GPINA-ISYE6501-HW9

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1 Retailer Shelf Space Optimization Analysis

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1.1 Overview

A big-box retailer faces a fundamental question: **How should limited shelf space be allocated among product types to maximize sales or profit?** This analysis systematically addresses the optimization challenge by testing the retailer's hypotheses and providing clear, data-driven recommendations.

1.1.1 Business Problem

The retailer must allocate finite shelf space across various product types while adhering to constraints, including: - **Minimum required shelf space** per product type. - **Maximum allowable shelf space** per product type. - **Total available shelf space** within each store.

The challenge is determining the optimal allocation that maximizes profitability within these operational constraints.

1.1.2 Retailer Hypotheses

The retailer proposes three hypotheses requiring analytical validation:

1. **Direct Space-Sales Relationship:** More shelf space leads to increased sales.
2. **Complementary Sales Effect:** Higher sales of one product type enhance sales of complementary products.
3. **Spatial Adjacency Benefit:** Placing complementary products adjacently amplifies their combined sales.

1.1.3 Analytical Objectives

This analysis aims to: - **Validate hypotheses** using rigorous statistical methods. - **Quantify relationships** between shelf space and sales outcomes. - **Measure effect sizes** to gauge po-

tential impact. - **Develop optimization models** based on validated relationships. - **Validate recommendations** via controlled testing.

1.1.4 Key Analytical Challenges

Major challenges include: - **Data complexity:** Gathering extensive, high-quality historical data. - **Distinguishing causality:** Identifying true causal relationships, not mere correlations. - **Accounting for non-linearity:** Considering diminishing returns and saturation effects. - **Controlling confounding factors:** Managing external influences like seasonality and promotions. - **Practical implementation:** Ensuring solutions are operationally feasible.

1.1.5 Analytical Approach Framework

The analysis follows the structured **{Given, Use, To}** framework: - **Given:** Clear data requirements and constraints. - **Use:** Appropriate statistical and optimization methods. - **To:** Deliver measurable business outcomes.

This ensures robust validation precedes optimization.

1.2 Data Requirements

1.2.1 Historical Sales and Space Data

- Sales by product type across multiple stores and periods.
- Historical shelf-space allocation.
- Seasonal sales and space data to capture trends.
- Store-specific data including size, location, and demographics.

1.2.2 Product and Layout Data

- Product profit margins.
- Current product placements and adjacency information.
- Shelf space constraints per product.
- Complementary product pair data with relationship strengths.

1.2.3 Transactional and Behavioral Data

- Customer purchase behavior and cross-selling trends.
- Foot traffic patterns and dwell times.
- Inventory turnover rates and stock levels.
- Pricing histories to control price impacts.

1.2.4 External Variables

- Seasonal and local event data.
- Competitor shelf-space allocation benchmarks.
- Store operations data.
- Economic conditions impacting consumer behavior.

1.2.5 Data Quality Requirements

- Detailed product-level granularity.
- Extensive multi-store data coverage.
- Complete data sets minimizing gaps.
- Consistent measurement standards.

1.3 Analytical Approach (Given, Use, To Framework)

1.3.1 Phase 1: Hypothesis Testing

Hypothesis 1 Testing: - **Given:** Historical sales, shelf-space allocations, store attributes. - **Use:** Linear regression models controlling for store and product-specific effects. - **To:** Validate direct shelf space-sales relationship.

Hypothesis 2 Testing: - **Given:** Sales data of complementary product pairs. - **Use:** VAR or simultaneous equation models. - **To:** Determine significant cross-product sales effects.

Hypothesis 3 Testing: - **Given:** Spatial layout and adjacency data. - **Use:** Spatial regression analyses. - **To:** Assess sales benefits from adjacency.

1.3.2 Phase 2: Sales Response Modeling

Direct Sales Effect: - **Given:** Validated space-sales relationships. - **Use:** Non-linear regression to model diminishing returns. - **To:** Quantify shelf-space impacts accurately.

Complementary Sales Effect: - **Given:** Cross-product elasticity measures. - **Use:** Multi-product demand modeling. - **To:** Estimate cross-product sales impacts.

Adjacency Effect: - **Given:** Product layout and sales data. - **Use:** Spatial interaction models. - **To:** Quantify incremental adjacency benefits.

1.3.3 Phase 3: Optimization Modeling

Shelf Space Optimization: - **Given:** Sales response functions, product margins, space constraints. - **Use:** Non-linear optimization. - **To:** Maximize profits while respecting operational limits.

1.3.4 Phase 4: Implementation and Validation

Model Validation: - **Given:** Optimized space recommendations. - **Use:** Controlled store-level A/B testing. - **To:** Validate and refine recommendations.

Continuous Improvement: - **Given:** Ongoing data updates. - **Use:** Adaptive optimization. - **To:** Maintain optimal allocation.

1.4 Mathematical Optimization Formulation

1.4.1 Objective Function

Maximize total profits considering direct, complementary, and adjacency effects:

$$\max \sum_i m_i S_i(x_i) + \sum_{i,j} m_j C_{ij}(x_i, x_j) + \sum_{i,j} m_j A_{ij}(x_i, x_j, a_{ij})$$

1.4.2 Constraints

- **Total Space Constraint:** $\sum_i x_i = \text{Total Shelf Space}$
- **Product-Level Constraints:** $\text{Min}_i \leq x_i \leq \text{Max}_i$
- **Adjacency Constraints:** $a_{ij} \in \{0, 1\}$, with practical adjacency limits.

1.5 Conclusion

By systematically testing hypotheses and leveraging robust optimization methods, this analysis provides actionable, data-driven shelf-space recommendations. Continuous validation and improvement ensure sustainable profitability and efficiency for the retailer.