Key Concepts:

- Either motion may be uniform or accelerated.
- The objects may be initially separated by a "gap" in distance.
- To solve chase equations, find the intersection point of the position-time graphs. If you equate the expressions for displacement, you will find the time at which they meet!

Example 1: *Identify the form of the displacement equations for each vehicle below and sketch the corresponding position-time graph.*

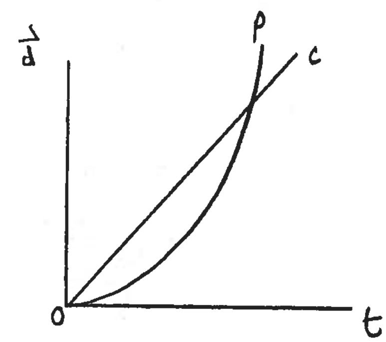
A speeding car moving at uniform velocity forward passes a police car at rest at a stoplight. At the moment the speeding car passes the police car, the police car begins accelerating uniformly forward until it catches up with the speeding car.

Speeding Car - Uniform Velocity

$\boxed{\Delta \vec{d}_c = \vec{v}_c \cdot \Delta t}$

Police car - uniform acceleration from rest.

$\boxed{\Delta \vec{d}_p = \frac{1}{2}\vec{a}\Delta t^2}$



Problem 1: In a swimming race, a father gives his 4-year old son a 10.0 second head-start. The pool is 25.0 meters long. The child swims at 0.80 m/s while the father swims at 1.20 m/sec. Find the displacement at which the father catches up with the child.

Ans: 24.0 m [Forward]

Let C = child F = Father

$\vec{v}_c = 0.80 m/s$ [F] ; $\Delta d_{c_1} = (10.0s)(0.80 m/s)$
$= 8.0 m$ [F].

$\vec{v}_F = 1.20 m/s$ [F]

$\Delta t = ?$ } at meeting point?
$\Delta d = ?$

Let t = 0 s be the time the Father starts swimming.

Child

$\Delta \vec{d}_c = 8.0m + \vec{v}_c \cdot \Delta t$

Father

$\Delta \vec{d}_F = \vec{v}_F \cdot \Delta t$

at meeting point,

$\Delta \vec{d}_c = \Delta d_F$

$8.0m + (0.80 m/s)\Delta t = (1.20 m/s)\Delta t$

$\therefore (0.40 m/s)(\Delta t) = -8.0m$

$\Delta t = \frac{-8.0m}{-0.40 m/s}$

$\boxed{\Delta t = 20.0s}$

Find the Father's displacement at meeting point.

$\Delta \vec{d}_F = (1.20 m/s [F])(20.0s)$
$= 24.0m$ [F]

$\therefore$ the father catches the son 24.0m from the end where he started swimming.

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Problem 2: A speeding car, moving at 25.0 m/s East, passes a police car with a radar speed detector. After the police officer sees the car's speed reading, she begins to accelerate after the car at 1.50 m/s² East. The car is already 50.0 m ahead when the police officer starts her acceleration. At what time, after starting her acceleration, does the policewoman catch the car?

Ans: 35.2 s

C- car p- police car

$\vec{V}_c = 25.0\,m/s\ [E]$

$\Delta d_{c\ head\ start} = 50.0\,m\ [E]$

$\vec{V}_p = 1.50\,m/s^2\ [E]$

$\left.\begin{matrix}\Delta d = ? \\ \Delta t = ?\end{matrix}\right\}$ at meeting point?

Let $[E] = +$

Car

$\Delta \vec{d}_c = 50.0\,m + 25.0\,m/s\,(\Delta t)$

Police

$\Delta \vec{d}_p = \frac{1}{2}(1.50\,m/s^2)\Delta t^2$

Set $\boxed{\Delta \vec{d}_c = \Delta \vec{d}_p}$

$50.0 + 25.0\,\Delta t = 0.75\,\Delta t^2$

$\therefore\ 0.75\,\Delta t^2 - 25.0\,\Delta t - 50 = 0$

Let $a = 0.75$, $b = -25.0$, $c = -50.0$ ~ $\boxed{\text{Solve for } \Delta t}$

$\Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{25.0 \pm \sqrt{(25.0)^2 - 4(0.75)(50)}}{2(0.75)}$

$= \dfrac{25 \pm \sqrt{775}}{1.50}$

$\therefore \Delta t = \dfrac{25 \pm 27.84}{1.50}$

$\therefore \Delta t = -1.89\,s$ OR $35.23\,s$

inadmissable.

$\therefore$ the police car catches the speeding car at 35.2s.

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Problem 3: The Easter Bunny runs along a straight and narrow path with a constant velocity of 25.0 m/s forward. He passes a sleeping tortoise, which immediately starts to chase the bunny with a constant acceleration of 3.0 x 10⁻³ m/s² forward. How much time does it take the tortoise to catch the bunny? (Express your answer in hours)

Ans: 4.6 hours

B- Bunny T- Tortoise

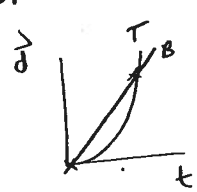
$\vec{V}_B = 25.0\,m/s\ [F]$

$\vec{a}_T = 3.0 \times 10^{-3}\,m/s^2\ [F]$

$\vec{V}_{1T} = 0.0\,m/s$

$\left.\begin{matrix}\Delta \vec{d} = ? \\ \Delta t = ?\end{matrix}\right\}$ at meeting point.

Let $[F] = +$



$\Delta \vec{d}_B = 25.0\,m/s\,[\Delta t]$

$\Delta d_T = \frac{1}{2}(3.00 \times 10^{-3}\,m/s)(\Delta t)^2$

Set $\Delta \vec{d}_B = \Delta \vec{d}_T$

$25.0\,m/s\,\Delta t = \frac{1}{2}(3.00 \times 10^{-3}\,m/s^2)\Delta t^2$

$\therefore 0 = 0.0015\,m/s^2 \cdot \Delta t^2 - 25.0\,m/s\,\Delta t$

$0 = 0.0015\,\Delta t(\Delta t - 16{,}666.67)$

$\therefore \Delta t = 0.0\,s$ OR $\Delta t = 16{,}666.67\,h \div 3600\,s/h$

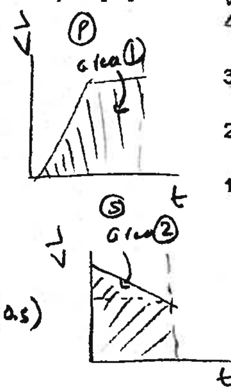
$= 4.63\,h$

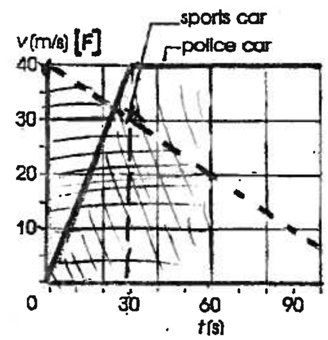
$\therefore$ it takes tortoise 4.6h to catch the bunny

Problem 4: At time t=0.0 seconds, a stationary police car stopped at a traffic light is passed by a speeding sports car. This occurs on a straight road. Their subsequent velocities are shown on the $\vec{v} - t$ graph below. (Assume 2 significant figures in values read from the graph.) P - police car S - sports car

a) Prove that the police car overtakes the sports car when t=60.0 seconds..
b) Determine the average velocity at which the police car travels over the 60.0 seconds.
c) How far ahead is the sports car when t=30.0 seconds?
d) Determine the acceleration of the sports car when t= 30.0 seconds.

Ans. b) 30. m/s [F] c) 450 m d) 0.33 m/s² [B]





a) Prove $\Delta \vec{d}_P = \Delta \vec{d}_S$ at t=60.0 s

$\Delta \vec{d}_P = \frac{1}{2}(30.s)(40.m/s [F]) + (40.0 m/s [F])(30.s)$
$= 600. [F] + 1200. m [F]$
$= 1800 m [F]$

$\Delta \vec{d}_S = (20. m/s [F])(60.s) + \frac{1}{2}(40.-20. m/s [F])(60.s)$
$= 1200. m [F] + 600. m [F]$
$= 1800. m [F]$

∴ Since the displacements are equal at 60 s, we can determine that the police car has caught the speeding car.

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b) $\vec{V}_{av} = ?$ $\quad \vec{V}_{av} = \frac{\Delta \vec{d}_R}{\Delta t} = \frac{1800 m [F]}{60.0 s} = 30. m/s [F]$

∴ the police car's average velocity was 30 m/s [F]

c) $\Delta \vec{d}_{P_{30}} = ?$
$\Delta \vec{d}_{S_{30}} = ?$

$\Delta \vec{d}_P = \frac{1}{2}(40. m/s [F])(30.s) = 600. m [F]$

$\Delta \vec{d}_c = (30. m/s)(30.0 s) + \frac{1}{2}(10.0 m/s)(30. s)$
$= 900. m [F] + 150. m [F]$
$= 1050. m [F]$

∴ $\Delta \vec{d} = \Delta \vec{d}_c - \Delta \vec{d}_P$
$= 1050. m [F] - 600. m [F]$
$= 450. m [F]$

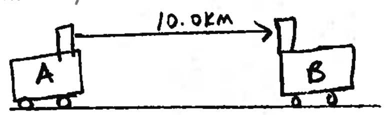
∴ the speeding car is 450. m ahead at 30. s

d) $\vec{a}_s = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{40. m/s [F] - 20. m/s [F]}{0 - 60.0 s}$
$= \frac{20. m/s [F]}{-60.0 s}$
$= -0.33 m/s^2 [F]$

∴ the car accelerates uniformly throughout at 0.33 m/s² [Back

Problem 5: Two trains are separated by 10.0 km and are heading towards each other on a single track. Train A is moving at 30.0 km/h [East] while train B is moving at 40.0 km/h [West]. If the engineers do not stop their trains, when and where will they collide? *Ans: 0.143 h, 4.29 km from Train A's initial position*

$\xrightarrow{E = +}$



Let the origin be located at train A's original location.

$\vec{V}_A = 30.0\,km/h\,[E]$

$\vec{V}_B = 40.0\,km/h\,[W]$

$\vec{d}_{B_{t=0}} = 10.0\,Km\,[W]$

$\left.\begin{matrix}\Delta\vec{d} = ? \\ \Delta t = ?\end{matrix}\right\}$ at collision point.

$\underline{A}$

$\Delta\vec{d}_A = 30.0\,km/h\,\Delta t$

$\underline{B}$

$\Delta\vec{d}_B = 10.0 - 40.0\,km/h \cdot \Delta t$

Set $\Delta\vec{d}_A = \Delta\vec{d}_B$

$30.\Delta t = 10.0 - 40.0\Delta t$

$70.0\Delta t = 10.0$

$\Delta t = \dfrac{10.0}{70.0} = 0.143\,h$

$\therefore$ they will collide in 0.143h (8.6 min) if they do not stop.

Solve for $\Delta\vec{d}_A$:

$\Delta\vec{d}_A = (30.0)(0.143)$

$= 4.29\,Km\,[E].$

$\therefore$ they will meet 4.29 km [E] from train A's original position.