases in the level of earnings with increases in dividends, keeping the dividend payout ratio within a range of 40 percent to 60 percent. Williams also notes:

- SON's most recent quarterly dividend (ex-dividend date: 14 August 2013) was \$0.31, consistent with a current annual dividend of $4 \times \$0.31 = \$1.24$ per year.
- A forecasted dividend growth rate of 4.0 percent per year.
- With a beta ($\beta_i$) of 0.95, given an equity risk premium (expected excess return of equities over the risk-free rate, $E(R_M) - R_F$) of 4.5 percent and a risk-free rate ($R_F$) of 3 percent, SON's required return on equity is $r = R_F + \beta_i[E(R_M) - R_F] = 3.0 + 0.95(4.5) = 7.3$ percent, using the capital asset pricing model (CAPM).

Williams believes the Gordon growth model may be an appropriate model for valuing SON.

1. Calculate the Gordon growth model value for SON stock.
2. The current market price of SON stock is \$38.10. Using your answer to Question 1, judge whether SON stock is fairly valued, undervalued, or overvalued.

*Solution to 1:* Using Equation 10,

$$V_0 = \frac{D_0(1+g)}{r-g} = \frac{\$1.24 \times 1.04}{0.073 - 0.04} = \frac{\$1.2896}{0.033} = \$39.08$$

*Solution to 2:* The market price of \$38.10 is \$0.98 or approximately 2.5 percent less than the Gordon growth model intrinsic value estimate of \$39.08. SON appears to be slightly undervalued, based on the Gordon growth model estimate.

The next example illustrates a Gordon growth model valuation introducing some problems the analyst might face in practice. The example refers to adjusted beta; the most common calculation adjusts raw historical beta toward the overall mean value of one for beta.

## EXAMPLE 6 Valuation Using the Gordon Growth Model (2)

As an analyst for a US domestic equity–income mutual fund, Roberta Kim is evaluating Middlesex Water Company (NASDAQ: MSEX), a publicly traded water utility, for possible inclusion in the approved list of investments. Kim is conducting the analysis in mid-2013.

Not all countries have traded water utility stocks. In the United States, about 85 percent of the population gets its water from government entities. A group of investor-owned water utilities, however, also supplies water to the public. With a market capitalization of about $327 million as of mid-2013, MSEX is among the ten largest publicly traded US water utilities. MSEX's historical base is the Middlesex System, serving residential, industrial, and commercial customers in a well-developed area of central New Jersey. Through various subsidiaries, MSEX also provides water and wastewater collection and treatment services to areas of southern New Jersey and Delaware.

Hampered by a decline in earnings during the recent recession, net income growth during the past five years has been somewhat less than 2 percent. During the last five years, MSEX's return on equity averaged 7.8 percent with relatively little variation, and its profit margins are above industry averages. Because MSEX obtains most of its revenue from the regulated business providing an important staple, water, to a relatively stable population, Kim feels confident in forecasting future earnings and dividend growth. MSEX appears to have a policy of small annual increases in the dividend rate, maintaining an average dividend payout ratio of approximately 80 percent. Other facts and forecasts include the following:

- MSEX's per-share dividends for 2012 ($D_0$) were $0.74.
- Kim forecasts a long-term earnings growth rate of 3.5 percent per year, somewhat above the 2.7 percent consensus 3–5-year earnings growth rate forecast reported by Zacks Investment Research (based on two analysts).
- MSEX's raw beta and adjusted beta are, respectively, 0.70 and 0.80 based on 60 monthly returns. The $R^2$ associated with beta, however, is under 20 percent.
- Kim estimates that MSEX's pretax cost of debt is 5.6 percent based on Standard & Poor's issuer rating for MSEX of A– and the current corporate yield curve.
- Kim's estimate of MSEX's required return on equity is 7.00 percent.
- MSEX's current market price is $20.50.

1. Calculate the Gordon growth model estimate of value for MSEX using Kim's required return on equity estimate.
2. State whether MSEX appears to be overvalued, fairly valued, or undervalued based on the Gordon growth model estimate of value.
3. Justify the selection of the Gordon growth model for valuing MSEX.
4. Calculate the CAPM estimate of the required return on equity for MSEX under the assumption that beta regresses to the mean. (Assume an equity risk premium of 4.5 percent and a risk-free rate of 3 percent as of the price quotation date.)
5. Calculate the Gordon growth estimate of value using A) the required return on equity from your answer to Question 4, and B) a bond-yield-plus-risk-premium approach with a risk premium of 2.5 percent.
6. Evaluate the effect of uncertainty in MSEX's required return on equity on the valuation conclusion in Question 2.

*Solution to 1:* From Equation 10,

$$V_0 = \frac{D_0(1+g)}{r-g} = \frac{\$0.74(1.035)}{0.07-0.035} = \$21.88$$

*Solution to 2:* Because the Gordon growth model estimate of \$21.88 is \$1.38 or about 6.7 percent higher than the market price of \$20.50, MSEX appears to be undervalued.

*Solution to 3:* The Gordon growth model, which assumes that dividends grow at a stable rate in perpetuity, is a realistic model for MSEX for the following reasons:

- MSEX profitability is stable as reflected in its return on equity. This stability reflects predictable demand and regulated prices for its product, water.
- Dividends bear an understandable and consistent relationship to earnings, as evidenced by the company's policy of annual increases and predictable dividend payout ratios.
- Historical earnings growth, at 2.5 percent a year, is somewhat below the long-term nominal annual GDP growth for the United States (3.2 percent for 1947–2013, according to the US Bureau of Economic Analysis).
- Forecasted earnings growth of 3.5 percent seems attainable, given a plausible forecast for nominal GDP growth, and does not include a period of forecasted very high or very low growth.

*Solution to 4:* The assumption of regression to the mean is characteristic of adjusted historical beta. The required return on equity as given by the CAPM is 3 percent + 0.80(4.5 percent) = 6.6 percent using adjusted beta, which assumes reversion to the mean of 1.0.

*Solution to 5:*

A. The Gordon growth value of MSEX using a required return on equity of 6.6 percent is

$$V_0 = \frac{D_0(1+g)}{r-g} = \frac{\$0.74(1.035)}{0.066-0.035} = \$24.71$$

B. The bond-yield-plus-risk-premium estimate of the required return on equity is 5.6 percent + 2.5 percent = 8.1 percent.

$$V_0 = \frac{D_0(1+g)}{r-g} = \frac{\$0.74(1.035)}{0.081-0.035} = \$16.65$$

*Solution to 6:* Using the CAPM estimate of the required return on equity (Question 5A), MSEX appears to be definitely undervalued. Beta explains less than 20 percent of the variation in MSEX's returns, however, according to the fact given concerning $R^2$. Using a bond-yield-plus-risk-premium approach, MSEX appears to be overvalued (\$16.65 is less than the market price of \$20.50 by more than 18 percent). No specific evidence, however, supports the particular value of the risk premium selected in the bond-yield-plus-risk-premium approach. In this case, because of the uncertainty in the required return on equity estimate, one has less confidence that MSEX is overvalued. Given the results of the other two approaches, the analyst may view MSEX as undervalued.

As mentioned earlier, an analyst needs to be aware that Gordon growth model values can be very sensitive to small changes in the values of the required rate of return and expected dividend growth rate. Example 7 illustrates a format for a sensitivity analysis.

## EXAMPLE 7 Valuation Using the Gordon Growth Model (3)

In Example 6, the Gordon growth model value for MSEX was estimated as \$21.88 based on a current dividend of \$0.74, an expected dividend growth rate of 3.5 percent, and a required return on equity of 7.00 percent. What if the estimates of $r$ and $g$ can each vary by 25 basis points? How sensitive is the model value to changes in the estimates of $r$ and $g$? Exhibit 3 provides information on this sensitivity.

EXHIBIT 3 Estimated Price Given Uncertain Inputs

| | $g = 3.25\%$ | $g = 3.50\%$ | $g = 3.75\%$ |
|---|---|---|---|
| $r = 6.75\%$ | \$21.83 | \$23.57 | \$25.59 |
| $r =$ **7.00%** | \$20.37 | **\$21.88** | \$23.62 |
| $r = 7.25\%$ | \$19.10 | \$20.42 | \$21.94 |

A point of interest following from the mathematics of the Gordon growth model is that when the spread between $r$ and $g$ is the widest ($r = 7.25$ percent and $g = 3.25$ percent), the Gordon growth model value is the smallest (\$19.10), and when the spread is the narrowest ($r = 6.75$ percent and $g = 3.75$ percent), the model value is the largest (\$25.59). As the spread goes to zero, in fact, the model value increases without bound. The largest value in Exhibit 3, \$25.59, is almost 34 percent larger than the smallest value, \$19.10. Two-thirds of the values in Exhibit 3 exceed MSEX's current market price of \$20.50, tending to support the conclusion that MSEX is undervalued. In summary, the best estimate of the value of MSEX given the assumptions is \$21.88, bolded in Exhibit 3, but the estimate is quite sensitive to rather small changes in inputs.

Examples 6 and 7 illustrate the application of the Gordon growth model to a utility, a traditional source for such illustrations because of the stability afforded by providing an essential service in a regulated environment. Before applying any valuation model, however, analysts need to know much more about a company than industry membership. For example, as of mid-2013, another water utility, Aqua America Inc. (NYSE: WTR), was expected to grow at 6.4 percent for the next five years as a result of an aggressive growth-by-acquisition strategy. Furthermore, many utility holding companies in the United States have major, nonregulated business subsidiaries, so the traditional picture of steady and slow growth often does not hold.

In addition to individual stocks, analysts have often used the Gordon growth model to value broad equity market indices, especially in developed markets. Because the value of publicly traded issues typically represents a large fraction of the overall corporate sector in developed markets, such indices reflect average economic growth rates. Furthermore, in such economies, a sustainable trend value of growth may be identifiable.

The Gordon growth model can also be used to value the noncallable form of a traditional type of preferred stock, **fixed-rate perpetual preferred stock** (stock with a specified dividend rate that has a claim on earnings senior to the claim of common stock, and no maturity date).

Perpetual preferred stock has been used particularly by financial institutions such as banks to obtain permanent equity capital while diluting the interests of common equity. Generally, such issues have been callable by the issuer after a certain period, so valuation must take account of the issuer's call option. Valuation of the noncallable form, however, is straightforward.

If the dividend on such preferred stock is $D$, because payments extend into the indefinite future a **perpetuity** (a stream of level payments extending to infinity) exists in the constant amount of $D$. With $g = 0$, which is true because dividends are fixed for such preferred stock, the Gordon growth model becomes

$$V_0 = \frac{D}{r} \tag{11}$$

The discount rate, $r$, capitalizes the amount $D$, and for that reason is often called a **capitalization rate** in this expression and any other expression for the value of a perpetuity.

## EXAMPLE 8 Valuing Noncallable Fixed-Rate Perpetual Preferred Stock

Kansas City Southern Preferred 4% (NYSE: KSU-P), issued 2 January 1963, has a par value of \$25 per share. Thus, a share pays 0.04(\$25) = \$1.00 in annual dividends. The required return on this security is estimated at 5.5 percent. Estimate the value of this issue.

*Solution:* According to the model in Equation 11, KSU-P preferred stock is worth $D/r = 1.00/0.055 = \$18.18$.

A perpetual preferred stock has a level dividend, thus a dividend growth rate of zero. Another case is a declining dividend—a negative growth rate. The Gordon growth model also accommodates this possibility, as illustrated in Example 9.

## EXAMPLE 9 Gordon Growth Model with Negative Growth

Afton Mines is a profitable company that is expected to pay a \$4.25 dividend next year. Because it is depleting its mining properties, the best estimate is that dividends will decline forever at a rate of 4 percent. The required rate of return on Afton stock is 9 percent. What is the value of Afton shares?

*Solution:* For Afton, the value of the stock is

$$\begin{aligned} V_0 &= \frac{4.25}{[0.09 - (-0.04)]} \\ &= \frac{4.25}{0.13} = \$32.69 \end{aligned}$$

The negative growth results in a \$32.69 valuation for the stock.

### 4.2. The Links Among Dividend Growth, Earnings Growth, and Value Appreciation in the Gordon Growth Model

The Gordon growth model implies a set of relationships for the growth rates of dividends, earnings, and stock value. With dividends growing at a constant rate $g$, stock value also grows at $g$ as well. The current stock value is $V_0 = D_1/(r - g)$. Multiplying both sides by $(1 + g)$ gives $V_0(1 + g) = D_1(1 + g)/(r - g)$, which is $V_1 = D_2/(r - g)$. So, both dividends and value have grown at a rate of $g$ (holding $r$ constant).[17] Given a constant payout ratio—a constant, proportional relationship between earnings and dividends—dividends and earnings grow at $g$.

To summarize, $g$ in the Gordon growth model is the rate of value or capital appreciation (sometimes also called the capital gains yield). Some textbooks state that $g$ is the rate of price appreciation. If prices are efficient (price equals value), price is indeed expected to grow at a rate of $g$. If there is mispricing (price is different from value), however, the actual rate of capital appreciation depends on the nature of the mispricing and how fast it is corrected, if at all. This topic is discussed in the reading on return concepts.

Another characteristic of the constant growth model is that the components of total return (dividend yield and capital gains yield) will also stay constant through time, given that price tracks value exactly. The dividend yield, which is $D_1/P_0$ at $t = 0$, will stay unchanged because both the dividend and the price are expected to grow at the same rate, leaving the dividend yield unchanged through time. For example, consider a stock selling for €50.00 with a **forward dividend yield** (a dividend yield based on the anticipated dividend during the next 12 months) of 2 percent based on an expected dividend of €1. The estimate of $g$ is 5.50 percent per year. The dividend yield of 2 percent, the capital gains yield of 5.50 percent, and the total return of 7.50 percent are expected to be the same at $t = 0$ and at any future point in time.

### 4.3. Share Repurchases

An issue of increasing importance in many developed markets is share repurchases. Companies can distribute free cash flow to shareholders in the form of share repurchases (also called buybacks) as well as dividends. In the United States currently, more than half of dividend-paying companies also make regular share repurchases.[18] Clearly, analysts using DDMs need to understand share repurchases. Share repurchases and cash dividends have several distinctive features:

- Share repurchases involve a reduction in the number of shares outstanding, all else equal. Selling shareholders see their relative ownership position reduced compared to nonselling shareholders.
- Whereas many corporations with established cash dividends are reluctant to reduce or omit cash dividends, corporations generally do not view themselves as committed to maintain share repurchases at any specified level.

[17] More formally, the fact that the value grows at a rate equal to $g$ is demonstrated as follows:

$$\frac{V_{t+1}-V_t}{V_t} = \frac{D_{t+2}/(r-g)-D_{t+1}/(r-g)}{D_{t+1}/(r-g)} == \frac{D_{t+2}-D_{t+1}}{D_{t+1}} = 1 + g - 1 = g$$

[18] See Skinner (2008), who also finds evidence that this group of companies increasingly has tended to distribute earnings increases via share repurchases rather than cash dividends.

- Cash dividends tend to be more predictable in money terms and more predictable as to timing.[19] Although evidence from the United States suggests that, for companies with active repurchase programs, the amount of repurchases during two-year intervals bears a relationship to earnings, companies appear to be opportunistic in timing exactly when to repurchase.[20] Thus, share repurchases are generally harder to forecast than the cash dividends of companies with an identifiable dividend policy.
- As a baseline case, share repurchases are neutral in their effect on the wealth of ongoing shareholders if the repurchases are accomplished at market prices.

The analyst could account for share repurchases directly by forecasting the total earnings, total distributions to shareholders (via either cash dividends or share repurchases), and shares outstanding. Experience and familiarity with such models is much less than for DDMs. Focusing on cash dividends, however, DDMs supply accurate valuations consistent with such an approach if the analyst takes account of the effect of expected repurchases on the per-share growth rates of dividends. Correctly applied, the DDM is a valid approach to common stock valuation even when the company being analyzed engages in share repurchases.

### 4.4. The Implied Dividend Growth Rate

Because the dividend growth rate affects the estimated value of a stock using the Gordon growth model, differences between estimated values of a stock and its actual market value might be explained by different growth rate assumptions. Given price, the expected next-period dividend, and an estimate of the required rate of return, the dividend growth rate reflected in price can be inferred assuming the Gordon growth model. (Actually, it is possible to infer the market-price-implied dividend growth based on other DDMs as well.) An analyst can then judge whether the implied dividend growth rate is reasonable, high, or low, based on what he or she knows about the company. In effect, the calculation of the implied dividend growth rate provides an alternative perspective on the valuation of the stock (fairly valued, overvalued, or undervalued). Example 10 shows how the Gordon growth model can be used to infer the market's implied growth rate for a stock.

### EXAMPLE 10 The Growth Rate Implied by the Current Stock Price

Suppose a company has a beta of 1.1. The risk-free rate is 5.6 percent, and the equity risk premium is 6 percent. The current dividend of $2.00 is expected to grow at 5 percent indefinitely. The price of the stock is $40.

1. Estimate the value of the company's stock.
2. Determine the constant dividend growth rate that would be required to justify the market price of $40.

[19] As discussed by Wanger (2007).
[20] See Skinner (2008).

*Solution to 1:* The required rate of return is 5.6 percent + 1.1(6 percent) = 12.2 percent. The value of one share, using the Gordon growth model, is

$$V_0 = \frac{D_0(1+g)}{r-g}$$
$$= \frac{2.00(1.05)}{0.122-0.05}$$
$$= \frac{2.10}{0.072} = \$29.17$$

*Solution to 2:* The valuation estimate of the model ($29.17) is less than the market value of $40.00; thus, the market price must be forecasting a growth rate above the assumed 5 percent. Assuming that the model and the required return assumption are appropriate, the growth rate in dividends required to justify the $40 stock price can be calculated by substituting all known values into the Gordon growth model equation except for *g*:

$$40 = \frac{2.00(1+g)}{0.122-g}$$
$$4.88 - 40g = 2 + 2g$$
$$42g = 2.88$$
$$g = 0.0686$$

An expected dividend growth rate of 6.86 percent is required for the stock price to be correctly valued at the market price of $40.

## 4.5. The Present Value of Growth Opportunities

The value of a stock can be analyzed as the sum of 1) the value of the company without earnings reinvestment, and 2) the **present value of growth opportunities** (PVGO). PVGO, also known as the **value of growth**, sums the expected value today of opportunities to profitably reinvest future earnings.[21] In this section, we illustrate this decomposition and discuss how it may be interpreted to gain insight into the market's view of a company's business and prospects.

Earnings growth may increase, leave unchanged, or reduce shareholder wealth depending on whether the growth results from earning returns in excess of, equal to, or less than the opportunity cost of funds. Consider a company with a required return on equity of 10 percent that has earned €1 per share. The company is deciding whether to pay out current earnings as a dividend or to reinvest them at 10 percent and distribute the ending value as a dividend in one year. If it reinvests, the present value of investment is €1.10/1.10 = €1.00, equaling its cost, so the decision to reinvest has a net present value (NPV) of zero. If the company were able to earn more than 10 percent by exploiting a profitable growth opportunity, reinvesting would have a

[21] More technically, PVGO can be defined as the forecasted total net present value of future projects. See Brealey, Myers, and Allen (2006), p. 259.

positive NPV, increasing shareholder wealth. Suppose the company could reinvest earnings at 25 percent for one year: The per-share NPV of the growth opportunity would be €1.25/1.10 – €1 ≈ €0.14. Note that any reinvestment at a positive rate below 10 percent, although increasing EPS, is not in shareholders' interests. Increases in shareholder wealth occur only when reinvested earnings earn more than the opportunity cost of funds (i.e., investments are in positive net present value projects).[22] Thus, investors actively assess whether and to what degree companies will have opportunities to invest in profitable projects. In principle, companies without prospects for investing in positive NPV projects should distribute most or all earnings to shareholders as dividends, so the shareholders can redirect capital to more attractive areas.

A company without positive expected NPV projects is defined as a **no-growth company** (a term for a company without opportunities for *profitable* growth). Such companies should distribute all their earnings in dividends because earnings cannot be reinvested profitably and earnings will be flat in perpetuity, assuming a constant return on equity (ROE). This flatness occurs because earnings equal ROE × Equity, and equity is constant because retained earnings are not added to it. $E_1$ is $t = 1$ earnings, which is the constant level of earnings or the average earnings of a no-growth company if return on equity is viewed as varying about its average level. The **no-growth value per share** is defined as $E_1/r$, which is the present value of a perpetuity in the amount of $E_1$ where the capitalization rate, $r$, is the required rate of return on the company's equity. $E_1/r$ can also be interpreted as the per-share value of assets in place because of the assumption that the company is making no new investments because none are profitable. For any company, the actual value per share is the sum of the no-growth value per share and the present value of growth opportunities (PVGO):

$$V_0 = \frac{E_1}{r} + \text{PVGO} \tag{12}$$

If prices reflect value ($P_0 = V_0$), $P_0$ less $E_1/r$ gives the market's estimate of the company's value of growth, PVGO. Referring back to Example 6, suppose that MSEX is expected to have average EPS of \$0.79 if it distributed all earnings as dividends. Its required return of 9.25 percent and a current price of \$18.39 gives

$$\begin{aligned} \$18.39 &= (\$0.79/0.0925) + \text{PVGO} \\ &= \$8.54 + \text{PVGO} \end{aligned}$$

where PVGO = \$18.39 – \$8.54 = \$9.85. So, 54 percent (\$9.85/\$18.39 = 0.54) of the company's value, as reflected in the market price, is attributable to the value of growth.

Exhibit 4, based on data from early August 2013, illustrates that the value of growth represented about 44 percent of the market value of technology company Google and a much smaller percentage of McDonald's value and Macy's value. The low value for McDonald's PVGO could be explained in several ways. The value could reflect increased competition in the fast food business, commodity cost pressures, and/or unfavorable foreign exchange (foreign operations contribute over 65 percent of revenues); it could reflect that the company has

[22] We can interpret this condition of profitability as ROE > $r$ with ROE calculated with the *market* value of equity (rather than the book value of equity) in the denominator. Book value based on historical cost accounting can present a distorted picture of the value of shareholders' investment in the company. The condition that ROE = $r$ would be consistent with an equilibrium in which investment opportunities were such that a company could just earn its opportunity cost of capital.

a much higher payout ratio than Google or Macy's (53 percent in 2012 versus zero and 23 percent for Google and Macy's) and, therefore, future growth is expected to be slower; or it might indicate that the estimated no-growth value per share was too high because the earnings estimate was too high and/or the required return on equity estimate was too low.

EXHIBIT 4 Estimated PVGO as a Percentage of Price

| Company | β | $r$ | $E_1$ | Price | $E_1/r$ | PVGO | PVGO/Price |
|---|---|---|---|---|---|---|---|
| Google, Inc. | 0.90 | 7.1% | \$35.80 | \$896.57 | \$504.23 | \$392.34 | 43.8% |
| McDonald's Corp | 0.60 | 5.7 | 5.70 | 102.14 | 100.00 | 2.14 | 2.1 |
| Macy's Inc. | 1.35 | 9.1 | 4.00 | 48.79 | 43.96 | 4.83 | 9.9 |

*Source:* Value Line Investment Survey for beta, earnings estimate, and price of each.

*Note:* The required rate of return is estimated using the CAPM with the following inputs: the beta from the Value Line Investment Survey, 3.0 percent (20-year US T-bond rate) for the risk-free rate of return, and 4.5 percent for the equity risk premium.

What determines PVGO? One determinant is the value of a company's options to invest, captured by the word "opportunities." In addition, the flexibility to adapt investments to new circumstances and information is valuable. Thus, a second determinant of PVGO is the value of the company's options to time the start, adjust the scale, or even abandon future projects. This element is the value of the company's **real options** (options to modify projects, in this context). Companies that have good business opportunities and/or a high level of managerial flexibility in responding to changes in the marketplace should tend to have higher values of PVGO than companies that do not have such advantages. This perspective on what contributes to PVGO can provide additional understanding of the results in Exhibit 4.

As an additional aid to an analyst, Equation 12 can be restated in terms of the familiar P/E ratio based on forecasted earnings:

$$\frac{V_0}{E_1} \text{ or } \frac{P_0}{E_1} \text{ or P/E} = \frac{1}{r} + \frac{\text{PVGO}}{E_1} \tag{13}$$

The first term, $1/r$, is the value of the P/E for a no-growth company. The second term is the component of the P/E value that relates to growth opportunities. For MSEX, the P/E is \$18.39/\$0.79 = 23.3. The no-growth P/E is 1/0.0925 = 10.8 and is the multiple the company should sell at if it has no growth opportunities. The growth component of \$9.85/\$0.79 = 12.5 reflects anticipated growth opportunities. Leibowitz and Kogelman (1990) and Leibowitz (1997) have provided elaborate analyses of the drivers of the growth component of P/E as a franchise-value approach.

As analysts, the distinction between no-growth and growth values is of interest because the value of growth and the value of assets in place generally have different risk characteristics (as the interpretation of PVGO as incorporating the real options suggests).

## 4.6. Gordon Growth Model and the Price-to-Earnings Ratio

The price-to-earnings ratio (P/E) is perhaps the most widely recognized valuation indicator, familiar to readers of newspaper financial tables and institutional research reports. Using the

Gordon growth model, an expression for P/E in terms of the fundamentals can be developed. This expression has two uses:

1. When used with forecasts of the inputs to the model, the analyst obtains a **justified (fundamental) P/E**—the P/E that is fair, warranted, or justified on the basis of fundamentals (given that the valuation model is appropriate). The analyst can then state his or her view of value in terms not of the Gordon growth model value but of the justified P/E. Because P/E is so widely recognized, this method may be an effective way to communicate the analysis.
2. The analyst may also use the expression for P/E to weigh whether the forecasts of earnings growth built into the current stock price are reasonable. What expected earnings growth rate is implied by the actual market P/E? Is that growth rate plausible?

The expression for P/E can be stated in terms of the current (or trailing) P/E (today's market price per share divided by trailing 12 months' earnings per share) or in terms of the leading (or forward) P/E (today's market price per share divided by a forecast of the next 12 months' earnings per share, or sometimes the next fiscal year's earnings per share).

Leading and trailing justified P/E expressions can be developed from the Gordon growth model. Assuming that the model can be applied for a particular stock's valuation, the dividend payout ratio is considered fixed. Define $b$ as the retention rate, the fraction of earnings reinvested in the company rather than paid out in dividends. The dividend payout ratio is then, by definition, $(1 - b)$ = Dividend per share/Earnings per share = $D_t / E_t$. If $P_0 = D_1/(r - g)$ is divided by next year's earnings per share, $E_1$, we have

$$\frac{P_0}{E_1} = \frac{D_1/E_1}{r-g} = \frac{1-b}{r-g} \tag{14}$$

This represents a leading P/E, which is current price divided by next year's earnings. Alternatively, if $P_0 = D_0(1 + g)/(r - g)$ is divided by the current-year's earnings per share, $E_0$, the result is

$$\frac{P_0}{E_0} = \frac{D_0(1+g)/E_0}{r-g} = \frac{(1-b)(1+g)}{r-g} \tag{15}$$

This expression is for trailing P/E, which is current price divided by trailing (current year) earnings.

## EXAMPLE 11 The Justified P/E Based on the Gordon Growth Model

Harry Trice wants to use the Gordon growth model to find a justified P/E for the French company Carrefour SA (NYSE Euronext: CA), a global food retailer specializing in hypermarkets and supermarkets. Trice has assembled the following information:

- Current stock price = €23.84.
- Trailing annual earnings per share = €1.81
- Current level of annual dividends = €0.58

- Dividend growth rate = 3.5 percent
- Risk-free rate = 2.8 percent
- Equity risk premium = 4.00 percent
- Beta versus the CAC index = 0.80

1. Calculate the justified trailing and leading P/Es based on the Gordon growth model.
2. Based on the justified trailing P/E and the actual P/E, judge whether CA is fairly valued, overvalued, or undervalued.

*Solution to 1:* For CA, the required rate of return using the CAPM is

$$\begin{aligned} r_i &= 2.80\% + 0.80(4.00\%) \\ &= 6.0\% \end{aligned}$$

The dividend payout ratio is

$$\begin{aligned} (1 - b) &= D_0/E_0 \\ &= 0.58/1.81 \\ &= 0.32 \end{aligned}$$

The justified leading P/E (based on next year's earnings) is

$$\frac{P_0}{E_1} = \frac{1-b}{r-g} = \frac{0.32}{0.06-0.035} = 12.8$$

The justified trailing P/E (based on trailing earnings) is

$$\frac{P_0}{E_0} = \frac{(1-b)(1+g)}{r-g} = \frac{0.32(1.035)}{0.06-0.035} = 13.2$$

*Solution to 2:* Based on a current price of €23.84 and trailing earnings of €1.81, the trailing P/E is €23.84/€1.81 = 13.2. Because the actual P/E of 13.2 is the same as the justified trailing P/E of 13.2 (to one decimal place), the conclusion is that CA appears to be fairly valued. The result can also be expressed in terms of price using the Gordon growth model. Using Trice's assumptions, the Gordon growth model assigns a value of 0.58(1.035)/(0.06 − 0.035) = €24.01, which is about the same as the current market value of €23.84.

Later in the reading, we will present multistage DDMs. Expressions for the P/E can be developed in terms of the variables of multistage DDMs, but the usefulness of these expressions is not commensurate with their complexity. For multistage models, the simple way to calculate a justified leading P/E is to divide the model value directly by the first year's expected earnings. In all cases, the P/E is explained in terms of the required return on equity, expected dividend growth rate(s), and the dividend payout ratio(s). All else being equal, higher prices are associated with higher anticipated dividend growth rates.

### 4.7. Estimating a Required Return Using the Gordon Growth Model

Under the assumption of efficient prices, the Gordon growth model has been used to estimate a stock's required rate of return, or equivalently, the market-price-implied expected return. The Gordon growth model solved for $r$ is

$$r = \frac{D_0(1+g)}{P_0} + g = \frac{D_1}{P_0} + g \tag{16}$$

As explained in the reading on return concepts, $r$ in Equation 16 is technically an internal rate of return (IRR). The rate $r$ is composed of two parts; the dividend yield ($D_1/P_0$) and the capital gains (or appreciation) yield ($g$).

#### EXAMPLE 12 Finding the Expected Rate of Return with the Gordon Growth Model

Bob Inguigiatto, CFA, has been given the task of developing mean return estimates for a list of stocks as preparation for a portfolio optimization. On his list is NextEra Energy, Inc. (NYSE: NEE), formerly FPL Group, Inc. On analysis, he decides that it is appropriate to model NEE using the Gordon growth model, and he takes prices as reflecting value. The company paid dividends of \$2.40 in 2012, and the current stock price is \$80.19. The growth rates of dividends and earnings per share have been 7.5 percent and 10.0 percent, respectively, for the past five years. Analysts' consensus estimate of the five-year earnings growth rate is 5.0 percent. Based on his own analysis, Inguigiatto has decided to use 5.50 percent as his best estimate of the long-term earnings and dividend growth rate. Next year's projected dividend, $D_1$, should be \$2.40(1.055) = \$2.532. Using the Gordon growth model, NEE's expected rate of return should be

$$\begin{aligned} r &= \frac{D_1}{P_0} + g \\ &= \frac{2.532}{80.19} + 0.055 \\ &= 0.0316 + 0.055 \\ &= 0.0866 = 8.66\% \end{aligned}$$

The expected rate of return can be broken into two components, the dividend yield ($D_1/P_0 = 3.16$ percent) and the capital gains yield ($g = 5.50$ percent).

### 4.8. The Gordon Growth Model: Concluding Remarks

The Gordon growth model is the simplest practical implementation of discounted dividend valuation. The Gordon growth model is appropriate for valuing the equity of dividend-paying companies when its key assumption of a stable future dividend and earnings growth rate is expected to be satisfied. Broad equity market indices of developed markets frequently satisfy the conditions of the model fairly well; as a result, analysts have used it to judge whether an equity market is fairly valued or not and for estimating the equity risk premium associated with the current market level. In the multistage models discussed in the next section, the Gordon growth model has often been used to model the last growth stage, when a previously high growth company matures and the growth rate drops to a long-term sustainable level. In any case in which the model is applied, the analyst must be aware that the output of the model is typically sensitive to small changes in the assumed growth rate and required rate of return.

The Gordon growth model is a single-stage DDM because all future periods are grouped into one stage characterized by a single growth rate. For many or even the majority of companies, however, future growth can be expected to consist of multiple stages. Multistage DDMs are the subject of the next section.

## 5. MULTISTAGE DIVIDEND DISCOUNT MODELS

Earlier we noted that the basic expression for the DDM (Equation 7) is too general for investment analysts to use in practice because one cannot forecast individually more than a relatively small number of dividends. The strongest simplifying assumption—a stable dividend growth rate from now into the indefinite future, leading to the Gordon growth model—is not realistic for many or even most companies. For many publicly traded companies, practitioners assume growth falls into three stages (see Sharpe, Alexander, and Bailey 1999):

- **Growth phase.** A company in its growth phase typically enjoys rapidly expanding markets, high profit margins, and an abnormally high growth rate in earnings per share (**supernormal growth**). Companies in this phase often have negative free cash flow to equity because the company invests heavily in expanding operations. Given high prospective returns on equity, the dividend payout ratios of growth-phase companies are often low or even zero. As the company's markets mature or as unusual growth opportunities attract competitors, earnings growth rates eventually decline.
- **Transition phase.** In this phase, which is a transition to maturity, earnings growth slows as competition puts pressure on prices and profit margins or as sales growth slows because of market saturation. In this phase, earnings growth rates may be above average but declining toward the growth rate for the overall economy. Capital requirements typically decline in this phase, often resulting in positive free cash flow and increasing dividend payout ratios (or the initiation of dividends).
- **Mature phase.** In maturity, the company reaches an equilibrium in which investment opportunities on average just earn their opportunity cost of capital. Return on equity approaches the required return on equity, and earnings growth, the dividend payout ratio, and the return on equity stabilize at levels that can be sustained long term. The dividend and earnings growth rate of this phase is called the **mature growth rate**. This phase, in fact, reflects the stage in which a company can properly be valued using the Gordon growth model, and that model is one tool for valuing this phase of a current high-growth company's future.

A company may attempt and succeed in restarting the growth phase by changing its strategic focuses and business mix. Technological advances may alter a company's growth prospects for better or worse with surprising rapidity. Nevertheless, this growth-phase picture of a company is a useful approximation. The growth-phase concept provides the intuition for multistage discounted cash flow (DCF) models of all types, including multistage dividend discount models. Multistage models are a staple valuation discipline of investment management firms using DCF valuation models.

In the following sections, we present three popular multistage DDMs: the two-stage DDM, the H-model (a type of two-stage model), and the three-stage DDM. Keep in mind that all these models represent stylized patterns of growth; they are attempting to identify the pattern that most accurately approximates an analyst's view of the company's future growth.

### 5.1. Two-Stage Dividend Discount Model

Two common versions of the two-stage DDM exist. Both versions assume constant growth at a mature growth rate (for example, 7 percent) in Stage 2. In the first version ("the general two-stage model"), the whole of Stage 1 represents a period of abnormal growth—for example, growth at 15 percent. The transition to mature growth in Stage 2 is generally abrupt.

In the second version, called the H-model, the dividend growth rate is assumed to decline from an abnormal rate to the mature growth rate during the course of Stage 1. For example, the growth rate could begin at 15 percent and decline continuously in Stage 1 until it reaches 7 percent. The second model will be presented after the general two-stage model.

The first two-stage DDM provides for a high growth rate for the initial period, followed by a sustainable and usually lower growth rate thereafter. The two-stage DDM is based on the multiple-period model

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{V_n}{(1+r)^n} \tag{17}$$

where $V_n$ is used as an estimate of $P_n$. The two-stage model assumes that the first $n$ dividends grow at an extraordinary short-term rate, $g_S$:

$$D_t = D_0 (1+g_S)^t$$

After time $n$, the annual dividend growth rate changes to a normal long-term rate, $g_L$. The dividend at time $n + 1$ is $D_n+_1 = D_n(1 + g_L) = D_0(1 + g_S)^n(1 + g_L)$, and this dividend continues to grow at $g_L$. Using $D_n+_1$, an analyst can use the Gordon growth model to find $V_n$:

$$V_n = \frac{D_0 (1+g_S)^n (1+g_L)}{r-g_L} \tag{18}$$

To find the value at $t = 0$, $V_0$, simply find the present value of the first $n$ dividends and the present value of the projected value at time $n$:

$$V_0 = \sum_{t=1}^{n} \frac{D_0 (1+g_S)^t}{(1+r)^t} + \frac{D_0 (1+g_S)^n (1+g_L)}{(1+r)^n (r-g_L)} \tag{19}$$

## EXAMPLE 13 Valuing a Stock Using the Two-Stage Dividend Discount Model

Carl Zeiss Meditec AG (Deutsche Börse XETRA: AFX), 65 percent owned by the Carl Zeiss Group, provides screening, diagnostic, and therapeutic systems for the treatment of ophthalmologic (vision) problems. Reviewing the issue as of mid-August 2013, when it is trading for €23.37, Hans Mattern, a buy-side analyst covering Meditec, forecasts that the current dividend of €0.40 will grow by 9 percent per year during the next 10 years. Thereafter, Mattern believes that the growth rate will decline to 5 percent and remain at that level indefinitely.

Mattern estimates Meditec's required return on equity as 7.1 percent based on a beta of 0.90 against the DAX, a 2.4 percent risk-free rate, and his equity risk premium estimate of 5.2 percent.

Exhibit 5 shows the calculations of the first ten dividends and their present values discounted at 7.1 percent. The terminal stock value at $t = 10$ is

$$
\begin{aligned}
V_{10} &= \frac{D_0\left(1+g_S\right)^n\left(1+g_L\right)}{r-g_L} \\
&= \frac{0.40(1.09)^{10}(1.05)}{0.071-0.05} \\
&= 47.3473
\end{aligned}
$$

The terminal stock value and its present value are also given.

EXHIBIT 5 Carl Zeiss Meditec AG

| Time | Value | Calculation | $D_t$ or $V_t$ | Present Values $D_t/(1.071)^t$ or $V_t/(1.071)^t$ |
|---|---|---|---|---|
| 1 | $D_1$ | €0.40(1.09) | €0.4360 | €0.4071 |
| 2 | $D_2$ | $0.40(1.09)^2$ | 0.4752 | 0.4143 |
| 3 | $D_3$ | $0.40(1.09)^3$ | 0.5180 | 0.4217 |
| 4 | $D_4$ | $0.40(1.09)^4$ | 0.5646 | 0.4291 |
| 5 | $D_5$ | $0.40(1.09)^5$ | 0.6154 | 0.4368 |
| 6 | $D_6$ | $0.40(1.09)^6$ | 0.6708 | 0.4445 |
| 7 | $D_7$ | $0.40(1.09)^7$ | 0.7312 | 0.4524 |
| 8 | $D_8$ | $0.40(1.09)^8$ | 0.7970 | 0.4604 |
| 9 | $D_9$ | $0.40(1.09)^9$ | 0.8688 | 0.4686 |
| 10 | $D_{10}$ | $0.40(1.09)^{10}$ | 0.9469 | 0.4769 |
| 10 | $V_{10}$ | $0.40(1.09)^{10}(1.05)/(0.071 - 0.05)$ | 47.3473 | 23.8452 |
| Total | | | | €28.2570 |

In this two-stage model, the dividends are forecast during the first stage, and then their present values are calculated. The Gordon growth model is used to derive the terminal value (the value of the dividends in the second stage as of the beginning of that stage). As shown in Exhibit 5, the terminal value is $V_{10} = D_{11}/(r - g_L)$. Ignoring rounding errors, the Period 11 dividend is €0.9943 (= $D_{10} \times 1.05$ = €0.9479 × 1.05). By using the standard Gordon growth model, $V_{10}$ = €47.3473 = €0.9943/(0.071 − 0.05). The present value of the terminal value is €23.8452 = €47.3473/$1.071^{10}$. The total estimated value of Meditec is €28.26 using this model. Notice that approximately 84 percent of this value, €23.85, is the present value of $V_{10}$, and the balance, €28.26 − €23.85 = €4.41, is the present value of the first ten dividends. Recalling the discussion of the sensitivity of the Gordon growth model to changes in the inputs, an interval for the intrinsic value of Meditec could be calculated by varying the mature growth rate through the range of plausible values.

The two-stage DDM is useful because many scenarios exist in which a company can achieve a supernormal growth rate for a few years, after which time the growth rate falls to a more sustainable level. For example, a company may achieve supernormal growth through possession of a patent, first-mover advantage, or another factor that provides a temporary lead in a specific marketplace. Subsequently, earnings will most likely descend to a level that is more consistent with competition and growth in the overall economy. Accordingly, that is why in the two-stage model, extraordinary growth is often forecast for a few years and normal growth is forecast thereafter. A possible limitation of the two-stage model is that the transition between the initial abnormal growth period and the final steady-state growth period is abrupt.

The accurate estimation of $V_n$, the **terminal value of the stock** (also known as its **continuing value**) is an important part of the correct use of DDMs. In practice, analysts estimate the terminal value either by applying a multiple to a projected terminal value of a fundamental, such as earnings per share or book value per share, or they estimate $V_n$ using the Gordon growth model. In the reading on market multiples, we will discuss using price–earnings multiples in this context.

In the examples, a single discount rate, $r$, is used for all phases, reflecting both a desire for simplicity and lack of a clear objective basis for adjusting the discount rate for different phases. Some analysts, however, use different discount rates for different growth phases.

The following example values E. I. DuPont de Nemours and Company by combining the dividend discount model and a P/E valuation model.

## EXAMPLE 14 Combining a DDM and P/E Model to Value a Stock

An analyst is reviewing the valuation of DuPont (NYSE: DD) as of the beginning of July 2013 when DD is selling for $52.72. In the previous year, DuPont paid a $1.70 dividend that the analyst expects to grow at a rate of 4 percent annually for the next four years. At the end of Year 4, the analyst expects the dividend to equal 35 percent of earnings per share and the trailing P/E for DD to be 13. If the required return on DD common stock is 9.0 percent, calculate the per-share value of DD common stock.

Exhibit 6 summarizes the relevant calculations. When the dividends are growing at 4 percent, the expected dividends and the present value of each (discounted at 9.0 percent) are shown. The terminal stock price, $V_4$, deserves some explanation. As shown in the table, the Year 4 dividend is $1.70(1.04)^4 = 1.9888$. Because dividends at that time are assumed to be 35 percent of earnings, the EPS projection for Year 4 is $EPS_4 = D_4/0.35 = 1.9888/0.35 = 5.6822$. With a trailing P/E of 13.0, the value of DD at the end of Year 4 should be 13.0(5.6822) = \$73.8682. Discounted at 9 percent for four years, the present value of $V_4$ is \$52.3301.

EXHIBIT 6 Value of DuPont Common Stock

| Time | Value | Calculation | $D_t$ or $V_t$ | Present Values $D_t/(1.09)^t$ or $V_t/(1.09)^t$ |
|---|---|---|---|---|
| 1 | $D_1$ | $\$1.70(1.04)^1$ | \$1.7680 | \$1.6220 |
| 2 | $D_2$ | $1.70(1.04)^2$ | 1.8387 | 1.5476 |
| 3 | $D_3$ | $1.70(1.04)^3$ | 1.9123 | 1.4766 |
| 4 | $D_4$ | $1.70(1.04)^4$ | 1.9888 | 1.4089 |
| 4 | $V_4$ | $13 \times [1.70(1.04)^4/0.35]$<br>$= 13 \times [1.9888/0.35]$<br>$= 13 \times 5.6822$ | 73.8682 | 52.3301 |
| Total | | | | \$58.3852 |

The present values of the dividends for Years 1 through 4 sum to \$6.06. The present value of the terminal value of \$73.87 is \$52.33. The estimated total value of DD is the sum of these, or \$58.39 per share.

## 5.2. Valuing a Non-Dividend-Paying Company

The fact that a stock is currently paying no dividends does not mean that the principles of the dividend discount model do not apply. Even though $D_0$ and/or $D_1$ may be zero, and the company may not begin paying dividends for some time, the present value of future dividends may still capture the value of the company. Of course, if a company pays no dividends and will never be able to distribute cash to shareholders, the stock is worthless.

To value a non-dividend-paying company using a DDM, generally an analyst can use a multistage DDM model in which the first-stage dividend equals zero. Example 15 illustrates the approach.

### EXAMPLE 15 Valuing a Non-Dividend-Paying Stock

Assume that a company is currently paying no dividend and will not pay one for several years. If the company begins paying a dividend of \$1.00 five years from now, and the dividend is expected to grow at 5 percent thereafter, this future dividend stream can be discounted back to find the value of the company. This company's required rate of return is 11 percent. Because the expression

$$V_n = \frac{D_{n+1}}{r-g}$$

values a stock at period $n$ using the next period's dividend, the $t = 5$ dividend is used to find the value at $t = 4$:

$$V_4 = \frac{D_5}{r-g} = \frac{1.00}{0.11-0.05} = \$16.67$$

To find the value of the stock today, simply discount $V_4$ back for four years:

$$V_0 = \frac{V_4}{(1+r)^4} = \frac{16.67}{(1.11)^4} = \$10.98$$

The value of this stock, even though it will not pay a dividend until Year 5, is \$10.98.

If a company is not paying a dividend but is very profitable, an analyst might be willing to forecast its future dividends. Of course, for non-dividend-paying, unprofitable companies, such a forecast would be very difficult. Furthermore, as discussed in Section 2.2 (Streams of Expected Cash Flows), it is usually difficult for the analyst to estimate the timing of the initiation of dividends and the dividend policy that will then be established by the company. Thus, the analyst may prefer a free cash flow or residual income model for valuing such companies.

### 5.3. The H-Model

The basic two-stage model assumes a constant, extraordinary rate for the supernormal growth period that is followed by a constant, normal growth rate thereafter. The difference in growth rates may be substantial. For instance, in Example 13, the growth rate for Carl Zeiss Meditec was 9 percent annually for 10 years, followed by a drop to 5 percent growth in Year 11 and thereafter. In some cases, a smoother transition to the mature phase growth rate would be more realistic. Fuller and Hsia (1984) developed a variant of the two-stage model in which growth begins at a high rate and declines linearly throughout the supernormal growth period until it reaches a normal rate at the end. The value of the dividend stream in the H-model is

$$V_0 = \frac{D_0(1+g_L)}{r-g_L} + \frac{D_0 H(g_S - g_L)}{r-g_L} \tag{20}$$

or

$$V_0 = \frac{D_0(1+g_L) + D_0 H(g_S - g_L)}{r-g_L}$$

Where

$V_0$ = value per share at $t = 0$
$D_0$ = current dividend
$r$ = required rate of return on equity
$H$ = half-life in years of the high-growth period (i.e., high-growth period = $2H$ years)
$g_S$ = initial short-term dividend growth rate
$g_L$ = normal long-term dividend growth rate after Year $2H$

The first term on the right-hand side of Equation 20 is the present value of the company's dividend stream if it were to grow at $g_L$ forever. The second term is an approximation of the extra value (assuming $g_S > g_L$) accruing to the stock because of its supernormal growth for Years 1 through $2H$ (see Fuller and Hsia 1984, for technical details).[23] Logically, the longer the supernormal growth period (i.e., the larger the value of $H$, which is one-half the length of the supernormal growth period) and the larger the extra growth rate in the supernormal growth period (measured by $g_S$ minus $g_L$), the higher the share value, all else being equal. To illustrate the expression, if the analyst in Example 13 had forecast a linear decline of the growth rate from 9 percent to 5 percent over the next 10 years, his estimate of value using the H-model would have been €23.81 (rather than €28.26 as in Example 13):

$$\begin{aligned} V_0 &= \frac{D_0(1+g_L) + D_0 H(g_S - g_L)}{r - g_L} \\ &= \frac{0.40(1.05) + 0.40(5)(0.09 - 0.05)}{0.071 - 0.05} \\ &= \frac{0.42 + 0.08}{0.021} \\ &= 23.81 \end{aligned}$$

Note that an $H$ of 5 corresponds to the 10-year high-growth period of Example 13. Example 16 provides another illustration of the H-model.

## EXAMPLE 16 Valuing a Stock with the H-Model

Françoise Delacour, a portfolio manager of a US-based diversified global equity portfolio, is researching the valuation of Vinci SA (NYSE Euronext: DG). Vinci is the world's largest construction company, operating chiefly in France (approximately two-thirds of revenue) and the rest of Europe (approximately one-quarter of revenue). Through 2003, DG paid a single regular cash dividend per fiscal year. Since 2004 it has paid two dividends per (fiscal) year, an interim dividend in December and a final dividend in May.

[23] We can provide some intuition on the expression. On average, the expected excess growth rate in the supernormal period will be $(g_S - g_L)/2$. Through $2H$ periods, a total excess amount of dividends (compared with the level given $g_L$) of $2HD_0(g_S - g_L)/2 = D_0H(g_S - g_L)$ is expected. This term is the H-model upward adjustment to the first dividend term, reflecting the extra expected dividends as growth declines from $g_S$ to $g_L$ during the first period. Note, however, that the timing of the individual dividends in the first period is not reflected by individually discounting them; the expression is thus an approximation.

Although during the past five years total annual dividends grew at less than 3 percent per year, Delacour foresees faster future growth.

Having decided to compute the H-model value estimate for DG, Delacour gathers the following facts and forecasts:

- The share price as of mid-August 2013 was €41.70.
- The current dividend is €1.77.
- The initial dividend growth rate is 7 percent, declining linearly during a 10-year period to a final and perpetual growth rate of 4 percent.
- Delacour estimates DG's required rate of return on equity as 9.5 percent.

1. Using the H-model and the information given, estimate the per-share value of DG.
2. Estimate the value of DG shares if its normal growth period began immediately.
3. Evaluate whether DG shares appear to be fairly valued, overvalued, or undervalued.

*Solution to 1:* Using the H-model expression gives

$$\begin{aligned} V_0 &= \frac{D_0(1+g_L)}{r-g_L} + \frac{D_0 H(g_S - g_L)}{r-g_L} \\ &= \frac{1.77(1.04)}{0.095-0.04} + \frac{1.77(5)(0.07-0.04)}{0.095-0.04} \\ &= 33.47 + 4.83 = €38.30 \end{aligned}$$

*Solution to 2:* If DG experienced normal growth starting now, its estimated value would be the first component of the H-model estimate, €33.47. Note that the faster initial growth assumption adds €4.83 to its value, resulting in an estimate of €38.30 for the value of a DG share.

*Solution to 3:* €38.30 is approximately 8 percent less than DG's current market price. Thus DG appears to be overvalued.

The H-model is an approximation model that estimates the valuation that would result from discounting all of the future dividends individually. In many circumstances, this approximation is very close. For a long extraordinary growth period (a high $H$) or for a large difference in growth rates (the difference between $g_S$ and $g_L$), however, the analyst might abandon the approximation model for the more exact model. Fortunately, the many tedious calculations of the exact model are made fairly easy using a spreadsheet program.

## 5.4. Three-Stage Dividend Discount Models

There are two popular versions of the three-stage DDM, distinguished by the modeling of the second stage. In the first version ("the general three-stage model"), the company is assumed to have three distinct stages of growth, and the growth rate of the second stage is typically constant. For example, Stage 1 could assume 20 percent growth for three years, Stage

2 could have 10 percent growth for four years, and Stage 3 could have 5 percent growth thereafter. In the second version, the growth rate in the middle (second) stage is assumed to decline linearly to the mature growth rate: essentially, the second and third stages are treated as an H-model.

The example below shows how the first type of the three-stage model can be used to value a stock, in this case IBM.

## EXAMPLE 17 The Three-Stage DDM with Three Distinct Stages

IBM (as of early 2013) pays a dividend of \$3.30 per year. A current price is \$194.98. An analyst makes the following estimates:

- the current required return on equity for IBM is 9 percent, and
- dividends will grow at 14 percent for the next two years, 12 percent for the following five years, and 6.75 percent thereafter.

Based only on the information given, estimate the value of IBM using a three-stage DDM approach.

*Solution:* Exhibit 7 gives the calculations.

EXHIBIT 7 Estimated Value of IBM

| Time | Value | Calculation | $D_t$ or $V_t$ | Present Values $D_t/(1.09)^t$ or $V_t/(1.09)^t$ |
|---|---|---|---|---|
| 1 | $D_1$ | 3.30(1.14) | \$3.7620 | \$3.4514 |
| 2 | $D_2$ | $3.30(1.14)^2$ | 4.2887 | 3.6097 |
| 3 | $D_3$ | $3.30(1.14)^2(1.12)$ | 4.8033 | 3.7090 |
| 4 | $D_4$ | $3.30(1.14)^2(1.12)^2$ | 5.3797 | 3.8111 |
| 5 | $D_5$ | $3.30(1.14)^2(1.12)^3$ | 6.0253 | 3.9160 |
| 6 | $D_6$ | $3.30(1.14)^2(1.12)^4$ | 6.7483 | 4.0238 |
| 7 | $D_7$ | $3.30(1.14)^2(1.12)^5$ | 7.5581 | 4.1346 |
| 7 | $V_7$ | $3.30(1.14)^2(1.12)^5(1.0675)/(0.09 - 0.0675)$ | \$358.5908 | 196.161 |
| Total | | | | \$222.8171 |

Given these assumptions, the three-stage model indicates that a fair price should be \$222.82, which is above the current market price by over 14 percent. Characteristically, the present value of the terminal value of \$196.16 constitutes the overwhelming portion (here, about 88 percent) of total estimated value.

A second version of the three-stage DDM has a middle stage similar to the first stage in the H-model. In the first stage, dividends grow at a high, constant (supernormal) rate for the whole period. In the second stage, dividends decline linearly as they do in the H-model. Finally, in Stage 3, dividends grow at a sustainable, constant growth rate. The process of using this model involves four steps:

1. Gather the required inputs:
   - the current dividend;
   - the lengths of the first, second, and third stages;
   - the expected growth rates for the first and third stages; and
   - an estimate of the required return on equity.
2. Compute the expected dividends in the first stage and find the sum of their present values.
3. Apply the H-model expression to the second and third stages to obtain an estimate of their value as of the beginning of the second stage. Then find the present value of this H-value as of today ($t = 0$).
4. Sum the values obtained in the second and third steps.

In the first step, analysts often investigate the company more deeply, making explicit individual earnings and dividend forecasts for the near future (often 3, 5, or 10 years), rather than applying a growth rate to the current level of dividends.

## EXAMPLE 18 The Three-Stage DDM with Declining Growth Rates in Stage 2

Elaine Bouvier is evaluating Energen (NYSE: EGN) for possible inclusion in a small-cap growth-oriented portfolio. Headquartered in Alabama, EGN is a diversified energy company involved in oil and gas exploration through its subsidiary, Energen Resources, and in natural gas distribution through its Alabama Gas Corporation subsidiary. In light of EGN's aggressive program of purchasing oil and gas producing properties, Bouvier expects above average growth for the next five years. Bouvier establishes the following facts and forecasts (as of the beginning of August 2013):

- The current market price is $56.18.
- The current dividend is $0.56.
- Bouvier forecasts an initial 5-year period of 11 percent per year earnings and dividend growth.
- Bouvier anticipates that EGN can grow 6.5 percent per year as a mature company, and allows 10 years for the transition to the mature growth period.
- To estimate the required return on equity using the CAPM, Bouvier uses an adjusted beta of 1.2 based on 2 years of weekly observations, an estimated equity risk premium of 4.2 percent, and a risk-free rate based on the 20-year Treasury bond yield of 3 percent.
- Bouvier considers any security trading within a band of ± 20 percent of her estimate of intrinsic value to be within a "fair value range."

1. Estimate the required return on EGN using the CAPM. (Use only one decimal place in stating the result.)
2. Estimate the value of EGN using a three-stage dividend discount model with a linearly declining dividend growth rate in Stage 2.
3. Calculate the percentages of the total value represented by the first stage and by the second and third stages considered as one group.
4. Judge whether EGN is undervalued or overvalued according to Bouvier's perspective.
5. Some analysts are forecasting essentially flat EPS and dividends in the second year. Estimate the value of EGN making the assumption that EPS is flat in the second year and that 11 percent growth resumes in the third year.

*Solution to 1:* The required return on equity is $r$ = 3 percent + 1.2(4.2 percent) = 8 percent.

*Solution to 2:* The first step is to compute the five dividends in Stage 1 and find their present values at 8 percent. The dividends in Stages 2 and 3 can be valued with the H-model, which estimates their value at the beginning of Stage 2. This value is then discounted back to find the dividends' present value at $t = 0$.

The calculation of the five dividends in Stage 1 and their present values are given in Exhibit 8. The H-model for calculating the value of the Stage 2 and Stage 3 dividends at the beginning of Stage 2 ($t = 5$) would be

$$V_5 = \frac{D_5(1+g_L)}{r-g_L} + \frac{D_5 H(g_S-g_L)}{r-g_L}$$

where

$D_5 = D_0(1 + g_S)^5 = 0.56(1.11)^5 = \$0.9436$
$g_S = 11.0\%$
$g_L = 6.5\%$
$r = 8.0\%$
$H = 5$ (the second stage lasts $2H = 10$ years)

Substituting these values into the equation for the H-model gives $V_5$ as:

$$\begin{aligned} V_5 &= \frac{0.9436(1.065)}{0.08-0.065} + \frac{0.9436(5)(0.11-0.065)}{0.08-0.065} \\ &= 66.9979 + 14.1545 \\ &= \$81.1524 \end{aligned}$$

The present value of $V_5$ is $\$81.1524/(1.08)^5 = \$55.2310$.

EXHIBIT 8 Energen

| Time | $D_t$ or $V_t$ | Explanation of $D_t$ or $V_t$ | Value of $D_t$ or $V_t$ | PV at 8% |
|---|---|---|---|---|
| 1 | $D_1$ | $0.564(1.11)^1$ | \$0.6216 | \$0.5756 |
| 2 | $D_2$ | $0.564(1.11)^2$ | 0.6900 | 0.5915 |
| 3 | $D_3$ | $0.564(1.11)^3$ | 0.7659 | 0.6080 |
| 4 | $D_4$ | $0.564(1.11)^4$ | 0.8501 | 0.6249 |
| 5 | $D_5$ | $0.564(1.11)^5$ | 0.9436 | 0.6422 |
| 5 | $V_5$ | H-model explained above | \$81.1524 | 55.2310 |
| Total | | | | \$58.2731 |

According to the three-stage DDM model, the total value of EGN is \$58.27.

*Solution to 3:* The sum of the first five present value amounts in the last column of Exhibit 8 is \$3.0422. Thus, the first stage represents \$3.0422/\$58.2731 = 5.2 percent of total value. The second and third stages together represent 100% – 5.2% = 94.8 percent of total value (check: \$55.2310/\$58.2731 = 94.8 percent).

*Solution to 4:* The band Bouvier is looking at is \$58.27 ± 0.20(\$58.27), which runs from \$58.27 + \$11.65 = \$69.92 on the upside to \$58.27 − \$11.65 = \$46.62 on the downside. Because \$56.18 is between \$46.62 and \$69.92, Bouvier would consider EGN to be fairly valued.

*Solution to 5:* The estimated value becomes \$52.56 with no growth in Year 2 as shown in Exhibit 9. The value of the second and third stages are given by

$$V_5 = \frac{0.8501(1.065)}{0.08-0.065} + \frac{0.8501(5)(0.11-0.065)}{0.08-0.065} = \$73.1103$$

EXHIBIT 9 Energen with No Growth in Year 2

| Time | $D_t$ or $V_t$ | Explanation of $D_t$ or $V_t$ | Value of $D_t$ or $V_t$ | PV at 8% |
|---|---|---|---|---|
| 1 | $D_1$ | $0.564(1.11)^1$ | \$0.6216 | \$0.5756 |
| 2 | $D_2$ | No growth in Year 2 | 0.6216 | 0.5329 |
| 3 | $D_3$ | $0.564(1.11)^2$ | 0.6900 | 0.5477 |
| 4 | $D_4$ | $0.564(1.11)^3$ | 0.7659 | 0.5629 |
| 5 | $D_5$ | $0.564(1.11)^4$ | 0.8501 | 0.5786 |
| 5 | $V_5$ | H-model explained above | \$73.1103 | 49.7576 |
| Total | | | | \$52.5553 |

In Problem 5 of Example 18, the analyst examined the consequences of 11 percent growth in Year 1 and no growth in Year 2, with 11 percent growth resuming in Years 3, 4, and 5. In the first stage, analysts may forecast earnings and dividends individually for a certain number of years.

The three-stage DDM with declining growth in Stage 2 has been widely used among companies using a DDM approach to valuation. An example is the DDM adopted by Bloomberg L.P., a financial services company that provides "Bloomberg terminals" to professional investors and analysts. The Bloomberg DDM is a model that provides an estimated value for any stock that the user selects. The DDM is a three-stage model with declining growth in Stage 2. The model uses fundamentals about the company for assumed Stage 1 and Stage 3 growth rates, and then assumes that the Stage 2 rate is a linearly declining rate between the Stage 1 and Stage 3 rates. The model also makes estimates of the required rate of return and the lengths of the three stages, assigning higher-growth companies shorter growth periods (i.e., first stages) and longer transition periods, and slower-growth companies longer growth periods and shorter transition periods. Fixing the total length of the growth and transition phases together at 17 years, the growth stage/transition stage durations for Bloomberg's four growth classifications are 3 years/14 years for "explosive growth" equities, 5 years/12 years for "high growth" equities, 7 years/10 years for "average growth" equities, and 9 years/8 years for "slow/mature growth" equities. Analysts, by tailoring stage specifications to their understanding of the specific company being valued, should be able to improve on the accuracy of valuations compared to a fixed specification.

## 5.5. Spreadsheet (General) Modeling

DDMs, such as the Gordon growth model and the multistage models presented earlier, assume stylized patterns of dividend growth. With the computational power of personal computers, calculators, and personal digital assistants, however, *any* assumed dividend pattern is easily valued.

Spreadsheets allow the analyst to build complicated models that would be very cumbersome to describe using algebra. Furthermore, built-in spreadsheet functions (such as those for finding rates of return) use algorithms to get a numerical answer when a mathematical solution would be impossible or extremely challenging. Because of the widespread use of spreadsheets, several analysts can work together or exchange information by sharing their spreadsheet models. The following example presents the results of using a spreadsheet to value a stock with dividends that change substantially through time.

### EXAMPLE 19 Finding the Value of a Stock Using a Spreadsheet Model

Yang Co. is expected to pay a $21.00 dividend next year. The dividend will decline by 10 percent annually for the following three years. In Year 5, Yang will sell off assets worth $100 per share. The Year 5 dividend, which includes a distribution of some of the proceeds of the asset sale, is expected to be $60. In Year 6, the dividend is expected to decrease to $40 and will be maintained at $40 for one additional year. The dividend is then expected to grow by 5 percent annually thereafter. If the required rate of return is 12 percent, what is the value of one share of Yang?

*Solution:* The value is shown in Exhibit 10. Each dividend, its present value discounted at 12 percent, and an explanation are included in the table. The final row treats the dividends from $t = 8$ forward as a Gordon growth model because after Year 7, the dividend grows at a constant 5 percent annually. $V_7$ is the value of these dividends at $t = 7$.

EXHIBIT 10 Value of Yang Co. Stock

| Year | $D_t$ or $V_t$ | Value of $D_t$ or $V_t$ | Present Value at 12% | Explanation of $D_t$ or $V_t$ |
|---|---|---|---|---|
| 1 | $D_1$ | \$21.00 | \$18.75 | Dividend set at \$21 |
| 2 | $D_2$ | 18.90 | 15.07 | Previous dividend × 0.90 |
| 3 | $D_3$ | 17.01 | 12.11 | Previous dividend × 0.90 |
| 4 | $D_4$ | 15.31 | 9.73 | Previous dividend × 0.90 |
| 5 | $D_5$ | 60.00 | 34.05 | Set at \$60 |
| 6 | $D_6$ | 40.00 | 20.27 | Set at \$40 |
| 7 | $D_7$ | 40.00 | 18.09 | Set at \$40 |
| 7 | $V_7$ | 600.00 | 271.41 | $V_7 = D_8/(r-g)$<br>$V_7 = (40.00 \times 1.05)/(0.12 - 0.05)$ |
| Total | | | \$399.48 | |

As the table in Example 19 shows, the total present value of Yang Co.'s dividends is \$399.48. In this example, the terminal value of the company ($V_n$) at the end of the first stage is found using the Gordon growth model and a mature growth rate of 5 percent. Several alternative approaches to estimating $g$ are available in this context:

- Use the formula $g = (b$ in the mature phase$) \times ($ROE in the mature phase$)$. We will discuss the expression $g = b \times \text{ROE}$ in Section 6. Analysts estimate mature-phase ROE in several ways, such as:
  - The DuPont decomposition of ROE based on forecasts for the components of the DuPont expression.
  - Setting ROE = $r$, the required rate of return on equity, based on the assumption that in the mature phase companies can do no more than earn investors' opportunity cost of capital.
  - Setting ROE in the mature phase equal to the median industry ROE.
- The analyst may estimate the growth rate, $g$, with other models by relating the mature growth rate to macroeconomic, including industry, growth projections.

When the analyst uses the sustainable growth expression, the earnings retention ratio, $b$, may be empirically based. For example, Bloomberg L.P.'s model assumes that $b = 0.55$ in the mature phase, equivalent to a dividend payout ratio of 45 percent, a long-run average payout ratio for mature dividend-paying companies in the United States. In addition, sometimes analysts project the dividend payout ratio for the company individually.

## EXAMPLE 20 A Sustainable Growth Rate Calculation

In Example 17, the analyst estimated the dividend growth rate of IBM in the final stage of a three-stage model as 6.75 percent. This value was based on the expression

$$g = (b \text{ in the mature phase}) \times (\text{ROE in the mature phase})$$

IBM's payout ratio has increased from 16.5 percent to 22.7 percent over the last 10 years. Assuming that in the final stage IBM has a payout ratio of 25 percent and achieves a ROE equal to its estimated required return on equity of 9 percent, the calculation is:

$$g = 0.75(9\%) = 6.75\%$$

### 5.6. Estimating a Required Return Using Any DDM

This reading has focused on finding the value of a security using assumptions for dividends, required rates of return, and expected growth rates. Given current price and all inputs to a DDM except for the required return, an IRR can be calculated. Such an IRR has been used as a required return estimate (although reusing it in a DDM is not appropriate because it risks circularity). This IRR can also be interpreted as the expected return on the issue implied by the market price—essentially, an efficient market expected return. In the following discussion, keep in mind that if price does not equal intrinsic value, the expected return will need to be adjusted to reflect the additional component of return that accrues when the mispricing is corrected, as discussed earlier.

In some cases, finding the IRR is very easy. In the Gordon growth model, $r = D_1/P_0 + g$. The required return estimate is the dividend yield plus the expected dividend growth rate. For a security with a current price of \$10, an expected dividend of \$0.50, and expected growth of 8 percent, the required return estimate is 13 percent.

For the H-model, the expected rate of return can be derived as[24]

$$r = \left(\frac{D_0}{P_0}\right)\left[\left(1+g_L\right)+H\left(g_S-g_L\right)\right]+g_L \tag{21}$$

When the short- and long-term growth rates are the same, this model reduces to the Gordon growth model. For a security with a current dividend of \$1, a current price of \$20, and an expected short-term growth rate of 10 percent declining over 10 years ($H = 5$) to 6 percent, the expected rate of return would be

$$r = \left(\frac{\$1}{\$20}\right)\left[\left(1+0.06\right)+5\left(0.10-0.06\right)\right]+0.06 = 12.3\%$$

For multistage models and spreadsheet models, finding a single equation for the rate of return can be more difficult. The process generally used is similar to that of finding the IRR for a series of varying cash flows. Using a computer or trial and error, the analyst must find the rate of return such that the present value of future expected dividends equals the current stock price.

[24] Fuller and Hsia (1984).

## EXAMPLE 21 Finding the Expected Rate of Return for Varying Expected Dividends

An analyst expects JNJ's (Johnson & Johnson) dividend of \$2.40 for 2012 to grow by 7.5 percent for six years and then grow by 6 percent into perpetuity. A recent price for JNJ as of late-August 2013 is \$86.97. What is the IRR on an investment in JNJ's stock?

In performing trial and error with the two-stage model to estimate the expected rate of return, having a good initial guess is important. In this case, the expected rate of return formula from the Gordon growth model and JNJ's long-term growth rate can be used to find a first approximation: $r = (\$2.40 \times 1.075)/\$86.97 + 0.06 = 9$ percent. Because the growth rate in the first six years is more than the long-term growth rate of 6 percent, the estimated rate of return must be above 9 percent. Exhibit 11 shows the value estimate of JNJ for two discount rates, 9 percent and 10 percent.

EXHIBIT 11 Johnson & Johnson

| Time | $D_t$ | Present Value of $D_t$ and $V_6$ at $r = 9\%$ | Present Value of $D_t$ and $V_6$ at $r = 10\%$ |
|---|---|---|---|
| 1 | \$2.5800 | \$2.3670 | \$2.3455 |
| 2 | 2.7735 | 2.3344 | 2.2921 |
| 3 | 2.9815 | 2.3023 | 2.2401 |
| 4 | 3.2051 | 2.2706 | 2.1891 |
| 5 | 3.4455 | 2.2393 | 2.1394 |
| 6 | 3.7039 | 2.2085 | 2.0908 |
| 7 | 3.9262 | | |
| Subtotal 1 | ($t = 1$ to 6) | \$13.7221 | \$13.2970 |
| Subtotal 2 | ($t = 7$ to $\infty$) | \$78.0347 | \$55.4054 |
| Total | | \$91.76 | \$68.70 |
| Market Price | | \$86.97 | \$86.97 |

In the exhibit, the first subtotal is the present value of the expected dividends for Years 1 through 6. The second subtotal is the present value of the terminal value, $V_6/(1 + r)^6 = [D_7/(r - g)]/(1 + r)^6$. For $r = 9$ percent, that present value is $[3.9262/(0.09 - 0.06)]/(1.09)^6 = \$78.0347$. The present value for other values of $r$ is found similarly.

Using 9 percent as the discount rate, the value estimate for JNJ is \$91.76, which is about 5.5 percent larger than JNJ's market price of \$86.97. This fact indicates that the IRR is greater than 9 percent. With a 10 percent discount rate, the present value of \$68.72 is significantly less than the market price. Thus, the IRR is slightly more than 9 percent. The IRR can be determined to be 9.16 percent, using a calculator or spreadsheet.

### 5.7. Multistage DDM: Concluding Remarks

Multistage dividend discount models can accommodate a variety of patterns of future streams of expected dividends.

In general, multistage DDMs make stylized assumptions about growth based on a lifecycle view of business. The first stage of a multistage DDM frequently incorporates analysts' individual earnings and dividend forecasts for the next two to five years (sometimes longer). The final stage is often modeled using the Gordon growth model based on an assumption of the company's long-run sustainable growth rate. In the case of the H-model, the transition to the mature growth phase happens smoothly during the first stage. In the case of the standard two-stage model, the growth rate typically transitions immediately to mature growth rate in the second period. In three-stage models, the middle stage is a stage of transition. Using a spreadsheet, an analyst can model an almost limitless variety of cash flow patterns.

Multistage DDMs have several limitations. Often, the present value of the terminal stage represents more than three-quarters of the total value of shares. Terminal value can be very sensitive to the growth and required return assumptions. Furthermore, technological innovation can make the lifecycle model a crude representation.

## 6. THE FINANCIAL DETERMINANTS OF GROWTH RATES

In a number of examples earlier in this reading, we have implicitly used the relationship that the dividend growth rate ($g$) equals the earning retention ratio ($b$) times the return on equity (ROE). In this section, we explain this relationship and show how it can be combined with a method of analyzing return on equity, called DuPont analysis, as a simple tool for forecasting dividend growth rates.

### 6.1. Sustainable Growth Rate

We define the **sustainable growth rate** as the rate of dividend (and earnings) growth that can be sustained for a given level of return on equity, assuming that the capital structure is constant through time and that additional common stock is not issued. The reason for studying this concept is that it can help in estimating the stable growth rate in a Gordon growth model valuation, or the mature growth rate in a multistage DDM in which the Gordon growth formula is used to find the terminal value of the stock.

The expression to calculate the sustainable growth rate is

$$g = b \times \text{ROE} \tag{22}$$

where

$g$ = dividend growth rate
$b$ = earnings retention rate (1 – Dividend payout ratio)
ROE = return on equity

More precisely, in Equation 22 the retention rate should be multiplied by the rate of return expected to be earned on new investment. Analysts commonly assume that the rate of return is well approximated by the return on equity, as shown in Equation 22; however, whether that is actually the case should be investigated by the analyst on a case-by-case basis.

Example 22 is an illustration of the fact that growth in shareholders' equity is driven by reinvested earnings alone (no new issues of equity and debt growing at the rate *g*).[25]

## EXAMPLE 22 Example Showing $g = b \times \text{ROE}$

Suppose that a company's ROE is 25 percent and its retention rate is 60 percent. According to the expression for the sustainable growth rate, the dividends should grow at $g = b \times \text{ROE} = 0.60 \times 25$ percent = 15 percent.

To demonstrate the working of the expression, suppose that, in the year just ended, a company began with shareholders' equity of \$1,000,000, earned \$250,000 net income, and paid dividends of \$100,000. The company begins the next year with \$1,000,000 + 0.60(\$250,000) = \$1,000,000 + \$150,000 = \$1,150,000 of shareholders' equity. No additions to equity are made from the sale of additional shares.

If the company again earns 25 percent on equity, net income will be 0.25 × \$1,150,000 = \$287,500, which is a \$287,500 – \$250,000 = \$37,500 or a \$37,500/\$250,000 = 0.15 percent increase from the prior year level. The company retains 60 percent of earnings, 60 percent × \$287,500 = \$172,500, and pays out the other 40 percent, 40 percent × \$287,500 = \$115,000 as dividends. Dividends for the company grew from \$100,000 to \$115,000, which is exactly a 15 percent growth rate. With the company continuing to earn 25 percent each year on the 60% of earnings that is reinvested in the company, dividends would continue to grow at 15 percent.

Equation 22 implies that the higher the return on equity, the higher the dividend growth rate, all else being constant. That relation appears to be reliable. Another implication of the expression is that the lower (higher) the earnings retention ratio, the lower (higher) the growth rate in dividends, holding all else constant; this relationship has been called *the dividend displacement of earnings*.[26] Of course, all else may not be equal—the return on reinvested earnings may not be constant at different levels of investment, or companies with changing future growth prospects may change their dividend policy. Arnott and Asness (2003) and Zhou and Ruland (2006), in providing US-based evidence that dividend-paying companies had higher future growth rates during the period studied, indicate that caution is appropriate in assuming that dividends displace earnings.

A practical logic for defining *sustainable* in terms of growth through internally generated funds (retained earnings) is that external equity (secondary issues of stock) is considerably more costly than internal equity (reinvested earnings), for several reasons including

[25] With debt growing at the rate *g*, the capital structure is constant. If the capital structure is not constant, ROE would not be constant in general because ROE depends on leverage.

[26] ROE is a variable that reflects underlying profitability as well as the use of leverage or debt. The retention ratio or dividend policy, in contrast, is not a fundamental variable in the same sense as ROE. A higher dividend growth rate through a higher retention ratio (lower dividend payout ratio) is neutral for share value in and of itself. Holding investment policy (capital projects) constant, the positive effect on value from an increase in *g* will be offset by the negative effect from a decrease in dividend payouts in the expression for the value of the stock in any DDM. Sharpe, Alexander, and Bailey (1999) discuss this concept in more detail.

the investment banker fees associated with secondary equity issues. In general, continuous issuance of new stock is not a practical funding alternative for companies.[27] Growth of capital through issuance of new debt, however, can sometimes be sustained for considerable periods. Further, if a company manages its capital structure to a target percentage of debt to total capital (debt and common stock), it will need to issue debt to maintain that percentage as equity grows through reinvested earnings. (This approach is one of a variety of observed capital structure policies.) In addition, the earnings retention ratio nearly always shows year-to-year variation in actual companies. For example, earnings may have transitory components that management does not want to reflect in dividends. The analyst may thus observe actual dividend growth rates straying from the growth rates predicted by Equation 22 because of these effects, even when his or her input estimates are unbiased. Nevertheless, the equation can be useful as a simple expression for approximating the average rate at which dividends can grow over a long horizon.

### 6.2. Dividend Growth Rate, Retention Rate, and ROE Analysis

Thus far we have seen that a company's sustainable growth, as defined in Section 6.1, is a function of its ability to generate return on equity (which depends on investment opportunities) and its retention rate. We now expand this model by examining what drives ROE. Remember that ROE is the return (net income) generated on the equity invested in the company:

$$\text{ROE} = \frac{\text{Net income}}{\text{Shareholders' equity}} \tag{23}$$

If a company has a ROE of 15 percent, it generates \$15 of net income for every \$100 invested in stockholders' equity. For purposes of analyzing ROE, it can be related to several other financial ratios. For example, ROE can be related to return on assets (ROA) and the extent of financial leverage (equity multiplier):

$$\text{ROE} = \frac{\text{Net income}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Shareholders' equity}} \tag{24}$$

Therefore, a company can increase its ROE either by increasing ROA or the use of leverage (assuming the company can borrow at a rate lower than it earns on its assets).

This model can be expanded further by breaking ROA into two components, profit margin and turnover (efficiency):

$$\text{ROE} = \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Shareholders' equity}} \tag{25}$$

[27] As a long-term average, about 2 percent of US publicly traded companies issue new equity in a given year, which corresponds to a secondary equity issue once every 50 years, on average. Businesses may be rationed in their access to secondary issues of equity because of the costs associated with informational asymmetries between management and the public. Because management has more information on the future cash flows of the company than the general public, and equity is an ownership claim to those cash flows, the public may react to additional equity issuance as possibly motivated by an intent to "share (future) misery" rather than "share (future) wealth."

The first term is the company's profit margin. A higher profit margin will result in a higher ROE. The second term measures total asset turnover, which is the company's efficiency. A turnover of one indicates that a company generates \$1 in sales for every \$1 invested in assets. A higher turnover will result in higher ROE. The last term is the equity multiplier, which measures the extent of leverage, as noted earlier. This relationship is widely known as the DuPont model or analysis of ROE. Although ROE can be analyzed further using a five-way analysis, the three-way analysis will provide insight into the determinants of ROE that are pertinent to our understanding of the growth rate. By combining Equations 22 and 25, it shows that the dividend growth rate is equal to the retention rate multiplied by ROE:

$$\begin{aligned} g = {} & \frac{\text{Net income} - \text{Dividends}}{\text{Net income}} \times \frac{\text{Net income}}{\text{Sales}} \\ & \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Shareholders' equity}} \end{aligned} \tag{26}$$

This expansion of the sustainable growth expression has been called the PRAT model (Higgins 2007). Growth is a function of profit margin (P), retention rate (R), asset turnover (A), and financial leverage (T). The profit margin and asset turnover determine ROA. The other two factors, the retention rate and financial leverage, reflect the company's financial policies. So, the growth rate in dividends can be viewed as determined by the company's ROA and financial policies. Analysts may use Equation 26 to forecast a company's dividend growth rate in the mature growth phase.

Theoretically, the sustainable growth rate expression and this expansion of it based on the DuPont decomposition of ROE hold exactly only when ROE is calculated using beginning-of-period shareholders' equity, as illustrated in Example 22. Such calculation assumes that retained earnings are not available for reinvestment until the end of the period. Analysts and financial databases more frequently prefer to use average total assets in calculating ROE and, practically, DuPont analysis is frequently performed using that definition.[28] The example below illustrates the logic behind this equation.

## EXAMPLE 23 ROA, Financial Policies, and the Dividend Growth Rate

Baggai Enterprises has an ROA of 10 percent, retains 30 percent of earnings, and has an equity multiplier of 1.25. Mondale Enterprises also has an ROA of 10 percent, but it retains two-thirds of earnings and has an equity multiplier of 2.00.

1. What are the sustainable dividend growth rates for (A) Baggai Enterprises and (B) Mondale Enterprises?
2. Identify the drivers of the difference in the sustainable growth rates of Baggai Enterprises and Mondale Enterprises.

[28] See Robinson, van Greuning, Henry, and Broihahn (2006).

*Solution to 1:*

A. Baggai's dividend growth rate should be $g = 0.30 \times 10\% \times 1.25 = 3.75\%$.
B. Mondale's dividend growth rate should be $g = (2/3) \times 10\% \times 2.00 = 13.33\%$.

*Solution to 2:*
Because Mondale has the higher retention rate and higher financial leverage, its dividend growth rate is much higher.

If growth is being forecast for the next five years, an analyst should use the expectations of the four factors driving growth during this five-year period. If growth is being forecast into perpetuity, an analyst should use very long-term forecasts for these variables.

To illustrate the calculation and implications of the sustainable growth rate using the expression for ROE given by the DuPont formula, assume the growth rate is $g = b \times \text{ROE} = 0.60$ (15 percent) = 9 percent. The ROE of 15 percent was based on a profit margin of 5 percent, an asset turnover of 2.0, and an equity multiplier of 1.5. Given fixed ratios of sales-to-assets and assets-to-equity, sales, assets, and debt will also be growing at 9 percent. Because dividends are fixed at 40 percent of income, dividends will grow at the same rate as income, or 9 percent. If the company increases dividends faster than 9 percent, this growth rate would not be sustainable using internally generated funds. Earning retentions would be reduced, and the company would not be able to finance the assets required for sales growth without external financing.

An analyst should be careful in projecting historical financial ratios into the future when using this analysis. Although a company may have grown at 25 percent a year for the last five years, this rate of growth is probably not sustainable indefinitely. Abnormally high ROEs, which may have driven that growth, are unlikely to persist indefinitely because of competitive forces and possibly other reasons, such as adverse changes in technology or demand. In the following example, an above-average terminal growth rate is plausibly forecasted because the company has positioned itself in businesses that may have relatively high margins on an ongoing basis.

## EXAMPLE 24 Forecasting Growth with the PRAT Formula

International Business Machines (NYSE: IBM), which currently pays a dividend of \$3.40 per share, has been the subject of two other examples in this reading. In one example, an analyst estimated IBM's mature phase growth rate at 6.75 percent, based on its mature phase ROE exactly equaling its estimated required return on equity of 9 percent. Another estimate can be made using the DuPont decomposition of ROE.

An analysis of IBM's ROE for the past ten years is shown in Exhibit 12. During the period shown, EPS grew at a compound annual rate of about 16 percent. IBM's retention ratio has declined from 83.5 percent in 2003 to 77.3 percent in 2012. Annual nominal investment in property, plant, and equipment has also declined from \$4.4 billion in 2003 to \$4.1 billion in 2012.

EXHIBIT 12 IBM Corporation

| Year | ROE (%) | Profit Margin (%) | Asset Turnover | Financial Leverage |
|---|---|---|---|---|
| 2012 | 87.5 = | 15.89 | × 0.88 | × 6.28 |
| 2011 | 78.4 = | 14.83 | × 0.92 | × 5.75 |
| 2010 | 64.0 = | 14.85 | × 0.88 | × 4.90 |
| 2009 | 59.0 = | 14.02 | × 0.88 | × 4.79 |
| 2008 | 90.8 = | 11.90 | × 0.95 | × 8.06 |
| 2007 | 36.6 = | 10.55 | × 0.82 | × 4.23 |
| 2006 | 33.3 = | 10.38 | × 0.89 | × 3.62 |
| 2005 | 24.0 = | 8.71 | × 0.86 | × 3.19 |
| 2004 | 23.6 = | 7.77 | × 0.87 | × 3.50 |
| 2003 | 22.2 = | 7.36 | × 0.84 | × 3.59 |

IBM's ROE has been much higher the last five years than it was in the preceding five year period. This change can largely be attributed to improved profit margins and a significant increase in leverage. However, it is unrealistic to assume that sustainable earnings growth (growth in perpetuity) will be equal to a recent average ROE (75.9 percent over the last five years) times a plausible assumption for the future retention rate (most recently 77.3 percent) given that world-wide economic growth is typically in the mid-single digits. That sort of estimate is also totally inconsistent with the noted historical EPS growth rate of 16.0 percent.

Profit margins are strongly mean-reverting. Suppose the analyst believes that IBM's recent superiority in profit margin in comparison to peers will be much reduced in the mature phase. The analyst forecasts a peer mean pretax profit margin of 5 percent during IBM's mature phase. With its strategy of searching for high-margined growth and its strong ability to compete in integrated hardware–software solutions for businesses, the analyst forecasts a long-run pretax profit margin of 6 percent for IBM, equal to a profit margin (after tax) of about 4.2 percent based on an effective tax rate of about 30 percent.

The analyst also believes that capital investment will continue to decline as IBM matures, and cash flow that was previously used for investment will be used to retire debt and pay dividends. The analyst forecasts a financial leverage ratio of 3.5, consistent with 2003–2007 values, but well below the ratio of 6.28 in 2012. The analyst also sees the dividend payout ratio continuing its recent rise and ultimately reaching a level of 40 percent.

Based on an asset turnover ratio of 0.88 (the mean value in Exhibit 12), but using a profit margin estimate of 4.2, a forecast of ROE in the maturity phase is (4.2 percent)(0.88)(3.5) = 12.9 percent. Therefore, based on this analysis, the estimate of the sustainable growth rate for IBM would be $g$ = (0.60)(12.9 percent) = 7.7 percent.

## 6.3. Financial Models and Dividends

Analysts can also forecast dividends by building more complex models of the company's total operating and financial environment. Because there can be so many aspects to such a model, a spreadsheet is used to build pro forma income statements and balance sheets. The company's ability to pay dividends in the future can be predicted using one of these models. The example below shows the dividends that a highly profitable and rapidly growing company can pay when its growth rates and profit margins decline because of increasing competition over time.

## EXAMPLE 25 A Spreadsheet Model for Forecasting Dividends

An analyst is preparing a forecast of dividends for Hoshino Distributors for the next five years. He uses a spreadsheet model with the following assumptions:

- Sales are \$100 million in Year 1. They grow by 20 percent in Year 2, 15 percent in Year 3, and 10 percent in Years 4 and 5.
- Operating profits (earnings before interest and taxes, or EBIT) are 20 percent of sales in Years 1 and 2, 18 percent of sales in Year 3, and 16 percent of sales in Years 4 and 5.
- Interest expenses are 10 percent of total debt for the current year.
- The income tax rate is 40 percent.
- Hoshino pays out 20 percent of earnings in dividends in Years 1 and 2, 30 percent in Year 3, 40 percent in Year 4, and 50 percent in Year 5.
- Retained earnings are added to equity in the next year.
- Total assets are 80 percent of the current year's sales in all years.
- In Year 1, debt is \$40 million, and shareholders' equity is \$40 million. Debt equals total assets minus shareholders' equity. Shareholders' equity will equal the previous year's shareholders' equity plus the addition to retained earnings from the previous year.
- Hoshino has 4 million shares outstanding.
- The required return on equity is 15 percent.
- The value of the company at the end of Year 5 is expected to be 10.0 times earnings.

The analyst wants to estimate the current value per share of Hoshino. Exhibit 13 adheres to the modeling assumptions above. Total dividends and earnings are found at the bottom of the income statement.

EXHIBIT 13 Hoshino Distributors Pro Forma Financial Statements (in Millions)

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---|---|---|---|---|
| Income statement | | | | | |
| Sales | \$100.00 | \$120.00 | \$138.00 | \$151.80 | \$166.98 |
| EBIT | 20.00 | 24.00 | 24.84 | 24.29 | 26.72 |
| Interest | 4.00 | 4.83 | 5.35 | 5.64 | 6.18 |
| EBT | 16.00 | 19.17 | 19.49 | 18.65 | 20.54 |
| Taxes | 6.40 | 7.67 | 7.80 | 7.46 | 8.22 |
| Net income | 9.60 | 11.50 | 11.69 | 11.19 | 12.32 |
| Dividends | 1.92 | 2.30 | 3.51 | 4.48 | 6.16 |
| Balance sheet | | | | | |
| Total assets | \$80.00 | \$96.00 | \$110.40 | \$121.44 | \$133.58 |
| Total debt | 40.00 | 48.32 | 53.52 | 56.38 | 61.81 |
| Equity | 40.00 | 47.68 | 56.88 | 65.06 | 71.77 |

Dividing the total dividends by the number of outstanding shares gives the dividend per share for each year shown below. The present value of each dividend, discounted at 15 percent, is also shown.

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Total |
|---|---|---|---|---|---|---|
| DPS | \$0.480 | \$0.575 | \$0.877 | \$1.120 | \$1.540 | \$4.59 |
| PV | 0.417 | 0.435 | 0.577 | 0.640 | 0.766 | 2.84 |

The earnings per share in Year 5 are \$12.32 million divided by 4 million shares, or \$3.08 per share. Given a P/E of 10, the market price in Year 5 is predicted to be \$30.80. Discounted at 15 percent, the required return on equity by assumption, the present value of this price is \$15.31. Adding the present values of the five dividends, which sum to \$2.84, gives a total stock value today of \$18.15 per share.

## 7. SUMMARY

This reading provided an overview of DCF models of valuation, discussed the estimation of a stock's required rate of return, and presented in detail the dividend discount model.

- In DCF models, the value of any asset is the present value of its (expected) future cash flows

$$V_0 = \sum_{t=1}^{n} \frac{\mathrm{CF}_t}{(1+r)^t}$$

  where $V_0$ is the value of the asset as of $t = 0$ (today), $\mathrm{CF}_t$ is the (expected) cash flow at time $t$, and $r$ is the discount rate or required rate of return. For infinitely lived assets such as common stocks, $n$ runs to infinity.
- Several alternative streams of expected cash flows can be used to value equities, including dividends, free cash flow, and residual income. A discounted dividend approach is most suitable for dividend-paying stocks in which the company has a discernible dividend policy that has an understandable relationship to the company's profitability, and the investor has a noncontrol (minority ownership) perspective.
- The free cash flow approach (FCFF or FCFE) might be appropriate when the company does not pay dividends, dividends differ substantially from FCFE, free cash flows align with profitability, or the investor takes a control (majority ownership) perspective.
- The residual income approach can be useful when the company does not pay dividends (as an alternative to a FCF approach) or free cash flow is negative.
- The DDM with a single holding period gives stock value as

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{P_1}{(1+r)^1} = \frac{D_1 + P_1}{(1+r)^1}$$

  where $D_1$ is the expected dividend at time 1 and $V_0$ is the stock's (expected) value at time 0. Assuming that $V_0$ is equal to today's market price, $P_0$, the expected holding-period return is

$$r = \frac{D_1 + P_1}{P_0} - 1 = \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

- The expression for the DDM for any given finite holding period $n$ and the general expression for the DDM are, respectively,

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \text{ and } V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

- There are two main approaches to the problem of forecasting dividends. First, an analyst can assign the entire stream of expected future dividends to one of several stylized growth patterns. Second, an analyst can forecast a finite number of dividends individually up to a terminal point and value the remaining dividends either by assigning them to a stylized growth pattern or by forecasting share price as of the terminal point of the dividend forecasts.
- The Gordon growth model assumes that dividends grow at a constant rate $g$ forever, so that $D_t = D_{t-1}(1 + g)$. The dividend stream in the Gordon growth model has a value of

$$V_0 = \frac{D_0(1+g)}{r-g}, \text{ or } V_0 = \frac{D_1}{r-g} \text{ where } r > g$$

- The value of noncallable fixed-rate perpetual preferred stock is $V_0 = D/r$, where $D$ is the stock's (constant) annual dividend.
- Assuming that price equals value, the Gordon growth model estimate of a stock's expected rate of return is

$$r = \frac{D_0(1+g)}{P_0} + g = \frac{D_1}{P_0} + g$$

- Given an estimate of the next-period dividend and the stock's required rate of return, the Gordon growth model can be used to estimate the dividend growth rate implied by the current market price (making a constant growth rate assumption).
- The present value of growth opportunities (PVGO) is the part of a stock's total value, $V_0$, that comes from profitable future growth opportunities in contrast to the value associated with assets already in place. The relationship is $V_0 = E_1/r + \text{PVGO}$, where $E_1/r$ is defined as the no-growth value per share.
- The leading price-to-earnings ratio ($P_0/E_1$) and the trailing price-to-earnings ratio ($P_0/E_0$) can be expressed in terms of the Gordon growth model as, respectively,

$$\frac{P_0}{E_1} = \frac{D_1/E_1}{r-g} = \frac{1-b}{r-g} \text{ and } \frac{P_0}{E_0} = \frac{D_0(1+g)/E_0}{r-g} = \frac{(1-b)(1+g)}{r-g}$$

  The above expressions give a stock's justified price-to-earnings ratio based on forecasts of fundamentals (given that the Gordon growth model is appropriate).
- The Gordon growth model may be useful for valuing broad-based equity indices and the stock of businesses with earnings that are expected to grow at a stable rate comparable to or lower than the nominal growth rate of the economy.
- Gordon growth model values are very sensitive to the assumed growth rate and required rate of return.
- For many companies, growth falls into phases. In the growth phase, a company enjoys an abnormally high growth rate in earnings per share, called supernormal growth. In the

transition phase, earnings growth slows. In the mature phase, the company reaches an equilibrium in which such factors as earnings growth and the return on equity stabilize at levels that can be sustained long term. Analysts often apply multistage DCF models to value the stock of a company with multistage growth prospects.

- The two-stage dividend discount model assumes different growth rates in Stage 1 and Stage 2

$$V_0 = \sum_{t=1}^{n} \frac{D_0(1+g_S)^t}{(1+r)^t} + \frac{D_0(1+g_S)^n(1+g_L)}{(1+r)^n(r-g_L)}$$

where $g_S$ is the expected dividend growth rate in the first period and $g_L$ is the expected growth rate in the second period.

- The terminal stock value, $V_n$, is sometimes found with the Gordon growth model or with some other method, such as applying a P/E multiplier to forecasted EPS as of the terminal date.
- The H-model assumes that the dividend growth rate declines linearly from a high supernormal rate to the normal growth rate during Stage 1, and then grows at a constant normal growth rate thereafter:

$$V_0 = \frac{D_0(1+g_L)}{r-g_L} + \frac{D_0 H(g_S-g_L)}{r-g_L} = \frac{D_0(1+g_L)+D_0 H(g_S-g_L)}{r-g_L}$$

- There are two basic three-stage models. In one version, the growth rate in the middle stage is constant. In the second version, the growth rate declines linearly in Stage 2 and becomes constant and normal in Stage 3.
- Spreadsheet models are very flexible, providing the analyst with the ability to value any pattern of expected dividends.
- In addition to valuing equities, the IRR of a DDM, assuming assets are correctly priced in the marketplace, has been used to estimate required returns. For simpler models (such as the one-period model, the Gordon growth model, and the H-model), well-known formulas may be used to calculate these rates of return. For many dividend streams, however, the rate of return must be found by trial and error, producing a discount rate that equates the present value of the forecasted dividend stream to the current market price.
- Multistage DDM models can accommodate a wide variety of patterns of expected dividends. Even though such models may use stylized assumptions about growth, they can provide useful approximations.
- Dividend growth rates can be obtained from analyst forecasts, statistical forecasting models, or company fundamentals. The sustainable growth rate depends on the ROE and the earnings retention rate, $b$: $g = b \times \text{ROE}$. This expression can be expanded further, using the DuPont formula, as

$$g = \frac{\text{Net income} - \text{Dividends}}{\text{Net income}} \times \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Shareholders' equity}}$$

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