Equilibrium

Ref. Gravelle & Rees, Henderson & Quandt

Dr. Anwesha Aditya

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IIT Kharagpur

Two approaches of equilibrium:

- Walrasian: A price $P_e \geq 0$ is a Walrasian equilibrium price if for such a price buyers' plan match sellers' plans, i.e., $x_d(P_e)= x_s(P_e)$, i.e., $E(P_e)=0$ (ED quantity function).

- Marshallian: A quantity $x_e \geq 0$ is a Marshallian equilibrium quantity if for such a quantity buyers' plan match sellers' plans, i.e., $P_d(x_e)= P_s(x_e)$, i.e., $F(x_e)=0$ (ED price function).

- No difference in Walrasian and Marshallian equilibrium price and quantity.

Example. $$x_d = a - bp$$

$$x_s = c + dp$$

At equilibrium in Walrasian approach:

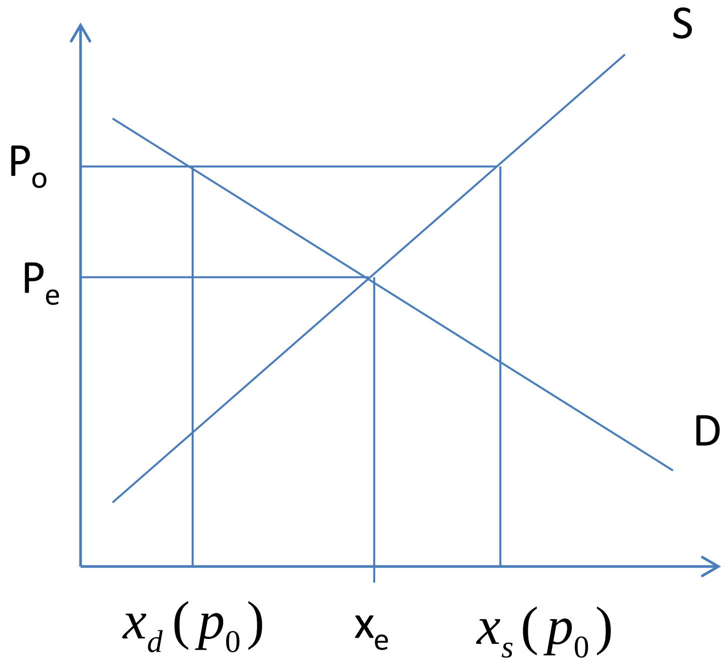
$$x_d(p_e) = x_s(p_e) \Rightarrow p_e = \frac{a-c}{b+d}, x_e = \frac{ad+bc}{b+d}$$

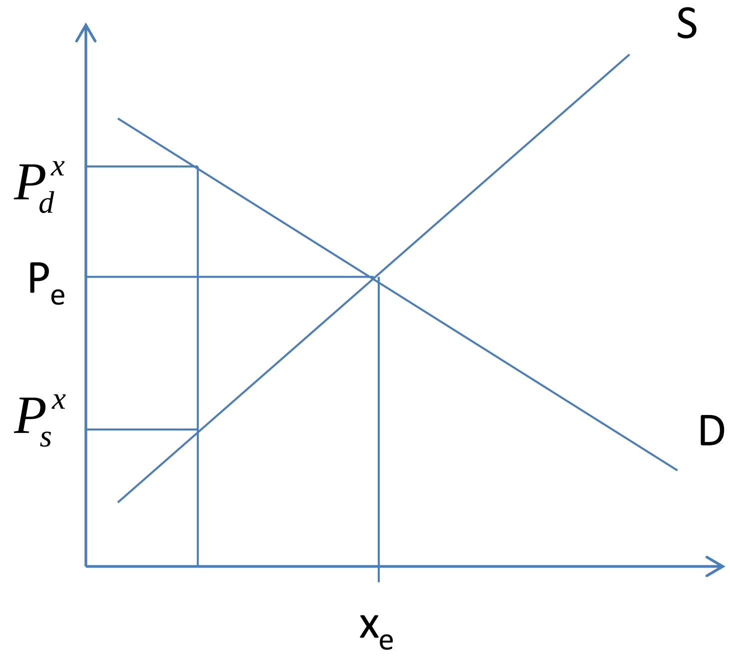
$$p_d = \frac{a}{b} - \frac{x}{b}$$

$$p_s = \frac{x}{d} - \frac{c}{d}$$

At equilibrium in Marshallian approach:

$$p_d(x_e) = p_s(x_e) \Rightarrow x_e = \frac{ad+bc}{b+d}, p_e = \frac{a-c}{b+d}$$


S
P_o
P_e
D
$x_d(p_0)$
x_e
$x_s(p_0)$


S
P_d^x
P_e
P_s^x
D
x_e

In disequilibrium, minimum will be realized.

If $x_d<x_s$ ➔ Plan of the buyer is realized

If $x_s<x_d$ ➔ Plan of the seller is realized

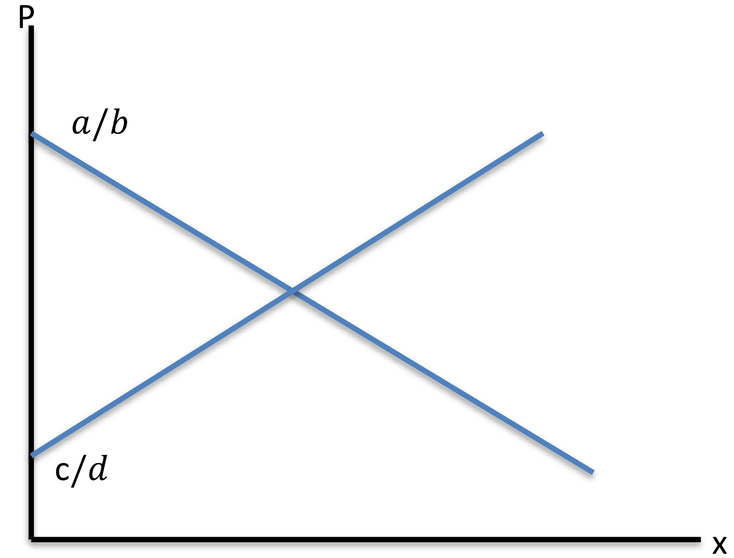
$$x_d = a - bp, x_s = c + dp$$

$$Eqm \Rightarrow a - bp_e = c + dp_e$$

$$\Rightarrow P_e = \frac{a-c}{b+d}$$

$$x_d^e = a - \frac{b(a-c)}{b+d} = \frac{ad+bc}{b+d}.$$

$$\therefore x_t = x_e = \frac{ad+bc}{b+d}$$



Existence

Can we have at least one

$$P_e \geq 0 \ni E(P_e) = 0$$

Or in Marshallian sense one

$$x_e \geq 0 \ni F(x_e) = 0$$

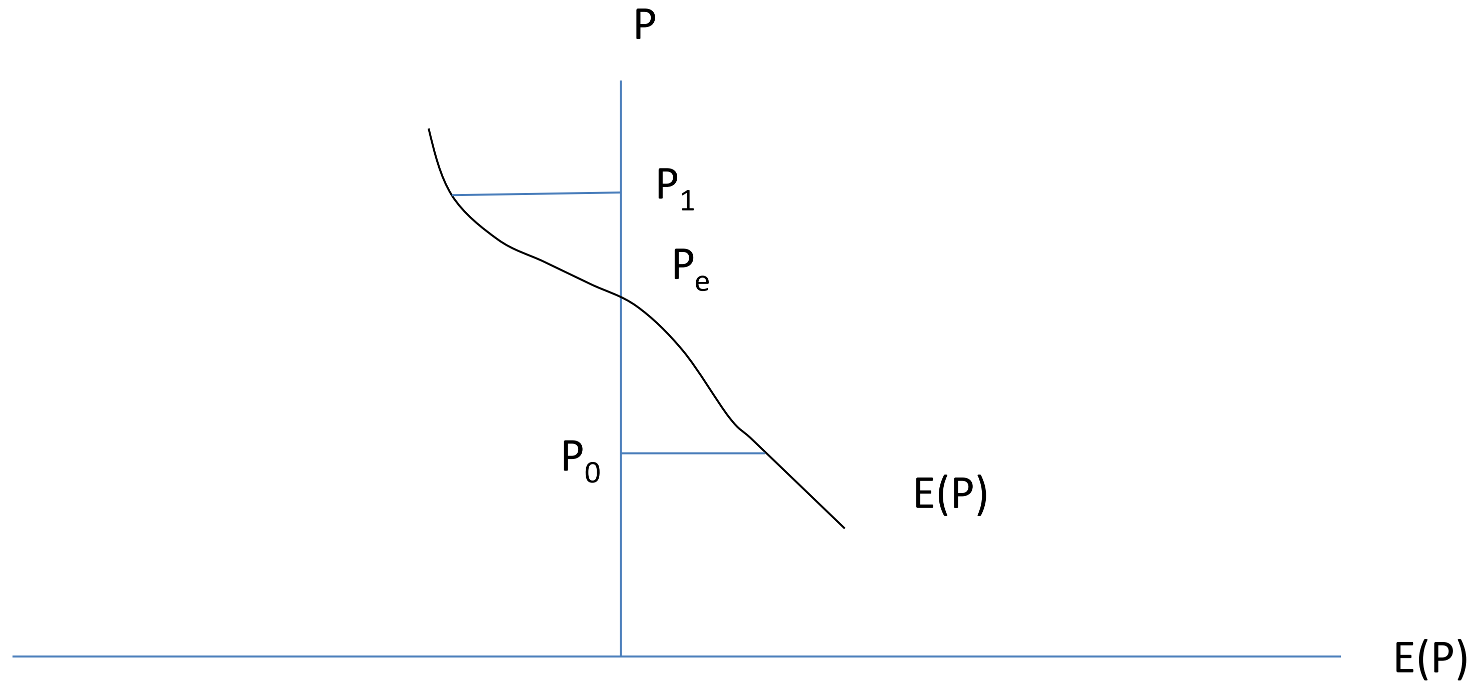
Condition for Existence of Walrasian equilibrium:

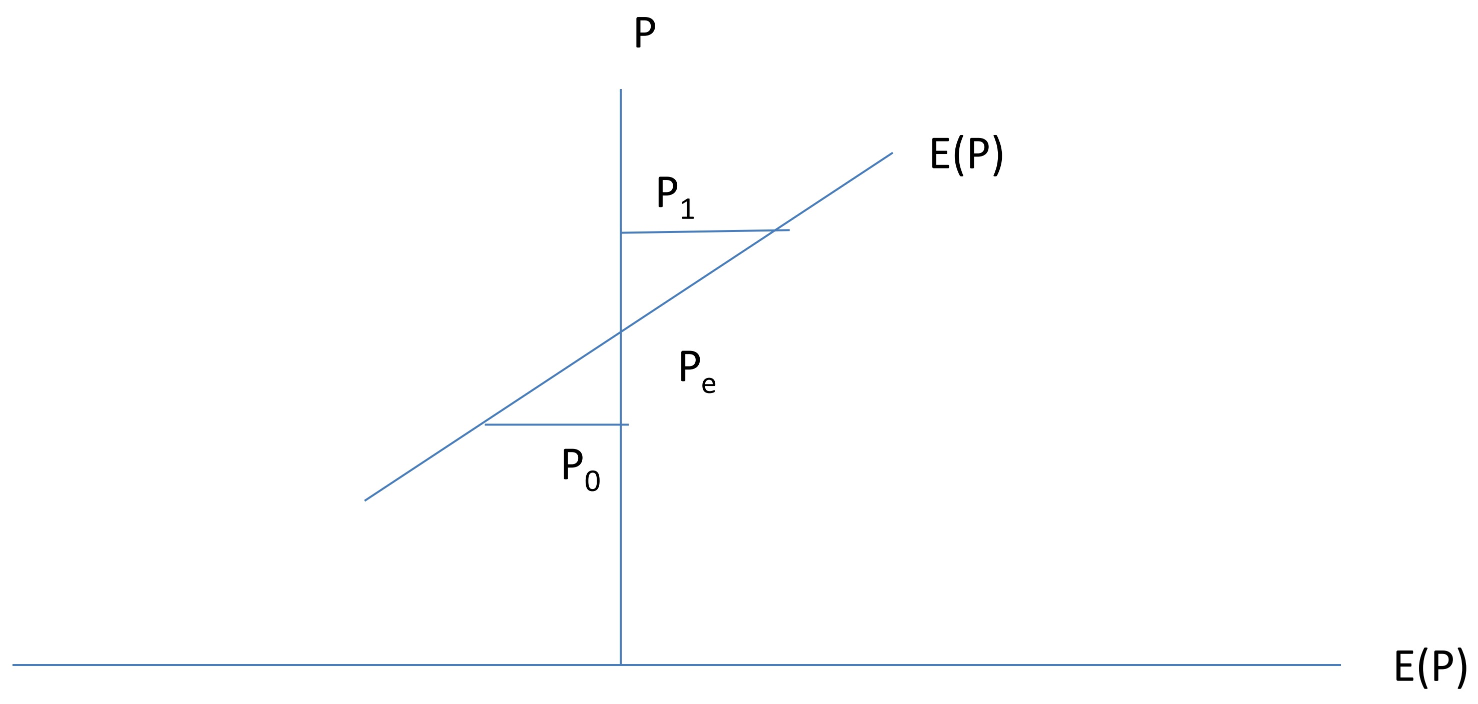
A price $P_e \geq 0$ exists if the following conditions hold:

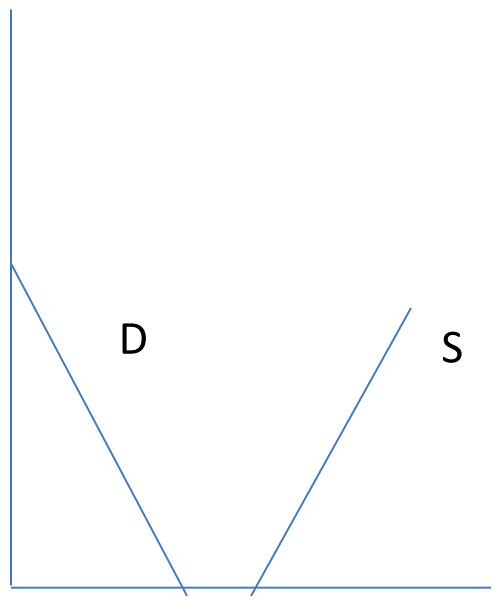
i) E(P) is continuous in p (SC, not NC);

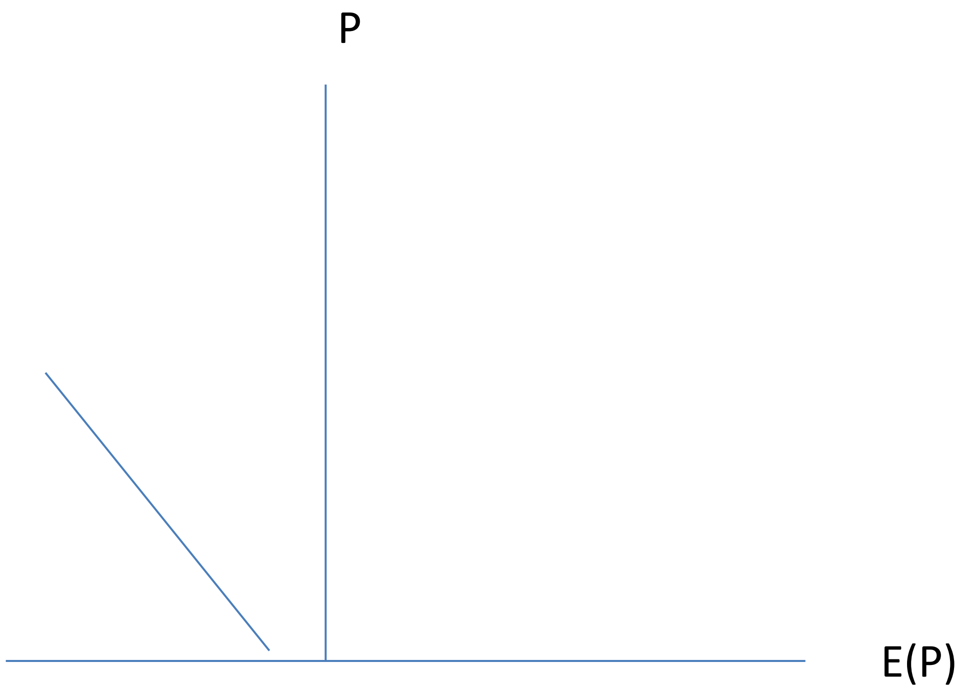
ii) $\exists a P_0 > 0 \ni E(P_0) > 0$

iii) $\exists a P_1 > 0 \ni E(P_1) < 0$


P
P1
Pe
P0
E(P)
E(P)


P
E(P)
P1
Pe
P0
E(P)


D
S


P
E(P)

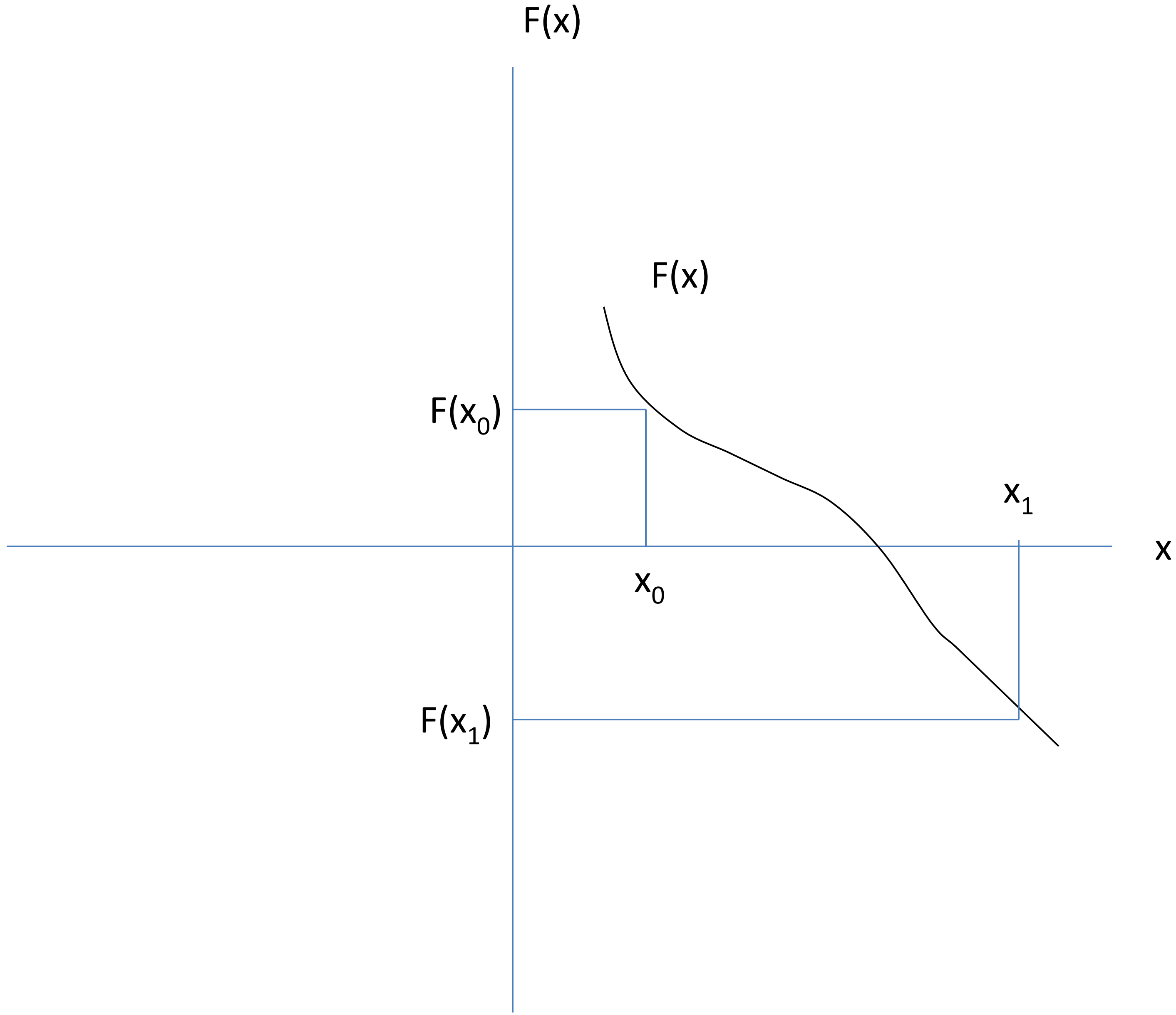
Condition for Existence of Marshallian equilibrium:

A quantity $x_e \geq 0$ exists if the following conditions hold:

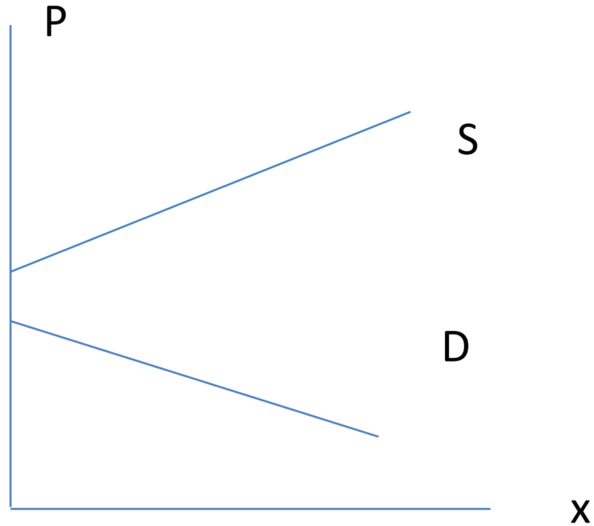
i) F(x) is continuous in x;

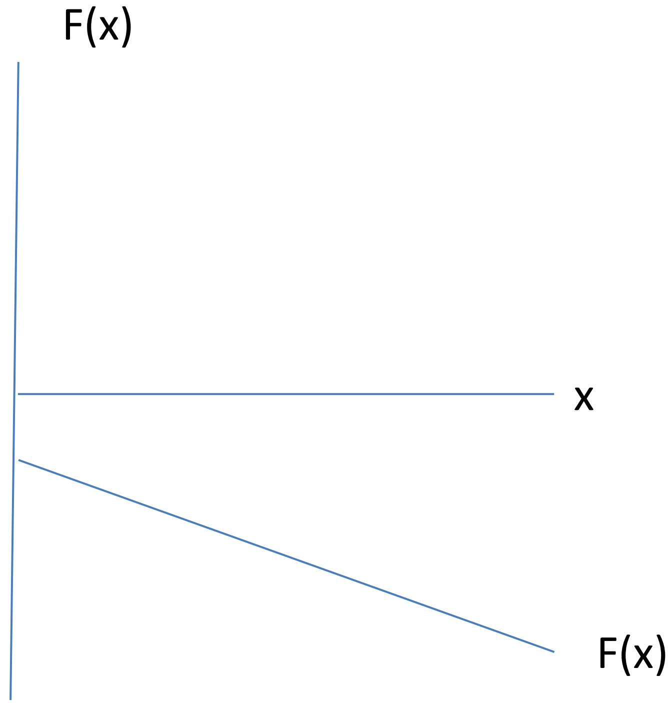
ii) $\exists a x_0 > 0 \ni F(x_0) > 0$

iii) $\exists a x_1 > 0 \ni F(x_1) < 0$



F(x)
F(x)
F(x$_0$)
x$_1$
x
x$_0$
F(x$_1$)


P
S
D
x


F(x)
x
F(x)

Uniqueness

If equilibrium exists, is it unique?

Can we have only one

$$P_e \geq 0 \ni E(P_e) = 0 \quad ?$$

Or in Marshallian sense one $x_e \geq 0 \ni F(x_e) = 0$?

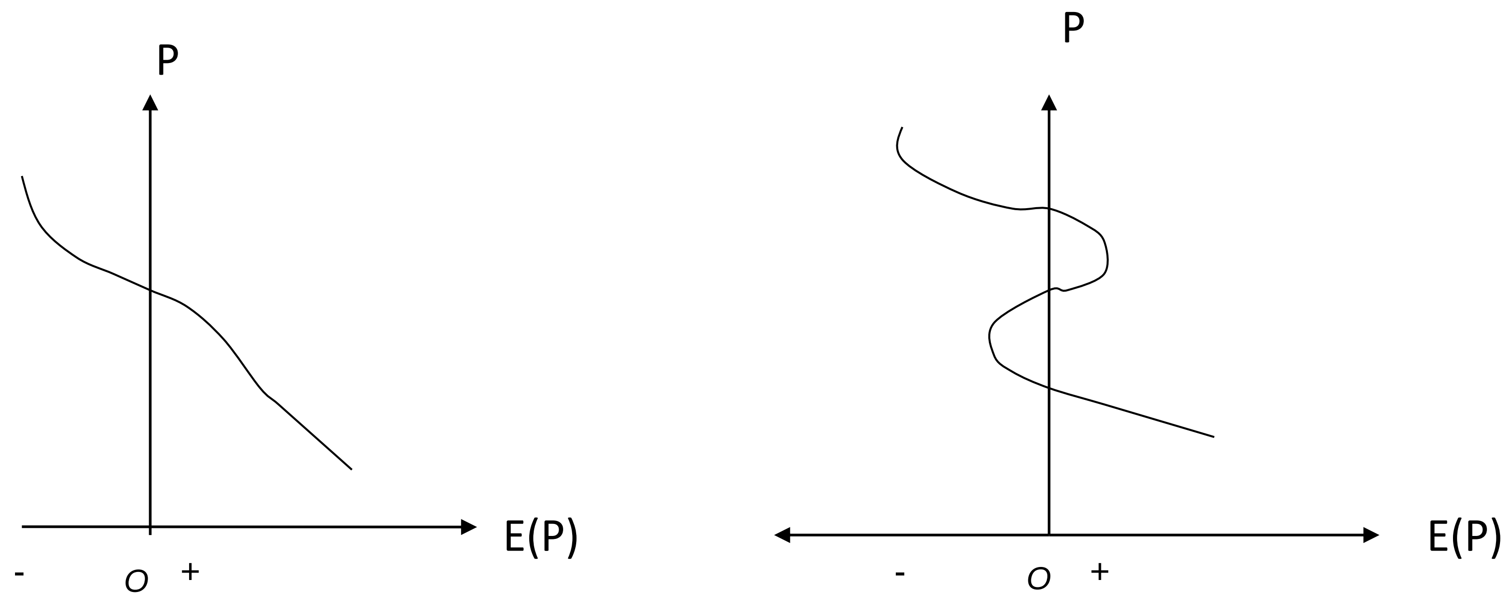
Condition:

If equilibrium exists, it will be unique iff

Either $E'(P) > 0 \forall p$

or $F'(x) > 0 \forall x$

i.e., E(P)/F(x)should be monotonic.



Unique and Multiple Equilibria

- Linearity of dd and ss doesn't necessarily imply existence and uniqueness of eqm.

Condition for Uniqueness of Walrasian equilibrium:

Proof. Let $P_e > 0$ be an equilibrium price.

Hence, by definition, $E(P_e)=0$ ………….(1)

Suppose also that

$$E'(P) > 0 \forall p \quad \text{........(2)}$$

Combining (1) and (2) we can conclude that

$$E(P) > 0 \forall p > p_e$$
$$E(P) < 0 \forall p < p_e$$

Hence there cannot be any "other" price $p' \ni E(P') = 0$

Stabilty

An equilibrium is stable if after an initial disturbance which causes price or quantity to deviate from its equilibrium value, the market adjusts itself towards the initial equilibrium.

Static vs dynamic Stability

$\downarrow$ studies direction of adjustment

$\downarrow$ tells about speed and magnitude

Local vs global Stability

$\downarrow$ disturbance within a small range

$\downarrow$ stable for any initial disturbance

Walrasian Static Stability

Adjustment mechanism: price

E(P)=D(P)-S (P)

Condition: Rule 1. If at a price P_0, $E(P_0)>0$, price increases further.

Rule 2. If at a price P_1, $E(P_1)<0$, price decreases further.

Given rules 1 and 2, Walrasian Static Stability requires that

$$\mathrm{E}^{'}(\mathrm{P}) < 0 \forall P$$

Proof.

If $E^{'}(P) < 0$

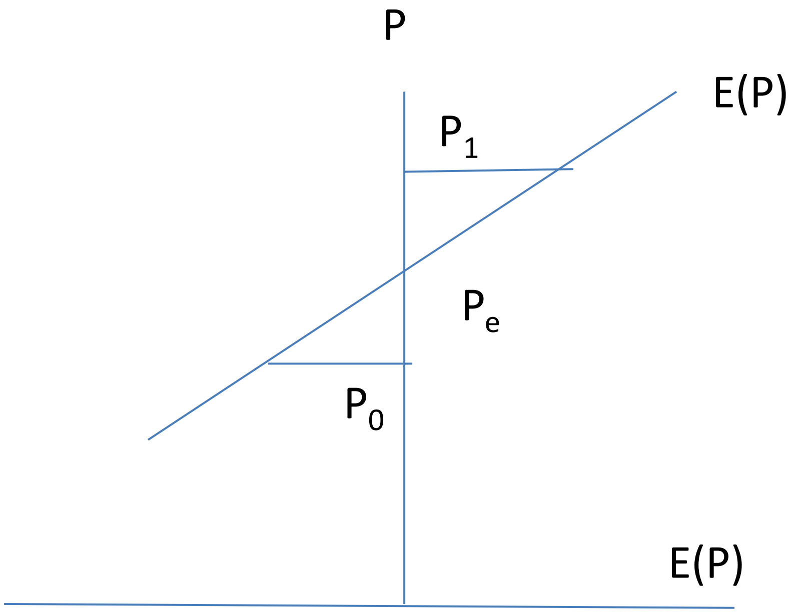
then

a) $E(P) < 0 \forall p > p_e$

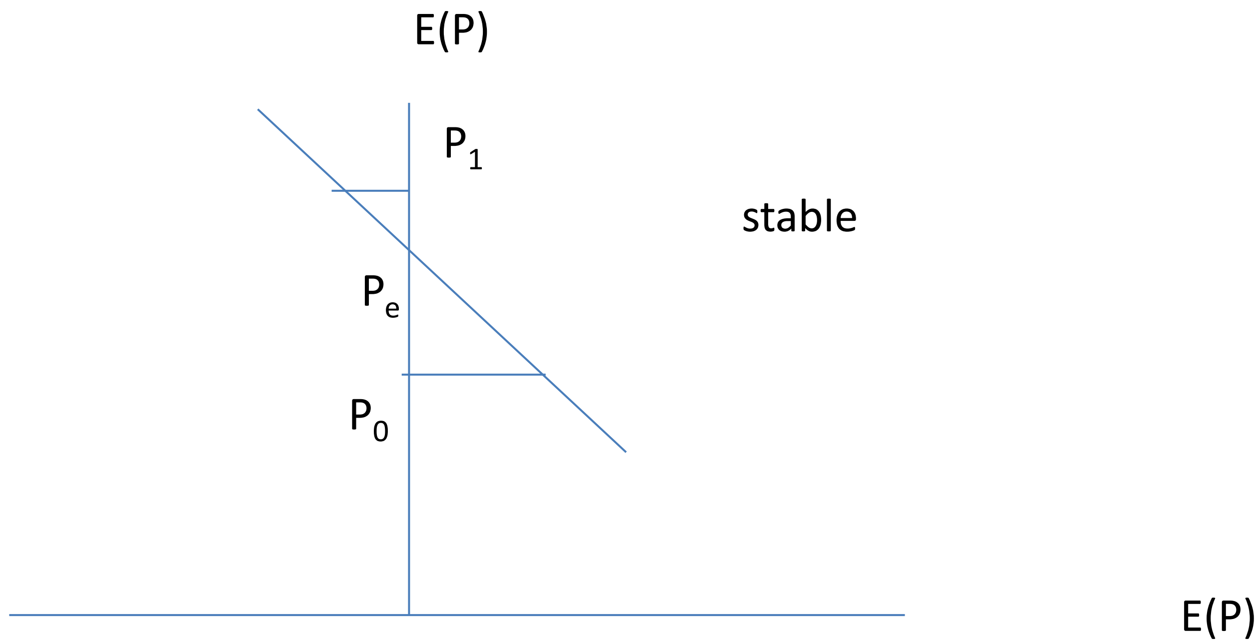
b) $E(P) > 0 \forall p < p_e$

If initially price deviates below P_e, there arises ED for the good which in turn increases price towards P_e.

If initially price increases above P_e, there arises ES for the good which in turn decreases price towards P_e.



unstable



stable

Marshallian Static Stability

Adjustment mechanism: quantity

Condition: Rule 1. If $P_d > P_s$, x increases. .

Rule 2. If $P_d < P_s$, x decreases.

Given rules 1 and 2, a market is Marshallian Static Stable

$$i)\, p^d > p^s \forall x < x_e$$

$$ii)\, p^d < p^s \forall x > x_e$$

Combining (i) and (ii) $\mathrm{F}^{'}(\mathrm{x}) < 0$

Marshallian Static Stability requires

a) $F(x) < 0 \forall x > x_e$

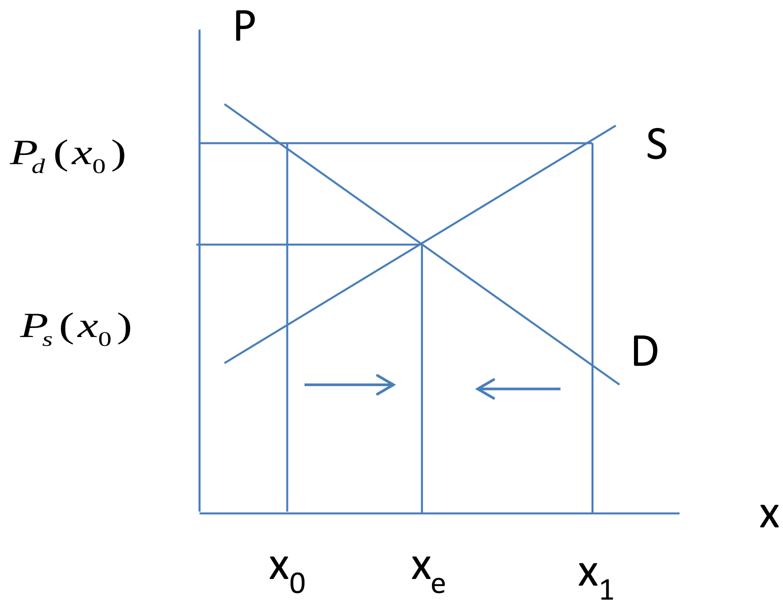
b) $F(x) > 0 \forall x < x_e$

Global stability:

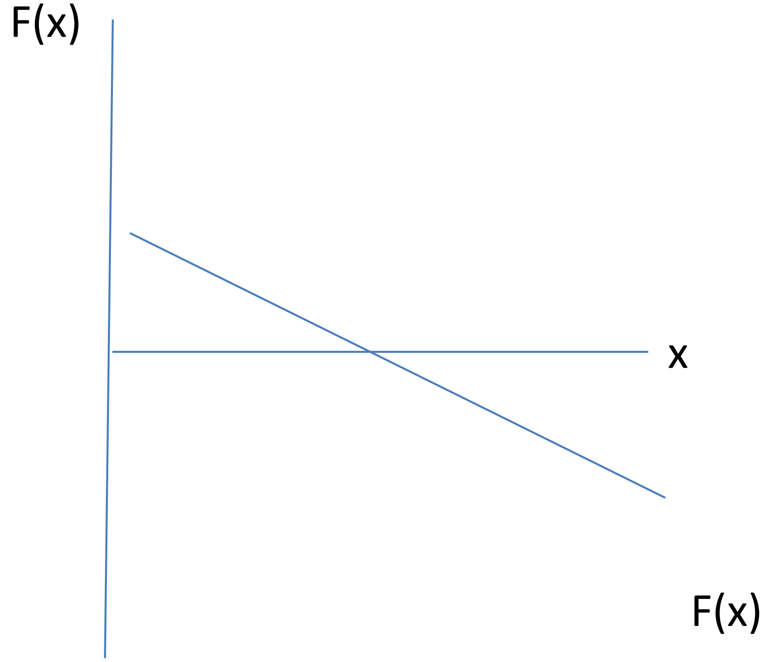
$$\mathrm{F}^{'}(\mathrm{x}) < 0 \forall x \geq 0$$

Local stability:

$$\mathrm{F}^{'}(\mathrm{x}) < 0 \forall x \in [x_e - \varepsilon, x_e + \varepsilon]$$



P
$P_d(x_0)$
S
$P_s(x_0)$
D
x
x_0
x_e
x_1



F(x)
x
F(x)

Difference between the two approaches

- When dd and ss curves slope in opposite direction the two concepts give us identical results.

- When dd and ss curves slope in same direction the two concepts give us polar opposite results.

Proof

Marshallian Static Stability requires

$$\mathrm{F}'(\mathrm{x}) < 0 \ldots\ldots (F)$$

i.e., $$\frac{\partial p^d}{\partial x} - \frac{\partial p^s}{\partial x} < 0 \Rightarrow \frac{\dfrac{\partial x^s}{\partial p} - \dfrac{\partial x^d}{\partial p}}{\dfrac{\partial x^d}{\partial p} \cdot \dfrac{\partial x^s}{\partial p}} < 0 \ldots\ldots (F')$$

If the denominator is >0

(which happens when the dd and ss curves slope in the same direction)

then (F')< 0 implies (E')>0.