QUADRILATERALS THAT ARE PARALLELOGRAMS
&
PROPERTIES OF PARALLELOGRAMS

Objectives

- recall the different quadrilaterals;
- determine the conditions that make a quadrilateral a parallelogram through an investigation using an online geometry tool;
- use the properties of parallelograms to find the measures of angles, sides and other quantities involving parallelograms; and
- appreciate parallelograms through examples and real-life applications.

Table 1: The Quadrilaterals

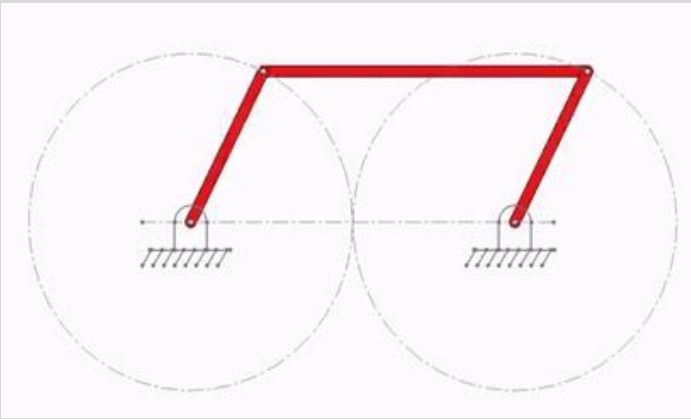
NAME/TYPE	DEFINITION/DESCRIPTION	FIGURE
Quadrilateral	- A four-sided figure having four straight sides.	
Parallelogram	- A quadrilateral that has equal and parallel opposite sides.	

Table 1: The Quadrilaterals

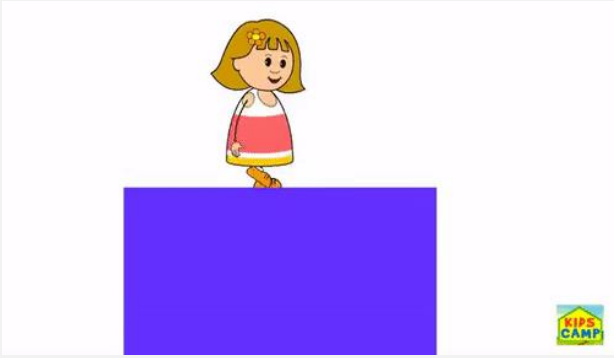
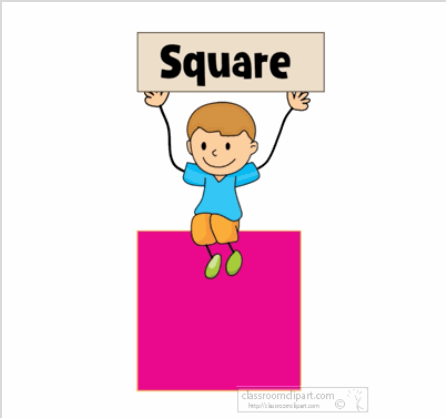
NAME/TYPE	DEFINITION/DESCRIPTION	FIGURE
Rectangle	- Is a quadrilateral with 4 straight sides and 4 right angles. The opposite sides have the same lengths and are parallel.	
Square	- A quadrilateral with 4 equal straight sides and 4 right angles. Opposite sides are parallel.	

Table 1: The Quadrilaterals

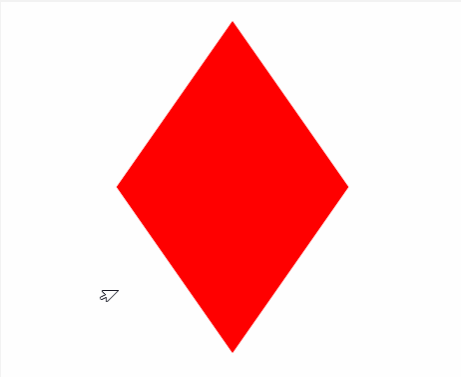
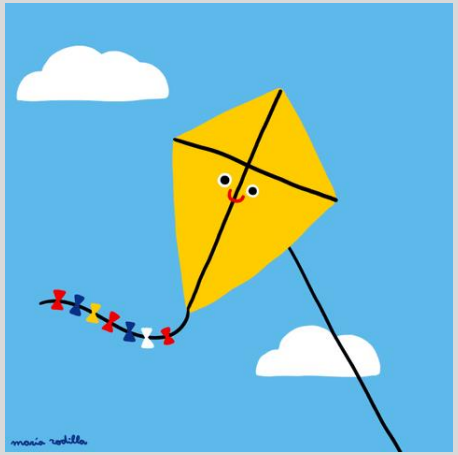
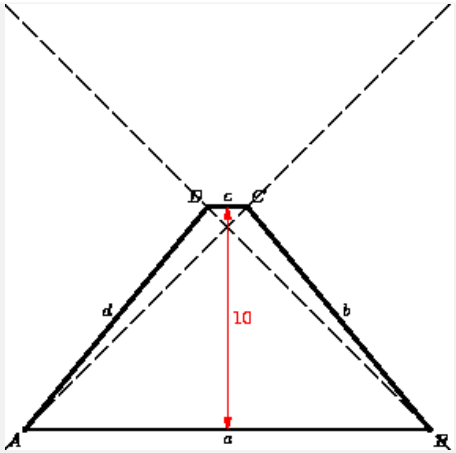
NAME/TYPE	DEFINITION/DESCRIPTION	FIGURE
Rhombus	- Is a quadrilateral with 4 straight sides that are all equal in length. Opposite sides are congruent.	
Kite	- Is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are adjacent to each other.	

Table 1: The Quadrilaterals

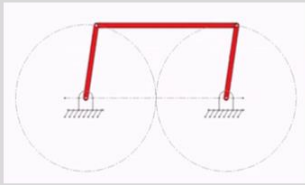
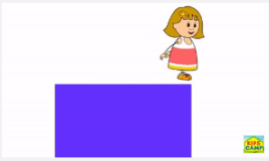
NAME/TYPE	DEFINITION/DESCRIPTION	FIGURE
Trapezoid	- Is a quadrilateral with only one pair of parallel sides.	

The Quadrilaterals

QUESTIONS to ponder:

Recall the definitions of the quadrilaterals: parallelogram, rectangle, square, rhombus, kite, and trapezoid.

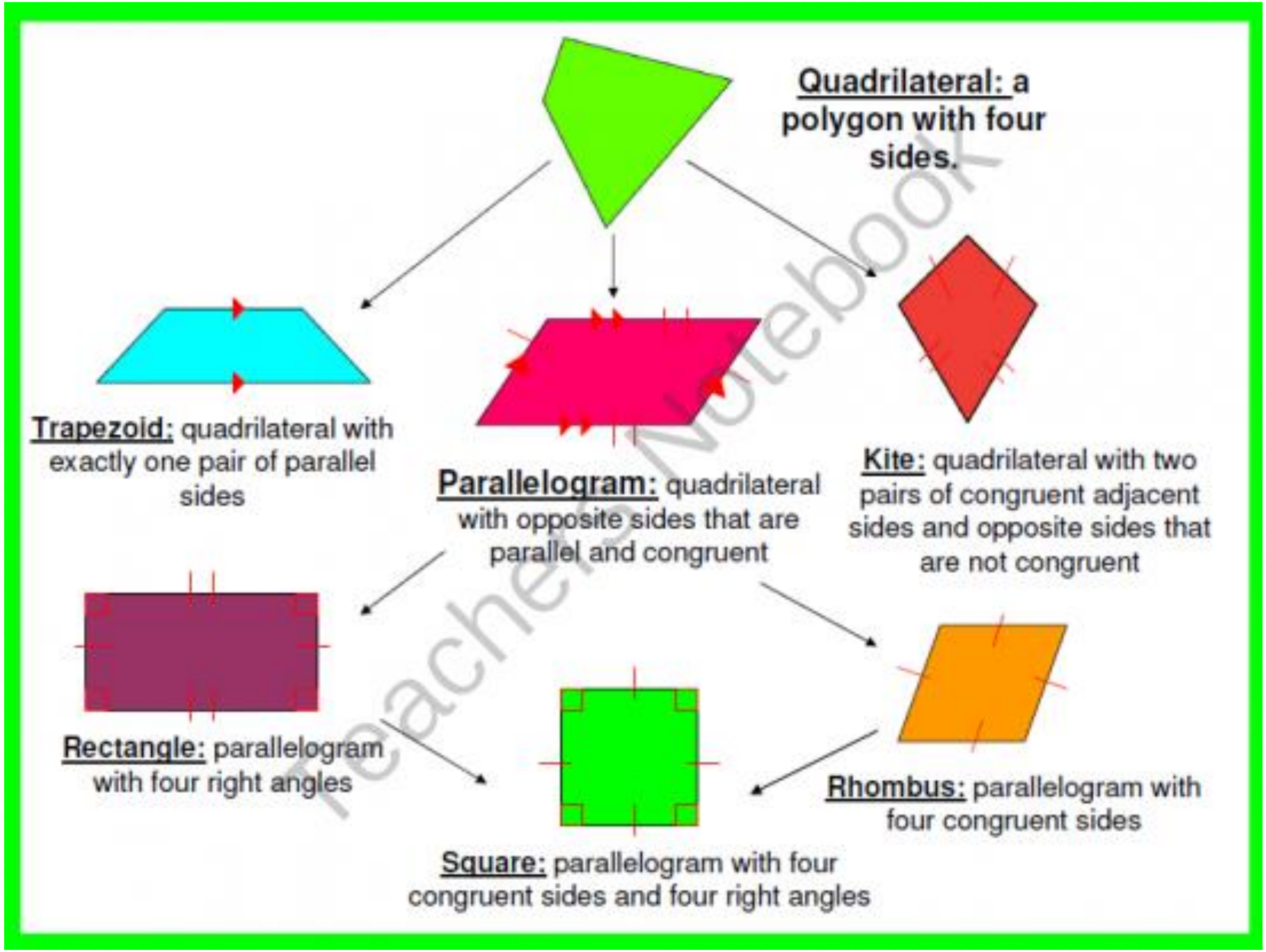
Are there similarities?
What is the implication?

Parallelogram	- A quadrilateral that has equal and parallel opposite sides.	
Rectangle	- Is a quadrilateral with 4 straight sides and 4 right angles. The opposite sides have the same lengths and are parallel.	
Square	- A quadrilateral with 4 equal straight sides and 4 right angles. Opposite sides are parallel.	
Rhombus	- Is a quadrilateral with 4 straight sides that are all equal in length. Opposite sides are congruent.	

Let's Summarize!

Description	QUADRILATERALS					
	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Kite
1. Opposite sides are parallel.						
2. Opposite sides are congruent.						
3. Opposite angles are congruent.						
4. Has at least 1 pair of consecutive sides congruent.						
5. Has exactly 1 pair of parallel sides.						

The Quadrilaterals

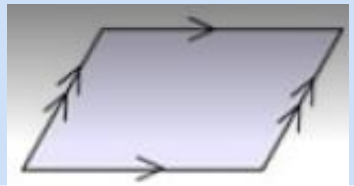
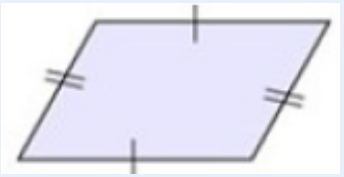
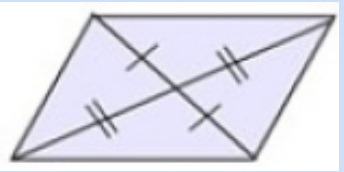
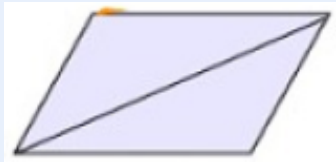


Quadrilaterals That Are Parallelograms

What properties are true to all parallelograms?

What are the properties of parallelograms?

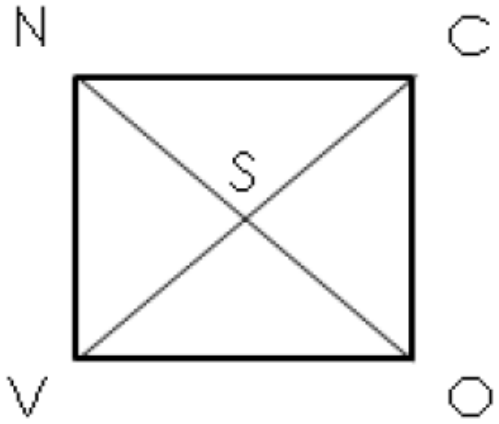
PROPERTIES OF PARALLELOGRAM

Definition: A parallelogram is a quadrilateral with both pairs of opposite sides parallel.		
SIDES	**In a parallelogram, any two opposite sides are congruent.**	
ANGLES	In a parallelogram, any two opposite angles are congruent.	
	In a parallelogram, any two consecutive angles are supplementary.	
DIAGONALS	The diagonals of a parallelogram bisect each other.	
	A diagonal of a parallelogram forms two congruent triangles.	

Activity 1

Directions: For each of the following, state the property that can support why NCOV with diagonals that meet at S, is a parallelogram.

For items 1-5:



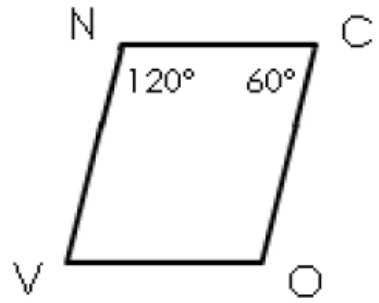
	Property
1. $\overline{NS} \cong \overline{OS}; \overline{VS} \cong \overline{CS}$	_____
2. $\overline{NC} \cong \overline{CO} \cong \overline{OV} \cong \overline{VN}$	_____
3. $\Delta NVO \cong \Delta OCN$	_____
4. $m\angle COV + m\angle OVN = 180°$	_____
5. $\angle CNV \cong \angle VOC; \angle OVN \cong \angle NCO$	_____

Activity 1

03:00

Directions: For each of the following, state the property that can support why NCOV with diagonals that meet at S, is a parallelogram.

1



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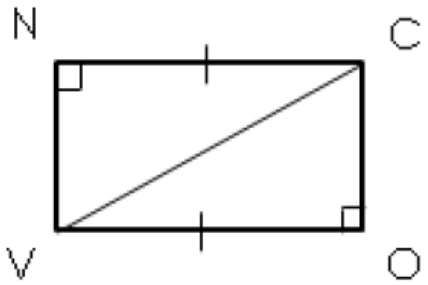

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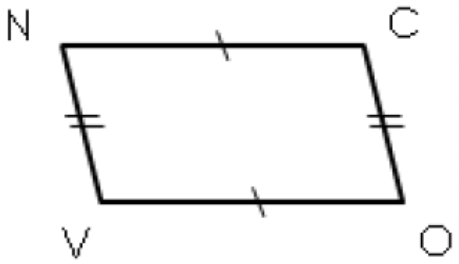

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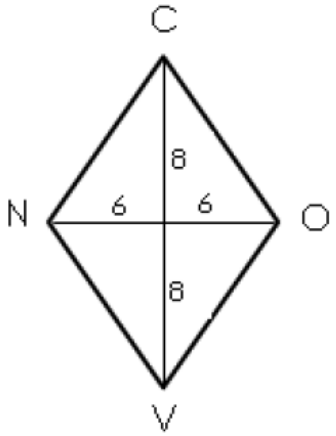
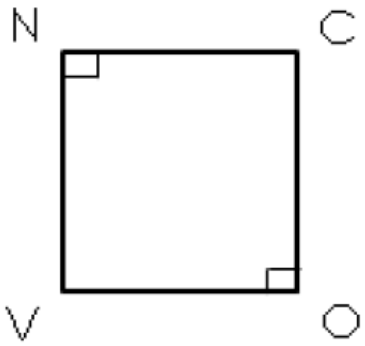


5

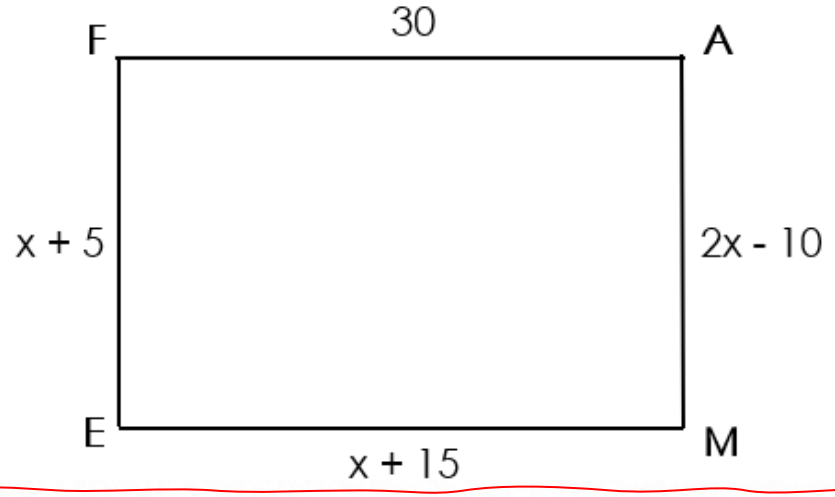


How to use the properties?

Example 1:

Quadrilateral FAME is a parallelogram. Find: (a) x; (b) EM; (c) EF; (d) MA; and the perimeter of FAME if FA = 30, EM = x + 15, EF = x + 5, and MA = 2x – 10.

STEP 1: Understand the problem.



What properties of parallelograms can be used?

❖ *In a parallelogram, any two opposite sides are congruent.*

STEP 2: Devise a plan.

FA = EM and EF = MA

P = FA + EM + EF + MA

STEP 3: Carry out the plan.

To solve for x, consider that FA = EM.

Solution: EM = FA;

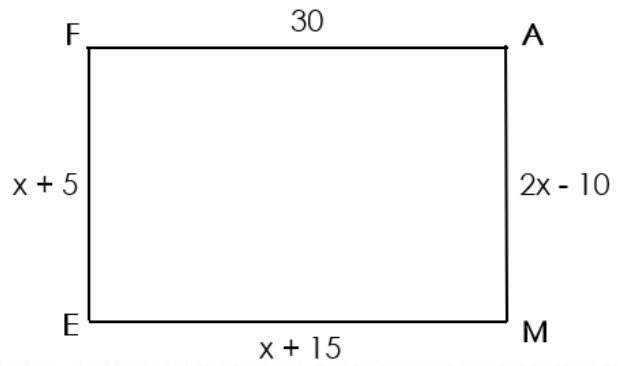
$x + 15 = 30$

$x = 15$

Example 1:

Quadrilateral FAME is a parallelogram. Find: (a) x; (b) EM; (c) EF; (d) MA; and the perimeter of FAME if FA = 30, EM = x + 15, EF = x + 5, and MA = 2x – 10.

FA = EM and EF = MA
P = FA + EM + EF + MA



To find EF, use substitution property.

Solution: $EF = x + 5$
$= 15 + 5$
$EF = 20$

MA = 2x – 10
= 2(15) – 10
= 30 – 10
= 20

STEP 3: Carry out the plan.

To solve for x, consider that FA = EM.

Solution: $EM = FA$;
$x + 15 = 30$
$x = 15$

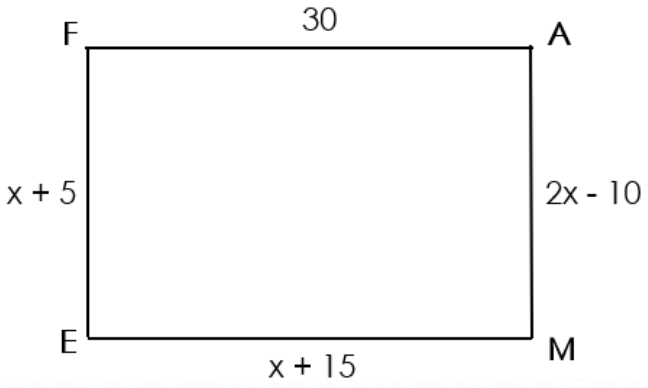
To find EM, use substitution property.

$EM = x + 15 = 15 + 15 = 30$

Example 1:

Quadrilateral FAME is a parallelogram. Find: (a) x; (b) EM; (c) EF; (d) MA; and the perimeter of FAME if FA = 30, EM = x + 15, EF = x + 5, and MA = 2x – 10.

FA = EM and EF = MA
P = FA + EM + EF + MA



STEP 3: Carry out the plan.

To find the perimeter, add all the side lengths.

Solution: P = FA + MA + EM + EF
P = 30 + 20 + 30 + 20
P = 100

STEP 4: Look Back.

❖ *In a parallelogram, any two opposite sides are congruent.*

FA = EM and EF = MA
P = FA + EM + EF + MA

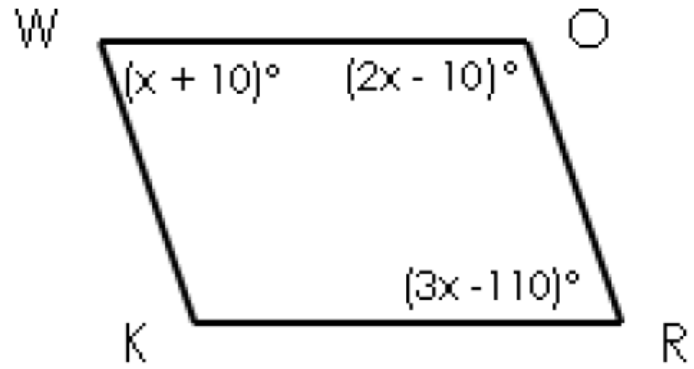
FA = 30
EM = 30

EF = 20
MA = 20

Example 2:

Quadrilateral WORK is a parallelogram. Find: (a) x; (b) $m\angle W$; (c) $m\angle O$; (d) $m\angle R$; (e) $m\angle K$; and (f) the sum of the interior angles of WORK if $m\angle W = (x+10)°$, $m\angle O = (2x-10)°$, and $m\angle R = (3x-110)°$.

STEP 1: Understand the problem.



What properties of parallelograms can be used?

STEP 2: Devise a plan.

$m\angle W = m\angle R$

$m\angle O = m\angle K$

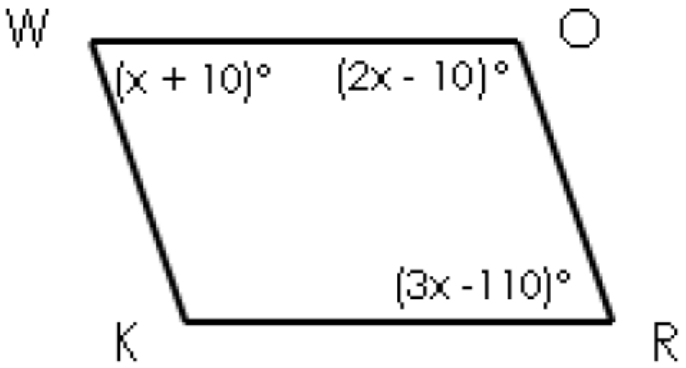
$m\angle W + m\angle O = 180°$

$m\angle O + m\angle K = 180°$

- *In a parallelogram, any two opposite angles are congruent.*
- *In a parallelogram, any two consecutive angles are supplementary.*

Example 2:

Quadrilateral WORK is a parallelogram. Find: (a) x; (b) $m\angle W$; (c) $m\angle O$; (d) $m\angle R$; (e) $m\angle K$; and (f) the sum of the interior angles of WORK if $m\angle W = (x+10)°$, $m\angle O = (2x-10)°$, and $m\angle R = (3x-110)°$.



$m\angle W = m\angle R$

$m\angle O = m\angle K$

$m\angle W + m\angle O = 180°$

$m\angle O + m\angle K = 180°$

To find the measure of angle O, use substitution property.

$$m\angle O = (2x-10)°$$
$$= [2(60)-10]°$$
$$m\angle O = 110°$$

STEP 3: CARRY OUT THE PLAN.

To solve for x, consider angles W and R.

Solution: $m\angle W = m\angle R$

$$x+10 = 3x-110$$
$$120 = 2x$$
$$60 = x$$
$$x = 60$$

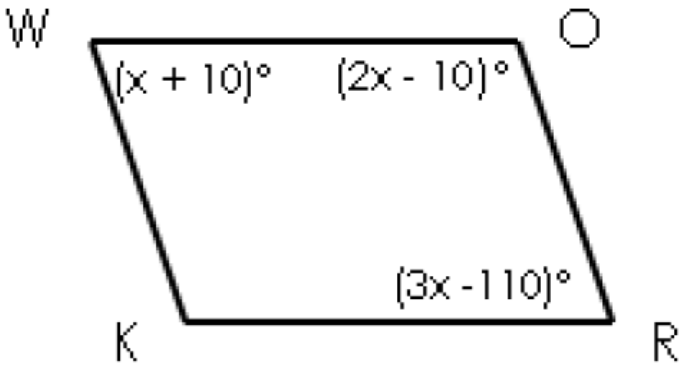
To find the measure of angle W, use substitution property.

Solution: $m\angle W = (x+10)°$

$$m\angle W = (60+10)°$$
$$m\angle W = 70°$$

Example 2:

Quadrilateral WORK is a parallelogram. Find: (a) x; (b) $m\angle W$; (c) $m\angle O$; (d) $m\angle R$; (e) $m\angle K$; and (f) the sum of the interior angles of WORK if $m\angle W = (x+10)°$, $m\angle O = (2x-10)°$, and $m\angle R = (3x-110)°$.



$m\angle W = m\angle R$

$m\angle O = m\angle K$

$m\angle W + m\angle O = 180°$

$m\angle O + m\angle K = 180°$

To find the sum of the interior angles of WORK:

$$m\angle W + m\angle O + m\angle R + m\angle K$$
$$= 70° + 110° + 70° + 110°$$
$$= 360°$$

STEP 3: CARRY OUT THE PLAN.

To find the measure of angle R, use substitution property.

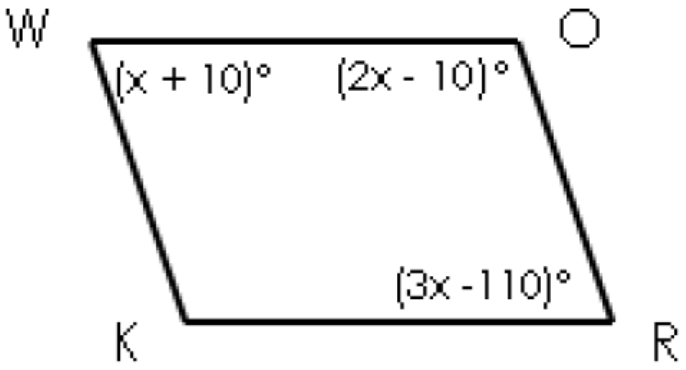
$$m\angle R = (3x-110)°$$
$$= [3(60)-110]°$$
$$= (180-110)°$$
$$= 70°$$

Since $m\angle O = m\angle K$,

$m\angle K = 110°$

Example 2:

Quadrilateral WORK is a parallelogram. Find: (a) x; (b) $m\angle W$; (c) $m\angle O$; (d) $m\angle R$; (e) $m\angle K$; and (f) the sum of the interior angles of WORK if $m\angle W = (x+10)°$, $m\angle O = (2x-10)°$, and $m\angle R = (3x-110)°$.



$m\angle W = m\angle R$

$m\angle O = m\angle K$

$m\angle W + m\angle O = 180°$

$m\angle O + m\angle K = 180°$

STEP 4: LOOK BACK.

- *In a parallelogram, any two opposite angles are congruent.*
- *In a parallelogram, any two consecutive angles are supplementary.*

$m\angle W = 70°$

$m\angle O = 110°$

$m\angle R = 70°$

$m\angle K = 110°$

Activity 2: **You try!**

03:00

□ BEAT is a parallelogram with diagonals $\overline{BA}$ and $\overline{TE}$ that meet at S. Find (a) AS, (b) ES, (c) TS, (d) $m\angle BSE$, and (e) $m\angle ESA$ if $BS = 8$, $TE = 12$, and $m\angle AST = 110°$.

Activity 2: **You try!**

03:00

□BEST is a parallelogram. Find (a) $m\angle EBS$, (b) $m\angle TES$, and (c) triangle that is congruent to ΔBES if $m\angle TSB = 28°$ and $m\angle ETB = 75°$.

Activity 3:

In your art class, your teacher asks you to draw parallelogram CAFE as one of the parts of the figure you are required to produce. To produce the actual drawing your teacher has in mind, he gave you the following conditions.

□CAFÉ is a parallelogram with diagonals $\overline{CF}$ and $\overline{EA}$ that meet at S. The perimeter of □CAFÉ is 192 units.

Given:

$CA = 3x - 1$
$CE = 2x + 7$
$m\angle ACF = 40°$
$m\angle CFA = 110°$
$m\angle AEF = 13°$
$m\angle CSE = 53°$
$CF = 27$
$ES = 46.5$

Activity 3:

Find the measure of the following:

GROUP 1

1. AS
2. CS
3. EA
4. x
5. EF
6. AF

GROUP 2

7. m∠EFC
8. m∠CAE
9. m∠ASF
10. m∠CSA
11. m∠CAF
12. m∠ECA

Give the congruent triangle of:

GROUP 3

1. ΔCAF
2. ΔECA
3. ΔEAF
4. ΔCFE
5. ΔEAC

Activity 3:

GROUP 4

Your sister planned to bake a cake in a shape of a parallelogram. She asks for your help with the measurements of the angles to make sure that she will have a perfect parallelogram-shaped cake.
If quadrilateral CAKE is a parallelogram, find (a) x, (b) $m\angle K$, and (c) $m\angle E$ if $m\angle A = 79°$ and $m\angle C = (4x + 1)°$.

GROUP 5

You ask a carpenter to make a table for you. For the table to hold a lot of things, you ask the carpenter to put diagonals at the back of the table's top.
If the top of the table illustrates parallelogram BREA with diagonals that meet at D, find (a) $m\angle ARE$, (b)$m\angle BER$, (c) $m\angle RDE$, and (d) $m\angle BDR$ if $m\angle BAR = 35°$ and $m\angle ABE = 70°$.

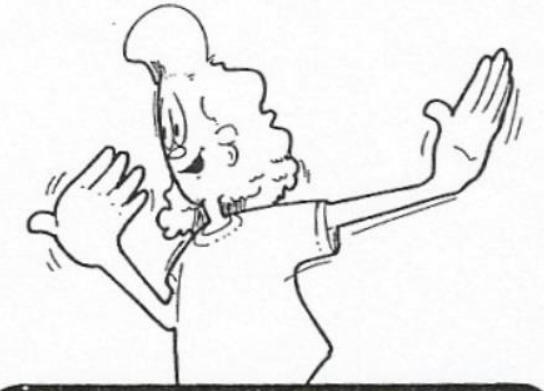
RUBRIC for the Group Activity

Criteria	5 (10 pts.)	4 (8 pts.)	3 (7 pts.)	2 (5 pts.)	1 (4 pts.)
Interpretation (Illustration)	The figure accurately illustrates the problem.	The figure illustrates the given problem.	The figure slightly illustrates the given problem.	The figure does not illustrate the given problem.	There is no illustration.
Answers	100% correct	With 1 error	With 2 errors	With 3 errors	With more than 3 errors
Presentation	Well-organized, very creative, easy to follow, presentable, and clean	Easy to follow, creative, presentable, and clean	Easy to follow, presentable, and clean	Not so easy to follow, presentable, and clean	Messy and difficult to follow

For this group task, you can be:

Encourager

Encourage teammates to participate and do well.

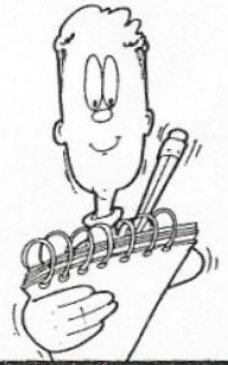


Gatekeeper

Make sure everyone is participating about equally.

Focus Keeper

Keep the team focused on task.



Recorder

Record the team's answers or ideas, or make sure they get recorded.

Checker

Check to make sure everyone has learned the material.

PowerPoint

EXPERT