Data Analysis Practice

1. Give the number of **significant digits** in each of the following measurements.

a) 12.04 m 4 b) 156.00 kg 5 c) 0.065 cm 2 d) 0.00320 ms 3

e) 129 g 3 f) 2.5×10^5 s 2 g) 6.300×10^5 km 4 h) 1070 kg 3

2. a) *State the "weakest link rule" for determining the number of significant digits in the resultant when* ***multiplying or dividing*** *measured values:*

– Keep same number of significant digits as measurement ~~with~~ with lowest significant digit count

State the "weakest link rule" for determining the precision of the resultant when ***adding or subtracting*** *measured values:*

– Keep same number of decimal places as measurement with lowest precision

b) Perform the following calculations, rounding off the answers to the appropriate number of significant digits. Include the correct units in your answer.

i) 136.06 g + 2.1 g = 138.2 g

ii) 1560.35 kg + 242 kg = 1802 Kg

iii) 8.5 m + 2.194 m – 3.45 m = 7.2 m

iv) 2.55 m ÷ 0.38 s = 6.7 m/s

v) 4.550 N × 1.2 m = 5.5 N.m

vi) 3.2 m × 1.455 m x 0.55m = $2.6\,m^3$

vii) 13.25 cm + 42.985 cm + 26.4 cm – 695 mm = 13.1 cm

viii) 467.28 g ÷ (10.6 cm x 3.7 cm x 2.75 cm) = $4.3\,g/cm^3$

3. Give the **units** that would result from each of the following calculations.

a) 15 m/s ÷ 5.0 s

$\frac{m}{s} \times \frac{1}{s} = \frac{m}{s^2}$

b) 4 kgm ÷ 6 m/s

$Kgm \times \frac{s}{m} = Kg \cdot s$

c) $5\ m/s^2 \times 4\ s^2$

$\frac{m}{\cancel{s^2}} \times \cancel{s^2} = m$

d) $5\ kgm/s^2 \div 3$ s

$\frac{Kgm}{s^2} \times \frac{1}{s} = \frac{Kgm}{s^3}$

e) $3\ N \div 2\ m^2$

$\frac{N}{m^2}$

f) $16\ kg/m^2 \div 4$ m

$\frac{Kg}{m^2} \times \frac{1}{m} = \frac{Kg}{m^3}$

4. Make the following unit conversions, using the "multiply by one" method. Keep the same number of significant digits after the conversion. Express the result in scientific notation if necessary! Show your work.

a) 3360 pm = ? nm $1 nm = 10^3 pm$

$3360\ pm \times \frac{1 nm}{10^3 pm} = 3.36\ nm$

b) 678.2 kg = ? Mg $1 Mg = 10^3 kg$ ①

② $678.2\ \cancel{kg} \times \frac{1 Mg}{10^3 \cancel{kg}} = 0.6782\ Mg$ ③

c) 0.00785 mm = ? µm $1 mm = 10^3 \mu m$

$7.85 \times 10^{-3} mm \times \frac{10^3 \mu m}{1 mm} = 7.85\ \mu m$

d) 1.5 year = ? seconds

$1.5 y \times \frac{365 day}{y} \times \frac{24 h}{day} \times \frac{3600 s}{h} = 4.7 \times 10^7 s$

e) 79 km/h = ? m/s $1 km = 1000 m$, $1 h = 3600 s$

$\frac{79 km}{h} \times \frac{1 h}{3600 s} \times \frac{1000 m}{1 km} = 21.94 \frac{m}{s} \sim 22 m/s$

f) 6.3 m/s² = ? km/h² $1 km = 1000 m$, $1 h^2 = (3600 s)^2$

$\frac{6.3 m}{s^2} \times \frac{1 km}{1000 m} \times \frac{(3600 s)^2}{1 h^2} = 8.2 \times 10^4 \frac{km}{h^2}$

g) 494 Mg = ? Gg $1 Gg = 10^3 Mg$

$494\ \cancel{Mg} \times \frac{1 Gg}{10^3 \cancel{Mg}} = 0.494\ Gg$

h) 0.000056 m³ = ? cm³ $1 m^3 = (10^2 cm)^3 = 10^6 cm^3$

$5.6 \times 10^{-5} m^3 \times \frac{10^6 cm^3}{1 m^3} = 5.6 \times 10^1 cm^3 = 56 cm^3$

5. a) *State the formula for finding* ***percent uncertainty*** *in a measurement given the* ***absolute uncertainty***:

$\%\ uncertainty = \frac{abs\ uncertainty}{measurement} \times 100\%$

b) Convert the following measurement uncertainties between absolute and percent uncertainty as indicated:

Quantity with absolute uncertainty	Quantity with percent uncertainty
24.0 ± 0.5 kg	= 24.0 kg ± 2 %
23.65 ± 0.05 s	23.65 s ± 0.2%
8.135 m ± 0.005m	8.135 m ± 0.06%
0.128 ± 0.005 g	0.128 g ± ~~[illegible]~~ 4%
4.8 ± 0.5 km	4.8 km ± 10%

Quantity with percent uncertainty	Quantity with absolute uncertainty
37.5 kg ± 2 %	37.5 ± 0.8 kg
156 m ± 1 %	156 ± 2 m
6.3 s ± 5 %	6.3 ± 0.3 s
23.6cm ± 0.5 %	23.6 ± 0.1 cm
18 g ± 3 %	18 ± 0.5 g

6.a) State the following rules for combining measurement uncertainties:

When adding or subtracting measured values we ADD ***the*** ABSOLUTE ***uncertainties.***

When multiplying or dividing measured values we ADD ***the*** RELATIVE ***uncertainties.***

b) Find the resultant for each calculation below and **determine the absolute uncertainty.**
In cases involving multiplication or division, you may need to convert to relative or percent uncertainties first.

i) $L_{total} = L_1 + L_2$, $L_1 = 5.35$ m $\pm$ 0.02 m , 9.87 m $\pm$ 0.02 m

$L_{TOTAL} = 5.35\text{m} + 9.87\text{m} \pm (0.02 + 0.02)\text{m}$
$= 15.22 \pm 0.04\text{m}$

ii) $m_{total} = m_1 + m_2 + m_3$, $m_1 = 14.3 \pm 0.2$ kg, $m_2 = 2.6 \pm 0.1$ kg, $m_3 = 7.4 \pm 0.1$ kg

$m_{TOTAL} = 14.3\text{ kg} + 2.6\text{ kg} + 7.4\text{ kg} \pm (0.2 + 0.1 + 0.1)\text{ kg}$
$= 24.3 \pm 0.4$ kg

iii) $\Delta t = t_2 - t_1$, $t_1 = 18.6 \pm 0.1$ s , $t_2 = 39.5 \pm 0.1$ s

$\Delta t = (39.5 - 18.6)\text{s} \pm (0.1 + 0.1)\text{s} = 20.9 \pm 0.2\text{s}$

iv) $A = L \times W$, $L = 1.64 \pm 0.05$ m , $W = 0.65 \pm 0.05$ m

$A = 1.64\text{m} \times 0.65\text{m}$
$= 1.0725\text{m}^2$
$\sim 1.1\text{m}^2$

$\%\text{ unc} = \left(\frac{0.05}{1.64}\right)100\% + \left(\frac{0.05}{0.65}\right)100\%$
$= 10.7\%$

$\boxed{A = 1.1 \pm 0.1\text{ m}^2}$

v) $D = m / V$, $m = 67.4$ g $\pm$ 1% , $V = 18.2\text{ cm}^3 \pm 2\%$

$D = \frac{67.4\text{g}}{18.2\text{cm}^3} \pm (1 + 2)\%$
$= 3.7 \pm 3\%$

$\boxed{D = 3.7 \pm 0.1\text{ g/cm}^3}$

vi) $v = \Delta d / \Delta t$, $\Delta d = 25.8 \pm 0.2$ m , $\Delta t = 6.3$ s $\pm$ 0.2 s

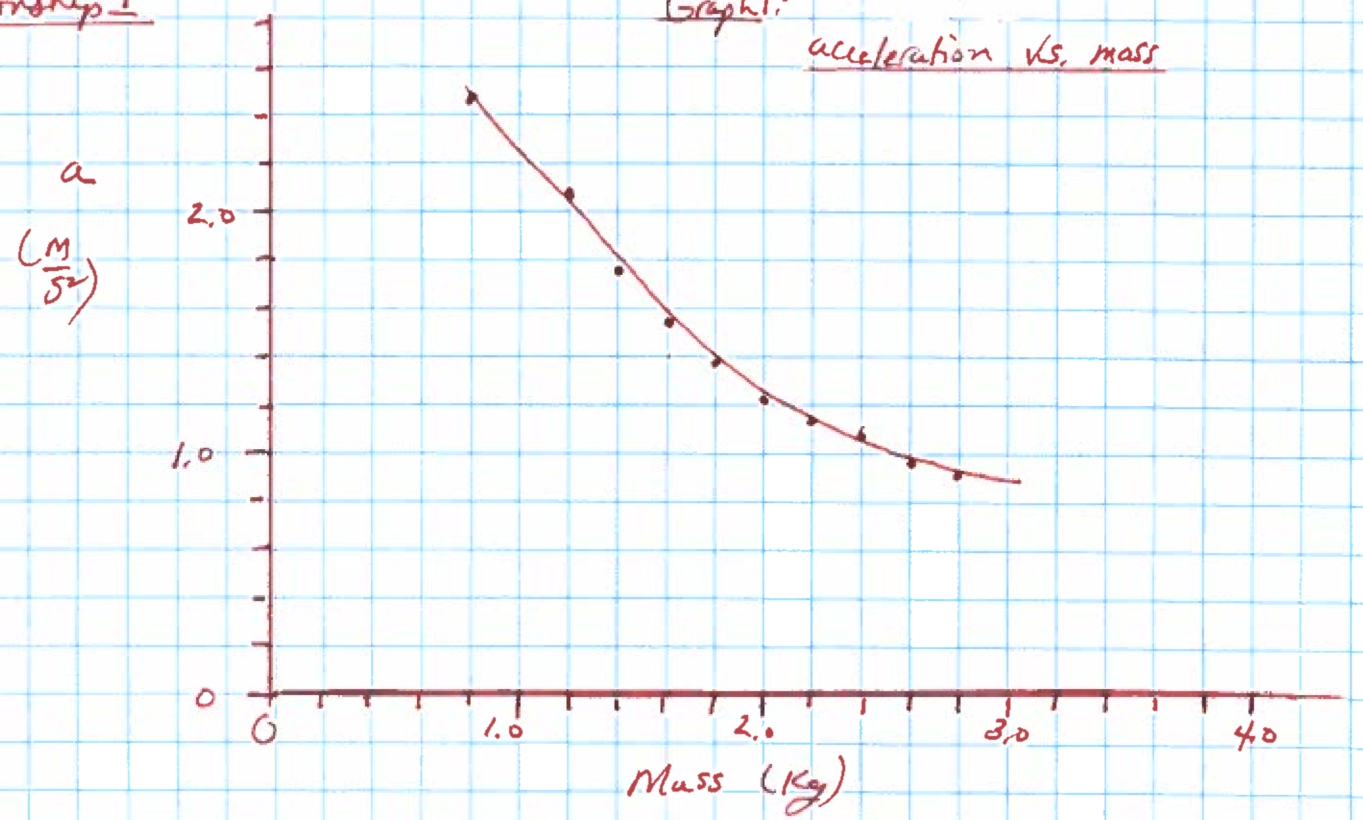
$v = \frac{25.8\text{m}}{6.3\text{s}}$
$= 4.095\text{ m/s}$

$\%\text{ unc} = \left(\frac{0.2}{25.8}\right)100\% + \left(\frac{0.2}{6.3}\right)100\%$
$= 3.95\%$

$\therefore \boxed{v = 4.1 \pm 0.2\text{ m/s}}$

GRAPHICAL ANALYSIS

Relationship 1

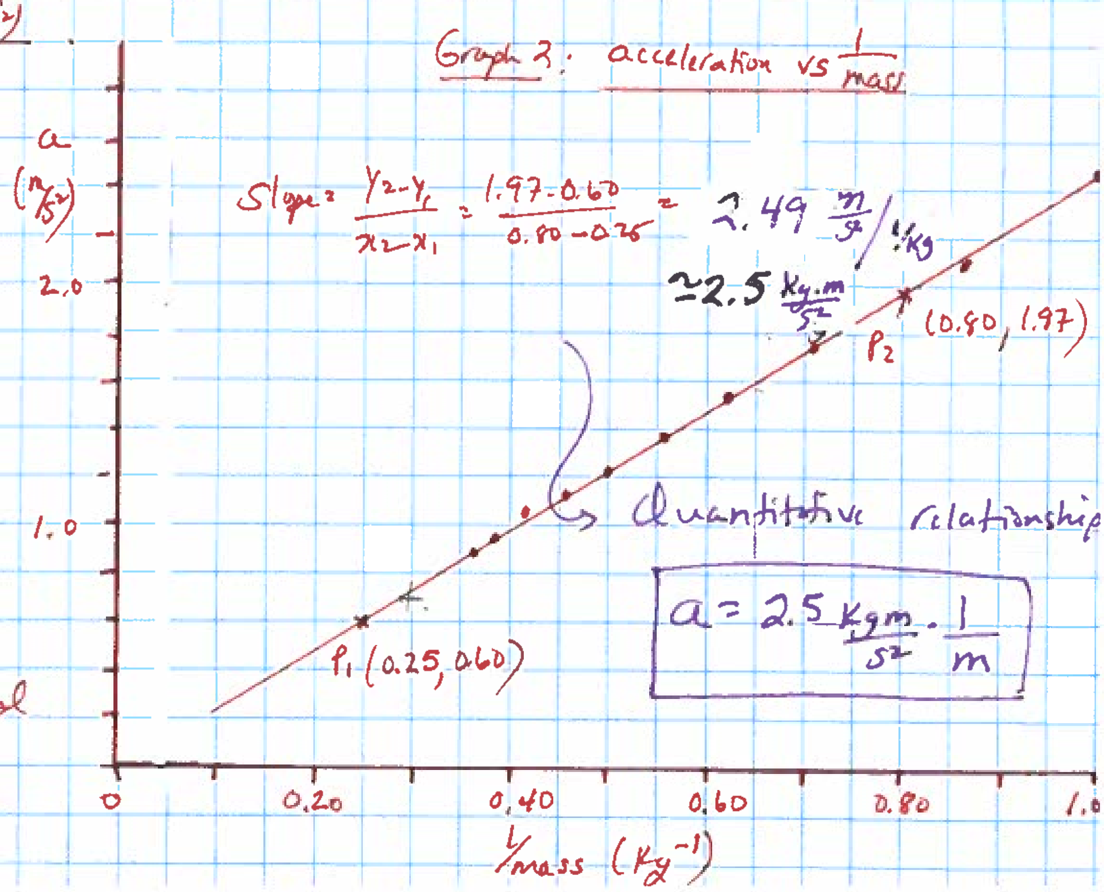


Analysis: · The curve appears to demonstrate an inverse relationship: $a \propto \frac{1}{m^n}$

· Test first if $a \propto \frac{1}{m}$ $(n=1)$; Plot a graph of a vs $1/m$.

Table 2:

Mass M (Kg)	Inverse Mass 1/m (1/Kg)	Acceleration a (m/s²)
1.00	1.00	2.46
1.20	0.833	2.08
1.40	0.714	1.76
1.60	0.625	1.56
1.80	0.556	1.39
2.00	0.500	1.22
2.20	0.455	1.13
2.40	0.417	1.05
2.60	0.385	0.94
2.80	0.357	0.89



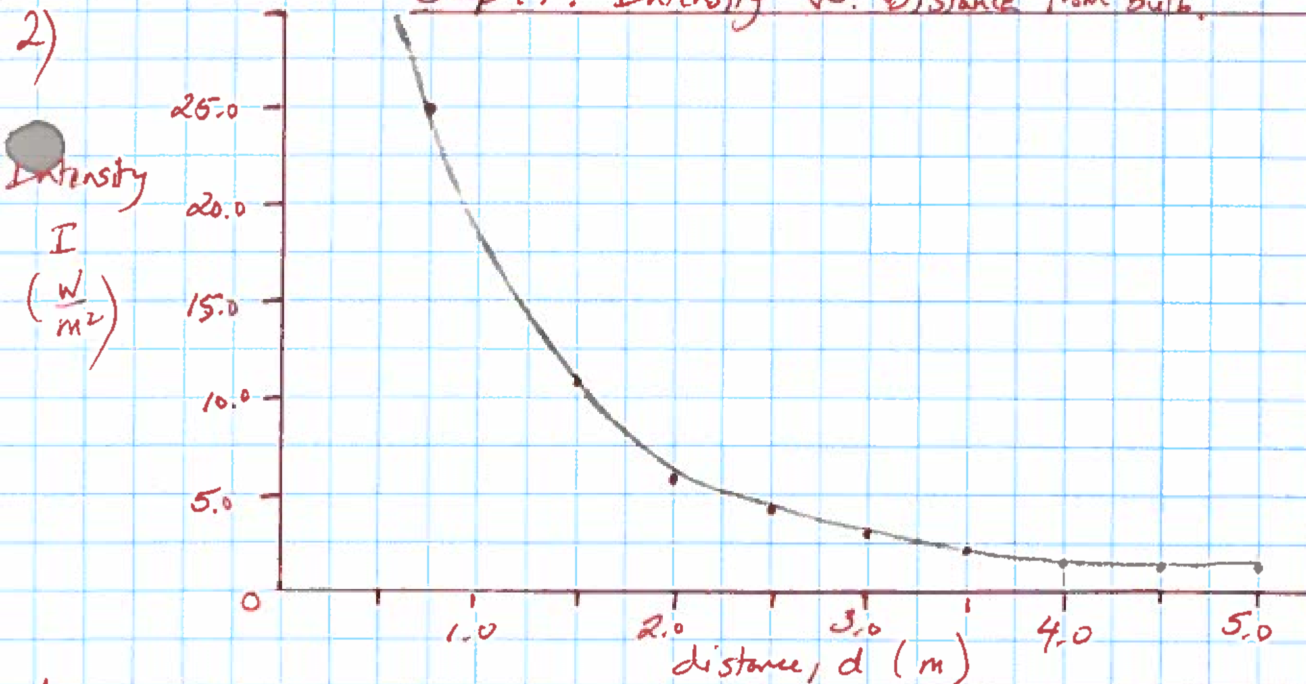
$$\boxed{a = 2.5\,\frac{Kg\,m}{s^2}\cdot\frac{1}{m}}$$

∴ acceleration is directly proportional to inverse mass

$a \propto 1/m$.

2)

Graph 1: Intensity vs. Distance from bulb



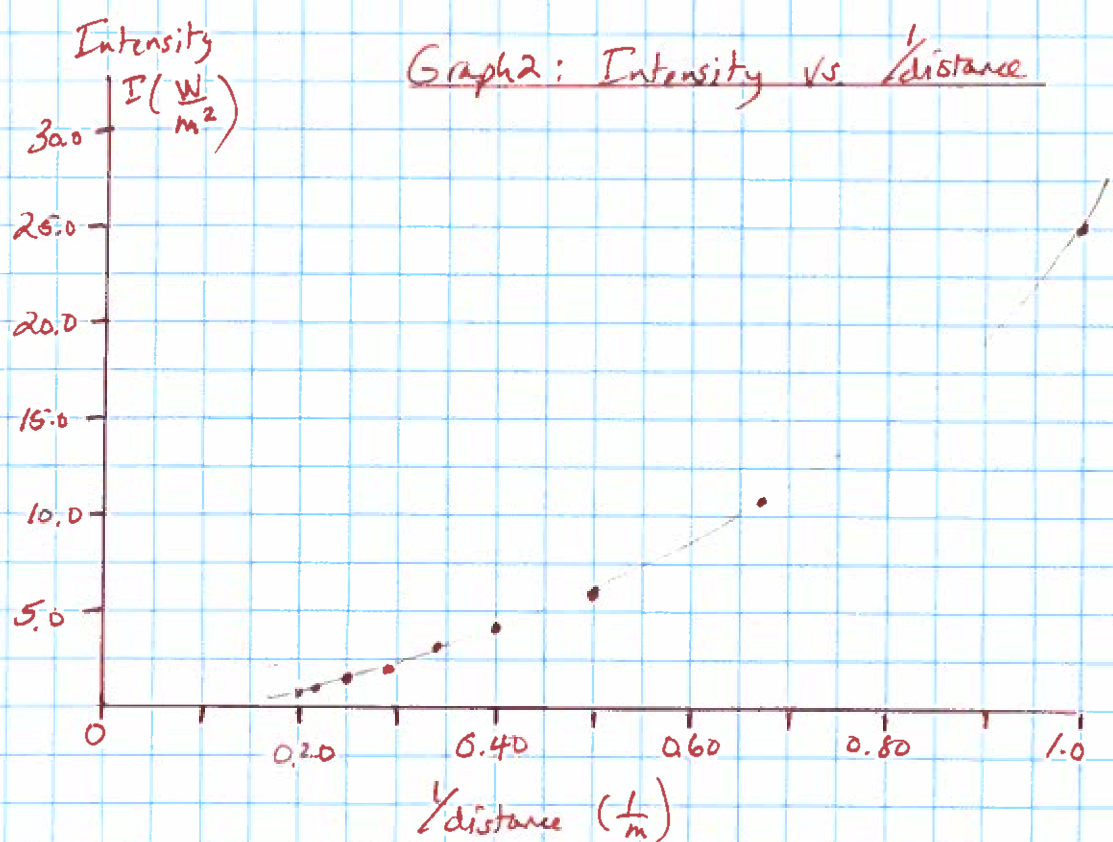
Analysis: • The curve appears to demonstrate an inverse relationship ($I \propto \frac{1}{d^n}$)

• Test first if $I \propto \frac{1}{d}$, where $n=1$. Plot a graph of I vs $\frac{1}{d}$.

Table 2:

Distance d (m)	Inverse distance $1/d$ ($1/m$)	Intensity I (W/m^2)
1.0	1.0	25.0
1.5	0.67	11.1
2.0	0.50	6.3
2.5	0.40	4.0
3.0	0.33	2.8
3.5	0.29	2.0
4.0	0.25	1.6
4.5	0.22	1.2
5.0	0.20	1.0

Graph 2: Intensity vs 1/distance



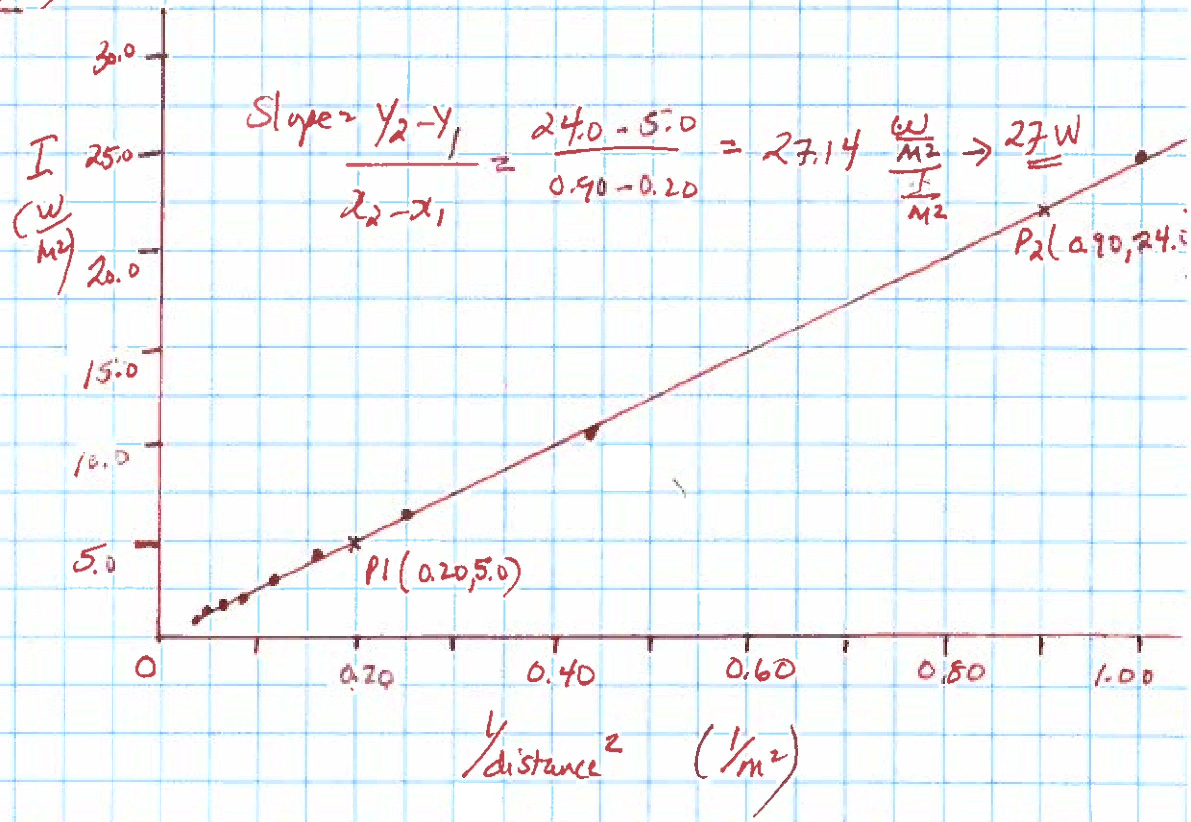
Analysis: Although this curve appears closer to a linear trend there is still a significant deviation from a straight line.

We will ~~[illegible]~~ try the next largest integer value of n ($n=2$) to test if this is an inverse square relationship.

Table 3

Distance d (m)	$1/d^2$ $(1/m^2)$	Intensity I (W/m^2)
1.0	1.0	25.0
1.5	0.44	11.1
2.0	0.25	6.3
2.5	0.16	4.0
3.0	0.11	2.8
3.5	0.082	2.0
4.0	0.062	1.6
4.5	0.049	1.2
5.0	0.040	1.0

Graph 3: Intensity vs 1/distance Squared



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{24.0 - 5.0}{0.90 - 0.20} = 27.14 \ \frac{W/M^2}{1/M^2} \rightarrow 27 W$$

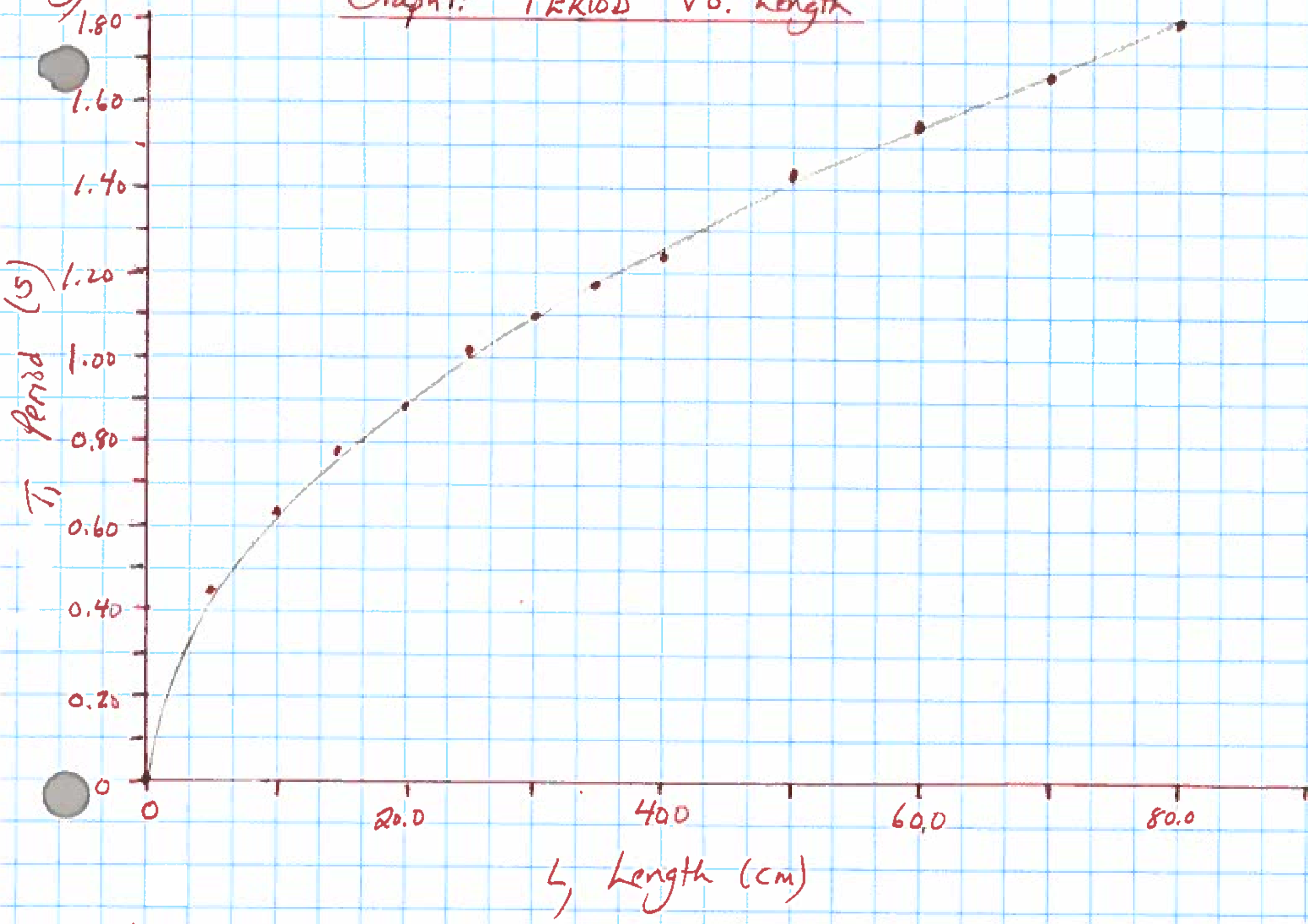
Analysis: The third graph appears linear. This verifies that intensity is proportional to the inverse square of distance.

$$I \propto \frac{1}{d^2}$$

The slope provides the constant of proportionality

$$I = (27\ W) \cdot \frac{1}{d^2}$$

3) Graph 1: PERIOD vs. Length



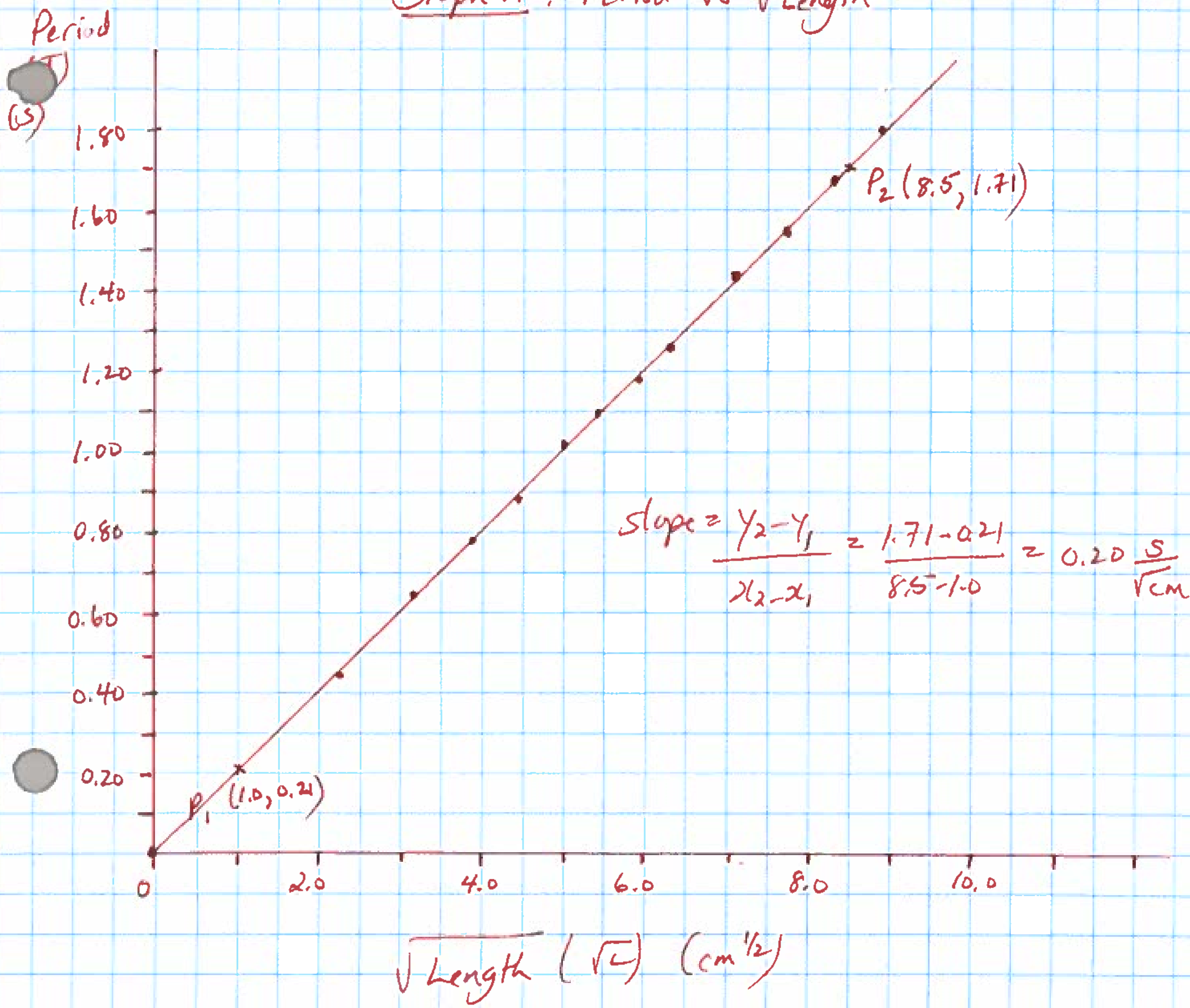
Analysis: The curve appears to demonstrate a root relationship. $(T \propto \sqrt[n]{L})$, $n > 1$

• Test first if $T \propto \sqrt{L}$ $(n = 2)$: Plot a graph of T vs $\sqrt{L}$.

Table 2:

Length (L), cm	$\sqrt{\text{Length}}$, $\sqrt{\text{cm}}$	Period (T) s
0.0	0.0	0.0
5.0	2.24	0.45
10.0	3.16	0.63
15.0	3.87	0.78
20.0	4.47	0.89
25.0	5.00	1.02
30.0	5.48	1.10
35.0	5.92	1.18
40.0	6.32	1.25
50.0	7.07	1.43
60.0	7.75	1.55
70.0	8.37	1.68
80.0	8.94	1.80

Graph 2 : Period Vs $\sqrt{\text{Length}}$



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.71 - 0.21}{8.5 - 1.0} = 0.20 \frac{s}{\sqrt{cm}}$$

<u>Analysis</u>: This graph is linear which verifies that period is directly proportional to the square root of pendulum length:

$$T \propto \sqrt{L}.$$

The constant of proportionality is determined from the slope which gives the complete relationship:

$$T = 0.20 \frac{s}{\sqrt{cm}} \sqrt{L}$$

$$\frac{2\pi}{\sqrt{g}}$$