$$G_p^2 + T_p^2 = 100$$

$$\text{MRT}: \; -\frac{dG_p}{dT_p} = \frac{T_p}{G_p} \quad \text{--- (1)}$$

If community consp^n occurs at a pt on a line which touches the frontier, then

$$-\frac{dG_p}{dT_p} = \frac{T_p}{G_p} = \frac{P_T}{P_G} \text{ (rel. pr.) } = \theta \quad \text{--- (2)}$$

$$\therefore \; G_p = \frac{T_p}{\theta} \quad \text{--- (3)}$$

Putting this value of G_p in the eq^n for PPF:

$$\frac{T_p^2}{\theta^2} + T_p^2 = 100 \;\Rightarrow\; T_p = \frac{10\theta}{\sqrt{(1+\theta^2)}} \quad \text{--- (4)}$$

$$G_p = \frac{T_p}{\theta} = \frac{10}{\sqrt{1+\theta^2}} \quad \text{--- (5)}$$

Trade balance cond^n (budget constraint):

$$\theta(T_C - T_P) = G_P - G_C \quad [\text{value of exp} = \text{value of imp.}]$$

$$\text{or, } \theta T_C + G_C = \theta T_P + G_P = \theta \cdot \frac{10\theta}{\sqrt{1+\theta^2}} + \frac{10}{\sqrt{1+\theta^2}}$$

$$= \frac{10(\theta^2+1)}{\sqrt{(\theta^2+1)}} = 10\left(\sqrt{\theta^2+1}\right) \quad \text{--- (6)}$$

Now community MRS:

$$\text{e. CIC}: \; \overline{u} = G_c T_c$$

$$G_c\, dG_c + T_c\, dG_c = 0$$

$$\text{or, } \frac{dG_c}{dT_c}\Big|_{IC} = \frac{T_C}{G_c}$$

Now MRS = pr ratio $\Rightarrow$ $\frac{G_c}{T_c} = \theta \Rightarrow G_c = \theta T_c$ --- (7)

Replace it in TB cond. (6):

$$\theta T_c + \theta T_c = 10\sqrt{(\theta^2+1)}$$

$$\Rightarrow T_c = \frac{5}{\theta}\sqrt{\theta^2+1}$$

$$\therefore G_c = \theta T_c = 5\sqrt{1+\theta^2}$$

Traded qt.s of 2 goods are

$$T_p - T_c = \frac{10\theta}{\sqrt{1+\theta^2}} - \frac{5}{\theta}\sqrt{1+\theta^2} = \frac{5(\theta^2-1)}{\theta\sqrt{1+\theta^2}}$$

$$G_c - G_p = 5\sqrt{1+\theta^2} - \frac{10}{\sqrt{1+\theta^2}} = \frac{5(\theta^2-1)}{\sqrt{1+\theta^2}}$$

~~If $\theta < 1$~~ $\Rightarrow$ Imp & exp acc to value of θ $(\theta \gtrless 1)$.