Special Cases in Assignment Problem

- Assignment Problem with Maximization objective
- Number of Assignees ≠ Number of Assignment (Unbalanced Assignment Problem)
- Reduced Matrix does not contain single zero in any row or column
- A particular job *i* should not be performed by resource *j*

Maximization type assignment problem

Unless otherwise stated, the assignment problem is a minimization type.

- If the problem is a maximization problem (Profit, sales, effectiveness, etc.), convert the problem into a minimization problem by multiplying each c_{ij} by -1.
- Then apply the usual procedure of an assignment problem.

Example : Assign 4 sales persons to four different sales regions such that total sales is maximized.

Sales person \ Sales region	1	2	3	4
1	10	22	12	14
2	16	18	22	10
3	24	20	12	18
4	16	14	24	20

Modified data , after multiplying the cell entries by -1.

Sales region / Sales person	1	2	3	4
1	-10	-22	-12	-14
2	-16	-18	-22	-10
3	-24	-20	-12	-18
4	-16	-14	-24	-20

Row Reduction

Sales region / Sales person	1	2	3	4
1	-10	**-22**	-12	-14
2	-16	-18	**-22**	-10
3	**-24**	-20	-12	-18
4	-16	-14	**-24**	-20

Row Reduction

Sales region / Sales person	1	2	3	4
1	12	0	10	8
2	6	4	0	12
3	0	4	12	6
4	8	10	0	**4**

Column reduction

Sales region / Sales person	1	2	3	4
1	12	0	10	4
2	6	4	0	8
3	0	4	12	2
4	8	10	0	0

Start making Assignments

Sales region / Sales person	1	2	3	4
1	12	[0]	10	4
2	6	4	0	8
3	0	4	12	2
4	8	10	0	0

Sales region / Sales person	1	2	3	4
1	12	[0]	10	4
2	6	4	[0]	8
3	0	4	12	2
4	8	10	~~0~~	0

Sales region / Sales person	1	2	3	4
1	12	[0]	10	4
2	6	4	[0]	8
3	[0]	4	12	2
4	8	10	~~0~~	0

Solution is feasible and optimal

Sales person \ Sales region	1	2	3	4
1	12	[0]	10	4
2	6	4	[0]	8
3	[0]	4	12	2
4	8	10	~~0~~	[0]

- **Result:**

Sales person	Sales region	Sales
1	2	22
2	3	22
3	1	24
4	4	20

Unable to get single zero in any row or column of the reduced matrix

- If there is no single zero in any row or column of the reduced matrix, then arbitrarily select a row or column having minimum number of zeros.
- Arbitrarily, choose a zero in the selected row or column for assignment and cross the remaining zeros in that row or column.
- Apply the usual procedure.
- Multiple Optimal Solution

Example :

Assignment \ Employee	A	B	C	D
I	**2**	3	4	5
II	**4**	5	6	7
III	**7**	8	9	8
IV	**3**	5	8	4

Row Reduction

Employee / Assignment	A	B	C	D
I	0	**1**	**2**	3
II	0	1	2	3
III	0	1	2	**1**
IV	0	2	5	1

Column Reduction

Employee / Assignment	A	B	C	D
I	0	0	0	2
II	0	0	0	2
III	0	0	0	0
IV	0	1	3	0

Employee / Assignment	A	B	C	D
I	~~0~~	0	0	2
II	~~0~~	0	0	2
III	~~0~~	0	0	0
IV	[0]	1	3	~~0~~

Employee / Assignment	A	B	C	D
I	~~0~~	0	0	2
II	~~0~~	0	0	2
III	~~0~~	~~0~~	~~0~~	[0]
IV	[0]	1	3	~~0~~

Assignment \ Employee	A	B	C	D
I	~~0~~	[0]	~~0~~	2
II	~~0~~	~~0~~	0	2
III	~~0~~	~~0~~	~~0~~	[0]
IV	[0]	1	3	~~0~~

Assignment \ Employee	A	B	C	D
I	~~0~~	[0]	~~0~~	2
II	~~0~~	~~0~~	[0]	2
III	~~0~~	~~0~~	~~0~~	[0]
IV	[0]	1	3	~~0~~

I → B, II → C, III → D, IV → A

Other optimal assignments are also possible each with cost 20.

I → A, II → B, III → C, IV → D

I → C, II → B, III → A, IV → D

I → C, II → B, III → D, IV → A

I → B, II → C, III → A, IV → D

Number of Assignees ≠ Number of Assignment (Unbalanced Assignment Problem)

- The assignment problem usually has a square matrix with *n* jobs to be assigned to *n* resources.
- Sometimes we may have fewer resources (rows) or fewer jobs (columns).
- In these cases, we make the matrix squire by creating additional dummy rows or dummy columns depending whether we have fewer rows or columns.
- For example,

  If the problem has four rows and six columns, we convert it to a 6x6 problem by adding two dummy rows.

  If it is a 7x5 problem, we create two additional dummy columns.
- The dummy rows and columns have zero cost.

Example

Person \ Job	1	2	3	4
A	7	5	8	4
B	5	6	7	4
C	8	7	9	8

Introduce a dummy person

Person \ Job	1	2	3	4
A	7	5	8	4
B	5	6	7	4
C	8	7	9	8
D (Dummy)	0	0	0	0

Row reduction

Job / Person	1	2	3	4
A	3	1	4	0
B	1	2	3	0
C	1	0	2	1
D (Dummy)	0	0	0	0

Job / Person	1	2	3	4
A	3	1	4	[0]
B	1	2	3	~~0~~
C	1	0	2	1
D (Dummy)	0	0	0	~~0~~

Job / Person	1	2	3	4
A	3	1	4	[0]
B	1	2	3	~~0~~
C	1	[0]	2	1
D (Dummy)	0	~~0~~	0	~~0~~

Person \ Job	1	2	3	4
A	3	1	4	[0]
B	1	2	3	~~0~~
C	1	[0]	2	1
D (Dummy)	[0]	~~0~~	~~0~~	~~0~~

Job / Person	1	2	3	4
A	3	1	4	[0]
B	1	2	3	~~0~~
C	1	[0]	2	1
D (Dummy)	[0]	~~0~~	~~0~~	~~0~~

√

Job / Person	1	2	3	4
A	3	1	4	[0]
B	1	2	3	~~0~~
C	1	[0]	2	1
D (Dummy)	[0]	~~0~~	~~0~~	~~0~~

√

√

Person \ Job	1	2	3	4	
A	3	1	4	[0]	√
B	1	2	3	~~0~~	√
C	1	[0]	2	1	
D (Dummy)	[0]	~~0~~	~~0~~	~~0~~	
					√

Person \ Job	1	2	3	4	
A	3	1	4	[0]	√
B	1	2	3	~~0~~	√
C	1	[0]	2	1	
D (Dummy)	[0]	~~0~~	~~0~~	~~0~~	
				√	

Job / Person	1	2	3	4
A	2	0	3	0
B	0	1	2	0
C	1	0	2	2
D (Dummy)	0	0	0	1

Job / Person	1	2	3	4
A	2	~~0~~	3	0
B	0	1	2	0
C	1	[0]	2	2
D (Dummy)	0	~~0~~	0	1

Job / Person	1	2	3	4
A	2	~~0~~	3	[0]
B	0	1	2	~~0~~
C	1	[0]	2	2
D (Dummy)	0	~~0~~	0	1

Job / Person	1	2	3	4
A	2	~~0~~	3	[0]
B	[0]	1	2	~~0~~
C	1	[0]	2	2
D (Dummy)	~~0~~	~~0~~	0	1

Person \ Job	1	2	3	4
A	2	~~0~~	3	[0]
B	[0]	1	2	~~0~~
C	1	[0]	2	2
D (Dummy)	~~0~~	~~0~~	[0]	1

Thus the optimum allocation is:

A → 4, B → 1 C → 2 D → 3 (Job 3 is not done by any real person)

Optimal cost = 4+5+7 = 16

A particular Assignment *i* should not be performed by resource *j*

- In this case, put c_{ij} = M (where M is a large positive number tends to infinity) in the minimization problem and proceed.
- Example

Salesman / Territory	A	B	C	D
1	20	-	32	27
2	15	20	17	18
3	16	18	-	20
4	-	20	18	24

Salesman / Territory	A	B	C	D
1	20	**M**	32	27
2	15	20	17	18
3	16	18	**M**	20
4	**M**	20	18	24

Row Reduction

Salesman / Territory	A	B	C	D
1	0	**M**	12	7
2	0	5	2	3
3	0	2	**M**	4
4	**M**	2	0	6

Column Reduction

Salesman / Territory	A	B	C	D
1	0	**M**	12	4
2	0	3	2	0
3	0	0	**M**	1
4	**M**	0	0	3

Salesman / Territory	A	B	C	D
1	[0]	**M**	12	4
2	~~0~~	3	2	0
3	~~0~~	0	**M**	1
4	**M**	0	0	3

Salesman / Territory	A	B	C	D
1	[0]	M	12	4
2	~~0~~	3	2	[0]
3	~~0~~	0	M	1
4	M	0	0	3

Salesman / Territory	A	B	C	D
1	[0]	M	12	4
2	~~0~~	3	2	[0]
3	~~0~~	[0]	M	1
4	M	~~0~~	0	3

Salesman / Territory	A	B	C	D
1	[0]	M	12	4
2	~~0~~	3	2	[0]
3	~~0~~	[0]	M	1
4	M	~~0~~	[0]	3

1 → A, 2 → D, 3 → B, 4 → C