Operations Research-I
Practice Problem 3

(1) Consider the following problem.

Maximize $Z = 5x_1 + 3x_2 + 4x_3,$

subject to

$$2x_1 + x_2 + x_3 \le 20$$
$$3x_1 + x_2 + 2x_3 \le 30$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

You are given the information that the nonzero variables in the optimal solution are x_2 and x_3. Describe how you can use this information to adapt the simplex method to solve this problem in the minimum possible number of iterations.

(2) Consider the following problem.

Maximize $Z = 5x_1 + x_2 + 3x_3 + 4x_4,$

subject to

$$x_1 - 2x_2 + 4x_3 + 3x_4 \le 20$$
$$-4x_1 + 6x_2 + 5x_3 - 4x_4 \le 40$$
$$2x_1 - 3x_2 + 3x_3 + 8x_4 \le 50$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0, \quad x_4 \ge 0.$$

Work through the simplex method to demonstrate that Z is unbounded.

(3) Consider the following problem.

Maximize $Z = x_1 + x_2 + x_3 + x_4,$

subject to

$$x_1 + x_2 \le 3$$
$$x_3 + x_4 \le 2$$

and

$$x_j \ge 0, \quad \text{for } j = 1, 2, 3, 4.$$

Work through the simplex method to find *all* the optimal BF solutions.

(4) Consider the following problem.

Maximize $Z = 2x_1 + 3x_2,$

subject to

$$x_1 + 2x_2 \le 4$$
$$x_1 + x_2 = 3$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

(a) Solve this problem graphically.
(b) Using the Big M method, construct the complete first (initial) simplex tableau for the simplex method and identify the corresponding initial BF solution. Also identify the initial entering basic variable and the leaving basic variable.
(c) Continue from part (b) to work through the simplex method step by step to solve the problem.

(5) Consider the following problem.

$$\text{Maximize} \quad Z = 4x_1 + 2x_2 + 3x_3 + 5x_4,$$

subject to

$$2x_1 + 3x_2 + 4x_3 + 2x_4 = 300$$
$$8x_1 + x_2 + x_3 + 5x_4 = 300$$

and

$$x_j \geq 0, \quad \text{for } j = 1, 2, 3, 4.$$

(a) Using the two-phase method, construct the complete first simplex tableau for phase 1 and identify the corresponding initial BF solution. Also identify the initial entering basic variable and the leaving basic variable.
(b) Work through phase 1 step by step.
(c) Construct the complete first simplex tableau for phase 2.
(d) Work through phase 2 step by step to solve the problem.

(6) Consider the following problem.

$$\text{Minimize} \quad Z = 2x_1 + 3x_2 + x_3,$$

subject to

$$x_1 + 4x_2 + 2x_3 \geq 8$$
$$3x_1 + 2x_2 \geq 6$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

(a) Using the Big *M* method, work through the simplex method step by step to solve the problem.
(b) Using the two-phase method, work through the simplex method step by step to solve the problem.
(c) Compare the sequence of BF solutions obtained in parts (a) and (b). Which of these solutions are feasible only for the artificial problem obtained by introducing artificial variables and which are actually feasible for the real problem?

(7) Consider the following problem.

$$\text{Maximize} \quad Z = 90x_1 + 70x_2,$$

subject to

$$2x_1 + x_2 \leq 2$$
$$x_1 - x_2 \geq 2$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

(a) Demonstrate graphically that this problem has no feasible solutions.
(b) Using the Big *M* method, work through the simplex method step by step to demonstrate that the problem has no feasible solutions.
(c) Repeat part (b) when using phase 1 of the two-phase method.

(8) Consider the following problem.

Maximize $Z = -x_1 + 4x_2$,

subject to

$$\begin{aligned} -3x_1 + x_2 &\le 6 \\ x_1 + 2x_2 &\le 4 \\ x_2 &\ge -3 \end{aligned}$$

(no lower bound constraint for x_1).

(a) Solve this problem graphically.

(b) Reformulate this problem so that it has only two functional constraints and all variables have nonnegativity constraints.

(c) Work through the simplex method step by step to solve the problem.

(9) Consider the following problem.

Maximize $Z = -x_1 + 2x_2 + x_3$,

subject to

$$\begin{aligned} 3x_2 + x_3 &\le 120 \\ x_1 - x_2 - 4x_3 &\le 80 \\ -3x_1 + x_2 + 2x_3 &\le 100 \end{aligned}$$

(no nonnegativity constraints).

(a) Reformulate this problem so that all variables have nonnegativity constraints.

(b) Work through the simplex method step by step to solve the problem.

(10) Consider the following problem.

Maximize $Z = 4x_1 + 5x_2 + 3x_3$,

subject to

$$\begin{aligned} x_1 + x_2 + 2x_3 &\ge 20 \\ 15x_1 + 6x_2 - 5x_3 &\le 50 \\ x_1 + 3x_2 + 5x_3 &\le 30 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

Work through the simplex method step by step to demonstrate that this problem does not possess any feasible solutions.

(11) You are given the following linear programming problem.

Maximize $Z = 4x_1 + 2x_2$,

subject to

$$\begin{aligned} 2x_1 &\le 16 \qquad \text{(resource 1)} \\ x_1 + 3x_2 &\le 17 \qquad \text{(resource 2)} \\ x_2 &\le 5 \qquad \text{(resource 3)} \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

(a) Solve this problem graphically.

(b) Use graphical analysis to find the shadow prices for the resources.

(c) Determine how many additional units of resource 1 would be needed to increase the optimal value of Z by 15.

Use the concept of the Revised Simplex to solve problems (12) to (14).

(12) Consider the following problem.

$$\text{Maximize} \quad Z = 8x_1 + 4x_2 + 6x_3 + 3x_4 + 9x_5,$$

subject to

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 3x_4 \quad &\le 180 \quad \text{(resource 1)} \\ 4x_1 + 3x_2 + 2x_3 + x_4 + x_5 &\le 270 \quad \text{(resource 2)} \\ x_1 + 3x_2 \quad + x_4 + 3x_5 &\le 180 \quad \text{(resource 3)} \end{aligned}$$

and

$$x_j \ge 0, \qquad j = 1, \ldots, 5.$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{27}\begin{bmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{bmatrix}.$$

You are given the facts that the basic variables in the optimal solution are x_3, x_1, and x_5 and that

(a) Use the given information to identify the optimal solution.

(b) Use the given information to identify the shadow prices for the three resources.

(13) Consider the following problem.

$$\text{Maximize} \quad Z = x_1 - x_2 + 2x_3,$$

subject to

$$\begin{aligned} 2x_1 - 2x_2 + 3x_3 &\le 5 \\ x_1 + x_2 - x_3 &\le 3 \\ x_1 - x_2 + x_3 &\le 2 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

Let x_4, x_5, and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1				1	1	0	
x_2	(1)	0				1	3	0	
x_6	(2)	0				0	1	1	
x_3	(3)	0				1	2	0	

Identify the missing numbers in the final simplex tableau. Show your calculations.

(14) Consider the following problem.

$$\text{Maximize} \quad Z = 20x_1 + 6x_2 + 8x_3,$$

subject to

$$\begin{aligned} 8x_1 + 2x_2 + 3x_3 &\le 200 \\ 4x_1 + 3x_2 + 3x_3 &\le 100 \\ 2x_1 + 3x_2 + x_3 &\le 50 \\ x_3 &\le 20 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

Let x_4, x_5, x_6, and x_7 denote the slack variables for the first through fourth constraints, respectively. Suppose that after some number of iterations of the simplex method, a portion of the current simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Right Side
Z	(0)	1				$\frac{9}{4}$	$\frac{1}{2}$	0	0	
x_1	(1)	0				$\frac{3}{16}$	$-\frac{1}{8}$	0	0	
x_2	(2)	0				$-\frac{1}{4}$	$\frac{1}{2}$	0	0	
x_6	(3)	0				$-\frac{3}{8}$	$\frac{1}{4}$	1	0	
x_7	(4)	0				0	0	0	1	

Identify the missing numbers in the current simplex tableau. Show your calculations.