Sol 1

(1) From the given set of final equations:

BV	x_1	x_2	x_3	x_4	x_5	RHS
x_3	$1/2$	0	1	$3/2$	$-1/2$	25
x_2	$1/2$	1	0	$-1/2$	$1/2$	25
$z_j - c_j$	3	0	0	4	1	300

(a) shadow price for sugar = 4

" " " chocolate = 1

(b) Since x_1 is non-basic variable, so only $z_1 - c_1$ will change

$$z_1 - c_1 = c_B B^{-1} a_1 - c_1 = [4 \quad 1]\begin{bmatrix} 1 \\ 2 \end{bmatrix} - c_1 = 6 - c_1 \geq 0 \Rightarrow \boxed{c_1 \leq 6}$$

Thus, if $c_1 \leq 6$, the current basis remains optimal

(c)

$$\underset{\sim}{B}^{-1} \underset{\sim}{b}' = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} b_1 \\ 100 \end{bmatrix} = \begin{bmatrix} \frac{3b_1}{2} - 50 \\ -\frac{b_1}{2} + 50 \end{bmatrix}$$

The current basis remains optimal if

$$\left.\begin{array}{l} \frac{3b_1}{2} - 50 \geq 0 \Rightarrow b_1 \geq \frac{100}{3} \\ \text{and } -\frac{b_1}{2} + 50 \geq 0 \Rightarrow b_1 \leq 100 \end{array}\right\} \Rightarrow \boxed{\frac{100}{3} \leq b_1 \leq 100}$$

(d) For $b_1 = 30$, the current basis would not be ~~optimal~~ feasible

To find new solution using dual simplex method

BV	x_1	x_2	x_3	x_4	x_5	RHS
x_3	$1/2$	0	1	$3/2$	$-1/2$	~~25~~ -5
x_2	$1/2$	1	0	$-1/2$	$1/2$	35
$z_j - c_j$	3	0	0	4	1	220
Ratio	—	—	—	—	$1/(-1/2)$	
x_5	-1	0	-2	-3	1	10
x_2	1	1	1	1	0	30
$z_j - c_j$	4	0	2	7	0	210

$$c_B B^{-1} b = \begin{bmatrix}5 & 7\end{bmatrix}\begin{bmatrix}-5\\35\end{bmatrix} = 220$$

$\boxed{\text{optimal sol}^n.\ x_1 = 0,\ x_2 = 30,\ x_3 = 0 \quad z = 210}$

(e) Let x_6 be the number of type 4 candy bars to be produced

$$z_6 - c_6 = c_B B^{-1} a_6 - c_6 = \begin{bmatrix}4 & 1\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix} - 17 = -1$$

Hence, the current basis is not optimal

BV	x_1	x_2	x_3	x_4	x_5	x_6	RHS	Ratio
x_3	$1/2$	0	1	$3/2$	$-1/2$	$5/2$	25	10
x_2	$1/2$	1	0	$-1/2$	$1/2$	$1/2$	25	50
$z_j - c_j$	3	0	0	4	1	-1	300	
x_6	5	0	$2/5$	$3/5$	$-1/5$	1	10	
x_2	-2	1	$-1/5$	$-4/5$	$3/5$	0	20	
$z_j - c_j$	8	0	$2/5$	$23/5$	$4/5$	0	310	

$$B^{-1} a_6 = \begin{bmatrix}3/2 & -1/2\\ -1/2 & 1/2\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}5/2\\1/2\end{bmatrix}$$

$\boxed{\text{Optimal sol}^n: \rightarrow x_2 = 20,\ x_1 = 0,\ x_3 = 0 \quad x_6 = 10 \quad z = 310}$

Sol 2)

(a) Max $Z = 2x_1 + 3x_2 + x_3$, s.t $\frac{2}{5}x_1 + \frac{1}{5}x_2 + \frac{3}{5}x_3 \leq 4/5$
and $\frac{1}{5}x_1 + \frac{3}{5}x_2 + \frac{4}{5}x_3 \leq 7/5$

(b) $C_B B^{-1}A - C \geq 0 \rightarrow c_3 \leq 5$
For $c_3 = 6 \rightarrow C_B B^{-1}A - C = [0 \quad 0 \; -1]$ x_3 will enter the basis
$[0, 1, 1, 0, 0]$ $Z = 9$

(c) $-2 \leq \lambda \leq 3$

(4) Equation (2) as shadow price is higher $(4 > 3)$
Increase in obj func $= 4$, Student needs to show $B^{-1}b \geq 0$ for this increase (for full marks)

(5) $B^{-1}b_{new} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$ need to apply Dual Simplex method

	x_1	x_2	x_3	x_4	x_5	RHS
Z	0	0	4	3	4	16
x_1	1	0	1	3	-1	-1
x_2	0	1	1	-1	2	6
Z	4	0	8	15	0	12
x_5	-1	0	-1	-3	1	1
x_2	2	1	3	5	0	4

$(0, 4, 0, 0, 1)$
$Z = 12$

Sol 3)

Q2) A

Vogel's Approximation method:- (minimize the total transportation cost / day).

	Delhi	Mumbai	Kolkata	Chennai	Supply
Mysore	20	6	17	5	5000
Nasik	12	2	18	16	5000
Dummy	0	0	0	0	2000
Demand	3000	3000	3000	3000	12000 / 12000

					Supply	penalty-1	penalty-2	penalty-3
	20	6 (2000)	17	5 (3000)	~~5000~~ 2000	1	1	(11)
	12 (3000)	2 (1000)	18 (1000)	16 X	5000	10	10	10
	0 X	0 X	0 (2000)	0 X	~~2000~~ X	0	0	0
Demand	3000	~~3000~~ 1000	~~3000~~ 1000	3000	12000 / 12000			
penalty-1	12	2	(17)	5				
penalty-2	8	4	1	(11)				
penalty-3	8	4	1	0				

optimality checking :-

					u_i
	[20] +4	[6] 2000	[17] -5	[5] 3000	0
	[12] 3000	[2] 1000	[18] 1000	[16] 15+	-4
	[0] +6	[0] +16	[0] 2000	[0] +17	-22
v_j –	16	6	22	5	

$c_{ij} - (u_i + v_j)$

Iteration :-2

u_i v_j :	Delhi 16	Mumbai 6	Kolkata ~~17~~	chennai 5
Mysore 0	[20] +4	[6] 1000 ✓	[17] 1000 ✓	[5] 3000 ✓
Nasik -4	[12] ✓ 3000	[2] 2000 ✓	[18] +5	[16] +15
Dummy -17	[0] +1	[0] +11	[0] 2000 ✓	[0] +12

$c_{ij} - (u_i + v_j) \geq 0$

It is the optimal solution

Total cost $= 6 \times 1000 + 17 \times 1000 + 5 \times 3000 + 12 \times 3000 + 2 \times 2000$
$+ 0 \times 2000$

$= 78{,}000$.

$\boxed{\text{Kolkata}}$ have shortage of $\boxed{2000}$ supply.

Sol 4)

Maximization (Hungarian algorithm).

	T1	T2	T3	T4
C1	-50	-60	-70	-80
C2	-70	-50	-20	-90
C3	-40	-70	-80	-60
C4	-100	-70	-60	-90

→ Row wise deduction.

	T1	T2	T3	T4
C1	30	20	10	0
C2	20	40	70	0
C3	40	10	0	20
C4	0	30	40	10

→ column wise deduction.

	T1	T2	T3	T4
C1	30	10	10	$\boxed{0}$
C2	20	30	70	~~0~~
C3	40	$\boxed{0}$	~~0~~	20
C4	$\boxed{0}$	20	40	10

min

$C^* = 10$

→

	T1	T2	T3	T4
C1	20	$\boxed{0}$	0	0
C2	10	20	60	$\boxed{0}$
C3	40	0	$\boxed{0}$	30
C4	$\boxed{0}$	20	40	20

Total ⇒ 60 + 90 + 100 + 80

⇒ 330

2

	T1	T2	T3	T4
C1	20	0	$\boxed{0}$	0
C2	10	20	60	$\boxed{0}$
C3	40	$\boxed{0}$	0	30
C4	$\boxed{0}$	20	40	20

Total = 70 + 90 + 70 + 100

= 330 .