Indian Institute of Technology Kharagpur
Department of Humanities and Social Sciences
BS-MS in Economics
Mid-Autumn Semester: 2022-2023
Time: 2 hours Full Marks: 30 Number of Students: 78
Subject Name: Econometric Analysis I Subject No: HS20202/EM20202
Instruction: Answer all the questions. Write the answers in sequence of the questions.

1. Comment on the following statement with justification/mathematical proof: 3×2=6

   (a) For the regression model $Y_i = \alpha + \beta X_i + u_i$, if u_i is independently, identically and normally distributed, so will be the distribution of Y_i.

   (b) While testing of a hypothesis, one should always prefer 1 percent significance level as compared to 5 percent level.

2. For the regression model $Y_i = \alpha + \beta X_i + u_i$, if the scale of Y_i and X_i are changed by w_1 and w_2 respectively, i.e., $Y_i^* = w_1 Y_i$ and $X_i^* = w_2 X_i$, prove that $\text{var}(\alpha^*) = w_1^2\, \text{var}(\hat{\alpha})$ and $\text{var}(\beta^*) = \left(\frac{w_1}{w_2}\right)^2 \text{var}(\hat{\beta})$. Here, $\hat{\alpha}$ and $\hat{\beta}$ are OLS estimators of the initial model and α^* and β^* are that of the new model (i.e., after change of scale of the variables). What will be the impact of the change of scale of the variables on the corresponding t-Statistics?

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3. For the regression model $Y_i = \alpha + u_i$, examine if the OLS estimator of α is unbiased. What will be the OLS estimator of the variance of this coefficient (α)?

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4. Consider the model $Y_i = \alpha + \beta X_i + u_i$. If Y_i is regressed on $\hat{Y}_i$, i.e., $Y_i = \theta_0 + \theta_1 \hat{Y}_i + v_i$, derive the OLS estimators of θ_0 and θ_1. How will the value of goodness-of-fit differ between the two models?

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5. For the regression model $Y_i = \alpha + \beta X_i + u_i$, prove that (i) $t_{\hat{\alpha}} = \frac{\hat{\alpha}}{SE(\hat{\alpha})} \sim t_{(n-2)}$, and (ii) $F = \frac{ESS/df_1}{RSS/df_2} \sim F_{(1,n-2)}$

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6. Consider the following models:
   (a) $Y_i = \alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i} + u_i$; (b) $Y_i = \beta_0 + \beta_1 X_{1i} + v_i$; (c) $Y_i = \gamma_0 + \gamma_1 X_{2i} + w_i$
   Examine if the statistical significance and sign of α_1 and α_2 can differ from that of β_1 and γ_1 respectively. Will it be a better approach to estimate model (b) and (c) separately instead of estimating model (a)? Justify your answer.

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7. Consider the production function $Q = AL^{\alpha}K^{\beta}e^{u}$. Discuss how one can examine if capital is more productive as compared to labour in the production process. Can this be tested by using Restricted F Test as well? Justify your answer.

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