

Indian Institute of Technology Kharagpur  
Mid-Semester Examination: Spring 2023

Date of Examination: 20/02/2023 (AN)

Duration: 2 Hrs

Subject. No: MA20208

Subject Name: Stochastic Processes and Applications

Department: Mathematics

TOTAL MARKS: 30

Specific Chart, graph paper log book etc. required: None

Special Instruction: None

**ANSWER ALL THE QUESTIONS**

1. State whether the following statements are **TRUE** or **FALSE**. Marks will not be awarded unless the answers are supported by proper justification (in the form of a proof of the statement or a counter example). [10 marks]

- (a) Sum of  $n$  independent exponential random variables with mean  $\lambda$  is a Gamma random variable with mean  $n\lambda$ .
- (b) Let  $N_1, N_2, N_3$  be independent Poisson random variables with mean  $\lambda_1, \lambda_2, \lambda_3$  respectively. Then  $N_1 + N_2 + N_3$  is also a Poisson random variable with mean  $\lambda_1 + \lambda_2 + \lambda_3$ .
- (c) Let  $X_1$  denote the waiting time for the first arrival in a Poisson process  $\{N(t)\}_{t \geq 0}$  with rate  $\lambda > 0$ . Then the conditional density of  $X_1$  given  $N(t) = 1$  is  $U(0, t)$ .
- (d) For discrete random variables  $X$  and  $Y$ ,  $E(X) = E(E(X|Y))$ .
- (e) Let  $Y_1, Y_2, \dots$  be a sequence of iid random variables and  $N$  denote a non-negative integer valued random variable. Then

$$E\left(\sum_{i=1}^N Y_i\right) = E(N)E(Y_1)$$

2. Let  $\{N(t)\}_{t \geq 0}$  be a Poisson process with rate  $\lambda = 3$ . Compute the conditional probability

$$P(N(3) = 2 \mid N(6) = 4)$$

[5 marks]

3. Let  $\{W(t)\}_{t \geq 0}$  be a compound Poisson process with

$$W(t) = \sum_{i=1}^{N(t)} Y_i$$

where  $\{N(t)\}_{t \geq 0}$  is a Poisson process with rate  $\lambda = 1$  and the random variables  $Y_i$  are iid and independent of  $\{N(t)\}$  with the pmf given by

$$P(Y_i = y) = \begin{cases} \frac{y}{10} & \text{for } y = 1, 2, 3, 4, \\ 0 & \text{otherwise} \end{cases}$$

Compute  $P(W(4) = 4)$ .

[5 marks]

4. If customers arrive at an ATM machine according to Poisson process with rate 10 customers per half an hour. The only service available at the machine is withdrawal of money. A customer withdraws a sum of greater than or equal to Rs. 10,000 with probability 0.1. Then what is the probability that exactly 4 customers will withdraw more than Rs. 10,000 from 9 AM to 12:00 noon? [5 marks]

A car driver breaks traffic rules according to a Poisson process with rate 1 rule break per two months. The driver's driving permit is suspended after breaking 10 traffic rules. Compute the mean and variance of the time (in months) before the driver's permit is suspended. [5 marks]

\*\*\*\*\* THE END \*\*\*\*\*