12 Physics

Data Analysis Principles

You are responsible for the following topics:

1) Units Conversions ("multiplying-by-one" method)
2) Determining significant digits in measurements
3) Propagating significant digits in calculations
   ("weakest link rules" for multiplication and division & addition and subtraction)
4) Working with absolute and relative experimental uncertainties
5) Percent difference and Percent Error
6) Graphical analysis to determine a relationship between two experimental variables

1) Units Conversions

When converting from one unit to another, a useful tool is the "multiplying-by-one" method. In this method, the conversion factor relating the two units is first determined. We then multiply by the conversion factor so that the units to be converted cancel and the desired units remain.
The result has the same value though the units have changed. Thus we have "multiplied by one".

Example 1: To convert 2.5 hours into minutes we multiply by the conversion factor 1h = 60 min.

$$2.5\,\cancel{h} \times \frac{60\,\text{min}}{1\,\cancel{h}} = 150\,\text{min}$$

Since 60 min is the same as 1 h, the ratio 60 min/ 1 h is equal to 1, so all we have done is multiply our original quantity (2.5 h) by 1.

Example 2: To convert 34.5 cm to mm we multiply by the conversion factor 1 cm = 10 mm.

$$34.5\,\cancel{\text{cm}} \times \frac{10\text{mm}}{1\,\cancel{\text{cm}}} = 345\text{ mm}$$

Example 3: To convert 75 kg to μg we multiply by the conversion factor 1kg = 10^9 μg.

$$75\,\cancel{\text{kg}} \times \frac{10^9\,\mu\text{g}}{1\,\cancel{\text{kg}}} = 75 \times 10^9\,\mu\text{g} = 7.5 \times 10^{10}\,\mu\text{g}$$

Alternatively, we could first replace the metric prefix "k" with 10^3 to convert from kg to g. We can then multiply by the conversion factor 1μg = 10^{-6} g.

$$75\text{ kg} = 75 \times 10^3\,\cancel{\text{g}} \times \frac{1\,\mu\text{g}}{10^{-6}\,\cancel{\text{g}}} = 75 \times 10^9\,\mu\text{g} = 7.5 \times 10^{10}\,\mu\text{g}$$

Example 4: To 42 km/h to m/s, we need to multiply by TWO conversion factors:
1 km = 1000 m and 1 h = 3600 s

$$42\,\frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{m}}{1\,\cancel{\text{km}}} \times \frac{1\cancel{\text{h}}}{3600\text{s}} = 11.7\text{ m/s} = 12\text{ m/s (rounded to two significant digits)}$$

If the unit is raised to some power, we also need to do the same to the conversion factor.
Example 5: Convert 1.5 m^2 into mm^2

$$1.5\,\text{m}^2 \times \left(\frac{1000\,\text{mm}}{1\,\text{m}}\right)^2 = 1.5\,\cancel{\text{m}}^2 \times \frac{10^6\,\text{mm}^2}{1\,\cancel{\text{m}}^2} = 1.5 \times 10^6\,\text{mm}^2$$

2) Significant Digits in Calculations Involving Measurements

Also refer to Appendix A pg 748-749 for more information on significant digits.

Recording Measurements

When making a measurement, you should use the "***certain plus one***" rule when recording the number of digits in your measured value:

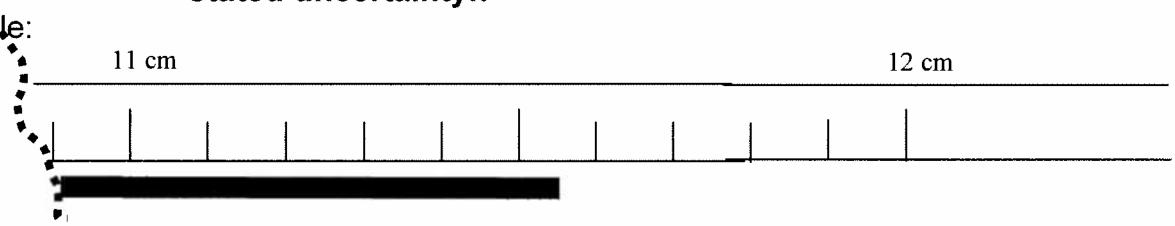
1. Record all certain digits plus one estimated digit

2. Determine the last estimated digit. (If your limiting factor is the precision of the measuring scale then this digit is determined by reading between the smallest division on the scale).

3. State the measurement uncertainty. This should include both the instrumental uncertainty and uncertainty due to other factors (e.g. human reaction time for time measurements).

GENERAL RULE: The measurement precision should match the precision of your stated uncertainty!!

Example:



The bar length is 11.55 cm ± 0.05 cm

certain digits — estimated digit — stated uncertainty (± half the smallest division in this case)

Rules for Determining Significant Digits in Stated Measurement Values

For measured values, significant digits are the digits that are certain plus one estimated (uncertain) digit. The following rules are used to decide the number of significant digits in a stated value:

All digits are significant EXCEPT:

- **leading zeros before any non-zero digit (as they are placeholders)**
- **trailing zeros for a whole number with no decimal place (as they are placeholders)**

**** For problems, we will assume that all stated whole numbers have at least 2 s.d.'s (e.g. 3000 m has 2 significant digits not 1)***

Examples: 0.000325 g has 3 s.d.   2500 m has 2 s.d.

1.280 kg has 4 s.d. 2.500×10^3 km has 4 s.d.

3) Propagating Significant Digits in Calculations

If we are performing mathematical operations on measurements, the precision of the results is going to reflect the precision of the numbers we begin with.

Multiplication and Division

In this case, the number of significant digits in the numbers used will tell us how many significant digits to keep in the result. **When multiplying or dividing, the result is rounded off to the same number of significant digits as the measurement with the fewest significant digits.**

e.g. 2.937 cm (4 sd) × 6.24 cm (3 sd) = 18.32688 cm² = 18.3 cm² (3 sd)

(6.24 cm is the "weakest link" with 3 sd so the result is rounded to 3 sd)

EXACT VALUES
Exact numbers (counted values, π, etc.) have infinite numbers of significant digits so we do not need to consider them when determining significant digits in a calculated result.

e.g. one marble has a mass of 8.3 g then 4 marbles have a mass of
$4 \times 8.3g = 33.2\ g = 33\ g$ (2 sd's because 8.3 has two sd's)

Addition or Subtraction
When adding or subtracting quantities, the result must be rounded off so that the decimal position of its last digit corresponds to the decimal position of the last digit in the number with the "leftmost" least significant digit. **Here it is the <u>position</u> of the least significant digit that matters, not the <u>number</u> of significant digits.**

e.g. 2.41 kg +1.7 kg + 0.025 kg = 4.135 kg = 4.1 kg
(the second measurement is the "weakest link"- it is only precise to the nearest 1/10th kg so the result is also rounded to the nearest 1/10th kg)
e.g. 450 m + 1.3 m = 451.3 m = 450 m
(the first number is only precise to the nearest 10 m, so the answer must also be rounded off to the nearest 10 m)

<u>4) Absolute and Relative Experimental Uncertainties</u>

Refer to Appendix A pg 755.
Below are some guidelines on how absolute and percent uncertainties are calculated and written.

Absolute uncertainty
- Expressed in the measured units.
- The precision of the measured value should match the stated uncertainty.
- Absolute uncertainties should have only one non-zero digit and are written before the units

  e.g. 2.65 ± 0.05 cm
- When calculations involve addition or subtraction, the absolute uncertainties are added.
- **absolute uncertainty= (percent uncertainty /100 %) × measured value**

Relative Uncertainty or Percent Uncertainty
- Determined by expressing the absolute uncertainty as a percentage of the measured value.
- Percent uncertainties should have no more than two significant digits and are written after the units.

  e.g. 5.65 cm ± 1.9%
- When calculations involve multiplication or division, the relative uncertainties are added.
- **percent uncertainty = (absolute uncertainty/measured value) × 100 %**

<u>5) Percent Difference and Percent Error</u>

Use percent error when comparing an experimental value with an accepted value:

$$\%\ \text{error} = \frac{|\text{experimental value-accepted value}|}{\text{accepted value}} \times 100\%$$

Use percent difference when comparing two experimental measurements:

$$\%\ \text{difference} = \frac{|\text{measurement1-measurement2}|}{(\text{measurement1+measurement2})/2} \times 100\%$$

e.g. the % difference between 7.2 m/s and 7.6 m/s is
$|7.6–7.2| / |(7.6+7.2)/2)| \times 100\ \% = 0.4 / 7.4 \times 100\ \% = 5\%$

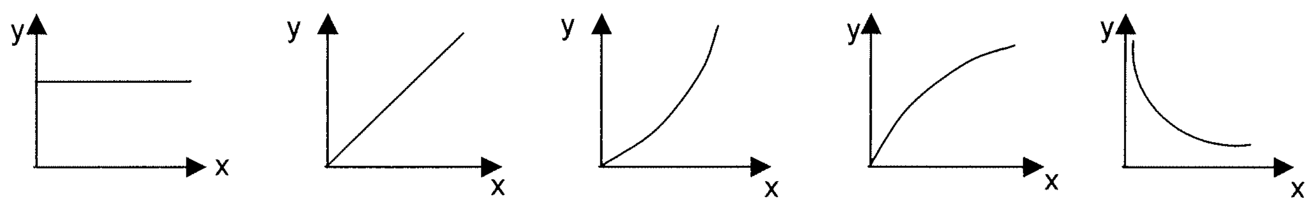
6) Graphical Analysis to Determine a Relationship Between Two Variables

Also refer to Appendix A pg 751-753 "Analyzing Experimental Data" for further examples of how to determine the relationship between an independent variable and a dependent variable.

Controlled experiments are performed to determine a relationship between two variables. After identifying the general type of relationship that exists between the two variables, graphical analysis can be used to verify the predicted relationship.

There are 5 basic cases or types of relationships that can exist between variables:

y independent of x / **no relationship** — **$y \propto x$** / **linear** — **$y \propto x^n$, n>1** / **power** — **$y \propto \sqrt[n]{x}$, n>1** / **root** — **$y \propto 1/x^n$, n>0** / **inverse**

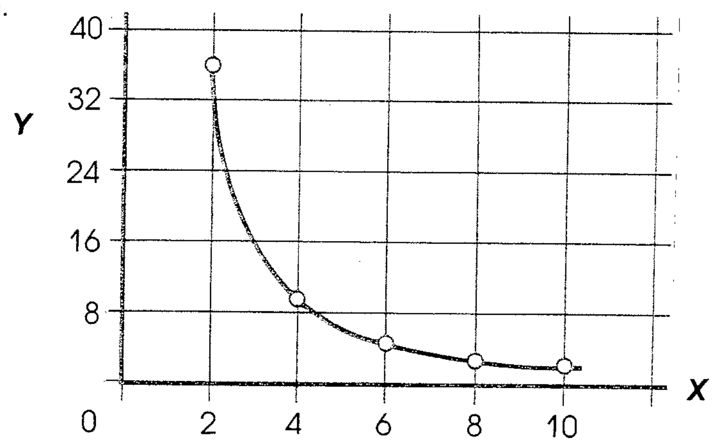


Sample Analysis to Determine the Relationship between Two Variables:

Example: Given the table of values below, find the relationship between *Y* and *X*.

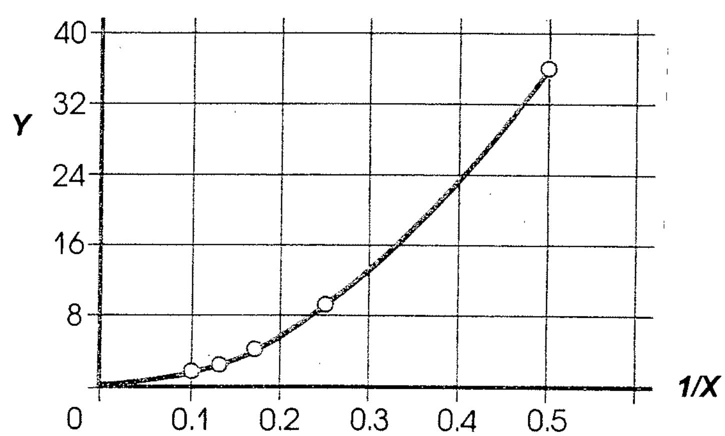
Step 1: A plot of ***Y versus X*** is made.

X	Y
2.0	36.0
4.0	9.0
6.0	4.0
8.0	2.3
10.0	1.4



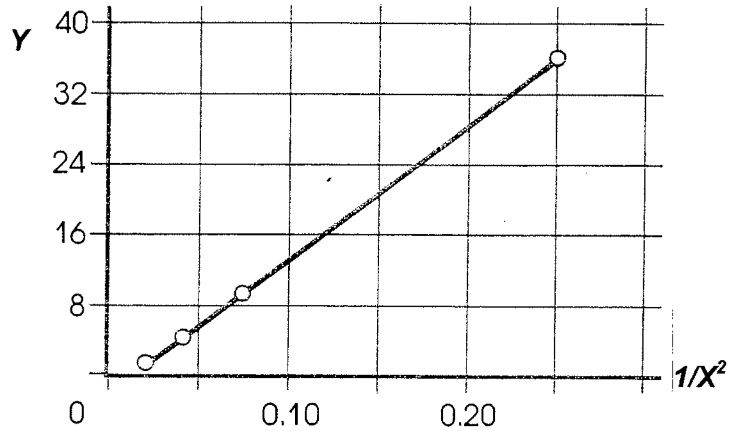
Step 2: From the shape of the graph of ***Y versus X***, it appears that ***Y*** is inversely proportional to X^n. ($Y \propto 1/X^n$). We first assume the simplest value of n, that is n=1. We will test this relationship by setting up a table of ***Y versus 1/X*** and then plotting a graph of ***Y versus 1/X***.

$\frac{1}{X}$	Y
0.50	36.0
0.25	9.0
0.17	4.0
0.13	2.3
0.10	1.4



Step 3: Since the plot of ***Y versus 1/X*** does not produce a straight line, we assume n=2 and set up a table of values of ***Y versus 1/X²***. We then plot a graph of ***Y versus 1/X²***.

$\frac{1}{X^2}$	Y
0.25	36.0
0.063	9.0
0.019	4.0
0.017	2.3
0.010	1.4



Step 4: Since this last plot of ***Y versus 1/X²*** shows a straight line, this verifies the nature of the relationship between ***Y*** and ***X*** ($Y \propto 1/X^2$). The equation relating them will be of the form $y = k\frac{1}{X^2}$ where ***k*** is the slope of the final graph. From this graph, we find the slope is 144 so the equation for the full relationship is: $y = 144\frac{1}{X^2}$

NOTE:
If the variables *Y* and *X* have units, the slope will also have units (e.g. if *Y* is measured in kg and *X* is measured in seconds, then the units of slope will be kg/(1/s²) or kg s².

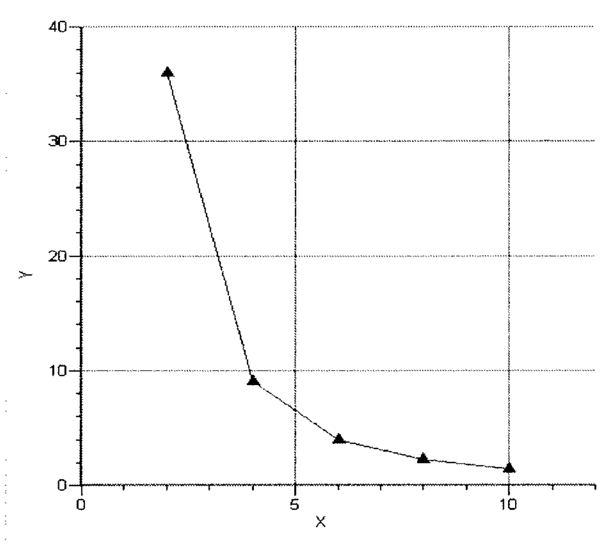
RELATIONSHIP ANALYSIS USING GRAPHING PROGRAMS
Graphing programs are powerful analysis tools that can help us to determine relationships between variables. In general, a graphing program will be able to "try" different possible fitting functions for a data set and will provide a "goodness of fit measure" which will indicate whether the trial function is a good match for the data set.

For example, the program "***Graphical Analysis***" was used to determine the relationship between the data set of ***y versus x*** analyzed above.

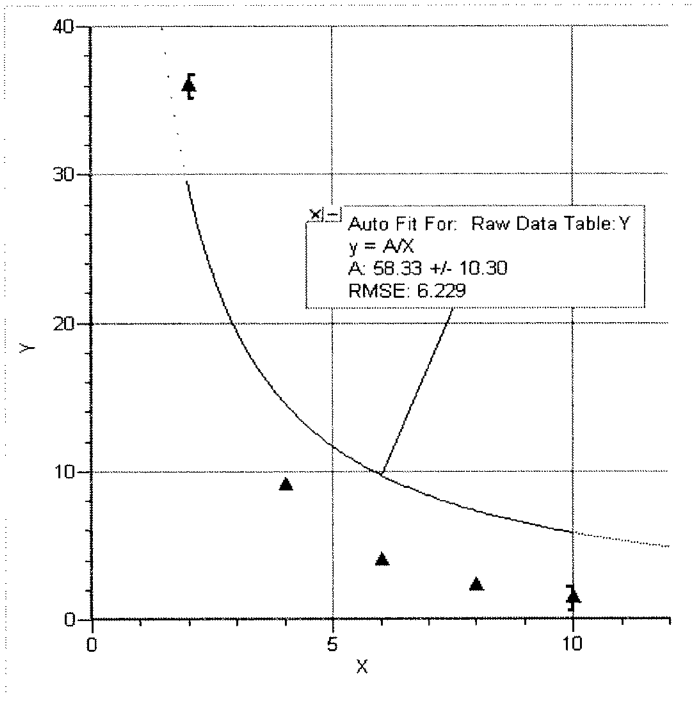
Step 1: The data values were typed into a data table and a graph was generated as shown below.

Raw Data Values

X	Y
2.0	36.0
4.0	9.0
6.0	4.0
8.0	2.3
10.0	1.4

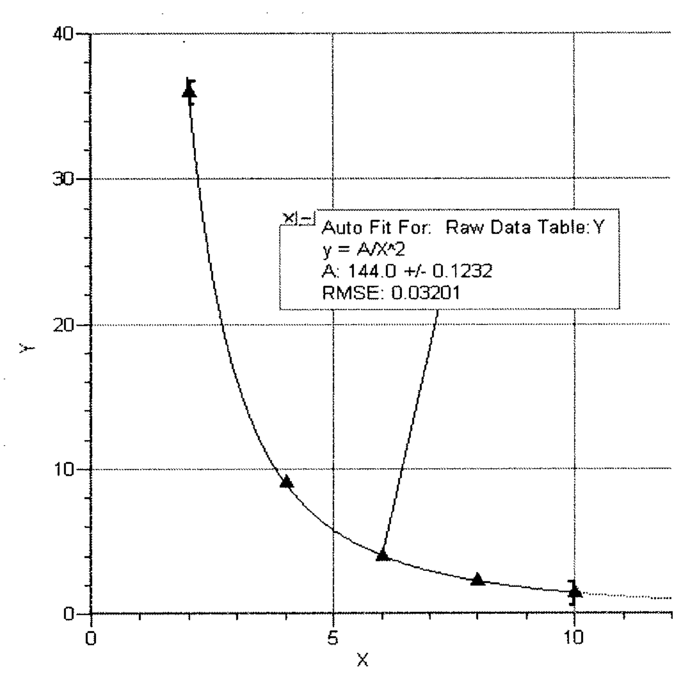


Step 2: The curved graph indicated that a there is some sort of inverse relationship between **y** and **x**. To test this, we select **"Analyze⇒ Curve Fit"** from the menu. The Curve Fit window allows us to choose a general function to try. We first try an Inverse relationship (y=k/x). The results of this trial are shown below:



* note that the Graphical Analysis program uses "A" rather than "k" as the proportionality constant

Step 3: The poor correspondence between the data curve and the inverse fit and the high value of the Root Mean Square Error parameter (6.229) indicate that this is **not a good fit**. We then go back to the curve fit menu and try an Inverse Square relationship ($y = k/x^2$). The results of this trial are shown below:



Step 4: The good correspondence between the data curve and the inverse square fit and the low value of the Root Mean Square Error parameter (0.032) indicate that this **is a good fit**. We can also determine the constant of proportionality from the fit data to be 144.0 ± 0.1. Thus the final statement of the relationship between **y** and **x** is : $\mathbf{y = 144.0 \bullet \frac{1}{x^2}}$ which agrees with our original analysis.