Gpina-ISYE6501-HW5

June 18, 2025

1 ISYE 6501 - Homework 5

1.1 Question 11.1: Regression Modeling on the `uscrime` Dataset

Goals: 1. Stepwise regression (forward, backward, both) 2. Lasso (= 1) 3. Elastic Net (0 1)

1.1.1 1. Setup

[11]:
```r
## 1. Setup -------------------------------------------------------------------

# 1.1 Load required libraries
library(MASS) # stepAIC
library(glmnet) # lasso / elastic net

# Install and load FrF2 with error handling
if (!require(FrF2, quietly = TRUE)) {
    cat("Installing FrF2 package...\n")
    install.packages("FrF2", dependencies = TRUE)
    library(FrF2)
}

# Check if FrF2 loaded successfully
if (!"FrF2" %in% loadedNamespaces()) {
    stop("FrF2 package failed to load. Please install manually.")
} else {
    cat("FrF2 package loaded successfully.\n")
}

# 1.2 Load data
crime <- read.table("uscrime.txt", header = TRUE)
head(crime)
str(crime)

# 1.3 Define response & predictors
response <- "Crime"
predictors <- setdiff(names(crime), response)
y <- crime[[response]]
x <- as.matrix(crime[, predictors])
```

```
FrF2 package loaded successfully.
```

A data.frame: 6 × 16

	M <dbl>	So <int>	Ed <dbl>	Po1 <dbl>	Po2 <dbl>	LF <dbl>	M.F <dbl>	Pop <int>	NW <dbl>	U1 <dbl>
1	15.1	1	9.1	5.8	5.6	0.510	95.0	33	30.1	0.108
2	14.3	0	11.3	10.3	9.5	0.583	101.2	13	10.2	0.096
3	14.2	1	8.9	4.5	4.4	0.533	96.9	18	21.9	0.094
4	13.6	0	12.1	14.9	14.1	0.577	99.4	157	8.0	0.102
5	14.1	0	12.1	10.9	10.1	0.591	98.5	18	3.0	0.091
6	12.1	0	11.0	11.8	11.5	0.547	96.4	25	4.4	0.084

```
'data.frame':   47 obs. of  16 variables:
 $ M      : num  15.1 14.3 14.2 13.6 14.1 12.1 12.7 13.1 15.7 14 …
 $ So     : int  1 0 1 0 0 0 1 1 1 0 …
 $ Ed     : num  9.1 11.3 8.9 12.1 12.1 11 11.1 10.9 9 11.8 …
 $ Po1    : num  5.8 10.3 4.5 14.9 10.9 11.8 8.2 11.5 6.5 7.1 …
 $ Po2    : num  5.6 9.5 4.4 14.1 10.1 11.5 7.9 10.9 6.2 6.8 …
 $ LF     : num  0.51 0.583 0.533 0.577 0.591 0.547 0.519 0.542 0.553 0.632 …
 $ M.F    : num  95 101.2 96.9 99.4 98.5 …
 $ Pop    : int  33 13 18 157 18 25 4 50 39 7 …
 $ NW     : num  30.1 10.2 21.9 8 3 4.4 13.9 17.9 28.6 1.5 …
 $ U1     : num  0.108 0.096 0.094 0.102 0.091 0.084 0.097 0.079 0.081 0.1 …
 $ U2     : num  4.1 3.6 3.3 3.9 2 2.9 3.8 3.5 2.8 2.4 …
 $ Wealth: int  3940 5570 3180 6730 5780 6890 6200 4720 4210 5260 …
 $ Ineq   : num  26.1 19.4 25 16.7 17.4 12.6 16.8 20.6 23.9 17.4 …
 $ Prob   : num  0.0846 0.0296 0.0834 0.0158 0.0414 …
 $ Time   : num  26.2 25.3 24.3 29.9 21.3 …
 $ Crime : int  791 1635 578 1969 1234 682 963 1555 856 705 …
```

1.2 2. Stepwise Regression

We fit three variants via AIC: forward, backward, and both.

[12]:
```r
## 2. Stepwise Regression Analysis ------------------------------------------

## 2.1 Full and Null Models
full_mod <- lm(Crime ~ ., data = crime)
null_mod <- lm(Crime ~ 1, data = crime)

## 2.2 Stepwise Methods
step_fwd <- stepAIC(null_mod,
    scope = list(lower = null_mod, upper = full_mod),
    direction = "forward", trace = FALSE
)
step_bwd <- stepAIC(full_mod, direction = "backward", trace = FALSE)
step_both <- stepAIC(null_mod,
    scope = list(lower = null_mod, upper = full_mod),
```

```r
    direction = "both", trace = FALSE
)

## 2.3 Results Summary
stepwise_results <- data.frame(
    Method = c("Full", "Forward", "Backward", "Both"),
    Variables = c(
        length(coef(full_mod)) - 1,
        length(coef(step_fwd)) - 1,
        length(coef(step_bwd)) - 1,
        length(coef(step_both)) - 1
    ),
    AIC = round(c(AIC(full_mod), AIC(step_fwd), AIC(step_bwd), AIC(step_both)),
 ↪1),
    Adj_R2 = round(c(
        summary(full_mod)$adj.r.squared,
        summary(step_fwd)$adj.r.squared,
        summary(step_bwd)$adj.r.squared,
        summary(step_both)$adj.r.squared
    ), 3)
)

print(stepwise_results)

# Best model
best_idx <- which.min(stepwise_results$AIC[-1]) + 1
# Exclude full model from comparison
cat(
    "\nBest model:", stepwise_results$Method[best_idx],
    "| AIC:", stepwise_results$AIC[best_idx],
    "| Adj R²:", stepwise_results$Adj_R2[best_idx], "\n"
)

# Selected variables for best model
best_model <- list(step_fwd, step_bwd, step_both)[[best_idx - 1]]
cat("Variables:", paste(names(coef(best_model))[-1], collapse = ", "), "\n")
```

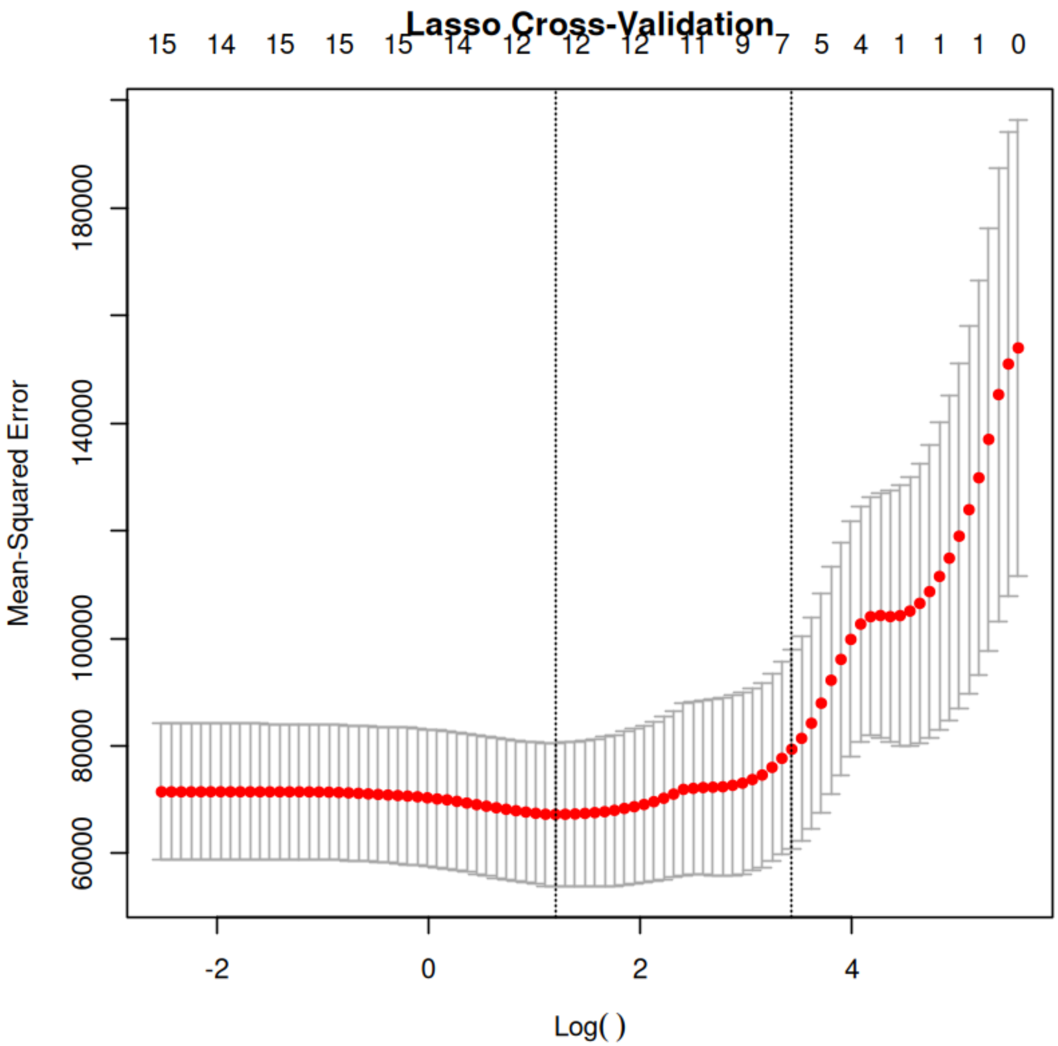
```
    Method Variables   AIC Adj_R2
1     Full        15 650.0  0.708
2  Forward         6 640.2  0.731
3 Backward         8 639.3  0.744
4     Both         6 640.2  0.731

Best model: Backward | AIC: 639.3 | Adj R²: 0.744
Variables: M, Ed, Po1, M.F, U1, U2, Ineq, Prob
```

1.3 3. Lasso Regression (= 1)

We'll standardize predictors and use `cv.glmnet` to pick via 10-fold CV.

[13]:
```
## 3. Lasso Regression ( = 1) ---------------------------------------------

## 3.1 Scale predictors and fit CV Lasso
x_scaled <- scale(x)
set.seed(123)
cv_lasso <- cv.glmnet(x_scaled, y, alpha = 1, nfolds = 10)

## 3.2 Results
best_lambda <- cv_lasso$lambda.min
lasso_mod <- glmnet(x_scaled, y, alpha = 1, lambda = best_lambda)

# Extract non-zero coefficients
lasso_coefs <- coef(lasso_mod)
selected_vars <- rownames(lasso_coefs)[lasso_coefs[, 1] != 0][-1] # Exclude␣
↪intercept

cat("Lasso Results:\n")
cat("Optimal  :", round(best_lambda, 4), "\n")
cat("Variables selected:", length(selected_vars), "\n")
cat("Selected variables:", paste(selected_vars, collapse = ", "), "\n")

# Optional: Show plot
plot(cv_lasso, main = "Lasso Cross-Validation")
```

```
Lasso Results:
Optimal  : 3.3199
Variables selected: 12
Selected variables: M, So, Ed, Po1, M.F, Pop, NW, U1, U2, Wealth, Ineq, Prob
```



1.4 4. Elastic Net Regression

We'll sweep from 0 (ridge) to 1 (lasso) in steps of 0.1, pick the best (,) by CV MSE, then refit.

[14]:
```
## 4. Elastic Net Regression ------------------------------------------------

## 4.1 Grid Search over
alpha_vals <- seq(0, 1, by = 0.1)
set.seed(123)

# Find best alpha-lambda combination
```

```r
best_cvm <- Inf
best_alpha <- NA
best_lambda <- NA

for (a in alpha_vals) {
  cv <- cv.glmnet(x_scaled, y, alpha = a, nfolds = 10)
  if (min(cv$cvm) < best_cvm) {
    best_cvm <- min(cv$cvm)
    best_alpha <- a
    best_lambda <- cv$lambda.min
  }
}

## 4.2 Final Elastic Net Fit
enet_mod <- glmnet(x_scaled, y, alpha = best_alpha, lambda = best_lambda)
enet_coefs <- coef(enet_mod)
selected_vars <- rownames(enet_coefs)[enet_coefs[, 1] != 0][-1] # Exclude
 ↪intercept

cat("Elastic Net Results:\n")
cat("Best  :", best_alpha, "| Best  :", round(best_lambda, 4), "\n")
cat("Variables selected:", length(selected_vars), "\n")
cat("Selected variables:", paste(selected_vars, collapse = ", "), "\n")
```

```
Elastic Net Results:
Best  : 0.8 | Best  : 12.6731
Variables selected: 12
Selected variables: M, So, Ed, Po1, LF, M.F, NW, U1, U2, Wealth, Ineq, Prob
```

1.5 Question 12.1

Describe a situation or problem from your job, everyday life, current events, etc., for which a design of experiments approach would be appropriate.

Answer:

- **Marketing Email Optimization (Job Context):** At GEICO, we might want to optimize the impact of promotional emails. We can use a factorial design varying **subject line length** (short vs. long), **send time** (morning vs. afternoon), and **call-to-action color** (blue vs. green). By systematically testing all combinations, we can measure open rates and click-through rates to identify the most effective factors and interactions.
- **Baking Recipe Experiment (Everyday Life):** When perfecting a cake, you could vary **oven temperature** (325°F vs. 350°F) and **bake time** (25 vs. 30 minutes) in a two-factor design. Measuring texture and moistness for each combination helps determine the optimal settings without testing every possible time–temperature pair ad hoc.
- **Vaccine Dosage Study (Current Events):** In clinical trials, a DOE approach can test

different **dosages** and **adjuvant types** to evaluate efficacy and side effects. A randomized block or factorial design ensures reliable estimation of main and interaction effects.

1.6 Question 12.2

To determine the value of 10 different yes/no features to the market value of a house (large yard, solar roof, etc.), a real estate agent plans to survey 50 potential buyers, showing a fictitious house with different combinations of features. To reduce the survey size, the agent wants to show just 16 fictitious houses. Use R's `FrF2` function (in the `FrF2` package) to find a fractional factorial design for this experiment: what set of features should each of the 16 fictitious houses have? Note: the output of `FrF2` is `1` (include) or `-1` (don't include) for each feature.

[15]:
```
# Define 10 yes/no factors
factors <- paste0("F", 1:10)

# Generate a 2^(10-6) fractional factorial design (16 runs)
design <- FrF2(nruns = 16, nfactors = 10, factor.names = factors)

# Display the design (1 = include feature, -1 = exclude feature)
print(design)
```

```
   F1 F2 F3 F4 F5 F6 F7 F8 F9 F10
1   1  1 -1 -1  1 -1 -1 -1  1   1
2  -1 -1  1  1  1 -1 -1 -1 -1   1
3  -1  1 -1 -1 -1  1 -1  1  1  -1
4  -1  1  1 -1 -1 -1  1  1 -1   1
5  -1 -1 -1 -1  1  1  1  1 -1   1
6   1 -1  1 -1 -1  1 -1 -1  1   1
7   1 -1 -1  1 -1 -1  1  1  1   1
8   1  1 -1  1  1 -1 -1  1 -1  -1
9  -1  1  1  1 -1 -1  1 -1  1  -1
10 -1 -1  1 -1  1 -1 -1  1  1  -1
11 -1  1 -1  1 -1  1 -1 -1 -1   1
12  1 -1 -1 -1 -1 -1  1 -1 -1  -1
13 -1 -1 -1  1  1  1  1 -1  1  -1
14  1  1  1 -1  1  1  1 -1 -1  -1
15  1  1  1  1  1  1  1  1  1   1
16  1 -1  1  1 -1  1 -1  1 -1  -1
class=design, type= FrF2
```

Answer

Based on the output from the cell above:

- **House 1**: Should include features F1, F2, F5, F9, and F10. It should exclude features F3, F4, F6, F7, and F8.
- **House 2**: Should include features F3, F4, F5, and F10. It should exclude features F1, F2, F6, F7, F8, and F9.

- **House 12**: Should only include feature F1 and exclude all others.
- **House 15**: Should include all 10 features.

1.7 Question 13.1

For each distribution below, give an example of data you would expect to follow it (besides the examples discussed in class):

- Binomial
- Geometric
- Poisson
- Exponential
- Weibull

Answer: - **Binomial**: The number of patients who respond positively to a new drug in a clinical trial of 20 participants. - **Geometric**: The number of job applications a person submits before receiving their first interview offer. - **Poisson**: The number of mutations in a specific gene sequence of a given length. - **Exponential**: The time until the next earthquake occurs in a specific region. - **Weibull**: The wind speed distribution for a particular location, often used for wind power assessment.