Utility

Intermediate Microeconomics

by

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Approaches of consumer Theory

I. Axiomatic approach

II. Utility approach- cardinal & ordinal

III. Revealed Preference approach

- Working with preference relations is not always convenient.

- Economists like to work with Utility Functions, which are simple and easy way of summarizing preferences.

Utility Defined

- “utility” is want-satisfying power. The utility of a good or service s is the satisfaction or pleasure it provides to a consumer.
  - Not a synonym for “usefulness”
  - Utility is subjective
  - Difficult to quantify (unit of measurement is called “utils”)

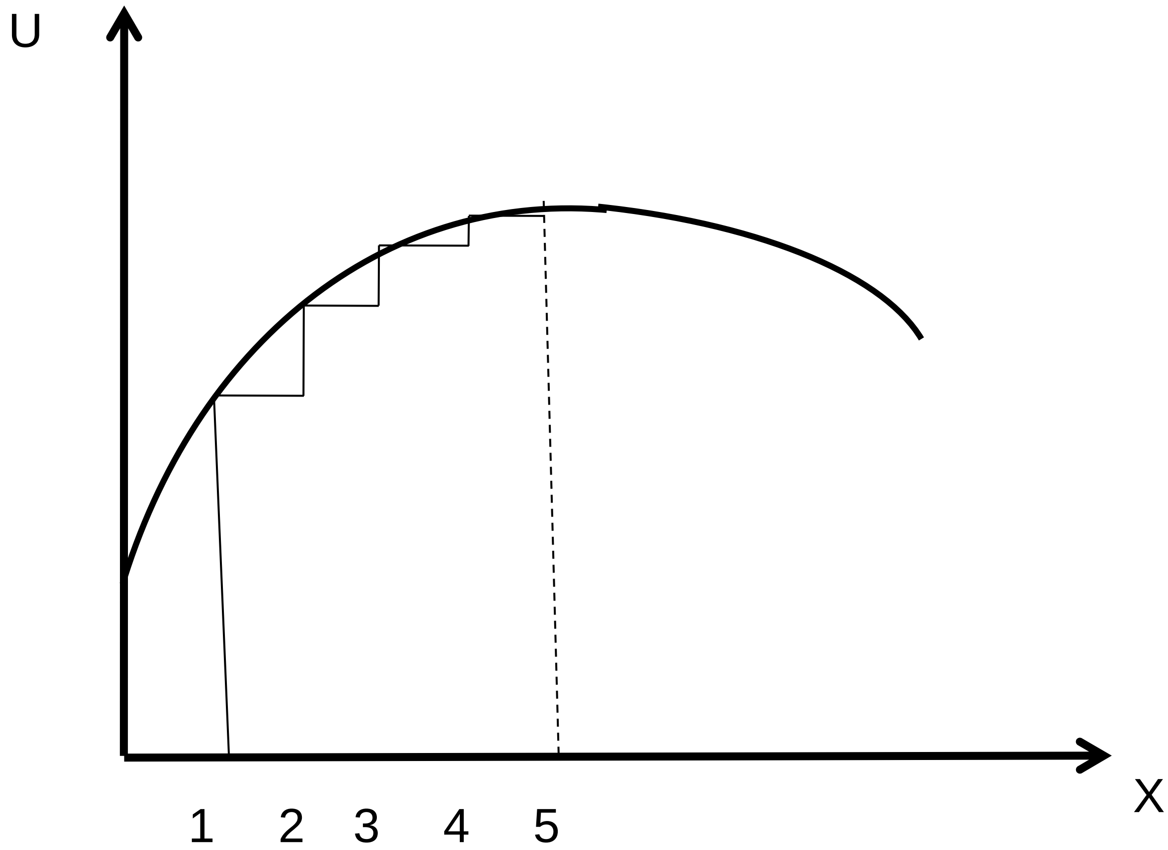
The cardinal utility analysis

- The consumer is able to measure the utility in cardinal numbers.
- Goods are comparable.

Concepts

- Total utility
- Marginal Utility—extra, additional, incremental
- **Law of Diminishing Marginal Utility—**beyond some point of consumption, utility will decline.

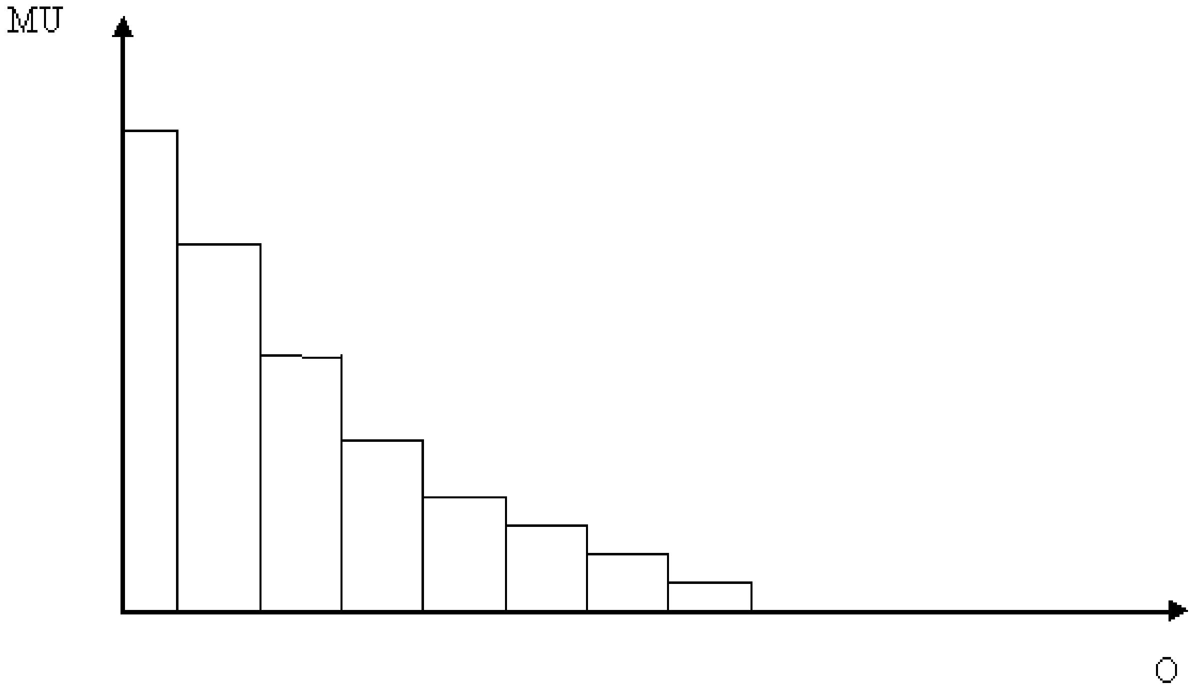
Total Utility (TU)



Marginal utility (MU)

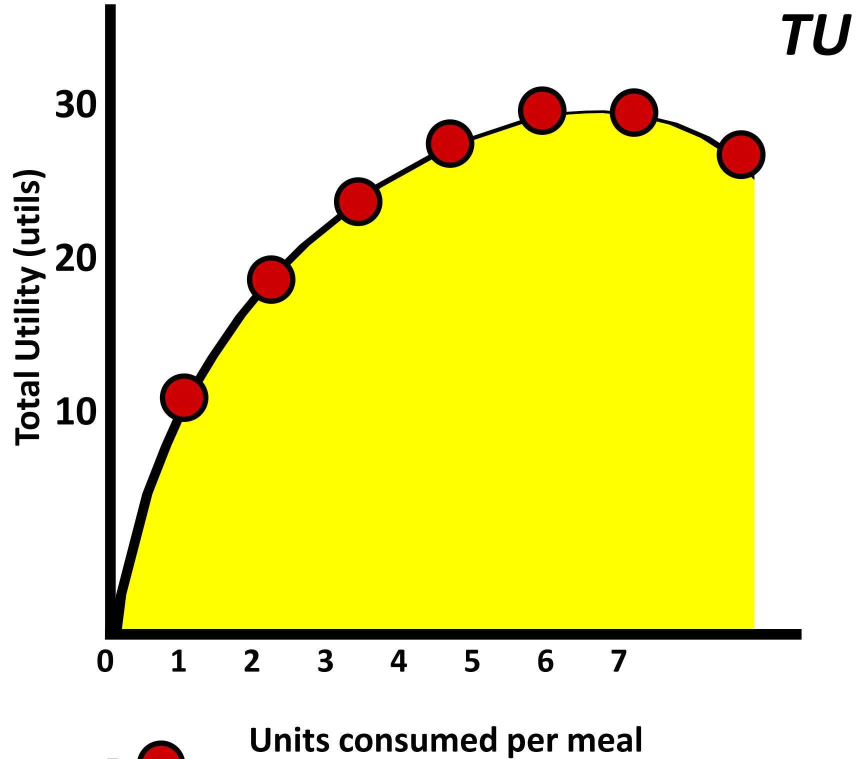
Marginal utility (MU): **The change in the utility resulting from the consumption of a subsequent piece of goods.**

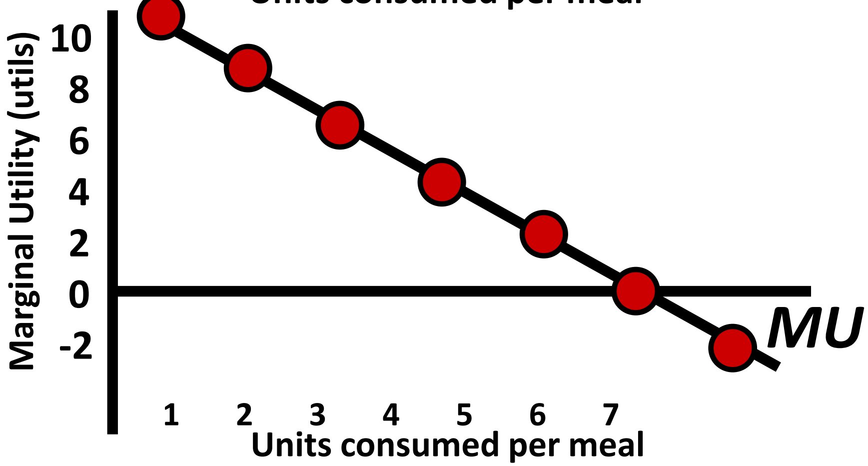
$$MU = \frac{\Delta TU}{\Delta Q}$$



TOTAL AND MARGINAL UTILITY

Chips consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
		10
1	10	
		8
2	18	
		6
3	24	
		4
4	28	
		2
5	30	
		0
6	30	
		-2
7	28	





Concept of MU

- Total utility: u =u(x)

$$MU = \frac{\partial U(X)}{\partial x_i} \underset{\text{good}}{\succ} \underset{\text{neutral}}{=} \underset{\text{bad}}{\prec} 0$$

Law of diminishing MU

$$\frac{\partial}{\partial x_i}\left(\frac{\partial U}{\partial x_i}\right) \prec 0$$

Preference Ordering & Ordinal utility

Sometimes we attach numerical numbers- construction of a utility function

X	1	30	100
X’	2	20	70
X”	3	10	50
X’”	4	5	2

An ordinal utility fn is numerical representation of preference ordering
A utility fn u(X) will be a representation of preference ordering if:

(i) u(X’)= u(X”)when X’IX”
(ii) u(X’”)= u(X””)whenever X’”PX””

Monotonic transformations of utility functions

An ordinal utility fn allows any positive monotonic transformation as they preserve the ordering.

$$V = \phi[u(X)], \phi' > 0$$

- Any monotonic transformation of a utility function will represent the same preferences. Some examples of monotonic transformations:
  - $\log(U(x_1, x_2, \ldots, x_n))$
  - $\exp\left(U(x_1, x_2, \ldots, x_n)\right)$
  - $\sqrt{U(x_1, x_2, \ldots, x_n)}$

Indifference curves are utility contours.

Proof.

Consider a particular level of utility, u_0, for which the utility contour is $u(X)=u_0$

Consider any two bundles X' & X" such that X'IX"

Therefore, by construction of utility fn (by condition (1))
$u(X')= u(X'')= u_0$ (say)

That means X' & X" lie on the same IC.

By definition of utility contour $u(X')= u(X'')$ is true.

They are on the same utility contour. This will be true for all such X' & X".

Therefore, all indifference curves are utility contours.

An ordinal utility fn is a rule by which we assign a real number to each particular commodity bundle.

Assume $X'IX''$

Therefore, $u(X')= u(X'')=3$

But $X'PX'''$

$u(X')> u(X''')$

By monotonic preference, **a higher IC should be assigned a higher utility index.**

MU & Interdependence of MRS

Let $u=u\ (x_1, x_2)$

$$\left.\frac{dx_2}{dx_1}\right|_{U_0} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}$$

$MRS=MU_1/MU_2$

$u_i = u_i\ (x_i)$

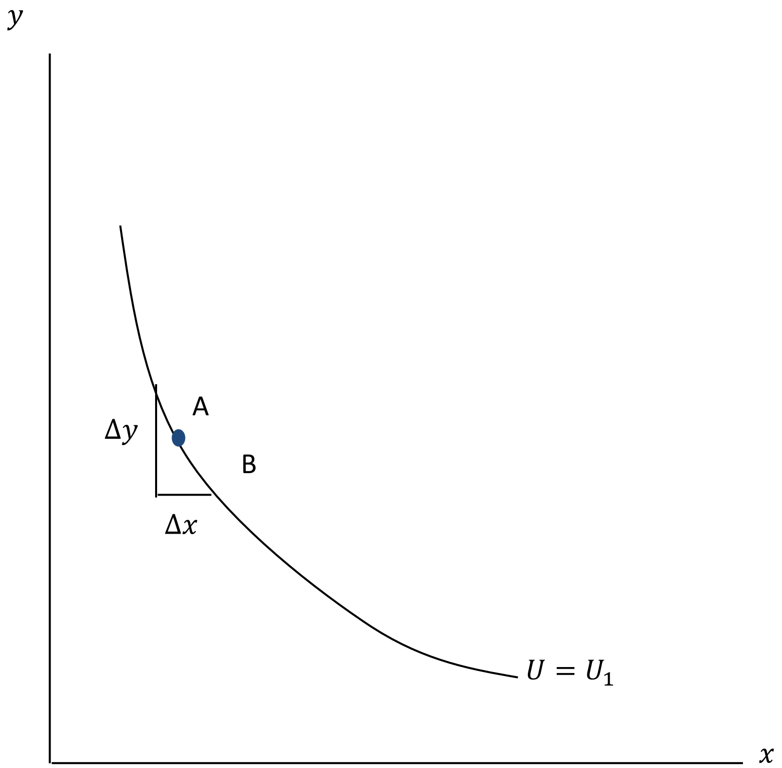
Let $u= u(x_1) + u(x_2)$

If Additively separable

$$\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1}\right) = 0$$

$$\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1}\right) \neq 0$$ Additively non-separable

Marginal utility



From point A to point B, the consumer loses Δx_1 and gains Δx_2. We also know that both A and B give the consumer the same utility U_1.

Marginal utility lost from less x_1 must be offset by marginal utility gained from more x_2

Different types of utility functions

1. $u = u(x_1, x_2)$, $u_1 > 0$, $u_2 > 0$

2. $u = \min(ax_1, bx_2)$

$$3. u = x_1^{\alpha} x_2^{\beta}$$

$$4. \ln u = \alpha \ln x_1 + \beta \ln x_2$$

$$5. u = x_1 + x_2^{\beta}$$

$$6. u = x_1^{\alpha} + x_2$$

- Cobb-Douglas preferences
  - $U(X,Y) = X^a Y^b$