

### Assignment - 1 (No need to submit)

*Note: Unless otherwise stated, notation used is as defined in the class.*

1. What is the converse, inverse and contrapositive for the following propositions?
  - (a) A right implies a responsibility.
  - (b) The right to search for the truth implies also a duty.
  - (c) Speak only if it improves upon the silence.
2. Find the negation of the proposition:  
"Alexander the Great was a Roman leader and  $2 + 1 = 3$ "
3. Rewrite the quote by Benjamin Franklin (1706-1790) as a conditional proposition:  
"Never put off till tomorrow what you can do today".
4. Construct a truth table for each of the following propositions, and classify if each proposition as either a tautology, a contradiction, or a contingencies:
  - (a)  $(p \rightarrow q) \vee (q \rightarrow p)$
  - (b)  $(p \vee q) \wedge (p \vee \neg q)$
  - (c)  $(p \vee q) \wedge (\neg p \wedge \neg q)$
5. Use the rules of inference to show that  $\neg(p \vee \neg(p \wedge q))$  is a contradiction.
6. Prove that  $[(p \wedge \neg(\neg p \vee q)) \vee (p \wedge q)] \rightarrow p$  is a tautology. Do not use truth table.
7. Find the negation of the implication:  
"If you are from the state of New York, then you will receive a scholarship."
8. For the following argument, translate it into symbolic logic, and determine whether or not the argument is valid.  
"If a country is developing, it cannot devote much of its financial resource to technology development. However, if a country cannot devote much of its financial resource to technology development, then its economy will not grow. Therefore, a developing country will not have an economic growth. "
9. Using modus ponens, what can you conclude from the following premises?  
"My improvement upon silence is necessary when I need to speak the truth. And after seeing these grave atrocities, I cannot restrain my tongue."
10. When the Roman Emperor Marcus Aurelius (121-180) persecuted early Christians, Tertullian (155-240) argued against the methods Aurelius employed.

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Either Christians have committed crimes or not.

If they are guilty of crimes, your refusal to permit a public inquiry is irrational.

If they have committed no offence, it is unjust to punish them.

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Therefore, your conduct is either unjust or irrational.

Show that Tertullian's arguments is valid.

——-PTO——-

11. Aladdin finds two trunks  $A$  and  $B$  in a cave. He knows that each of them either contains a treasure or a fatal trap.

*On trunk A is written:* “At least one of these two trunks contains a treasure.”

*On trunk B is written:* “In  $A$  there’s a fatal trap.”

Aladdin knows that either both the inscriptions are true, or they are both false.

Can Aladdin choose a trunk being sure that he will find a treasure? If this is the case, which trunk should he open? Formalize the puzzle in propositional logic and find the solution using a truth table.

12. Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:

Box 1: “The gold is not here”

Box 2: “The gold is not here”

Box 3: “The gold is in Box 2”

Only one message is true; the other two are false. Which box has the gold?

Formalize the puzzle in propositional logic and find the solution using a truth table.

13. Show that the following is a valid argument:

Everyone shouts or cries.
Not everyone cries.
So some people shout and does not cry.

Which rules of inference are used to establish the above conclusion?

14. Use resolution to show that the following is a valid argument:

All students go to parties.
Some students drink too much.
Therefore, some people who drink too much must go to parties.

15. Write down the converse of the following statement.

“If  $n$  is a multiple of 3 then  $n$  is not a multiple of 7.”

Say whether the original statement and its converse are true or false, and justify your answers.

16. Consider the statement.

$u$  : if  $n$  is the square of an even integer then  $n$  is the sum of two successive odd integers.  
(Here “successive odd integers” means odd integer of the form  $k, k + 2$ .)

- (a) Show that  $u$  is true by using a construction based on the following examples:

$$4^2 = 7 + 9, 6^2 = 17 + 19, 8^2 = 31 + 33, 10^2 = 49 + 51$$

- (b) Write down the converse of the statement  $u$  and show that it is false.

- (c) Write down the contrapositive of the statement  $u$ . Is it true or false?

17. Obtain disjunctive normal (dnf) form, conjunctive normal (cnf) form, principal disjunctive normal (pdnf) form and principal conjunctive normal (pcnf) form of  $(\neg P \vee \neg Q) \longrightarrow (P \longleftrightarrow \neg Q)$ .