Department of Industrial and Systems Engineering

Indian Institute of Technology, Kharagpur

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  Timings – Wednesday (10.00-11.00AM) Thursday (9.00-10.00AM) and Friday (11:00AM-01:00PM)

  Venue – Online MS Team
- Individual meetings: Email/phone

BACKGROUND, OBJECTIVE AND SCOPE

Operations Research – I is one of the compulsory courses in the 3rd Semester of B. Tech. and dual program in Industrial Engineering, QEDM, MF and elective course for other branches of engineering.

- ✓ Operations Research (OR) involves "research on operations". In other words, OR is applied to the problems that concern how to conduct and coordinate the operations within an organization.
- ✓ OR techniques have been widely applied to diverse areas such as manufacturing, transportation, telecommunication, military, finance, health care, and public services.

This course is intended

- ➢ to give an appreciation of the problems which can be approached through basic OR techniques
- ➢ to familiarize the students with the various techniques available
- ➢ to develop an understandings of the mechanics of such techniques, and
- ➢ to develop a general awareness of the applicability and limitations of OR methods and techniques.

At the end of the course, the students are expected to have a reasonable foundation in OR, based on which they can further build up and broaden their knowledge-base in order to deal with the much more complex real-world problems in their-related research and subsequent jobs.

TEXT BOOK AND REFERENCES

- **Introduction to Operations Research by F.S. Hillier and G.J. Lieberman**
- **Operations Research: an Introduction by H.A. Taha**
- Operations Research: Applications and Algorithms by W.L. Winston
- Operations Research: Principles and Practice by Ravindran, Phillips and Solberg

TENTATIVE SESSION PLAN

Topic	Readings from Hillier and Lieberman
Introduction to Operations Research	Chapter 1
Operations research Modeling Approach	Chapter 2
Introduction to Linear Programming	Chapter 3
Simplex Method	Chapter 4
Theory of Linear Programming	Chapter 5
Duality Theory and Sensitivity Analysis	Chapter 6
Dual Simplex Method	Chapter 7
Transportation, Assignment, Transshipment Problems	Taha
Integer Programming	Taha
Additional Topics	If time permits

Scope and Applicability of Operations Research (OR)

The Scope and applicability of OR as a field: Enormous

In this course:

Introductory

Applied side

Somewhat basic: Variety of backgrounds

Pre-requisites: Some knowledge of Matrix Algebra

Genesis of OR

World War II (WW II)

- UK, USA
- Military applications in transportation and logistics

Military Operations Research

Post - WW II, other organization (industrial, government)

- Industrial revolution (Boom)
- Size of firms ↑
- Complexity of operations ↑
- Subdivisions within organization

Later

Operations Research

(Management Science, Decision Science)

Basic Question

How to allocate scarce resources to various activities of an organization ?

→ Same military like problem in different contexts

MIT was one of the birthplaces of OR

Professor Philip M Morse at MIT was a pioneer in the US

Founded MIT OR Center and helped found ORSA

Definition of OR

OR is the branch of science dealing with *techniques* for *optimizing* (maximizing/minimizing) the *performance* (profit, market share, cost etc.) of *systems* (manufacturing company, service industry, Government organization, etc.)

Factors Influencing growth of OR

Initial

- Substantial research in OR (by military scientists mainly)
- Simplex method for solving Linear Programming (LP) Problem (by George Dantzig, 1947)
- LP, DP, Queuing theory, inventory theory 1950's

Later

Computer revolution

- Personal Computer (PC) with high capability
- Software packages for LP/IP/NLP: CPLEX, LINGO, SAS
- Current areas: SCM, Logistics, Healthcare, ERP

Features of OR

(i) Use of scientific method ("research")

- Careful observation of a phenomenon: also called data gathering, useful in problem definition
- Model for hypothesis: model should be good representation of reality
- Experiments: to test hypothesis
- Model validation: verify against fresh data

(ii) Search for optimality

find a "best" solution w.r.t. measure of performance

e.g. Typical problem structure (e.g. LP, IP, Transportation, Assignment)

Maximize/Minimize: OBJECTIVE FUNCTION

Subject to: CONSTRAINTS

However, not all problems in OR may follow this structure e.g. Forecasting, Queuing, Simulation (these are advanced topics which will not be covered in this course)

Features of OR Contd…

(iii) OR techniques rely on algorithms:

- iterative procedures
- computer application natural choice

(iv) For a full-fledged implementation, OR applications go beyond many discipline boundaries e.g. Mathematics, Statistics, Economics, Computer Science, Engineering

A team-like approach works best here

(v) Broadly applicable

- Different organizations (manufacturing, transportation, construction, telecom, military)
- Diverse problems:

Complex (airline scheduling)

Important (safety, health care): inventory management in blood bank

Unprecedented (New Product Development): Forecasting demand, choosing advertising levels for a new product to maximize awareness s.t. budget constraint

Steps in OR Modeling Approach

To Solve a real-world problem

Step I: Extract the important features of the problem

Step II: Construct a mathematical model

Step III: Solve the model

Step IV: Test the model

Step V: Apply the model

Steps in OR Modeling Approach

Step I: Extract the important features of the problem

(a) Define the problem

(b) Gather relevant data

(a) Define the problem

- Decision variables: unknown quantities whose value will provide solution to the problem
- Parameters: constant (given, fixed) values in objective functions/constraints
- Constraints: restrictions on the values of the decision variables
- Objective function: Target to be achieved

Optimizing: maximizing – profits, sales

minimizing – costs

Should give unique best solution

Satisfying: Good enough

Satisfactory + optimizing

Close to optimal, above a threshold

Different Objectives can lead to different solutions even for the same set of constraints

- A company manufactures two types of product

Product	Selling price per unit	Cost price per unit	Minimum production requirement
1	p_1	c_1	δ_1
2	p_2	c_2	δ_2

- Production capacity B units
- Suppose

\- $\delta_{1,}$ δ_2 much smaller than B

\- Selling price of product 1 is less than product 2, i.e. $p_1 < p_2$

\- Profit /unit on product 1 is greater than product 2, i.e. $p_1 - c_1 > p_2 - c_2$

- Parameters: p_1, p_2, c_1, c_2, B, δ_1, δ_2
- Decision Variables:

x_1 : number of units of products 1 to be produced

x_2: number of units of products 2 to be produced

- Objective: Maximize profit Vs. Maximize Sales price

  Max $(p_1 - c_1)x_1 + (p_2 - c_2)x_2$ Vs. Max $p_1x_1 + p_2x_2$

- Constraints:

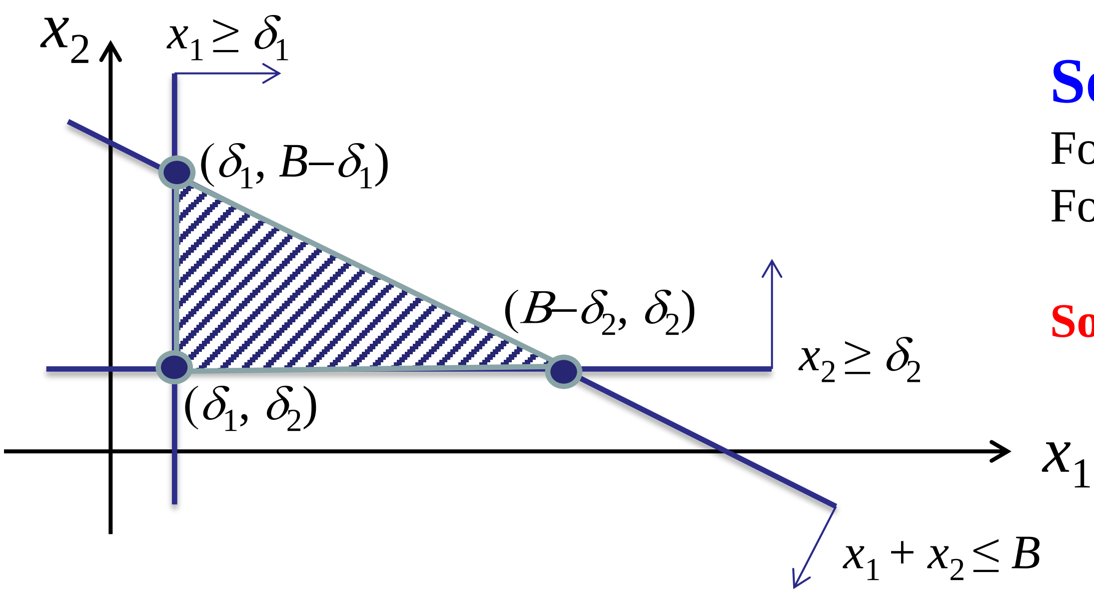
  Subject to

  $x_1 + x_2 \leq B$ (Production capacity constraint)

  $x_1 \geq \delta_1$ (Minimum production constraint for product 1)

  $x_2 \geq \delta_2$ (Minimum production constraint for product 2)

What picture emerges in x_1-x_2 plane?



Solution:

For profit maximization: $(B-\delta_2, \delta_2)$

For sales maximization: $(\delta_1, B-\delta_1)$

Solution for Cost minimization?

Steps in OR Modeling Approach contd…

(b) Gather relevant data

- Available / Secondary data: manager provides the data
- MIS
- Soft / qualitative / subjective data – rough, educated guess

Different types of organization may have different objectives

NPOs: usage or awareness maximization

Airline Industries: Multiple objectives

- customer satisfaction
- employee morale
- cost minimization/revenue maximization

Step II: Construct a mathematical model

- idealized representation of reality
- should be good (in terms of representation) and solvable (tractable). Otherwise model might be as complex as the reality
- Advantages of Model: Concise, comprehensive, programmable on computer
- Components: Parameters, Decision variables, Objective Function, Constraints,

Example: Product Mix Problem

Tech Edge Company imports electronic components that are used to assemble two different models (Deskpro and Portable) of personal computers. The company is currently interested in developing a weekly production plan for both models. The other relevant data is given below:

Model	**Assembly time (hour)**	**Special component (unit)**	**Warehouse Space (Sq m)**	**Profit per unit ($)**
Deskpro	3	0	8	50
Portable	5	1	5	40
Maximum Availability	150	20	300	

Decision Variables:

x_1 = Number of units of Deskpro produced

x_2 = Number of units of Portable produced

Max Z = 50 x_1 + 40 x_2

$3 x_1 + 5 x_2 \leq 150$ (Assembly time)

$x_2 \leq 20$ (Special component)

$8 x_1 + 5 x_2 \leq 300$ (Space)

$x_1, x_2 \geq 0$ (Non-negativity constraint)

Example: Diet Problem

The goal of the **diet problem** is to select a set of foods that will satisfy a set of daily nutritional requirement at minimum cost.

Parameters:

n : number of food items, $j = 1, 2, \ldots, n$

m: number of nutrients, , $i = 1, 2, \ldots, m$

c_j : cost per unit of food j

a_{ij}: amount of nutrient i available in per unit of food j

b_i: minimum nutritional requirement for nutrient i

Diet Problem: Formulation

Decision Variable:

x_j: amount of food j to be included in the diet, $j = 1, 2, \ldots, n$

Objective Function:

$$\text{Min } Z = \sum_{j=1}^{n} c_j x_j$$

Constraints

$$\text{S.t.} \quad \sum_{j=1}^{n} a_{ij} x_j \geq b_i, \qquad \forall i = 1, 2, \ldots, m$$

$$x_j \geq 0, \qquad \forall j = 1, 2, \ldots, n$$

Step III: Solve the model

- **Standard algorithm** can usually give optimal solution. Example: Simplex
- **Heuristic Solution:** decision based on intuition, can sometimes give good sub-optimal solution. **Example:**

Knapsack Problem –Given a set of items (j=1, 2, …, n) with a weight (w_j) and value (v_j), select the items so that the total weight is less than a given limit (W) and the total value is as large as possible.

- Decision variable: x_j =1 if item is included in knapsack; 0 Otherwise

Formulation

$$\max \sum_{j=1}^{n} v_j x_j ,$$

$$\text{subject to } \sum_{j=1}^{n} w_j x_j \leq W$$

$$x_j \in \{0,1\}, \forall j = 1,2,\ldots,n$$

Efficient Heuristic:
- Sort v_j / w_j in descending order
- Starting with the item with highest v_j / w_j ratio, pick items to include in knapsack till the total weight $\leq$ W

-Sensitivity analysis: How sensitive solution is to changes in parameter values

Step IV: Test the model

- Does model solution translate to problem solution
- Since model is representation of reality: what is the role of assumptions
- Retrospective test: use past, reconstruct history, test model recommendations
- Face validity: Does it make sense?

Step V: Apply the model in actual decision making

- Model should be part of a large system used to help managers in making better decisions: DSS approach
- Top management support
- Careful documentation: solution procedures, operating procedures

Other Linear Programming Formulations

Steps in LP Model Formulation

Step 1: Identify the decision variables

Unknown quantities whose value will provide solution of the problem

Step 2: Identify the objective function

Target to be achieved

Optimizing: maximizing – profits, sales

minimizing – costs

Should give unique best solution

Step 3: Identify the constraints

restrictions on the values of the decision variables

Step 4: Construct mathematical expressions for objective function and constraints

Example: Advertising Media Selection

An advertising company wishes to plan an advertising campaign in three different media - television, radio and magazines. The purpose of the advertising program is to reach as many potential customers as possible. Results of a market study are given below:

	Television			
	Daytime	Prime time	Radio	Magazines
Cost of an advertising unit	\$40,000	\$75,000	\$30,000	\$15,000
Number of potential customers reached per unit	400,000	900,000	500,000	200,000
number of women customers reached per unit	300,000	400,000	200,000	100,000

The company does not want to spend more than \$800,000 on advertising. Further

(1) at least 2 million exposures take place among women;

(2) advertising on television be limited to \$500,000;

(3) at least 3 advertising units be bought on daytime television, and 2 units during prime time; and

(4) the number of advertising units on radio and magazine should each be between 5 and 10.

Advertising Media Selection: LP Formulation

Decision Variables :

x_1 = Number of advertising units bought in daytime television

x_2 = Number of advertising units bought in prime time television

x_3 = Number of advertising units bought in radio

x_4 = Number of advertising units bought in magazines

Objective: Maximize $Z = 400{,}000x_1 + 900{,}000x_2 + 500{,}000x_3 + 200{,}000x_4$

Subject to:

$40{,}000x_1 + 75{,}000x_2 + 30{,}000x_3 + 15{,}000x_4 \leq 800{,}000$ (advertising budget)

$300{,}000x_1 + 400{,}000x_2 + 200{,}000x_3 + 100{,}000x_4 \geq 2{,}000{,}000$ (women customers)

$40{,}000x_1 + 75{,}000x_2 \leq 500{,}000$ (television advertisement)

$x_1 \geq 3$ (advertising units on daytime)

$x_2 \geq 2$ (advertising units on prime time)

$x_3 \geq 5$ (advertising units on radio)

$x_3 \leq 10$ (advertising units on radio)

$x_4 \geq 5$ (advertising units on magazines)

$x_4 \leq 10$ (advertising units on magazines)

Example: Load Balancing Problem

A machine shop has one drill and five milling machines, which are to be used to produce an assembly of two parts, 1 and 2. The productivity of each machine for the two parts is given below:

	Production time in minutes per piece	
Part	Drill	Milling
1	3	20
2	5	15

It is desired to maintain a balanced loading on all machines such that no machine runs more than 30 minutes per day longer than any other machine (assume that the milling load is distributed equally among all five milling machines).

Divide the work time of each machine to obtain the maximum number of completed assemblies assuming an 8-hour working day.

Load Balancing Problem: LP Formulation

Decision Variables:

x_1 = number of part 1 produced per day

x_2 = number of part 2 produced per day

y = number of completed assemblies, then y = minimum of $(x_{1,} x_2)$

Objective: Maximize y = Maximize (minimum of $(x_{1,} x_2)$)

Equivalent to this: Maximize y,

Subject to $y \leq x_1$, $y \leq x_2$

Load on each milling m/c (in minutes)= $(20x_1 + 15x_2)/5 = 4x_1 + 3x_2$

Load on the drill m/c (in minutes)= $3x_1 + 5x_2$

Total time available per day = 8x60 = 480 minute

Thus, the time restriction on each machine

For each milling m/c, $4x_1 + 3x_2 \leq 480$

For the drill press, $3x_1 + 5x_2 \leq 480$

The load balance constraint can be represented by

$$|(4x_1 + 3x_2) - (3x_1 + 5x_2)| \leq 30$$

or, $| x_1 - 2x_2 | \leq 30$ (non-linear constraint)

equivalent to this: $x_1 - 2x_2 \leq 30$ and $-x_1 + 2x_2 \leq 30$

Load Balancing Problem: Complete Formulation

Objective:

Maximize y

Subject to:

$y \leq x_1$

$y \leq x_2$

$4x_1 + 3x_2 \leq 480$ (Milling Machine)

$3x_1 + 5x_2 \leq 480$ (Drill Press)

$x_1 - 2x_2 \leq 30$ (Balance Constraint)

$-x_1 + 2x_2 \leq 30$ (Balance Constraint)

$x_1 \geq 0$,

$x_2 \geq 0$

$y \geq 0$

Example: Marketing Research

A market survey company (MSC) specializes in evaluating consumer reaction to new products, services, and advertising campaigns. A client firm requested MSC's assistance in ascertaining consumer reaction to a recently marketed household product. During meetings with the client, MSC agreed to conduct door-to-door personal interviews to obtain responses from households with children and households without children. In addition, MSC agreed to conduct both day and evening interviews. Specifically, the client's contract called for MSC to conduct 1000 interviews under the following quota guidelines.

i.Interview at least 400 households with children.

ii.Interview at least 400 households without children.

iii.The total number of households interviewed during the evening must be at least as great as the number of households interviewed during the day.

iv.At least 40% of the interviews for households with children must be conducted during the evening.

v.At least 60% of the interviews for households without children must be conducted during the evening.

Household	**Cost per interview (in $)**	
	Day	**Evening**
Children	20	25
No children	18	20

Face validity check

Per unit cost of interviewing a household with children in evening higher, because

evening -> overtime to interview

HH/C/E -> take longer to interview

What is the household, time-of-day interview plan that will satisfy the contract requirements at a minimum total interviewing cost?

Marketing Research: LP Formulation

Decision variable:

DC = number of daytime interviews of households with children

EC = number of evening interviews of households with children

DNC = number of daytime interviews of households without children

ENC = number of evening interviews of households without children

Minimize $Z = 20DC + 25EC + 18DNC + 20ENC$

Subject to

$DC + EC + DNC + ENC = 1000$ (Total Interviews) (1)

$DC + EC \geq 400$ (Households with children) (2)

$DNC + ENC \geq 400$ (Households without children) (3)

$EC + ENC \geq DC + DNC$ (Evening interviews) (4)

$EC \geq 0.4(DC + EC)$ (Evening interviews in households with children) (5)

$ENC \geq 0.6(DNC + ENC)$ (Evening interviews in households without children) (6)

$DC, EC, DNC, ENC \geq 0$

Bus Scheduling Problem

City bus service is planning to introduce a mass-transit bus system. The number of buses required during each 4-hour shift is given below. To carry out the required daily maintenance, each bus can operate 8 successive hours a day only. Schedule the buses to meet the each 4-hr shift requirement while minimizing the total number of buses in operation.

Shift	12am-4am	4am-8am	8am-12pm	12pm-4pm	4pm-8pm	8pm-12am
Busses needed	6	8	10	7	12	4

Bus Scheduling Problem: Formulation

- **Decision variables:**

x_1= number of buses starting at 12:00 am

x_2= number of buses starting at 4:00 am

x_3= number of buses starting at 8:00 am

x_4= number of buses starting at 12:00 pm

x_5= number of buses starting at 4:00 pm

x_6= number of buses starting at 8:00 pm

Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

Subject to

$x_1 + x_6 \geq 6$	(12:00 am - 4:00 am)
$x_1 + x_2 \geq 8$	(4:00 am - 8:00 am)
$x_2 + x_3 \geq 10$	(8:00 am - 12:00 noon)
$x_3 + x_4 \geq 7$	(12:00 pm - 4:00 pm)
$x_4 + x_5 \geq 12$	(4:00 pm - 8:00 pm)
$x_5 + x_6 \geq 4$	(8:00 pm - 12:00 am)

$x_j \geq 0, j = 1,2,..,6$

Generalized Product Mix Problem

Company can manufacture a set of products using a set of resources. What should be the production quantity of each product so that the total profit is maximized.

Parameters:

n : number of products, $j = 1, 2, \ldots, n$

m: number of resources, $i = 1, 2, \ldots, m$

c_j : profit from per unit of product j

a_{ij}: amount of resource i required to manufacture per unit of product j

b_i: availability of resource i

Generalized formulation: Product Mix Problem

Decision Variable:

x_j: amount of product j to be manufactured, $j = 1, 2, \ldots, n$

Objective Function:

$$\text{Max } Z = \sum_{j=1}^{n} c_j x_j$$

Constraints

S.t. $\sum_{j=1}^{n} a_{ij} x_j \le b_i, \forall i = 1, 2, \ldots, m$ (1)

$x_j \ge 0, \forall j = 1, 2, \ldots, n$ (2)

Terminology

Eq. (1): resource availability or Functional or technological constraints

Eq. (2): Non-negativity constraint

c_j: Profit coefficient (cost coefficient in case of cost)

a_{ij}: Technological coefficient

b_j: resource availability