Model Specification

This section outlines the mathematical formulation of the MMX model.

Aggregate Revenue Model

We model the total observed revenue Y_t at time t as a function of organic and paid components:

$$Y_t = \left(S_t^{\text{org}} \cdot \nu + S_t^{\text{paid}} \cdot \sum_c \beta_c H_c\left(A_c\left(X_{t,c}\right)\right)\right) \cdot T_t + \epsilon_t$$

Latent States and Shared Trend

The shared latent state S_t is decomposed into paid and organic components via channel-specific deltas:

$$S_t^{\text{paid}} = S_t \cdot \delta_t^{\text{paid}}$$

$$S_t^{\text{org}} = S_t \cdot \delta_t^{\text{org}}$$

The shared state S_t is modeled as a softplus-transformed ARMA(3,1) process:

$$S_t = \text{softplus}\left(\sigma_s \cdot \left(\rho_1 z_{t-1} + \rho_2 z_{t-2} + \rho_3 z_{t-3} + \theta e_{t-1}\right)\right)$$

Media Effects

Each channel's media effect is modeled with a Hill function applied to an adstocked input:

$$H_c\left(A_c\left(X_{t,c}\right)\right) = \frac{1}{1 + \left(\frac{A_c\left(X_{t,c}\right)}{k_c}\right)^{-slope_c}}$$

Event and Seasonality Effects

A local linear trend is used to capture seasonality and long-run changes:

$$T_t = 1 + \mu_t + \beta_h H_t + \beta_p P_t + \beta_s S_t + \ldots$$

SKAN Adjustment

The SKAN signal $Y_{t,c}^{\text{SKAN}}$ is modeled with bias terms for cannibalization and halo:

$$Y_{t,c}^{\text{SKAN}} = \text{base}_{t,c} + \text{Cannib}_{t,c} + \text{Halo}_{t,c} + \eta_{t,c}$$

$$\text{Cannib}_{t,c} = \sum_{j \neq c} \gamma_j \log\left(1 + X_{t,j}\right) + \omega_c \log\left(1 + X_{t,c}\right) \log\left(1 + S_t^{\text{org}}\right)$$

$$\mathrm{Halo}_{t,c} = -\lambda_c \log\left(1 + X_{t,c}\right)$$