

### Measurement Error

Measurements are **NEVER** exact. There is always an uncertainty or "error" in any measured value. If several people measure the length of the same table using a metre stick, it is normal for them to get slightly different values.

**RANDOM ERRORS**- are due to random and unknown changes in experimental conditions and include factors such as fluctuations in the quantity being measured, and external electrical noise. Limitations in the resolution of the measuring device can also contribute to estimation error.

**SYSTEMATIC ERRORS** are due to an inherent problem with the measuring device, such as an error in instrument calibration, or an error in how the device is used.

### Measurement Uncertainty

Measurements should be made carefully and should indicate how confident you are of the measurement by stating the **measurement uncertainty**. The measurement uncertainty indicates the range of error in your measurement.

For example, a measurement of  $5.4 \pm 0.1$  cm indicates that the actual value could lie between 5.3 cm and 5.5 cm. Notice that the uncertainty of  $\pm 0.1$  cm shows we are not certain of the "4"-it is an estimated digit. Similarly, a measurement of mass of  $16.25 \pm 0.01$  g indicates we are uncertain of the last "5"-the actual value could lie between 16.24 g and 16.26 g.

**Practice 1:** State the range in possible values for each measurement below:

a)  $56.85 \pm 0.01$  g : 56.84 to 56.86

c)  $21.6 \pm 0.2$  s : 21.4 to 21.8

b)  $15.9 \pm 0.1$  cm : 15.8 to 16.0

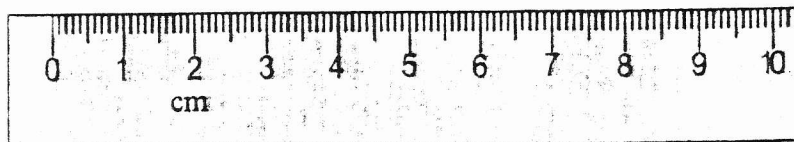
d)  $7.200 \pm 0.001$  g : 7.199 to 7.201

**Determining Uncertainty:** When performing a measurement, the uncertainty should reflect the resolution of the instrument and the conditions under which you are performing the measurement.

Both a ruler and a metre stick have scale markings of 1 mm but the uncertainty associated with measurements made with these two devices may differ. When using a ruler to measure a small object with sharply defined edges, you may be able to use a ruler to estimate *between* the millimetre scale markings to the nearest  $\frac{1}{2}$  mm ( $\pm 0.05$ cm).

If you are using a metre stick to measure a large object with edges that are not clearly defined, you may be only able to estimate to the nearest 2 mm to give a measurement uncertainty of  $\pm 0.2$  cm.

**Practice 2:** Determine the length of the arrow below using the given ruler. Make sure to state your uncertainty!



Measurement:

$9.2 \pm 0.1$  cm

Answers

## Significant Digits or "Sig Digs"

Significant digits are all the digits in a measurement of which we are reasonably certain. This includes:

ALL OF THE DIGITS ABOUT WHICH WE ARE CERTAIN  
PLUS THE LAST ESTIMATED OR UNCERTAIN DIGIT!

All digits in a measured number are significant EXCEPT  
*leading zeros for decimal numbers (as they are placeholders)*  
*trailing zeros for whole numbers (as they are placeholders)*

Examples: 45.005 m – has 5 Sig Digs  
0.0629 g – has 3 s.d.  
7.8450 s – has 5 s.d.  
5670 km – has 3 s.d.

## Avoiding Confusion with Whole Numbers:

Confusion regarding the number of significant digits in a measurement can occur if the number is recorded as a whole number. For example, the measurement of 5670 km could have 4 significant digits if it was measured to the nearest kilometre. It could also have 3 significant digits if it was measured to the nearest 10 kilometres.

Scientific notation is used to avoid confusion!

Ex.  $5.670 \times 10^3$  km indicates the measurement was made to the nearest kilometre.

$5.67 \times 10^3$  km To indicate a measurement made to the nearest 10 kilometres:

## Counted and Defined Values

These kind of values are **exact** and are considered to have an **INFINITE** number of significant digits.

Examples: 27 students in class      conversion factors: 100 cm/m  
10 cycles of a pendulum      defined values :  $\pi$  (pi)

**Practice 3:** Represent a measurement of mass which was written down as 5200 g to indicate that it was:

- i) Made to the nearest gram :  $5.200 \times 10^3$  g  
ii) Made to the nearest 10 grams:  $5.20 \times 10^3$  g  
iii) Made to the nearest 100 grams:  $5.2 \times 10^3$  g

## Practice 4:

Indicate the number of significant digits in the following measured quantities:

- a) 5.2 m 2      b) 245 kg 3      c) 999 s 3      d) 0.3 cm 1  
e) 0.125 mm 3      f) 0.0035 km 2      g) 0.608 g 3      h) 3005 m 4  
i) 450 m 2      j)  $7.80 \times 10^2$  m 3      k)  $1.030 \times 10^3$  km 4

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**Calculations with Measured Values: Weakest Link Rules**

When performing calculations with measured values, it is important that we keep only the digits that are significant in our measurements. A chain is only as strong as the "weakest link"!

We first use all of the given significant digits in each measurement when performing the calculation, but then round off the final answer to the appropriate number of significant digits determined by the "weakest link".

**Multiplying or dividing measured values:**

The final answer should contain the same number of significant digits as the number with least number of significant digits.

ex.  $2.85 \text{ m} \times 1.347 \text{ m} = \underline{3.83895}$  (calculator answer)  
 $= \underline{3.84}$  (answer rounded to the correct number of significant digits)

Weakest link? 2.85 HAS ONLY 3 SIG DIGS.

**Adding or subtracting measured values:**

The final answer should contain the same number of decimal places as the least precise number.

e.g.  $5.23 \text{ m} + 1.225 \text{ m} + 41.7 \text{ m} = \underline{48.155}$  (calculator answer)  
 $= \underline{48.2}$  (answer rounded to the correct precision)

Weakest link? 41.7 HAS ONLY ONE DECIMAL PLACE

**Practice 1:** Perform the following calculations, keeping the correct number of significant digits in your answer.

a)  $50.0 \text{ m} / 5.5 \text{ s} = \underline{9.1 \text{ m/s}}$       b)  $1.00 \times 10^2 \text{ m} + 17.35 \text{ m} - 41.7 \text{ m} = \underline{76 \text{ m}}$

\*  $\downarrow$   
 $100 = \text{NO DECIMAL PLACE}$

**Note on Rounding Numbers**

The simple rounding rule, taught in junior grades, says that you "round up" if the number to the right of the digit to be retained is **5 or more**. This rule works fine in almost all cases, but can lead to bias or round-off error if you are rounding a set of numbers that are **exactly half-way** between the lower and upper possibilities. This bias occurs because you will always "round up" using the simple rule.

Examples:  $7.85 \text{ g}$  is exactly half-way between  $7.80 \text{ g}$  and  $7.90 \text{ g}$ .  
 $11.5 \text{ km}$  is exactly half-way between  $11.0 \text{ km}$  and  $12.0 \text{ km}$ .  
 $1.450 \text{ g}$  is exactly halfway between  $1.400 \text{ g}$  and  $1.500 \text{ g}$ .

To avoid this bias, use the ODD-EVEN rule when rounding a number which is EXACTLY half-way between the lower and upper possibility:

- round up if the digit to be retained is **odd**
- round down if the digit to be retained is **even**.

**Practice 2:** Round each measurement below to **TWO** significant digits:

$1.850 \text{ m}$	<u>1.8</u>	$9.85001 \text{ s}$	<u>9.9</u>
$6.75 \text{ kg}$	<u>6.8</u>	$0.08749 \text{ km}$	<u>0.087</u>
$9.85000 \text{ s}$	<u>9.8</u>	$754 \text{ g}$	<u>750</u>
$7.35 \text{ cm}$	<u>7.4</u>		