

class test - 1

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Ques 2

(a) To prove: If a relation R has only one key, it is in BCNF if and only if it is in 3NF.
let us assume F^+ denote the closure of the set of functional dependencies satisfied by a relation R which is assumed to be in 3NF.

We need to show that for each non-trivial dependency $X \rightarrow A$ in F^+ , X is a superkey.

To this end, considering a dependency. If X is not a superkey, the 3NF property guarantees that the attribute A is a part of a key.

Since all keys are simple by assumption, we have that A is a key.

This last fact together with the dependencies $X \rightarrow A$ implies that X is a superkey (this follows, from transitive axiom) which is a contradiction.

Ques 2

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(b) (i)

$R(W, X, Y, Z)$

$X \rightarrow Y, W \rightarrow Z$ and $Y \subseteq W$

$Y \subseteq W \Rightarrow W \rightarrow Y$ (by reflexivity)

$$X^+ = XY$$

So $X \not\rightarrow Z$ (disproved)

(ii) $XZ \rightarrow Y, X \rightarrow W$ & $Z \subseteq W$

$Z \subseteq W \Rightarrow W \rightarrow Z$ (by reflexivity)

Now, $X^+ = XWZY \Rightarrow X \rightarrow Y$ (proved)

Ques 2

(c)

$R(A, B, C, D, E, X, Y)$

The set of FD's

$\{D \rightarrow A, XD \rightarrow C, DA \rightarrow B, A \rightarrow X, XE \rightarrow B, E \rightarrow A, B \rightarrow D, EB \rightarrow C, AB \rightarrow C, Y \rightarrow B, C \rightarrow B\}$

First we find out Key

$$YE^+ = YEBA XCD$$

Hence, YE is the Key

To find 3NF lossless decomposition

Step 1 Since all the FD's has only one attribute on the right side, no need to do anything

Step 2 checking Redundant FD's

$$D \rightarrow A$$

$$D^+ = D \quad \text{NR}$$

$$XD \rightarrow C$$

$$XD^+ = XDABC \quad \text{NR}$$

$$DA \rightarrow B$$

$$DA^+ = DAX \quad \text{NR}$$

$$A \rightarrow X$$

$$A^+ = A \quad \text{NR}$$

$$XE \rightarrow B$$

$$XE^+ = XEA \quad \text{NR}$$

$$E \rightarrow A$$

$$E^+ = E \quad \text{NR}$$

$$B \rightarrow D$$

$$B^+ = B \quad \text{NR}$$

$$EB \rightarrow C$$

$$EB^+ = EBAXDC$$

~~Redundant~~

$$AB \rightarrow C$$

$$AB^+ = ABXDC \quad \text{NR}$$

$$Y \rightarrow B$$

$$Y^+ = Y \quad \text{NR}$$

$$C \rightarrow B$$

$$C^+ = C \quad \text{NR}$$

Now,

$$F' = \{D \rightarrow A, DA \rightarrow B, A \rightarrow X, XE \rightarrow B, E \rightarrow A, B \rightarrow D, EB \rightarrow C, AB \rightarrow C, Y \rightarrow B, C \rightarrow B\}$$

So 3NF decomposition is

$$R_1(DA)$$

$$R_2(DAB)$$

$$R_3(A, X)$$

$$R_4(X, E, B)$$

$$R_5(E, A)$$

$$R_6(B, D)$$

~~$$R_7(A, B, C)$$~~

~~$$R_8(Y, B)$$~~

~~$$R_9(C, B)$$~~

after removing subsets

$$R_2(DA, B)$$

$$R_3(A, X)$$

$$R_4(X, E, B)$$

$$R_5(E, A)$$

~~$$R_7(A, B, C)$$~~

~~$$R_8(Y, B)$$~~

and adding

one more for key

$$R_9(Y, E)$$

Ques 3 (a) To calculate natural join

- (i) Compute $R \times S$
 (ii) write only tuples which are having common attributes in R and S

$R \times S \Rightarrow$

A	B	C	A	B	D

$R \bowtie S \Rightarrow$

A	B	C	D

Relational algebra representation \Rightarrow

$$\pi_{R.A, R.B, R.C, S.D} \left(\sigma_{\substack{R.A = S.A \wedge \\ R.B = S.B}} (R \times S) \right)$$

$$R \bowtie S = \{ t \mid \exists u \exists v (R(u) \wedge S(v) \wedge \\ u[A] = v[A] \wedge u[B] = v[B] \wedge t[A] = u[A] \\ \wedge t[B] = u[B] \wedge t[C] = u[C] \\ \wedge t[D] = v[D]) \}$$

Ques 3

(b) (i) $T = \pi_{u\text{-name}} (\text{user} \bowtie \text{Borrow})$

$$S = \pi_{u\text{-name}} (\text{user})$$

$S - T$ gives the desired result

(ii) $\pi_{s\text{-name}} \left(\sigma_{\text{cardno} = 'A11'} (\text{user} \bowtie \text{Borrow} \bowtie \text{supply} \bowtie \text{suppliers}) \right)$

(iii) $P = \pi_{\text{price}} (\text{book} \bowtie \text{supply})$

$$Q = \pi_1 \left(\sigma_{1 < 2} (P \times P) \right)$$

$$R = P - Q$$

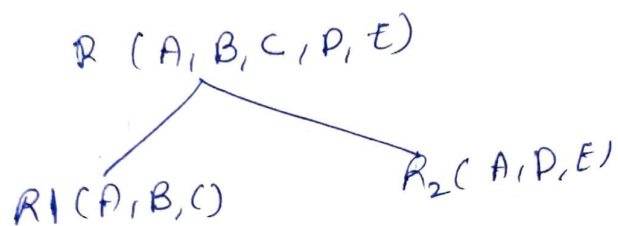
$$\pi_{\text{title}} \left(\sigma_{3 > 8} (\text{supply} \bowtie \text{book} \bowtie R) \right)$$

Entity set	Key
Student	student - id
Department	(name, code)
Course	Course - no.
section	Section number
grade report	student number

Student

Ques 4

(a) Given that a relational scheme
 $R(A, B, C, D, E)$ with FD's
 $\{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$



This decomposition $R = (R_1, R_2)$ is a loss less join decomposition. Since we know that a decomposition

$R = (R_1, R_2)$ is a lossless join decomposition — (1)

iff $R_1 \cap R_2 \rightarrow R_1 - R_2$ or $R_1 \cap R_2 \rightarrow R_2 - R_1$

i.e. ~~if~~ $R_1 \cap R_2$ should be either key of R_1
or R_2 or both

for $R_1(A, B, C)$

$A \rightarrow BC$

since, $A^+ = ABC$ so A is key for relation R_1

Now $R_1 \cap R_2 = \{A\}$

Since A is a key of R_1 so

$R_1 \cap R_2 \rightarrow \{A\} \rightarrow R_1$

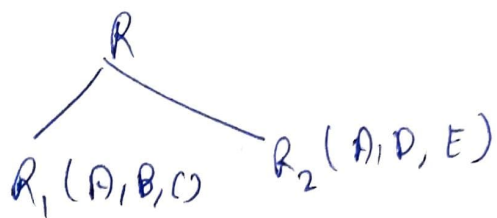
$R_1 \cap R_2 \rightarrow R_1$

from (1)

$R = (R_1, R_2)$ is a loss less join decomposition

(ii) Decomposition is not a dependency preserving decomposition

Given that $R_1 = (R_1, R_2)$



For $R_1(A, B, C)$ possible FD's will be as

$$A \rightarrow BC$$

$$B \rightarrow AC$$

$$C \rightarrow AB$$

$$AB \rightarrow C$$

$$BC \rightarrow A$$

$$AC \rightarrow B$$

Now we will check which are valid

$$A^+ = ABCDE \quad (\text{from } R)$$

$$B^+ = BD \quad (\quad)$$

$$C^+ = C \quad (\quad)$$

$$AB^+ = ABC \quad (\quad)$$

$$BC^+ = BCDE \quad (\quad)$$

So valid FD's for R_1 will be

$$A \rightarrow BC$$

$$AB \rightarrow C$$

$$BC \rightarrow A$$

$$AC \rightarrow B$$

} — (2)

for $R_2(A, D, E)$ possible FD's

$$A \rightarrow DE, D \rightarrow AE, E \rightarrow AD, AD \rightarrow E, AE \rightarrow D$$

$$DE \rightarrow A$$

Now, $A^T = ABCF$
 $D^T = D$
 $E^T = ABCDE$ } form R

Now valid FD's for R_2 will be
 $A \rightarrow ADE$ $AE \rightarrow D$ $E \rightarrow AD$ $DE \rightarrow A$
 $AD \rightarrow E$ — (3)

All the valid FD's for $R_1 \cup R_2$ will be
 $F' = \{ A \rightarrow BC, AB \rightarrow C, BC \rightarrow A, AC \rightarrow B,$
 $A \rightarrow DE, E \rightarrow AD, AD \rightarrow E, AE \rightarrow D,$
 $DE \rightarrow A \}$

From FD's of R
 we will check whether we can derive FD's of
 R from F'

~~So~~ $CD^+ = CD$
 $E \subseteq CD^+$

So $CD \not\rightarrow E$ (can't be written)

Hence decomposition $R = R_1 \cup R_2$ is not
 dependency preserving decomposition.