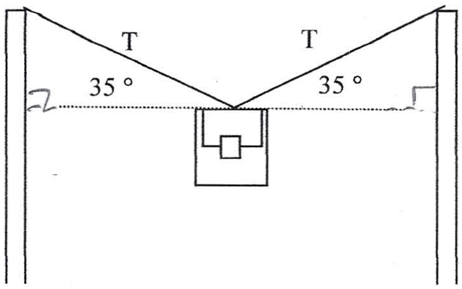
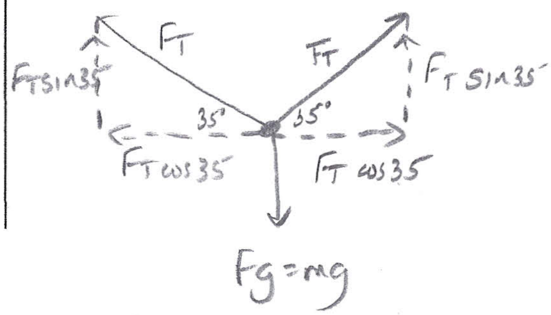
DYNAMICS SAMPLE PROBLEMS

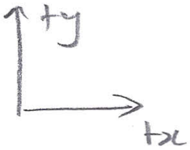
Assignment: Section 2.2: P6,7 pg 81   Section 2.3 P5-7 pg 92, P10-11 pg 94, S3-11 pg 95-96

1) STATIC EQUILIBRIUM PROBLEM:

A 32.0 kg hiker's backpack is suspended by a single rope between two trees so that the pack is in the centre of the rope as shown below. Find the tension, T, in the rope. (Ans: 273 N)







$m = 32.0\ kg$

$g = 9.80\ m/s^2$

$F_T = ?$

Conditions:

x dir:

$$\Sigma F_x = 0$$

$$\Sigma F_x = F_T \cos 35 - F_T \cos 35$$

$$0 = 0$$

Not helpful in our analysis!

y dir:

$$\Sigma F_y = 0$$

$$\Sigma F_y = 2F_T \sin 35 - F_g$$

$$2F_T \sin 35 - F_g = 0$$

$$2F_T \sin 35 - mg = 0$$

$$2F_T \sin 35 = mg$$

$$\therefore F_T = \frac{mg}{2 \sin 35}$$

$$F_T = \frac{(32.0\ kg)(9.80\ m/s^2)}{2 \sin 35}$$

$$= 273.37\ N$$

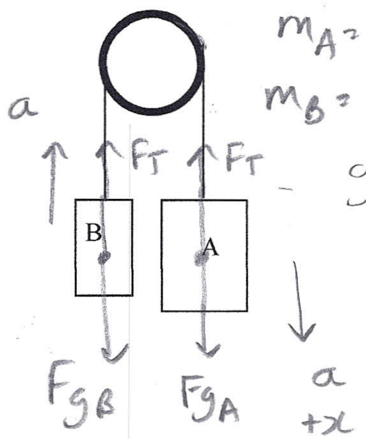
$\therefore$ The tension in the rope is 273 N.

2) *PULLEY PROBLEM 1: ATWOOD'S MACHINE*

Two blocks , A and B, (m_A = 3.5 kg and m_B=2.0 kg) are attached by a light string which is placed over a frictionless pulley as shown below. The blocks are released from rest as shown below.

Find: a) the acceleration of the blocks

b) the tension in the string (Ans: a) 2.7 m/s^2, b) 25 N)



$m_A = 3.5$ kg

$m_B = 2.0$ kg

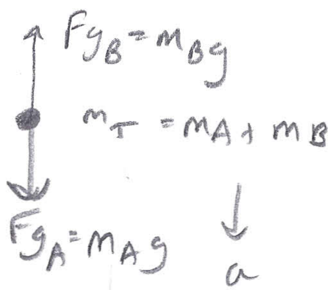
$g = 9.80$ m/s^2

$a = ?$

$F_T = ?$

a) Consider the two blocks as a single system to find acceleration:

System FBD:



$$\begin{cases} \Sigma F = m_T a \\ \Sigma F = F_{gA} - F_{gB} \end{cases}$$

$m_T a = F_{gA} - F_{gB}$

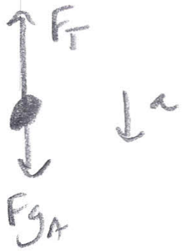
$m_T a = m_A g - m_B g$

$$a = \frac{g(m_A - m_B)}{m_T}$$

$$a = \frac{(9.80 \text{ m/s}^2)(3.5 - 2.0 \text{ kg})}{(3.5 + 2.0 \text{ kg})} = 2.673 \text{ m/s}^2$$

$\therefore$ the acceleration is 2.7 m/s^2.

b) Isolate block A to solve for Tension:



$$\begin{cases} \Sigma F = m_A a \\ \Sigma F = F_{gA} - F_T \end{cases}$$

$F_{gA} - F_T = m_A a$

$m_A g - F_T = m_A a$

$F_T = m_A g - m_A a$

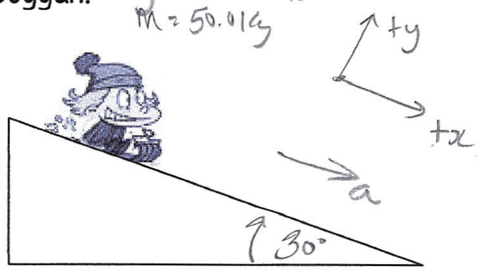
$= (3.5 \text{ kg})(9.80 - 2.673 \text{ m/s}^2) = 24.95 \text{ N} \simeq 25 \text{ N}$

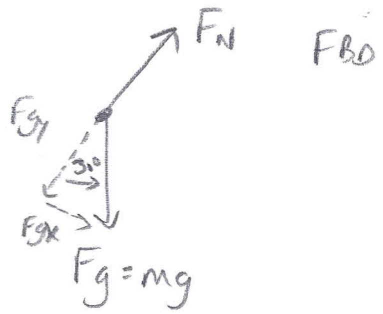
$\therefore$ the tension in the rope is 25N.

3) INCLINED PLANE PROBLEM

A girl on a toboggan (total mass 50.0 kg) slides down a frictionless hill (inclination angle 30.0°) as shown below. Find the girl's acceleration down the hill and the normal force of the hill on the toboggan. (Ans: 4.90 m/s^2, 424 N)

$g = 9.80\,m/s^2$
$m = 50.0\,kg$





x dir:

$$\Sigma F_x = ma$$
$$\Sigma F_x = F_{gx}$$
$$F_{gx} = ma$$
$$F_g \sin 30 = ma$$
$$\cancel{m} g \sin 30 = \cancel{m} a$$
$$\therefore a = g \sin 30$$
$$= (9.80\,m/s^2) \sin 30$$
$$= 4.90\,m/s^2$$

y dir:

$$\Sigma F_y = 0$$
$$\Sigma F_y = F_N - F_{gy}$$
$$F_N - F_{gy} = 0$$
$$F_N - mg \cos 30 = 0$$
$$F_N = mg \cos 30$$
$$F_N = (50.0\,kg)(9.80\,\tfrac{m}{s^2}) \cos 30$$
$$= 424.35\,N$$

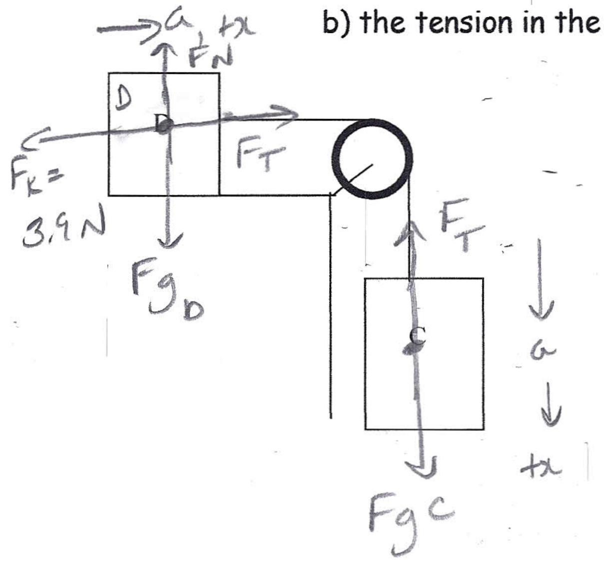
$\therefore$ the girl accelerates at 4.90 m/s^2 down the hill. The normal force of the hill on the toboggan is 424 N.

4) *PULLEY PROBLEM 2: TWO BLOCKS, A DESK AND A PULLEY*

Two blocks , C and D, (m_C = 1.5 kg and m_D=1.0 kg) are attached by a light string which is placed over a fricionless pulley as shown below. The blocks are accelerating so that D is approaching the edge of the desk. There is a frictional force of 3.9 N on block D. Find:

a) the acceleration of the blocks

b) the tension in the string (Ans: a) 4.3 m/s², b) 8.2 N)



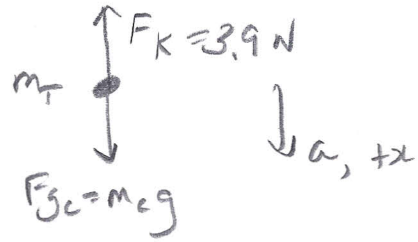
$m_C = 1.5$ kg a = ?

$m_D = 1.0$ kg F_T = ?

a) Treat the two blocks as a system to solve for acceleration:

System FBD.

$m_T = (1.5 + 1.0)$ kg
$= 2.5$ kg



$$\left.\begin{array}{l}\Sigma F = m_T a \\ \Sigma F = m_c g - F_K\end{array}\right\} \quad m_T a = m_c g - F_K$$

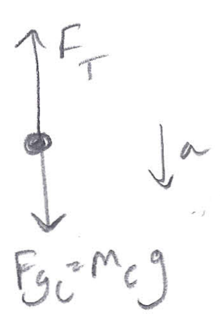
$$a = \frac{m_c g - F_K}{m_T}$$

$$= \frac{(1.5\,kg)(9.80\,\frac{m}{s^2}) - 3.9\,N}{(2.5\,kg)}$$

$$= 4.32\,m/s^2$$

the acceleration is 4.3 m/s².

b) Isolate block C to solve for tension.



$$\left.\begin{array}{l}\Sigma F = m_c a \\ \Sigma F = F_{gc} - F_T\end{array}\right\} \quad m_c g - F_T = m_c a$$

$$F_T = m_c g - m_c a$$

$$= m_c (g - a)$$

$$= 1.5\,kg\,(9.80 - 4.32\,m/s^2)$$

$$= 8.22\,N$$

the tension in the string is 8.2 N.

5) THIRD LAW PROBLEM: LINKED OBJECTS

A train locomotive pulls three boxcars (each of mass 2.5×10^4 kg) with an applied force of 5.0×10^4 N forward. There is a frictional force of 450 N on each boxcar. Find:

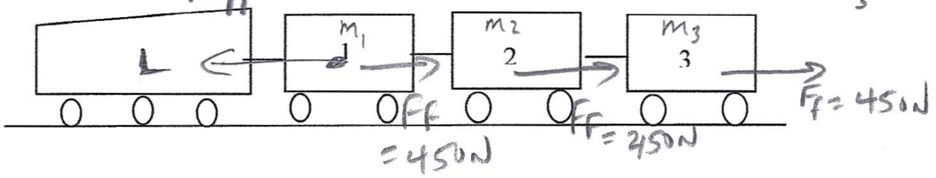
a) The acceleration of the train.

b) The force exerted by the second boxcar on the third boxcar.

c) The force exerted by the first boxcar on the second boxcar.

(Ans: a) 0.65 m/s² b) 1.7×10^4 N, c) 3.3×10^4 N)

$F_{app} = 5.0\times10^4$ N $\qquad m_1 = m_2 = m_3 = 2.5\times10^4$ kg



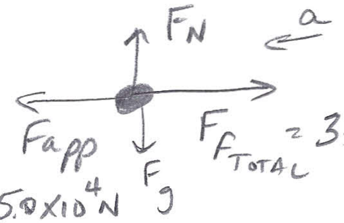
$a = ?$

$F_{23} = ?$

$F_{12} = ?$

a) Treat the 3 box cars as a system to find acceleration.

System FBD



$F_{f_{TOTAL}} = 3 \times 450\text{N} = 1350\text{N}$

$m_T = 3(2.5\times10^4 \text{kg}) = 7.5\times10^4 \text{kg}$

$\Sigma F = m_T a$

$\Sigma F = F_{app} - F_{f_{TOTAL}}$

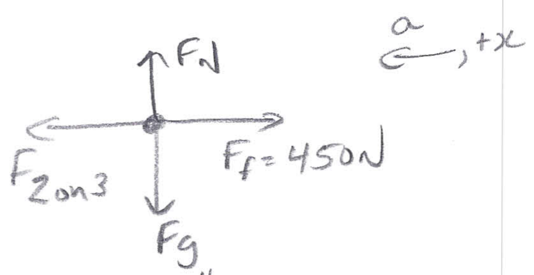
$m_T a = F_{app} - F_{f_{TOTAL}}$

$a = \dfrac{F_{app} - F_{f_{TOTAL}}}{m_T}$

$a = \dfrac{5.0\times10^4\text{N} - 1350\text{N}}{7.5\times10^4\text{kg}} = 0.649 \text{ m/s}^2$

∴ the acceleration is 0.65 m/s^2

b) Isolate car 3:



$m = 2.5\times10^4$ kg

$\Sigma F = ma$

$\Sigma F = F_{2on3} - F_f$

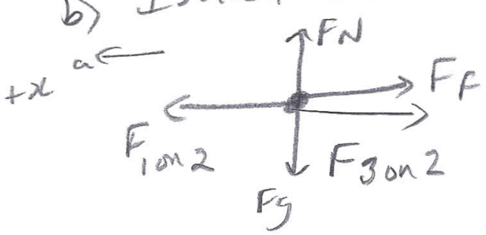
$F_{2on3} - F_f = ma$

$F_{2on3} = ma + F_f$

$F_{2on3} = (2.5\times10^4 \text{kg})(0.649 \text{ m/s}^2) + 450\text{N}$

$= 1.6667\times10^4 \text{N}$

b) Isolate car 2:



$\Sigma F = ma$

$\Sigma F = F_{1on2} - F_{3on2} - F_f$

$F_{1on2} - F_{3on2} - F_f = ma$

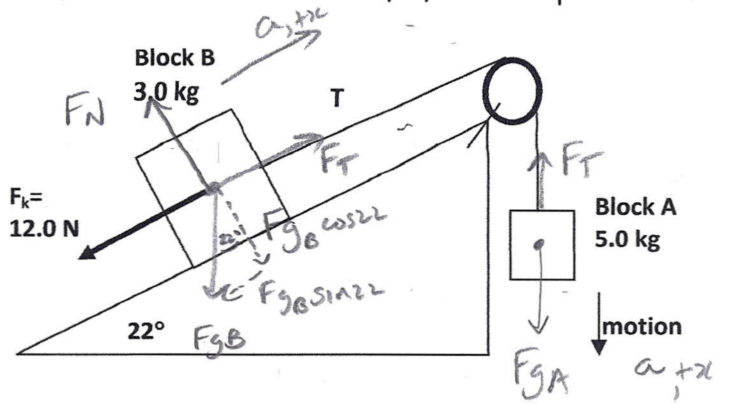
$F_{1on2} = ma + F_{3on2} + F_f$

$= (2.5\times10^4 \text{kg})(0.649 \text{ m/s}^2) + 1.6667\times10^4 \text{N} + 450\text{N} = 3.333\times10^4 \text{N}$

∴ the force of car 2 on 3 is 1.7×10^4N and the force of car 1 on 2 is 3.3×10^4N.

6) *THE COMBO PROBLEM: INCLINED PLANE PLUS PULLEY*

A system of two linked blocks slides along an inclined plane as shown. Find the acceleration of the system of blocks and the tension, T, in the rope. (Ans: 3.2 m/s², 33 N)



$g = 9.80\,m/s^2$

$m_T = m_A + m_B = 5.0\,kg + 3.0\,kg$
$= 8.0\,kg$

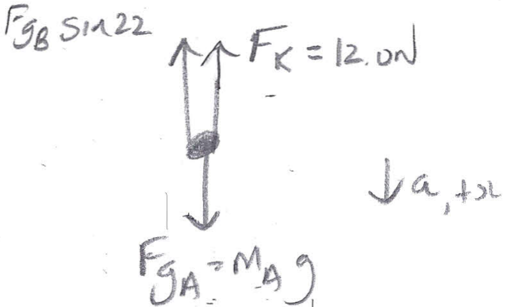
$a = ?$

$F_T = ?$

Treat the blocks as a system to find acceleration:

System FBD:

m_T



$\downarrow a, +x$

$$\left\{\begin{array}{l}\Sigma F = m_T a \\ \Sigma F = F_{gA} - F_K - F_{gB}\sin 22\end{array}\right.$$

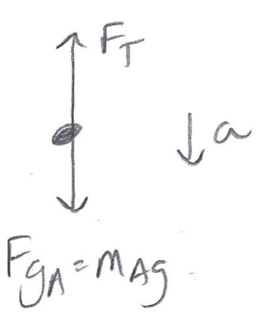
$$m_T a = m_A g - F_K - m_B g \sin 22$$

$$a = \frac{(5.0kg)(9.80\,m/s^2) - 12.0N - (3.0kg)(9.80\,m/s^2)\sin 22}{8.0kg}$$

$$a = 3.2485\,m/s^2$$

the blocks accelerate at $3.2\,m/s^2$.

Isolate block A to find the tension:



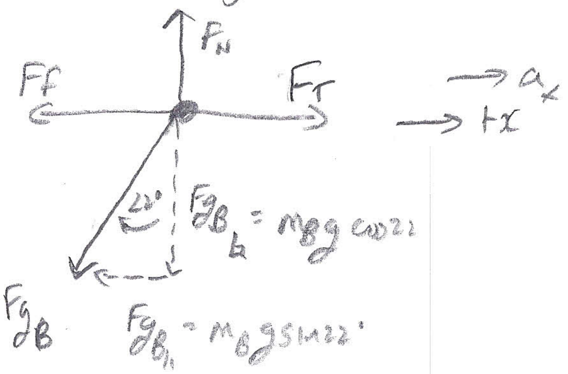
$\downarrow a$

$$\left.\begin{array}{l}\Sigma F = m_A a \\ \Sigma F = F_{gA} - F_T\end{array}\right\} \; m_A g - F_T = m_A a$$

$$F_T = m_A g - m_A a$$
$$= (5.0kg)(9.80\,m/s^2 - 3.248\,m/s^2)$$
$$= 32.76\,N$$
$$\approx 33\,N$$

the tension in the rope is 33 N.

Alternatively, isolate block B:



$$\left.\begin{aligned}\Sigma F_x &= m_B a_x \\ \Sigma F_x &= F_T - F_f - F_{g_{B\parallel}}\end{aligned}\right\} F_T - F_f - F_{g_{B\parallel}} = m_B a_x$$

$$F_T = m_B a_x + F_f + F_{g_{B\parallel}}$$

$$= (3.0\,kg)\,3.248\,\tfrac{m}{s^2} + 12.0\,N + (3.0\,kg)(9.8\,\tfrac{N}{kg})\sin 22°$$

$$= 32.757\,N$$

$$\sim 33\,N$$