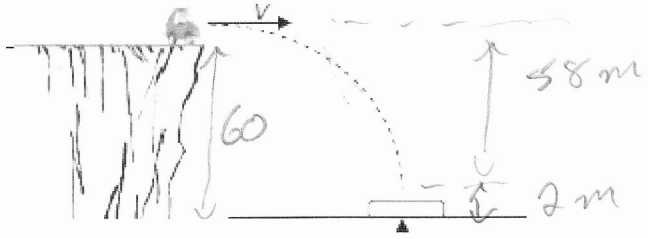
Review Problems - Projectile Solutions

1. A stunt car drives off a 60 m high, level cliff with a speed of 60 km/h. How from the base of the cliff should the centre of a 2 m high crash mat be placed to ensure the car and more importantly the driver survive the fall? (Ans: $\Delta d_x \cong 57$ m)



(1) COMPONENTS

$V_{iy} = 0$ $\quad V_{ix} = 60 \text{ km/h} = 16.7 \text{ m/s}$.

(2) HOW LONG DOES IT TAKE TO DROP 58 m

USE $V_{iy} = 0$, $a = +9.81$, $\Delta d_x = 58$ m.

$\Delta d_x = V_{iy}\Delta t + \frac{1}{2} a \Delta t^2$.

$58 = 0 + 4.91 \Delta t^2 \Rightarrow$ $\quad \Delta t^2 = 11.8$

$\boxed{\Delta t = 3.44 \text{ s}}$.

(3) HOW FAR DOES THE CAR TRAVEL IN 3.44 s IN THE HORIZONTAL DIRECTION.

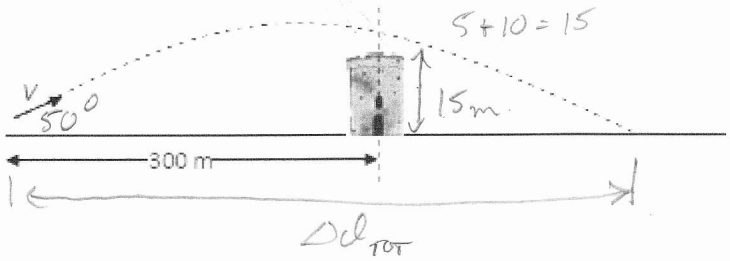
$\Delta d_x = V_{ix} \Delta t$

$\Delta d_x = (16.7)(3.44)$.

$\boxed{\Delta d_x = 57.4 \text{ m}}$

Review Problems - Projectile Solutions

2. A cannon is discharged towards a castle tower (near the time when the ancient world was meeting the modern world). The cannon ball flies 5 m above the top of the 10 metre tall tower and lands on level ground (same level as cannon) behind the tower. The cannon was positioned 300 m from the tower and was angled at 50° to the horizontal. What was the muzzle velocity of the cannon ball? How far behind the tower did it land? [Ans: $V \cong 56\frac{m}{s}$; $\Delta d \cong 15$ m] Hint: up + and right +



① USE COMPONENTS.

$V_{iy} = V\sin 50° = 0.77V$ $\quad V_{ix} = V\cos 50° = 0.64V$

② SOLVE AT 300m, 15m HEIGHT.

$\Delta d_y = 15m$, $V_{iy} = 0.77V$, $a = -9.81$

USE $\Delta d_y = V_{iy}\Delta t + \frac{1}{2}a\Delta t^2$

$15 = 0.77V\Delta t + \frac{1}{2}(-9.81)\Delta t^2$

(A) $15 = 0.77V\Delta t - 4.91\Delta t^2$

$\Delta d_x = 300m$, $V_{ix} = 0.64V$

USE $\Delta d_x = V_x \Delta t$ OR $\Delta t = \frac{\Delta d_x}{V_x}$

(B) $\Delta t = \frac{300}{0.64V} = \frac{468.8}{V}$

③ TWO EQUATIONS, TWO UNKNOWNS. SUBSTITUTE Δt (EQU'N B) TO SOLVE FOR V (EQU'N A)

$15 = 0.77V\Delta t - 4.91\Delta t^2$

$15 = 0.77V\left(\frac{468.8}{V}\right) - 4.91\left(\frac{468.8}{V}\right)^2$

$15 = 361 - \frac{1.08\times10^6}{V^2}$

$-346 = -\frac{1.08\times10^6}{V^2}$

$V^2 = \frac{1.08\times10^6}{346}$

$V^2 = 3103 \Rightarrow \boxed{V = 55.7}$

④ FIND LANDING TIME.

$\Delta d_y = V_{iy}\Delta t + \frac{1}{2}a\Delta t^2$

$0 = 0.77V\Delta t - 4.91\Delta t^2$

$0 = 0.77(55.7)\Delta t - 4.91\Delta t^2$

$0 = 42.9\Delta t - 4.91\Delta t^2$

$0 = \Delta t(42.9 - 4.91\Delta t)$

$\Delta t = 0s$ OR $\Delta t = 8.76s$

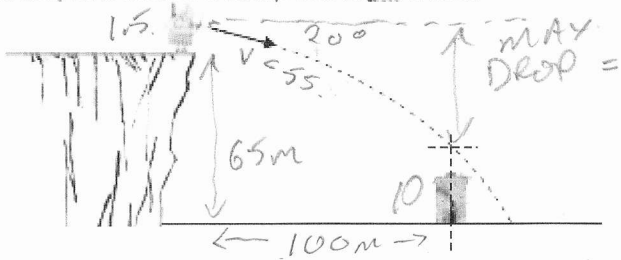
⑤ FIND Δd_x TOTAL $\boxed{313m}$

$\Delta d_x = V_x \Delta t$

$= 0.64(55.7)(8.76)$

Review Problems - Projectile Solutions

3. A 1.5 m tall archer is standing on a 65 m cliff. Trying to maximize impact force while clearing a castle wall below he fires his arrow with a speed of 55 m/s at an angle of 20° below the horizontal. The castle wall is 100 m from the cliff and is 10 m high. Does the arrow clear the castle wall (and by how much if it does)? [Ans: $\Delta d_{above} \geq 1.8$ m]



(1) COMPONENTS.

$V_{iy} = 55 \sin 20°$
$= 18.8 m/s$

$V_{ix} = 55 \cos 20°$
$= 51.7 m/s.$

(2) FIND HOW LONG IT TAKES TO TRAVEL 100m IN HORIZONTAL DIRECTION.

USE $\Delta d_x = V_x \Delta t$ OR $\Delta t = \frac{\Delta d_x}{V_x} = \frac{100}{51.7}$

$\Delta t = 1.93 s.$

(3) FIND HOW FAR IT DROPS IN 1.93 SECONDS.

USE $a = +9.81$, $V_{iy} = 18.8$, $\Delta t = 1.93$

$\Delta d = V_{iy} \Delta t + \frac{1}{2} a \Delta t^2$

$\Delta d = (18.8)(1.93) + \frac{1}{2}(9.81)(1.93)^2$

$\Delta d = 36.3 + 18.1$

$\Delta d = 54.4 m.$

(4) THE DROP OF 54.4 m IS LESS THAN THE MAX DROP OF 56.5 m

SO IT CLEARS THE WALL BY. $56.5 - 54.4 = 2.1 m$