Assignment 3

Symbols: $p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}, r \equiv \frac{\partial^2 z}{\partial x^2}, s \equiv \frac{\partial^2 z}{\partial x \partial y}, t \equiv \frac{\partial^2 z}{\partial y^2};\ u_x \equiv \frac{\partial u}{\partial x}, u_{xx} \equiv \frac{\partial^2 u}{\partial x^2},$

$u_{xy} = u_{yx} \equiv \frac{\partial^2 u}{\partial x \partial y};\ f'(v) \equiv \frac{\mathrm{d}f}{\mathrm{d}v}\ , f''(v) \equiv \frac{\mathrm{d}^2 f}{\mathrm{d}v^2};$

Operators: $D \equiv \frac{\partial}{\partial x}\ , D' \equiv \frac{\partial}{\partial y}\ , DD' \equiv \frac{\partial^2}{\partial x \partial y}\ , D^2 \equiv \frac{\partial^2}{\partial x^2}\ , D'^2 \equiv \frac{\partial^2}{\partial y^2}$

Topics: First order PDE→ Compatibility of two PDEs. Charpit's Method to find CI of PDE. To find GI (from CI), PI, SI (if exists) of PDE.

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1. Check compatibility of following pairs of 1st order PDEs, and if compatible, find common solution $z(x, y)$:
    a) $xp - yq = 0\ ,\quad z(xp + yq) - 2xy = 0$
    b) $xp - yq = x,\quad x^2 p + q = xz$
    c) $xpq - yq = xy\ ,\quad x^2 p + q^2 = z$

2. Solve following 1st order PDEs by Charpit's method:
    a) $q + \ln p = 2 \ln 2$
    b) $pz + q = 1$

3. Find complete integral (CI) of following PDE by Charpit's method:
    a) $p^3 y(1 + x^2) = qx^3$
    b) $z = px + qy + \sin(pq)$

5. Find singular integral, if any, of $(p^2 + q^2)y - qz = 0$

END