Product Differentiation I

Background

- Consider an industry with fairly large number of firms (electronic goods, banks and financial institutions, newspapers, medicines etc..)

- Theories suggest they will earn zero profit (perfect competition, Bertrand oligopoly) or near zero profits (competitive selection) in the long-run

- Evidence: all of them earn fairly handsome profits in the long-run, which can't exit.

- One of the main source of such profit despite of large number of players is differentiated products.

- Basic logic: different tastes, different varieties and hence become a price maker in at least one variety and have your own niche.

Background

- oligopolies producing differentiated products

- consumers could not recognize or did not bother to learn the producers' names or logos of homogeneous products

- consumers are able to distinguish among the different producers and to treat the products (brands) as close but imperfect substitutes

- Several important observations make the analysis of differentiated products highly important.

Background

1. Most industries produce a large number of similar but not identical products.

2. Only a small subset of all possible varieties of differentiated products are actually produced. For example, most products are not available in all colors.

3. Most industries producing differentiated products are concentrated, in the sense that it is typical to have two to five firms in an industry.

4. Consumers purchase a small subset of the available product varieties.

Types and Modeling

- Horizontal: Different types of consumers prefer different types of products (preferring Coke over Pepsi) – consumers have brand preferences. Includes location or address models (buyers buy only preferred products) and non-address Chamberlinian models.

- Vertical: Unanimous ranking of products by the consumers (Mac is better than any other laptop brands) – consumers can rank quality and buy the best if prices are equal

- Non-address models: all consumers derive utility from consuming a variety of products – no brand loyalty

- Characteristics: Consumers prefer not the products(s) but embedded characteristics. (1) more characteristics better (2) some characters are preferred (3) varieties

Types

- **Product Differentiation**
  - **Comes from Differences in Purchasing Power (Vertical)**
  - **Comes from Differences in Preferences (Horizontal)**
  - **Comes from Love for Variety (Characteristics)**

Characteristics Approach

- Two types of consumers – $i = 1,2$
- Two products – $k = 1,2$
- Four characteristics – $j = 1, \cdots, 4$

Characteristics **Models**	**HP/W**	**AC**	**MP$**	**Size**	**Price**
Geo	0.3	0	64	0.9	4000
Porsche	1	1	12	1.2	68000

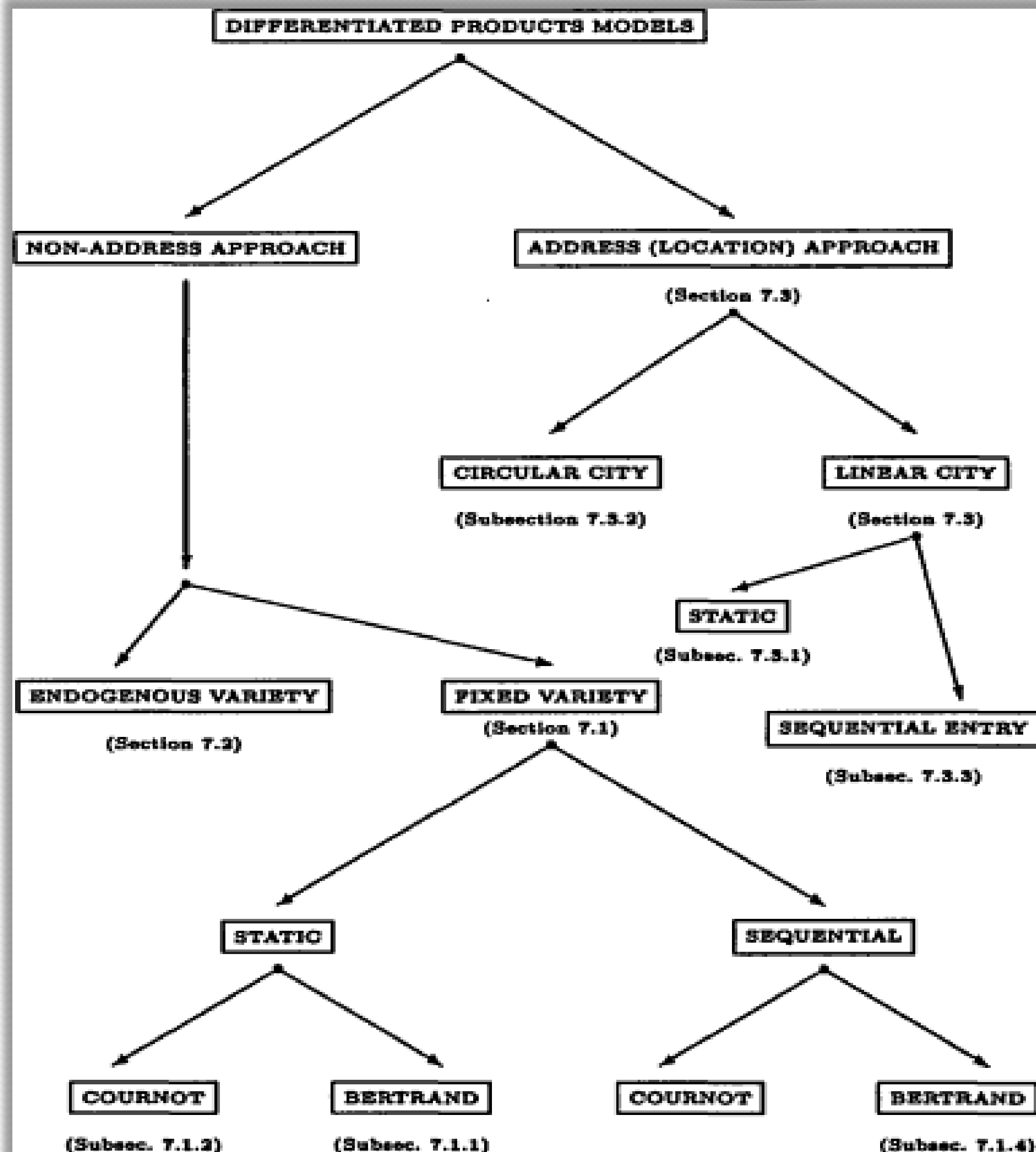
Characteristics **Buyers**	**HP/W**	**Air**	**MP$**	**Size**	**Price**
Graduate	5	0.5	0.1	1	-1
CEO	40	40	0	20	-1

Characteristics Approach

- Net utility: $u_{ik} = [(\sum_{j=1}^{4} b_{ij} c_{kj}) - p_k]$

Models Buyers	Geo	Porsche
Graduate		
CEO		

Alternative Classification



Simple (non-Address) Models

- Consider a two-firm industry producing two differentiated products indexed by i = 1, 2.
- To simplify the exposition, we assume that production is costless.
- Let the inverse demand functions to be:

$$p_1 = \alpha - \beta q_1 - \gamma q_2;\ p_2 = \alpha - \gamma q_1 - \beta q_2;\ \beta > 0;\ \beta^2 > \gamma^2$$

- Thus, we assume that that there is a fixed number of two brands and that each is produced by a different firm facing an inverse demand curve
- The last condition implies own price elasticity larger than cross price elasticity.

Simple (non-Address) Models

- Direct demand functions:

$$q_1 = a - bp_1 + cp_2; \; q_2 = a + cp_1 - bp_2$$

- Find values of a, b and c.

$$a = \frac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2}$$

$$b = \frac{\beta}{\beta^2 - \gamma^2} > 0$$

$$c = \frac{\gamma}{\beta^2 - \gamma^2}$$

Measure of Differentiation

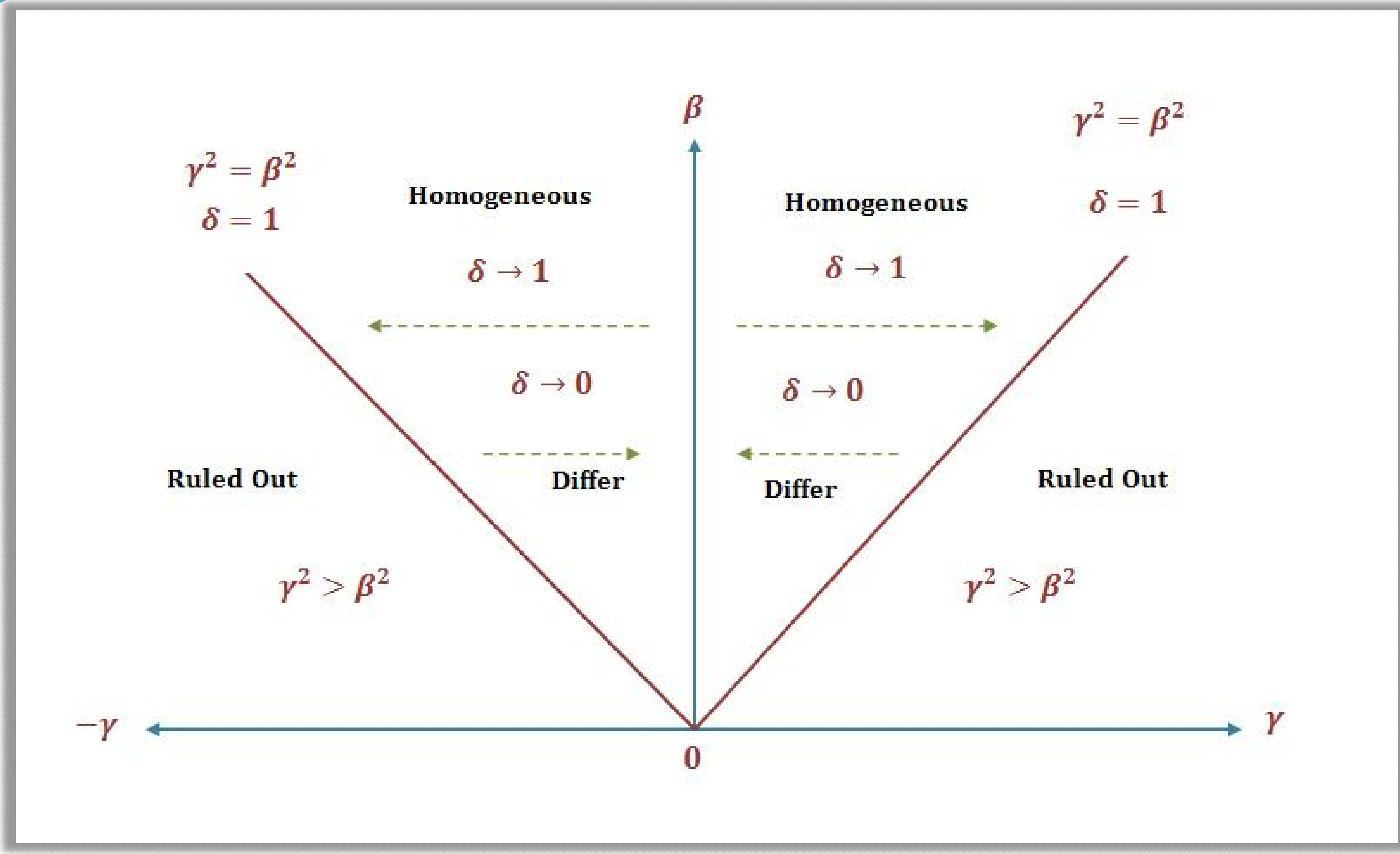
- Brand's measure of differentiation is given by

$$\delta = \frac{\gamma^2}{\beta^2}$$

- Brands are said to be highly differentiated when the change in one brand's price has negligible effect on the demand of the other brand, that is, when $\gamma \to 0$ and therefore, $\delta \to 0$.

- Brands are quite homogeneous when the cross and own price effects are almost the same. That is, price of both the brands affect each others prices significantly. Technically $\gamma^2 \to \beta^2$ and hence, $\delta \to 1$.

Zones of Differentiation



Quantity Game

- Cournot market structure – firms choose quantity produced as strategies
- we look for a Nash equilibrium in firms' output levels
- zero production costs
- inverse demand functions:

$$p_1 = \alpha - \beta q_1 - \gamma q_2$$

$$p_2 = \alpha - \gamma q_1 - \beta q_2$$

$$\beta > 0;\ \beta^2 > \gamma^2$$

Quantity Game

- Each firm i takes q_j as given and chooses q_i to

$$\max_{q_i} \pi_i(q_i, q_j) = (\alpha - \beta q_i - \gamma q_j) q_i ;\ i = 1, 2;\ i \neq j$$

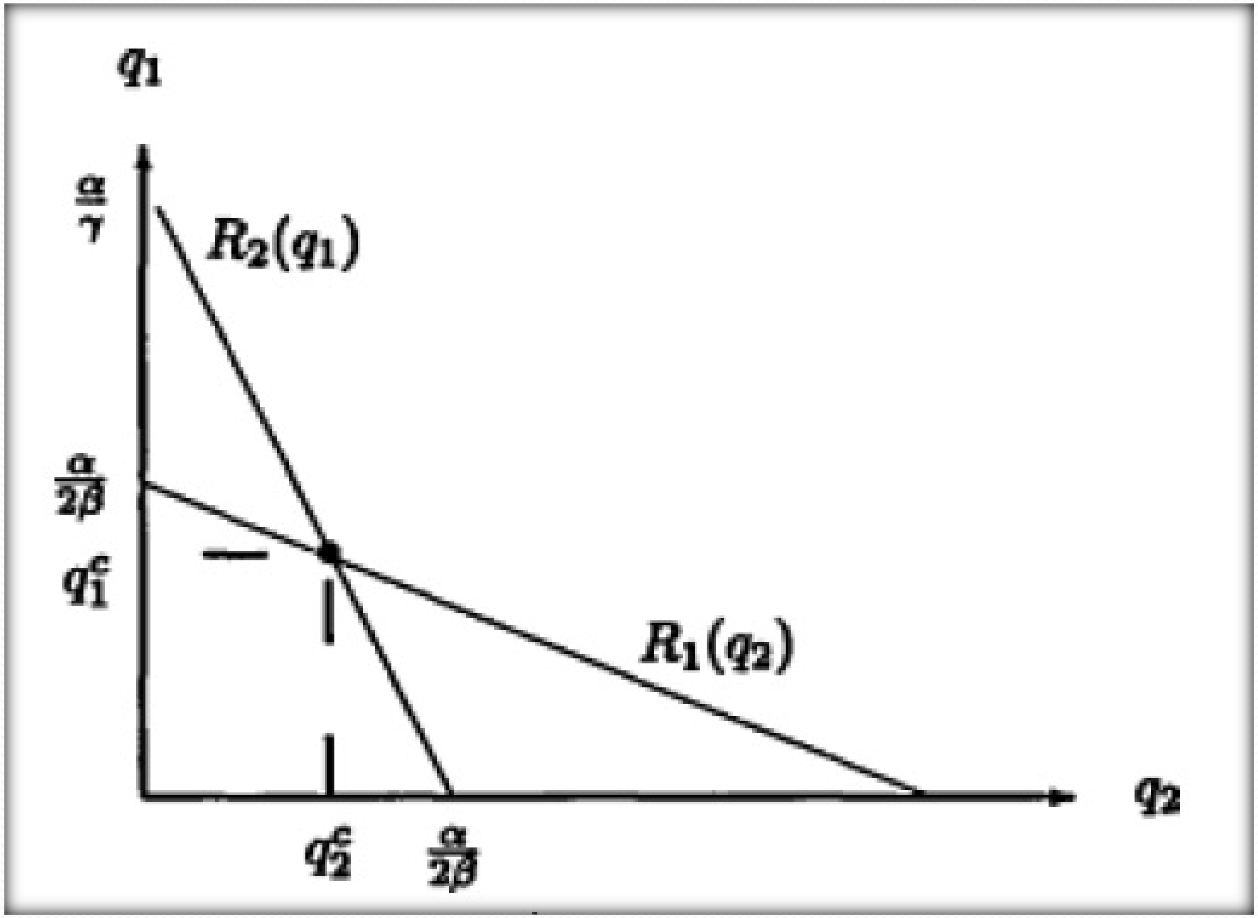
- FOCs: $$\frac{\partial \pi_i}{\partial q_i}(q_i, q_j) = (\alpha - 2\beta q_i - \gamma q_j) = 0$$

- Hence, the best response functions can be written as –

$$q_i = R_i(q_j) = \frac{\alpha - \gamma q_j}{2\beta}$$

Quantity Game

- Depiction of the best response functions:



Quantity Game

- the best response functions are:

$$R_1(q_2) = \frac{\alpha}{2\beta} - \frac{\gamma}{2\beta} q_2$$

$$R_2(q_1) = \frac{\alpha}{2\beta} - \frac{\gamma}{2\beta} q_1$$

- Note that as $\gamma \to \beta$ (the products are more homogeneous), the best-response function becomes steeper, thereby making the profit-maximizing output level of firm i more sensitive to changes in the output level of firm j (due to stiffer competition).

- When $\gamma \to 0$ (the products are more differentiated), the best-response function becomes constant (zero sloped)

Quantity Game

- Solving the best-response functions we have –

$$q_i^c = \frac{\alpha}{2\beta+\gamma};\ p_i^c = \frac{\alpha\beta}{2\beta+\gamma};\ \pi_i^c = \frac{\alpha^2\beta}{(2\beta+\gamma)^2}$$

- Note that as γ increases (the products are less differentiated), the individual and aggregate quantity produced, the prices, and the profits all decline

- ***In a Cournot game with differentiated products, the profits of firms increase when the products become more differentiated***

Quantity Game

- This result can explain why firms tend to spend large sums of money to advertise their brands
- Firms would like the consumers to believe that the brands are highly differentiated from the competing brands for the purpose of increasing their profits.
- In other words, differentiation increases the monopoly power of brand-producing firms

Price Game

- Bertrand market structure, firms choose prices as their strategies
- we look for a Nash equilibrium in firms' price levels
- zero production costs
- direct demand functions: $q_1 = a - bp_1 + cp_2;\ q_2 = a + cp_1 - bp_2$
- Remember: $a = \dfrac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2};\ b = \dfrac{\beta}{\beta^2 - \gamma^2} > 0;\ c = \dfrac{\gamma}{\beta^2 - \gamma^2}$

Price Game

- Each firm *i* takes p_j as given and chooses p_i to

$$\max_{p_i} \pi_i(p_i, p_j) = (a - bp_i + cp_j)p_i;\ i = 1, 2;\ i \neq j$$

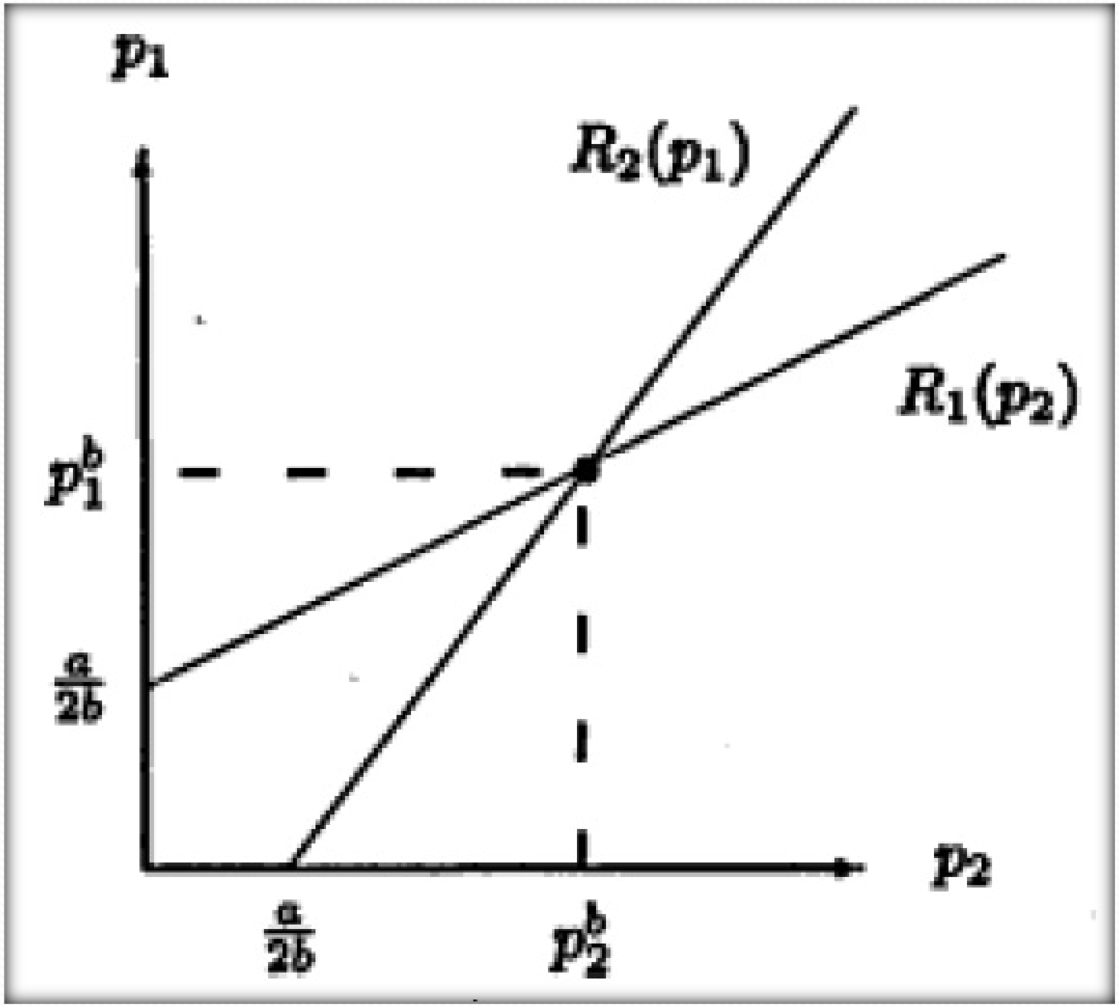
- FOCs: $\dfrac{\partial \pi_i}{\partial p_i}(p_i, p_j) = (a - 2bp_i + cp_j) = 0$

- Hence, the best response functions can be written as –

$$p_i = R_i(p_j) = \frac{a + cp_j}{2b}$$

Price Game

- Depiction of the best response functions:



Price Game

- Note that there is something different between the best response functions obtained in the Bertrand game vis-à-vis the Cournot game.

- In price games, the best-response functions are upward sloping, implying that, if one firm raises its price, the other would respond by raising its price as well

- Definitions –

1. Players' strategies are said to be ***strategic substitutes*** if the best-response functions are downward sloping
2. Players' strategies are said to be ***strategic complements*** if the best-response functions are upward sloping

Price Game

- Hence, in a quantity game the quantities are strategic substitutes, whereas in a price game prices are strategic complements.

- Quite intuitive: both are rational players maximizing profit

- Now, solving the best-response functions we get –

$$p_i^b = \frac{a}{2b-c} = \frac{\alpha(\beta-\gamma)}{2\beta-\gamma}$$

$$q_i^b = \frac{ab}{2b-c} = \frac{\alpha\beta}{(2\beta-\gamma)(\beta+\gamma)}$$

$$\pi_i^b = \frac{a^2 b}{(2b-c)^2} = \frac{\alpha^2\beta(\beta-\gamma)}{(2\beta-\gamma)^2(\beta+\gamma)}$$

Price Game

- The profit levels decline when the products become less differentiated (γ increases).

- In the limit, when $\gamma = \beta$, the products become homogeneous, and the profits drop to zero

- ***In a Bertrand game with differentiated products, the profits of firms increase when the products become more differentiated.***

- As with the Cournot case, product differentiation increases the monopoly power of brand-producing firms by loosening up price competition among the brand-producing firms

Cournot versus Bertrand

- Which market structure, a Cournot or a Bertrand, would yield a higher market price?

- How would changing the degree of product differentiation affect the relative difference between the two market structure outcomes?

- Formal comparison yields –

$$p_i^c - p_i^b = \frac{\alpha\beta}{2\beta+\gamma} - \frac{\alpha(\beta-\gamma)}{(2\beta-\gamma)} = \frac{\alpha}{4\dfrac{\beta^2}{\gamma^2}-1} > 0 \quad \cdots [\because \beta^2 > \gamma^2]$$

Cournot versus Bertrand

- Thus, in a differentiated products industry –

1. The market price under Cournot is higher than it is under Bertrand. Formally, $p_i^c > p_i^b$

2. The more differentiated the products are, the smaller the difference between the Cournot and Bertrand prices.

   Formally, $\dfrac{\partial(p_i^c - p_i^b)}{\partial\gamma} > 0$

3. This difference in prices is zero when the products become independent.

   Formally, $\lim_{\gamma\to 0}[p_i^c - p_i^b] = 0$

Cournot versus Bertrand

- Under Cournot market structure each firm expects the other firm to hold its output level constant.

- Hence, each firm would maintain a low output level since it is aware that a unilateral output expansion would result in a drop in the market price.

- In contrast, under the Bertrand market structure each firm assumes that the rival firm holds its price constant, hence output expansion will not result in a price reduction.

- Therefore, more output is produced under the Bertrand market structure than under the Cournot market structure.

Sequential Game

- So far we have analyzed industries where firms strategically choose their output/price levels.
- Both the games were static in the sense that players simultaneously choose their price/quantity produced.
- Let's we assume that the firms move in sequence.
- Best known example is the Stackelberg game which is a sequential version of the Cournot game (in a two-firm, sequential moves game, firm 1 will choose its output level before firm 2 does. Then, firm 2, after observing the output level chosen by firm 1, will choose its output level, and only then will output be sold and profits collected by the two firms).
- This type of market structure is often referred to as Leader-Follower structure

Sequential Game

- However we try to solve for a sequential game in prices (for the original Stackelberg game see 6.2)

- We analyze a two-stage game, where firm 1 (the leader) chooses the price in the first stage .
- The price chosen in the first stage is irreversible and cannot be adjusted in the second stage.
- In the second stage, only firm 2 (the follower) chooses its price after observing the price chosen by firm 1 in the first stage.
- Here, the game ends after the second stage, and each firm collects its profit.

Sequential Game

- Our main questions are –

1. Is there any advantage for moving in the first stage rather than the second? (One related and important question is who moves first? That we will analyze later with an established firm and a new entrant)

2. How would the outcomes (price, profit, quantity) compare to the static outcomes?

- Approach – backward induction (SPNE)

Sequential Game

- Direct demand functions:

$$q_1 = 168 - 2p_1 + p_2$$
$$q_2 = 168 + p_1 - 2p_2$$

- For these demand functions the static Bertrand outcomes are –

$$p_i^b = 56;\ q_i^b = 112;\ \pi_i^b = 6272$$

- In the first period, firm 1 takes firm 2's best-response function as given, and chooses p_1 that solves

$$\max_{p_1} \pi_1(p_1, R_2(p_1)) = (168 - 2p_1 + \frac{168 + p_1}{4})p_1$$

Sequential Game

- FOC: $$\frac{\partial \pi_1}{\partial p_1} = 210 - \frac{7}{2} p_1 = 0$$
$$\Rightarrow p_1^s = 60$$
$$\Rightarrow p_2^s = 57$$
$$\Rightarrow q_1^s = 105;\ q_2^s = 114$$
$$\Rightarrow \pi_1^s = 6300;\ \pi_2^s = 6498$$

- Therefore, $\pi_1^s > \pi_1^b;\ \pi_2^s > \pi_2^b$

Sequential Game

- Under a sequential-moves price game (or more generally, under any game where actions are strategically complements):

1. Both firms collect a higher profit under a sequential-moves game than under the single-period Bertrand game. Formally, $\pi_i^s > \pi_i^b$

2. The firm that sets its price first (the leader) makes a lower profit than the firm that sets its price second (the follower).

3. Compared to the Bertrand profit levels, the increase in profit to the first mover (the leader) is smaller than the increase in profit to the second mover (the follower). Formally, $\pi_1^s - \pi_1^b < \pi_2^s - \pi_2^b$

Sequential Game

- Thus, first to move is not always an advantage.

- Here, each firm would want the other firm to make the first move.

- The intuition behind this result is as follows.
- When firm 1 sets its price in period 1, it calculates that firm 2 will slightly undercut p_1 in order to obtain a larger market share than firm 1.
- This calculation puts pressure on firm 1 to maintain a high price to avoid having firm 2 set a very low market price.
- Hence, both firms set prices above the static Bertrand price levels.
- Now, firm 1 always makes a lower profit than firm 2, since firm 2 slightly undercuts firm 1 and captures a larger market share.

Sequential Game

- Finally, note that we could have predicted that the profit of firm 1 will increase beyond the static Bertrand profit level even without resorting to the precise calculations.

- Using a revealed profitability argument, we can see clearly that firm 1 can always set $p_1 = p_1^b$ and make the same profit as under the static Bertrand game.
- However, given that firm 1 chooses a different price, its profit can only increase.
- The profit of firm 2 (the follower's) is higher under the sequential-moves price game than its profit under the static Bertrand game.
- In contrast, under the sequential-moves quantity game the followers' profit is lower than it is under the static Cournot game.

Reference

- Oz Shy (1995). Industrial Organization. MIT Press. Chapter – 7.