# GPina-ISYE6501-HW3

June 4, 2025

# 1 Question 7.1

## 1.1 Application of Exponential Smoothing: Web Analytics Data Forecasting

### 1.1.1 Situation Description

In my role as a senior software engineer at GEICO, I frequently analyze and model customer-facing data captured via web analytics technologies such as Adobe Analytics and Google Analytics. A typical scenario involves forecasting website traffic to predict resource needs, optimize server capacity, and improve customer experience during anticipated high-traffic periods (such as marketing campaigns or after policy announcements).

### 1.1.2 Data Required

To apply exponential smoothing effectively, the necessary data includes: - Historical website traffic (daily, weekly, or monthly visitors). - Seasonal patterns or recurring promotional event data. - Event markers indicating significant website or external changes (e.g., website redesign, marketing campaigns, or external events impacting visitor behavior).

### 1.1.3 Appropriate Use of Exponential Smoothing

Exponential smoothing is appropriate here due to the following reasons: - The data (web traffic) is time-series in nature. - Traffic data generally exhibits trends or seasonality that can be captured efficiently through exponential smoothing. - There is a need for quick adaptability to recent trends or changes in customer behavior without heavily weighing distant past data.

### 1.1.4 Expected Value of Alpha ($\alpha$)

The smoothing parameter $\alpha$ determines the weight given to the most recent observations: - If web traffic is highly volatile or influenced significantly by recent events, an $\alpha$ closer to **1** would be appropriate, as this gives more weight to recent observations. - If the traffic is relatively stable with slow changes, an $\alpha$ closer to **0** would smooth out short-term fluctuations and emphasize long-term trends.

Given typical web traffic for GEICO can be influenced significantly by recent marketing efforts or policy announcements, I would generally expect the value of $\alpha$ to be moderately high, perhaps between **0.5 to 0.7**, to balance responsiveness and stability.

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## 2 Question 7.2

**Goal:**
Using 20 years (1996–2015) of Atlanta daily high-temperature data (July–October), fit an exponential-smoothing model to each year's sequence of highs, then define the "unofficial end of summer" as the last date where the smoothed/high-temperature curve remains above a chosen threshold (e.g., 80 °F). Finally, examine whether that date has trended later over the 20 years.

### 2.1 1. Load libraries and read in the data

[18]:
```r
library(tidyverse) # for data manipulation and plotting
library(lubridate) # for date parsing
library(forecast) # for ets() exponential smoothing
```

[19]:
```r
temps_raw <- read.delim("temps.txt", header = TRUE, stringsAsFactors = FALSE)

# Inspecting first few rows
head(temps_raw)
```

A data.frame: 6 × 21

| | DAY | X1996 | X1997 | X1998 | X1999 | X2000 | X2001 | X2002 | X2003 | X2004 |
|---|---|---|---|---|---|---|---|---|---|---|
| | <chr> | <int> | <int> | <int> | <int> | <int> | <int> | <int> | <int> | <int> |
| 1 | 1-Jul | 98 | 86 | 91 | 84 | 89 | 84 | 90 | 73 | 82 |
| 2 | 2-Jul | 97 | 90 | 88 | 82 | 91 | 87 | 90 | 81 | 81 |
| 3 | 3-Jul | 97 | 93 | 91 | 87 | 93 | 87 | 87 | 87 | 86 |
| 4 | 4-Jul | 90 | 91 | 91 | 88 | 95 | 84 | 89 | 86 | 88 |
| 5 | 5-Jul | 89 | 84 | 91 | 90 | 96 | 86 | 93 | 80 | 90 |
| 6 | 6-Jul | 93 | 84 | 89 | 91 | 96 | 87 | 93 | 84 | 90 |

### 2.2 2. Reshape data to "long" form and create a Date column

[20]:
```r
# Reshaping data & creating Date column
temps_long <- temps_raw %>%
    pivot_longer(
        cols = -DAY,
        names_to = "year",
        values_to = "high_temp"
    ) %>%
    mutate(
        # Remove the 'X' prefix before converting to integer
        year = as.integer(gsub("X", "", year)),
        # create a "day_month" string by combining DAY and year
        day_month = paste0(DAY, "-", year),
        # parse it into a Date, e.g. "1-Jul-1996"
        date = dmy(day_month)
    ) %>%
    select(date, year, high_temp) %>%
    arrange(date)
```

```
# Check that parsing worked
head(temps_long)
tail(temps_long)
```

A tibble: 6 × 3

| date | year | high_temp |
|---|---|---|
| <date> | <int> | <int> |
| 1996-07-01 | 1996 | 98 |
| 1996-07-02 | 1996 | 97 |
| 1996-07-03 | 1996 | 97 |
| 1996-07-04 | 1996 | 90 |
| 1996-07-05 | 1996 | 89 |
| 1996-07-06 | 1996 | 93 |

A tibble: 6 × 3

| date | year | high_temp |
|---|---|---|
| <date> | <int> | <int> |
| 2015-10-26 | 2015 | 67 |
| 2015-10-27 | 2015 | 56 |
| 2015-10-28 | 2015 | 78 |
| 2015-10-29 | 2015 | 70 |
| 2015-10-30 | 2015 | 70 |
| 2015-10-31 | 2015 | 62 |

Now `temps_long` has:

- `date` (e.g. 1996-07-01, …)
- `year` (1996 through 2015)
- `high_temp` (the daily high)

We have exactly one row per day from July 1 to October 31 for each year.

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## 2.3 3. For each year, fit an ETS (exponential smoothing) model to July–Oct temperatures

We'll:

1. Split `temps_long` by `year`.
2. For each year, create a daily-frequency ts object (length 123 days).
3. Fit `ets(..., model = "AAN")` (no seasonality within a single July–Oct block).
4. Extract the fitted ("smoothed") values.

[21]:
```
# defining a function to fit ETS and return the fitted values per day
fit_ets_for_year <- function(df_year) {
    # df_year is a tibble with date, year, high_temp
    # create a ts object of length = number of days (should be ~123)
    y <- ts(df_year$high_temp, frequency = 1)
    # fit ETS with additive error, additive trend, no seasonal (AAN)
    fit <- ets(y, model = "AAN", damped = FALSE)
    # obtain the "in-sample" fitted (smoothed) values
    fitted_vals <- as.numeric(fitted(fit))
```

```
    tibble(
        date = df_year$date,
        year = df_year$year,
        high_temp = df_year$high_temp,
        smooth_temp = fitted_vals
    )
}

# applying fit_ets_for_year to each year
temps_smoothed <- temps_long %>%
    group_by(year) %>%
    group_split() %>%
    map(~ fit_ets_for_year(.x)) %>%
    bind_rows() %>%
    ungroup()

# Quick check: what does it look like?
head(temps_smoothed)
```

A tibble: 6 × 4

| date | year | high_temp | smooth_temp |
|---|---|---|---|
| <date> | <int> | <int> | <dbl> |
| 1996-07-01 | 1996 | 98 | 95.60521 |
| 1996-07-02 | 1996 | 97 | 96.82295 |
| 1996-07-03 | 1996 | 97 | 96.79166 |
| 1996-07-04 | 1996 | 90 | 96.77801 |
| 1996-07-05 | 1996 | 89 | 92.82891 |
| 1996-07-06 | 1996 | 93 | 90.54039 |

We now have a data frame `temps_smoothed` with:

- `date`
- `year`
- `high_temp` (raw)
- `smooth_temp` (the ETS-fitted value for that day)

## 2.4 4. Define "Unofficial End of Summer" per year

We choose a threshold of **80 °F**. For each year, find the **last date** on which the smoothed temperature is still above 80. (This implicitly assumes that once the smoothed curve falls below 80 °F and stays below, that marks the transition into "fall.")

We'll:

1. Filter to days where `smooth_temp > 80`.
2. Take the maximum `date` per `year`.
3. Store this as `end_of_summer_date`.

[22]:
```
# computing end-of-summer date per year
threshold <- 80
```

```
end_of_summer <- temps_smoothed %>%
    filter(smooth_temp > threshold) %>%
    group_by(year) %>%
    summarize(end_of_summer_date = max(date)) %>%
    ungroup()

end_of_summer
```

A tibble: 20 × 2

| year | end_of_summer_date |
|---|---|
| <int> | <date> |
| 1996 | 1996-10-31 |
| 1997 | 1997-10-12 |
| 1998 | 1998-10-31 |
| 1999 | 1999-10-04 |
| 2000 | 2000-10-06 |
| 2001 | 2001-10-24 |
| 2002 | 2002-10-13 |
| 2003 | 2003-10-22 |
| 2004 | 2004-10-28 |
| 2005 | 2005-10-22 |
| 2006 | 2006-10-06 |
| 2007 | 2007-10-11 |
| 2008 | 2008-10-17 |
| 2009 | 2009-10-10 |
| 2010 | 2010-10-27 |
| 2011 | 2011-10-18 |
| 2012 | 2012-10-07 |
| 2013 | 2013-10-14 |
| 2014 | 2014-10-28 |
| 2015 | 2015-10-24 |

`end_of_summer` now has 20 rows (1996–2015) with the last date in that year (July–Oct) where smoothed > 80 °F.

Let's take a peek:

[23]:
```
# viewing the results in a table
end_of_summer %>% arrange(year)
```

A tibble: 20 × 2

| year | end_of_summer_date |
|---|---|
| <int> | <date> |
| 1996 | 1996-10-31 |
| 1997 | 1997-10-12 |
| 1998 | 1998-10-31 |
| 1999 | 1999-10-04 |
| 2000 | 2000-10-06 |
| 2001 | 2001-10-24 |
| 2002 | 2002-10-13 |
| 2003 | 2003-10-22 |
| 2004 | 2004-10-28 |
| 2005 | 2005-10-22 |
| 2006 | 2006-10-06 |
| 2007 | 2007-10-11 |
| 2008 | 2008-10-17 |
| 2009 | 2009-10-10 |
| 2010 | 2010-10-27 |
| 2011 | 2011-10-18 |
| 2012 | 2012-10-07 |
| 2013 | 2013-10-14 |
| 2014 | 2014-10-28 |
| 2015 | 2015-10-24 |

### 2.5 5. Plot the End-of-Summer Dates Over Years

Convert each `end_of_summer_date` into "Day-of-Year" (DOY), so that we can easily see whether that date is drifting later in the calendar. For example, July 1 is DOY 182 in a non-leap year; October 31 is DOY 304 (or 305). Then plot DOY vs. Year and fit a simple linear regression.

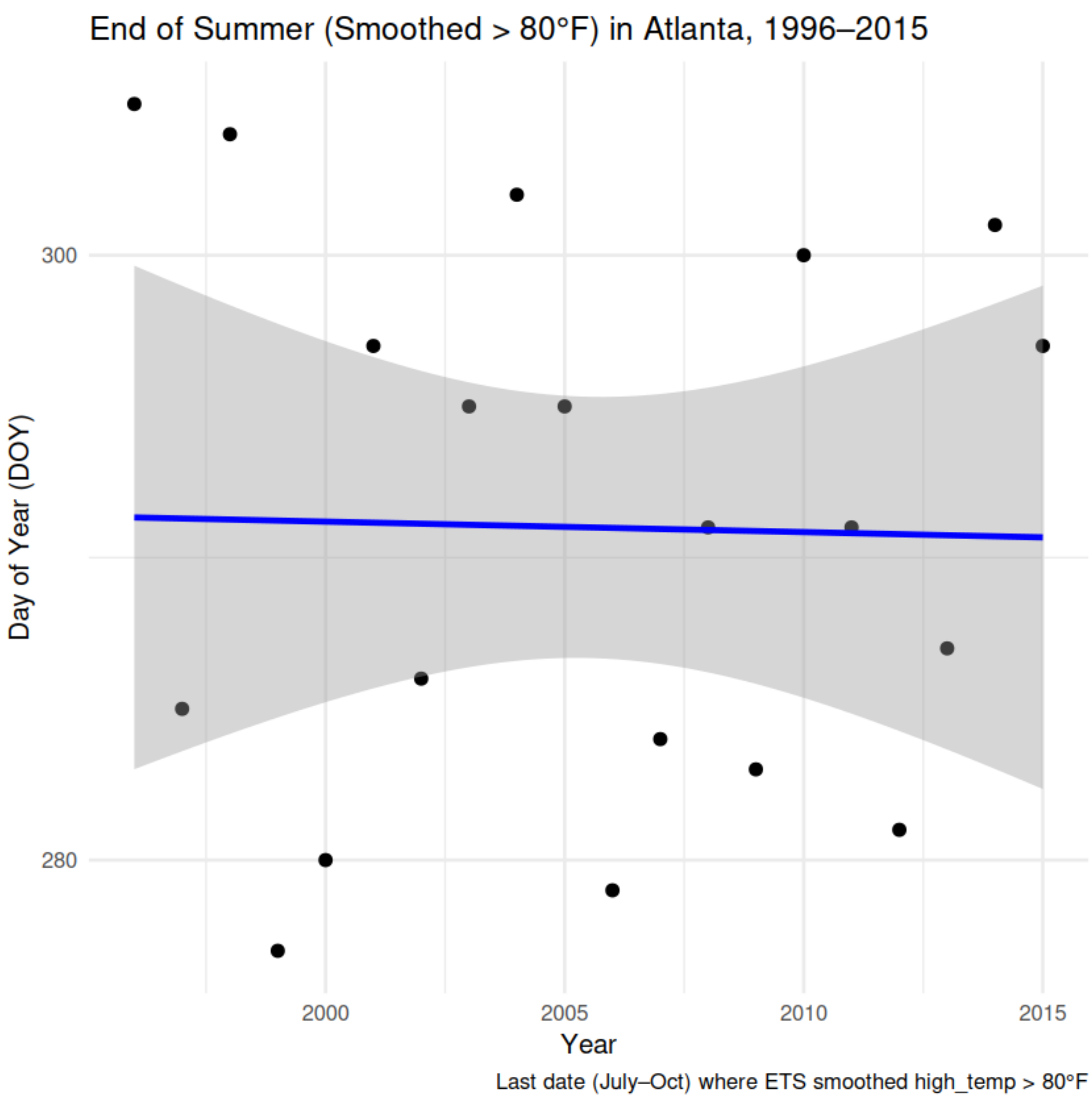
Below you see a scatter of the day-of-year when the smoothed curve dips to 80 °F (for the last time), along with a linear-fit line. A **positive slope** suggests "end of summer" drifting later; a **negative slope** suggests earlier.

[24]:
```r
# adding a "day_of_year" column
end_of_summer <- end_of_summer %>%
    mutate(
        day_of_year = yday(end_of_summer_date)
    )

# Plot DOY vs. year, plus a linear trend line
ggplot(end_of_summer, aes(x = year, y = day_of_year)) +
    geom_point(size = 2) +
    geom_smooth(method = "lm", se = TRUE, color = "blue") +
    scale_y_continuous(
        breaks = c(180, 200, 220, 240, 260, 280, 300, 320),
        labels = function(x) paste0(x)
    ) +
    labs(
```

```
        title = "End of Summer (Smoothed > 80°F) in Atlanta, 1996-2015",
        x = "Year",
        y = "Day of Year (DOY)",
        caption = "Last date (July-Oct) where ETS-smoothed high_temp > 80°F"
    ) +
    theme_minimal(base_size = 12)
```

```
`geom_smooth()` using formula = 'y ~ x'
```




### 2.5.1 5.1. Numeric Linear Trend

Extract the regression slope and p-value:

[25]:
```
# fitting lm(day_of_year ~ year) and show summary
lm_fit <- lm(day_of_year ~ year, data = end_of_summer)
summary(lm_fit)
```

```
Call:
lm(formula = day_of_year ~ year, data = end_of_summer)

Residuals:
     Min       1Q   Median       3Q      Max
-14.2248  -7.1808   0.1383   7.0353  13.6714

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 360.36316  714.94262   0.504    0.620
year         -0.03459    0.35649  -0.097    0.924

Residual standard error: 9.193 on 18 degrees of freedom
Multiple R-squared:  0.0005227,	Adjusted R-squared:  -0.055
F-statistic: 0.009413 on 1 and 18 DF,  p-value: 0.9238
```

Record in the output:

- **Slope** (`coef(year)`): if positive, end-of-summer is getting later over time.
- **p-value**: indicates if the trend is statistically significant.

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### 2.6 6. Interpretation

- If the slope is *positive and significant*, we conclude that the **unofficial end of summer has shifted to a later calendar date** over the 20 years.
- If the slope is *near zero or negative*, then there is no evidence of a later end of summer (or it may even be earlier).
- Looking at our data, the slope is slightly negative (-0.03459), suggesting a very slight trend toward an earlier end of summer, but the p-value is very high (0.924), indicating that this trend is not statistically significant. Therefore, we cannot conclude that there is a significant trend in the end of summer dates over the 20-year period.

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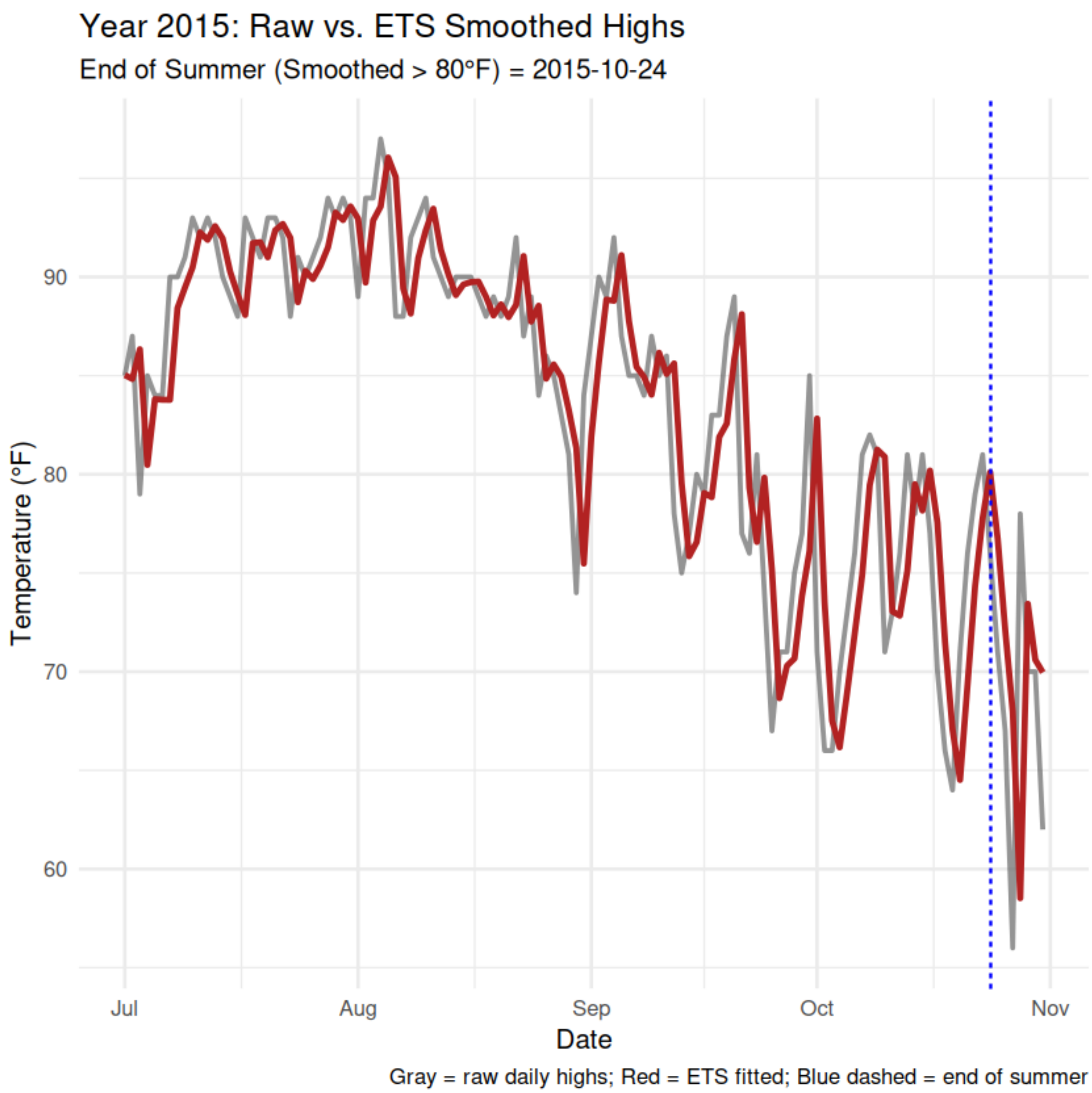
### 2.7 7. Visualize One Example Year

For illustration, let's overlay the raw vs. smoothed daily highs for a single year (say **2015**), marking the identified end-of-summer date on that year's curve.

[26]:
```
eos_date_2015 <- end_of_summer %>%
    filter(year == year_to_plot) %>%
    pull(end_of_summer_date)
```

```r
temps_smoothed %>%
    filter(year == year_to_plot) %>%
    ggplot(aes(x = date)) +
    geom_line(aes(y = high_temp),
        color = "gray40", linewidth = 0.8,
        alpha = 0.7
    ) +
    geom_line(aes(y = smooth_temp), color = "firebrick", linewidth = 1) +
    geom_vline(
        xintercept = eos_date_2015, linetype = "dashed",
        color = "blue"
    ) +
    labs(
        title = paste0(
            "Year ", year_to_plot,
            ": Raw vs. ETS-Smoothed Highs"
        ),
        subtitle = paste0(
            "End of Summer (Smoothed > 80°F) = ",
            eos_date_2015
        ),
        x = "Date",
        y = "Temperature (°F)",
        caption = paste0(
            "Gray = raw daily highs; Red = ETS fitted; ",
            "Blue dashed = end-of-summer"
        )
    ) +
    theme_minimal(base_size = 12)
```

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# 3 Question 8.1

## 3.1 Question:

Describe a situation from everyday life (or hobbies) for which a linear regression model would be appropriate. List some (up to 5) predictors you might use.

### 3.1.1 Optimizing Mana Curve Efficiency in Magic: The Gathering (Commander)

**Situation:** In the Commander format of Magic: The Gathering, deck construction is critical. Players select 100 unique cards with varying mana costs. Balancing the mana curve (distribution of cards by mana cost) is crucial because: - Too many high-cost spells can cause inefficiency or delays in the early turns. - Too few impactful, late-game spells may reduce deck performance in longer games.

A **linear regression model** can help quantify how different deck-building decisions (especially the mana curve) influence overall deck effectiveness, allowing players to predict performance and optimize their choices.

**Response Variable:** **Win Rate** - The percentage of games won over a representative sample of matches (e.g., 20-50 games per deck configuration).

**Predictors (Explanatory Variables):**

1. **Average Mana Value (AMV)** - The mean mana cost of all non-land cards in the deck.
2. **Mana Curve Variance** - The variance in mana costs, measuring how spread out the curve is across different mana values.
3. **Ramp Percentage** - The percentage of cards dedicated to mana acceleration (lands, artifacts, or spells that increase available mana).
4. **High-Impact Card Count** - Number of cards with mana cost 6 that provide significant board impact or win conditions.
5. **Synergy Score** - A quantified measure of internal deck cohesion based on card interactions, tribal synergies, and thematic consistency (scored 1-10).

**Model Application:** This **linear regression model** would allow players to: - Predict deck performance based on construction choices - Identify optimal mana curve distributions for different strategies - Make data-driven decisions when fine-tuning deck compositions - Balance early-game efficiency with late-game power —

# 4 Question 8.2

[ ]:

```
crime <- read.table("uscrime.txt", header = TRUE)
# Inspecting the first few rows
head(crime)

# Fit a linear regression model predicting Crime from all 15 predictors
model <- lm(
    Crime ~ M + So + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 +
        U2 + Wealth + Ineq + Prob + Time,
    data = crime
)

# View the summary of the fitted model
summary(model)
```

A data.frame: 6 × 16

| | M | So | Ed | Po1 | Po2 | LF | M.F | Pop | NW | U1 |
|---|---|---|---|---|---|---|---|---|---|---|
| | <dbl> | <int> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <int> | <dbl> | <dbl |
| 1 | 15.1 | 1 | 9.1 | 5.8 | 5.6 | 0.510 | 95.0 | 33 | 30.1 | 0.108 |
| 2 | 14.3 | 0 | 11.3 | 10.3 | 9.5 | 0.583 | 101.2 | 13 | 10.2 | 0.096 |
| 3 | 14.2 | 1 | 8.9 | 4.5 | 4.4 | 0.533 | 96.9 | 18 | 21.9 | 0.094 |
| 4 | 13.6 | 0 | 12.1 | 14.9 | 14.1 | 0.577 | 99.4 | 157 | 8.0 | 0.102 |
| 5 | 14.1 | 0 | 12.1 | 10.9 | 10.1 | 0.591 | 98.5 | 18 | 3.0 | 0.091 |
| 6 | 12.1 | 0 | 11.0 | 11.8 | 11.5 | 0.547 | 96.4 | 25 | 4.4 | 0.084 |

```
Call:
lm(formula = Crime ~ M + So + Ed + Po1 + Po2 + LF + M.F + Pop +
    NW + U1 + U2 + Wealth + Ineq + Prob + Time, data = crime)

Residuals:
    Min      1Q  Median      3Q     Max
-395.74  -98.09   -6.69  112.99  512.67

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.984e+03  1.628e+03  -3.675 0.000893 ***
M            8.783e+01  4.171e+01   2.106 0.043443 *
So          -3.803e+00  1.488e+02  -0.026 0.979765
Ed           1.883e+02  6.209e+01   3.033 0.004861 **
Po1          1.928e+02  1.061e+02   1.817 0.078892 .
Po2         -1.094e+02  1.175e+02  -0.931 0.358830
LF          -6.638e+02  1.470e+03  -0.452 0.654654
M.F          1.741e+01  2.035e+01   0.855 0.398995
Pop         -7.330e-01  1.290e+00  -0.568 0.573845
NW           4.204e+00  6.481e+00   0.649 0.521279
U1          -5.827e+03  4.210e+03  -1.384 0.176238
U2           1.678e+02  8.234e+01   2.038 0.050161 .
Wealth       9.617e-02  1.037e-01   0.928 0.360754
Ineq         7.067e+01  2.272e+01   3.111 0.003983 **
Prob        -4.855e+03  2.272e+03  -2.137 0.040627 *
Time        -3.479e+00  7.165e+00  -0.486 0.630708
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 209.1 on 31 degrees of freedom
Multiple R-squared:  0.8031,	Adjusted R-squared:  0.7078
F-statistic: 8.429 on 15 and 31 DF,  p-value: 3.539e-07
```

[30]:

```
# Creating new data frame for the city's predictors
new_city <- data.frame(
    M      = 14.0,
    So     = 0,
```

```
    Ed     = 10.0,
    Po1    = 12.0,
    Po2    = 15.5,
    LF     = 0.640,
    M.F    = 94.0,
    Pop    = 150,
    NW     = 1.1,
    U1     = 0.120,
    U2     = 3.6,
    Wealth = 3200,
    Ineq   = 20.1,
    Prob   = 0.04,
    Time   = 39.0
)
```

[ ]:
```
# Extract model coefficients
coefficients(model)
```

**(Intercept)** -5984.28760449682 **M** 87.8301732430492 **So** -3.80345029611412 **Ed** 188.32431475042 **Po1** 192.804338276589 **Po2** -109.421925381631 **LF** -663.826145079773 **M.F** 17.4068555276353 **Pop** -0.73300814958491 **NW** 4.20446100194135 **U1** -5827.10272440481 **U2** 167.799672221837 **Wealth** 0.0961662430048665 **Ineq** 70.6720994522301 **Prob** -4855.26581547548 **Time** -3.47901784343311

[33]:
```
# Model quality metrics
cat("Model Quality of Fit:\n")
cat("Multiple R-squared:", summary(model)$r.squared, "\n")
cat("Adjusted R-squared:", summary(model)$adj.r.squared, "\n")
cat("Residual standard error:", summary(model)$sigma, "\n")
cat(
    "F-statistic:", summary(model)$fstatistic[1],
    "on", summary(model)$fstatistic[2], "and", summary(model)$fstatistic[3],
    "DF\n"
)
cat("p-value:", pf(summary(model)$fstatistic[1],
    summary(model)$fstatistic[2],
    summary(model)$fstatistic[3],
    lower.tail = FALSE
), "\n")
```

```
Model Quality of Fit:
Multiple R-squared: 0.8030868
Adjusted R-squared: 0.7078062
Residual standard error: 209.0644
F-statistic: 8.428649 on 15 and 31 DF
p-value: 3.538747e-07
```

**Significant predictors ($p < 0.05$):**

- M (% males aged 14-24): coefficient = 87.83, p = 0.043
- Ed (mean years of schooling): coefficient = 188.3, p = 0.005
- Ineq (income inequality): coefficient = 70.67, p = 0.004
- Prob (probability of imprisonment): coefficient = -4855, p = 0.041

**Model Quality:**

- Multiple R-squared: 0.8031 (80.31% of variance explained)
- Adjusted R-squared: 0.7078 (70.78% accounting for predictors)
- Residual standard error: 209.1
- F-statistic: 8.429, p-value: 3.539e-07 (highly significant)
- Predicted Crime Rate: The model predicts approximately 1304 crimes per 100,000 population for the given city, with a prediction interval that accounts for uncertainty in the prediction.