

SPH4U0

Acceleration Problems

1. Determine the vector operation to solve the following questions and sketch the resultant vectors.

- a) A large cruise boat moving at 5.00 km/h [NE] speeds up and turns over a 15.00 minute time interval to reach a new velocity of 8.00 km/h [S]. Find the acceleration of the boat.

$$\vec{v}_1 = 5.00 \text{ km/h [NE]}$$

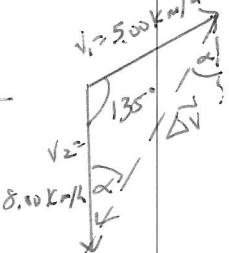
$$\vec{v}_2 = 8.00 \text{ km/h [S]}$$

$$\Delta\vec{v} = ?$$

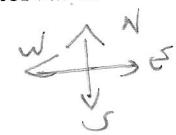
$$\Delta t = 0.250 \text{ h}, \vec{a} = ?$$

Analysis:

$$\vec{\Delta v} = \vec{v}_2 - \vec{v}_1$$



$$\Delta v^2 = \sqrt{5.00^2 + 8.00^2 - 2(5.00)(8.00) \cos 135^\circ} = 12.07 \text{ km/h}$$



$$\frac{\sin 135^\circ}{5.00} = \frac{\sin 135^\circ}{12.07} \rightarrow \alpha = 17^\circ$$

$$\vec{a} = \frac{\vec{\Delta v}}{\Delta t} = \frac{12.07 \text{ km/h}}{0.250 \text{ h}} = 48.3 \text{ km/h} [51.7^\circ \text{ W}]$$

- b) A jogger running at 5.0 m/s [N] accelerates at 0.25 m/s^2 [W] for 10.0 seconds. What is her final velocity?

$$\vec{v}_1 = 5.0 \text{ m/s [N]}$$

$$\vec{a} = 0.25 \text{ m/s}^2 [\text{W}]$$

$$\Delta t = 10.0 \text{ s}$$

$$\vec{\Delta v} = \vec{a} \Delta t = 2.50 \text{ m/s [W]}$$

$$\vec{v}_2 = ?$$

Analysis:

$$\vec{v}_2 = \vec{v}_1 + \vec{\Delta v}$$

$$\vec{\Delta v} = 2.50 \text{ m/s}$$

$$\vec{v}_1 = 5.0 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{2.50}{5.00}\right) = 26.6^\circ$$



$$v_2 = \sqrt{5.00^2 + 2.50^2} = 5.59 \text{ m/s}$$

$$\therefore \vec{v}_2 = 5.6 \text{ m/s [N}26.6^\circ\text{ W}]$$

- c) A remote controlled car reaches a final velocity of 2.00 m/s [N20.0°E] after accelerating for 5.00 seconds at 0.850 m/s^2 [East]. What was its original velocity?

$$\vec{v}_2 = 2.00 \text{ m/s [N}20.0^\circ\text{ E}]$$

$$\vec{a} = 0.850 \text{ m/s}^2 [\text{E}]$$

$$\Delta t = 5.00 \text{ s}$$

$$\vec{\Delta v} = \vec{a} \Delta t = 4.25 \text{ m/s [E]}$$

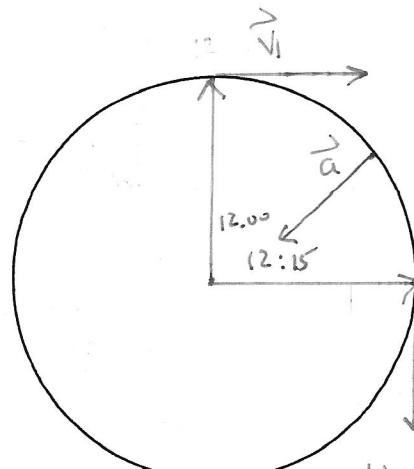
$$\vec{v}_1 = ? \quad \text{analysis: } \vec{v}_1 = \vec{v}_2 - \vec{\Delta v}$$

$$v_{1x} = v_{2x} - \Delta v_x \\ = 2.00 \sin 20^\circ - 4.25 \\ = -3.57 \text{ m/s}$$

$$v_{1y} = v_{2y} - \Delta v_y \\ = 2.00 \cos 20^\circ - 0 \\ = 1.88 \text{ m/s}$$

2. Clock Acceleration Vectors: The minute hand of a clock is 5.00 cm long.

- a) Sketch the positions of the minute hand on the clock below at times 12:00 pm and 12:15 pm.
 b) Calculate the instantaneous speed of minute hand (it takes one hour to go fully around) and sketch in the instantaneous velocity vectors at 12 and 12:15 pm.
 c) Find the average acceleration of the tip of the minute hand over the given time interval using vector subtraction. First find Δv by translating v_1 and v_2 and then calculate the value of the acceleration. Sketch this acceleration vector at the midpoint between 12 and 12:15 pm. How does the acceleration vector point? \rightarrow Radially inward.
 d) Calculate the instantaneous acceleration of the tip of the minute hand using the formula for centripetal acceleration.
 e) Compare the instantaneous acceleration value to the average acceleration you found in part (c).



$$b) v = \frac{2\pi r}{T} = \frac{2\pi (5.00 \text{ cm})}{60.0 \text{ min}} = 0.524 \text{ m/min}$$

$$v_1 = 0.524 \text{ m/min}$$

$$v_2 = 0.524 \text{ m/min}$$

$$\Delta v = 0.740 \text{ m/min}$$

$$\Delta v = \sqrt{0.524^2 + 0.524^2} = 0.740 \text{ m/min}$$

$$\vec{a} = \frac{\vec{\Delta v}}{\Delta t} = \frac{0.740 \text{ m/min}}{15 \text{ min}} = 0.049 \text{ m/min}^2$$

$$d) a = \frac{v^2}{r} = \frac{(0.524 \text{ m/min})^2}{5.00 \text{ cm}} = 0.055 \text{ m/min}^2$$

c) Continued

Diagram: A right-angled triangle representing a vector decomposition. The vertical leg is labeled $+1.88 \text{ m/s}$, the horizontal leg is labeled -3.57 m/s , and the hypotenuse is labeled \vec{V}_i . The angle between the horizontal leg and the hypotenuse is labeled θ .

$$\begin{aligned} V_i &= \sqrt{3.57^2 + 1.88^2} \\ &\approx 4.03 \text{ m/s} \\ \theta &= \tan^{-1}\left(\frac{1.88}{3.57}\right) = 27.8^\circ \end{aligned}$$

$$\therefore \vec{V}_i = 4.03 \text{ m/s [W} 27.8^\circ \text{ N]}$$