Technology, Production Cost and Demand

Basics of Microeconomics

1. Technology and Cost (Single product production function and cost function)
2. Basic properties of demand function

- Production Function: the know-how of a certain entity (we call firm) which enables it to transform factors of production to final goods

- Assumption 1: Only 1 final good and two factors of production (labour and capital) $Q = f(l, k)$

- Assumption 2: function *f* is twice continuously diffentiable wrt both *l* and *k*

Basics of Microeconomics

- Define marginal products

$$MP_L(l,k) \equiv \frac{\partial f(l,k)}{\partial l};\ MP_K(l,k) \equiv \frac{\partial f(l,k)}{\partial k}$$

- Example: $Q = (l^\alpha + k^\alpha)^\beta;\ \alpha, \beta > 0$

- Marginal products:

$$MP_L(l,k) = \alpha\beta(l^\alpha + k^\alpha)^{\beta-1} l^{\alpha-1}$$

$$MP_K(l,k) = \alpha\beta(l^\alpha + k^\alpha)^{\beta-1} k^{\alpha-1}$$

- Implication?

Basics of Microeconomics

- Definitions –

1. Supporting Factors: $$\frac{\partial MP_L(l,k)}{\partial k} = \frac{\partial MP_K(l,k)}{\partial l} > 0$$
2. Substitute Factors: $$\frac{\partial MP_L(l,k)}{\partial k} = \frac{\partial MP_K(l,k)}{\partial l} < 0$$

- Example: $Q = (l^{\alpha} + k^{\alpha})^{\beta};\ \alpha, \beta > 0$

- Question: L and K are substitutes or supporting?

Basics of Microeconomics

- The cost function is a mapping from the rental prices of the factors of production and the production level to the total production cost.

- The cost function is a technological relationship that can be derived from the production function.

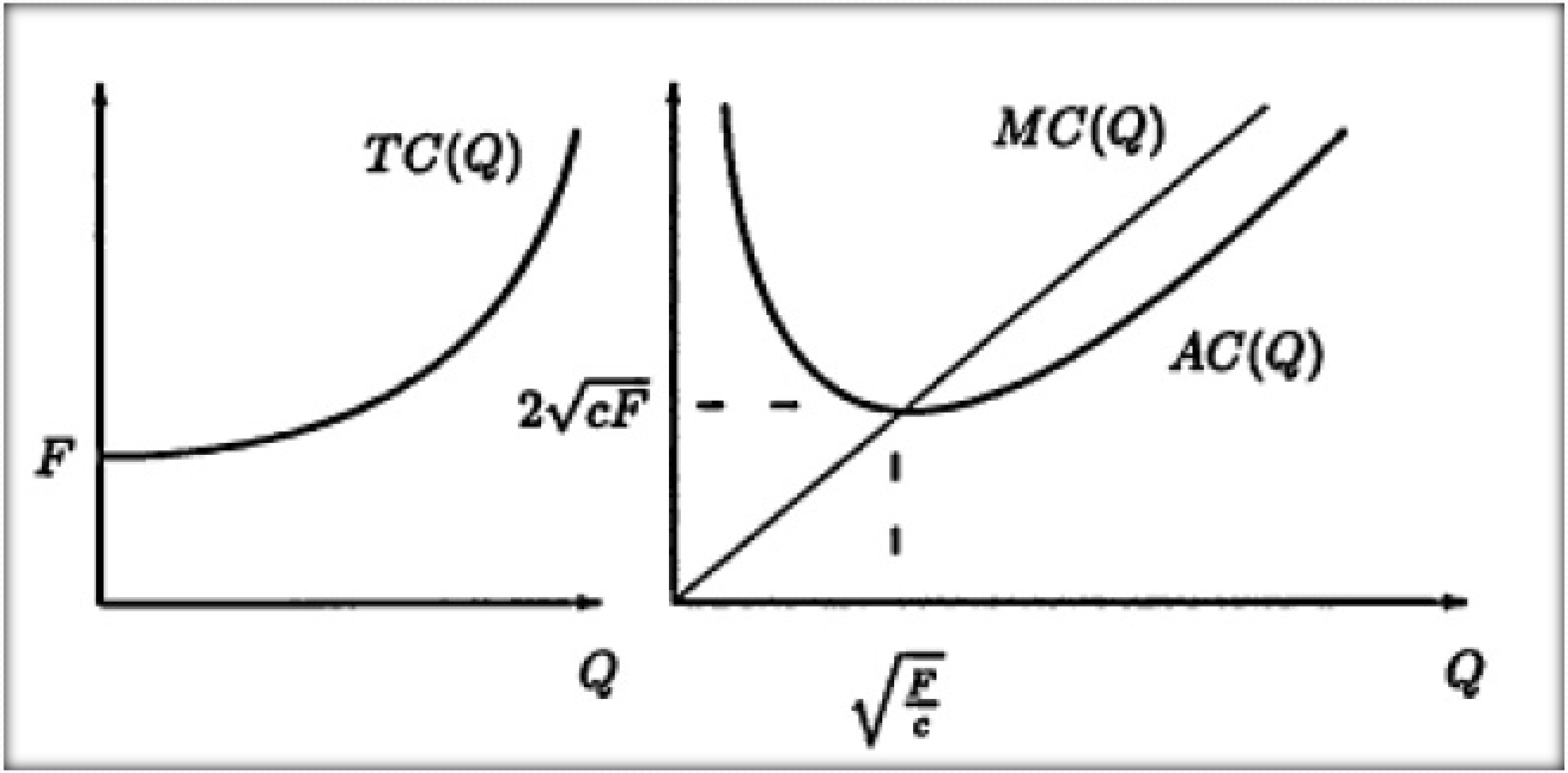
- Let W denote wage rate, and R the rental price for one unit of capital.

- The cost function is denoted by the function TC(W, R; Q) measures the total production cost of producing Q units of output, when factor prices are W (for labor) and R (for capital).

Basics of Microeconomics

- Average cost function (cost per unit of output): $AC(Q) = TC(Q)/Q$
- Marginal cost: $MC(Q) = \partial TC(Q)/\partial Q$
- Example: $TC(Q) = F + cQ^2;\ F, c > 0$
- $AC(Q) = F/Q + cQ;\ MC(Q) = 2cQ$
- Where does AC reach minimum?
- If the average cost function reaches a minimum at a strictly positive output level, then at that particular output level the average cost equals the marginal cost. Formally, if $Q_{min} > 0$ minimizes AC(Q), then AC(Q_{min} = MC(Q_{min}).

Basics of Microeconomics



Basics of Microeconomics

- Let $\lambda > 1$. Then a production technology $Q = f(k, l)$ will exhibit

1. IRS: $f(\lambda k, \lambda l) > \lambda f(k, l)$
2. CRS: $f(\lambda k, \lambda l) = \lambda f(k, l)$
3. DRS: $f(\lambda k, \lambda l) < \lambda f(k, l)$

- Example: $Q = f(k, l) = (l^{\alpha} + k^{\alpha})^{\beta}$

- The above production function exhibits IRS *iff* $\alpha\beta > 1$

Basics of Microeconomics

- Duality between production and cost functions

- Suppose that only labor is required for producing the final good, and let the production technology be given by $Q = f(l) = l^{\gamma}, \gamma > 0$.

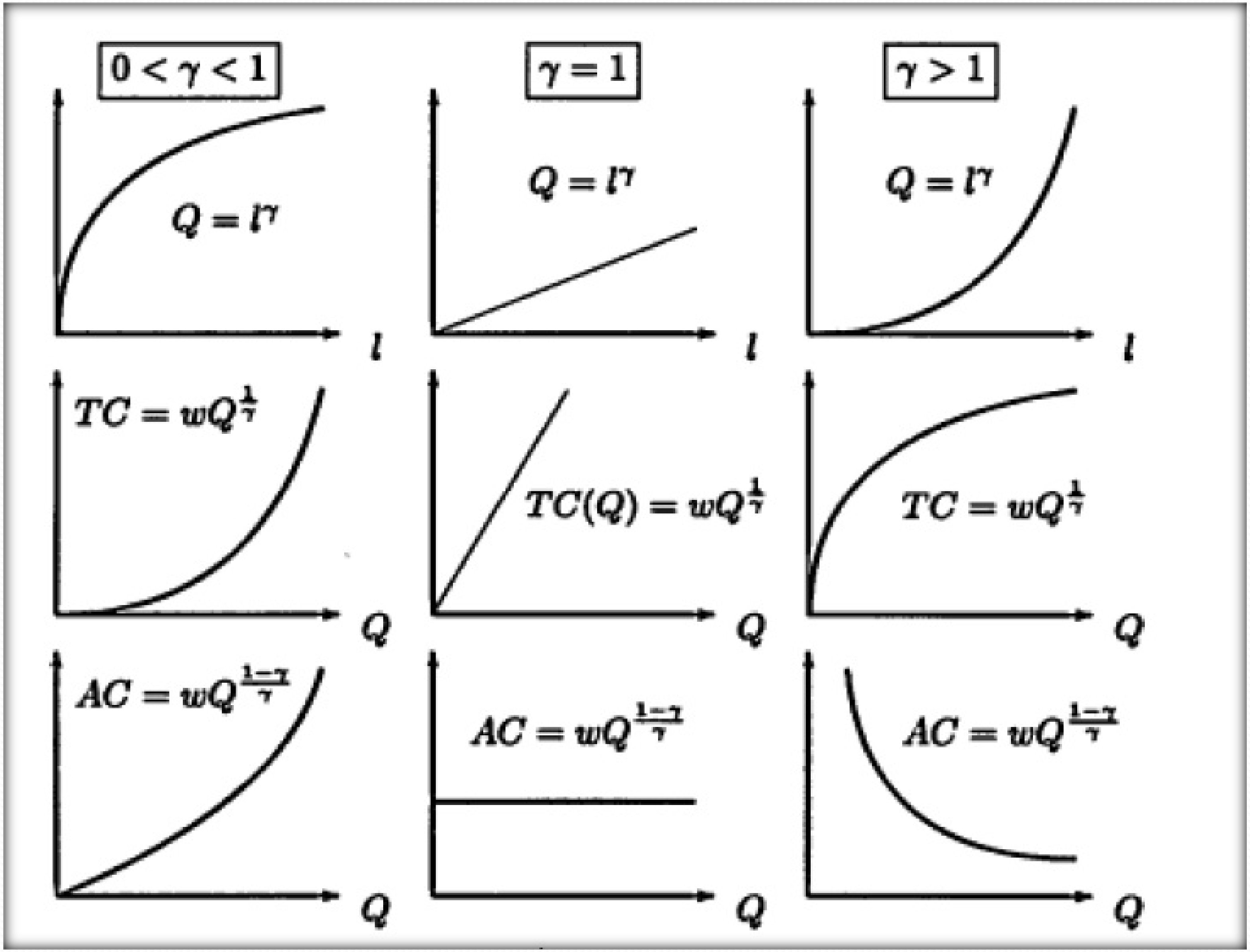
- 3 cases: $0 < \gamma < 1$, $\gamma = 1$, and $\gamma > 1$.

- Let ω denote the wage rate. Inverting the production function we obtain $l = Q^{1/\gamma}$. Hence, TC = $wl = wQ^{1/\gamma}$

- $(\lambda l)^{\gamma} > \lambda l^{\gamma}$ if and only if $\gamma > 1$.

- Hence, $Q = l^{\gamma}$ exhibits IRS, CRS or DRS according to $\gamma \gtreqless 1$

Basics of Microeconomics



Basics of Microeconomics

- Lets consider $Q = f(k, l) = (l^{\alpha} + k^{\alpha})^{\beta}$.
- This production function exhibits IRS *iff* $\alpha\beta > 1$
- Lets consider: $TC(W, R; Q) = \phi Q^{1/\alpha\beta}$
- $AC(Q) = \phi Q^{\frac{1}{\alpha\beta} - 1}$
- AC falling with Q (implying IRS) if $\frac{1}{\alpha\beta} - 1 < 0$ or $\alpha\beta > 1$ and vice-versa

Basics of Microeconomics

- Linear demand function: $Q(p) = \frac{a}{b} - \frac{1}{b}p$

- Inverse demand function: $p(Q) = a - bQ$

- Non-linear (Constant elasticity) demand function: $Q(p) = ap^{-\epsilon}$

- Price Elasticity: $\eta_p(Q) \equiv \frac{\partial Q(p)}{\partial p}\frac{p}{Q}$

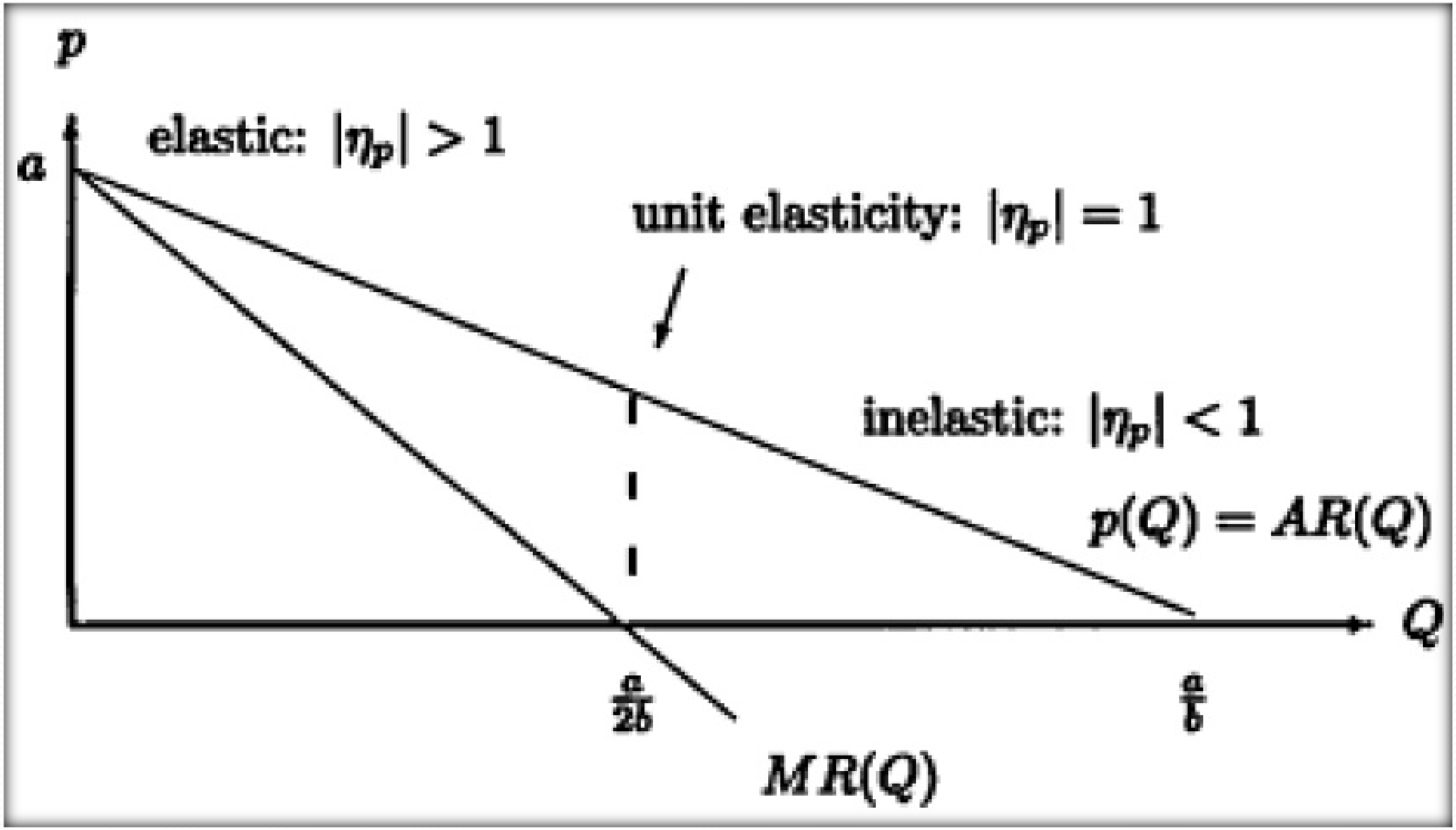
- At a given Q, the demand is called

1. Elastic if $\eta_p(Q) < -1$ or $|\eta_p(Q)| > 1$
2. Inelastic if $-1 < \eta_p(Q) < 0$ or $|\eta_p(Q)| < 1$
3. Unit elastic if $\eta_p(Q) = -1$ or $|\eta_p(Q)| = 1$

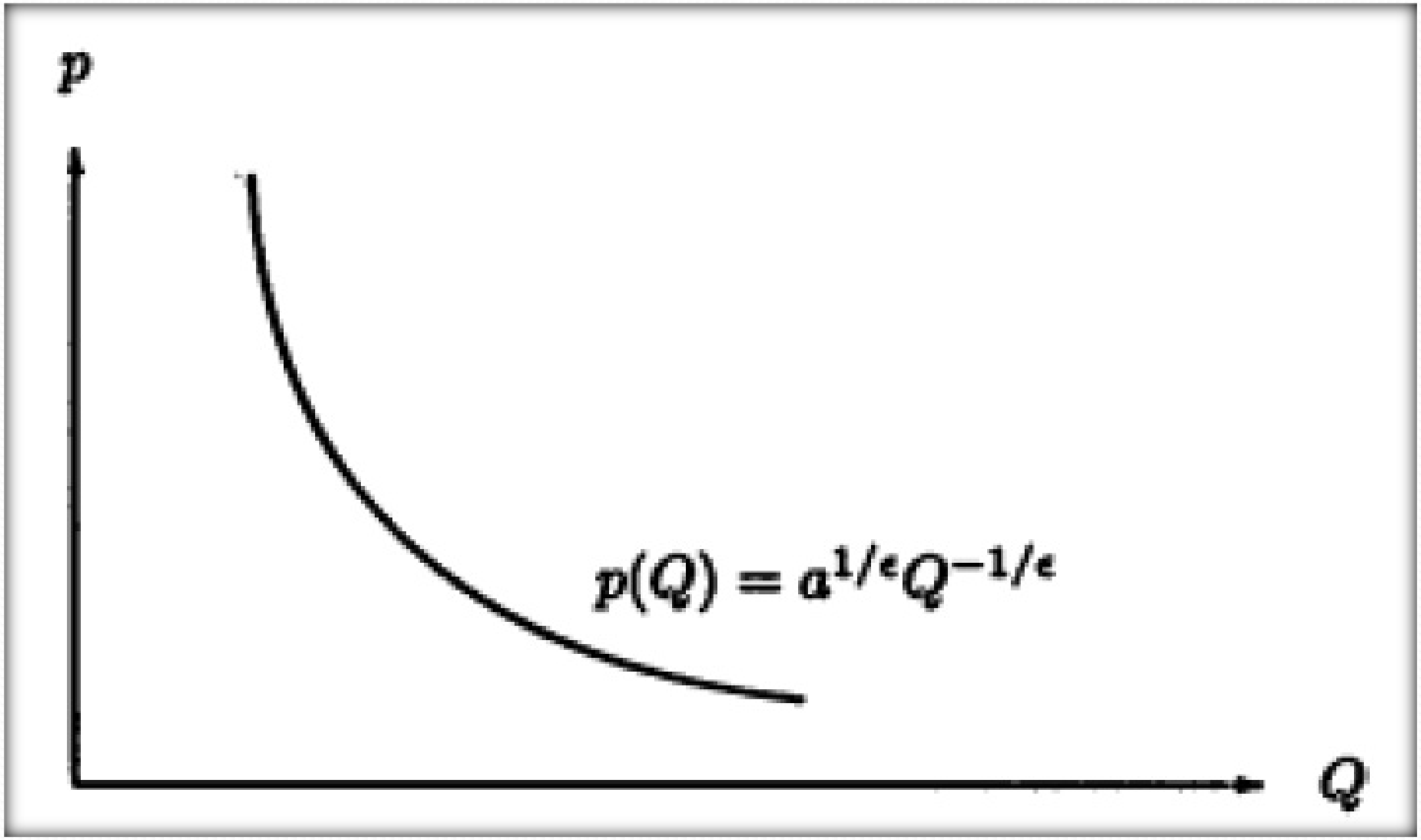
Basics of Microeconomics

- Linear demand function: $Q(p) = \frac{a}{b} - \frac{1}{b}p$
- Elasticity: $\eta_p(Q) = 1 - {}^{a}/_{bQ}$
- Unit elastic if $Q = \frac{a}{2b}$; elastic when $Q < \frac{a}{2b}$ and inelastic when $Q > \frac{a}{2b}$
- Non-linear (Constant elasticity) demand function: $Q(p) = ap^{-\epsilon}$
- $\eta_p(Q) = -\epsilon$

Basics of Microeconomics



Basics of Microeconomics



Basics of Microeconomics

- Revenue: $TR(Q) = p(Q).Q$

- Linear demand function: $TR(Q) = aQ - bQ^2$

- Constant elasticity demand function: $TR(Q) = a^{\frac{1}{\epsilon}}Q^{1-\frac{1}{\epsilon}}$

- Note that a more suitable name for the revenue function would be to call it the total expenditure function since we actually refer to consumer expenditure rather than producers' revenue.
- That is, consumers' expenditure need not equal producers' revenue, for example, when taxes are levied on consumption.
- Thus, the total revenue function measures how much consumers spend at every given market price, and not necessarily the revenue collected by producers.

Basics of Microeconomics

- The marginal-revenue function (again, more appropriately termed the "marginal expenditure") shows the amount by which total revenue increases when the consumers slightly increase the amount they buy

- $MR(Q) \equiv \frac{\partial TR(Q)}{\partial Q}$

- Linear demand function: $MR(Q) = a - 2bQ$

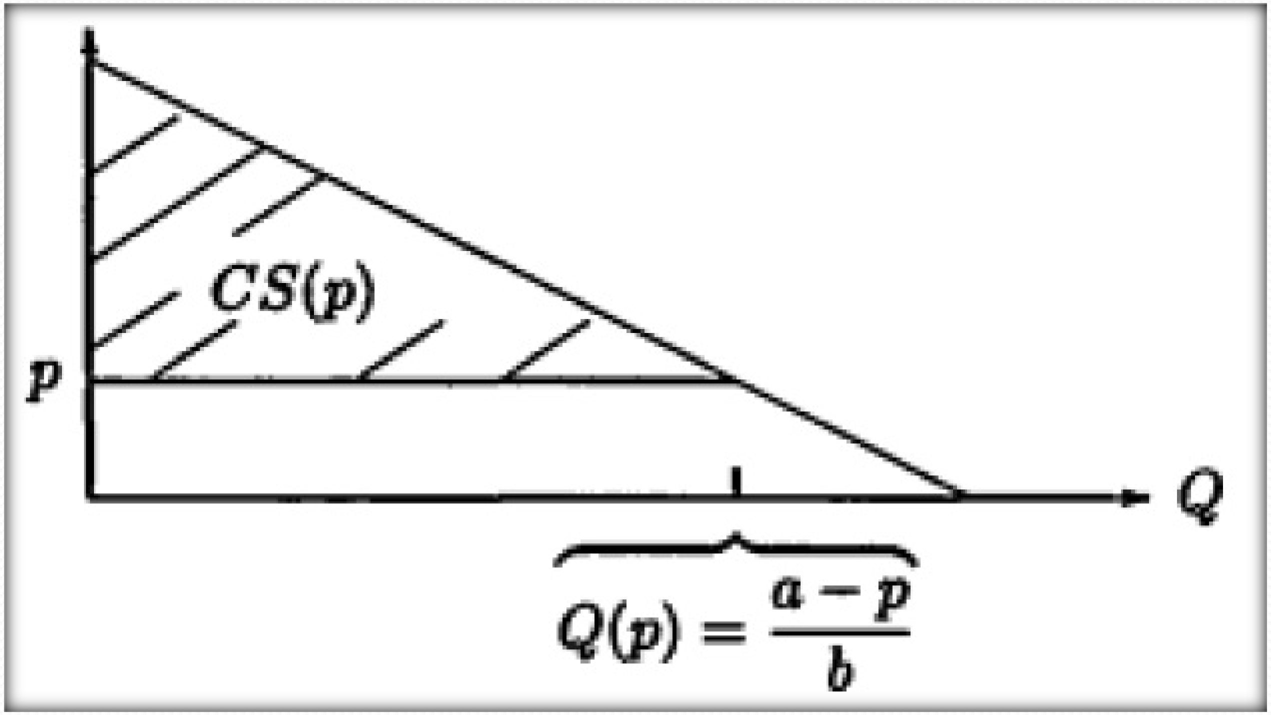
- Relationship between MR and elasticity –

$$MR(Q) = p(Q)\left[1 + \frac{1}{\eta_p(Q)}\right]$$

Basics of Microeconomics

- Consumer Surplus: The welfare measure that approximates the welfare gain associated with the opening of the market (of a good) - the area beneath the demand curve above the market price

- Linear demand curve: $CS(p) \equiv \frac{(a-p)Q(p)}{2}$



Reference

- Oz Shy (1995). Industrial Organization. MIT Press. Chapter 3.