

# Artificial Intelligence Foundations and Applications

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# Planning

Deterministic Planning

# Planning

- **Planning** is one of the classic AI problems
- E.g. how can a robot figure out a sequence of actions to solve some problem?
- It is part of the logic-oriented tradition of AI

Autonomous vehicle navigation

Process control

Assembly line

Military operations

Travel planning

design and manufacturing environments

military operations, games, space exploration

- Military operations
- Autonomous space operations
- Construction tasks
- Machining tasks
- Mechanical assembly
- Design of experiments in genetics
- Command sequences for satellite

# What is Planning?

The task of finding a course of action to achieve goals

- Given

- a logical description of the **world states**
- a logical description of a set of **possible actions**
- a logical description of the **initial situation**, and
- a logical description of the **goal conditions**

- Find

- a **sequence of actions** (a **plan of actions**) that brings us from the initial situation to a situation in which the goal conditions hold.

Classical planning:

Environment

- Fully observable
- Deterministic
- Static
- Discrete

# Example: A Robot that Makes Tea

1. put water in the kettle
2. heat the kettle
3. get a cup
4. pour hot water into the cup (after the water is hot enough)
5. get a tea bag
6. leave the tea bag in the water for enough time
7. remove the tea bag
8. add milk
9. add sugar
10. stir until mixed

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- **Sub-actions**, are often needed
- e.g. “get a cup” might consist of the actions
  - A. Move to the cupboard
  - B. Open the cupboard door
  - C. Grasp a cup
  - D. Take it out of the cupboard
  - E. Close the cupboard
- Any of these actions could further broken down into smaller actions
  - What are the exact muscle and finger movements needed to grasp a cup?

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- **Exceptional** situations might trigger entire **sub-plans**
  - E.g. if there's no milk, the “get milk” action might trigger one of the following sub-plans
    - Go to the store to buy milk
    - Borrow some milk from a neighbor
    - Find a substitute for milk
    - Make something other than tea
    - Etc.
- Actions might also **fail**, e.g. the cup could slip and fall to the floor, or opening the sugar could spill the sugar on the floor
  - Must fix, or re-do, failed actions

# Example: A Robot that Makes Tea

- **Timing** and **sensing** are also important
  - The kettle can't be heated for too short a time or too long a time.
  - Does the order in which milk and sugar are added matter? Could they be added at the same time?
  - The robot might need to taste the tea to decide if more milk/sugar is needed.
  - How does the robot know when the mixing is done?

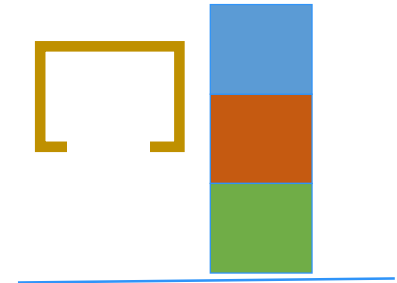


# Example: Dialog Planning

- Supermarket conversation:
  - Customer: Can you tell me where I can find the bread?
  - Employee: It's in aisle 2.
  - Customer: Thanks!
- A dialog like this can be modelled as a planning problem
  - The customer's goal is to get bread
  - He could do that in various ways, e.g. walking around the store searching for bread, finding a map that says where bread is, asking someone for help, etc.
  - In this dialog, the customer has chosen to try the “ask someone for help” action

# Represent this Blocks World

- A robot arm (yellow) can **pick up** and **put down** blocks to form stacks.
- It cannot pick up a block that has another block on top of it.
- It cannot pick up more than one block at a time.
- Any number of blocks can sit on the table.

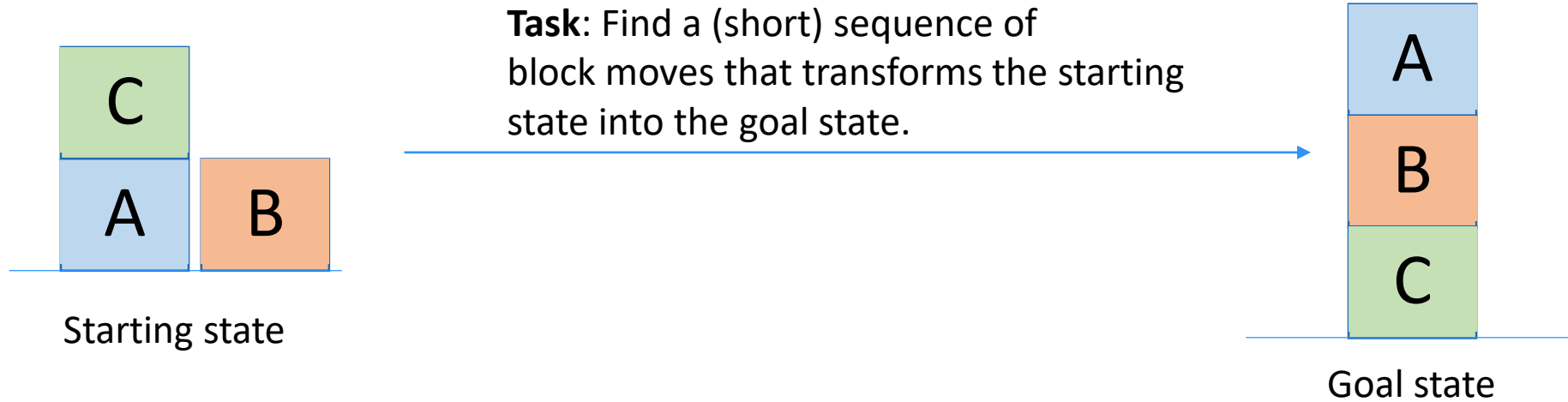


- How would you represent this block world to use logic to find a plan?
- You need to represent the states, actions, goals, transitions.

## Classical Planning:

- + concise object representation and clearer action definitions
- — only works for deterministic fully observable worlds

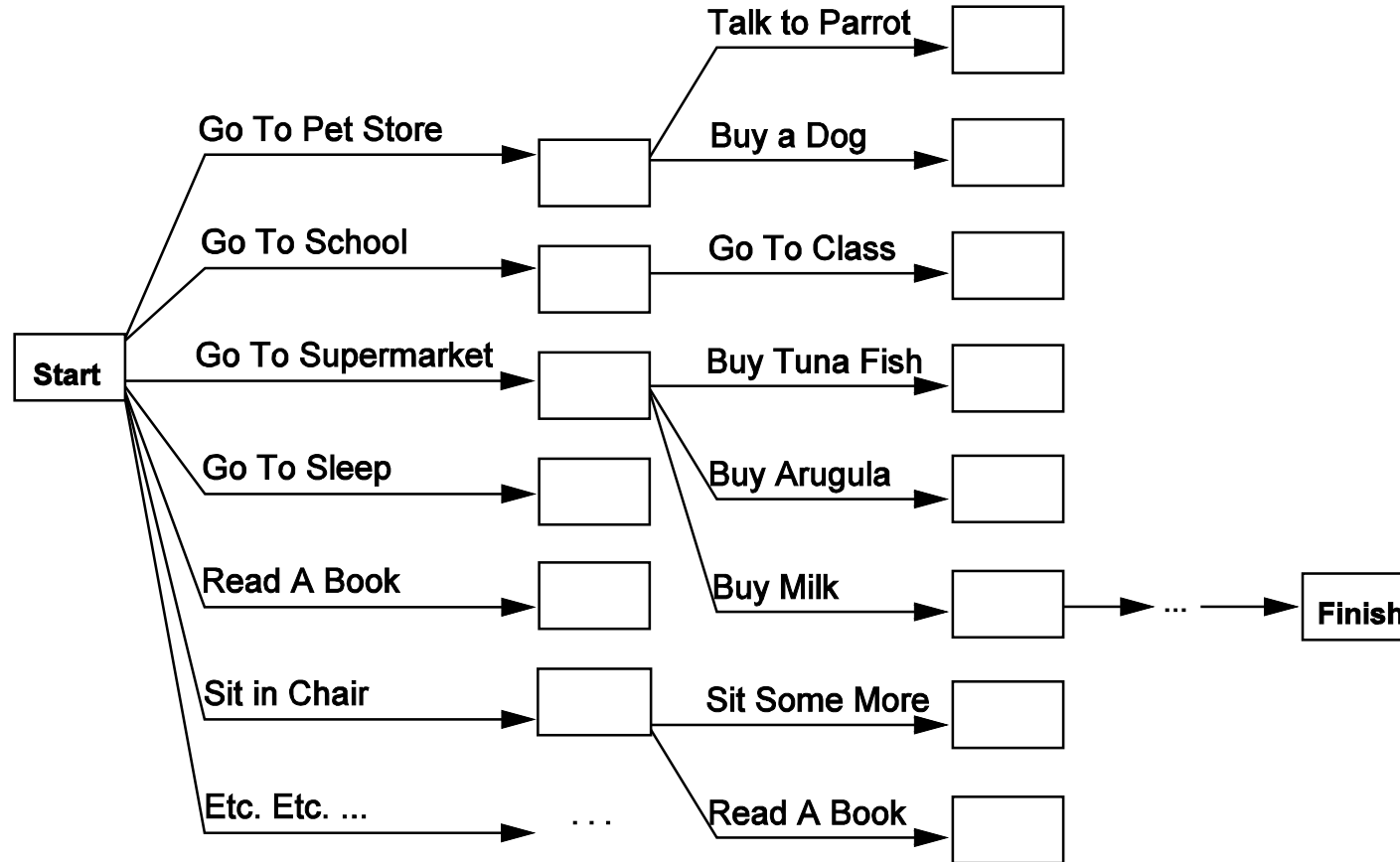
# Blocks World



# Planning vs. General Search

- Basic difference: Explicit, logic-based representation
- **States/Situations:** descriptions of the world by logical formulae
  - Agent can explicitly reason about and communicate with the world.
- **Operators/Actions:** Axioms or transformation on formulae in a logical form
  - Agent can gain information about the effects of actions by inspecting the operators.
- **Goal conditions** as logical formulae vs. goal test (black box)
  - Agent can reflect on its goals.

# Challenges in Planning – too many choices!



# Languages for Planning Problems

- STRIPS
  - Stanford Research Institute Problem Solver
  - Historically important
- ADL
  - Action Description Languages
- PDDL
  - Planning Domain Definition Language
  - Revised & enhanced for the needs of the International Planning Competition

# State of the world (STRIPS language)

- State of the world = conjunction of positive, ground, function-free literals
- `At(Home)` AND `IsAt(Umbrella, Home)` AND `CanBeCarried(Umbrella)` AND `IsUmbrella(Umbrella)` AND `HandEmpty` AND `Dry`
- Not OK as part of the state:
  - `NOT(At(Home))` (negative)
  - `At(x)` (not ground)
  - `At(Bedroom(Home))` (uses the function `Bedroom`)
- Any literal not mentioned is assumed false
  - Other languages make different assumptions, e.g., negative literals part of state, unmentioned literals unknown

# Action Representation

- Action Schema

- Action name
- Preconditions
- Effects

- Example

*Action*(Fly(p,from,to),

PRECOND:  $\text{At}(p,\text{from}) \wedge \text{Plane}(p) \wedge \text{Airport}(\text{from}) \wedge \text{Airport}(\text{to})$

EFFECT:  $\neg \text{At}(p,\text{from}) \wedge \text{At}(p,\text{to})$ )

- Sometimes, Effects are split into ADD list and DELETE list

$\text{At}(\text{WHI},\text{LNK}), \text{Plane}(\text{WHI}),$   
 $\text{Airport}(\text{LNK}), \text{Airport}(\text{OHA})$

$\text{Fly}(\text{WHI},\text{LNK},\text{OHA})$

$\text{At}(\text{WHI},\text{OHA}), \neg \text{At}(\text{WHI},\text{LNK})$



# Action TakeObject

TakeObject(location, x)

## Preconditions:

- HandEmpty
- CanBeCarried(x)
- At(location)
- IsAt(x, location)

**Effects** (“NOT something” means that that something should be removed from state):

- Holding(x)
- NOT(HandEmpty)
- NOT(IsAt(x, location))

## WalkWithUmbrella

(location1, location2, umbr)

- Preconditions:
  - At(location1)
  - Holding(umbr)
  - IsUmbrella(umbr)
- Effects:
  - At(location2)
  - NOT(At(location1))

## WalkWithoutUmbrella

(location1, location2)

- Preconditions:
  - At(location1)
- Effects:
  - At(location2)
  - NOT(At(location1))
  - NOT(Dry)

# A goal and a plan

Goal: At(Work) AND Dry

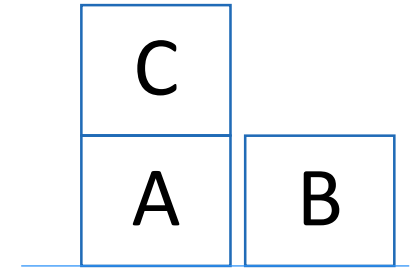
Initial state:

- At(Home) AND IsAt(Umbrella, Home) AND CanBeCarried(Umbrella) AND IsUmbrella(Umbrella) AND HandEmpty AND Dry
- TakeObject(Home, Umbrella)
  - At(Home) AND CanBeCarried(Umbrella) AND IsUmbrella(Umbrella) AND Dry AND Holding(Umbrella)
- WalkWithUmbrella(Home, Work, Umbrella)
  - At(Work) AND CanBeCarried(Umbrella) AND IsUmbrella(Umbrella) AND Dry AND Holding(Umbrella)

# Classical Planning Model

- Classical planning model is a tuple  $S = \langle S, s_0, S_G, A, f, c \rangle$ 
  - Finite and discrete state space  $S$
  - A known initial state  $s_0 \in S$
  - A set  $S_G \subseteq S$  of goal states
  - Actions  $A(s) \subseteq A$  applicable in each  $s \in S$
  - A deterministic transition function
$$s' = f(a, s)$$
  - Non-negative action costs  $c(a, s)$

# Input Representation



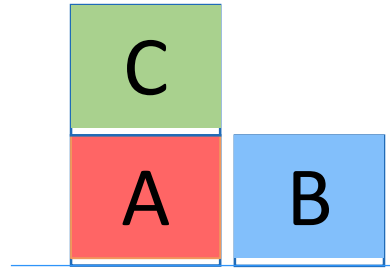
- Description of initial state of world
  - Conjunction of propositions:

block (a), block (b), block (c),  
on-table (a), on-table(b), clear (a), clear (b), clear (c), arm-empty()

Generalize with variables

- **block(x)** means object x is a block
  - the table is an object, but not a block
- **on(x, y)** means object x is on top of object y
  - on(x,x) is not allowed, i.e. an object can't be on top of itself
  - on(x,y) and on(y,x) cannot *both* be true at the same time
- **clear(x)** means there is nothing on top of object x
  - without this, we'd have to use quantified statements like "for all blocks y, on(y,x) is false"

# Blocks World Representation

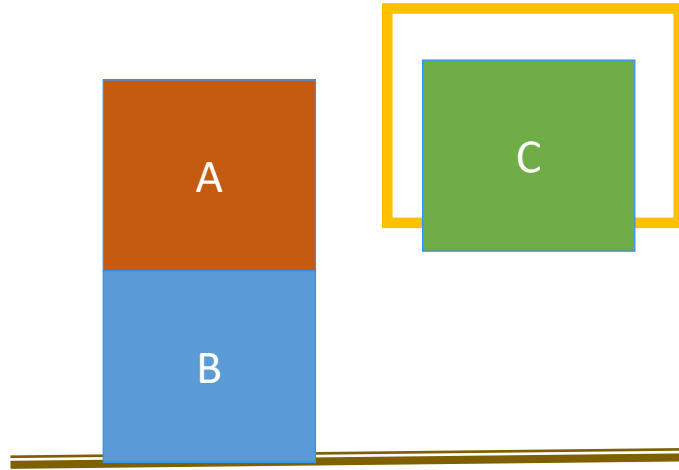


## Representation of this State

- `block(A)`, `block(B)`, `block(C)`
- `on(C, A)`
- `on-table (A)`, `on-table(B)`
- `clear(C)`, `clear(B)`

All terms are and-ed together

# How to represent this state?



- `block(A)`, `block(B)`, `block(C)`
- `on-block (A, B)`
- `on-table(B)`
- `clear(A)`
- `In-hand(C)`

# Goal Description

Description of goal: i.e. set of worlds

- E.g., Logical conjunction
- Any world satisfying conjunction is a goal  
and (on-block (a, b) , on-block (b ,c))



# Actions / Operators

Operators **change** the state by adding/deleting predicates

Preconditions:

Actions can be applied only if all precondition predicates are true in the current state

Effects:

New state is a copy of the current predicates with the addition or deletion of specified predicates

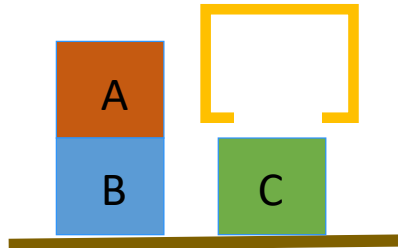
# Pickup Block C from Table (State Transition)

## Instances:

Blocks A, B, C

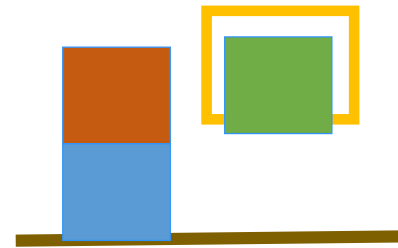
## Possible Predicates:

HandEmpty()  
On-Table(block)  
On-Block(b1,b2)  
Clear(block)  
In-Hand(block)



### State:

HandEmpty()  
On-Table(B)  
On-Table(C)  
On-Block(A,B)  
Clear(A)  
Clear(C)



### State:

In-Hand(C)  
On-Table(B)  
On-Block(A,B)  
Clear(A)  
Clear(C)

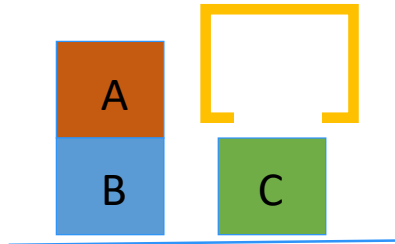
# Pickup Block C from Table (Preconditions, Effects)

## Instances:

Blocks A, B, C

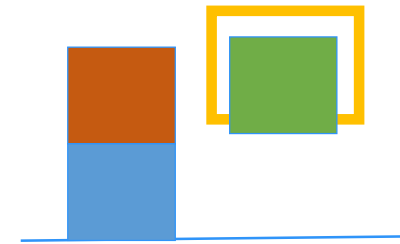
## Possible Predicates:

HandEmpty()  
On-Table(block)  
On-Block(b1,b2)  
Clear(block)  
In-Hand(block)



## State:

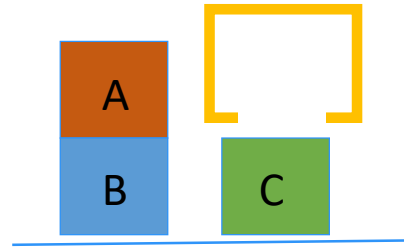
HandEmpty()  
On-Table(B)  
On-Table(C)  
On-Block(A,B)  
Clear(A)  
Clear(C)



## State:

In-Hand(C)  
On-Table(B)  
On-Block(A,B)  
Clear(A)  
Clear(C)  
Delete HandEmpty()  
Delete On-Table(C)

# Operator: Pickup-Block-C from Table

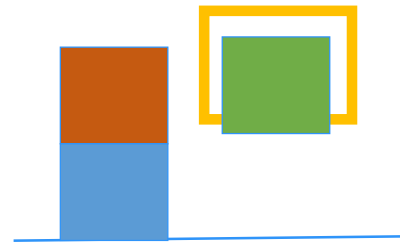


## Preconditions

HandEmpty()

Clear(C)

On-Table(C)



## Effects

Add In-Hand(C)

Delete HandEmpty()

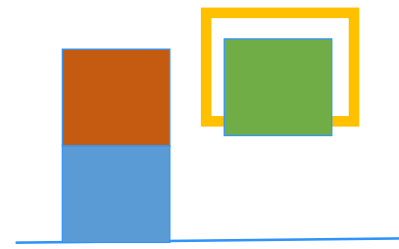
On-Table(C)

# Operator: Pickup-Block from Table



## Preconditions

HandEmpty() Add  
Clear(block)  
On-Table(block)



## Effects

In-Hand(block)  
Delete HandEmpty()  
On-Table(block)

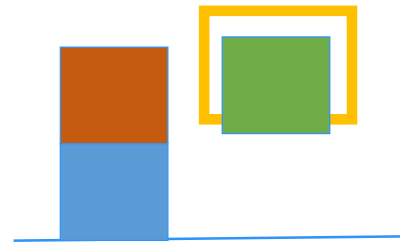
Create a **variable** that takes on the value of a particular instance for **all times** it appears in an operator.

# Operator: PutDown-Block on Table



Preconditions

In-Hand(block)



Effects

Add HandEmpty()

On-Table(block)

Delete In-Hand(block)

Why do we not need to check if  $\sim$ HandEmpty() is true?

# Operators for Block Stacking

## Pickup\_Table(b):

Pre: HandEmpty(), Clear(b), On-Table(b)

Add: In-Hand(b)

Delete: HandEmpty(), On-Table(b)

## Pickup\_Block(b,c):

Pre: HandEmpty(), On-Block(b,c),  $b \neq c$

Add: In-Hand(b), Clear(c)

Delete: HandEmpty(), On-Block(b,c)

## Putdown\_Table(b):

Pre: In-Hand(b)

Add: HandEmpty(), On-Table(b)

Delete: In-Hand(b)

## Putdown\_Block(b,c):

Pre: In-Hand(b), Clear(c)

Add: HandEmpty(), On-Block(b,c)

Delete: Clear(c), In-Hand(b)

Why do we need separate operators for table vs on a block?

# Example Matching Operators

HandEmpty() & On-Table(O) & On-Block(B,O) & Clear(B) & On-Table(G) & Clear(G)

Pickup\_Block(b,c):

Pre: HandEmpty(), On-Block(b,c),  $b \neq c$

Add: In-Hand(b), Clear(c)

Delete: HandEmpty(), On(b,c)

Pickup\_Table(b):

Pre: HandEmpty, Clear(b), On-Table(b)

Add: In-Hand(b)

Delete: HandEmpty(), On-Table(b)





# Finding Plans with Symbolic Representations

## Breadth-First Search

Sound?      **Yes**

Complete?   **Yes**

Optimal?     **Yes**

**Soundness** - all solutions found are legal plans

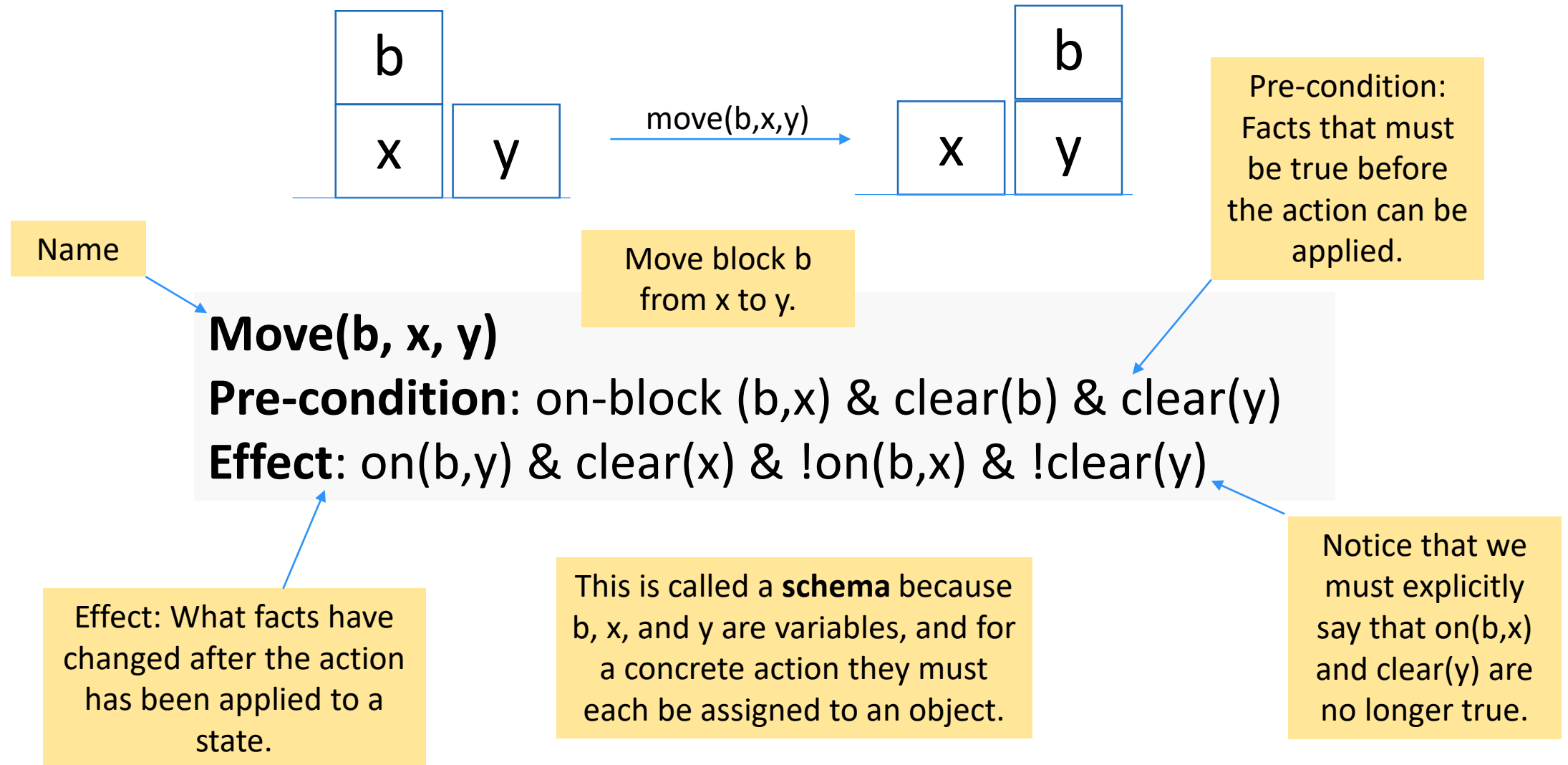
**Completeness** - a solution can be found whenever one actually exists

**Optimality** - the order in which solutions are found is consistent with some measure of plan quality

# Linear Planning

- Since we have a conjunction of goal predicates, let's try to solve one at a time
  - Maintain a stack of achievable goals
  - Use BFS (or anything else) to find a plan to achieve that single goal
  - Add a goal back on the stack if a later change makes it violated

# Blocks World: Action Schemas



# Blocks World

MoveToTable(b,x) moves  
block b from the top of x  
onto the table

**MoveToTable(b, x)**

**Pre-condition:**  $\text{on}(b,x) \ \& \ \text{clear}(b)$

**Effect:**  $\text{on}(b,\text{table}) \ \& \ \text{clear}(x) \ \& \ \neg \text{on}(b,x)$

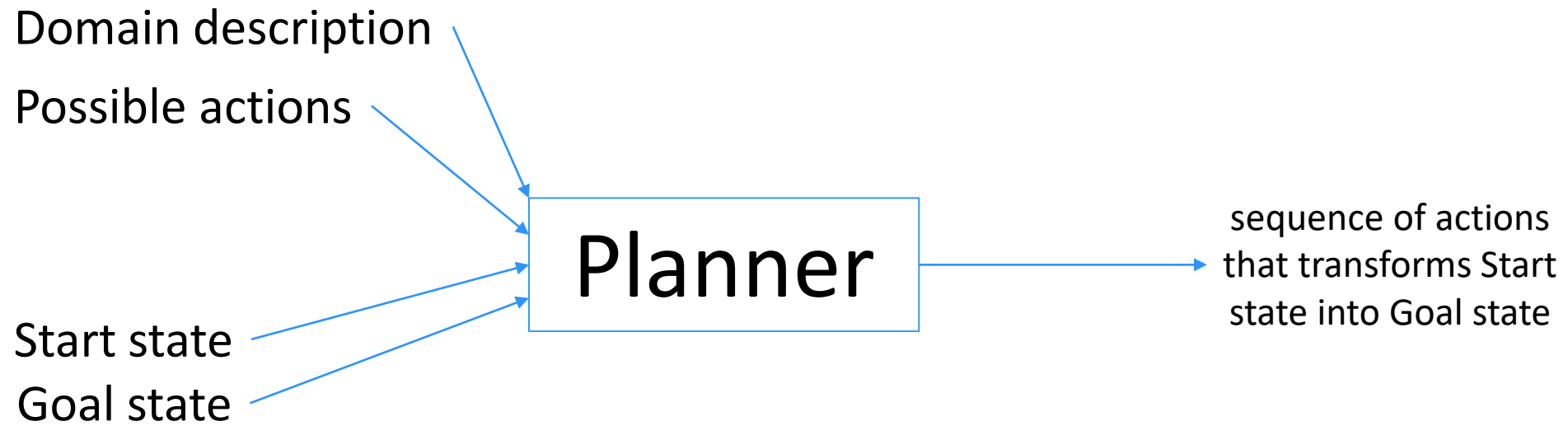
**Move(b, x, y)**

**Pre-condition:**  $\text{on}(b,x) \ \& \ \text{clear}(b) \ \& \ \text{clear}(y)$

**Effect:**  $\text{on}(b,y) \ \& \ \text{clear}(x) \ \& \ \neg \text{on}(b,x) \ \& \ \neg \text{clear}(y)$

The need for these two action schemas shows that coming up with the right actions in a planning problem can be tricky, even for relatively simple domains like blocksworld.

# A General-purpose Planner



# Planning to write a paper

**Goal:** Paper AND Contributed(You)

## **LearnAbout(x,y)**

Preconditions: HasTimeForStudy(x)

Effects: Knows(x,y),  
NOT(HasTimeForStudy(x))

## **HaveNewIdea(x)**

Preconditions: Knows(x,AI),  
Creative(x)

Effects: Idea, Contributed(x)

## **FindExistingOpenProblem(x)**

Preconditions: Knows(x,AI)

Effects: Idea

## **ProveTheorems(x)**

Preconditions: Knows(x,AI),  
Knows(x,Math), Idea

Effect: Theorems, Contributed(x)

## **PerformExperiments(x)**

Preconditions: Knows(x,AI),  
Knows(x,Coding), Idea

Effect: Experiments, Contributed(x)

## **WritePaper(x)**

Preconditions: Knows(x,AI),  
Knows(x,Writing), Idea, Theorems,  
Experiments

Effect: Paper, Contributed(x)

# Some start states

**Start1:** HasTimeForStudy(You) AND Knows(You,Math) AND Knows(You,Coding) AND Knows(You,Writing)

**Start2:** HasTimeForStudy(You) AND Creative(You) AND Knows(Advisor,AI) AND Knows(Advisor,Math) AND Knows(Advisor,Coding) AND Knows(Advisor,Writing)  
(Good luck with that plan...)

**Start3:** Knows(You,AI) AND Knows(You,Coding) AND Knows(OfficeMate,Math) AND HasTimeForStudy(OfficeMate) AND Knows(Advisor,AI) AND Knows(Advisor,Writing)

**Start4:** HasTimeForStudy(You) AND Knows(Advisor,AI) AND Knows(Advisor,Math) AND Knows(Advisor,Coding) AND Knows(Advisor,Writing)

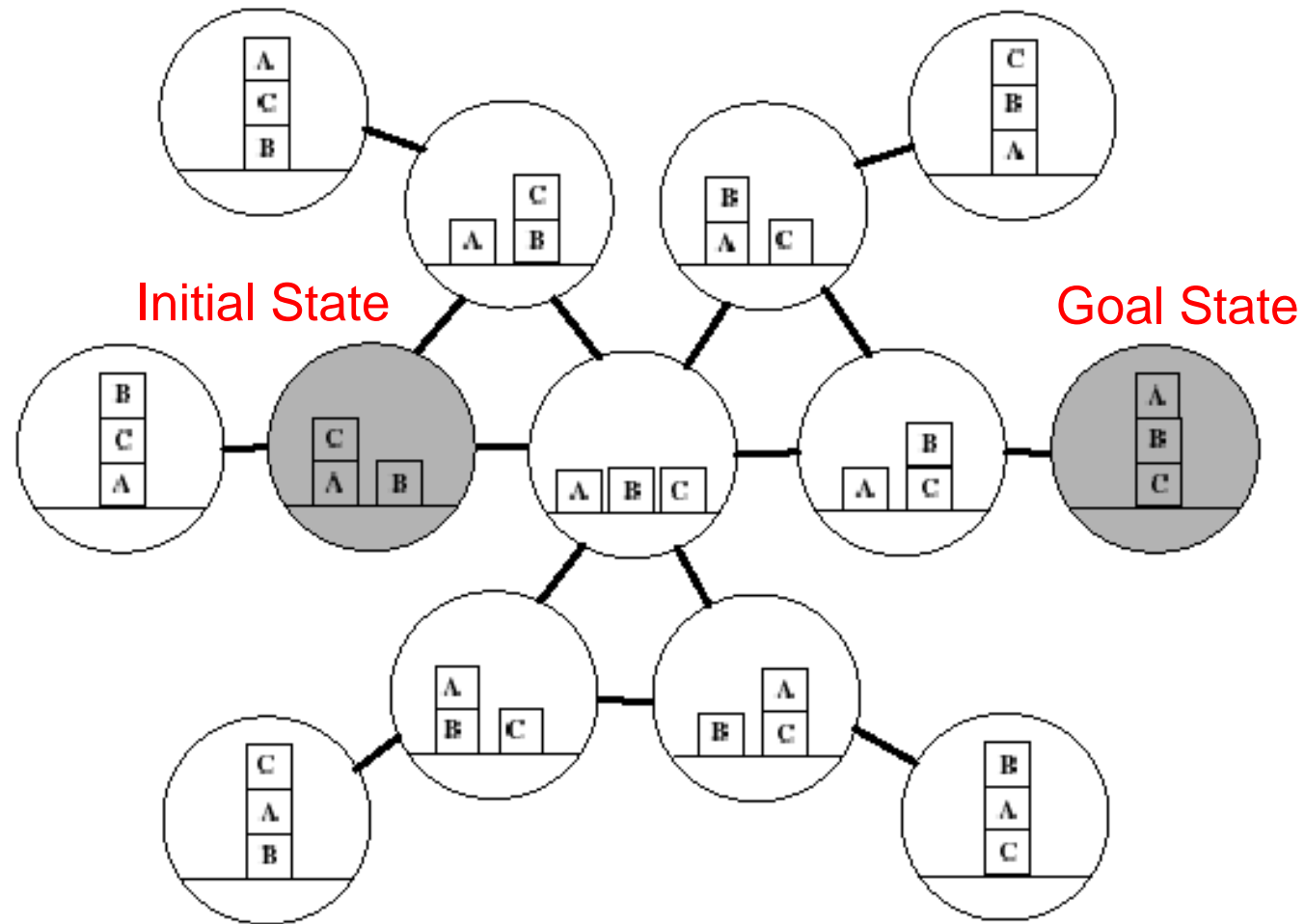
# Planning as Graph Search

- It is easy to view planning as a graph search problem
- Nodes/vertices = possible states
- Directed Arcs = STRIPS actions
- Solution: path from the initial state (i.e. vertex) to one state/vertices that satisfies the goal



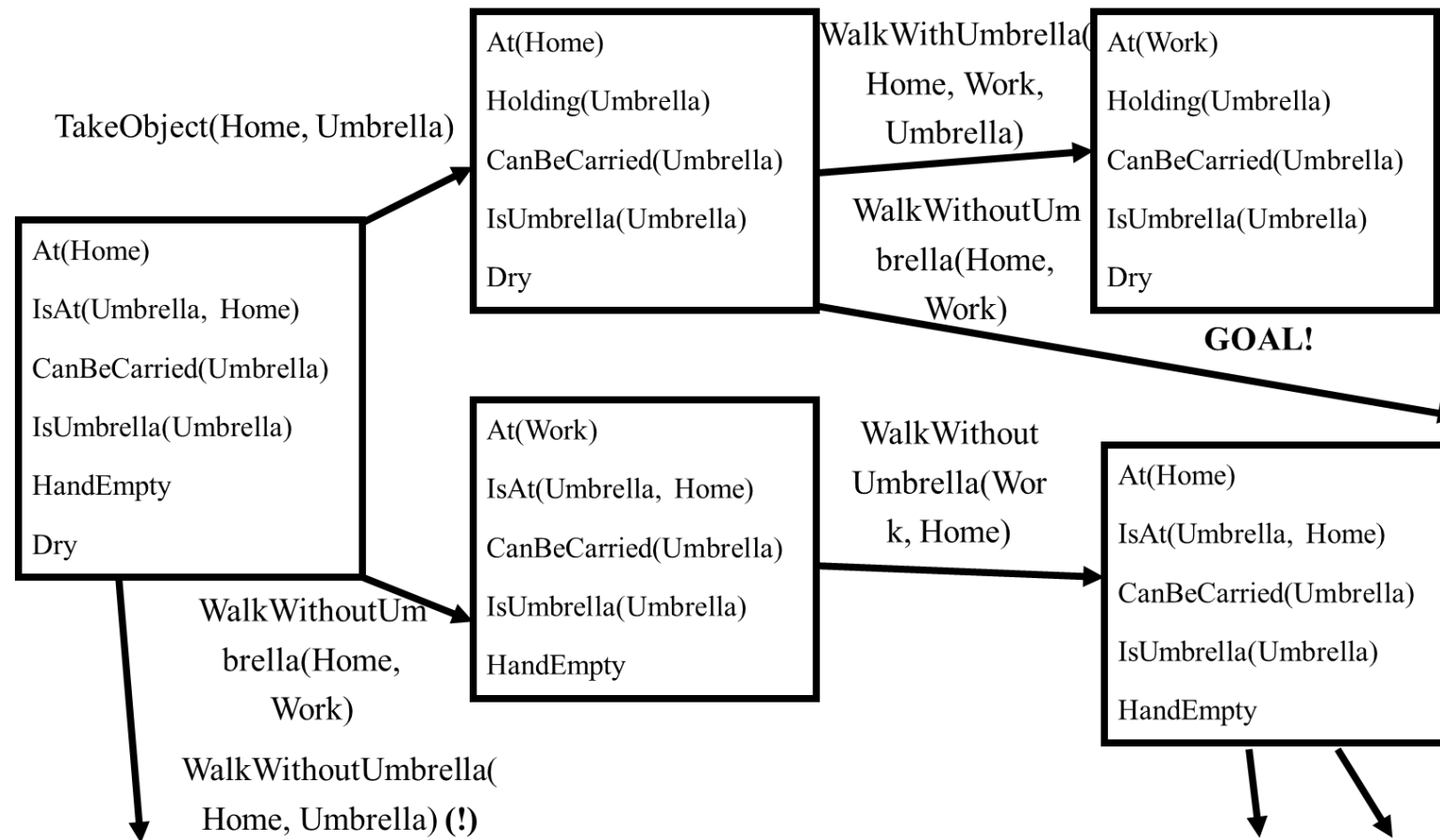
# Search Space: Blocks World

Graph is finite



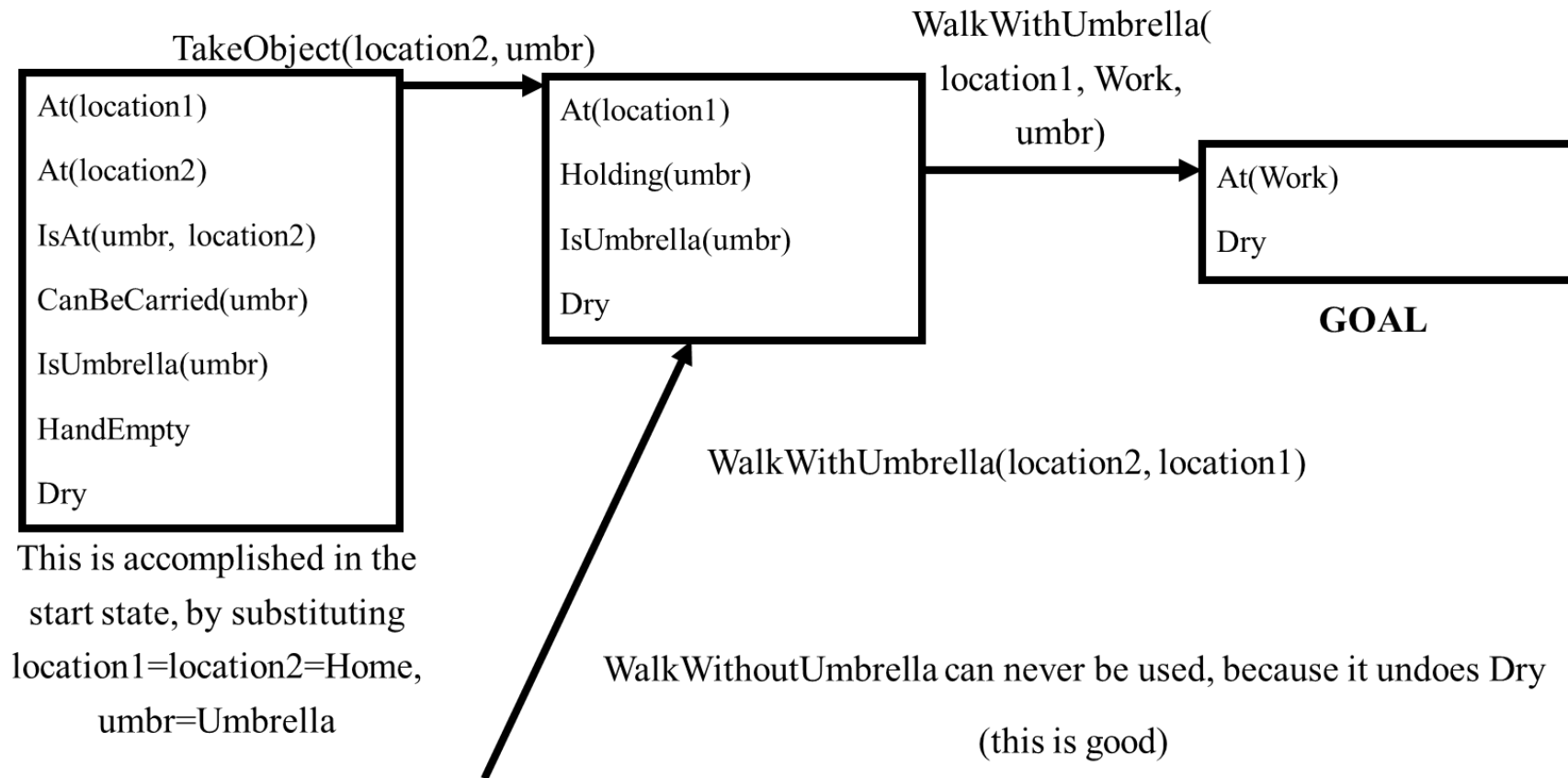
# Forward state-space search (progression planning)

- Successors: all states that can be reached with an action whose preconditions are satisfied in current state



# Backward state-space search (regression planning)

Predecessors: for every action that accomplishes one of the literals (and does not undo another literal), remove that literal and add all the preconditions



# Other Kinds of Planners

- **Graphplan** is an influential planner from the 1990s that converts a planning problem (the actions and initial/goal state) into a kind of graph that can be efficiently searched
- **SATplan** is another influential planner that converts a planning problem into a SAT problem, and then uses a SAT solver to find a plan
  - The textbook describes how to do this transformation if you are curious
- **Partial Order Planners** are a kind of planner that searches the space of *plans* instead of the space of *states*
  - A partial order planner starts with an empty plan, and then inserts actions into the plan to make it better
  - For some applications (like dialog planning), this kind of planning makes a lot of sense, since it reasons directly about the actions, In general, though, they get outperformed by forward planners

# Planning

Deterministic Planning

# GraphPlan: Basic idea

- Construct a planning graph that encodes constraints on possible plans
- Use graph to constrain search for a valid plan
- Planning graph can be built for each problem in a relatively short time
- Extract a solution from planning graph

# Planning as Graph Search

- Planning is just finding a path in a graph
  - Why not just use standard graph algorithms for finding paths?
- **Answer:** graphs are exponentially large in the problem encoding size (i.e. size of STRIPS problems).
  - But, standard algorithms are poly-time in graph size
  - So standard algorithms would require exponential time
- Can we do better than this?

# Graphplan



# Planning graph

- Directed, leveled graph with alternating layers of nodes
- Odd layers (state levels) represent candidate propositions that could possibly hold at step  $i$
- Even layers (action levels) represent candidate actions that could possibly be executed at step  $i$ , including maintenance actions [do nothing]
- Arcs represent preconditions, adds and deletes
- Can only execute one real action at a step, but the data structure keeps track of all actions & states that are possible

# Simple planning problem

**Initial state:** Have(cake)

**Goal:** Have(cake), Eaten(cake)

**Action Eat(cake):**

Preconditions: Have(cake)

Effects:  $\neg$ Have(cake), Eaten(cake)

**Action Bake(cake):**

Preconditions:  $\neg$ Have(cake)

Effects: Have(cake)

Solution:

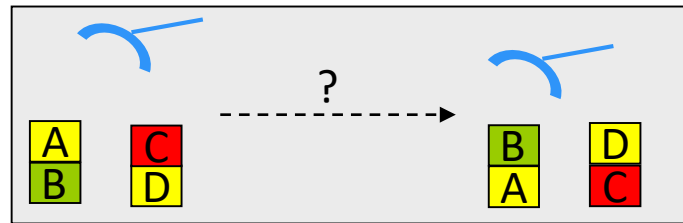
Eat(cake)

Bake(cake)

- Phase 1 – Create a Planning Graph
  - ▲ built from initial state
  - ▲ contains actions and propositions that are possibly reachable from initial state
  - ▲ does not include unreachable actions or propositions
- Phase 2 - Solution Extraction
  - ▲ Backward search for the solution in the planning graph
    - backward from goal

# Layered Plans

- Graphplan searches for **layered plans** (often called parallel plans)
- A layered plan is a sequence of **sets** of actions
  - actions in the same set must be **compatible**
    - a1 and a2 are compatible iff a1 does not delete preconditions or positive effects of a2 (and vice versa)
  - all sequential orderings of compatible actions gives same result



**Layered Plan:** (a two layer plan)

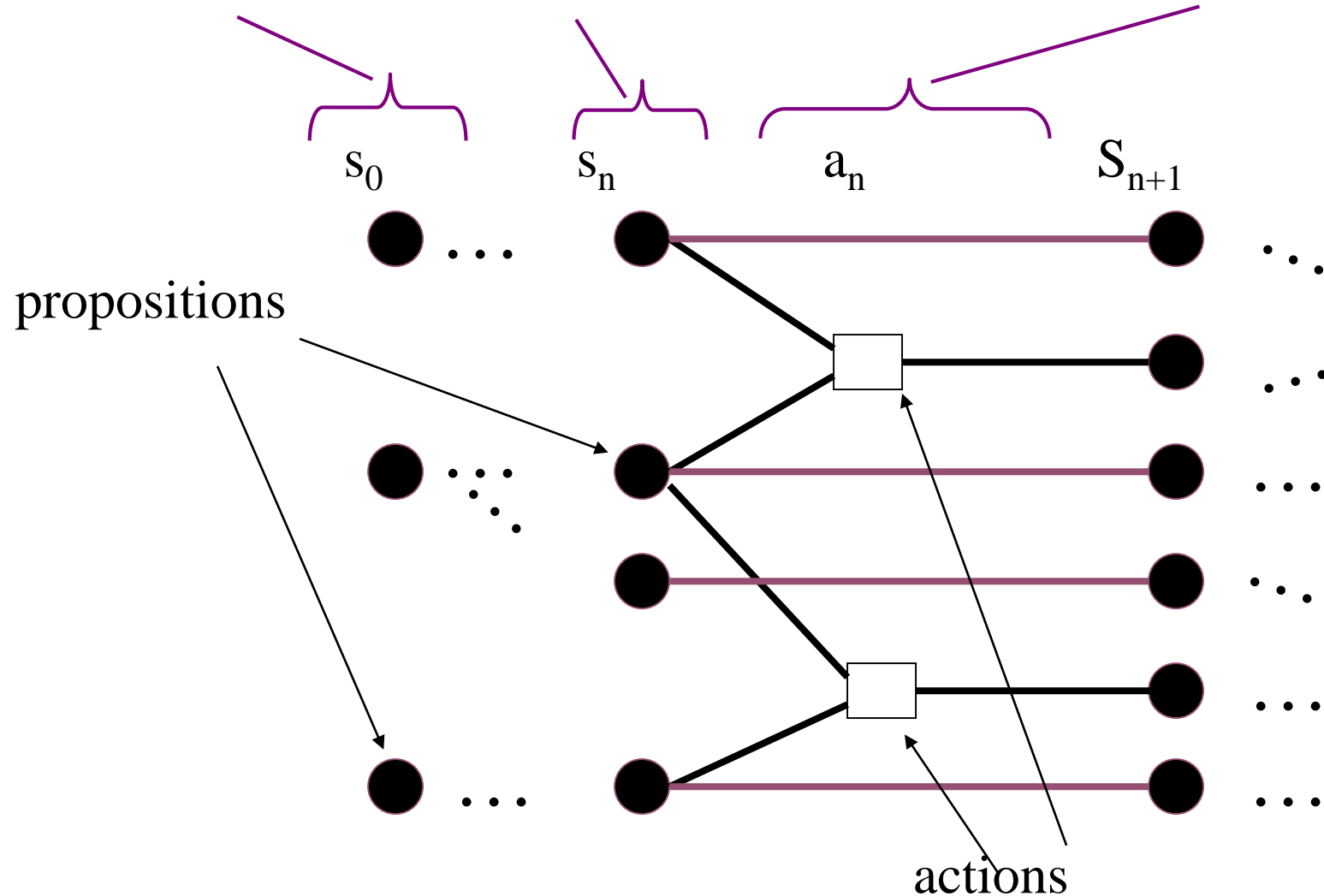
$$\left\{ \begin{array}{l} \text{move(A,B,TABLE)} \\ \text{move(C,D,TABLE)} \end{array} \right\} \cdot \left\{ \begin{array}{l} \text{move(B,TABLE,A)} \\ \text{move(D,TABLE,C)} \end{array} \right\}$$

# Planning Graph

**state-level 0:**  
propositions true  
in  $s_0$

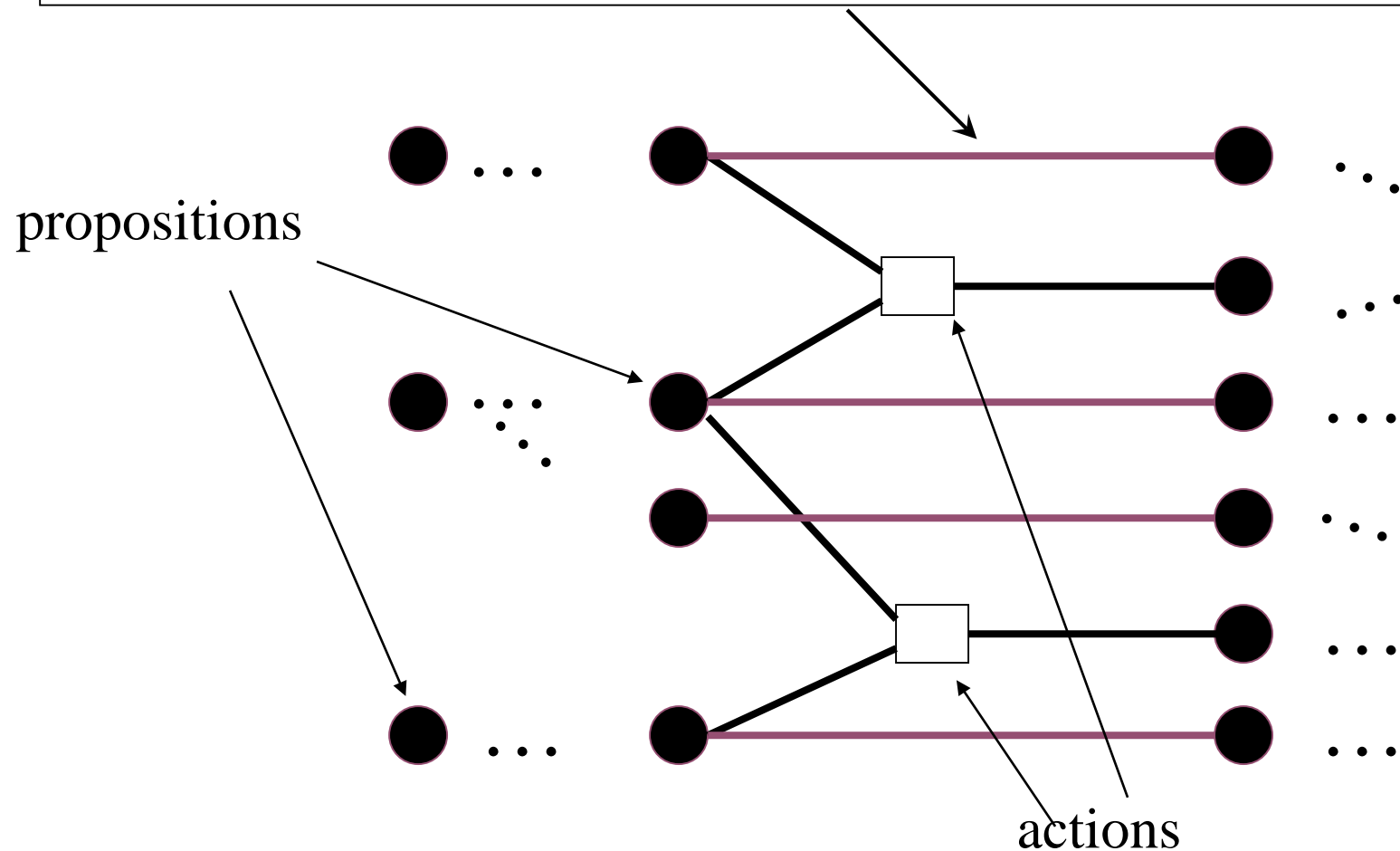
**state-level  $n$ :** literals that  
may possibly be true after  
some  $n$  level plan

**action-level  $n$ :** actions that  
may possibly be applicable  
after some  $n$  level plan



# Planning Graph

- maintenance action (persistence actions)
  - ▲ represents what happens if no action affects the literal
  - ▲ include action with precondition  $c$  and effect  $c$ , for each literal  $c$



# Graph expansion

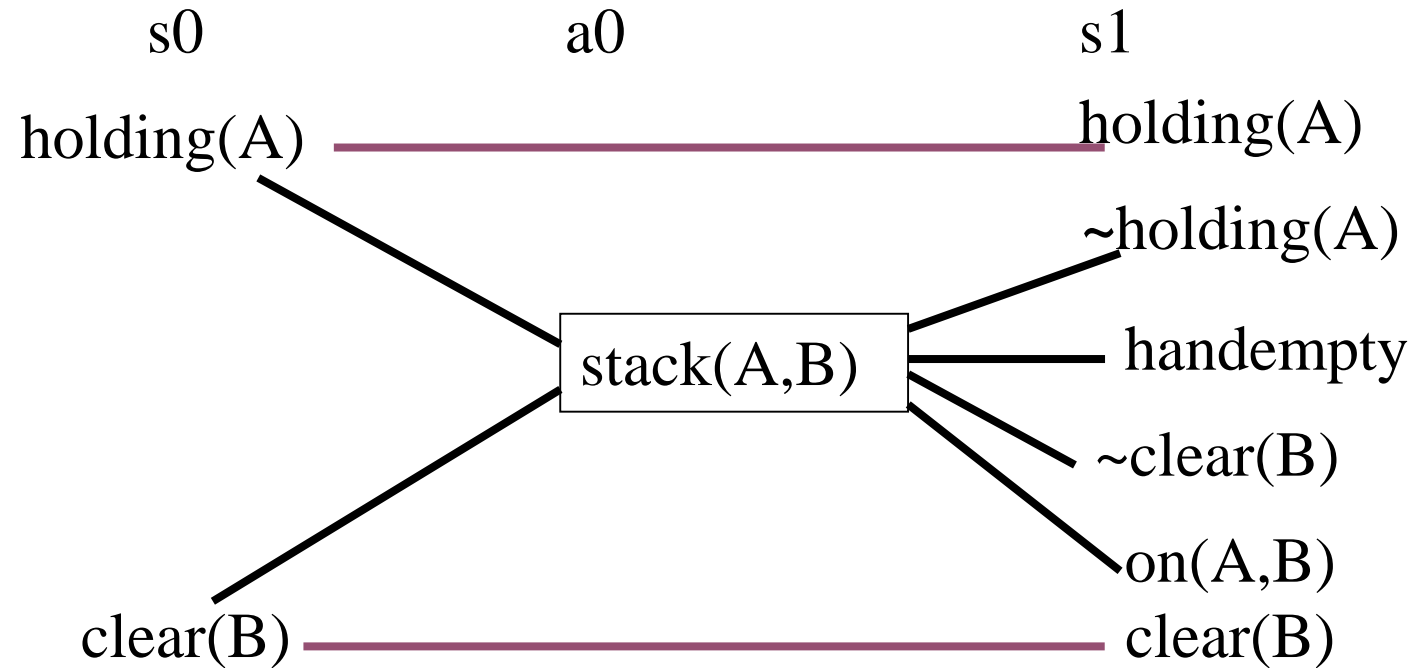
- Initial proposition layer
  - Just the propositions in the initial state
- Action layer  $n$ 
  - If all of an action's preconditions are in proposition layer  $n$ , then add action to layer  $n$
- Proposition layer  $n+1$ 
  - For each action at layer  $n$  (including persistence actions)
  - Add all its effects (both positive and negative) at layer  $n+1$   
(Also allow propositions at layer  $n$  to persist to  $n+1$ )
- Propagate mutex information  
(we'll talk about this in a moment)

# Example

stack(A,B)

precondition: holding(A), clear(B)

effect:  $\sim$ holding(A),  $\sim$ clear(B), on(A,B), clear(B), handempty



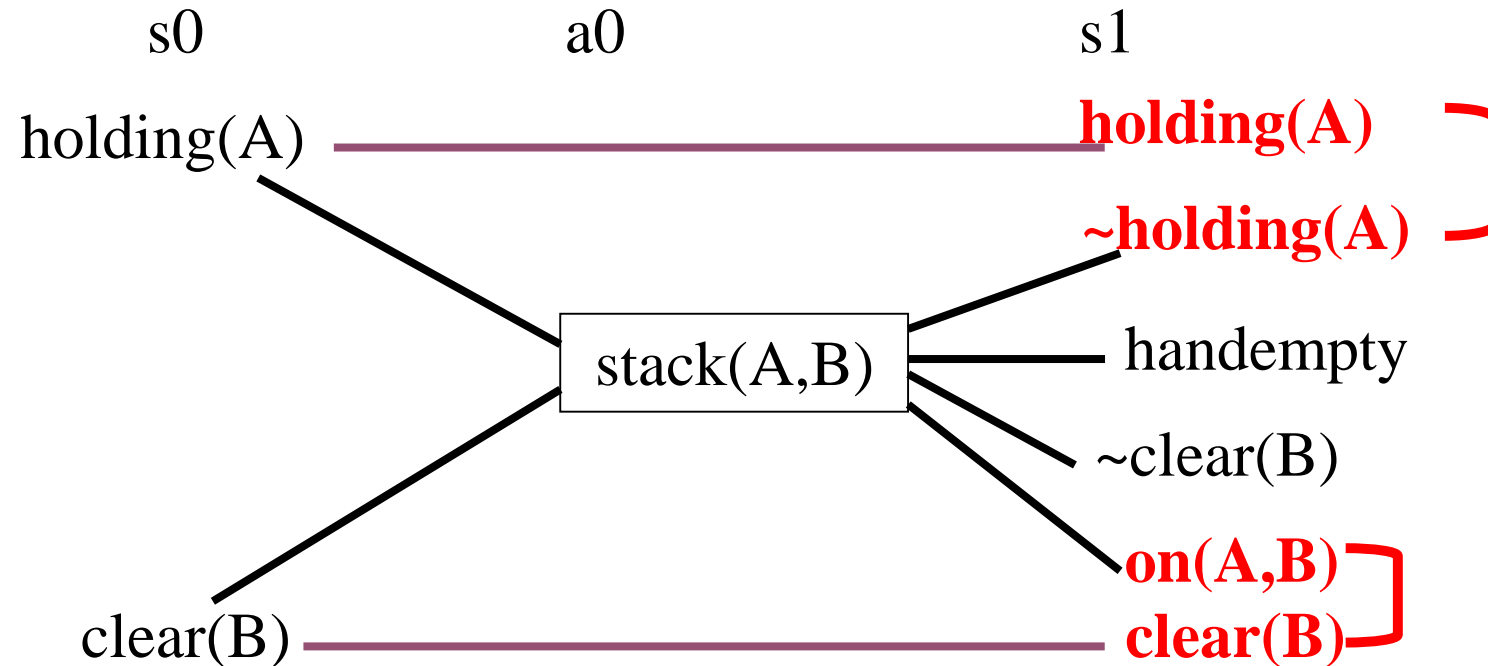


# Example

stack(A,B)

precondition: holding(A), clear(B)

effect:  $\sim$ holding(A),  $\sim$ clear(B), on(A,B), clear(B), handempty



Notice that not all literals in `s1` can be made true simultaneously after 1 level:  
e.g. `holding(A)`,  `$\sim$ holding(A)` and `on(A,B)`, `clear(B)`

# Mutual Exclusion (Mutex)

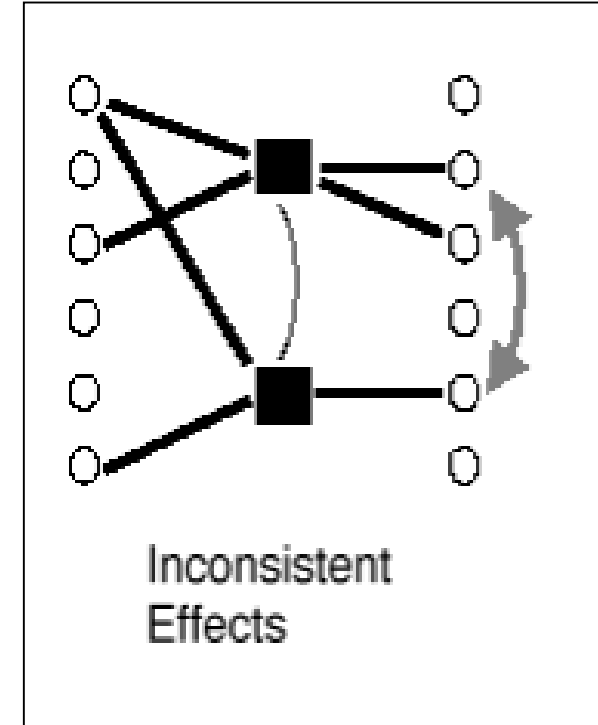
- Mutex between pairs of actions at layer  $n$  means
  - no valid plan could contain both actions at layer  $n$
  - E.g., `stack(a,b)`, `unstack(a,b)`
- Mutex between pairs of literals at layer  $n$  means
  - no valid plan could produce both at layer  $n$
  - E.g., `clear(a)`, `~clear(a)`  
`on(a,b)`, `clear(b)`
- GraphPlan checks pairs only
  - mutex relationships can help rule out possibilities during search in phase 2 of Graphplan

# Mutual exclusion links between actions

- **Inconsistent effects:** one action negates an effect of another.
- **Interference:** one of the effects of one action is the negation of the precondition for another.
- **Competing needs:** one of the preconditions of one action is mutually exclusive with a precondition of the other.
- **Negation:** one proposition is the negation of the other.
- **Inconsistent support:** all the actions for establishing one proposition are mutually exclusive with the actions of establishing the other proposition.

# Action Mutex: condition 1

- Inconsistent effects
  - an effect of one negates an effect of the other
- E.g., stack(a,b) & unstack(a,b)
  - ↓
  - add handempty
  - ↓
  - delete handempty  
(add ~handempty)



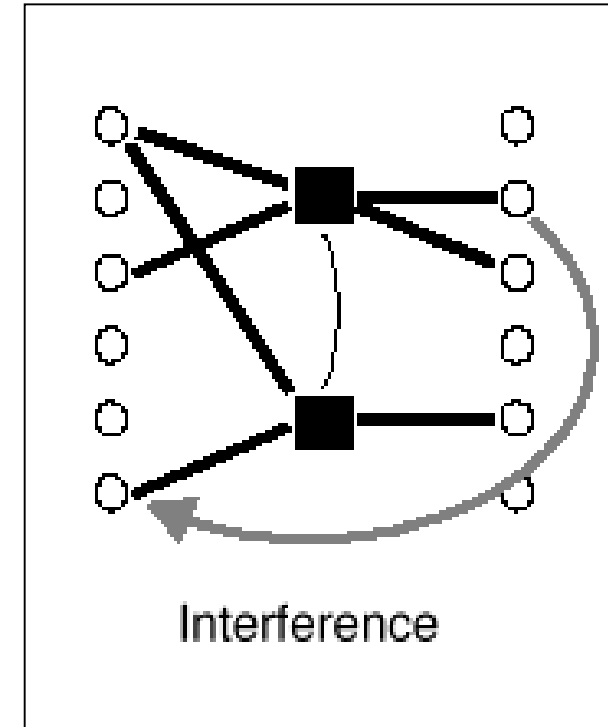
# Action Mutex: condition 2

- *Interference :*

- ▲ one deletes a precondition of the other

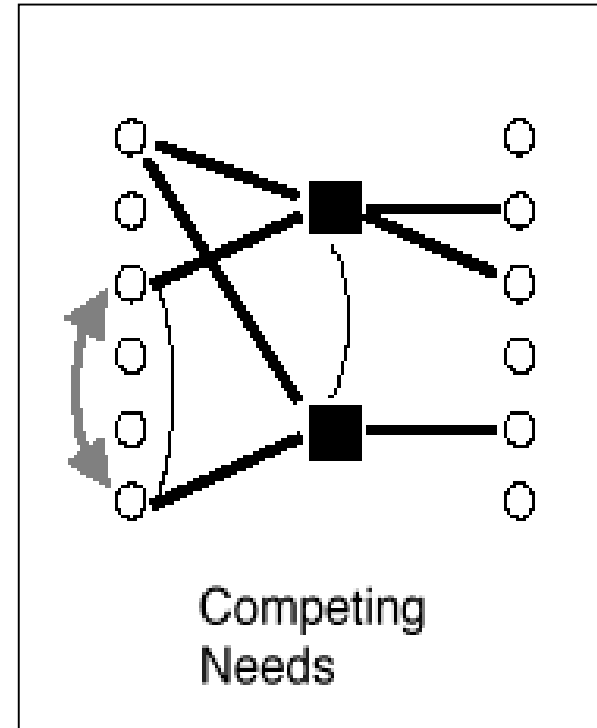
- E.g., `stack(a,b)` & `putdown(a)`

↓	↓
deletes <code>holding(a)</code>	needs <code>holding(a)</code>



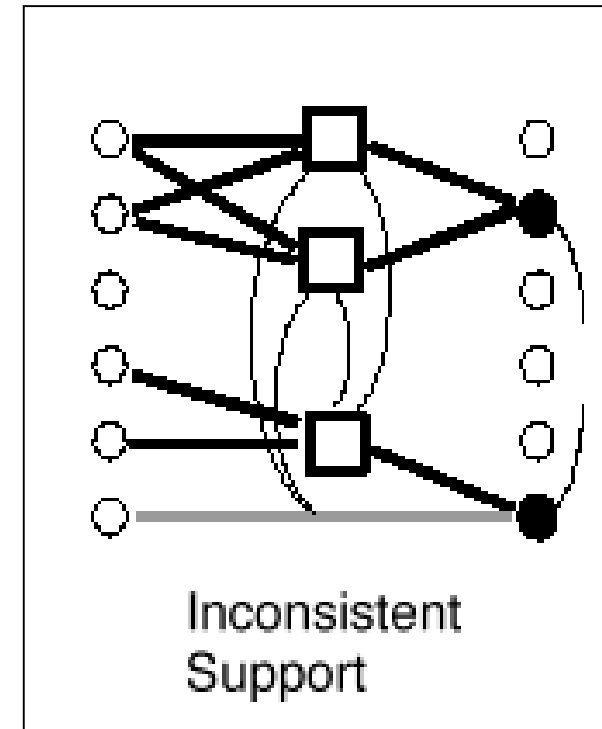
# Action Mutex: condition 3

- *Competing needs:*
  - ▶ they have mutually exclusive preconditions
  - ▶ Their preconditions can't be true at the same time



# Literal Mutex: two conditions

- *Inconsistent support* :
  - ▶ one is the negation of the other  
E.g., handempty and  $\sim$ handempty
  - ▶ or all ways of achieving them via actions are pairwise mutex



- **Inconsistent effects:** one action negates an effect of another.
  - Eat(cake) deletes Have(cake) so is inconsistent with persistence of Have(cake)
  - Eat(cake) adds Eaten(cake) so is inconsistent with persistence of  $\neg$ Eaten(Cake)
- **Interference:** one of the effects of one action is the negation of the precondition for another.
  - Eat(cake) negates the preconditions of the persistence actions for Have(cake) and  $\neg$ Eaten(Cake)
- **Competing needs:** one of the preconditions of one action is mutually exclusive with a precondition of the other.
  - Bake(cake) requires  $\neg$ Have(cake) while Eat(cake) requires Have(cake)
- **Negation:** one proposition is the negation of the other.
  - Have(cake) and  $\neg$ Have(cake) are mutually exclusive.
  - **Inconsistent support:** all the actions for establishing one proposition are mutually exclusive with the actions of establishing the other proposition.
    - Have(cake) and Eaten(cake) are mutex in  $S_1$  because all their establishing actions are mutex
    - Have(cake) and Eaten(cake) are not mutex in  $S_2$



# Simple planning problem

**Initial state:** Have(cake)

**Goal:** Have(cake), Eaten(cake)

**Action Eat(cake):**

Preconditions: Have(cake)

Effects:  $\neg$ Have(cake), Eaten(cake)

**Action Bake(cake):**

Preconditions:  $\neg$ Have(cake)

Effects: Have(cake)

Solution:

Eat(cake)

Bake(cake)

# Planning Graph for Cake Example

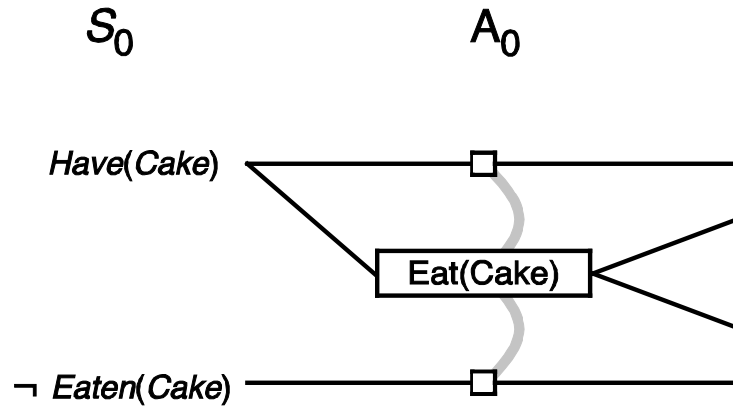
$S_0$

*Have(Cake)*

$\neg$  *Eaten(Cake)*

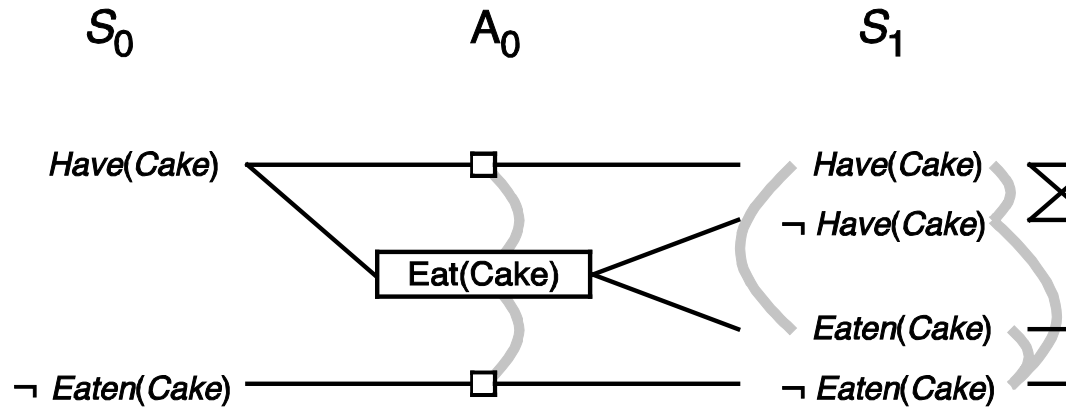
- Level  $S_0$  has all literals from initial state

# Planning Graph for Cake Example



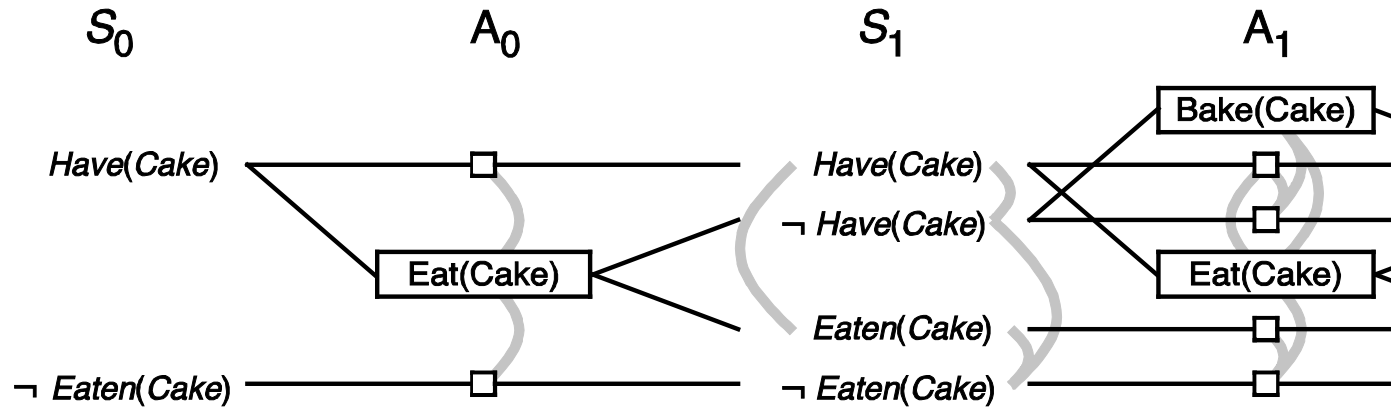
- Level  $S_0$  has all literals from initial state
- **Level  $A_0$  has all actions whose preconditions are satisfied in  $S_0$ , including no-ops**

# Planning Graph for Cake Example



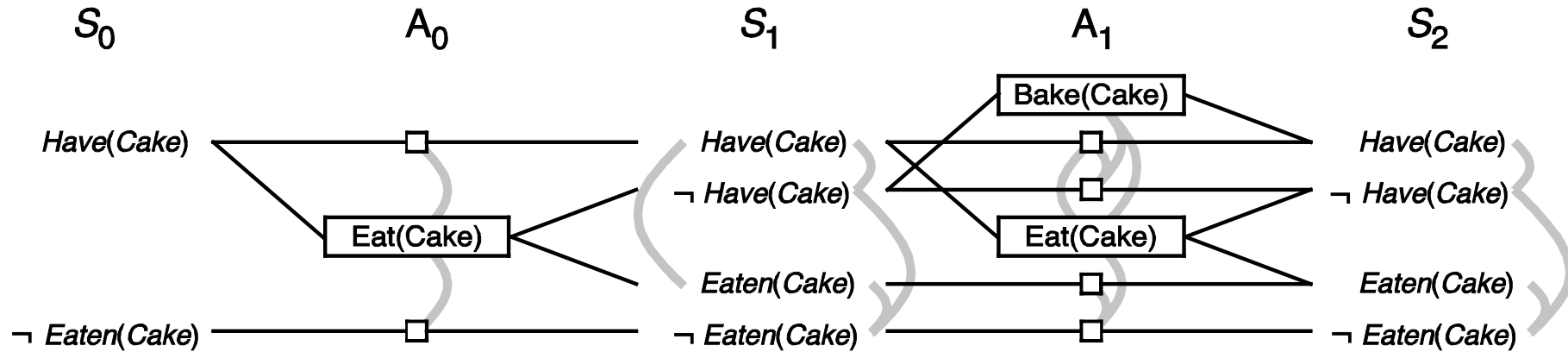
- Level  $S_0$  has all literals from initial state
- Level  $A_0$  has all actions whose preconditions are satisfied in  $S_0$ , including no-ops
- **Actions connect preconditions to effects**
- **Gray arcs connect propositions that are mutex (mutually exclusive) & actions that are mutex**

# Planning Graph for Cake Example



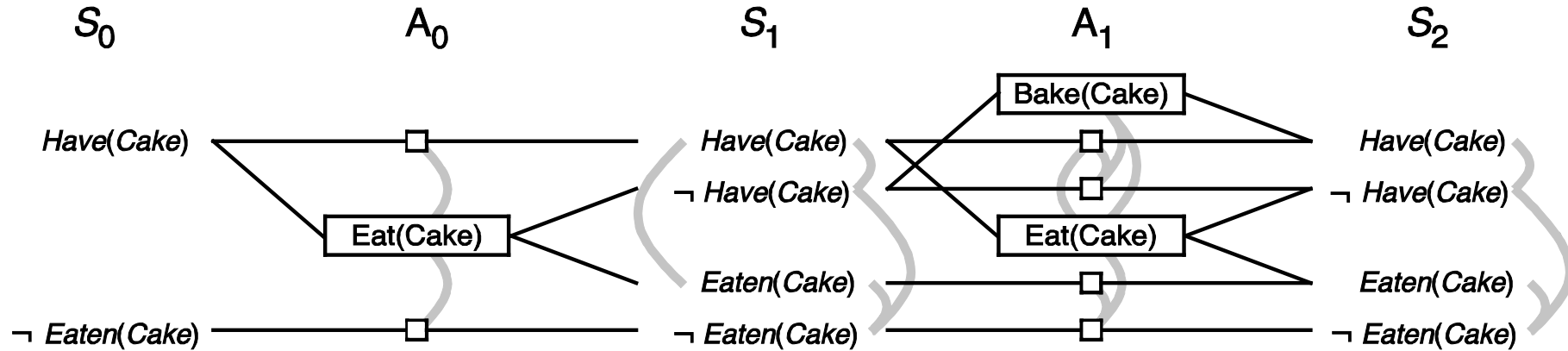
- Actions connect preconditions to effects
- Gray arcs connect propositions that are mutex
- **Actions at level  $A_i$  must have support from a set of literals in state  $S_i$  that have no mutex relations among themselves**

# Planning Graph for Cake Example



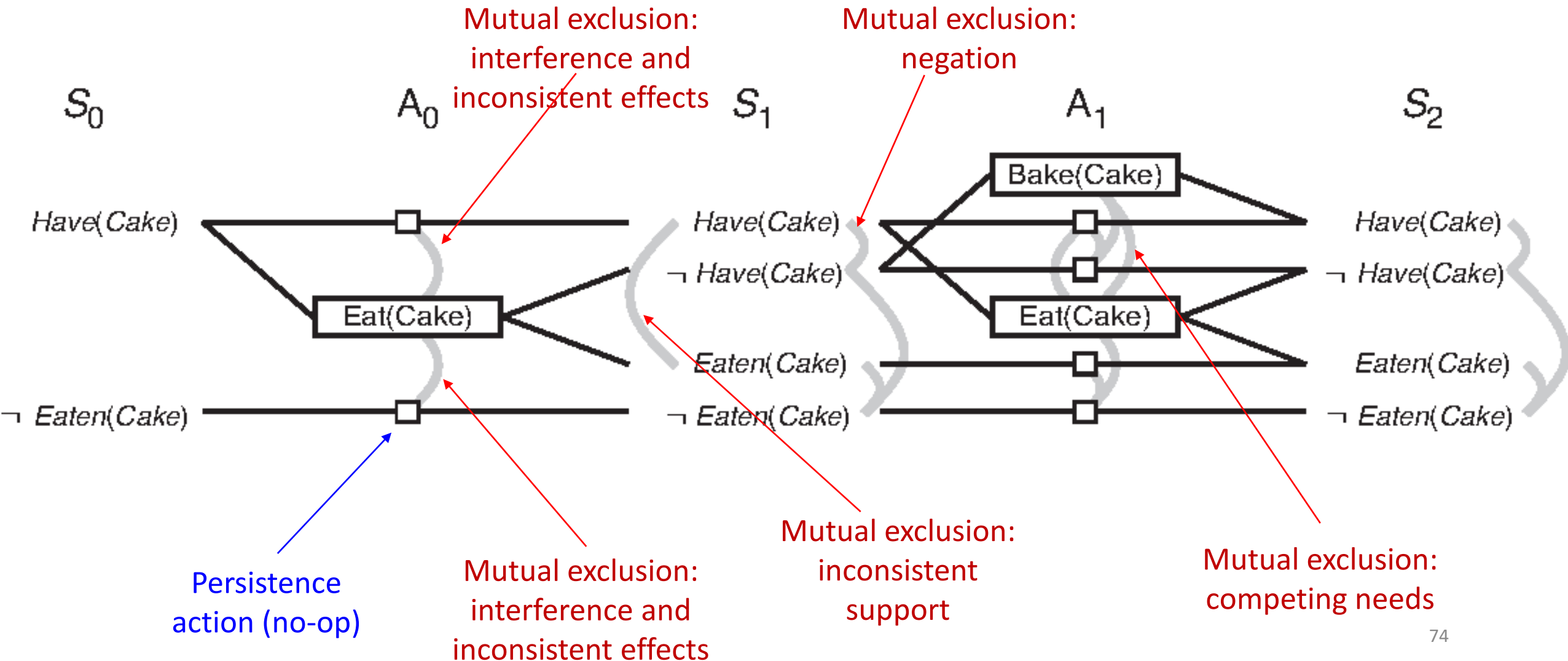
- Actions at level  $A_i$  must have support from a set of literals in state  $S_i$  that have no mutex relations among themselves
- **Stop when the set of literals does has not changed**

# Planning Graph for Cake Example



- If all of the literals in the goal are in the final state and are non-mutex ...
- We can try to extract a plan from the plan graph

# Planning graph representation of cake problem





# GraphPlan

**function** GRAPHPLAN(*problem*) **returns** solution or failure

$$graph \leftarrow \text{INITIAL-PLANNING-GRAPH}(problem)$$

```
goals ← CONJUNCTS(problem.GOAL)
```

*nogoods*  $\leftarrow$  an empty hash table

**for  $t = 0$  to  $\infty$  do**

**if** *goals* all non-mutex in  $S_t$  of *graph* **then**

$$solution \leftarrow \text{EXTRACT-SOLUTION}(graph, goals, \text{NUMLEVELS}(graph), nogoods)$$

**if *graph* and *nogoods* have both leveled off then return *failure***

$$graph \leftarrow \text{EXPAND-GRAPH}(graph, problem)$$

# Solution Extraction: Backward Search

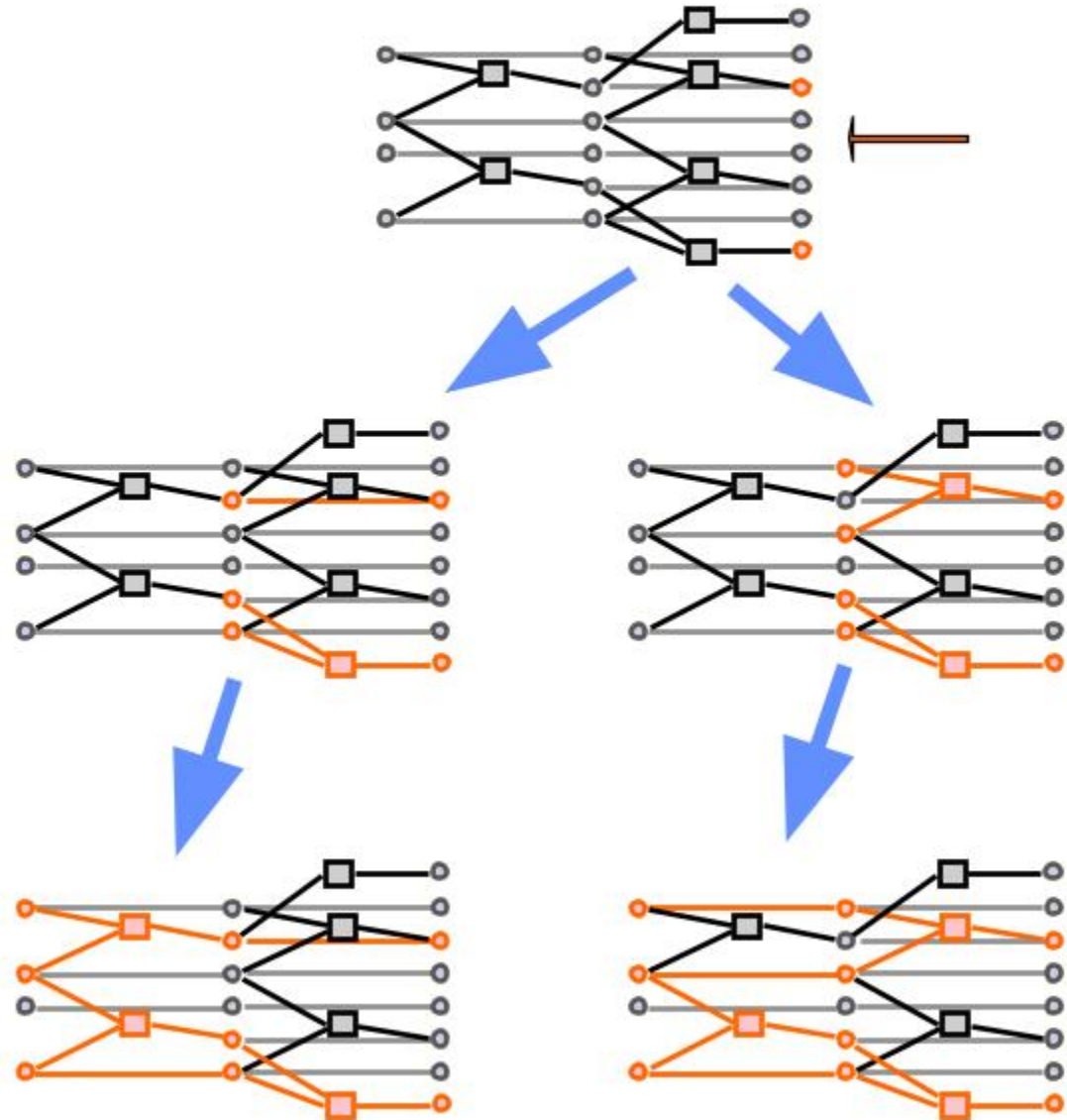
Search problem:

Start state: goal set at last level

Actions: conflict-free ways of  
achieving the current goal set

Terminal test: at S0 with goal set  
entailed by initial planning state

Note: may need to start much  
deeper than the leveling-off point!  
Caching, good ordering is important



# Important Ideas

- Plan graph construction is polynomial time
  - Though construction can be expensive when there are many “objects” and hence many propositions
- The plan graph captures important properties of the planning problem
  - Necessarily unreachable literals and actions
  - Possibly reachable literals and actions
  - Mutually exclusive literals and actions
- Significantly prunes search space compared to previously considered planners
- Plan graphs can also be used for deriving admissible (and good non-admissible) heuristics

# Spare Tire Problem

Init(Tire(Flat)  $\wedge$  Tire(Spare)  $\wedge$  At(Flat,Axle)  $\wedge$  At(Spare,Trunk))

Goal(At(Spare,Axle))

Action(Remove(obj,loc),  
    PRECOND: At(obj,loc),  
    EFFECT:  $\neg$ At(obj,loc)  $\wedge$  At(obj,Ground))

Action(PutOn(t, Axle),  
    PRECOND: Tire(t)  $\wedge$  At(t,Ground)  $\wedge$   $\neg$ At(Flat,Axle),  
    EFFECT:  $\neg$ At(t,Ground)  $\wedge$  At(t,Axle))

Action(LeaveOvernight,  
    PRECOND:  $\emptyset$ ,  
    EFFECT:  $\neg$ At(Spare,Ground)  $\wedge$   $\neg$ At(Spare,Axle)  $\wedge$   $\neg$ At(Spare,Trunk)  $\wedge$   $\neg$ At(Flat,Ground)  $\wedge$   
     $\neg$ At(Flat,Axle)  $\wedge$   $\neg$ At(Flat,Trunk))

# SATPlan

- Formulate the planning problem as a CSP
- Assume that the plan has  $k$  actions
- Create a binary variable for each possible action  $a$ :
  - $\text{Action}(a,i)$  (TRUE if action  $a$  is used at step  $i$ )
- Create variables for each proposition that can hold at different points in time:
  - $\text{Proposition}(p,i)$  (TRUE if proposition  $p$  holds at step  $i$ )

# Constraints

- Only one action can be executed at each time step (XOR constraints)
- Constraints describing effects of actions
- Persistence: if an action does not change a proposition  $p$ , then  $p$ 's value remains unchanged
- A proposition is true at step  $i$  only if some action (possibly a maintain action) made it true
- Constraints for initial state and goal state

Now apply our favorite CSP solver!

# SATPLAN

