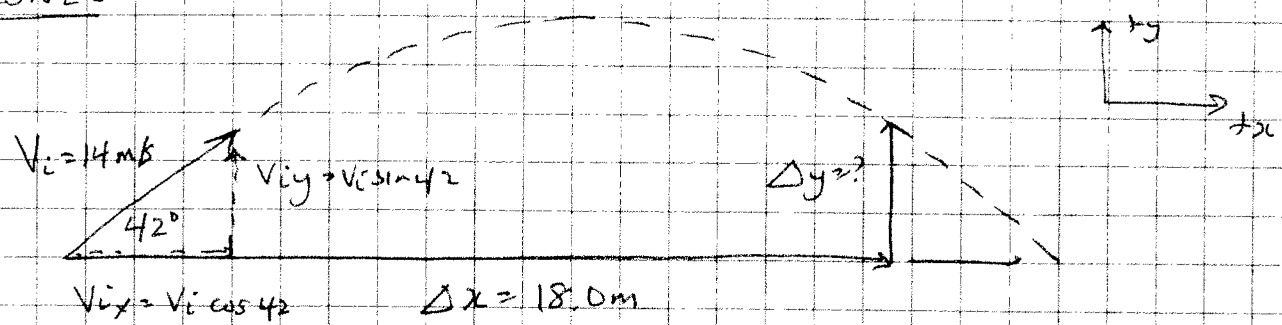
PROJECTILE MOTION OLYMPICS

BRONZE



a) Solve for time for ball to reach fence:

$\Delta x = V_{ix} \Delta t$

$\Delta t = \frac{\Delta x}{V_{ix}} = \frac{18.0\,m}{14 \cos 42°} = 1.730\ s$

Solve for time ball was in air:

at landing $\Delta y = 0$ $\quad \Delta y = V_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$

$0 = 14\,\frac{m}{s} \cdot \sin 42\ \Delta t - 4.9\,\frac{m}{s^2} \Delta t^2$

$0 = 9.368\ \Delta t - 4.9\ \Delta t^2$

$0 = \Delta t\,(9.368 - 4.9\,\Delta t)$

$\therefore \Delta t = 0$ or $9.368 - 4.9\,\Delta t = 0$

$\therefore \Delta t = 1.912\,s \sim \underline{1.9\,s}$

b) Solve for height of fence

$\Delta y = ?$
$\Delta t = 1.730\,s$

$\Delta y = V_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$

$\Delta y = 14\ m/s \cdot \sin 42\ (1.730\,s) - 4.9\,\frac{m}{s^2}(1.730\,s)^2$

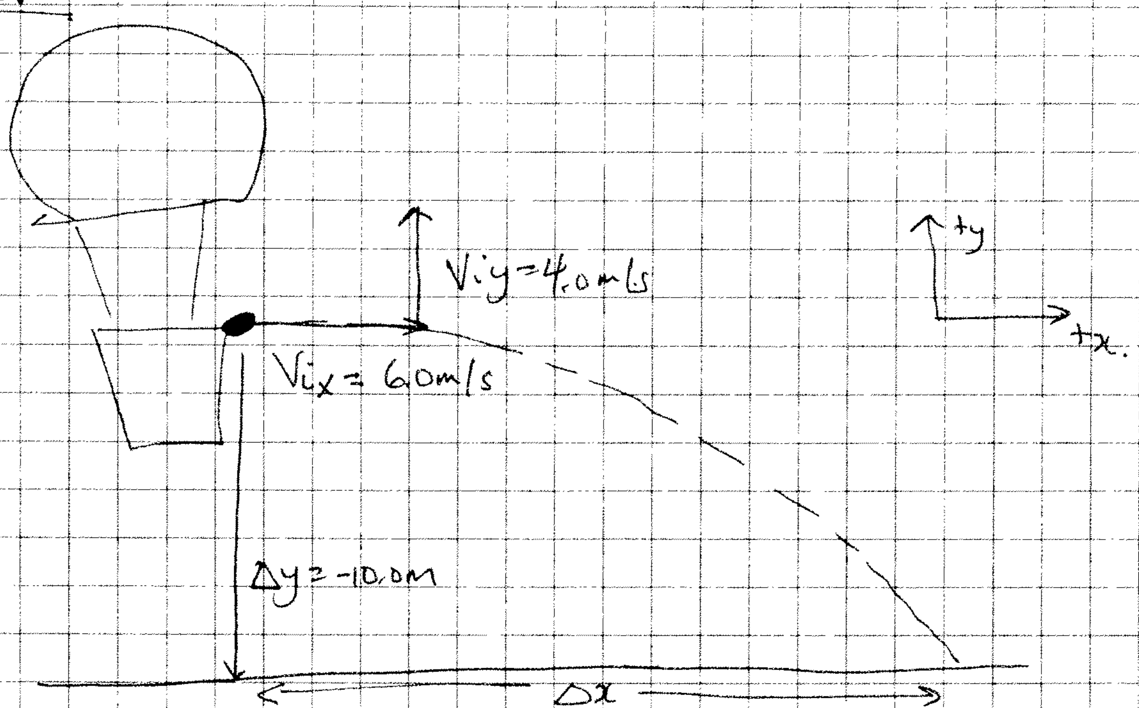
$\Delta y = 1.540\,m \sim \underline{1.5\,m}$

c) $\Delta x_{max} = V_{ix} \Delta t_{max}$

$= 14\,m/s \cdot \cos 42 \cdot (1.9125) = 19.89\,m$

$\therefore$ distance beyond fence $= 19.89\,m - 18.0 = 1.89\,m \sim \underline{\underline{1.9\,m}}$

SILVER



a) Solve for time to land

$\Delta y = -10.0\,m$
$V_{iy} = 4.0\,m/s$
$\Delta t = ?$
$a_y = -9.8\,m/s^2$

$$\Delta y = V_{iy}\Delta t + \frac{1}{2}a_y \Delta t^2$$

$$-10.0\,m = 4.0\,m/s\,\Delta t - 4.9\,\Delta t^2$$

$$\therefore 4.9\Delta t^2 - 4.0\Delta t - 10.0 = 0$$

$$a = 4.9,\quad b = -4.0,\quad c = -10.0$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4.0 \pm \sqrt{16.0 - 4(4.9)(-10.0)}}{2(4.9)}$$

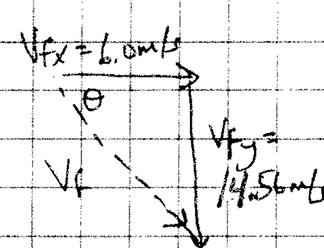
$$= \frac{4.0 \pm 14.56}{9.8} = 1.894\,s \text{ OR } \cancel{-1.078\,s}$$

b) $\Delta x = ?$

$$\Delta x = V_{ix}\Delta t$$
$$= (6.0\,m/s)(1.894\,s) = 11.364\,m \sim 11\,m.$$

c) $V_{fy} = ?$
$\vec{V_f} = ?$
$V_{iy} = 4.0\,m/s$
$V_{fx} = V_{ix} = 6.0\,m/s$

$$V_{fy} = V_{iy} + a_y\Delta t$$
$$= 4.00\,m/s - 9.8\frac{m}{s^2}(1.894\,s)$$
$$= -14.56\,m/s$$



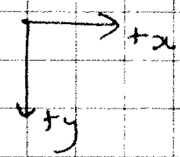
$$V_f = \sqrt{6.0^2 + 14.56^2}$$
$$= 15.7\,m/s$$

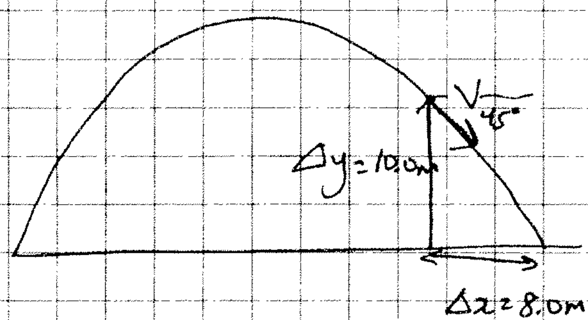
$$\theta = \tan^{-1}\left(\frac{14.56}{6.0}\right) = 67.$$

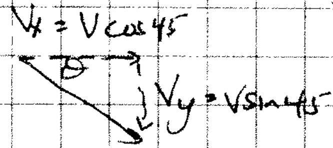
$$\therefore \vec{V_f} = 16\,m/s\ [68^\circ \text{ below horizontal}]$$

Projectile Motion. Problem Olympics

GOLD







* $\boxed{V_x = V_y = V_c = V\sin 45°}$ *

At the point shown we need to solve for V:

x dir: $\Delta x = V\cos 45\,\Delta t$

y dir: $\Delta y = V\sin 45\,\Delta t + \frac{1}{2} g \Delta t^2$

$\therefore \Delta x = V_c \Delta t$ —— ①

$\Delta y = V_c \Delta t + \frac{1}{2} g \Delta t^2$ —— ②

$\Delta t = \Delta x / V_c$

Rearrange ① to solve for Δt and substitute in ②:

$$\therefore \Delta y = V_c\left(\frac{\Delta x}{V_c}\right) + \frac{1}{2} g \left(\frac{\Delta x}{V_c}\right)^2$$

$$\therefore 10.0\,m = 8.0\,m + \frac{1}{2}\left(9.8\,\frac{m}{s^2}\right)\frac{(8.0\,m)^2}{V_c^2}$$

$$2.0\,m = \frac{313.6\ m^3/s^2}{V_c^2}$$

$$V_c^2 = \frac{313.6\ m^3/s^2}{2.0\,m}$$

$$V_c^2 = 156.8\ m^2/s^2$$

$$V_c = 12.52\ m/s$$

Solve for time for ball to land from given point:

$$\Delta t = \frac{\Delta x}{V_c} = \frac{8.0\,m}{12.52\,m/s} = 0.6389\,s$$

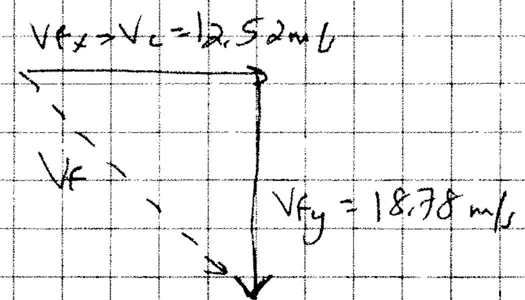
Solve for V_{fy} at landing:

$$V_{fy} = V_c + g\,\Delta t$$

$$V_{fy} = 12.52\,m/s + 9.8\frac{m}{s^2}(0.6389\,s) = 18.78\,m/s$$

Solve for final velocity:

$\vec{V}_f = ?$



$$V_f = \sqrt{12.52^2 + 18.78^2}$$

$$\boxed{V_f = 22.6\,m/s}$$

By symmetry, the magnitude of the initial launch velocity is the same as the final landing velocity.