INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Mid-Autumn Semester 2022-23

Date of examination: 20.09.2022 **Session (FN / AN):** FN **Duration:** 2 hours **Full marks:** 60
Subject No.: AI61005 **Subject:** Artificial Intelligence: Foundations and Applications
Department/Center/School: Centre of Excellence in Artificial Intelligence (CoEAI)
Instructions: ***Answer all questions. All parts of a question must be answered in the same place.***

1. State whether the statements are True or False and justify your answer with a proper proof or a clear counterexample:

   a) Algorithm A* for a minimization problem with all heuristics 0 and all edge costs positive except one edge which may have negative edge cost, may produce a non-optimal solution.

   b) If the problem is one of minimization, then Algorithms A* and IDA* will always give the same solution cost irrespective of whether the heuristics overestimate or not. Assume positive edge costs and non-negative heuristic estimates.

   c) In case of a maximization problem with no cycles in the graph, positive edge costs and overestimating heuristics, IDA* may expand more nodes than DFBB considering a total count of nodes expanded including re-expansions.

   d) There is a two-payer game of at least 16 leaf nodes where we may have no pruning if we use alpha-beta pruning using a left to right ordering.

   e) There is a two-payer game of at least 16 leaf nodes where exactly the same set of leaf nodes is pruned by alpha-beta pruning algorithm when we use left to right ordering and when we use right to left ordering.

   [(1+2) X 5 = 15 marks]

2. State whether the statements are True or False and justify your answer with a proper proof or a clear counterexample:

   a) A propositional formula of at least four propositional (Boolean) variables which is non-valid can never be satisfiable.

   b) Given a binary predicate $p(x, y)$, is the following first order predicate logic formula valid?

   $$\forall_x \exists_y p(x, y) \Rightarrow \exists_x \forall_y p(x, y)$$

   c) Given a binary predicate $p(x, y)$ and the fact that the domain is a finite set of 2 elements only, the predicate logic formula $\exists_x \forall_y p(x, y)$ be written in using propositional logic constructs only.

d) Vipin either ate ice-cream or drank hot coffee before going to bed but not both. If he ate ice-cream, he had a sore throat next morning. Vipin did not have a sore throat next morning. Therefore, he drank hot coffee before going to bed. [Use propositional logic based modelling to solve this]

[(1+2) X 3 + (1+5) = 15 marks]

3. Use Resolution Refutation to prove or disprove **Goal** from **Statement 1**.

> **Statement 1:** Every student in this class is an atheist or a theist.
>
> **Goal:** There is a student in this class who is an atheist or a theist.

Does the conclusion of the above reasoning of the goal change when **Statement 2** is added in the knowledge base?

> **Statement 2:** There is a student in this class.

Make use of the following steps to answer the question.

a) Translate the statements into first order logic using predicates: `studentInClass(s)`, `atheist(s)`, `theist(s)`.
b) Convert the translated sentences into Conjunctive Normal Form if required.
c) Try to derive the **Goal** with **Statement 1** only and state your conclusion.
d) Revise your conclusion (if any) after adding **Statement 2** into knowledge base. Show steps of resolution.

[2+2+2+2=8 Marks]

4. Answer the following questions related to the first order logic:
   a) Convert the following first order logic statements into corresponding Conjunctive Normal Forms.
      i. $\forall_x \left[\left(\forall_y p(y) \Rightarrow r(x,y)\right) \Rightarrow \exists_y r(y,x)\right]$
      ii. $\neg\exists_x \forall_y \forall_z [(p(y) \Rightarrow q(z)) \Rightarrow (p(x) \Rightarrow q(x))]$
      iii. $\forall_x \forall_y [p(x,y) \vee r(x,y) \Rightarrow \exists_z q(x,y,z)]$
   b) Consider the following first order logic knowledge base

$\forall_x p(x) \Rightarrow q(x)$	$\forall_x \neg p(x) \Rightarrow r(x)$
$\forall_x q(x) \Rightarrow s(x)$	$\forall_x r(x) \Rightarrow s(x)$

Can we use Resolution Rule to infer $s(a)$ from the knowledge base? If so, show inference steps; otherwise explain.

[(3+2+2)+3=10 Marks]

5. Write Prolog program for the following problems:
   a) Check whether two sets are equivalent or not:
      Implement the Prolog predicate `set_equiv(S1,S2)`, where S1 and S2 are two sets represented as Prolog list. For example, `set_equiv([8,3,4,1,9], [3,8,4,9,1])` will return true.

   b) Implement Prolog predicate `zip_concat(L1,L2,L3)`:
      The program takes two lists (L1 and L2) as input and outputs another list L3. The list L3 will place each pair of elements (one from L1 and other from L2 in the same index) consecutively. The remaining part of the bigger list will be appended to the final list (L3). For example, `zip_concat([3,4,7,1], [7,9,6,10,2,5], L3)` will output L3=[3,7,4,9,7,6,1,10,2,5]

   c) Implement Prolog predicate `split_pivot(P, L, U1, U2)`:
      The program splits a list L in two lists U1 and U2 with respect to a pivot P where all the elements in U1 are less than or equal to P and all the elements in U2 are greater than P with original order preserved. For example, `split_pivot(2, [5,6,2,8,7], U1, U2)` will output U1=[5,2] and U2=[6,8,7]

[4+4+4=12 Marks]