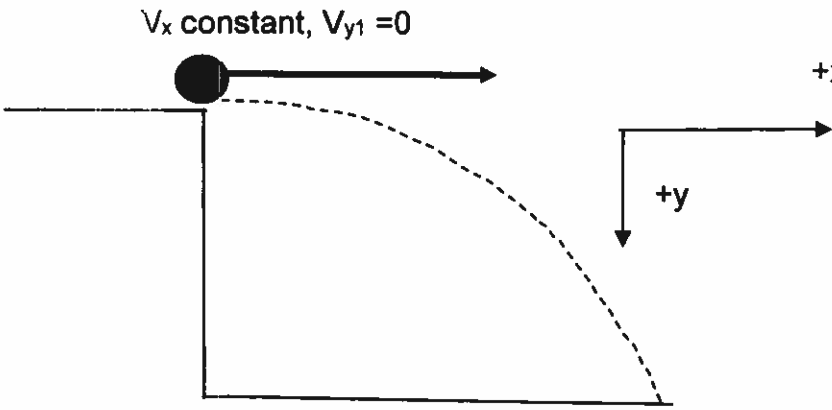
Projectile Motion Conditions

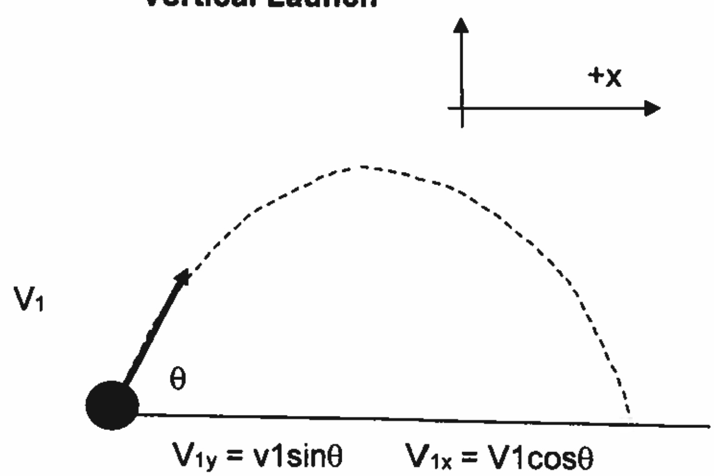
- **Motion in the horizontal (x) and vertical (y) component directions are independent**

- **The object is moving freely in a gravitational field**
- **The object follows a parabolic trajectory**
- **Motion Conditions:**
  Vertical: constant acceleration **Horizontal: constant velocity**

Horizontal Launch



Vertical Launch



Horizontal- Constant Velocity	Vertical- Constant Acceleration
V_x constant	V_y variable
$\Delta dx = v_x \Delta t$	$v_{2y} = v_{1y} + a_y\Delta t$
	$\Delta d_y = v_{y1}\Delta t + \frac{1}{2} a_y\Delta t^2$

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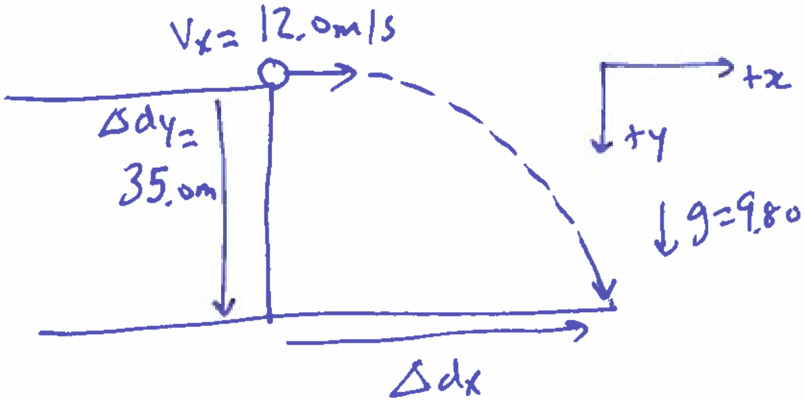
Projectile Motion Sample Problems

1. **Horizontal Launch**
A ball is kicked with a horizontal velocity of 12.0 m/s [Forward] off the top of a cliff 35.0 m high.
Find:
    a) The time for the ball to land on the ground below.
    b) The horizontal range.
    c) The final velocity of the ball as it lands.

(Ans: 2.67 s, 32.1 m, 28.8 m/s [65.4° below the horizontal])



$\downarrow g = 9.80\,m/s^2\,[\downarrow]$

x constant velocity	y constant acceleration
$V_x = 12.0\ m/s$	$V_{y_1} = 0.0\,m/s$
$\Delta d_x = ?$	$a_y = 9.80\ m/s^2$
$\Delta t = ?$	$\Delta d_y = 35.0\,m$
	$\Delta t = ?$
	$V_{y_2} = ?$
	$\vec{V}_f = ?$

a) Solve for time using y information.

$$\Delta d_y = \cancel{V_{y_1}}^{0} \Delta t + \frac{1}{2} a_y \Delta t^2$$

$$\Delta d_y = \frac{1}{2} a_y \Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$\Delta t = \sqrt{\frac{2(35.0\,m)}{(9.80\,m/s^2)}} = 2.673\,s$$

b) Solve for Δd_x:

$$\Delta d_x = V_x \Delta t$$

$$= (12.0\,m/s)(2.673\,s)$$

$$= 32.071\,m$$

$$\simeq 32.1\,m$$

c) $V_{y2} = ?$

$\vec{V}_f = ?$

$\Delta t = 2.673\,s$

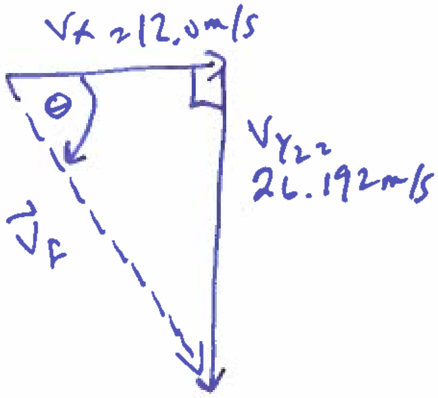
$$a = \frac{V_2 - V_1}{\Delta t}$$

$$V_2 = V_1 + a\,\Delta t$$

$$V_{y_2} = V_{y_1} + a_y\,\Delta t$$

$$V_{y_2} = 0 + (9.80\,m/s^2)(2.673\,s)$$

$$= 26.192\ m/s$$



$$V_f = \sqrt{12.0^2 + 26.192^2}$$

$$= 28.81\ m/s$$

$$\theta = \tan^{-1}\left(\frac{26.192}{12.0}\right) = 65.38^\circ$$

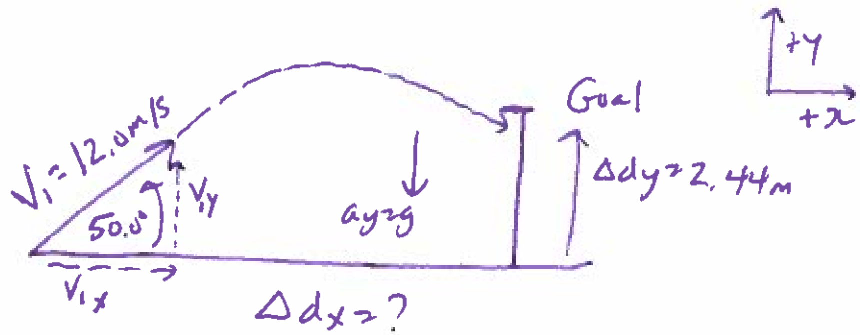
$\vec{V}_f = 28.8\ m/s$ [65.4° below horizontal]

2. **Angled Launch**

A soccer player kicks a soccer ball up in the air giving it an initial velocity of 12.0 m/s [50.0° above the horizontal] on a level playing field. The ball rises up and then drops into the net on its way down. If the top of the net is 2.44 m from the ground find:

a) The time for the ball to reach the net.

b) How far away from the net the player was when she kicked the ball.



constant vel. x	constant acc. y
$V_x = 12.0\cos 50.0°$ m/s	$V_{1y} = 12.0\sin 50.0°$ m/s
$\Delta t = ?$	$\Delta d_y = 2.44$ m
$\Delta d_x = ?$	$a_y = g = -9.80$ m/s^2
	$\Delta t = ?$

a) Solve for time using the y information

$$\Delta d_y = V_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$2.44 = 12.0\sin 50.0°\Delta t + \frac{1}{2}(-9.80)\Delta t^2$$

$$2.44 = 9.19\,\Delta t - 4.90\,\Delta t^2$$

$$4.90\Delta t^2 - 9.19\,\Delta t + 2.44 = 0$$

use quadratic formula:

$a = 4.90$, $b = -9.19$, $c = 2.44$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-9.19) \pm \sqrt{9.19^2 - 4(4.90)(2.44)}}{2(4.90)}$$

$$= \frac{9.19 \pm 6.06}{9.80}$$

$\therefore \Delta t = 0.320$ s OR 1.56 s

choose the second time as the ball is on the way down.

b) Solve for the horizontal range.

$\Delta t = 1.56$ s

$V_x = 12.0\cos 50.0°$ m/s

$\Delta d_x = ?$

$$\Delta d_x = V_x\Delta t$$

$$= (12.0\cos 50.0° \text{ m/s})(1.56 \text{ s})$$

$$= 12.0 \text{ m}$$

$\therefore$ the ball reached the net in 1.56 s and the player was 12.0 m from the net when she kicked the ball.