Indian Institute of Technology, Kharagpur
Department of Industrial & Systems Engineering
Mid-Semester Examination: Autumn 2022-23
IM21201 (Old: IM21003): Operations Research-I

Time: 2 Hours **Maximum Marks: 30**

Instruction: Assume any missing data suitably and state all your assumptions clearly.
Write your OR-I Section Number on your answer script.
All questions are compulsory.

Question (1) [5]

(a) Consider the following problem, where the value of c_1 has not yet been ascertained.

Maximize $Z = c_1 x_1 + 2x_2$
Subject to
$4x_1 + x_2 \leq 12$
$x_1 - x_2 \geq 2$
$x_1, x_2 \geq 0$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1.

(b) Carry out the full tableau implementation of the simplex method to find the optimal solution of the following problem:

Maximize z
Subject to
$z \leq 5$
$z \geq 0$

Question (2) [5]

ISE Department of IIT maintains its OR analytics lab for research use by its faculty and students. During all working hours, an operator must be available to maintain the lab. The operators are available to work only a limited number of hours each day, as shown in the following table.

Operators	Wage rate (Rs./hour)	Maximum hours of availability				
		Mon	Tue	Wed	Thu	Fri
S_1	250	6	0	6	0	6
S_2	260	0	6	0	6	0
S_3	240	4	8	4	0	4
S_4	230	5	5	5	0	5
J_1	280	3	0	3	8	0
J_2	300	0	0	0	6	2

There are six operators (four senior S_1, S_2, S_3, S_4 and two junior J_1, J_2). The above table shows their wage rates, along with the maximum number of hours that each operator can work each day. Each senior operator is guaranteed a minimum 8 hours per week and each junior operator is guaranteed a minimum 7 hours per week. The lab is to be open for departmental use from 8 A.M. to 10 P.M. Monday through Friday with exactly one operator on duty during these hours, and it remains closed on Saturdays and Sundays. Formulate a linear programming model to determine the number of hours that should be assigned to each operator on each day to minimize cost.

Question (3) [5]
Consider the following problem.

Maximize $Z = 5x_1 + 3x_2 + 4x_3$

subject to

$$2x_1 + x_2 + x_3 \leq 20$$
$$3x_1 + x_2 + 2x_3 \geq 30$$
$$x_1, x_2, x_3 \geq 0$$

You are given the information that the nonzero variables in the optimal solution are x_2 and x_3. Use the simplex method to solve this problem in the minimum possible number of iterations.

Question 4 [5]
The following simplex tableau shows the optimal solution of a linear programming problem. It is known that X_1, X_2 and X_3 are original variables and X_4 and X_5 are the slack variables in the first and second constraints of the original problem. All constraints are of $\leq$ type.

Iteration	BV	Equation	Z	X_1	X_2	X_3	X_4	X_5	RHS
Final	Z	0	-1	0	2	0	3	2	-35
	X_3	1	0	0	1/4	1	1/2	0	5/2
	X_1	2	0	1	-1/2	0	-1/6	1/3	5/2

Write the formulation of the original problem. Show all your calculations required to derive **A**, **b** and **c** matrix/vectors.

Question (5) [5]
Solve the following LPP using the two-phase method.

Maximize $Z = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$$3x_1 + 4x_2 + 2x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0.$$

Question (6) [3+2]
Consider the following primal linear program

Maximize $Z = x_1 + 2x_2 + x_3$

Subject to $x_1 + x_2 - x_3 \leq 2$

$$x_1 - x_2 + x_3 = 1$$
$$2x_1 + x_2 + x_3 \geq 2$$
$$x_1 \geq 0, x_2 \leq 0 \text{ and } x_3 \text{ is unrestricted}$$

(a) Write the dual of the above problem.

(b) Using duality theory prove that $Z \leq 1$.

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