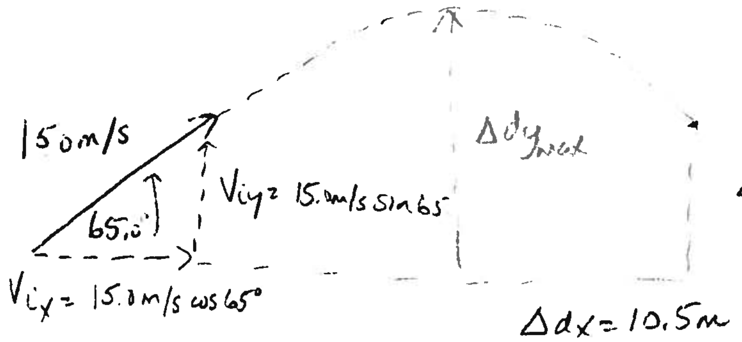
Sample problem in which the projectile lands at a specified horizontal distance (at less than maximum range) away from the launch point:

1. In yet another plot to catch the Roadrunner, Wile E. Coyote launches a rocket from level ground at an initial velocity of 15.0 m/s [65.0° above the horizontal]. The rocket rises up and on the way down, lands on top of a small hill 10.5 m away from the launch position (just missing the Roadrunner of course!). Find:
   a) The total time the rocket was in the air.
   b) The height of the hill.
   c) The final velocity of the rocket when it landed on the hill.
   d) The maximum height the rocket reached.



$a_y = g = 9.81 \, m/s^2 \, [\downarrow]$

a) $\Delta t = ?$ $\quad \Delta t = \frac{\Delta d_x}{V_{ix}} = \frac{10.5 m}{15.0 m/s \cos 65} = 1.656 s \sim 1.66 s$

b) $\Delta d_y = ?$ $\quad \Delta d_y = V_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$

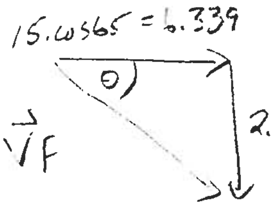
$= (15.0 \sin 65 \, m/s)(1.656 s) + \frac{1}{2}(-9.81 \, m/s^2)(1.656 s)^2$

$= 9.061 m$

$\sim 9.06 m$

c) $V_{fy} = ?$
$\vec{V}_f = ?$

$V_{fy} = V_{iy} + g \Delta t$
$= 15 \sin 65 - (9.81)(1.656) = -2.654$



$V_f = \sqrt{6.339^2 + 2.654^2} = 6.87 \, m/s$ $\quad \theta = \tan^{-1}\left(\frac{2.65}{6.34}\right) = 22.7°$

$\therefore \vec{V}_f = 6.87 \, m/s$ [22.7° below the horizontal]

d) $\Delta d_{y max} = ?$
$V_{y max} = 0$

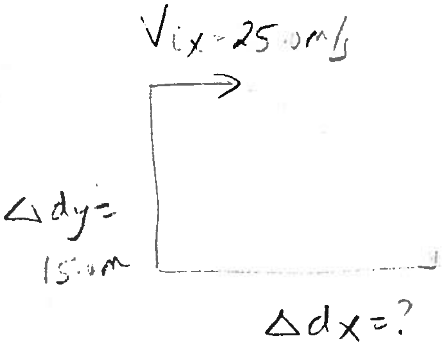
$\Delta d_{y max} = \frac{V_{y max}^2 - V_{iy}^2}{2g} = \frac{0 - (15 \sin 65)^2}{2(-9.81)} = 9.42 m$

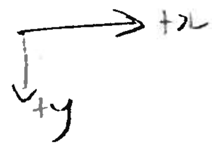
Sample problem in which the projectile is being carried by a horizontally moving object and then released:

A helicopter flying horizontally at 25.0 m/s drops a mailbag from a height of 15.0 m to a letter carrier waiting on the ground below.

a) How long will the ball take to fall to the ground?

b) How far in advance of the letter carrier must the bag be released so that it falls at her feet?





$a_y = g = 9.81\ m/s^2\ (\downarrow)$

$\Delta t = ?$

$\Delta d_x = ?$

a) $\Delta d_y = \frac{1}{2} g \Delta t^2$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{g}} = \sqrt{\frac{2(15.0\ m)}{(9.81\ m/s^2)}} = 1.7485 \sim 1.75\ s$$

b) $\Delta d_x = V_{ix} \cdot \Delta t$

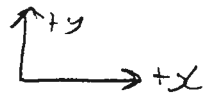
$= (25.0\ m/s)(1.7485\ s)$

$= 43.72\ m$

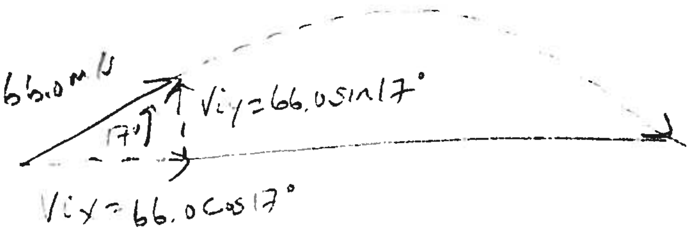
$\sim 43.7\ m$

Sample problem in which the launch velocity is known and the maximum time of flight and range must be determined:

3. A golfer strikes a golf ball off the tee with a velocity of 66.0 m/s [17° above the horizontal]. How long is the ball in the air if it lands on the green which is level with the tee? How far away is the green from the tee?



$a_y = g = -9.81\ m/s^2$



$\Delta d_y = 0$

$\Delta t = ?$

$\Delta d_x = ?$

Find time of flight:

$$\Delta d_y = V_{iy}\Delta t + \frac{1}{2} g \Delta t^2$$

$$0 = 66.0 \sin 17° \cancel{\Delta t} - 4.905 \Delta t^2 \quad (\div \Delta t)$$

$$\Delta t = \frac{66.0 \sin 17}{4.905} = 3.934\ s$$

Find range horizontally:

$$\Delta d_{max_x} = V_i \cos 17° \cdot \Delta t$$

$$= (66.0)(\cos 17)(3.934\ s)$$

$$= 248.3\ m$$

$$\sim 248\ m$$

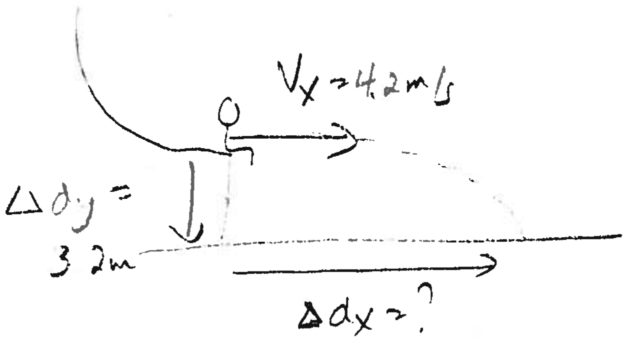
Sample problem in which the projectile is launched with initial horizontal velocity only and then undergoes projectile motion:

4. A child travels down a water slide, leaving it with a velocity of 4.2 m/s horizontally. The child then experiences projectile motion, landing in a swimming pool 3.2 m below the slide.

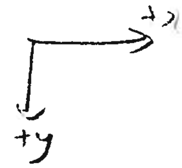
a) For how long is the child airborne?

b) Determine the child's horizontal displacement while in the air.

c) Determine the child's velocity upon entering the water.



$\Delta t = ?$



$a_y = g = 9.81\ m/s^2$ [↓]

$V_f = ?$

Solve for time in air.

a) $\Delta d_y = \frac{1}{2} g \Delta t^2$

$3.2m = \frac{1}{2}(+9.81\ m/s^2)(\Delta t^2)$

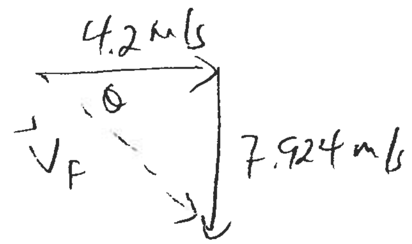
$\Delta t = \sqrt{\frac{2(3.2m)}{(9.81\ m/s^2)}} = 0.8077s \sim 0.81s$

b) $\Delta d_x = V_x \Delta t$

$= (4.2m/s)(0.81s) = 3.392m \sim 3.4m$

c) $V_{fy} = ?$ $\quad V_{fy} = g\Delta t = (9.81\ m/s^2)(0.8077s)$

$= 7.924\ m/s$



$V_f = \sqrt{(4.2)^2 + (7.924)^2} = 8.968\ m/s$

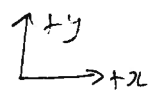
$\theta = Tan^{-1}\left(\frac{7.924}{4.2}\right) = 62.07°$

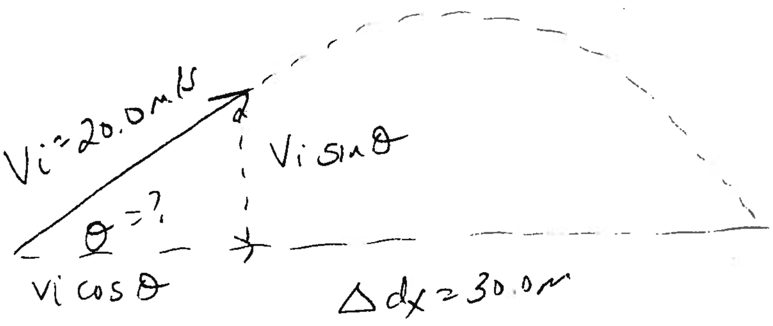
$\therefore \vec{V}_F = 9.0\ m/s$ [62° below horizontal]

Sample problem in which the launch angle is to be determined given initial launch speed, time of flight and maximum range :

5. Wile E. Coyote never gives up!! He now has an ACME projectile launcher which can launch pumpkins at a speed of 20.0 m/s. He sees Roadrunner 30.0 m away, dozing in the sun, and carefully aims his pumpkin launcher at his target. The pumpkin sails through the air landing 30.0 m away* after being in the air for 1.83 seconds. Find the launch angle that Coyote used.

   ** Of course Coyote failed again-Roadrunner was awoken by the shadow of the pumpkin and made a speedy getaway!*





$\Delta t = 1.83\,s$

$a_y = g = -9.81\,m/s^2$

Given time and horizontal distance, solve for launch angle.

$$\Delta d_x = v_x \Delta t$$

$$\Delta d_x = V_i \cos\theta \, \Delta t$$

$$\cos\theta = \frac{\Delta d_x}{V_i \Delta t}$$

$$\theta = \cos^{-1}\left(\frac{\Delta d_x}{V_i \Delta t}\right)$$

$$= \cos^{-1}\left(\frac{30.0\,m}{20.0\,m/s \cdot 1.83\,s}\right)$$

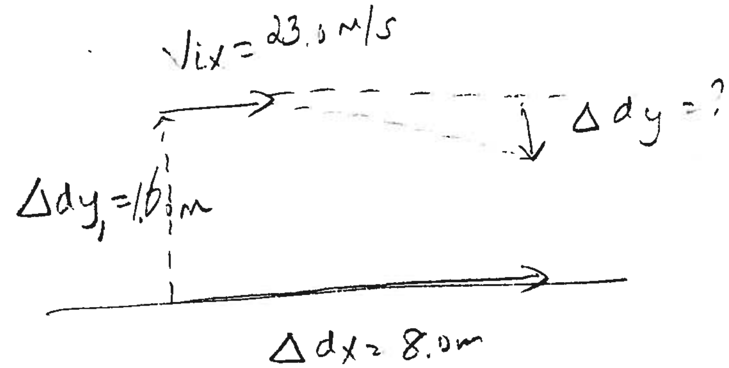
$$= \cos^{-1}(0.8197)$$

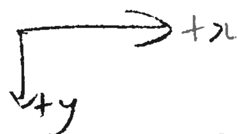
$$= 34.948^\circ$$

$$\sim 34.9^\circ$$

Sample problem in which the height at a specific time in the flight of the projectile is to be found given the initial launch velocity :

6. In a play-off match, a LASS tennis player strikes the tennis ball at a height of 1.60 m off the ground giving it an initial horizontal velocity of 23.0 m/s. If the player is standing 8.0 m from the net, determine if the ball clears the net which is 0.915 m high. (Hint: Determine how far down the ball has dropped in the time it takes to reach the net).





$a_y = g = 9.81\,m/s^2$

$\Delta t = ?$

$\Delta d_y = ?$

Use horizontal range + initial velocity to find time:

$$\Delta d_x = V_{ix} \cdot \Delta t$$

$$\Delta t = \frac{\Delta d_x}{V_{ix}} = \frac{8.0\,m}{23.0\,m/s} = 0.3478\,s$$

use time to find, Δd_y, amount ball has dropped!

$$\Delta d_y = \frac{1}{2} g \Delta t^2$$

$$= \frac{1}{2}(9.81\,m/s^2)(0.3478\,s)^2$$

$$= 0.593\,m$$

Height above ground:

$$1.60 - 0.593\,m = 1.0066\,m \approx 1.01\,m$$

$\therefore$ the ball just clears the net as $1.01\,m > 0.915\,m$ (the net height at centre tape).