

# Solutions Universal Gravitation Worksheet

Useful constants:

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

$$\text{Mass of Earth} = 5.98 \times 10^{24} \text{ kg}$$

$$\text{Radius of Earth} = 6.38 \times 10^6 \text{ m}$$

1. State the formula for Newton's Law of Universal Gravitation. Define all terms and state the standard units for each term!!

$$F_G = \frac{G m_1 m_2}{r^2}$$

$F_G$  = gravitational force in Newtons

$m_1$  = mass of object 1 in kg

$m_2$  = mass of object 2 in kg

$r$  = separation distance in metres

$G$  = Universal Gravitation constant

$$(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)$$

2. What is the force of attraction between two objects, masses  $2.0 \times 10^3 \text{ kg}$  and  $1.5 \times 10^5 \text{ kg}$  respectively, if they are positioned so that their centres are 45.0 m apart?

$$m_1 = 2.0 \times 10^3 \text{ kg}$$

$$m_2 = 1.5 \times 10^5 \text{ kg}$$

$$r = 45.0 \text{ m}$$

$$F_G = ?$$

$$F_G = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) (2.0 \times 10^3 \text{ kg}) (1.5 \times 10^5 \text{ kg})}{(45.0 \text{ m})^2}$$

$$= 9.9 \times 10^{-6} \text{ N}$$

3. A satellite, mass 550 kg, is orbiting the Earth at a radial distance of 6550 km (measured from the Earth's centre). What is the force of gravity acting on the satellite due to the Earth?

$$m = 550 \text{ kg}$$

$$r = 6550 \text{ km}$$

$$= 6550000 \text{ m}$$

$$F_G = ? = 6.65 \times 10^6 \text{ N}$$

$$M_E = 5.98 \times 10^{24} \text{ kg}$$

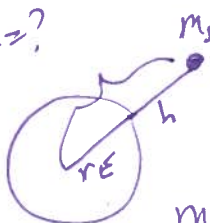
$$F_G = \frac{G M_E m}{r^2} = \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) (5.98 \times 10^{24} \text{ kg}) (550 \text{ kg})}{(6.55 \times 10^6 \text{ m})^2}$$

$$= 5113.4 \text{ N}$$

$$\approx 5.1 \times 10^3 \text{ N}$$

4. The space shuttle (mass  $2.0 \times 10^5 \text{ kg}$ ) is orbiting the Earth at an altitude of 240 km above the Earth's surface. What is the force of gravitational attraction between the shuttle and Earth? (Hint: Draw a picture!!!)

$$F_G = ?$$



$$h = 240 \text{ km}$$

$$= 240000 \text{ m}$$

$$= 2.40 \times 10^5 \text{ m}$$

$$m_s = 2.0 \times 10^5 \text{ kg}$$

$$r = r_E + h$$

$$= 6.38 \times 10^6 \text{ m} + 2.40 \times 10^5 \text{ m}$$

$$= 6.62 \times 10^6 \text{ m}$$

$$F_G = \frac{G M_E m_s}{r^2}$$

$$= \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) (5.98 \times 10^{24} \text{ kg}) (2.0 \times 10^5 \text{ kg})}{(6.62 \times 10^6 \text{ m})^2}$$

$$= 1.8 \times 10^6 \text{ N}$$

5. a) Calculate the force of gravitational attraction ( $F_G$ ) between a boulder (mass 25.0 kg) and the Earth when the boulder is sitting on the Earth's surface.

$$m = 25.0 \text{ kg}$$

$$M_E = 5.98 \times 10^{24} \text{ kg}$$

$$r = 6.38 \times 10^6 \text{ m}$$

$$F_G = \frac{G M_E m}{r^2} = \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) (5.98 \times 10^{24} \text{ kg}) (25.0 \text{ kg})}{(6.38 \times 10^6 \text{ m})^2}$$

$$= 244.9 \text{ N}$$

b) Now find the weight ( $F_g$ ) of the boulder when it is sitting on the Earth's surface.

$$F_g = ? \quad m = 25.0 \text{ kg} \\ g = 9.80 \text{ N/kg}$$

$$F_g = mg = (25.0 \text{ kg}) (9.80 \frac{\text{N}}{\text{kg}}) = 245 \text{ N}$$

c) How do your answers for (a) and (b) compare? Why?

The answers are the same as  $F_g$  is a simple way to find the force of gravitational attraction between the Earth + objects on the Earth's surface.

### Using Proportionalities with the Universal Gravitation Equation

6. a) How does the magnitude of the gravitational force between two objects change as the distance between them is increased?

As distance is increased, gravitational force decreases.

b) What specific relationship describes this variation?

$$F_g \propto \frac{1}{r^2}$$

c) A gravitational attractive force of 25.0 N exists between two objects when they are located a distance of 100.0 m apart. Determine the new force of gravitational attraction between them if they are now moved a distance of:

i) 200.0 m apart

ii) 25.0 m apart

$$F_g = 25.0 \text{ N} \Rightarrow F_g \propto \frac{Gm_1 m_2}{r^2} \\ r = 100.0 \text{ m}$$

$$\begin{aligned} \text{i) } r_2 &= 200.0 \text{ m} \\ r_2 &= 2r \\ F_{g2} &= \frac{Gm_1 m_2}{r_2^2} \\ &= \frac{Gm_1 m_2}{(2r)^2} \\ &= \frac{1}{4} \frac{Gm_1 m_2}{r^2} \\ &= \frac{1}{4} F_g \\ &= \frac{25.0 \text{ N}}{4} \\ &= 6.25 \text{ N} \\ &= \end{aligned}$$

$$\begin{aligned} \text{i) } r_3 &= 25.0 \text{ m} \\ r_3 &= \frac{r}{4} \\ F_{g3} &= \frac{Gm_1 m_2}{r_3^2} \\ &= \frac{Gm_1 m_2}{(\frac{r}{4})^2} \\ &= \frac{Gm_1 m_2}{\frac{r^2}{16}} \\ &= 16 F_g \\ &= 16 (25.0 \text{ N}) \\ &= 400. \text{ N} \\ &= \end{aligned}$$