Nov 2, 2012

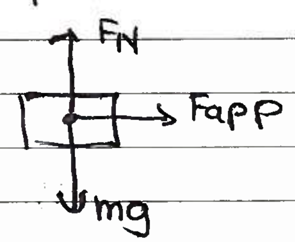
$W = \Delta E$

Kinetic Energy [scalar?]
recall: How did we define energy? [capacity to do work]

→ can a moving object do work? Can it apply a force and cause a displacement?

→ what might affect the amount of energy a moving object has?
→ speed [what if the object is moving quicker?]
→ mass [what if you have a heavy object?] can it do more work?!

Consider: you are pushing on a curling stone across the ice [which we will consider frictionless] you change the speed of the stone from v_1 to v_2 over a distance Δd.



$W_{done} = F_{app}\Delta d$ ← what force is doing the work

what is the force

$F_{net} = \boxed{F_{app} = \boxed{m \cdot a}}$ from newton's 2nd law

So $W_{done} = m \cdot a \cdot \Delta d$

what is a? use famous five $a = \frac{v_2^2 - v_1^2}{2\Delta d}$

$\Rightarrow W_{done} = m \cdot \left(\frac{v_2^2 - v_1^2}{2\cancel{\Delta d}}\right) \cdot \cancel{\Delta d}$

$W_{done} = \frac{mv_2^2 - mv_1^2}{2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

we also said work was ΔE, what energy is changing?

$\Delta E_K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

kinetic energy

~~How do~~ This is HUGE!

$$\boxed{W = \Delta E_K = \underbrace{\tfrac{1}{2}mv_2^2}_{E_{K2}} - \underbrace{\tfrac{1}{2}mv_1^2}_{E_{K1}}}$$ (this is what Δ means)

→ what would E_K by itself be?

$$\boxed{E_K = \tfrac{1}{2}mv^2}$$

Let's check units!

E_K	$m \cdot v^2$
J	$kg \cdot \left(\frac{m}{s}\right)^2$
	$= kg \, \frac{m^2}{s^2}$

$= \frac{kg\,m}{s^2} \cdot m$

$= N \cdot m$

$= J$

→ m in kg!

→ v in m/s!

E_K is a scalar as well!

Sample problems:

1.) find mass given E_K (sample 1)

2.) find v given E_K, and check units (sample 2)

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Kinetic Energy Sample Problems:

Sample Problem 1:

You watch your sister on the toboggan and realize that she has a kinetic energy of 250 J. You know your sister has a mass of 45kg so what speed is she currently travelling at? (work out the units to show that they are correct!)

$E_k = 250\,J \quad v = ?$

$m = 45\,kg$

$\left(E_k = \frac{1}{2}mv^2\right) \times 2$

$2E_k = mv^2$

$v^2 = \frac{2E_k}{m}$

$v = \sqrt{\frac{2E_k}{m}}$

$= \sqrt{\frac{2(250)}{46}}$

$= 3.33\ m/s$

$\boxed{\therefore \text{ velocity is } 3.33\ m/s}$

units

$E_k = J = N \cdot m$

$= \frac{kg\,m}{s^2} \cdot m$

$= kg\,m^2/s^2$

$m = kg$

$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{kg\,m^2/s^2}{\cancel{kg}}}$

$= \sqrt{\frac{m^2}{s^2}} = m/s$

Sample Problem 2

You know that a certain comet has a speed of 400 m/s and kinetic energy of 16 000 kJ. Find the mass of the comet.

$v = 400\ m/s \qquad E_k = 16000\ kJ \times \frac{1000\,J}{1\,kJ} = 1.6 \times 10^7\ J$

$m = ?$

$\left(E_k = \frac{1}{2}mv^2\right) \times 2$

$2E_k = mv^2$

$m = \frac{2E_k}{v^2} = \frac{2(1.6 \times 10^7\ J)}{(400)^2} = \frac{3.2 \times 10^7}{400^2} = 200$

$\therefore$ the mass of the object is 200 kg

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Sample Problem 3:

A force does 30 J of work in moving an object from rest to a final speed. Determine the final speed of the object if it has a mass of 150 g.

$W = \Delta E_k$ $\quad W = 30\,J \quad v_1 = 0 \quad m = 150g$

$W = \frac{1}{2}mv_2^2 - \cancel{\frac{1}{2}mv_1^2} \rightarrow 0 \quad v_2 = ? \quad = 0.150\,kg$

$30 = \frac{1}{2}(0.150)v_2^2$

$\sqrt{v_2^2} = \sqrt{400}$ $\quad \therefore$ the final speed is 20 m/s

$v_2 = 20$

Sample Problem 4: (tricky!)

A force accelerates an object of mass 10 kg from an initial speed of 4m/s to a final speed of 10 m/s. Over a distance of 5 m.

a.) Find the initial kinetic energy of the object

$E_{k_1} = \frac{1}{2}mv_1^2 = \frac{1}{2}(10)(4)^2 = 80\,J$

b.) Find the final kinetic energy of the object.

$E_{k_2} = \frac{1}{2}mv_2^2 = \frac{1}{2}(10)(10)^2 = 500\,J$

c.) Find the work done on the object.

$W = E_{k_2} - E_{k_1} = 500 - 80 = 420\,J$

d.) Find the magnitude of the net force that resulted in this acceleration.

$W = F \cdot \Delta d \quad F = \frac{W}{\Delta d} = \frac{420}{5} = 84\,N$