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For finite dimensional inner product space it is true.

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$x^t(A^t - A)y = 0.$ This is true for all $x, y \in \mathbb{R}^2$.

Take $x = (1,0)^t$ and $y = (0,1)^t$.

$$(1,0)\begin{pmatrix} 0 & a_{21} - a_{12} \\ a_{12} - a_{21} & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$a_{21} - a_{12} = 0 \implies a_{12} = a_{21}$.

Hence $A = A^t$.

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