

Using Component Analysis to Solve for Resultant Displacement:

Analysis approach:

In this analysis approach, we are breaking up displacements into two component directions (x and y).

STEPS:

1. Write down your given information and create a vector analysis statement. E.g. If you are given three displacements and you want to find resultant displacement, your analysis statement is:

$$\vec{\Delta d_R} = \vec{\Delta d_1} + \vec{\Delta d_2} + \vec{\Delta d_3}$$

2. Choose "x" and "y" directions based on the given vectors. E.g. If the directions given are mostly East and North, choose East as +x and choose North as +y.
3. Sketch the given displacement vectors and break up each vector into their x and y component values. Some vectors may have negative components. For example, if E= +x and N=+y, a vector directed southwest will have both -x and -y components!
4. Add up the displacements in each component direction to find the total "x" displacement and the total "y" displacement.

$$\Delta d_{Rx} = \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x} \qquad \Delta d_{Ry} = \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}$$
5. Sketch the resultant x and y displacements forming a right angled triangle. The hypotenuse is the overall displacement vector $\vec{\Delta d_R}$.
6. Use the Pythagorean Theorem and Trig to find the magnitude and direction of the resultant vector.

The following question illustrates how components can be used to analyze motion in two dimensions.

We will break the motion of the given object into two perpendicular component directions and find the displacement in each component direction. We will then combine the components together to find the overall resultant magnitude and direction of the displacement.

Question: A sailboat sails 40.0 km [E 20.0° N] and then 25 km [N 30.0° W]. The total trip took 8.0 hours.
Find: a) the resultant displacement b) the average velocity over the trip.

Answer: a) 43 km [E 55° N] b) 5.4 km/h [E 55° N]

$$\Delta \vec{d}_1 = 40.0 \text{ km [E } 20.0^\circ \text{ N]}$$

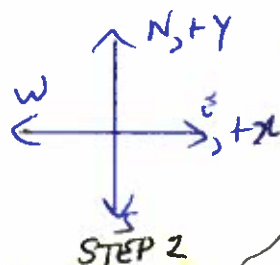
$$\Delta \vec{d}_2 = 25 \text{ km [N } 30.0^\circ \text{ W]}$$

$$\Delta t = 8.0 \text{ h}$$

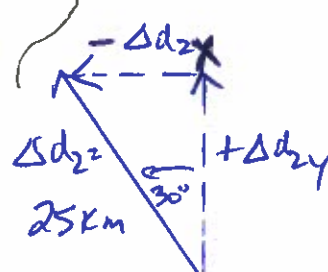
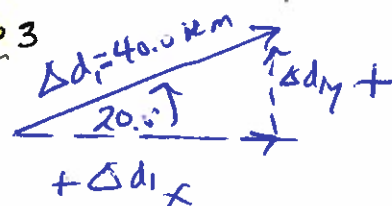
$$\Delta \vec{d}_R = ?$$

$$\vec{v}_{av} = ?$$

Analysis: $\Delta \vec{d}_R = \Delta \vec{d}_1 + \Delta \vec{d}_2$



STEP 3



STEP 1

STEP 4

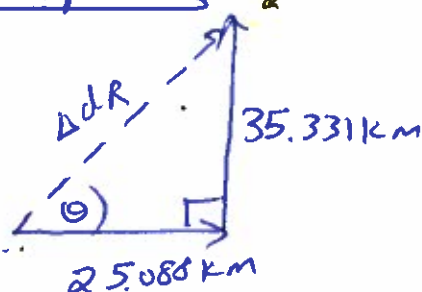
$$\begin{aligned} \Delta d_{Rx} &= \Delta d_{1x} + \Delta d_{2x} \\ &= 40.0 \cos 20.0^\circ - 25 \sin 30.0^\circ \\ &= +25.088 \text{ km} \\ &\quad (\text{East}) \end{aligned}$$

Y

$$\begin{aligned} \Delta d_{Ry} &= \Delta d_{1y} + \Delta d_{2y} \\ &= 40.0 \sin 20.0^\circ + 25 \cos 30.0^\circ \\ &= +35.331 \text{ km} \\ &\quad (\text{North}) \end{aligned}$$

Combine components

STEP 5



STEP 6

$$\begin{aligned} \Delta d_R &= \sqrt{25.088^2 + 35.331^2} \\ &= 43.33 \text{ km} \end{aligned}$$

$$\theta = \tan^{-1} \left(\frac{35.331}{25.088} \right) = 54.6^\circ$$

$$\Delta \vec{d}_R = 43 \text{ km [E } 55^\circ \text{ N]}$$

$$\vec{v}_{av} = \frac{\Delta \vec{d}_R}{\Delta t} = \frac{43 \text{ km [E } 55^\circ \text{ N]}}{8.0 \text{ h}} = 5.4 \frac{\text{km}}{\text{h}} [\text{E } 55^\circ \text{ N}]$$