Relative Velocity Homework - Part I

Pg. 48 P1, 5, 7, 8

P1] T - teenagers B - boat w - water

a) $\vec{V}_{BW} = 2.8\,m/s$ [F]

$\vec{V}_{TB} = 1.1\,m/s$ [F]

$\vec{V}_{TW} = ?$

analysis: $\vec{V}_{TW} = \vec{V}_{TB} + \vec{V}_{BW}$ let For = +

$\vec{V}_{TW} = 1.1\,m/s + 2.8\,m/s$

$= 3.9\,m/s$ [F].

b) $\vec{V}_{TB} = 1.1\,m/s$ [B]

$\vec{V}_{TW} = -1.1\,m/s + 2.8\,m/s = 1.7\,m/s$ [F]

5] C - child, B - boat, w - water

a) $\vec{V}_{BW} = 4.0\,m/s$ [N]

$\vec{V}_{CB} = 3.0\,m/s$ [N]

$\vec{V}_{CW} = ?$

$\vec{V}_{CW} = \vec{V}_{CB} + \vec{V}_{BW}$ let N = +

$= 3.0\,m/s + 4.0\,m/s$

$= 7.0\,m/s$ [N]

b) $\vec{V}_{CB} = 3.0\,m/s$ [S]

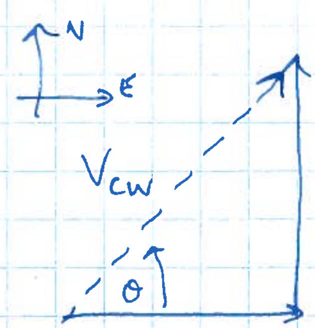
$\vec{V}_{CW} = \vec{V}_{CB} + \vec{V}_{BW}$

$= -3.0\,m/s + 4.0\,m/s = 1.0\,m/s$ [N]

c) $\vec{V}_{CB} = 3.0\,m/s$ [E]

$\vec{V}_{BW} = 4.0\,m/s$ [N]

$\vec{V}_{CW} = ?$



$V_{BW} = 4.0\,m/s$

$V_{CB} = 3.0\,m/s$

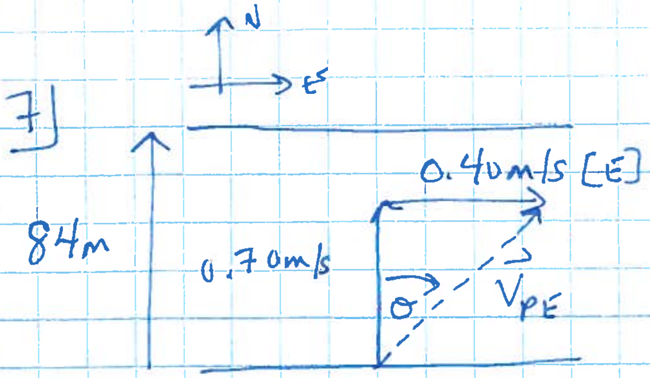
$V_{CW} = 5.0\,m/s$ (3-4-5 Triangle)

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°$

analysis: $\vec{V}_{CW} = \vec{V}_{CB} + \vec{V}_{BW}$

$\therefore \boxed{\vec{V}_{CW} = 5.0\,m/s\ [E\ 53°\ N]}$

7)



P-person, W-water, E-Earth

$\vec{V}_{WE} = 0.40\,m/s\ [E]$

$\vec{V}_{PW} = 0.70\,m/s\ [N]$

$\vec{V}_{PE} = ?$

Analysis: $\vec{V}_{PE} = \vec{V}_{PW} + \vec{V}_{WE}$

a) $V_{PE} = \sqrt{0.40^2 + 0.70^2}$
$= 0.81\ m/s$

$\theta = \tan^{-1}\left(\frac{0.40}{0.70}\right) = 29.7°$

$\therefore \vec{V}_{PE} = 0.81\ m/s\ [N30.°E]$.

b) $\Delta d_{NORTH} = 84m$

$V_{NORTH} = \vec{V}_{PW} = 0.70m/s\ [N]$

$\Delta t = ?$

$\Delta t = \frac{\Delta d_{NORTH}}{\vec{V}_{NORTH}} = \frac{84m}{0.70m/s} = 120.s = 2.0\,min$

c) $\Delta d_{East} = ?$

$V_{EAST} = \vec{V}_{WE} = 0.40m/s\ [E]$

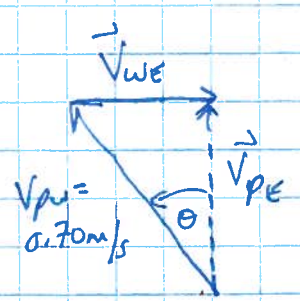
$\Delta t = 120.s$

$\Delta \vec{d}_{East} = (\vec{V}_{EAST})(\Delta t) = (0.40m/s\ [E])(120.s)$
$= 48.m\ [E]$

d) $\vec{V}_{WE} = 0.40m/s\ [E]$

$\vec{V}_{PW} = 0.70m/s\ [?]$

$\vec{V}_{PE} = ?\ [N]$



$\theta = \sin^{-1}\left(\frac{0.40}{0.70}\right) = 34.8°$

$\therefore$ they head [N 35° W].

* They maintain the same speed relative to the water but now head northwest to compensate for the current.

8] Let downstream = +

W = Water E = Earth U = upstream person D = downstream person

$|\vec{V}_{UW}| = |\vec{V}_{DW}| = V$, $\vec{V}_{UE} = -1.2\,m/s$, $\vec{V}_{DE} = +2.9\,m/s$

$\vec{V}_{UE} = \vec{V}_{UW} + \vec{V}_{WE}$ $\vec{V}_{DE} = \vec{V}_{DW} + \vec{V}_{WE}$

$-1.2\,m/s = -V + V_{WE}$ — ① $2.9\,m/s = V + V_{WE}$ — ②

Add the two equations to solve for V_{WE}:

$$\begin{aligned} -1.2\,m/s &= -V + V_{WE} \\ +\ 2.9\,m/s &= V + V_{WE} \\ \hline 1.7\,m/s &= 2V_{WE} \end{aligned}$$

$\therefore V_{WE} = 0.85\,m/s$ [Downstream]

b) Sub result into ② to solve for V_{DW}:

$$\begin{aligned} \vec{V}_{DW} &= \vec{V}_{DE} - \vec{V}_{WE} \\ &= 2.9\,m/s - 0.85\,m/s \\ &= 2.05\,m/s \\ &\simeq 2.0\,m/s \end{aligned}$$

Relative Velocity Part II

Pg 48 P4, 6, 9   Pg 49 # 6, 9

P4) Let P = plane, W = wind, G = ground

$\Delta\vec{d}_{PG} = 450\ km\ [S]$

$\Delta t = 3.0\ h$

$\vec{V}_{PG} = ?$

$\vec{V}_{WG} = 50.0\ km/h\ [E]$

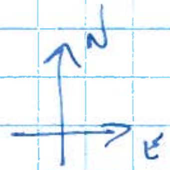
$\vec{V}_{PW} = ?$

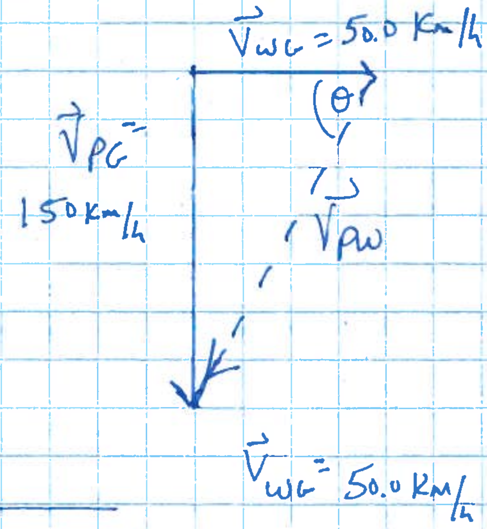
<u>Analysis</u>:

$$\boxed{\vec{V}_{PG} = \vec{V}_{PW} + \vec{V}_{WG}} \Rightarrow \boxed{\vec{V}_{PW} = \vec{V}_{PG} - \vec{V}_{WG}}$$

$$\vec{V}_{PG} = \frac{\Delta\vec{d}_{PG}}{\Delta t} = \frac{450\ km\ [S]}{3.0\ h} = 150.\ \frac{km}{h}\ [S]$$

Use vector subtraction:





$$V_{PW} = \sqrt{150^2 + 50^2}$$
$$= 158.1\ km/h$$

$$\theta = \tan^{-1}\left(\frac{150}{50}\right) = 71.6^\circ$$

$$\therefore \boxed{\vec{V}_{PW} = 1.6 \times 10^2\ \frac{km}{h}\ [W\,72^\circ S]}$$

Pb) Let P = Plane A = Air G = ground

$\vec{V}_{PA} = 350\ km/h\ [N35°W]$

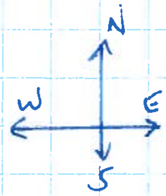
2/5

$\vec{V}_{AG} = 62\ km/h\ [S]$

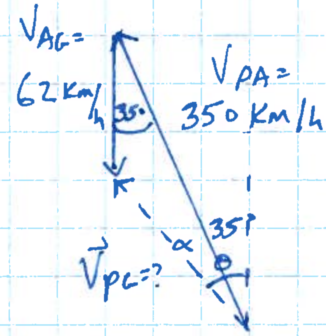
$\vec{V}_{PG} = ?$

$\Delta t = 1.2h$

$\Delta\vec{d}_{PG} = ?$



Analysis: $\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$



$$V_{PG} = \sqrt{62^2 + 350^2 - 2(62)(350)\cos 35°}$$

$$= 301.3\ km/h$$

$$\frac{\sin\alpha}{62} = \frac{\sin 35}{301.3} \rightarrow \alpha = \sin^{-1}\left(\frac{62 \cdot \sin 35}{301.3}\right) = \underline{\underline{6.78°}} \qquad \theta = 35° + 6.78° \doteq 42°$$

$$\therefore\ \vec{V}_{PG} = \underline{3.0 \times 10^2\ km/h\ [N42°W]}$$

b) $\Delta\vec{d}_{PG} = (\vec{V}_{PG})(\Delta t)$

$$= (3.0 \times 10^2\ km/h)(1.2h)$$

$$= 3.6 \times 10^2\ km\ [N42°W].$$

P9] P-plane, A-air G-ground

a) $\vec{V}_{PA} = 630\ Km/h\ [N]$

$\vec{V}_{AG} = 35\ Km/h\ [S]$

$\Delta \vec{d}_{PG} = 750\ Km/h\ [N]$

$\Delta t = ?$

$\vec{V}_{PG} = ?$

$\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$ let N=+

a) $= 630\frac{Km}{h} - 35\frac{Km}{h} = 595\frac{Km}{h}\ [N]$

$\Delta t = \frac{\Delta \vec{d}_{PG}}{\vec{V}_{PG}} = \frac{750\ Km\ [N]}{595\frac{Km}{h}\ [N]} = 1.26h = \underline{\underline{1.3\ h}}$

b) $\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$

$= 630\frac{Km}{h} + 35\frac{Km}{h} = 665\frac{Km}{h}\ [N]$

$\Delta t = \frac{750\ Km\ [N]}{665\frac{Km}{h}\ [N]} = 1.13h \doteq \underline{\underline{1.1h}}$

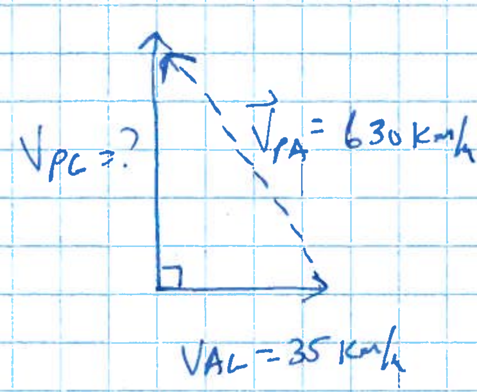
c) $\vec{V}_{AG} = 35 Km/h\ [E]$

$\vec{V}_{PG} = ?\ [N]$

$\vec{V}_{PA} = 630\frac{Km}{h}\ [?]$

analysis: $\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$

$\vec{V}_{PA} = \vec{V}_{PG} - \vec{V}_{AG}$



$V_{PG} = \sqrt{630^2 - 35^2} = 629.0\ Km/h$

$\therefore \Delta t = \frac{\Delta d_{PG}}{V_{PG}} = \frac{750\ Km}{629\ Km/h} = 1.19h = \underline{\underline{1.2\ h}}$

#6] P = plane, W = wind, G = ground

$\Delta\vec{d}_{PG} = 220\text{ Km [N]}$

$\vec{V}_{WG} = 42\text{ Km/h [N 36° E]}$

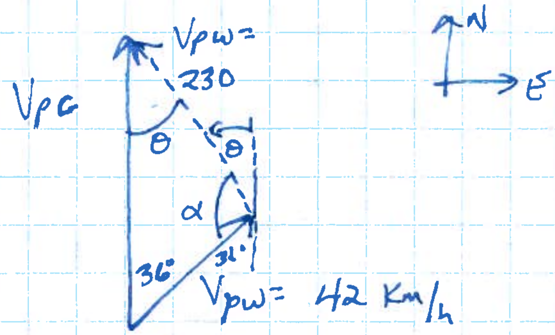
$\vec{V}_{PW} = 230\text{ Km/h [?]}$

$\vec{V}_{PG} = \text{ ? [N]}$

$\Delta t = ?$

Analysis: $\vec{V}_{PG} = \vec{V}_{PW} + \vec{V}_{WG}$

$\vec{V}_{PW} = \vec{V}_{PG} - \vec{V}_{WG}$



a)

Solve for θ, the plane heading, using sin law:

$$\frac{\sin\theta}{42} = \frac{\sin 36}{230} \rightarrow \theta = \sin^{-1}\left(\frac{42\cdot\sin 36}{230}\right) = \underline{6.16°}$$

∴ the plane heading is [N 6.2° W]

b) Apply sine law again to solve for V_{PG}:

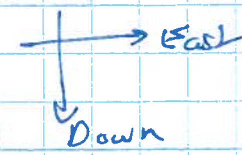
$\alpha = 180° - 36° - 6.16° = \underline{137.8°}$

$$\therefore \frac{V_{PG}}{\sin 137.8°} = \frac{230}{\sin 36°} \rightarrow V_{PG} = \frac{(\sin 137.8°)(230)}{\sin 36°} = 262.6\text{ Km/h}$$

$$\therefore \Delta t = \frac{\Delta d_{PG}}{V_{PG}} = \frac{220\text{ Km}}{262.6\text{ Km/h}} = 0.838\text{ h} \simeq \underline{0.84\text{ h}}$$

pg 49

#9] C - car, R - rain, E - Earth



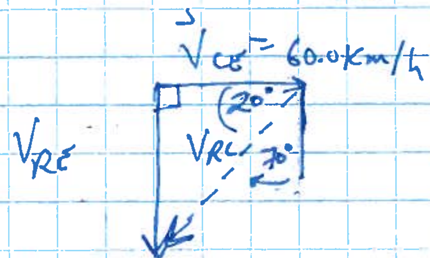
$\vec{V}_{CE} = 60.0\,km/h$ [E]

$\vec{V}_{RE} = ?$ [Down]

$\vec{V}_{RC} = ?$ [D70°W]

$\vec{V}_{RE} = \vec{V}_{RC} + \vec{V}_{CE}$

$\vec{V}_{RC} = \vec{V}_{RE} - \vec{V}_{CE}$



b) $\tan 30 = \dfrac{V_{RE}}{V_{CE}} \rightarrow V_{RE} = V_{CE} \cdot \tan 20° = 60.0\,km/h \tan 20° = 21.8\,km/h$

$\therefore \vec{V}_{RE} = 21.8\,km/h$ [Down]

c) $\cos 30 = \dfrac{V_{CE}}{V_{RC}} \rightarrow V_{RC} = \dfrac{V_{CE}}{\cos 20°} = \dfrac{60.0\,km/h}{\cos 20.°} = 63.9\,km/h$

$\therefore \vec{V}_{RC} = 63.9\,km/h$ [Down 70° West]