11

SENIOR HIGH SCHOOL

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SCHOOLS DIVISION OF NEGROS ORIENTAL
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STATISTICS and PROBABILITY

Quarter 3 - Module 4
Normal Distribution

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Alternative Delivery Mode
Quarter 3 – Module 4: Normal Distribution
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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

The module is intended for you to identify regions under the normal curve corresponding to different standard normal values, and convert a normal random variable to a standard normal variable and vice versa.

After going through this module, you are expected to compute probabilities and percentiles using the standard normal table.

What I Know

PRE-ASSESSMENT

Read and analyze the statements below. In your activity notebook/activity sheet, write TRUE if the statement is true and write FALSE if the statement is false.

___1. The total area under the normal curve is equal 1 or 100%.

___2. If the z-score is equal to -2.51, then the area of the region is 0.0060

___3. The area between the z-scores -2.0 and 1.56 is 0.9178

___4. If the area is 0.9332, then the z-score is 0.00

___5. Every normal distribution can be transformed to standard normal distribution.

___6. The scores on scholarship aptitude exam are normally distributed with a mean of 72 and a standard deviation of 8. If the score is 60, then the z score is -1.5

___7. The area between $x = 45$ and $x = 62$ of a given normal distribution with mean $\mu = 50$ and $\sigma = 10$ is 0.9764

___8. The value of z in the expression $P(Z < z) = 0.0075$ is 2.43

___9. The notation $P(Z > 1.81) = 0.0351$.

___10. The z-score of 33^{rd} percentile is -0.44

Lesson 1 Regions Under the Normal Curve

What's In

Task 1.1
Enumerate 5 properties of a normal distribution.
1.

2.

3.

4.

5.

What's New

In probability distribution, the frequently asked question is on finding the area or percentage between two points.

Abraham DeMoivre (1667- 1754) introduced to us the formula to answer the question.

$$y = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where:

$\pi = 3.1416$
$e = 2.7183;$
$\sigma =$ standard deviation of the population
$x =$ score in the distribution
$\mu =$ population mean

However, the formula above is now rarely used because of the readily available z-table. The z-table gives the area from $-\infty$ to any z-score.

The z-table

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.063	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247

-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

What is It

Steps in finding the area under the standard normal curve.

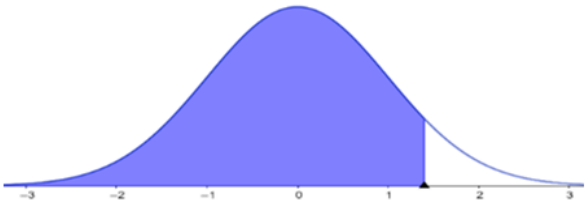
1) Sketch the standard normal curve and shade the appropriate area under the curve.
2) Express the given z-score into a three-digit number.
3) Look in the z-table and find the first two-digit (from the left) at the leftmost column of z-table and third digit at the topmost row.
4) Locate the intersection of the first two-digit and the third digit. The point of convergence indicates the area of the given z-score. (Larson, et al,2012)

Example 1.1

Find the area when $= 1.4$ **.**

Solution:

i) The given says when $z = 1.4$ which means from negative infinity to 1.4. Hence the shaded region is



ii) Express 1.4 into a three-digit number. Thus, we have **1.40**

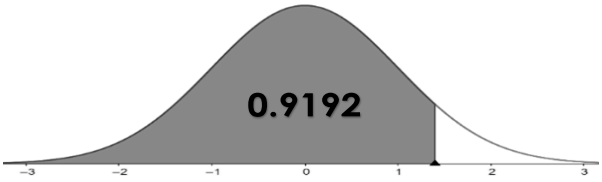
iii) In the z-table, find the first two-digit, **1.4**, at the leftmost column and the third digit, 0.0**0**, at the topmost row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Therefore, the area when $z = 1.4$ is 0.9192.

or

$$\boldsymbol{P(Z < 1.4) = 0.9192}$$

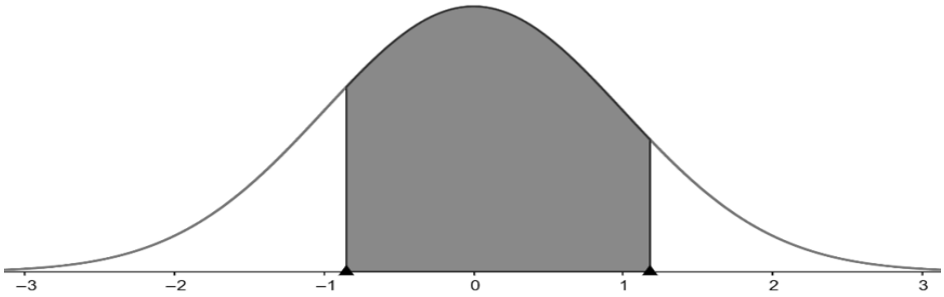


Example 1.2

Find the area between $z = -0.85$ and $z = 1.18$.

Solution:

i) The given says the area between $z = -0.85$ and $z = 1.18$, thus the shaded region must be



ii) As observed, the two z-scores, **-0.85** and **1.18**, are already expressed as three-digit number.

iii) For -0.85, find the first two-digit, **-0.8**, at the leftmost column of the z-table and the third digit, 0.0**5**, at the topmost row. Then find the intersection of the two points, **0.1977**.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.166	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	**0.1977**	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776

iv) Moreover, for 1.18 we will perform the same process, that is, find the first two-digit, **1.1**, at the leftmost column of the z-table and the third digit, 0.0**8**, at the topmost row. Then find the intersection of the two points, **0.8810**.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	**0.8810**	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

v) Since we are looking for the area between the two z-scores, we need to subtract the area that corresponds $z = -0.85$ (with smaller area) from the area that corresponds $z = 1.18$ (with bigger area). Thus, we have,

$$0.8810 - 0.1977 = \mathbf{0.6833}$$

Therefore, the area between $z = -0.85$ and $z = 1.18$ is 0.6833.

or

$$\boldsymbol{P(-0.85 < Z < 1.18) = 0.6833}$$

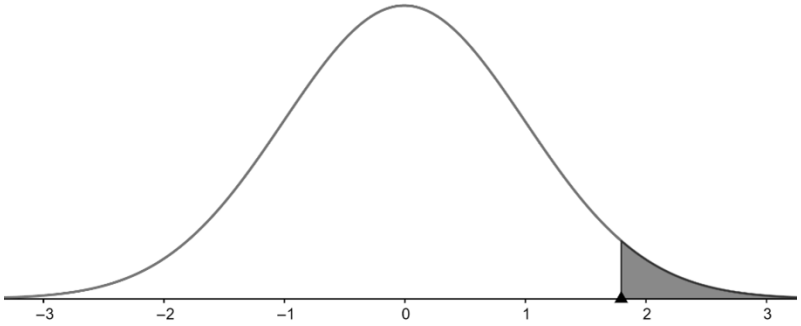
https://tinyurl.com/2p8sn5ht

Find the area to the right of $z = 1.8$

Example 1.3

Solution:

i) The problem says to the right of $z = 1.8$ which means greater than $= 1.8$. Hence, the shaded region is



ii) Express the given, **1.8**, into a three-digit number, **1.80.**

iii) In the z-table, find the first two-digit, **1.8**, at the leftmost column and the third digit, 0.0**0**, at the topmost row. Then find the intersection of the two points, **0.9641**.

https://www2.math.upenn.edu/~chhays/zscoretable.pdf

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	**0.9641**	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

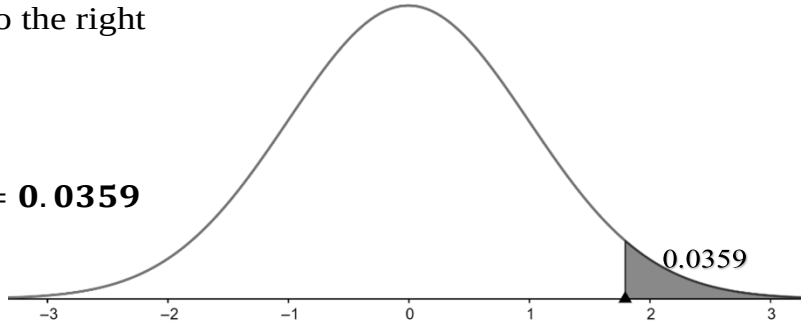
iv) Always remember that the area of the entire normal curve is 1 or 100%, and the area in z-table is from $-\infty$ to any z-score. Since the given says area to the right of $z = 1.8$, subtract the area that corresponds $z = 1.8$ from 1. Thus,

$$1 - 0.9641 = \mathbf{0.0359}$$

Therefore, the area to the right of $z = 1.8$ is 0.0359

or

$$\boldsymbol{P(Z > 1.8) = 0.0359}$$

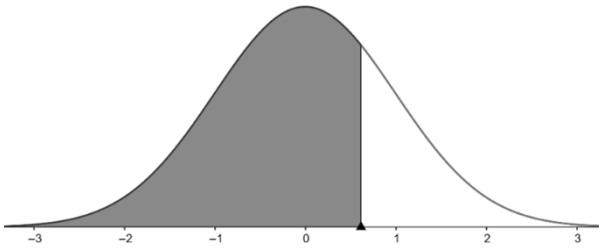


Example 1.4

Find the z-score when $P(Z < z) = 0.7389$

Solution:

i) Sketch the standard normal curve and the shaded region that satisfies the given



ii) As a review, the area reflected in the z-table starts from $-\infty$ to any z-score. As shown in the figure above, the shaded region starts from $-\infty$ to a certain z-score. In this case, we can directly use the z-table in finding the z-value. That is, by finding the given area **0.7389** in the z-table. Once found, identify the z-score to its leftmost column and to its topmost row. Thus the z-value is $\mathbf{0.64}$

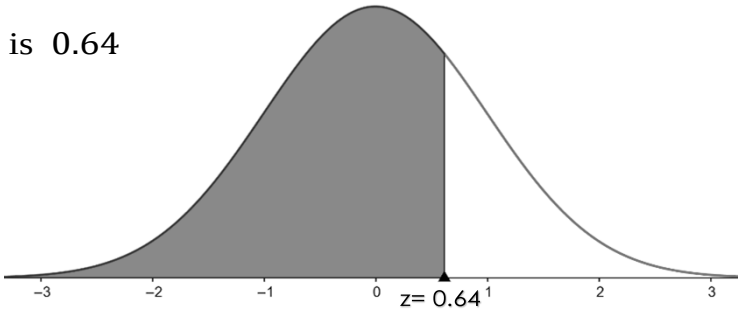
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389

Therefore, the z-score when

$$P(Z < z) = 0.7389 \text{ is } 0.64$$

or

$$P(Z < \mathbf{0.64}) = 0.7389$$



Example 1.5

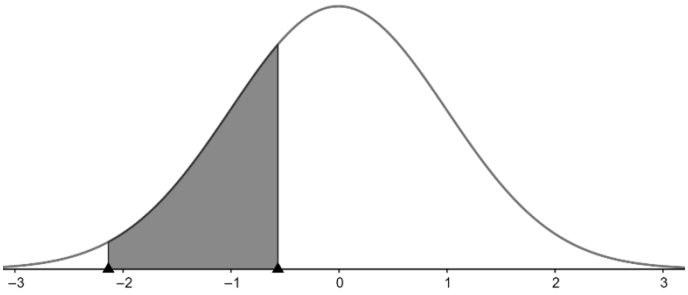
Find the z-score that will satisfy the given

$$P(-2.13 < Z < z) = 0.2711$$

Solution:

i) Draw the standard normal curve and show an estimated shaded region that satisfies the expression

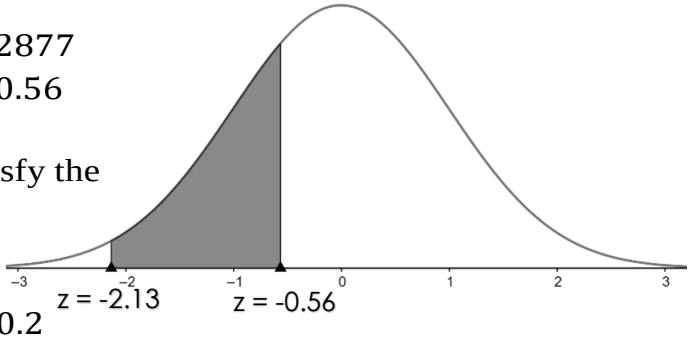
$$P(-2.13 < Z < z) = 0.2711$$



ii) In the given, it says that the area of the shaded region is **0.2711** and the z-score at the left is **-2.13**. To get the z-score at the right, first get the area of the region from -∞ to -2.13, then add it to the area of the shaded region. The sum you will get will be used in identifying the other z-score. That is,

$$P(Z < -2.13) + 0.2711$$
$$= 0.0166 + 0.2711$$
$$= 0.2877$$

$$P(Z < z) = 0.2877$$
$$z = -0.56$$



Therefore, the z-score that will satisfy the

$P(-2.13 < Z < z) = 0.27$ is $z = -0.56$

or

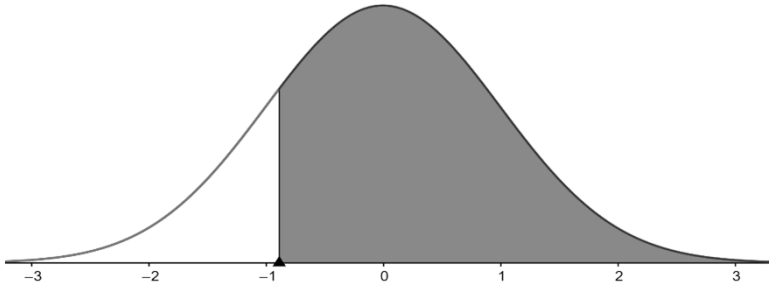
$$P(-2.13 < Z < -\mathbf{0.56}) = 0.2$$

Example 1.6

Find the z-score of the notation $P(Z > z) = 0.8106$.

Solution:

i) Draw the standard normal curve and the region based on the given.



ii) Again, remember that in the z-table the area is from $-\infty$ to any z-score. As you can observe, the above figure shows that the shaded region starts from a certain z-score to $+\infty$. Hence, directly using the z-table is not appropriate. Instead, subtract the given area from 1. That is,

$$1 - 0.8106 = 0.1894$$

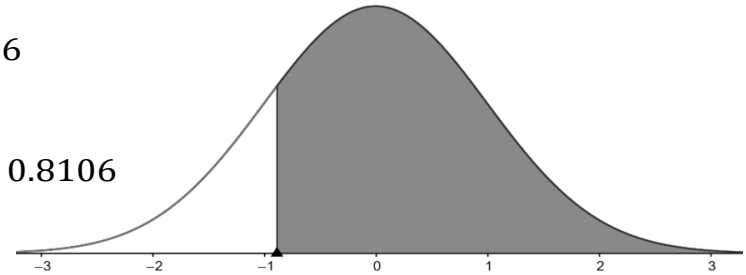
iii) This time use the z-table to find the exact z-score, and that is, -0.88.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.166	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776

Therefore, the z-score in the notation $P(Z > z) = 0.8106$ is $z = -0.88$

or

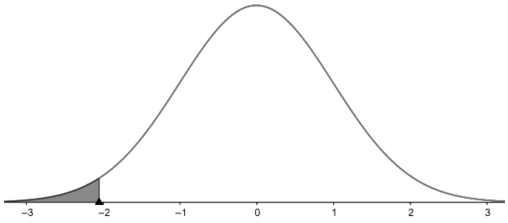
$P(Z > -\mathbf{0.88}) = 0.8106$



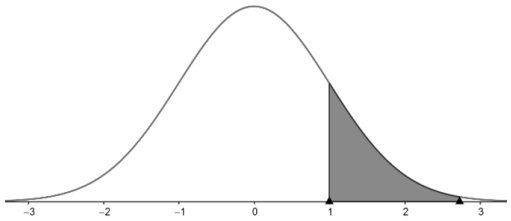
What's More

Task 1.2

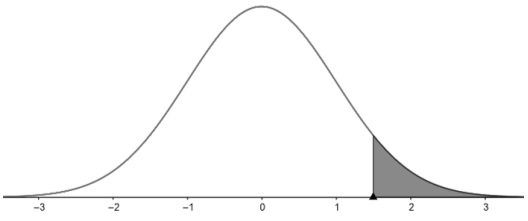
A. Match the figure to its corresponding notation or phrase. Write only the letter of your answer on your activity notebook/activity sheet.



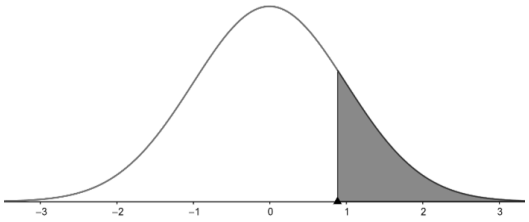
A



B

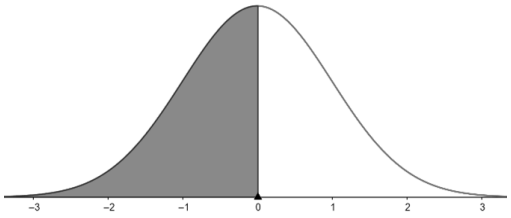


C



D

E



F

1. $P(Z > 1.5)$
2. $P(-1.29 < Z < 0.17)$
3. $P(Z < -2.05)$
4. to the right of $z = 0.89$
5. to the left of $z = 0.01$
6. between $z = 1.25$ and $z = 2.73$

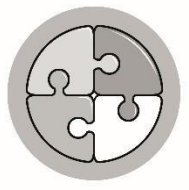
B. Find the probability (area) of:

7. $P(Z > 1.5)$
8. between $z = 1.25$ and $z = 2.73$

C. Give the z-score when:

9. $P(Z < z) = 0.0202$
10. $P(-1.29 < Z < z) = 0.4690$

Lesson 2 Conversion of Normal Random Variable into a Standard Normal Curve



What's In

Task 2

Match the probability notation in column A to its area in column B. Write only the letter of your choice in your notebook.

Column A	Column B
1. $P(Z < 0)$	A. 0.6826
2. $P(Z > 0)$	B. 0.0228
3. $P(-1 < Z < 1)$	C. 0.5000
4. $P(Z < 2)$	D. 0.9772
5. $P(Z > 2)$	

What's New

In Lesson 1, you learned how to find the area or probability of any two points and even find the unknown z-score given the area or the probability in the standard normal curve. So, what is meant by the standard normal distribution? There are infinitely many normal distributions, each with its own mean and standard deviation. The normal distribution with a mean of 0 and a standard deviation of 1 is called **the standard normal distribution**: the focused in Lesson 1.

However, in gathering data, raw scores may compose of large values. In which the standard normal curve cannot accommodate. Consequently, the raw scores must be transformed into z-scores to get meaningful decisions relative to the concepts of finding the equivalent probability of the given measure of value from the mean.

This gives the way to understand that for every raw score X, there corresponds an exactly one z-score value, and vice versa. Therefore, if we wish to find the percentage associated with X, we must look at its equivalent z-score using the z formula.

$$z = \frac{x-\mu}{\sigma}$$

where:

$z =$ the standard normal value
$x =$ the value from the given normal distribution
$\mu =$ the mean of the given normal distribution
$\sigma =$ the standard deviation of the given normal distribution

What is It

Steps in transforming *a normal distribution* into *the standard normal distribution.*

1) Use the formula: $z = \frac{x-\mu}{\sigma}$
2) Substitute the given values x, μ, and σ into the formula.
3) Simplify the expression.

Example 2.1

Let X be normally distributed random variable with mean $\mu = 85$ and $\sigma = 4$. If $x = 73$, find its standard value.

Solution:

$$z = \frac{x-\mu}{\sigma}$$
$$= \frac{73-85}{4}$$
$$\boldsymbol{z = -3}$$

Therefore, the standard value of $x = 73$ is $z = -3$.

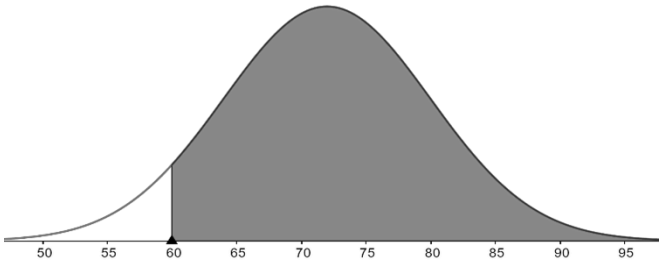
Example 2.2

Scores on scholarship aptitude exam are normally distributed with a mean of 72 and a standard deviation of 8.

a. What is the probability that a randomly selected applicant will score above 60?
b. What is the probability that a randomly selected applicants will score between 68 and 84?

***Solution for (*a)**

i) Sketch the normal curve reflecting the region described in (a).



ii) As we can see, the given is not the standard normal distribution. Thus, we need to transform the value from the given normal distribution, **60**, into the standard normal distribution.

$$z = \frac{x-\mu}{\sigma}$$
$$= \frac{60-72}{8}$$

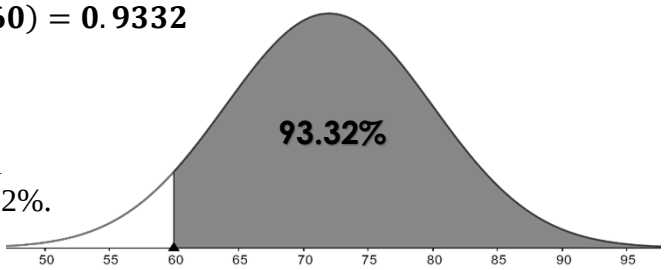
$$\boldsymbol{z = -1.5}$$

Thus, the standard value of $x = 60$ is $z = -1.5$.

iii) Based on the figure above, and the transformed value, it can be expressed as (take not of the keyword 'above' so we will use greater than >)

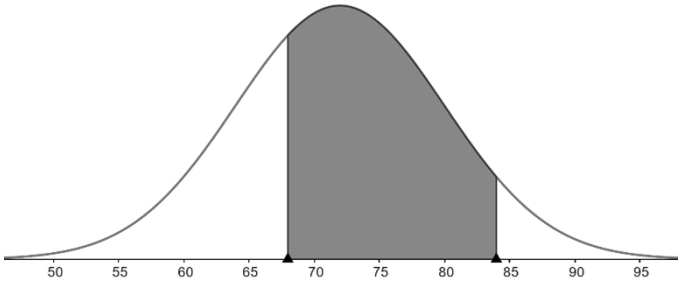
$$P(X > 60) = P(Z > -1.5)$$
$$= 1 - 0.0668$$
$$P(\boldsymbol{X > 60}) = \boldsymbol{0.9332}$$

Therefore, the probability that a randomly selected applicant will score above 60 is 0.9332 or 93.32%.



***Solution for (*b)**

i) Sketch the normal curve reflecting the region described in (b).



ii) As observed, the given is not the standard normal distribution. Thus, we need to transform the value from the given normal distribution, **68** and **84**, into the standard normal distribution.

when $x = 68$

$$z = \frac{x-\mu}{\sigma}$$
$$= \frac{68-72}{8}$$
$$\boldsymbol{z = -0.5}$$

when $x = 84$

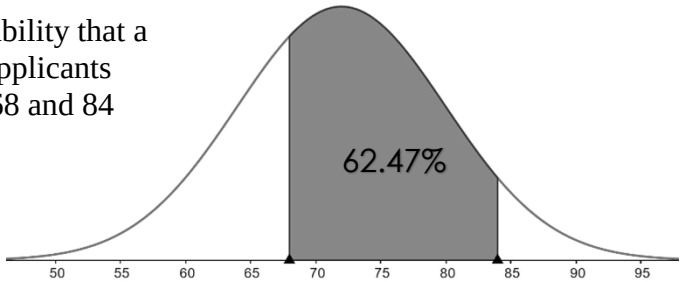
$$z = \frac{x-\mu}{\sigma}$$
$$= \frac{84-72}{8}$$
$$\boldsymbol{z = 1.5}$$

Thus, the standard values when $x = 68$ and $x = 84$ are $z = -0.5$ and $z = 1.5$ respectively.

iii) As shown in the figure above, and the transformed value, it can be expressed as

$$P(68 < X < 84) = P(-0.5 < Z < 1.5)$$
$$= 0.9332 - 0.3085$$
$$\boldsymbol{P(68 < X < 84) = 0.6247}$$

Therefore, the probability that a randomly selected applicants will score between 68 and 84 is 0.6247 or 62.47



Example 2.3

Assume X is normally distributed with a mean of 15 and a standard deviation of 3. Determine the value for x that solves each of the following:

a) $P(X < x) = 0.2810$
b) $P(x < X < 21) = 0.8185$
c) $P(X > x) = 0.9222$

Solution for (a)

I)In the given we only have the μ and the σ, we lack the z-score. Thus, the first thing to find is the z-score. Hence,

$$P(X < x) = 0.2810$$
$$P(X < x) = P(Z < z) = 0.2810$$
$$P(Z < z) = 0.2810$$
$$\boldsymbol{z = -0.58}$$

ii) Now we have the z-score, compute for x by using the transformed z-formula. We have,

$$x = \mu + z\sigma$$
$$= 15 + (-0.58)(5)$$
$$\boldsymbol{x = 17.9}$$

Therefore, with $\mu = 15$ and $\sigma = 5$, the x value that solves $P(X < x) = 0.2810$ is **17.9** or $P(X < \mathbf{17.9}) = 0.2810$

Solution for (b)

i)In this problem, there are two x-values, one is given while the other one is unknown. In this case, let us first transform the given x-value into z-score. We have,

when x= 21

$$z = \frac{x - \mu}{\sigma}$$
$$= \frac{21-15}{3}$$
$$\boldsymbol{z = 2}$$

ii) At this time, let us find the z-score of the unknown x-value. Thus,

$$P(x < X < 21) = 0.8185$$
$$P(x < X < 21) = P(z < Z < 2) = 0.8185$$
$$P(z < Z < 2) = P(Z < 2) - P(Z < z) = 0.8185$$
$$P(Z < z) = 0.9772 - 0.8185$$
$$P(Z < z) = 0.1587$$
$$\boldsymbol{z = -1.00}$$

iii) Let us now compute for the unknown x-value

$$x = \mu + z\sigma$$
$$= 15 + (-1.00)(5)$$
$$\boldsymbol{x = 10}$$

Therefore, with $\mu = 15$ and $\sigma = 5$, the x-value that solves $P(x < X < 21) = 0.8185$ is **10** or $P(\mathbf{10} < X < 21) = 0.8185$

Solution for (c)

i)First, let us identify the z-score that solves $P(X > x) = 0.9222$.

$$P(X > x) = 0.9222$$
$$P(X > x) = P(Z > z) = 0.9222$$
$$P(Z > z) = 0.9222$$
$$P(Z > z) = 1 - P(Z < z) = 0.9222$$
$$P(Z < z) = 1 - 0.9222$$
$$P(Z < z) = 0.0778$$
$$\boldsymbol{z = -1.42}$$

ii)With $\mu = 15$, $\sigma = 5$, and $z = -1.42$, we can now solve for x-value. Thus,

$$x = \mu + z\sigma$$
$$= 15 + (-1.42)(5)$$
$$\boldsymbol{x = 7.9}$$

Therefore, with $\mu = 15$ and $\sigma = 5$, the value of x that solves $P(X > x) = 0.9222$ is **7.9** or $P(X > \mathbf{7.9}) = 0.9222$

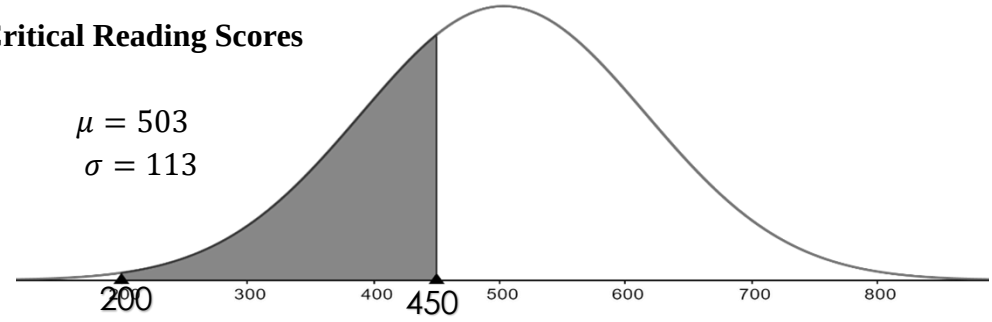
What's More

A. Assume the random variable X is normally distributed with mean $\mu = 86$ and standard deviation $\sigma = 5$. Find the probability and write your solution in your notebook.

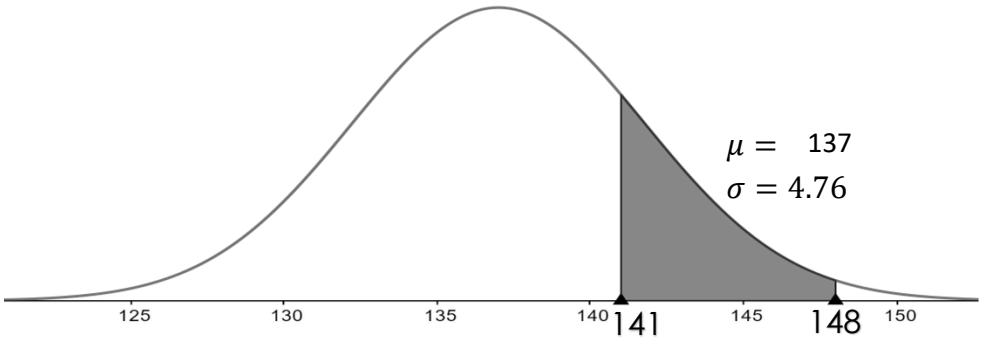
1. $P(X > 92)$
2. $P(70 < X < 80)$

B. Assume a member is selected at random from the population represented by the graph. Find the probability that the member selected at random is from the shaded area of the graph. Assume the variable X is normally distributed.

3. **Critical Reading Scores**



4. **Braking Distance on a Dry Surface**



C. 5. Suppose that the distribution of weights of Filipino females, 20-30 years old, is normal and has a mean weight of 65 kg with a standard deviation of 10 kg.

a) What is the probability that a randomly selected Filipino female 20 to 30 years old weighs less 50 kg?
b) What is the probability that she weighs more than 80 kg?
c) What is the probability that randomly selected individual from the population, has weight outside the range 55 to 70 kg?

6. Scores for civil service exam are normally distributed, with mean of 75 and a standard deviation of 6.5. to be eligible for civil service employment, you must score in the top 5%. What is the lowest score you can earn and still be eligible for employment?

Lesson 3 Solving Percentile with Normal Distribution

What's In

Task 3

Rewrite each of the following expressions in a card of size 2" x 1". Using the z-table, make 5 pairs of cards and creatively glue these in a long bond paper.

$z = -1.01$	$z = 0.19$	$z = -2.73$
$z = 2.00$	$z = 3.05$	
0.9772	0.0032	0.1562
0.9989	0.5753	

Rubrics for the output:

Perfect matched	= 10 points
Neatness	= 3 points
Timeliness	= 3 points
Creativity (Presentation)	= 4 points
Total	= 20 points

What's New

A **percentile** is a measure used in statistics indicating the value below which a given percentage of observations in a group of observations fall. It is a measure of relative standing as it measures the relationship of a measurement of the rest of the data.

For instance, your Statistics grade for the quarter is 89 and is at the 90th percentile. It means 90% of your classmates were graded lower than 89 and putting you in the top 10%. The alternative to percentiles would be the raw score, say "14 out of 20." But that would not tell you much without knowing what other people scored. Is 14/20 a good score, a bad score, or an average? We do not know unless we compare it with scores other people achieved. Thus, this is what a percentile score does for us.

What is It

Steps in Finding the z-score given the percentile:

1) Rewrite the percentile as probability.
2) Locate the newly found figure (area) in the z-table.
3) Identify the z-score.
4) Illustrate the normal curve reflecting the region being described.

Example 3.1

Find the 95th percentile of the normal curve.

Solution:

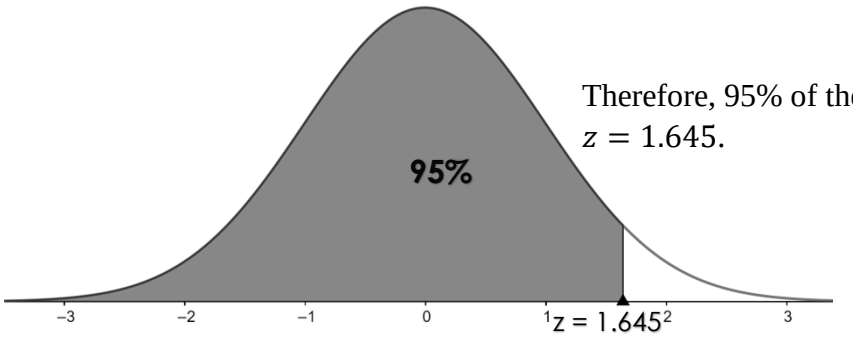
i) By definition, P_{95} means locating an area before or below the point. We want to know what the z-value is at this point. Thus, let us rewrite the 95th percentile into probability, that is**, 0.9500**.

ii) Refer to the z-table and look for 0.9500. Once found, identify the z-value by considering the leftmost column and the topmost row. Unfortunately, there is no 0.9500 in the table. It is between

0.9495 and 0.9505 with corresponding z-scores of 1.64 and 1.65. Since 0.9500 is halfway between 0.9495 and 0.9505, we will take the average of their z-scores. Thus,

$$z = \frac{1.64 + 1.65}{2} = \mathbf{1.645}$$

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767



Therefore, 95% of the distribution lies below $z = 1.645$.

Example 3.2

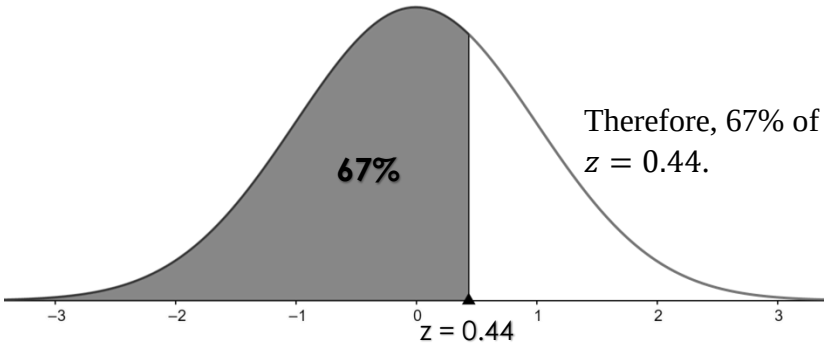
Find the 67th percentile of the normal curve.

Solution:

i) P_{67} means locating an area below the point. We want to know what the z-value is at this point. Thus, let us rewrite the 67th percentile into probability, that is**, 0.6700**.

ii) Use the z-table and look for 0.6700. Once located, identify the z-score from the leftmost column and the topmost row. Thus, it is **0.44.**

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879



Therefore, 67% of the distribution lies below $z = 0.44$.

Example 3.3

The height of 12-month-old boys is normally distributed with μ=76.4, σ=2.9 cm. What is the 10th percentile for height?

Solution:

i) Find the z-score for 10th percentile. Again, there is no $\mathbf{0.1000}$ in the z-table. Instead, it lies one-fifth of the way from 0.1003 to 0.0985. that corresponds to $\mathbf{-1.28}$ and $\mathbf{-1.29}$. Thus, to find the one-fifth of the way,

$$z = -1.28 + \frac{1}{5}(-1.29 - (-1.28))$$

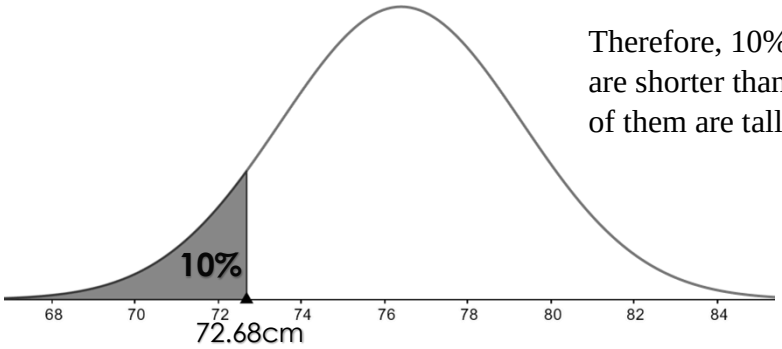
$$\boldsymbol{z = -1.282}$$

ii) At this time, let us find the 10th percentile for height by using the z-score found and the z-formula. We have,

$$x = \mu + z\sigma$$

$$= 76.4 + (-1.282)(2.9)$$

$$\boldsymbol{x = 72.68}$$



Therefore, 10% of the 12-month-old boys are shorter than 72.68 cm. Moreover, 90% of them are taller than 72.68cm.

What's More

Task 3.2

A. Solve each of the percentile below and sketch its graph. Write your answer in your notebook.
   1. 5^{th} percentile
   2. 40^{th} percentile
   3. P_{60}
   4. P_{39}

What I Have Learned

Task 4

Generalization

Write your learning of the lesson in your notebook by completing the unfinished statement below.

1. I have realized that provided with the z-table _____ _____.
2. To find the area of the shaded region of the standard normal distribution_____ _____.
3. After reading and doing the different activities in this lesson, I can now deduce the difference between "a normal distribution" and "the standard normal distribution". That is _____ _____.
4. If the given is a normal distribution and to have a meaningful analysis of the data, we need to _____ _____.
5. Percentile is used in stating results especially for standardized test like National Achievement Test (NAT) because _____ _____.

What I Can Do

Task 5

In your activity notebook/activity sheet, answer the following tasks.

A. Solve for the area of the following probability notations. Then, sketch the normal curve shading the required region of each given notation.

1. $P(Z < -1.1)$
2. $P(Z > -1.1)$
3. $P(0.01 < Z < 3.19)$

B. Using Normal Distributions. Answer the questions about the specified normal distribution in your notebook.

Heights of Women. In a survey of women in the Philippines (ages 20-29), the mean height was 64.1 inches with a standard deviation of 2.71 inches.

a) What height represents the 95^{th} percentile?
b) What height represents the first quartile?

C. Draw a normal curve with a mean of 60 and a standard deviation of 12. Describe how you constructed the curve and discuss its features.

Rubric for Performance Task 1.This performance task is worth 40 points.

	Unsatisfactory (0 point)	**Needs Improvement (5points)**	**Satisfactory (7 points)**	**Exemplary (10 points)**
Completeness of solution	0-25% solution complete	26-50% solution complete	51-75% solution complete	76-100% solution complete
Answer accuracy	0-25% answer accuracy	26-50% answer accuracy	51-75% answer accuracy	76-100% answer accuracy
On the graph/ normal curve	0-25% Correct direction of shading and region are shown	26-50% Correct direction of shading and region are shown	51-75% Correct direction of shading and region are shown	76-100% Correct direction of shading and region are shown

(Note to teacher: The final say as to the number of credit points in each column still depends on you).

Assessment

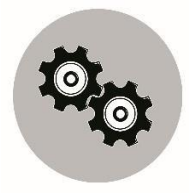
Task 6

Read and analyze each statement carefully. In your notebook write only the letter of your choice.

1. What is the area under the normal curve?
   A. 0 B. 1 C. 2 D. 3
2. Which of the following is the curve symmetrical to?
   A. Mean B. Variance C. Standard Deviation D. Area
3. What is the area when the z-score is 0.00?
   A. 0.00 B. 0.50 C. 0.10 D. 100
4. Find the area when the z-score is above 1.92?
   A. 0.0274 B. 0.9726 C. 0.5129 D. 0.0329
5. Which of the following z-score makes the notation a TRUE statement.
   $$P(z < Z < 3) = 0.1574$$
   A. 0 B. 1 C. 2 D. 3
6. Given that X is normally distributed with $\mu = 85$ and $\sigma = 6$. Find the z-score when $x = 97$.
   A. $z = 1$ B. $z = 2$ C. $z = 3$ D. $z = 4$
7. What is the probability of $P(X < 208)$ when $\mu = 178$ and $\sigma = 15$?
   A. 2.00 B. 0.0228 C. 0.5000 D. 0.9772
8. Assuming that the heights of college male students are normally distributed with a mean of 69 inches and standard deviation of 3 inches. If a student is chosen at random, what is the probability that his height is less than 65 inches?
   A. 0.0918 B. 0.3707 C. 0.9082 D. 0.9991
9. The weights of the contents of a cereal box are normally distributed with a mean weight of 20 ounces and a standard deviation of 0.07 ounce. Boxes in the lower 6.3% do not meet the minimum weight requirements and must be repackaged. What is the minimum weight requirement for a cereal box?
   A. 20 oz B. 19.99 oz C. 19.89 oz D. 19.79 oz
10. In a large section of a statistics class, the points for the final exam are normally distributed with a mean of 72 and a standard deviation of 9. A student will receive an A grade if he/she is in the top 10%. What will be the possible lowest score on the final exam that would qualify a student for an A?
    A. 72 B. 84 C. 60 D. 54
11. What is the value of the 82nd percentile?
    A. $z = 0.92$ B. $z = 0.715$ C. $z = 0.515$ D. $z = 0.315$
12. Solving for P_{48}, its z-score lies between what two z-scores?
    A. $z = -0.48$ and $z = -0.49$ B. $z = -0.47$ and $z = -0.48$
    C. $z = -0.04$ and $z = -0.05$ D. $z = -0.05$ and $z = -0.06$
13. When the probability you are looking at is not in the z-table, how will you find its z-score?
    A. Randomly choose any probability from the z-table.
    B. Look for another z-table, maybe it is in that new z-table.
    C. Choose the closest probability.
    D. Do not answer the problem.

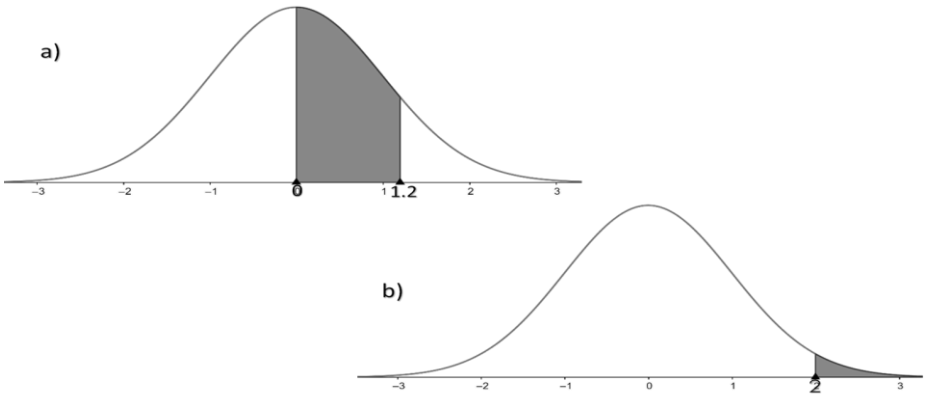
14. Which of the following statistics always corresponds to the 75^{th} percentile in a distribution?
    A. 3^{rd} decile B. 3^{rd} quartile C. 7^{th} decile D. 7th quartile

15. In a survey of men in the Philippines (ages 20-29), the mean height was 69.6 inches with a standard deviation of 3.0 inches. What height represents the first quartile?
    A. 68.57 inches B. 67.59 inches C. 66.59 inches D. 65.57 inches



Additional Activities

A. Graphical Analysis. Find the area of the indicated region under the standard normal curve.



B. Writing a Guarantee. You sell a brand of automobile tire that has a life expectancy that is normally distributed, with a mean life of 300,000 miles and a standard deviation of 2500 miles. You want to give a guarantee for free replacement of tires that don't wear well. How should you word your guarantee if you are willing to replace approximately 10% of the tires you sell? Write your answer in your notebook.

C. Getting at the Concept: If a z-score is zero, which of the following must be true? Write your reasoning in your notebook.

a) The mean is zero.
b) The corresponding x-value is zero.
c) The corresponding x-value is equal to the mean.

Answer Key

What I Need to Know

Pretest

1. T
2. T
3. T
4. F
5. T
6. T
7. F
8. F
9. T
10. T

What's In

Task 1.1

1.Bell-shaped
2.mean=median=mode
3.symmetrical about the mean
4.area=1 or 100%
5. asymptotic to the x-axis

Task 2.1

1. C
2. C
3. A
4. D
5. B

Task 3.1

1. $z = -1.01$; 0.1562
2. $z = 0.19$; 0.5753
3. $z = -2.73$; 0.0032
4. $z = 2.00$; 0.9772
5. $z = 3.05$; 0.9989

What's More

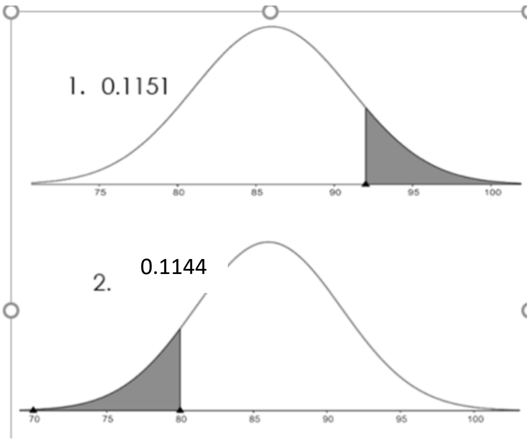
Task 1.2

A.1.C 2. E 3. A
4. D 5. F 6. B

B. 7. 0.0668
8. 0.1024

C. 9. -2.05
10. 0.17

Task 2.2 A



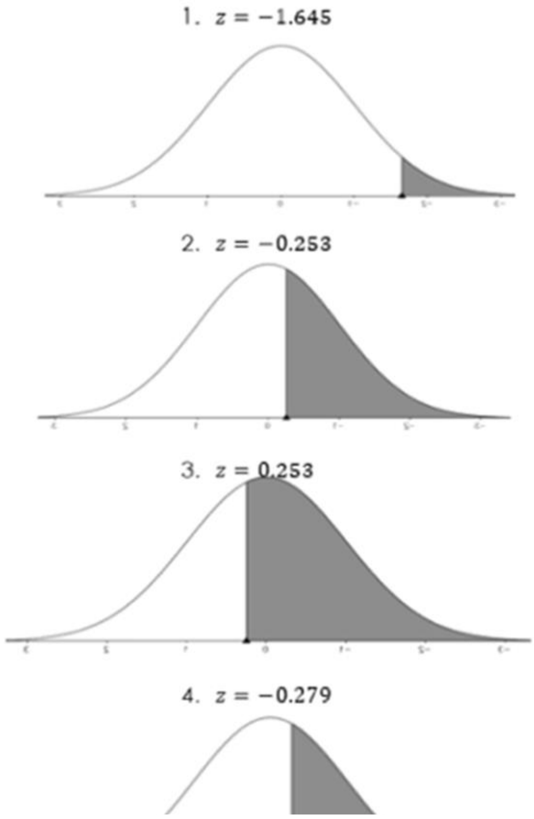
Task 2.2B

3. 0.3155
4. 0.1901

Task 2.2C

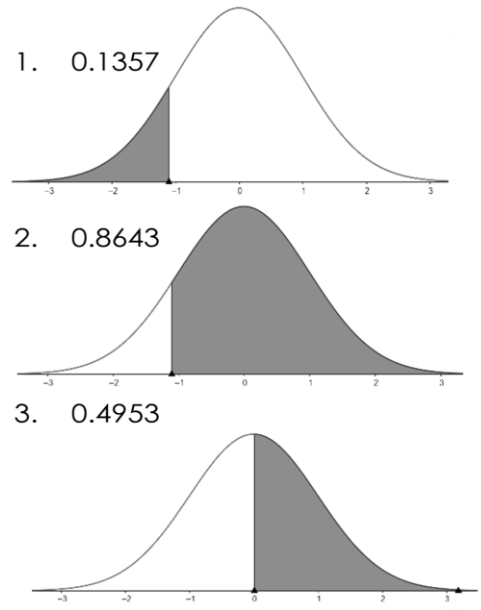
5. a) 0.0668 ; b) 0.0668 ; c) 0.5328
6. 86

Task 3.2



What I Have Leaned
Answers may vary

What I Can Do
Task 5 A



Task 5 B

1. a) 68.56 inches
b)62.28 inches

Task 5 C

In your notebook, draw a normal curve with a mean of 60 and a standard deviation of 12. Describe how you constructed the curve and discuss its features.

Assessment

1.B	6.B	11.A
2.A	7.D	12.D
3.B	8.A	13.C
4.A	9.C	14.B
5.B	10.B	15.B

Additional Activities
Lesson 1.

1. 0.3849
2. 0.0228

Lesson 2
Tires that wear out by 26,800miles (Tech: 26,796 miles) will be replaced free of charge.
Lesson 3

References

Books

Mamhot, M.R., et al, *Statistics for General Education (A Keypad Approach),* Purelybooks Trading &Publishing Corp., (2013)

Larson, Ron and Faber, Betsy. "Chapter 5. Normal Probability Distributions." In *Elementary Statistics ,5th edition*. Pearson Education, Inc. , 2012.

Websites

aanda.org.https://www.aanda.org/articles/aa/full_html/2017/04/aa29344-16/aa29344-16-fig27.pdf

chegg.com. n.d. https://tinyurl.com/2p8sn5ht (accessed January 10, 2022).

www2.math.upenn.edu. n.d. https://www2.math.upenn.edu/~chhays/zscoretable.pdf.

La Morte, Wayne W. *chegg.com.* April 21, 2021. https://tinyurl.com/5frzmm67 (accessed January 10, 2022).

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