Spend Optimization Framework

We define a framework for optimizing media spend that is rooted in probabilistic profitability rather than mean expectations. The goal is to estimate, for each channel and each discretized spend level, the probability that the revenue outcome exceeds cost, and to conditionally estimate expected returns given success.

Let $R(s)$ denote the estimated causal revenue at spend level s, sampled from the posterior predictive distribution. Let $c(s)$ denote the associated cost (often s itself, or a scaled version). Define:

$$\text{Profitability Indicator: } I_s^{(b)} = \left\{ \begin{matrix} 1 & \text{if } R^{(b)}(s) > c(s) \\ 0 & \text{otherwise} \end{matrix} \right)$$

where $R^{(b)}(s)$ is the b-th posterior draw of revenue at spend s.

Across B posterior samples, define:

$$P_{\text{profitable}}(s) = \frac{1}{B} \sum_{b=1}^{B} I_s^{(b)}$$

$$E\left[R(s) \mid R(s) > c(s)\right) = \frac{\sum_{b=1}^{B} I_s^{(b)} \cdot R^{(b)}(s)}{\sum_{b=1}^{B} I_s^{(b)}}$$

These quantities allow us to frame the optimization as a choice over spend levels that maximize the conditional expected return subject to a minimum profitability threshold:

$$s^{¿} = \underset{s \in S}{arg\,max}\, E\left[R(s) \mid R(s) > c(s)\right) \text{subject to } P_{\text{profitable}}(s) \geq \tau$$

where τ is a business-defined threshold (e.g., 60%).

This method addresses the non-convergent nature of real-world ad spend decisions: each spend level is a one-time action, not an expectation over many repeated trials. As such, using the probability of success and conditional gains provides a more realistic and robust decision framework.