# Optimum Choice of a consumer: Problem of Utility maximisation

Intermediate
Microeconomics
by
Hal R. Varian

- Max U= u(X,Y)................(1)

Subject to

- $M = P_X X + P_Y Y$.............(2)

## The Method of Lagrange Multipliers

- **method of Lagrange multipliers** Technique to maximize or minimize a function subject to one or more constraints.

- **Lagrangian** Function to be maximized or minimized, plus a variable (the Lagrange multiplier) multiplied by the constraint.

1. **Stating the Problem** First, we write the Lagrangian for the problem.

$$L = \mathrm{U(X,Y)} - \lambda(P_X X + P_Y Y - M) \qquad \textbf{(3)}$$

Note that we have written the budget constraint as

$$P_X X + P_Y Y - M = 0$$

2. **Differentiating the Lagrangian** We choose values of *X* and *Y* that satisfy the budget constraint, then the second term in equation (3) will be zero. By differentiating with respect to *X*, *Y*, and *M* and then equating the derivatives to zero, we can obtain the necessary conditions for a maximum.

$$\frac{\partial L}{\partial \mathrm{X}} = \mathrm{MU}_X(X, Y) - \lambda P_X = 0$$
$$\frac{\partial L}{\partial \mathrm{Y}} = \mathrm{MU}_Y(X, Y) - \lambda P_Y = 0 \qquad \textbf{(4)}$$
$$\frac{\partial L}{\partial \lambda} = M - P_X X - P_Y Y = 0$$

3. **Solving the Resulting Equations** The three equations in (4) can be rewritten as

$$\mathrm{MU}_X = \lambda P_X$$
$$\mathrm{MU}_Y = \lambda P_Y$$
$$P_X X - P_Y Y = M$$

## The Equal Marginal Principle

We combine the first two conditions above to obtain the *equal marginal principle*:

$$\lambda = \frac{\mathrm{MU}_X(X,Y)}{P_X} = \frac{\mathrm{MU}_Y(X,Y)}{P_Y} \qquad \textbf{(5)}$$

To optimize, *the consumer must get the same utility from the last rupee spent by consuming either X or Y*. To characterize the individual's optimum in more detail, we can rewrite the information in (5) to obtain

$$\frac{\mathrm{MU}_X(X,Y)}{\mathrm{MU}_Y(X,Y)} = \frac{P_X}{P_Y} \qquad \textbf{(6)}$$

## Duality in Consumer Theory

- Rather than choosing the highest indifference curve, given a budget constraint, the consumer chooses the lowest budget line that touches a given indifference curve.

Minimizing the cost of achieving a particular level of utility:

Minimize $P_X X + P_Y Y$ subject to the constraint that $U(X,Y) = U^*$

The corresponding Lagrangian is given by

$$L = P_X X + P_Y Y - \mu(U(X,Y) - U^*) \qquad \textbf{(7)}$$

Differentiating with respect to *X*, *Y*, and $\mu$ and setting the derivatives equal to zero, we find the following necessary conditions for expenditure minimization:

$$P_X - \mu \mathrm{MU}_X(X,Y) = 0$$

$$P_Y - \mu \mathrm{MU}_Y(X,Y) = 0$$

and

$$U(X,Y) = U^*$$

# Direct & indirect utility function

- Direct utility function: $u=u(x_1, x_2)$

  Now at optimum: $x_i = f(M, P_1, P_2)$

  $U = u[x_1(M, P_1, P_2), x_2(M, P_1, P_2)] = u[M, P_1, P_2]$

- This is called the indirect utility function.