Department of Industrial & Systems Engineering
Indian Institute of Technology, Kharagpur

Class Test II

Operations Research –I (IM21003)

Time: 1 Hr **Full Mark: 50**

(Assume missing data suitably)

Question 1 **[1+3+3+4+4]**

A company Candyco can manufacture three types of candy bar using sugar and chocolate. If x_j be the number of Type j candy bars manufactured, the company should solve the following LP:

$$\text{Max } Z = 3x_1 + 7x_2 + 5x_3$$
$$x_1 + x_2 + x_3 \le 50 \quad \text{(Sugar constraint)}$$
$$2x_1 + 3x_2 + x_3 \le 100 \quad \text{(Chocolate constraint)}$$
$$x_1, x_2, x_3 \ge 0$$

After adding slack variables x_4 and x_5 for the respective constraints, the simplex method yields the following final set of equations:

$$Z + 3x_1 + 4x_4 + x_5 = 300$$
$$\frac{1}{2}x_1 + x_3 + \frac{3}{2}x_4 - \frac{1}{2}x_5 = 25$$
$$\frac{1}{2}x_1 + x_2 - \frac{1}{2}x_4 + \frac{1}{2}x_5 = 25$$

Using the above final set of equations, answer the following questions (extend the simplex/dual simplex method if required):

(a) Find the shadow price for both the constraints.

(b) For what values of Type 1 candy bar profit does the current basis remain optimal?

(c) For what amount of available sugar would the current basis remain optimal?

(d) If 30 ounce of sugar were available, how many of each candy bar should the company make?

(e) Candyco is considering making Type 4 candy bars. A Type 4 candy bar earns Rs 17 profit and requires 3 ounce of sugar and 4 ounce of chocolate. Find the new optimal solution.

Question 2 **[4+4+4+3+5]**

The simplex method yields the following final set of equations to a standard linear program of form $Max\ Z = \boldsymbol{cx}$, $Subject\ to\ \boldsymbol{Ax} \le \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}$

$Z + 4x_3 + 3x_4 + 4x_5 = 8$ … (0)

$x_1 + x_3 + 3x_4 - x_5 = 1$ … (1)

$x_2 + x_3 - x_4 + 2x_5 = 2$ … (2)

Assume that x_4 and x_5 form the initial basis. Using the concept of sensitivity analysis, answer the following questions.

(a) Write the formulation of the original LP.

(b) How much can c_3 be increased before the current basis is no longer optimal? Find an optimal solution when $c_3 = 6$.

(c) Find the range of λ for which the given solution is still optimal if the coefficient in the $(0)^{th}$ equation is replaced by $\boldsymbol{c} + \lambda\boldsymbol{c}^*$ where $\boldsymbol{c}^* = (0, 0, 1, -1, 2)$ and $-\infty < \lambda < \infty$.

(d) If you were to choose between increasing (one unit) the right hand side of the first and second constraints, which one would you choose and why? What is the effect of this increase on the optimal value of the objective function?

(e) Find the optimal solution when b_2 is increased by 2 units.

Question 3 **[10]**

In the light of the recent ban on Rs. 500 and Rs. 1000 notes, Reserve Bank of India (RBI) plans a model of supplying new notes to its four regional offices located at Delhi, Mumbai, Kolkata and Chennai from its two printing presses located at Mysore and Nasik. The transportation cost (in Rupees) per note is given in the Table. Supply and demand are expressed in number of notes per day.

RBI wants to minimize the transportation cost for 1-day planning period. Use Vogel's Approximation Method (row penalty) to find an initial basic feasible solution and apply transportation Simplex method to obtain the optimal solution. Which city (or cities) will fall short of supply and by how much?

	Delhi	Mumbai	Kolkata	Chennai	Supply
Mysore	20	6	17	5	5000
Nasik	12	2	18	16	5000
Demand	3000	3000	3000	3000	

Question 4 **[5]**

Indian Army plans a surgical strike on its enemy and identifies four key personnel in the rank of Colonel to lead the strike. Four troops of soldiers of different specializations are formed and each colonel will lead exactly one troop. Based on the experience of the Colonels, Army estimates their leadership scores of leading a particular troop as shown in the following matrix.

	Troop 1	Troop 2	Troop 3	Troop 4
Colonel 1	50	60	70	80
Colonel 2	70	50	20	90
Colonel 3	40	70	80	60
Colonel 4	100	70	60	90

Army would like to find the optimal assignment to maximize the total leadership scores thereby maximally increasing the chance of a successful mission. Use Hungarian algorithm to solve this assignment problem.

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