Assignment 6: ILP and TSP (10 marks each)

Q1) Three jobs are to be processed using four machines. The technological sequence and processing time on machines for the three jobs are given in table below. For example: Job 1 is first processed in machine 1 with a processing time of 10 minutes, then processed on machine 2 with a processing time of 8 minutes and finally on machine 3 with a processing time of 4 minutes.

Job	Technological sequence	Processing time ($P_{machine, job}$)
1	1, 2,	P_{11}=10, P_{21}=8, P_{31}=4
2	1, 4, 3	P_{12}=3, P_{42}=5, P_{32}=6
3	1, 4	P_{13}=4, P_{43}=3

Assume that each machine can process only one job at a time. Formulate an Integer program to determine the sequence in which the various jobs are processed on the machines so as to complete all the jobs in the least possible time.

Q2) A manager has a total of 10 employees working on six projects. Projects are reviewed weekly with each employee. A project may employ more than one employee resulting in assignment overlaps, as the following table shows:

Employees \ Projects	1	2	3	4	5	6
1		x		x	x	
2	x		x		x	
3		x	x	x		x
4			x	x	x	
5	x	x	x			
6	x	x	x	x		x
7	x	x			x	x
8	x		x	x		
9					x	x
10	x	x		x	x	x

Note: x represents employee working on that project

Currently, the manager meets individually once a week with each employee. Each meeting lasts about 20 minutes for a total of 3 hours and 20 minutes for all 10 employees. To reduce the total time, the manager wants to hold group meeting depending on shared projects. The objective is to schedule the meetings in a way that will reduce the traffic (number of employees) in and out of the meeting room. Formulate this problem as a TSP by defining the cities and the distance matrix.

Q3)

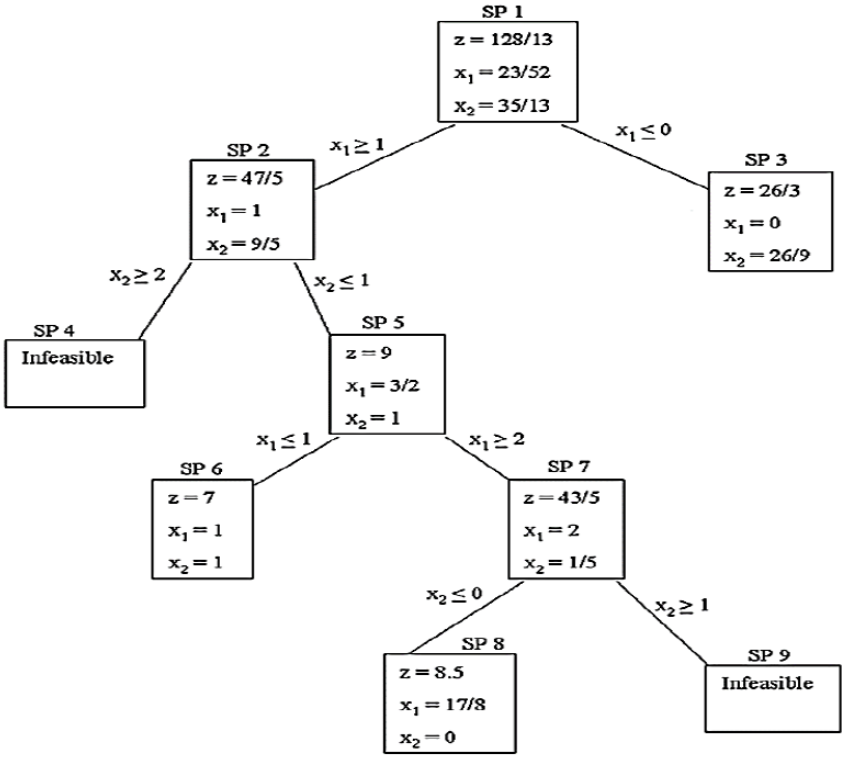
$$Z = 4x_1 + 3x_2$$
$$\text{s.t. } 4x_1 + 9x_2 \leq 26$$
$$8x_1 + 5x_2 \leq 17$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

While solving above problem with branch and bound technique, we have a branch and bound solution tree at a certain stage as shown in the figure below:

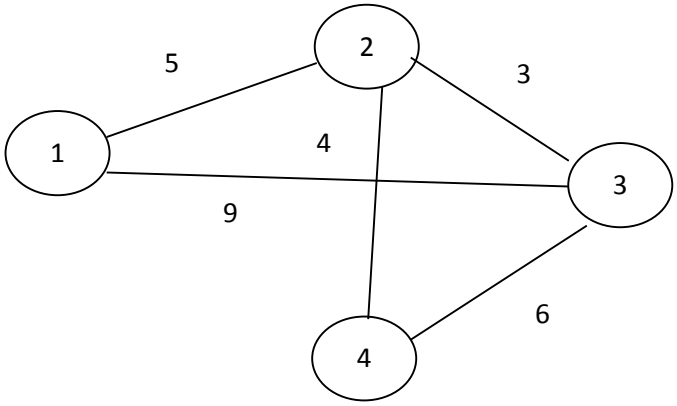
(a) Is it a maximization or minimization problem? What is the incumbent (current best) solution?

(b) Indicate the node(s) that have been fathomed and explain why? Identify the node(s) that have not been fathomed and explain why not.

(c) Have we reached an optimal solution to the integer program? If yes, then explain why? If not, formulate the subsequent sub problem(s) and solve using graphical method to find the optimal solution.



Q4) The cost between city pairs are given as



Find the tour of the traveling salesman using branch and bound method so that cost of travel is minimum. Given that he starts journey from city 2.

Q5) Suppose that you have 7 full wine bottles, 7 half-full, and 7 empty. You would like to divide the 21 bottles among three individuals so that each will receive exactly 7. Additionally, each individual must receive the same quantity of wine. Express the problem as ILP constraints, and find a solution. (Hint: Use a dummy objective function in which all the objective coefficients are zeros.)

Q6) ABC is an LTL (less-than-truckload) trucking company that delivers loads on a daily basis to five customers. The following list provides the customers associated with each route:

Route	Customers served on the route
1	1,2,3,4
2	4,3,5
3	1,2,5
4	2,3,5
5	1,4,2
6	1,3,5

The segments of each route are dictated by the capacity of the truck delivering the loads. For example, on route 1, the capacity of the truck is sufficient to deliver the loads to customers 1,2,3, and 4 only. The following table lists distances (in miles) among the truck terminal (ABC) and the customers.

	ABC	1	2	3	4	5
ABC	0	10	12	16	9	8
1	10	0	32	8	17	10
2	12	32	0	14	21	20
3	16	8	14	0	15	18
4	9	17	21	15	0	11
5	8	10	20	18	11	0

The objective is to determine the least distance needed to make daily deliveries to all five customers. Though the solution may result in a customer being served by more than one route, the implementation phase will use only one such route. Formulate the problem as an ILP and find the optimum solution.

Q7) Professor Yataha needs to schedule six round-trips between Boston and Washington, D.C. The route is served by three airlines: Eastern, US Air, and Continental and there is no penalty for the purchase of one-way tickets. Each airline offers bonus miles for frequent fliers. Eastern gives 1000 miles per (one-way) ticket plus 5000 extra miles if the number of tickets in a month reaches 2 and another 5000 miles if the number exceeds 5. US Air gives 1500 miles per trip plus 10,000 extra for each 6 tickets. Continental gives 1800 miles plus 7000 extra for each 5 tickets. Professor Yataha wishes to allocate the 12 one-way tickets among the three airlines to maximize the total number of bonus miles earned.

Q8) Gapco manufactures three products, whose daily labor and raw material requirements are given in the following table.

Product	Required daily labor (hr/unit)	Required daily raw material (lb/unit)
1	3	4
2	4	3
3	5	6

The profits per unit of the three products are $25, $30, and $22, respectively. Gapco has two options for locating its plant. The two locations differ primarily in the availability of labor and raw material, as shown in the following table:

Location	Available daily labor (hr/unit)	Available daily raw material (lb/unit)
1	100	100
2	90	120

Formulate the problem as an ILP, and determine the optimum location of the plant.

Q 9) Develop the B&B tree for each of the following problems. For convenience, always select Xl as the branching variable at node O.

Maximize $z = 3x1+ 2x2$

subject to,

$$2x1+5x2 \leq 9$$

$$4x1+2x2 \leq 9$$

$$x1,x2 \geq 0 \text{ and integer}$$