1)

Let $X$ be a random variable having $\text{Poisson}(2)$ distribution. Then $E\left(\frac{1}{1+X}\right)$ equals

(A) $1-e^{-2}$ (B) $e^{-2}$ (C) $\frac{1}{2}(1-e^{-2})$ (D) $\frac{1}{2}e^{-1}$

2)

The mean and the standard deviation of weights of ponies in a large animal shelter are 20 kg and 3 kg, respectively. A pony is selected at random from the shelter. Using Chebyshev's inequality, the value of the lower bound of the probability that the weight of the selected pony is between 14 kg and 26 kg is

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $0$ (D) $1$

3)

Let $X_1, X_2, \ldots, X_{10}$ be a random sample from $N(1,2)$ distribution. If

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i \quad \text{and} \quad S^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2,$$

then $\text{Var}(S^2)$ equals

(A) $\frac{2}{5}$ (B) $\frac{4}{9}$ (C) $\frac{11}{9}$ (D) $\frac{8}{9}$

4)

Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that $E(X_i) = 1$ and $\text{Var}(X_i) = 1$, $i = 1, 2, \ldots$. Then the approximate distribution of $\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_{2i} - X_{2i-1})$, for large $n$, is

(A) $N(0,1)$ (B) $N(0,2)$ (C) $N(0,0.5)$ (D) $N(0,0.25)$

5)

Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables having $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$. Define

$$W = \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} (X_i - X_j)^2$$

Then $W$, as an estimator of $\sigma^2$, is

(A) biased and consistent

(B) unbiased and consistent

(C) biased and inconsistent

(D) unbiased and inconsistent

6)

Let $X_1, X_2, X_3, X_4$ be i.i.d. random variables having a continuous distribution. Then $P(X_3 < X_2 < \max(X_1, X_4))$ equals

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

7)

Let $Z_1$ and $Z_2$ be i.i.d. $N(0, 1)$ random variables. If $Y = Z_1^2 + Z_2^2$, then $P(Y > 4)$ equals

(A) $e^{-2}$

(B) $1 - e^{-2}$

(C) $\frac{1}{2}e^{-2}$

(D) $e^{-4}$

8)

Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{3}$. Let $X$ denote the number of trials required to get the second success. Then $P(X \geq 5)$ equals

(A) $\frac{3}{7}$

(B) $\frac{16}{27}$

(C) $\frac{16}{21}$

(D) $\frac{9}{13}$

9)

Let the joint probability density function of $(X, Y)$ be

$$f(x, y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Then $P\left(X < \frac{Y}{2}\right)$ equals

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{1}{2}$

10)

Let $X_1, X_2, X_3, X_4, X_5$ be a random sample from $N(0,1)$ distribution and let $W = \frac{X_1^2}{X_2^2+X_3^2+X_4^2+X_5^2}$. Then $E(W)$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

11)

Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a random sample from $N(\mu_2, \sigma^2)$ distribution, where $\mu_i \in \mathbb{R}, i = 1, 2$ and $\sigma > 0$. Suppose that the two random samples are independent. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad W = \frac{\sqrt{mn}\ (\bar{X} - \mu_1)}{\sqrt{\sum_{i=1}^{m}(Y_i - \mu_2)^2}}$$

Then which one of the following statements is TRUE for all positive integers $m$ and $n$?

(A) $W \sim t_m$ (B) $W \sim t_n$

(C) $W^2 \sim F_{m,1}$ (D) $W^2 \sim F_{m,n}$

12)

Let $X$ be a random variable having $U(0,10)$ distribution and $Y = X - [X]$, where $[X]$ denotes the greatest integer less than or equal to $X$. Then $P(Y > 0.25)$ equals _______