

# How to prove you have built a quantum computer

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Gregory D. Kahanamoku-Meyer

November 2, 2023

# Introduction

... or, how did I get here?



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- Currently living on O‘ahu while continuing my research remotely as a postdoc



# Quantum computing: motivation

How hard is simulating quantum systems with (regular) computers?

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Single quantum system with two states

Ex: spin-1/2 particle

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Single quantum system with two states

Ex: spin-1/2 particle

$|1\rangle$  —————

$|0\rangle$  —————

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Quantum state represented by  
2 complex numbers

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How hard is simulating quantum systems with (regular) computers?

15 quantum particles with two states each

Ex: 15 spin-1/2 particles

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Quantum state represented by  
 $2^{15} \approx 30,000$  complex numbers

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How hard is simulating quantum systems with (regular) computers?

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Ex: 30 spin-1/2 particles

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Quantum state represented by  
 $2^{30} \approx 1,000,000,000$  complex numbers

# Quantum computing: motivation

Complexity grows exponentially with the number of particles!

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Complexity grows exponentially with the number of particles!

Can we use that complexity to perform computations?

## Quantum computing: history

Early 90s: Theoretical algorithms for “bespoke” problems built for quantum computers

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## Simon's problem

Given a function (implemented by a black box or oracle)  $f : \{0, 1\}^n \rightarrow \{0, 1\}^n$  with the promise that, for some unknown  $s \in \{0, 1\}^n$ , for all  $x, y \in \{0, 1\}^n$ ,

$$f(x) = f(y) \text{ if and only if } x \oplus y \in \{0^n, s\},$$

where  $\oplus$  denotes bitwise XOR. The goal is to identify  $s$  by making as few queries to  $f(x)$  as possible. Note that

$$a \oplus b = 0^n \text{ if and only if } a = b$$

Furthermore, for some  $x$  and  $s$  in  $x \oplus y = s$ ,  $y$  is unique (not equal to  $x$ ) if and only if  $s \neq 0^n$ . This means that  $f$  is two-to-one when  $s \neq 0^n$ , and one-to-one when  $s = 0^n$ . It is also the case that  $x \oplus y = s$  implies  $y = s \oplus x$ , meaning that

$$f(x) = f(y) = f(x \oplus s)$$

## Quantum computing: history

**Mid 90s:** Theoretical algorithms for real problems!

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Mid 90s: Theoretical algorithms for real problems!

Grover search

Faster searching



Shor's algorithm

Faster integer factorization

$$pq \rightarrow p \cdot q$$

## Framing the question

Goal: construct a physical system that can actually run these algorithms!

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Thursday, August 4, 2022  
Today's Paper

# The New York Times

World U.S. Politics N.Y. Business Opinion Tech Science Health Sports Arts Books

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**Theranos Leaves Biotech Business, Turns to Building Quantum Computers**

- CEO Elizabeth Holmes states the emerging field of quantum computing will be a "new start" for the company
- Despite extensive fraud at previous company, investors inexplicably believe it's a good idea to dump millions of dollars into this new venture



This is not a real headline! It is a joke.

## Framing the question

Goal: construct a physical system that can actually run these algorithms!

Suppose someone opens a cloud service to perform quantum computations.

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How do we test if they are really doing anything quantum?

With only classical questions and answers?

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Goal: construct a physical system that can actually run these algorithms!

Suppose someone opens a cloud service to perform quantum computations.

How do we test if they are really doing anything quantum?

With only classical questions and answers?

Maybe we can use those algorithms I just mentioned?

# Grover search

Cost to find the “good” value from  $N$  indices



Quantum

$\mathcal{O}(\sqrt{N})$  operations

Classical

$\mathcal{O}(N)$  operations



# Grover search

Cost to find the “good” value from  $N$  indices

X	X	X	X	X	X	X	✓	X	X	X	X
0	1	2	3	4	5	6	7	8	9	10	

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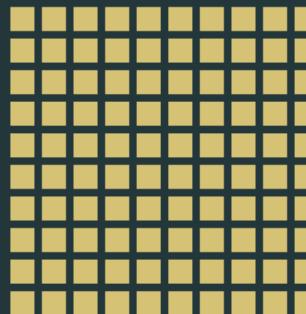
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# Quantum speedups

Task	Theoretical speedup	Practical in 2023?
Grover search	Somewhat fewer ops.	Quantum computers too slow

# Shor's algorithm

Goal: factor numbers  $pq = p \cdot q$

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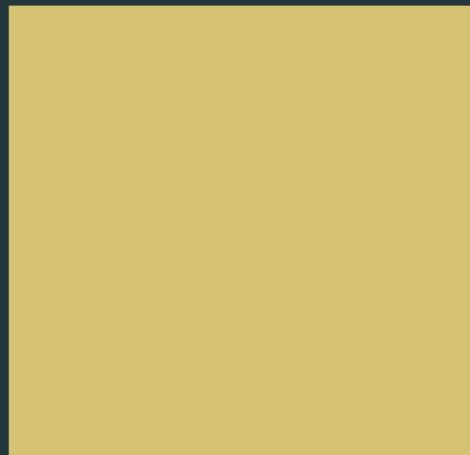
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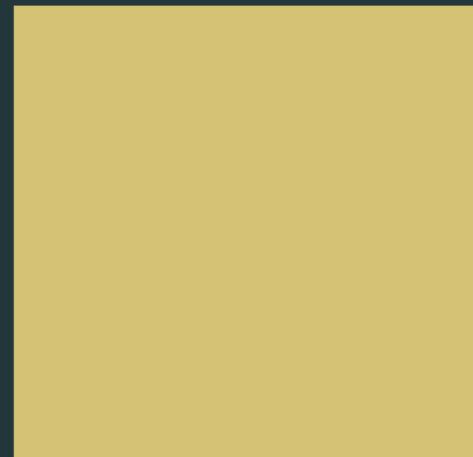
# Challenge: quantum computers too noisy

Quantum information very fragile!

Quantum



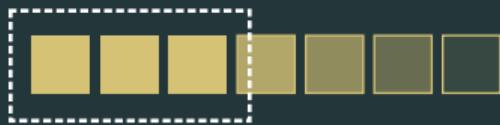
Classical



## Challenge: quantum computers too noisy

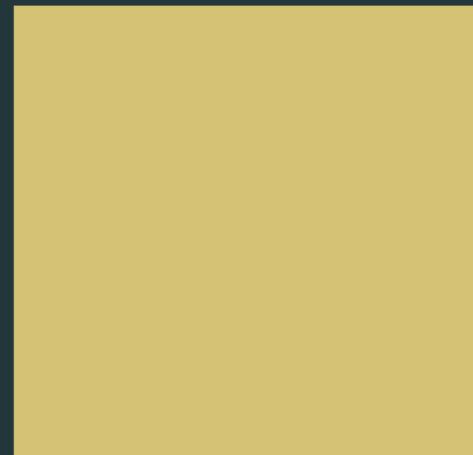
Quantum information very fragile! And devices are small!

Quantum



size of largest existing device

Classical



# Quantum advantage in practice

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Chemistry	Depends	Too small, slow and noisy

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Is there *anything* current quantum computers can do that classical ones can't?

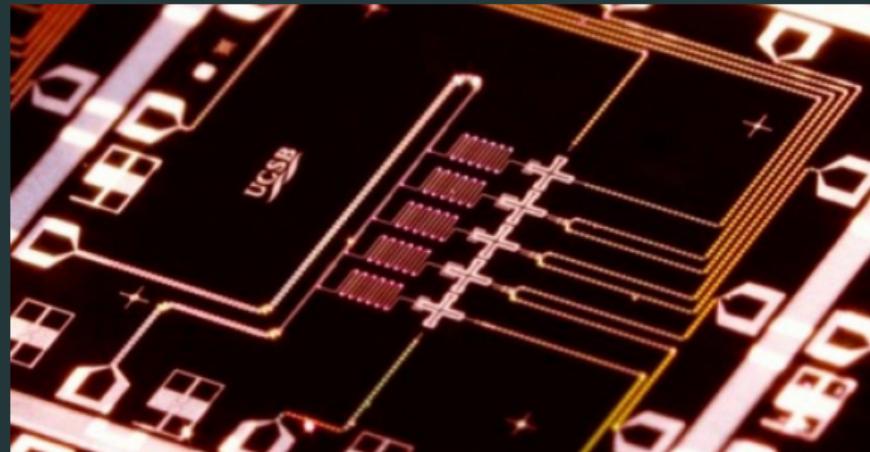
# Demonstrating “quantum advantage”

To prove we have built a quantum computer, the problem doesn't have to be *useful*

Is there *anything* current quantum computers can do that classical ones can't?

10 years ago: nope!

Trivial to simulate!



Google/UCSB's 5-qubit chip

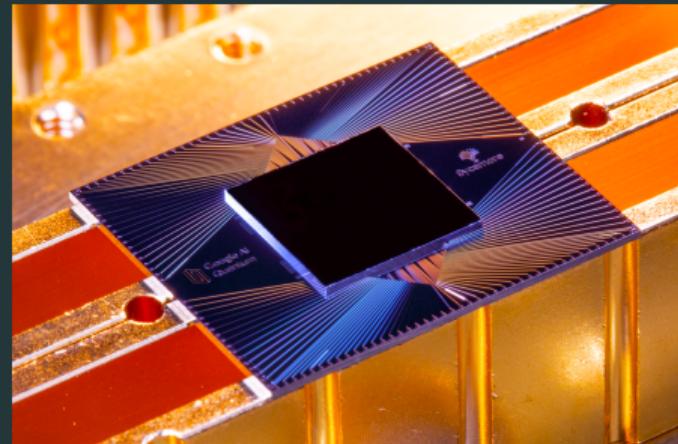
# Demonstrating “quantum advantage”

To prove we have built a quantum computer, the problem doesn't have to be *useful*

Is there *anything* current quantum computers can do that classical ones can't?

Since 4 years ago: maybe??

Very hard to simulate!



Google's 53-qubit chip

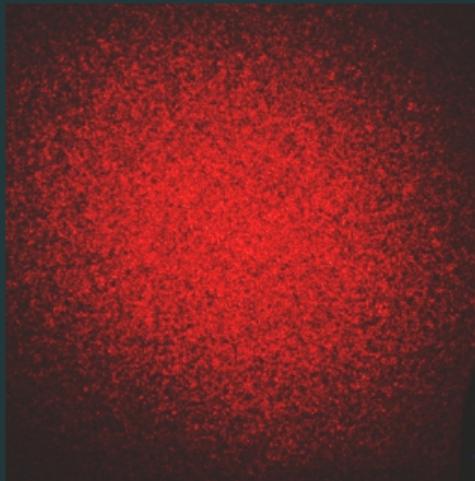
## Random circuit sampling

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Sampling from an “speckle” (interference pattern)



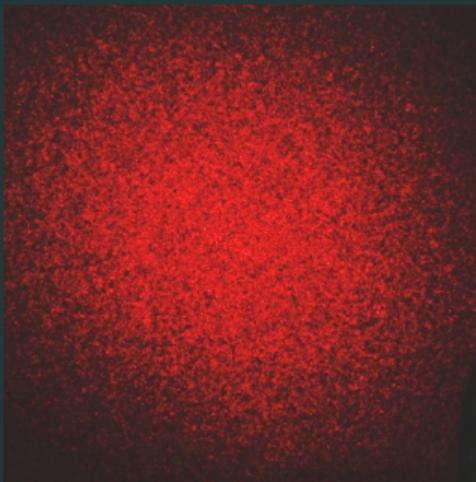
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**Mathematical problem:**

1. Define some operations that generate a complicated quantum state

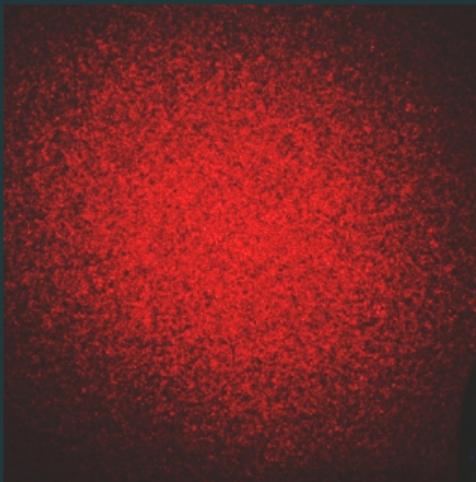
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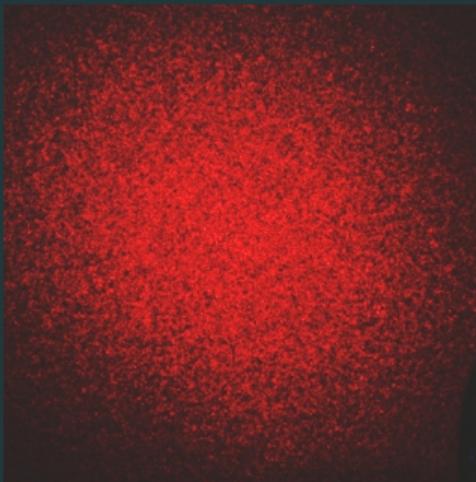
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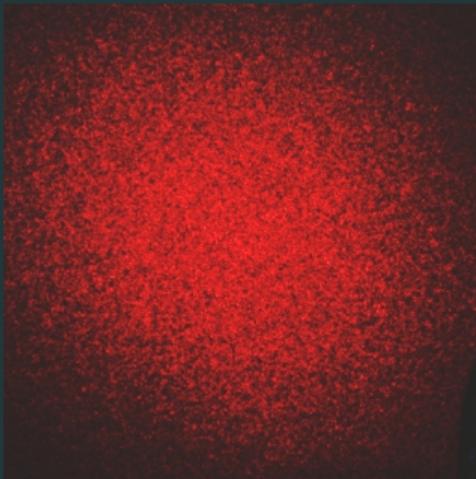
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If distribution is complicated enough, generating samples is classically hard

# Google's results

The screenshot shows the homepage of the **nature** journal website. At the top, there is a navigation bar with three dropdown menus: "Explore content", "About the journal", and "Publish with us". Below the navigation bar, the URL "nature > articles > article" is displayed. Underneath, it says "Article | Published: 23 October 2019". The main title of the article is "**Quantum supremacy using a programmable superconducting processor**". Below the title, the authors are listed: Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando G. S. L. Brandao, David A. Buell, Brian Burkett, Yu Chen, Zijun.

## A subtle challenge: verification

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Google published a bunch of samples from a “hard” probability distribution.

How do we confirm they actually came from that distribution?

1. “Benchmark” quantum device by sampling from related but easy distributions
2. Assume nothing weird happens when you switch to the hard distribution

# Quantum advantage

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Grover search	Somewhat fewer ops.	Too small, slow and noisy
Shor's factoring	Exponentially fewer ops.	Too small and noisy
Machine learning	Depends	Too small, slow and noisy
Chemistry	Depends	Too small, slow and noisy
Random sampling	Exponentially fewer ops.	Yes, but can't check answer

## Verifiable quantum advantage

We want a problem that is **hard to classically solve**, but **easy to classically check**

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Factoring and search are such problems!

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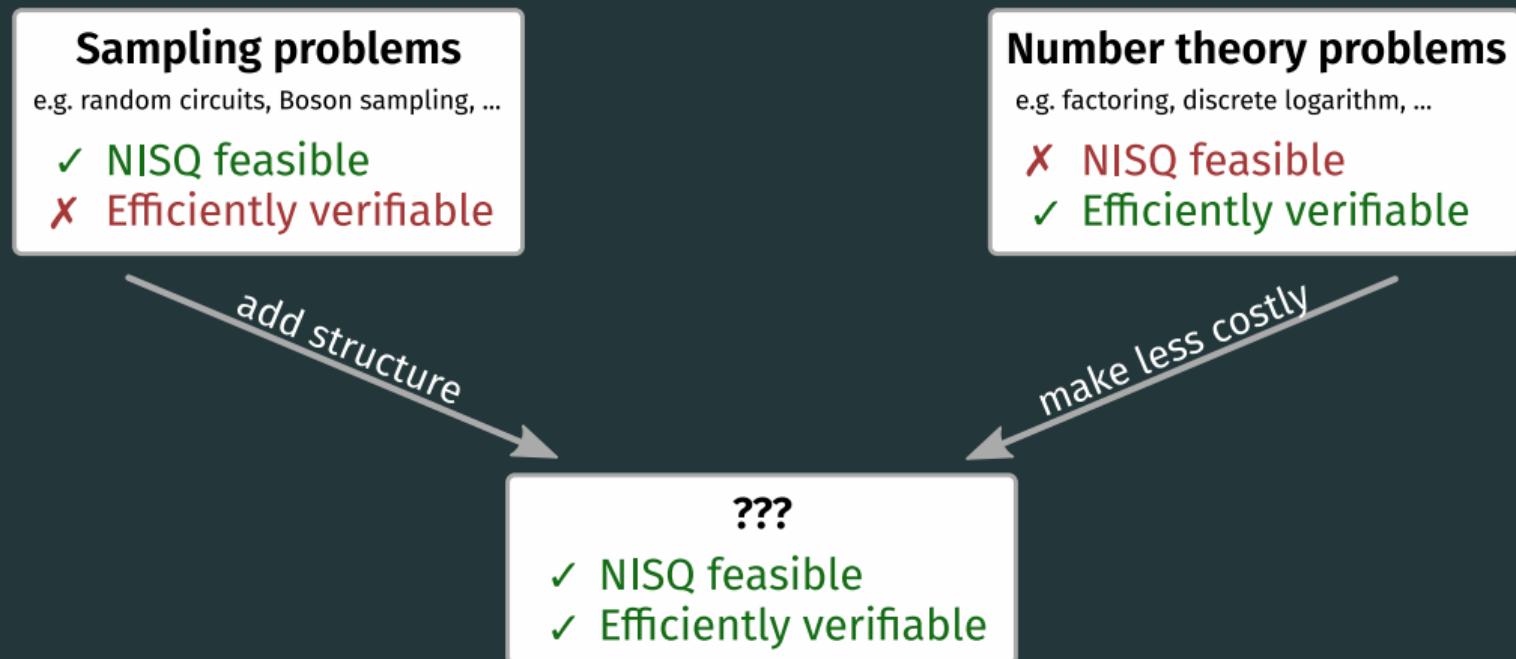
We want a problem that is **hard to classically solve**, but **easy to classically check**

Factoring and search are such problems!

But we also want **achievable on near-term quantum device**

# NISQ verifiable quantum advantage

NISQ = “noisy intermediate-scale quantum”



## Adding structure to sampling problems

**Example:** evolve a quantum system under “IQP” Hamiltonians (products of Pauli  $X$ ’s)

$$H = X_0X_1X_3 + X_1X_2X_4X_5 + \dots \quad (1)$$

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But how sure are we that the secret is really hidden?

# The \$25 challenge

Alice's quantum challenge  
C'mon Bob, show us how quantum you really are

"Bob,  
do u haz  
qwantum?"

Alice

If u iz qwantum u can  
run this program

I haz data.  
Iz I qwantum?

Bob

Challenge    Code

Alice's \$25 quantum challenge    Posted by: mick | September 4, 2008

Hi I'm Alice (and by alice we mean mick and Dan) and this is my new blog.

My friend Bob says that he has a quantum computer and I'm not really sure I believe him, and, in a lot of ways I'm not so sure that Bob believes himself either.

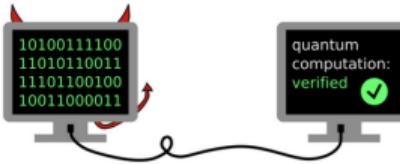
It's a good thing that I came across [this paper](#) (by Dan Shepherd and Michael Bremner) which

PAGES

- Challenge
- Code

# Classical algorithm to extract secret

PAPER



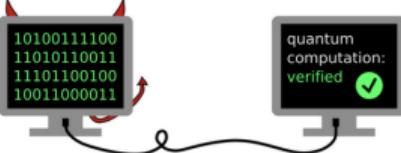
Forging quantum data: classically defeating an IQP-based quantum test

Gregory D. Kahanamoku-Meyer,  
[Quantum 7, 1107 \(2023\)](#).

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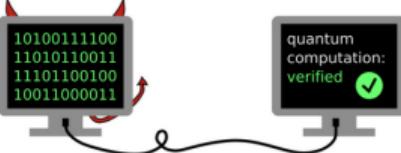
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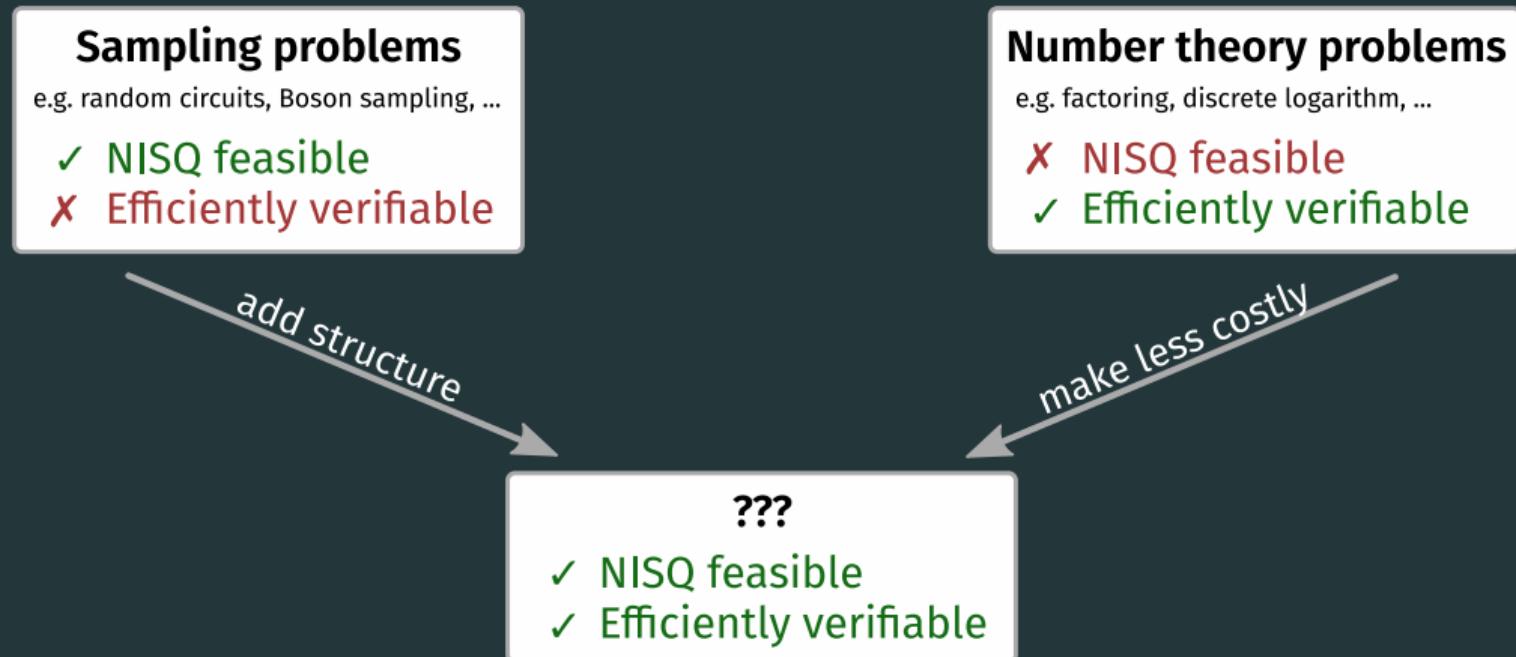
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[Bremner, Cheng, Ji 2023]: New scheme where the secret is (hopefully) hidden better

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## Making number theory problems less costly

Generating a quantum state that involves the factors is easy—  
getting the factors out as classical values is the hard part!

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getting the factors out as classical values is the hard part!

Idea from cryptography: zero-knowledge proof

## Zero-knowledge proofs: differentiating colors

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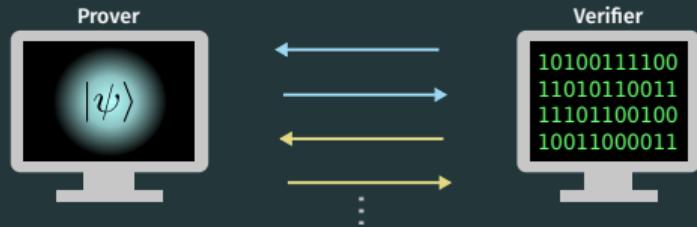
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Seeing color  $\Leftrightarrow$  Quantum capability

**Goal:** find protocol as verifiable and classically hard as factoring—but less expensive than actually finding factors (via Shor)

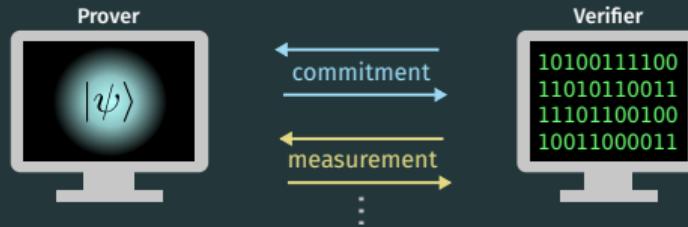
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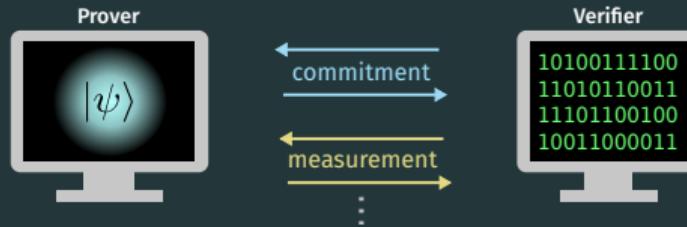


Round 1: Prover commits to holding a specific quantum state

Round 2: Verifier asks for measurement in random basis, prover performs it

# Interactive proofs of quantumness

Multiple rounds of interaction between the prover and verifier



Round 1: Prover commits to holding a specific quantum state

Round 2: Verifier asks for measurement in random basis, prover performs it

By randomizing choice of basis and repeating interaction,  
can ensure prover actually has the promised quantum state

Brakerski, Christiano, Mahadev, Vidick, Vazirani '18 (arXiv:1804.00640).

Can be extended to verify arbitrary quantum computations! (arXiv:1804.01082)

## Commitment: a secret quantum state

How does the prover commit to a state?

Consider a **2-to-1** function  $f$ :

for all  $y$  in range of  $f$ , there exist  $(x_0, x_1)$  such that  $y = f(x_0) = f(x_1)$ .

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Generate entangled superposition

$$\sum_x |x\rangle |f(x)\rangle$$

$$\xleftarrow{f} \xrightarrow{f}$$



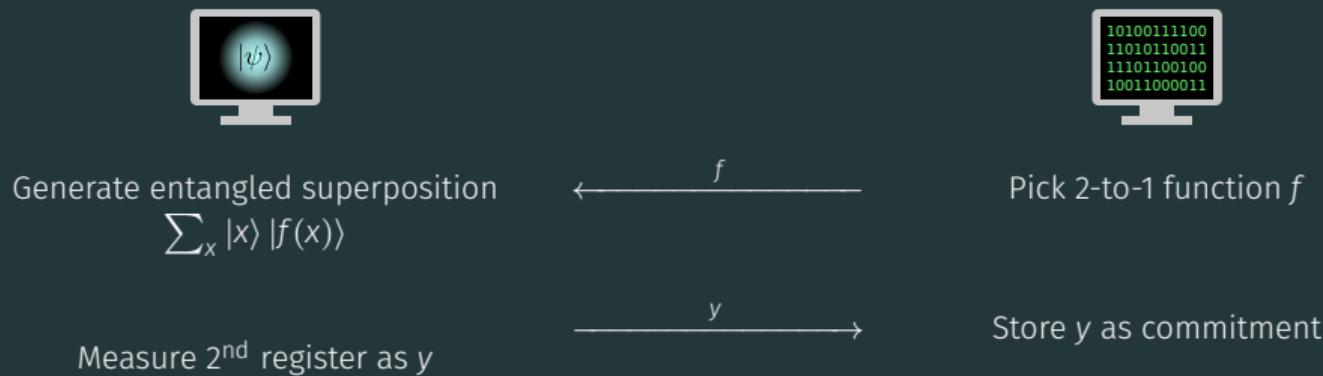
Pick 2-to-1 function  $f$

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Prover has committed to the state  $(|x_0\rangle + |x_1\rangle) |y\rangle$

## State commitment (round 1): trapdoor claw-free functions

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Generating a valid state without trapdoor uses  
superposition + wavefunction collapse—Inherently quantum!

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We have  $4^2 \equiv 11^2 \equiv 16 \pmod{35}$ ; and  $11 - 4 = 7$

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Idea: use the same circuits that we do in classical computers?

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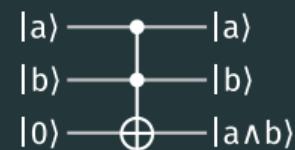
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Quantum AND (Toffoli)

If you're not careful, you will use up all of your precious qubits  
storing this “garbage data”!

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Applications include proving “quantumness” but also factoring and other algorithms!

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Using the new protocol:

~ 2,000,000 quantum operations

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Trapped ions at the University of Maryland

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For interactive protocol, need to **measure a subset** of the quantum particles!



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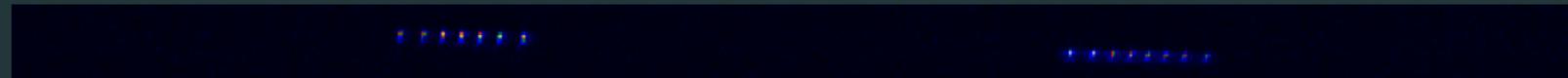
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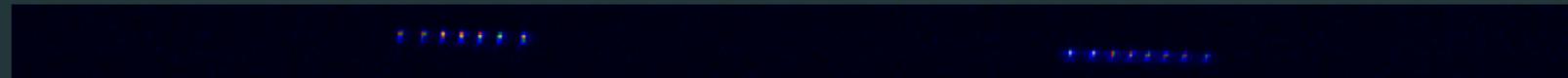
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Thank you!