Class test - 1

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Ques 2

(a) To prove: If a relation R has only one key, it is in BCNF if and only if it is in 3NF

let us assume F^+ denote the closure of the set of functional dependencies satisfied by a relation R which is assumed to be in 3NF.

We need to show that for each non-trivial dependancy $X \to A$ in F^+, X is a super key.

To this end, considering a dependency. If X is not a superkey, the 3NF property gurantees that the attribute A is a part of a key.

Since all keys are simple by assumption, we have that A is a key.

This last fact together with the dependancies $X \to A$ implies that X is a superkey (this follows, from transitive axiom) which is a contradiction

Ques 2

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(b) (i) $R(W, X, Y, Z)$

$X \to Y$, $W \to Z$ and $Y \subseteq W$

$Y \subseteq W$ $\Rightarrow$ $W \to Y$ (by reflexivity)

$X^+ = XY$

So $X \not\to Z$ (disproved)

(ii) $XZ \to Y$, $X \to W$ & $Z \subseteq W$

$Z \subseteq W$ $\Rightarrow$ $W \to Z$ (by reflexivity)

Now, $X^+ = XWZY$ $\Rightarrow$ $X \to Y$ (proved)

Ques 2

(C) $R(A, B, C, D, E, X, Y)$

The set of FD's

$\{D \to A,\ XD \to C,\ DA \to B,\ A \to X,\ XE \to B,\ E \to A,\ B \to D,\ EB \to C,\ AB \to C,\ Y \to B,\ C \to B\}$

First we find out key

$YE^+ = YEBAXCD$

Hence, YE is the key

To find 3NF loss less decomposition

Step 1 Since all the FD's has only one attribute on the right side, no need to do anything

Step 2 checking Redundant FD's

$D \to A$	$D^+ = D$	NR
$XD \to C$	$XD^+ = XDABC$	NR
$DA \to B$	$DA^+ = DAX$	NR

$A \to X$	$A^+ = A$	NR
$XE \to B$	$XE^+ = XEA$	NR
$E \to A$	$E^+ = E$	NR
$B \to D$	$B^+ = B$	NR
$EB \to C$	$EB^+ = EBAXDC$	Redundant
$AB \to C$	$AB^+ = ABXDC$	NR
$Y \to B$	$Y^+ = Y$	NR
$C \to B$	$C^+ = C$	NR

Now,

$F' = \{D \to A,\ DA \to B,\ A \to X,\ XE \to B,\ E \to A,\ B \to D,$
$EB \to C,\ AB \to C,\ Y \to B,\ C \to B\}$

So 3NF decomposition is

$R_1(D A)$ $R_2(D A B)$ $R_3(A X)$ $R_4(X, E, B)$ $R_5(E, A)$
$R_6(B, D)$ ~~[illegible]~~ $R_7(A, B, C)$ $R_8(Y, B)$ $R_9(C, B)$

after removing subsets

$R_2(D, A, B)$ $R_3(A, X)$, $R_4(X, E, B)$ $R_5(E, A)$
~~[illegible]~~ $R_7(A, B, C)$, $R_8(Y, B)$

and adding one more for key

$R_9(Y, E)$

Ques 3 (a) To calculate natural join

(i) Compute $R \times S$

(ii) write only tuples which are having common attributes in R and S

$R \times S \Rightarrow$

A	B	C	A	B	D

$R \bowtie S \Rightarrow$

A	B	C	D

Relational algebra representation $\Rightarrow$

$$\pi_{R.A,\, R.B,\, R.C,\, S.D} \left(\sigma_{R.A = S.A \,\wedge\, R.B = S.B} (R \times S) \right)$$

$$R \bowtie S = \{ t \mid \exists u \ \exists v \ (R(u) \wedge S(v) \wedge t[A] = u[A]$$
$$u[A] = v[A] \wedge u[B] = v[B]$$
$$\wedge\ t[B] = u[B] \wedge t[C] = u[C]$$
$$\wedge\ t[D] = v[D] \}$$

Ques 3

(b) (i) $T = \pi_{u\text{-}name}(user \bowtie Borrow)$

$S = \pi_{u\text{-}name}(user)$

S-T gives the desired result

(ii) $\pi_{s\text{-}name}\left(\sigma_{cardno='A11'}(user \bowtie Borrow \bowtie supply \bowtie SUPPLIER)\right)$

(iii) $P = \pi_{price}(Book \bowtie supply)$

$Q = \pi_{1}\left(\sigma_{1<2}(P \times P)\right)$

$R = P - Q$

$\pi_{title}\left(\sigma_{3 > 8}(supply \bowtie Book \bowtie R)\right)$

Entity set	Key
Student	student -Id
Department	(name, code)
Course	Course -no.
section	Section number
grade report	student number

Student

Ques 4

(a) Given that a relational scheme $R(A, B, C, D, E)$ with FD's
$\{A \to BC,\ CD \to E,\ B \to D,\ E \to A\}$

$R(A, B, C, D, E)$

$R1(A, B, C)$ $\quad$ $R_2(A, D, E)$

This decomposition $R = (R_1, R_2)$ is a loss less join decomposition. Since we know that a decomposition $R = (R_1, R_2)$ is a lossless join decomposition — (1)

iff $R_1 \cap R_2 \to R_1 - R_2$ or $R_1 \cap R_2 \to R_2 - R_1$

i.e. ~~eit~~ $R_1 \cap R_2$ should be either key of R_1 ~~or~~ R_2 or both

for $R_1(A, B, C)$

$A \to BC$

since, $A^+ = ABC$ so A is key for relation R_1

Now $R_1 \cap R_2 = \{A\}$

Since A is a key of R_1 so

$R_1 \cap R_2 \to \{A\} \to R_1$

$R_1 \cap R_2 \to R_1$

from (1)

$R = (R_1, R_2)$ is a loss less join decomposition

(iii) Decomposition is not a dependancy preserving decomposition

Given that $R_1 = (R_1, R_2)$

R

$R_1(A, B, C)$ $\quad$ $R_2(A, D, E)$

For $R_1(A, B, C)$ possible FD's will be as

$A \rightarrow BC$
$B \rightarrow AC$
$C \rightarrow AB$
$AB \rightarrow C$
$BC \rightarrow A$
$AC \rightarrow B$

Now we will check which are valid

$A^+ = ABCDE$ (from R)
$B^+ = BD$ (")
$C^+ = C$ (")
$AB^+ = ABC$ (")
$BC^+ = BCDE$ (")

so valid FD's for R_1 will be

$$\left.\begin{array}{l} A \rightarrow BC \\ AB \rightarrow C \\ BC \rightarrow A \\ AC \rightarrow B \end{array}\right\} \text{— (2)}$$

for $R_2(A, D, E)$ possible FD's

$A \rightarrow DE$, $D \rightarrow AE$, $E \rightarrow AD$, $AD \rightarrow E$, $AE \rightarrow D$

$DE \rightarrow A$

Now, $A^+ = ABCF$
$D^+ = D$
$E^+ = ABCDE$ } from R

Now valid FD's for R_2 will be
$A \to ADE$ $\quad AE \to D \quad E \to AD \quad DE \to A$
$AD \to E$ —— (3)

All the valid FD's for $R_1 \cup R_2$ will be
$F' = \{A \to BC,\ AB \to C,\ BC \to A,\ AC \to B,\ A \to BC,\ AE \to D$
$A \to DE,\ E \to AD,\ AD \to E,$
$DE \to A\}$

From FD's of R
we will check whether we can derive FD's of R from F'

$CD^+ = CD$
$E \nsubseteq CD^+$
So $CD \not\to E$ (can't be written)
Hence decomposition $R = R_1 \cup R_2$ is not dependency preserving decomposition.