Math 9A
Q4_Week 5

CHERRY LOU G. LAGRIMAS
Subject-Teacher

OBJECTIVES

- determine the Midline Theorem and the theorems on trapezoids and kite
- prove the Midline Theorem and the theorems on trapezoid and kite
- appreciate the theorems through application

Theorems to Prove

THEOREM 6: The median of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

THEOREM 7: The base angles of an isosceles trapezoid are congruent.

THEOREM 8: Opposite angles of an isosceles trapezoid are supplementary.

THEOREM 9: The diagonals of an isosceles trapezoid are congruent.

THEOREM 10: In a kite, the perpendicular bisector of at least one diagonal is the other diagonal.

THEOREM 11: The area of a kite is half the product of the lengths of its diagonals.

Rubric for the Group Task

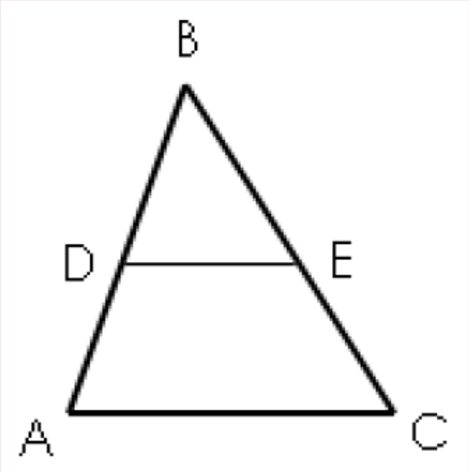
Criteria	5 (15 pts.)	4 (14 pts.)	3 (12 pts.)	2 (9 pts.)	1 (5 pts.)
Answer	No error.	One minor error.	Two errors.	Three errors.	Four or more errors.
Presentation	Clear and understandable with visual aid.	Understandable but not so clear and with visual aid.	Understandable with visual aid.	Understandable and no visual aid.	Not clear and no visual aid.
Cooperation	All members can answer the questions and explain the work clearly.	One member cannot answer the question or explain the group output clearly.	Two members cannot answer the question or explain the group output clearly.	Three members cannot answer the question or explain the group output clearly.	Four or more members cannot answer the question.
TOTAL					

The Midline Theorem

The Midline in Triangle (*Theorem 5*)

In ΔABC, if D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{BC}$, then $\overline{DE} \parallel \overline{AC}$ and

$$DE = \frac{1}{2}AC$$



CONCEPT:

In a triangle, the midline (or *mid-segment*) is any of the three lines joining the midpoints of any two of its sides. Each of these midlines is parallel to the third side and half as long.

Example 1: Solve for what is/are asked using the Midline Theorem

Given: ΔAGE

N is the midpoint of $\overline{AG}$

L is the midpoint of $\overline{GE}$

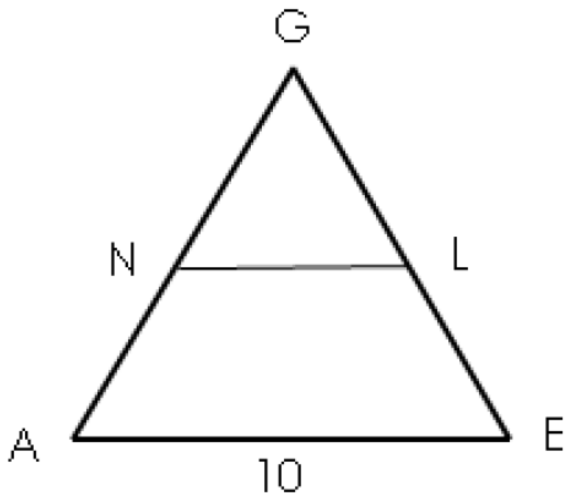
AE = 10

Find: NL

Solution: $NL = \frac{1}{2}AE$

$= \frac{1}{2}(10)$

$NL = 5$



Example 2: Solve for what is/are asked using the Midline Theorem

Given: ΔVAE

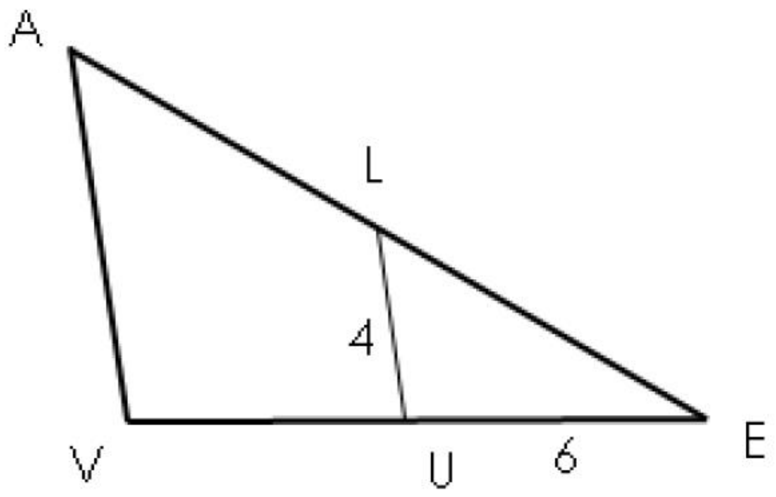
L is the midpoint of $\overline{AE}$

U is the midpoint of $\overline{VE}$

LU = 4

UE = 6

Find: AV and VE



Solution: $LU = \frac{1}{2}AV$

$4 = \frac{1}{2}AV$

$AV = 8$

VE = 12 ; VE is twice UE

Activity 1: Solve for what is/are asked using the Midline Theorem

03:00

Given: ΔSOV

L is the midpoint of $\overline{OV}$

E is the midpoint of $\overline{SV}$

$LE = 4$

$SO = 3x - 1$

$SV = 17$

Find: x and SE

Activity 1: Solve for what is/are asked using the Midline Theorem

03:00

Given: ΔLAR

E is the midpoint of $\overline{LA}$

N is the midpoint of $\overline{LR}$

$LE = 5y + 1$

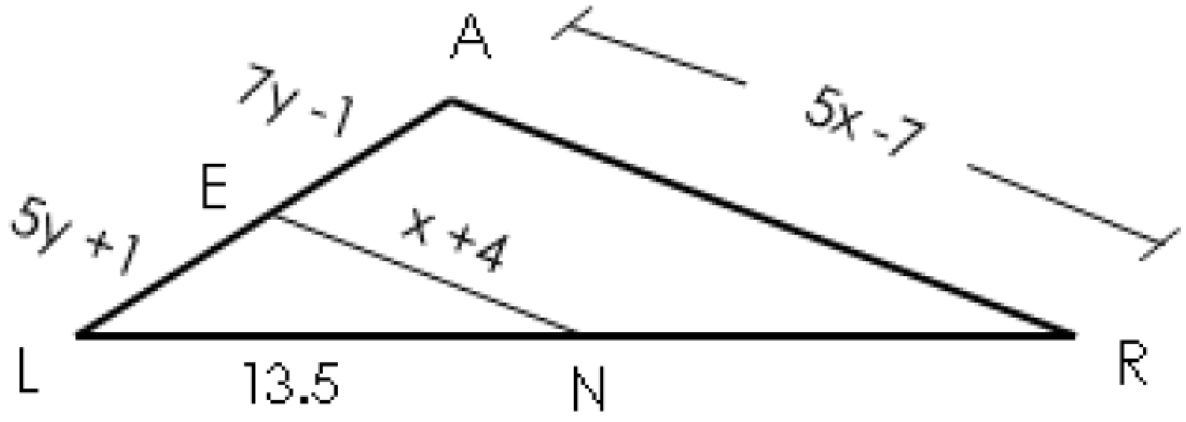
$EA = 7y - 1$

$AR = 5x - 7$

$EN = x + 4$

$LN = 13.5$

Find: y, LA, x and NR

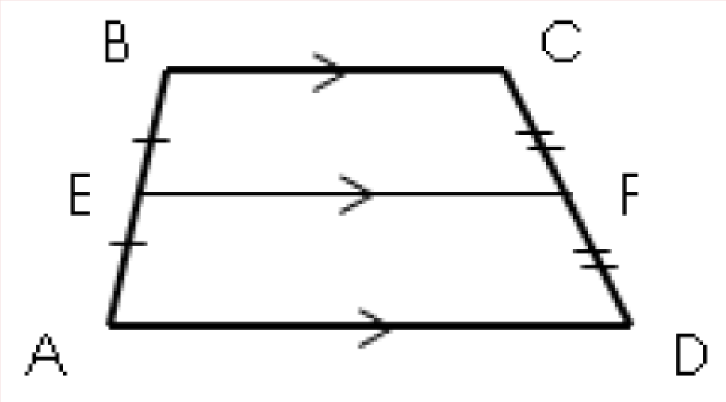


Theorems on Trapezoids

A trapezoid is a quadrilateral with only one pair of parallel sides. These parallel sides are also called *bases*. The non-parallel sides, on the other hand, are also called the *legs*. If the legs are congruent, the trapezoid is specifically called an isosceles trapezoid.

The Midsegment Theorem of Trapezoid
(or *Trapezoid Median Theorem*)
Theorem 6

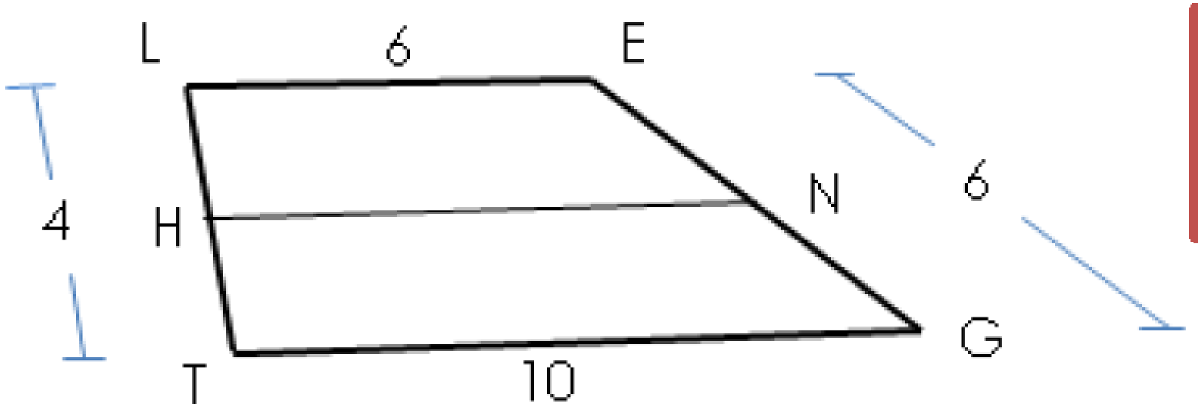
The *median* (the segment formed by connecting the midpoints of the legs of a trapezoid) is parallel to the bases, and its length is one-half the sum of the lengths of the bases.



Since $\overline{EF}$ is a median, then $\overline{BC} \parallel \overline{EF} \parallel \overline{AD}$, and

$$EF = \frac{1}{2}(BC + AD)$$

Example 1: Trapezoid TLEG with midsegment $\overline{HN}$



Find:

a. HN
b. LH
c. EN
d. NG

Solution: $HN = \frac{1}{2}(LE + TG)$

$$= \frac{1}{2}(6 + 10)$$

$$= \frac{1}{2}(16)$$

$$HN = 8$$

b. LH = 2; $\overline{LH}$ is half of $\overline{LT}$

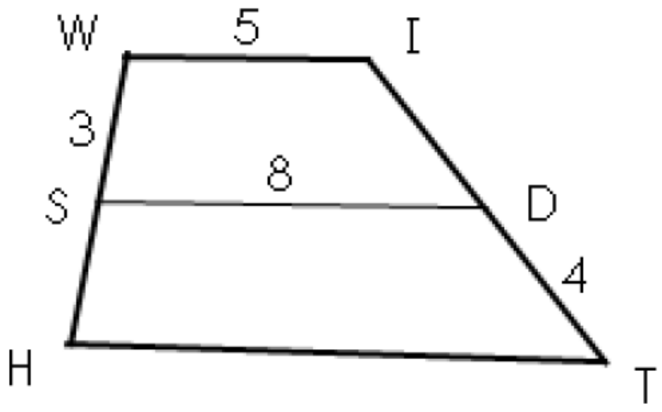
c. EN = 3; $\overline{EN}$ is half of $\overline{EG}$

d. NG = 3; EN = NG

03:00

Activity 2: Use the Midsegment Theorem of trapezoid in solving.

Trapezoid WITH with midsegment $\overline{SD}$



Find:

a. HT

b. HS

c. ID

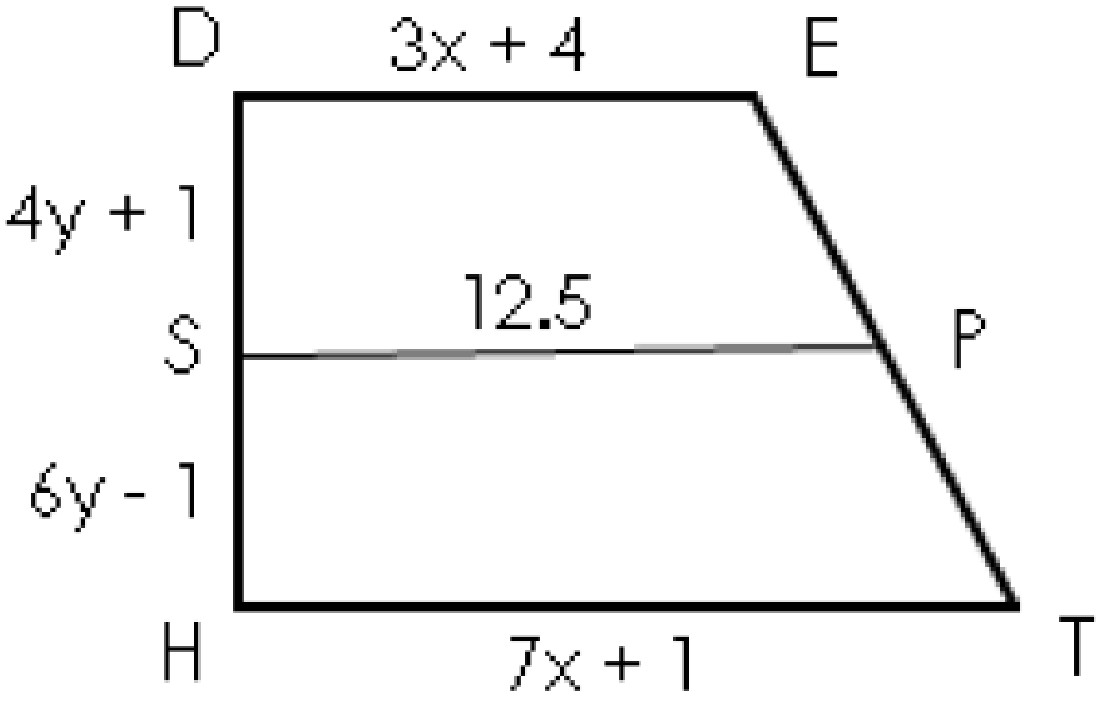
d. IT

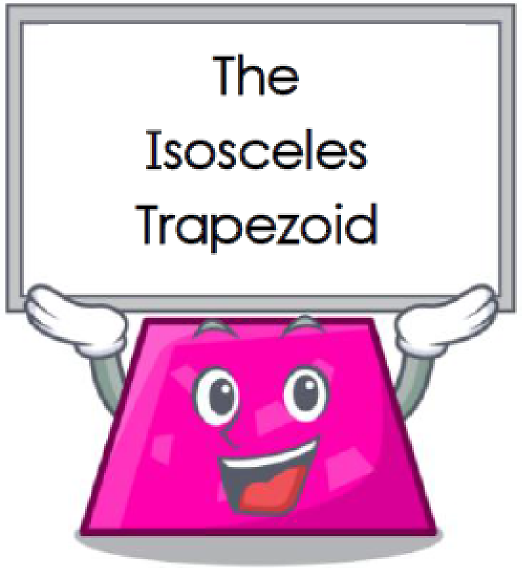
03:00

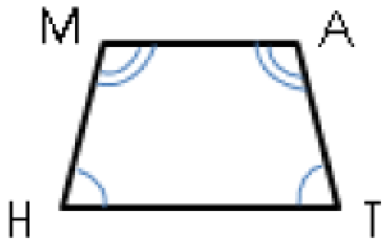
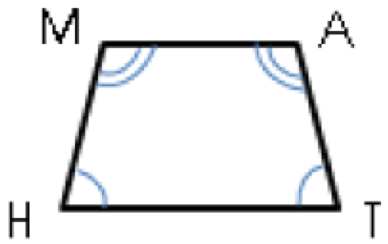
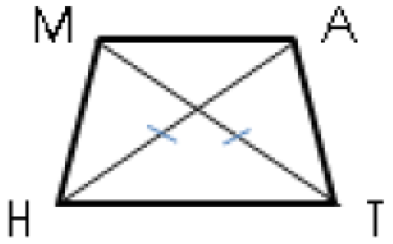
Activity 2: Use the Midsegment Theorem of trapezoid in solving.

Find:

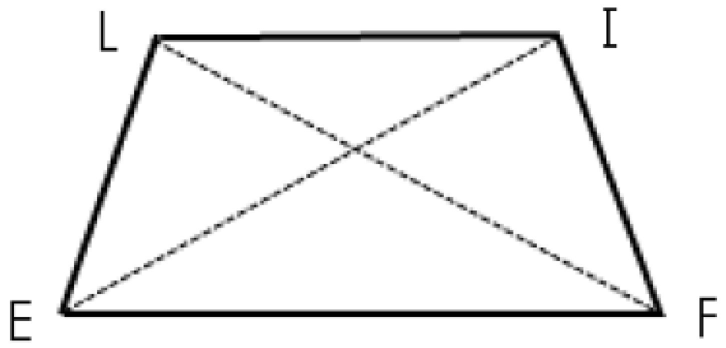
a. x
b. y
c. HD





Theorems on Isosceles Trapezoid		
Theorem 7	The two *base angles* (consecutive angles with a base as common side) that make up a pair in an isosceles trapezoid are congruent.	 $\angle H \cong \angle T; \angle M \cong \angle A$
Theorem 8	The opposite angles of an isosceles trapezoid are supplementary.	 $m\angle H + m\angle A = 180°$ $m\angle M + m\angle T = 180°$
Theorem 9	The diagonals of an isosceles trapezoid are congruent.	 $\overline{HA} \cong \overline{MT}$

Example 1: Isosceles trapezoid LIFE has $m\angle E = 70^\circ$ and LF = 15.



Find:
a. $m\angle F$
b. $m\angle I$
c. EI

a. $m\angle F = 70^\circ$ Base angles are equal

b. $m\angle I = 110^\circ$

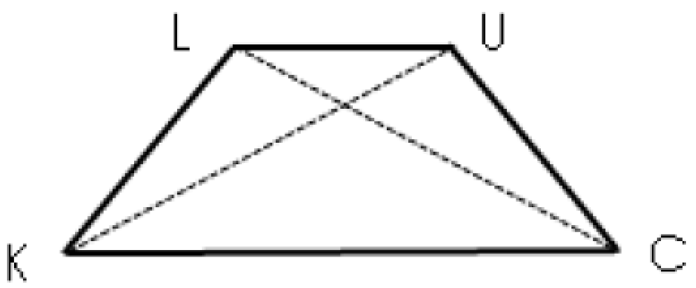
Solution: $m\angle E + m\angle I = 180^\circ$ Opposite angles are supplementary

$70^\circ + m\angle I = 180^\circ$

$m\angle I = 110^\circ$

C. EI = 15 Diagonals are equal

Example 2: Isosceles trapezoid LUCK with KU = 5w + 6, LC = 31, LK = 7x + 5, UC = 19, $m\angle LKC = (3y+5)^\circ$ and $m\angle LUC = (9y-5)^\circ$.



Find:

a. w

b. x

c. y

d. $m\angle KLU$

e. $m\angle KCU$

a. w = 5

Solution: $KU = LC$ — Diagonals are congruent

$$5w + 6 = 31$$
$$5w = 25$$
$$w = 5$$

b. x = 2

Solution: $LK = UC$ — The legs of an isosceles trapezoid are equal

$$7x + 5 = 19$$
$$7x = 14$$
$$x = 2$$

c. y = 15

Solution: $m\angle LKC + m\angle LUC = 180°$ Opposite angles are congruent

$$(3y+5)° + (9y-5)° = 180°$$

$$12y = 180$$

$$y = 15$$

d. $m\angle KLU = 130°$

Solution: $m\angle LUC = m\angle KLU$ Base angles are congruent

$$m\angle LUC = (9y-5)°$$

$$m\angle LUC = \left[9(15)-5\right]°$$

$$m\angle LUC = 130°$$

e. $m\angle KCU = 50°$

Solution: $m\angle KCU = m\angle LKC$ Base angles are equal

$$m\angle LKC = (3y+5)°$$

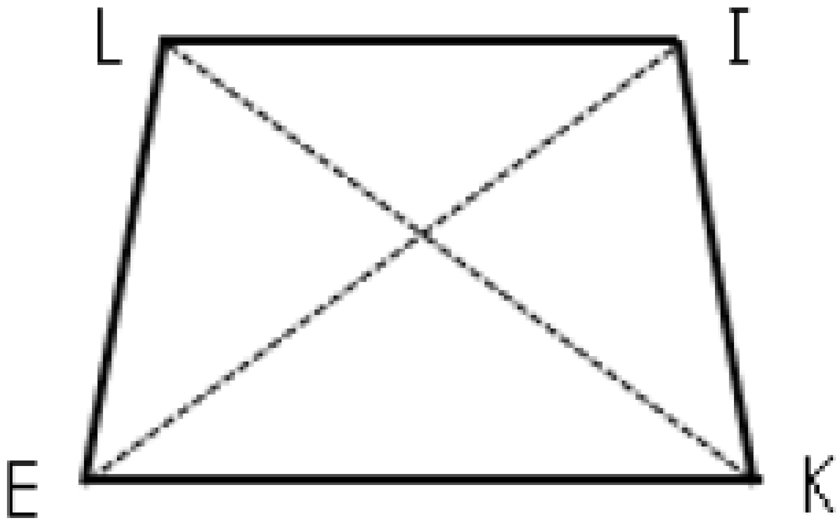
$$m\angle LKC = \left[3(15)+5\right]°$$

$$m\angle LKC = 50°$$

03:00

Activity 3: Use the theorems on isosceles trapezoid in answering.

Example 3: Isosceles trapezoid LIKE with $EI = 2x + 1$, $LK = 3x - 5$, $m\angle LEK = (3y + 14)^\circ$, $m\angle EKI = (4y - 8)^\circ$, and $m\angle LIK = (5y - 10)^\circ$.



Find:

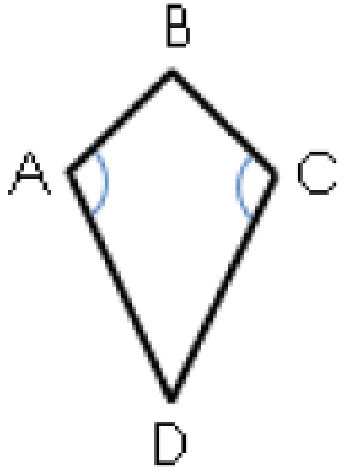
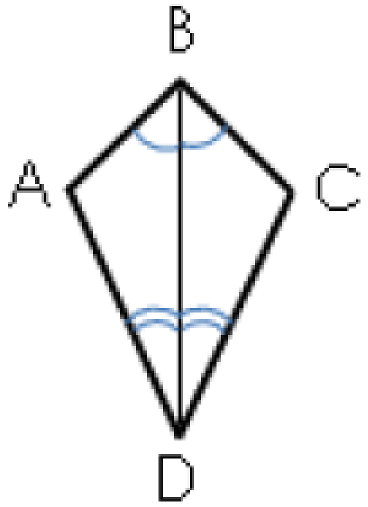
a. x

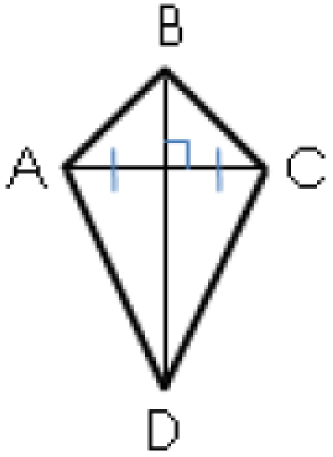
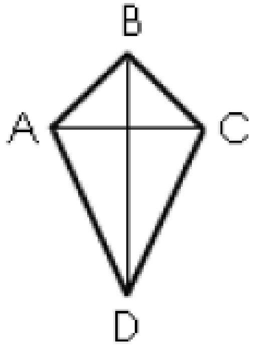
b. y

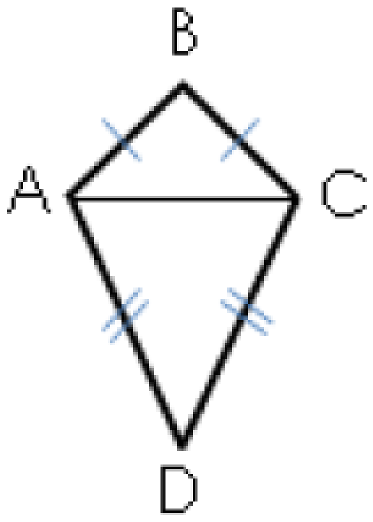
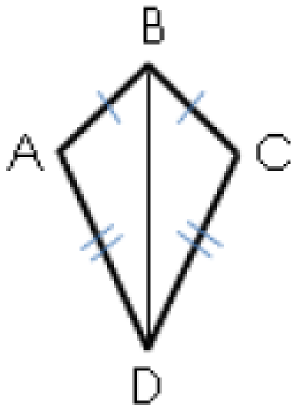
c. $m\angle ELI$

Theorems on Kite

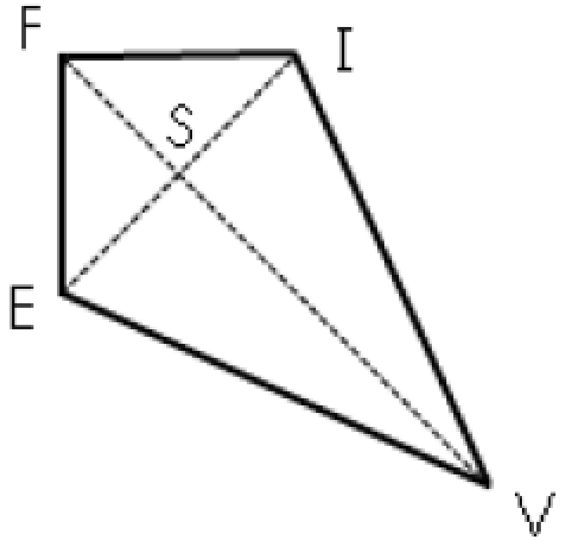
A kite is a trapezium in which two pairs of adjacent sides are congruent. The angles formed between each pair of congruent sides are called *vertex angles* and the angles between the non-congruent sides are called *nonvertex angles*.

Theorems on Kite		
Theorem 10	The nonvertex angles of a kite are congruent.	
Theorem 11	The vertex angles of a kite are bisected by a diagonal.	

Theorem 12	The diagonal connecting the vertex angles of a kite is the perpendicular bisector of the other diagonal.	
Theorem 13	The area of a kite is half the product of the lengths of its diagonals.	 $A_k = \frac{1}{2}(AC)(BD)$

Theorem 14	A kite has one diagonal forming two isosceles triangles.	
Theorem 15	A kite has one diagonal forming two congruent triangles.	 $\Delta BAD \cong \Delta BCD$

Example 1: In kite FIVE, name the congruent of:



a. FI
b. EV
c. ES
d. ∠EFV

e. ∠FVI
f. ∠FEV
g. ∠FIE
h. ΔFEV

Answers:

a. FE

b. IV

c. SI — $\overline{EI}$ is bisected by $\overline{FV}$

d. $\angle IFV$ — $\angle EFI$ is bisected by $\overline{FV}$

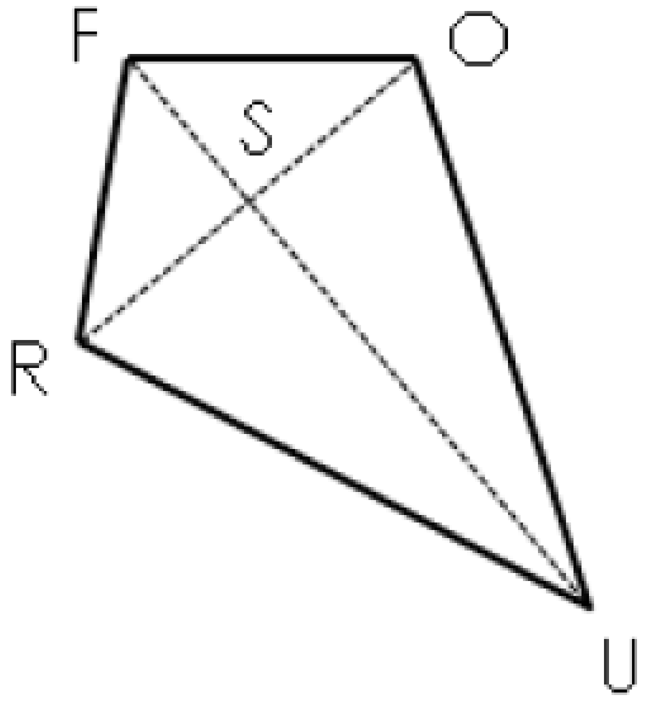
e. $\angle FVE$

f. $\angle FIV$ — Nonvertex angles of a kite are congruent

g. $\angle FEI$ — Corresponding parts of $\Delta FES \cong \Delta FIS$ are congruent

h. ΔFIV — $\overline{FV}$ divides the kite onto two congruent triangles

Example 2: Consider kite FOUR.



a. If $FO = 5$, what is FR?

Answer: FR = 5 FO and FR are equal

b. If $RO = 8$, what is RS?

Answer: RS = 4 RS is half RO

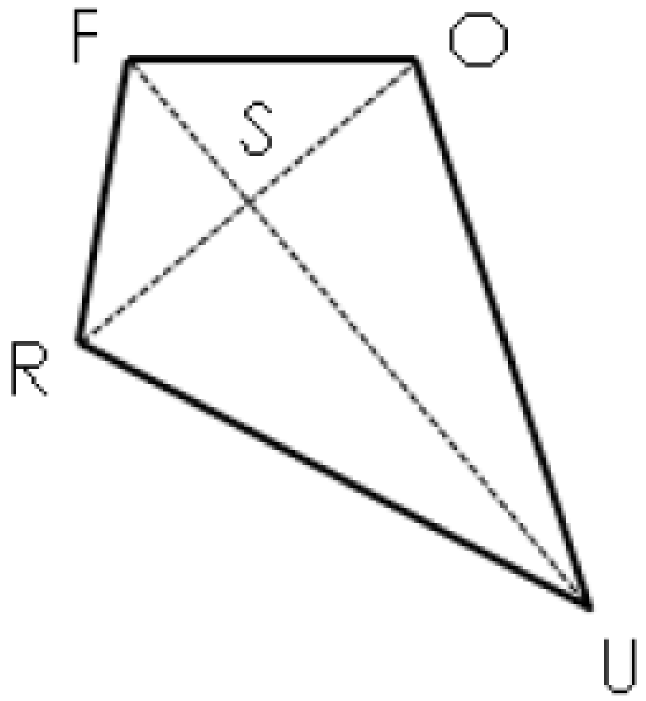
c. If $m\angle RFO = 100^\circ$, what is $m\angle OFU$?

Answer: $m\angle OFU = 50^\circ$ $\overline{FU}$ bisects $\angle RFO$

d. If $m\angle FRU = 107.5^\circ$, what is $m\angle FOU$?

Answer: $m\angle FOU = 107.5^\circ$ $m\angle FOU = m\angle FRU$

Example 2: Consider kite FOUR.



e. If $m\angle RUO = 45°$, what is $m\angle RUF$?

Answer: $m\angle RUF = 22.5°$

$\overline{FU}$ bisects $\angle RUO$

f. What is $m\angle FSO$?

Answer: $m\angle FSO = 90°$

$\overline{FU}$ and $\overline{RO}$ are perpendicular

g. If $FU = 12$ and $RO = 8$, what is the area of kite FOUR?

Answer: $A_{KiteFOUR} = 48$ square units

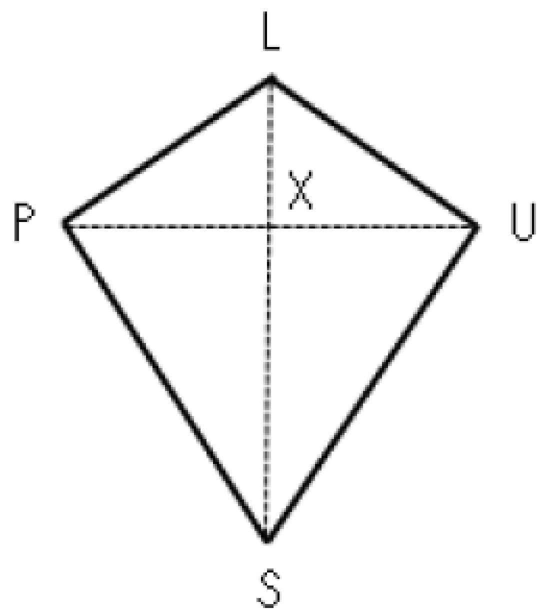
Solution: $A_{KiteFOUR} = \frac{1}{2}(FU)(RO)$

$= \frac{1}{2}(12)(8)$

$A_{KiteFOUR} = 48$

Activity 4: Use the theorems on kite in answering.

Consider kite PLUS



a. If $m\angle LPS = (3x+11)^\circ$ and $m\angle LUS = 92^\circ$, what is x?

b. If $PX = x + \frac{9}{2}$ and $XU = 3x + \frac{1}{2}$, what is x ? What is PU?

c. If $m\angle PLX = 7x - 8$ and $m\angle ULX = 6x + 1$, what is x?