Review of Vector Operations and Analyzing 2D Displacement

<u>Vectors</u>: are quantities which have both magnitude and direction.

Examples:	position -	$\vec{d}$
	displacement-	$\overrightarrow{\Delta d}$
	velocity -	$\vec{v}$
	acceleration-	$\vec{a}$

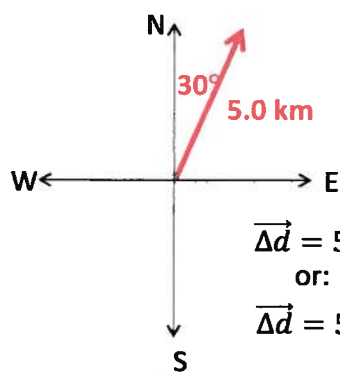
Solving kinematics problems in a plane involves adding and subtracting vectors in 2D!

$\overrightarrow{\Delta d_R} = \overrightarrow{\Delta d_1} + \overrightarrow{\Delta d_2} + \overrightarrow{\Delta d_3} + \ldots$ -finding resultant displacement

$\overrightarrow{\Delta v} = \overrightarrow{v_2} - \overrightarrow{v_2}$ - finding change in velocity to determine acceleration

Vector Notation

Examples of displacement vectors :



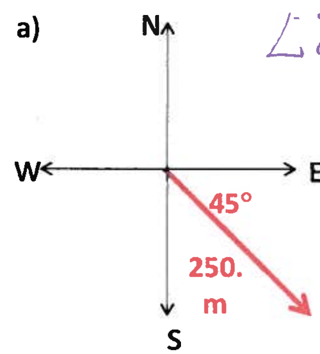
Examples of displacement vectors :

$\overrightarrow{\Delta d} = 5.0\ km[N30°E]$

or:

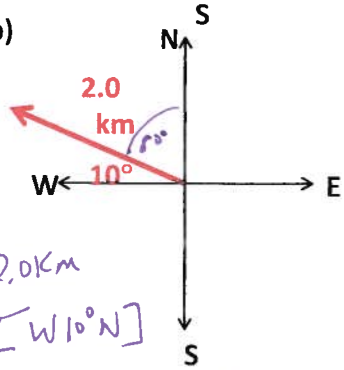
$\overrightarrow{\Delta d} = 5.0\ km[E60°N]$

a)



$\vec{\Delta d} = 250.\ m\ [SE]$

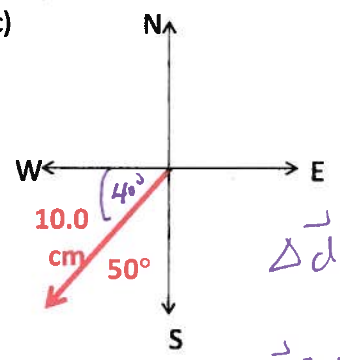
b)



$\vec{\Delta d} = 2.0\ km$ [W10°N]

or $\vec{\Delta d} = 2.0\ km$ [N 80° W]

c)



$\vec{\Delta d} = 10.0\ cm$ [S 50° W]

or

$\vec{\Delta d} = 10.0\ cm$ [W 40° S]

Drawing Scale Vector Diagrams

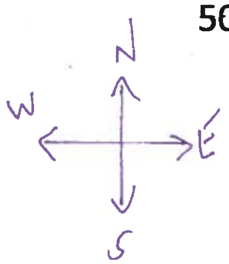
To represent position, displacement and velocity vectors in diagram form, we must choose a scale and draw a reference coordinate system.

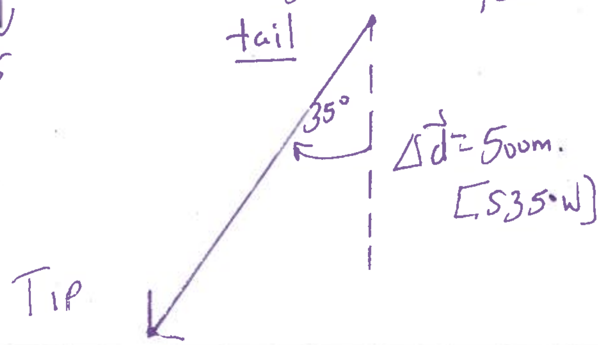
Example: Draw vectors representing

a) A displacement of 500. m [S 35°W]

Scale: 1cm = 100m

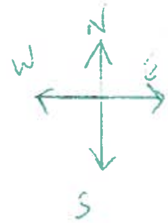
Vector length = $\frac{500m}{100m/cm}$ = 5.0cm

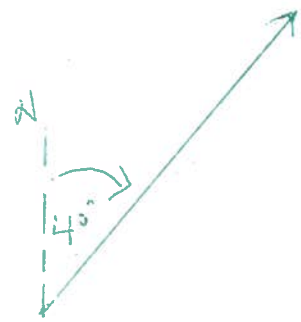




b) A velocity of 25 m/s [N40 °E]

Scale: 1cm = 5m/s



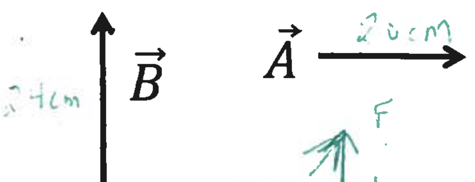


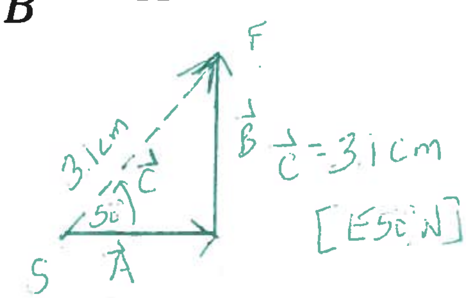
Rules for Adding and Subtracting Vectors

Vector Addition

$\vec{A} + \vec{B} = \vec{C}$

1. **Place vectors *"tail to tip"***
2. **Draw resultant from *start to finish***



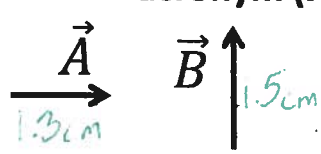


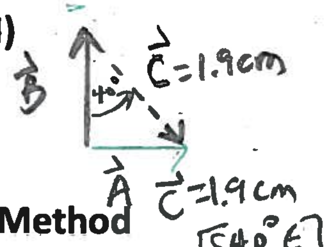
Vector Subtraction

$\vec{A} - \vec{B} = \vec{C}$

Method 1:

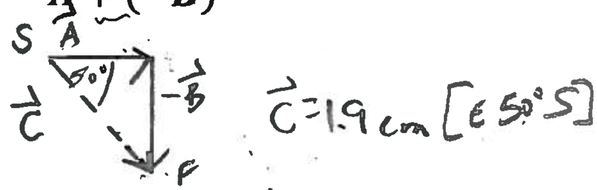
1. **Place vectors *"tail to tail"***
2. **Resultant *"points to initial"* acronym (PIN)**





Method 2: Addition Method

$\vec{C} = \vec{A} + (-\vec{B})$



Scalar Multiples of Vectors

- **If vector $\vec{A}$ is multiplied by a scale factor, k, its length changes but its direction remains the same.**

Example: $\vec{A}$=1.0 km [E]

$\overrightarrow{3A}$= 3.0 km [E]

- **If vector $\vec{A}$ is multiplied by -1, the vector direction is changed by 180° (the vector takes on the opposite direction).**

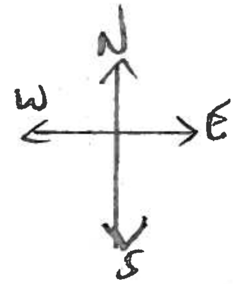
Example: $\vec{B}$=2.0 km [W]

$-\vec{B}$= 2.0 km [E] or -2.0 km [W]

Adding Displacement Vectors using a Scale Diagram

Example 1: A bird travels 4.8 km [N 50.°E] and then 3.9 km[E35 °S]. Find the bird's resultant displacement using a scale diagram.

Let: 1 cm = 1 km



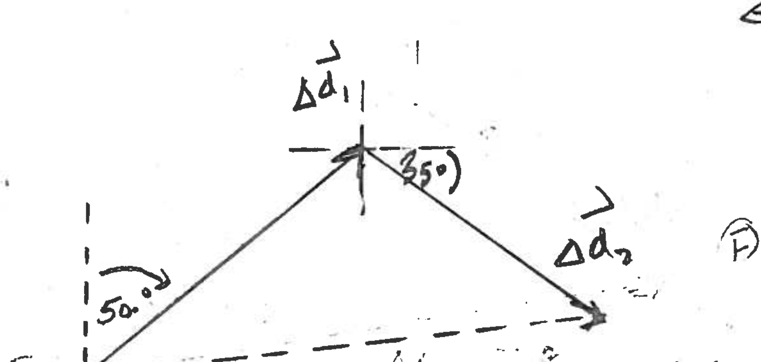
$\Delta\vec{d}_1$ = 4.8 km [N 50.° E]

$\Delta\vec{d}_2$ = 3.9 km [E 35° S]

$\Delta\vec{d}_R$ = ?

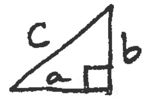
analysis:

$\Delta\vec{d}_R = \Delta\vec{d}_1 + \Delta\vec{d}_2$



$\Delta\vec{d}_R$ = 6.8 cm [N 81° E]

$\Delta\vec{d}_R$ = 6.8 km [N 81° E]



$a^2+b^2=c^2$

Adding Displacement Vectors using a Sketch Pythagorean Theorem

Example 2: A jogger runs 3.9 km [North] and 5.7 km [West]. Find the jogger's resultant displacement using a sketch and algebraic analysis.

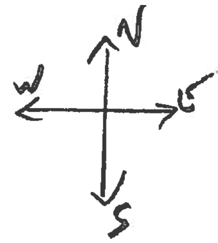
$\Delta\vec{d}_1 = 3.9\,km[N]$

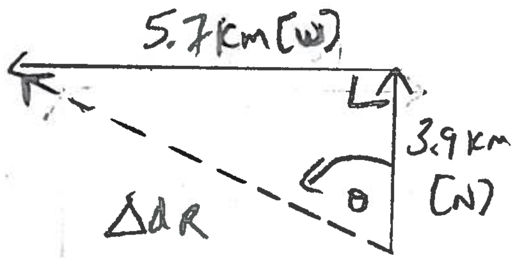
$\Delta\vec{d}_2 = 5.7\,km[W]$

$\Delta\vec{d}_R = ?$

analysis:

$\boxed{\Delta\vec{d}_R = \Delta\vec{d}_1 + \Delta\vec{d}_2}$





$$\Delta d_R = \sqrt{5.7^2 + 3.9^2}$$
$$= 6.9\,km$$

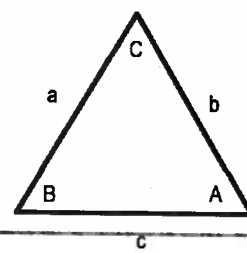
$$\theta = \tan^{-1}\left(\frac{5.7}{3.9}\right) = 56^\circ$$

$\Delta\vec{d}_R = 6.9\,km\,[N56^\circ W]$

Adding Displacement Vectors using a Sketch and Cosine/Sine Laws

Cosine Law: $c^2 = a^2 + b^2 - 2ab\cos C$

Sine Law: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$



Example 3: A bird travels 4.8 km [N 50.°E] and then 3.9 km[E35 °S]. Find the bird's resultant displacement algebraically by applying the Cosine and Sine Laws.

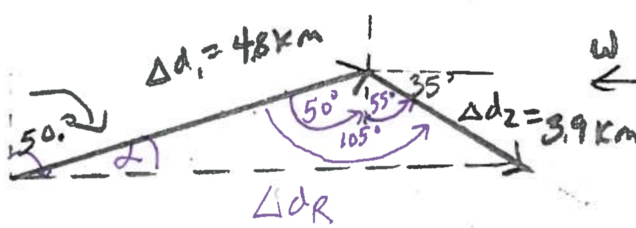
$\Delta\vec{d}_1 = 4.8\,km[N50.^\circ E]$

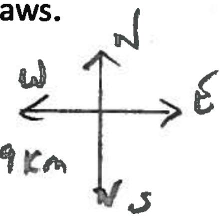
$\Delta\vec{d}_2 = 3.9\,km[E35^\circ S]$

$\Delta\vec{d}_R = ?$

analysis:

$\Delta\vec{d}_R = \Delta\vec{d}_1 + \Delta\vec{d}_2$





$$\Delta d_R = \sqrt{\Delta d_1^2 + \Delta d_2^2 - 2(\Delta d_1)(\Delta d_2)\cos 105^\circ}$$
$$= \sqrt{(4.8\,km)^2 + (3.9\,km)^2 - 2(4.8\,km)(3.9\,km)\cos 105^\circ}$$
$$= 6.9\,km$$

$$\frac{\sin\alpha}{3.9} = \frac{\sin 105^\circ}{6.924}$$

$$\alpha = \sin^{-1}\left(\frac{(3.9)\sin 105^\circ}{6.924}\right) = 32.96^\circ \simeq 33^\circ$$

$$\therefore \Delta \vec{d}_R = 6.9\,\text{Km}\ [N 83^\circ E]$$