Homework 1: Categorical Data

Nicholas G Reich, for Biostats 743 at UMass-Amherst

Your assignment should be submitted in two separate files by 5pm on Friday September 21st. The first, should be an RMarkdown (.Rmd) or LaTeX (.tex) file. The second should be the PDF file that was reproducibly compiled using the first file. Before submitting your files, create a folder on Google Drive with the name "[lastname]-[firstname]-cda" (e.g. "Reich-Nicholas-cda") and share it with the TA at `zhengfanwang@umass.edu`. The homework files should then be submitted by copying the files into this folder.

Question 1

Say you flip a coin 100 times and get 53 heads.

a. Write down the Score test statistic and by hand (i.e. don't used a canned R function to calculate the test statistic for you) conduct a test of the null hypothesis $H_0 : \pi$=0.5.
b. Draw a picture of the likelihood and show what features of the likelihood are used for the Score test statistic.
c. Explain (using words, pictures, and equations) how the Score test uses different information than the likelihood ratio test.

Question 2

Conduct a simulation study to evaluate the coverage probabilities of different methods for computing confidence intervals for π. You may vary the true π, the sample size, and must choose at least 4 methods to compare. Your results should be summarized with a one paragraph description of the implementation of the simulation study, a one paragraph descirption of the results, and one figure (multi-panel is ok) summarizing the results.

Question 3

Prove that if $Y \sim Binom(n, \pi)$ and $\pi \sim beta(\alpha_1, \alpha_2)$ (for $\alpha_1 > 0$ and $\alpha_2 > 0$) then the posterior $h(\pi|y)$ is a beta distribution, specifically, h follows a $beta(y + \alpha_1, n - y + \alpha_2)$.

Question 4

Complete problem 1.17 in CDA.

Question 5

Complete problem 1.3 in CDA.

Question 6

Complete problem 1.22 in CDA.

Question 7

Complete problem 1.29 in CDA.

Question 8

Complete problem 1.30 in CDA.