

## Projectile Motion "SIN" Problems

Δt (G)	Δy (N)
1	30
2	80
3	?



In general  $\Delta y = V_i \Delta t + \frac{1}{2} a \Delta t^2$

$$\Delta t = 1G \rightarrow 30N = V_i(1G) + \frac{a}{2}(1G)^2 \rightarrow 30 = V_i + \frac{a}{2} \quad \text{--- (1)}$$

$$\Delta t = 2G \rightarrow 80N = V_i(2G) + \frac{a}{2}(2G)^2 \rightarrow 80 = 2V_i + 2a \quad \text{--- (2)}$$

Solve for a:

$$\begin{aligned} & [ (2) - 2 \times (1) ] \\ & \frac{80 - 2 \times 30}{2} = 2V_i + 2a \\ & \underline{\underline{[ 60 = 2V_i + a ]}} \\ & 20 = a \\ & a = 20 \frac{N}{G^2} \end{aligned}$$

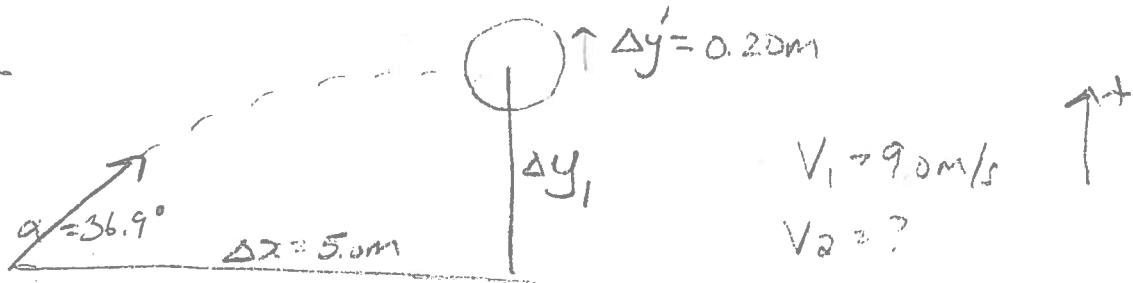
Solve for  $V_i$  from Eq (1):

$$\begin{aligned} V_i &= 30 - \frac{a}{2} \\ V_i &= 30 - \frac{20}{2} = 20 \frac{N}{G} \end{aligned}$$

$\Delta y_{\text{Total}}$  at  $\Delta t = 3G$ ?

$$\begin{aligned} \Delta y &= V_i \Delta t_3 + \frac{1}{2} a \Delta t_3^2 \\ &= \left(20 \frac{N}{G}\right)(3G) + \frac{1}{2} \left(20 \frac{N}{G^2}\right)(3G)^2 \\ &= 60N + 90N \\ &= 150 N \end{aligned}$$

91-11



Solve for time of Flight:

$$\Delta t = \frac{\Delta x}{V_{1x}} = \frac{5.0 \text{ m}}{9.0 \text{ m/s} \cdot \cos 36.9^\circ} = 0.6947 \text{ s}$$

Solve for original vertical displacement  $\Delta y_1$ :

$$\Delta y_1 = V_{1y} \Delta t - \frac{1}{2} g \Delta t^2$$

$$\Delta y_1 = (9.0 \text{ m/s}) (\sin 36.9^\circ) (0.6947 \text{ s}) - \frac{9.8 \text{ m/s}^2}{2} (0.6947 \text{ s})^2 \\ = 1.3892 \text{ m}$$

Second vertical displacement  $\Delta y_2 = \Delta y_1 + 0.20 \text{ m} = 1.59 \text{ m}$

Second time of flight  $\Delta t_2 = \frac{\Delta y_2}{V_{2y}}$

Solve for  $V_2$ :

$$\Delta y_2 = V_{2y} \Delta t_2 - \frac{1}{2} g \Delta t_2^2$$

$$1.59 \text{ m} = V_2 \cdot \sin 36.9^\circ \left( \frac{5.0 \text{ m}}{V_2 \cdot \cos 36.9^\circ} \right) - \frac{9.8 \text{ m/s}^2}{2} \left( \frac{5.0 \text{ m}}{V_2 \cdot \cos 36.9^\circ} \right)^2$$

$$1.59 \text{ m} = 3.754 \text{ m} - \frac{191.557 \text{ m}^{3/2}}{V_2^2}$$

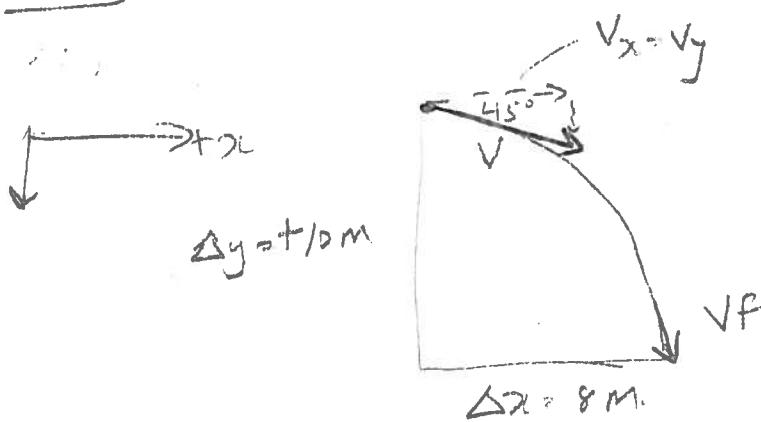
$$-2.164 \text{ m} = -\frac{191.557 \text{ m}^{3/2}}{V_2^2}$$

$$V_2 = 7.409 \text{ m/s}$$

$$\% \text{ increase} = \frac{V_2 - V_1}{V_1} \times 100\%$$

$$= \frac{7.409 - 9.0}{9.0} \times 100\% \\ = 4.5\%$$

94-2



\* by symmetry final  
velocity is the  
same as initial  
velocity

$$\Delta y = V_y \Delta t + \frac{1}{2} g \Delta t^2 \quad \text{--- (1)}$$

$$\Delta x = V_x \Delta t \quad \text{--- (2)} \quad \text{but } V_x = V_y \text{ at that point}$$

$$\therefore \Delta t = \frac{\Delta x}{V_x} = \frac{\Delta x}{V_y} \quad \text{--- (2b)}$$

Substitute (2b) into (1) to solve for  $\Delta t$ :

$$\Delta y = V_y \Delta t + \frac{1}{2} g \Delta t^2$$

$$\Delta y = V_y \cdot \frac{\Delta x}{V_y} + \frac{1}{2} g \Delta t^2$$

$$10 \text{ m} = 8 \text{ m} + \frac{1}{2} (9.8 \frac{\text{m}}{\text{s}^2}) \Delta t^2$$

$$2 \text{ m} = 4.9 \frac{\text{m}}{\text{s}^2} \cdot \Delta t^2$$

$$\Delta t = 0.639 \text{ s}$$

Solve for  $V_x + V_y$ :

$$V_x = \frac{\Delta x}{\Delta t} = \frac{8 \text{ m}}{0.639 \text{ s}} = 12.52 \text{ m/s} \quad V_y = V_x = 12.52 \text{ m/s}$$

Solve for  $V_f$ :

$$V_f = V_y + g \Delta t = 12.52 \text{ m/s} + (9.8 \frac{\text{m}}{\text{s}^2})(0.639 \text{ s}) = 18.78 \text{ m/s}$$

$$V_{fx} = V_x = 12.52 \text{ m/s}$$

$$\begin{array}{c} V_{fx} = 12.52 \text{ m/s} \\ V_f = ? \\ V_f = \sqrt{V_x^2 + V_y^2} \end{array}$$

$$V_f = \sqrt{12.52^2 + 18.78^2} = 22.6 \text{ m/s}$$

$$\underline{\underline{= 22.6 \text{ m/s}}}$$