

**Acceleration Problems**

1. Determine the vector operation to solve the following questions and sketch the resultant vectors.

- a) A large cruise boat moving at 5.00 km/h [NE] speeds up and turns over a 15.00 minute time interval to reach a new velocity of 8.00 km/h [S]. Find the acceleration of the boat.

$$\vec{v}_1 = 5.00 \text{ km/h [NE]}$$

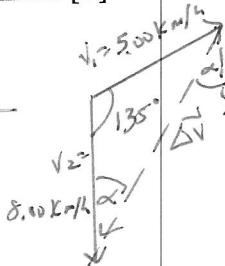
$$\vec{v}_2 = 8.00 \text{ km/h [S]}$$

$$\Delta \vec{v} = ?$$

$$\Delta t = 0.250 \text{ h}, \vec{a} = ?$$

Analysis:

$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$



$$\Delta v = \sqrt{5.00^2 + 8.00^2 - 2(5.00)(8.00)\cos 135^\circ}$$

$$= 12.07 \text{ km/h}$$

$$\sin \alpha = \frac{9.12135^\circ}{12.07} \rightarrow \alpha = 17^\circ$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{12.07 \text{ km/h}}{0.250 \text{ h}} = 48.3 \text{ km/h}^2$$

$$[517^\circ \text{W}]$$

- b) A jogger running at 5.0 m/s [N] accelerates at 0.25 m/s<sup>2</sup> [W] for 10.0 seconds. What is her final velocity?

$$\vec{v}_1 = 5.0 \text{ m/s [N]}$$

$$\vec{a} = 0.25 \text{ m/s}^2 \text{ [W]}$$

$$\Delta t = 10.0 \text{ s}$$

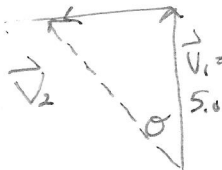
$$\Delta \vec{v} = \vec{a} \Delta t = 2.50 \text{ m/s [W]}$$

$$\vec{v}_2 = ?$$

Analysis:

$$\vec{v}_2 = \vec{v}_1 + \Delta \vec{v}$$

$$\Delta \vec{v} = 2.50 \text{ m/s [W]}$$



$$v_2 = \sqrt{5.00^2 + 2.50^2} = 5.59 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{2.50}{5.00}\right) = 26.6^\circ$$

$$\vec{v}_2 = 5.6 \text{ m/s [N} 27^\circ \text{W]}$$

- c) A remote controlled car reaches a final velocity of 2.00 m/s [N20.0°E] after accelerating for 5.00 seconds at 0.850 m/s<sup>2</sup> [East]. What was its original velocity?

$$\vec{v}_2 = 2.00 \text{ m/s [N} 20.0^\circ \text{E]}$$

$$\vec{a} = 0.850 \text{ m/s}^2 \text{ [E]}$$

$$\Delta t = 5.00 \text{ s}$$

$$\Delta \vec{v} = \vec{a} \Delta t = 4.25 \text{ m/s [E]}$$

$$\vec{v}_1 = ? \text{ analysis: } \vec{v}_1 = \vec{v}_2 - \Delta \vec{v}$$

$$v_{1x} = v_{2x} - \Delta v_x$$

$$= 2.00 \sin 20^\circ - 4.25$$

$$= -3.57 \text{ m/s}$$

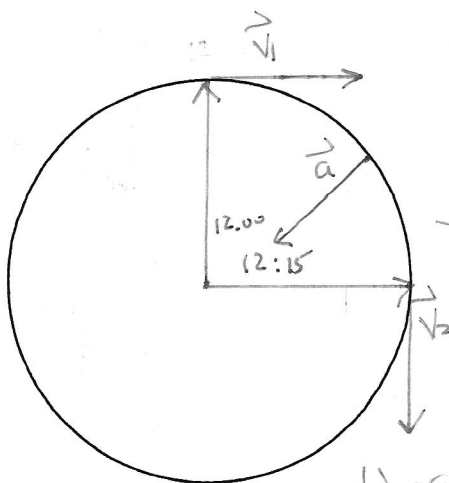
$$v_{1y} = v_{2y} - \Delta v_y$$

$$= 2.00 \cos 20^\circ - 0$$

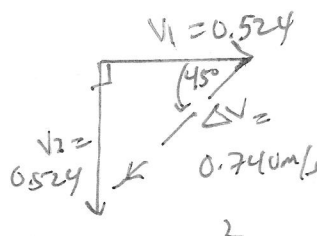
$$= 1.88 \text{ m/s}$$

2. **Clock Acceleration Vectors:** The minute hand of a clock is 5.00 cm long.

- Sketch the positions of the minute hand on the clock below at times 12:00 pm and 12:15 pm.
- Calculate the instantaneous speed of minute hand (it takes one hour to go fully around) and sketch in the instantaneous velocity vectors at 12 and 12:15 pm.
- Find the average acceleration of the tip of the minute hand over the given time interval using vector subtraction. First find  $\Delta \vec{v}$  by translating  $\vec{v}_1$  and  $\vec{v}_2$  and then calculate the value of the acceleration. Sketch this acceleration vector at the midpoint between 12 and 12:15 pm. How does the acceleration vector point?  $\rightarrow$  Radially inward.
- Calculate the instantaneous acceleration of the tip of the minute hand using the formula for centripetal acceleration.
- Compare the instantaneous acceleration value to the average acceleration you found in part (c).



$$b) v = \frac{2\pi r}{T} = \frac{2\pi(5.00 \text{ cm})}{60.0 \text{ min}} = 0.524 \text{ m/min}$$



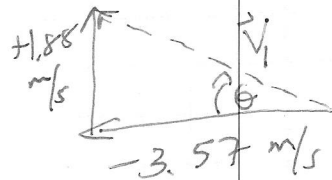
$$\Delta v = \sqrt{0.524^2 + 0.524^2}$$

$$= 0.740 \text{ m/min}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{0.740 \text{ m/min}}{15 \text{ min}} = 0.049 \text{ m/min}^2$$

$$d) a = \frac{v^2}{r} = \frac{(0.524 \text{ m/min})^2}{5.00 \text{ cm}} = 0.055 \text{ m/min}^2$$

1c) Continued



$$V_1 = \sqrt{3.57^2 + 1.88^2} \\ = 4.03 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{1.88}{3.57}\right) = 27.8^\circ$$

$$\therefore \vec{V}_1 = 4.03 \text{ m/s } [W 28^\circ N]$$