Operations Research-I
Practice Problem 2

(1) Use the graphical method to solve the problem:

Maximize $Z = 10x_1 + 20x_2$,

subject to

$$\begin{aligned} -x_1 + 2x_2 &\le 15 \\ x_1 + x_2 &\le 12 \\ 5x_1 + 3x_2 &\le 45 \end{aligned}$$

and

$x_1 \ge 0, \quad x_2 \ge 0.$

(2) Consider the following problem, where the value of c_1 has not yet been ascertained.

Maximize $Z = c_1x_1 + x_2$,

subject to

$$\begin{aligned} x_1 + x_2 &\le 6 \\ x_1 + 2x_2 &\le 10 \end{aligned}$$

and

$x_1 \ge 0, \quad x_2 \ge 0.$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of $c1$ $(\infty < c_1 < \infty)$.

(3) Consider the following problem, where the value of k has not yet been ascertained. The solution currently being used is $x_1 = 2$, $x_2 = 3$. Use graphical analysis to determine the values of k such that this solution actually is optimal.

Maximize $Z = x_1 + 2x_2$,

subject to

$$\begin{aligned} -x_1 + x_2 &\le 2 \\ x_2 &\le 3 \\ kx_1 + x_2 &\le 2k + 3, \quad \text{where } k \ge 0 \end{aligned}$$

and

$x_1 \ge 0, \quad x_2 \ge 0.$

(4) Use the graphical method to demonstrate that the following model has no feasible solutions.

Maximize $Z = 5x_1 + 7x_2$,

subject to

$$\begin{aligned} 2x_1 - x_2 &\le -1 \\ -x_1 + 2x_2 &\le -1 \end{aligned}$$

and

$x_1 \ge 0, \quad x_2 \ge 0.$

(5) Suppose that the following constraints have been provided for a linear programming model.

$$\begin{aligned} -x_1 + 3x_2 &\le 30 \\ -3x_1 + x_2 &\le 30 \end{aligned}$$

and

$x_1 \ge 0, \quad x_2 \ge 0.$

(a) Demonstrate that the feasible region is unbounded.

(b) If the objective is to maximize *Z* = -*x*1 +*x*2, does the model have an optimal solution? If so, find it. If not, explain why not.
(c) Repeat part (*b*) when the objective is to maximize *Z* = *x*1 - *x*2.
(d) For objective functions where this model has no optimal solution, does this mean that there are no good solutions according to the model? Explain. What probably went wrong when formulating the model?

(6) Consider the following model:

Minimize $Z = 40x_1 + 50x_2,$

subject to

$$2x_1 + 3x_2 \geq 30$$
$$x_1 + x_2 \geq 12$$
$$2x_1 + x_2 \geq 20$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

(a) Use the graphical method to solve this model.
(b) How does the optimal solution change if the objective function is changed to $Z = 40x_1 + 70x_2$?
(c) How does the optimal solution change if the third functional constraint is changed to $2x_1 + x_2 \geq 15$?

(7) Consider the following problem.

Maximize $Z = 3x_1 + 2x_2,$

subject to

$$2x_1 + x_2 \leq 6$$
$$x_1 + 2x_2 \leq 6$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

(a) Use the graphical method to solve this problem. Circle all the corner points on the graph.
(b) For each CPF solution, identify the pair of constraint boundary equations it satisfies.
(c) For each CPF solution, identify its adjacent CPF solutions.
(d) Calculate *Z* for each CPF solution. Use this information to identify an optimal solution.
(e) Describe graphically what the simplex method does step by step to solve the problem.

(8) Describe graphically what the simplex method does step by step to solve the following problem.

Maximize $Z = 2x_1 + 3x_2,$

subject to

$$-3x_1 + x_2 \leq 1$$
$$4x_1 + 2x_2 \leq 20$$
$$4x_1 - x_2 \leq 10$$
$$-x_1 + 2x_2 \leq 5$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

(9) Work through the simplex method (in algebraic form) step by step to solve the following problem.

Maximize $\quad Z = 4x_1 + 3x_2 + 6x_3,$

subject to

$$3x_1 + x_2 + 3x_3 \leq 30$$
$$2x_1 + 2x_2 + 3x_3 \leq 40$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

(10) Work through the simplex method step by step (in tabular form) to solve the following problem.

Maximize $\quad Z = 2x_1 - x_2 + x_3,$

subject to

$$3x_1 + x_2 + x_3 \leq 6$$
$$x_1 - x_2 + 2x_3 \leq 1$$
$$x_1 + x_2 - x_3 \leq 2$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$