11

SENIOR HIGH SCHOOL

DEPARTMENT OF EDUCATION
SCHOOLS DIVISION OF NEGROS ORIENTAL
REGION VII
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STATISTICS and PROBABILITY

Quarter 3 – Module 3

Mean and Variance of a Discrete Random Variable and Normal Random Variable

ALTERNATIVE DELIVERY MODE

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Quarter 3 – Module 3: <u>Mean and Variance of a Discrete Random Variable and Normal Random Variable</u>
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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

The module is intended for you to interpret the mean and the variance of a discrete random variable and illustrate a normal random variable and its characteristics.

After going through this module, you are expected to solve problems involving mean and variance of probability distributions.

What I Know

PRETEST

Find what is asked. Write your answers on your activity notebook/activity sheet.

A. Consider the problem below.

A certain university medical research center finds out that treatment of skin cancer by the use of chemotherapy has a success rate of 70%. Suppose five patients are treated with chemotherapy. The probability distribution of x successful cures of the five patients is given in the table below:

X	0	1	2	3	4	5
P(x)	0.002	0.029	0.132	0.309	0.360	0.168

Probability distribution of cancer cures of five patients.

1. Find μ
2. Find σ^2
3. Find σ
4. Graph *p(x)* and explain how μ and σ can be used to describe *p(x).*

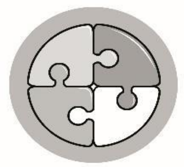
B. Directions: Do the following activity to check your understanding of the probability distribution.

Determine whether the statement is *True* or *False*. If False, modify the statement to make it True. Write your answer in your notebook.

Statement

1. The area under a probability distribution correspond to the probabilities of a random variable X.
2. Data in a continuous type are presented through a normal curve or normal probability distribution.
3. It is the variance of a normal distribution that locates the center of the distribution.
4. The normal curves vary in their center and spread which is dependent to the values of the variance and standard deviation.
5. It is the standard deviation that is in control of the geometry of the normal curve.
6. The area under the curve is 1.
7. The area under the curve represents the probability or proportion or the percentage associated with specific sets of measurement values.
8. A standard normal curve is a normal probability distribution that has a mean equal to 0 and a standard deviation equal to 1.
9. The curve is symmetrical about its center.
10. The mean, the median, and the mode coincide at the center.

LESSON 1 Mean and Variance of a Discrete Random Variable



What's In

Task 1.1

Match Column A with Column B. Write the letter of your chosen answer in your notebook.

Column A

1. also called probability mass function
2. a numerical description of the outcome of a statistical experiment
3. a weighted average of the values the random variable may assume
4. weighted average of the squared deviations from the mean
5. data that can take certain values, counted

Column B

A. random variable
B. variance
C. discrete probability distribution
D. discrete data
E. expected value or mean
F. continuous data

What's New

In a specified discrete random variable X, the mean, denoted by μ, is the summation of the products formed from multiplying the possible values of X with their corresponding probabilities. It is also called the expected value of X, and given a symbol *E(X)*.

$$\mu = E(X) = X_1 P_1 + X_2 P_2 + \cdots + X_k P_k$$

$$= \Sigma X_i P_i$$

where; Σ is Sigma notation, and means to sum up.

Note, that the empirical probabilities lean towards theoretical probabilities and, in consequence, the mean is also a long-run average, or the expected average outcome over many observations. That is, as the number of trials of a statistical experiment increases, the empirical average also gets closer and closer to the value of the theoretical average. Mean is the value that we expect the long-run average to approach and it is not the value of the random variable X that we expect to observe.

The variance of a discrete random variable X measures the spread, or variability, of the distribution. The variance, usually denoted by the symbol σ^2, and is also denoted as *Var (X)* and formally defined as

$$\sigma^2 = Var\ (X) = \Sigma\ (X_i - \mu)^2\ P_i$$

The variance gives a measure of how far the values of X are from the mean. Note, that in nontrivial cases (i.e. when there is more than one possible distinct value of X), the variance will be a positive value. The bigger the value of the variance, the farther the values of X get from the mean.

The standard deviation is defined as the square root of the variance of X. That is,

$$\sigma = \sqrt{Var\ (X)}$$

What is It

Since we already reviewed the equation for calculating the mean, variance and standard deviation, let us now solve some problems and interpret the results.

<u>Example 1.1</u>

GROCERY ITEMS

The probabilities that a customer will buy 1,2,3,4, and 5 items in a grocery store are $\frac{3}{10}, \frac{1}{10}, \frac{1}{10}, \frac{2}{10}, and\ \frac{3}{10}$ respectively. What is the average number of items that a customer will buy?

Solution:

$$\mu = E(X) = X1\ P1 + X2\ P2 + \cdots + Xk\ Pk$$
$$= \Sigma\ Xi\ Pi$$
$$\mu = (1)(\frac{3}{10}) + (2)(\frac{1}{10}) + (3)(\frac{1}{10}) + (4)(\frac{2}{10}) + (5)(\frac{3}{10})$$

$$\mu = \frac{3}{10} + \frac{2}{10} + \frac{3}{10} + \frac{8}{10} + \frac{15}{10}$$
$$\mu = 3.1$$

So, the mean of the probability distribution is 3.1. This implies that the average number of items that the customer will buy is 3.1.

<u>Example 1.2</u>

The probabilities that a surgeon operates on 3,4,5,6, or 7 patients in any day are 0.15, 0.10, 0.20, 0.25, and 0.30, respectively. Find the average number of patients that a surgeon operates on a day.

$$\mu = E(X) = X1\ P1 + X2\ P2 + \cdots + Xk\ Pk$$
$$= \Sigma\ Xi\ Pi$$
$$\mu = 3(0.15) + 4(0.10) + 5(0.20) + 6(0.25) + 7(0.30)$$
$$\mu = 0.45 + 0.40 + 1 + 1.50 + 2.10$$
$$\mu = 5.45$$

So, the average number of patients that a surgeon will operate in a day is 5.45 or 6.

What's More

TASK 1.2

NUMBER OF CARS SOLD

The number of cars sold per day at a local car dealership, along with its corresponding probabilities, is shown in the succeeding table. Compute the variance and the standard deviation of the probability distribution.

Number of Cars Sold, X	Probability, P(X)
0	$\frac{1}{10}$
1	$\frac{2}{10}$
2	$\frac{3}{10}$
3	$\frac{2}{10}$
4	$\frac{2}{10}$

Solution:
Solving for the variance; utilize the formula

$$\sigma^2 = Var(X) = \Sigma (Xi - \mu)^2 Pi$$

a) Solving for the mean first;

$$\mu = E(X) = X_1 P_1 + X_2 P_2 + \cdots + X_k P_k$$
$$= \Sigma Xi Pi$$
$$\mu = (0)(\frac{1}{10}) + (1)(\frac{2}{10}) + (2)(\frac{3}{10}) + (3)(\frac{2}{10}) + (4)(\frac{2}{10})$$
$$\mu = 0 + \frac{2}{10} + \frac{6}{10} + \frac{6}{10} + \frac{8}{10}$$
$$\mu = 2.2$$

Now solving for variance;

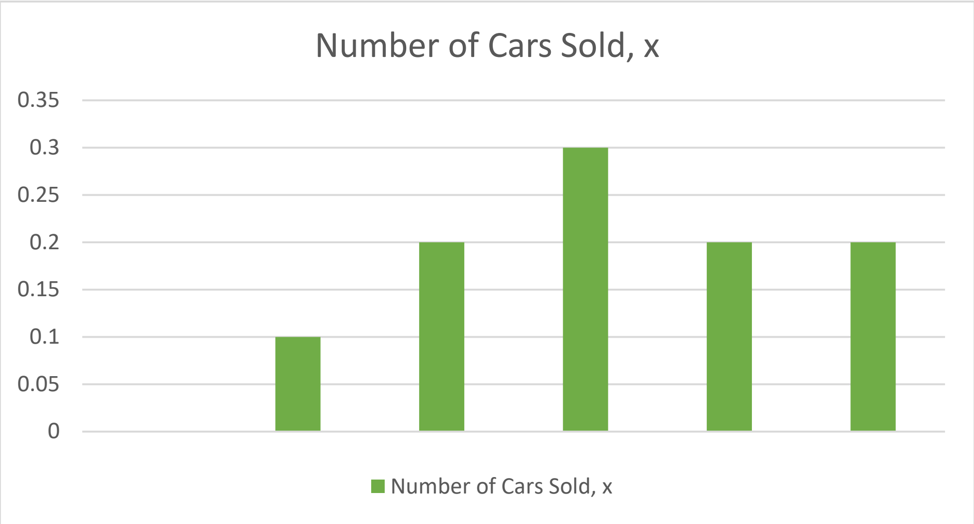
$$\sigma^2 = Var(X) = \Sigma (Xi - \mu)^2 Pi$$

$$\sigma^2 = (0-2.2)^2 (\frac{1}{10}) + (1-2.2)^2 (\frac{2}{10}) + (2-2.2)^2 (\frac{3}{10}) + (3-2.2)^2 (\frac{2}{10}) + (4-2.2)^2 (\frac{2}{10})$$

$$\sigma^2 = 1.56$$

Therefore, the standard deviation is $\sigma = \sqrt{1.56} = 1.25$

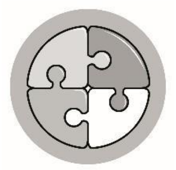
Plotting the graph, we have;



Notice that $\mu = 2.2$ is the center of probability distribution. We can conclude, that the number of cars sold per day in a local car dealership is 2.2 or approximately 3 cars. The standard deviation which is $\sigma = 1.25$ in this case, measures the spread of the probability distribution p(x).

The variance gives a measure of how far the values of X are from the mean. The bigger the value of the variance, the farther the values of X get from the mean.

LESSON 2 Normal Random Variable and its Characteristics



What's In

Review of Continuous Random Variables

Recall the definition of a continuous random variable. It is a random variable that can take any real value within a specified range whereas a discrete random variable takes some on a countable number of values). A continuous variable involves the measurement of something, such as height of a randomly selected student, the weight of a newborn baby, or the length of time that the battery of a cellphone lasts. (Pungtilan, n.d.)

What's New

There are many events in real life that generate random variables that have the natural tendency to approximate the shape of a bell. For example, the heights of a large number of seedlings that we see in fields *normally*, consist of a few tall ones, a few short ones, and most of them having heights in between tall and short. If a well-prepared test is administered to a class of 100 students, there will be a few high scores, as well as a few low scores. Most of the scores will be found in between these two extremes scores. In reality, if a distribution consists of a very large number of cases and the three measures of averages (mean, median, mode) are equal, then the distribution is symmetrical and the skewness is zero. In Statistics, such distribution is called *normal distribution* or simply *normal curve*.

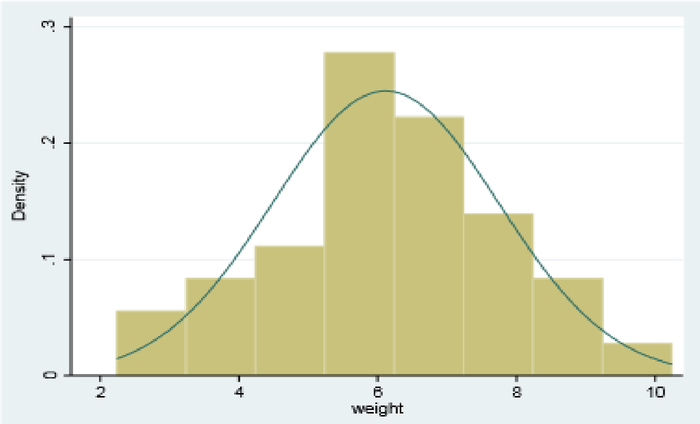
The normal curve has a very important role in inferential statistics. It provides a graphical representation of statistical values that are needed in describing the characteristics of populations as well as in making decisions. It is defined by an equation that uses the population mean, μ and the standard deviation, σ. There is no single curve, but rather a whole family of normal curves that have the same basic characteristics but have different means and standard deviations.

Consider the following data pertaining to hospital weights (in pounds) of all the 36 babies that were born in the maternity ward of a certain hospital.

4.94	4.69	5.16	7.29	7.19	9.47	6.61	5.84	6.83
3.45	2.93	6.38	4.38	6.76	9.01	8.47	6.8	6.4
8.6	3.99	7.68	2.24	5.32	6.24	6.19	5.63	5.37
5.26	7.35	6.11	7.34	5.87	6.56	6.18	7.35	4.21

The data have an average of 6.11 pounds and a standard deviation of 1.61 pounds.

Shown below is the histogram for this data set. Observe that the histogram is approximately bell-shaped.

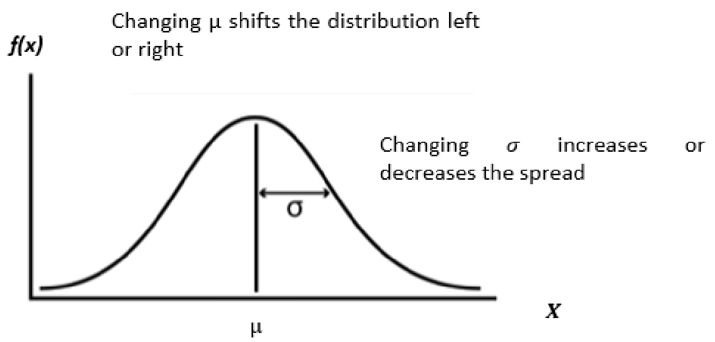


What is It

Many variables, such as weight, shoe sizes, foot lengths, and other human physical characteristics, exhibit these properties. The symmetry indicates that the variable is just as likely to take a value a certain distance below its mean as it is to take a value that same distance above its mean. The bell shape indicates that values closer to the mean are more likely, and it becomes increasingly unlikely to take values far from the mean in either direction.

We use a mathematical model with smooth bell-shaped curve to describe these bell-shaped data distributions. These models are called **normal curves** or **normal distributions**. The normal distribution is a continuous distribution just like the uniform and triangular distribution. However, the left and right tails of the normal distribution extend indefinitely but come infinitely close to the x-axis.

The general shape of the mathematical model used to generate a normal curve looks like this:



(Llego, 2017)

Figure 1: Mathematical Model for a Normal Curve

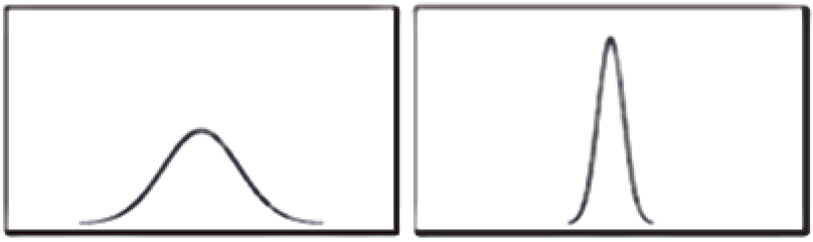
The graph of the normal distribution depends on two factors: the mean μ, and the standard deviation σ. In fact, the mean and standard deviation characterize the whole distribution. That is, we can get areas under the normal curve given information about the mean and standard deviation.

The mean determines the location of the center of the bell -shaped curve. Thus, a change in the value of the mean shifts the graph of the normal curve to the right or to the left.

Recall what the mean, median, mode of a distribution represent, (a) the mean represents the balancing point of the graph of the distribution; (b) the mode represents the "high point" of the probability density function (i.e. the graph of the distribution), (c) the median represents the point where 50% of the area under the distribution is to the left and 50% of the area under the distribution is to the right.

For symmetric distributions with a single peak, such as the normal curve, take note that; Mean = Median = Mode.

The standard deviation determines the shape of the graphs (particularly, the height and width of the curve). When the standard deviation is large, the normal curve is short and wide, while a small value for the standard deviation yields a skinnier and taller graph.



(Llego, 2017)

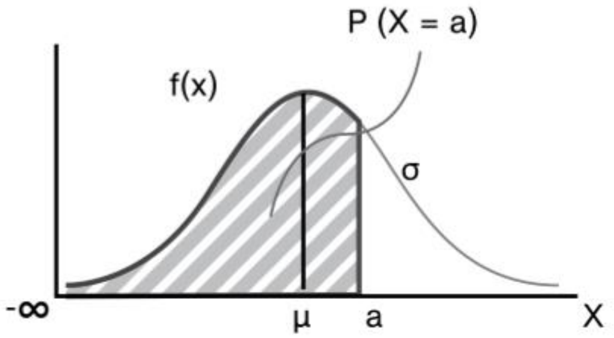
Figure 2: Height and Width of the Curve

The cure above on the left, is shorter and wider than the curve on the right, because the curve on the left has a bigger standard deviation. Note, that a normal curve is symmetric about its mean and is **more concentrated in the middle** rather than in the tails.

Properties of the Normal Probability Distribution

Use Figure 3 to understand the properties of a normal probability distribution.

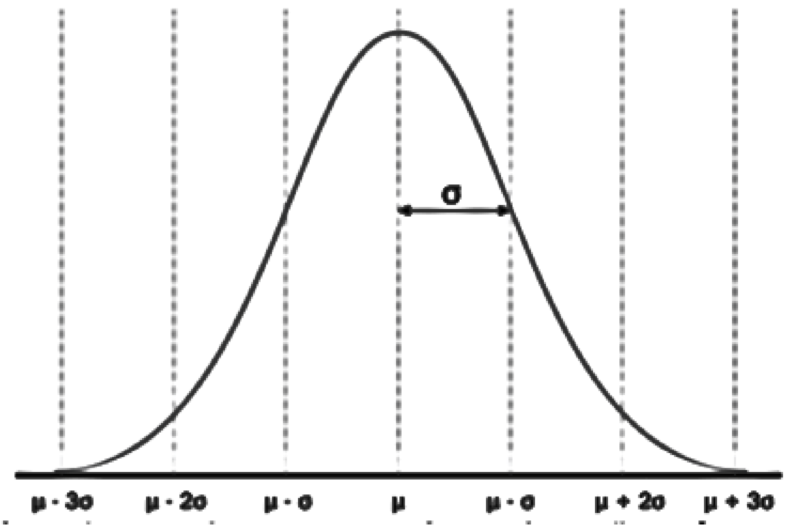
- The total area under the normal curve is equal to 1.
- The probability that a normal random variable X equals any particular value a, P(X=a) is zero (0) (since it is a continuous random variable).
- The probability that X is less than a, equals the area under the normal curve bounded by a and minus infinity (as indicated by the shaded area in figure 3 below)
- The probability that X is greater than some value a, equals the area under the normal curve bounded by a and plus infinity (as indicated by the non-shaded area in figure 3)
- Since the normal curve is symmetric about the mean, the area under the curve to the right of μ equals the area under the curve to the left of μ which equals ½, i.e. the mean $\mu$is the median.
- The probability density function is maximized at μ, i.e. the mode is also the mean
- The normal curve has inflection points (i.e. point at which a change in the direction of curvature occurs) at μ - s and at μ + s
- As x increases without bound (gets larger and larger), the graph approaches but never reaches, the horizontal axis. As x decreases without bound (gets larger and larger in the negative direction), the graph approaches, but never reaches, the horizontal axis



(Llego, 2017)

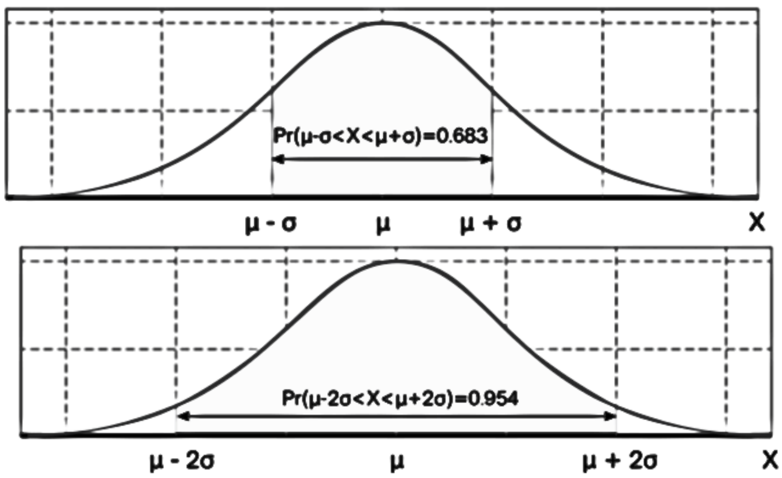
Figure 3

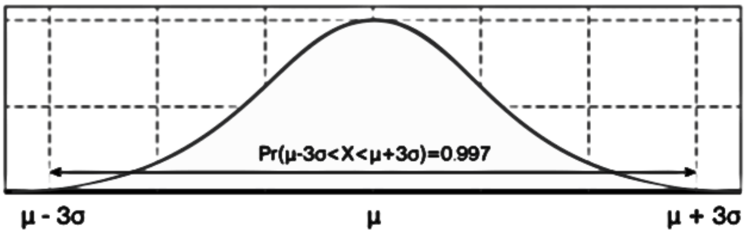
- Take note that every normal curve (regardless of its mean or standard deviation) conforms to the following "empirical rule" (also called the 68-95-99.7 rule):



(Llego, 2017)

- About 68% of the area under the curve falls within 1 standard deviation of the mean.
- About 95% of the area under the curve falls within 2 standard deviations of the mean.
- Nearly the entire distribution (About 99.7% of the area under the curve) falls within 3 standard deviations of the mean.





(Llego, 2017)

Explanatory Note: The empirical rule is actually a theoretical result based on an analysis of the normal distribution. In the first chapter, it was pointed out that the importance of the mean and standard deviation as summary measures is due to Chebyshev's Inequality, which guarantees that the area under a distribution within two standard deviations from the mean is at least 75%. For nearly all sets of data, the actual percentage of data may be much greater than the bound specified by Chebyshev's Inequality. In fact, for a normal curve, the area within two standard deviations from the mean is about 95%. Also, about two thirds of the distribution lie within one standard deviation from the mean and nearly the entire distribution (99.7%) is within three standard deviations from the mean.

Example 2.1

Consider the following data pertaining to hospital weights (in pounds) of all the 36 babies that were born in the maternity ward of a certain hospital.

4.94	4.69	5.16	7.29	7.19	9.47	6.61	5.84	6.83
3.45	2.93	6.38	4.38	6.76	9.01	8.47	6.8	6.4
8.6	3.99	7.68	2.24	5.32	6.24	6.19	5.63	5.37
5.26	7.35	6.11	7.34	5.87	6.56	6.18	7.35	4.21

The data have an average of 6.11 pounds and a standard deviation of 1.61 pounds.

Validating the Empirical Rule

Determine what frequency and relative frequency of babies' weights that are within:

a) One standard deviation from the
b) Two standard deviations from the mean
c) Three standard deviations from the mean

Solution:

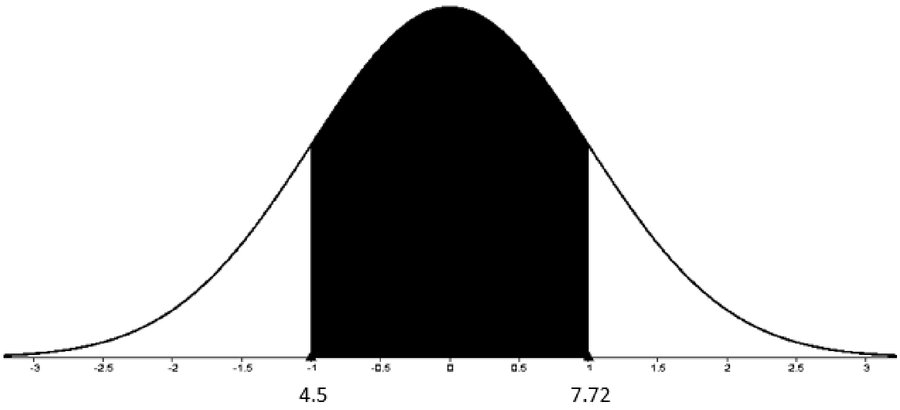
a) To find the frequencies and relative frequency within/under 1 standard deviation, we need to add 1 standard deviation from the mean and subtract 1 standard deviation from the mean to identify the range/area under 1 standard deviation.

$$\mu = 6.11 \qquad and \qquad \sigma = 1.61$$

Range: $\mu - \sigma < x < \mu + \sigma$

$$6.11 - 1.61 < x < 6.11 + 1.61$$

$$4.5 < x < 7.72$$


-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
4.5
7.72

These are the weights of babies in bold face that are within/under 1 standard deviation from 4.5 lbs. to 7.72 lbs.

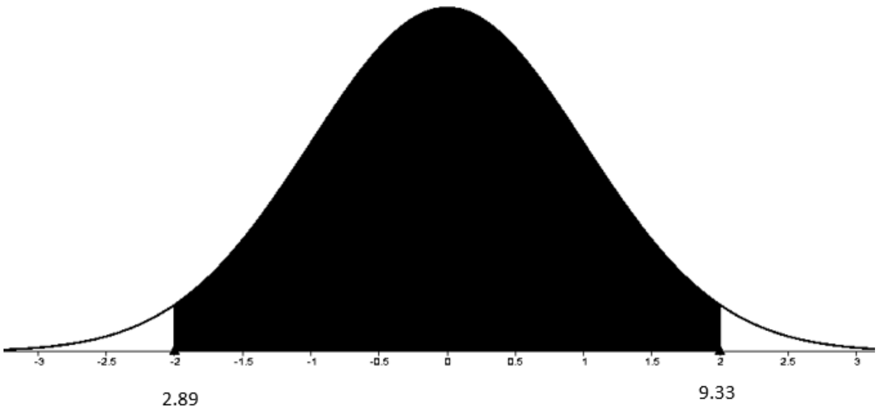
4.94	**4.69**	**5.16**	**7.29**	**7.19**	9.47	**6.61**	**5.84**	**6.83**
3.45	2.93	**6.38**	4.38	**6.76**	9.01	8.47	**6.8**	**6.4**
8.6	3.99	**7.68**	2.24	**5.32**	**6.24**	**6.19**	**5.63**	**5.37**
5.26	**7.35**	**6.11**	**7.34**	**5.87**	**6.56**	**6.18**	**7.35**	4.21

Therefore, there are 26 babies out of 36 babies or about 72% of the babies are within 1 standard deviation from the mean.

b) To find the frequencies and relative frequency within/under 2 standard deviation, we need to add 2 standard deviation from the mean and subtract 2 standard deviation from the mean to identify the range/area under 2 standard deviation.

Range: $\mu - 2\sigma < x < \mu + 2\sigma$

$$6.11 - 2(1.61) < x < 6.11 + 2(1.61)$$
$$6.11 - 3.22 < x < 6.11 + 3.22$$
$$2.89 < x < 9.33$$



These are the weights of babies in bold face that are within/under 2 standard deviation from 2.89 lbs. to 9.33 lbs.

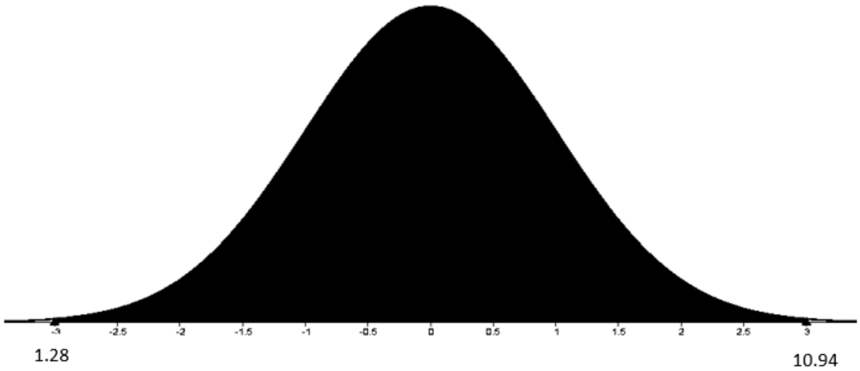
4.94	**4.69**	**5.16**	**7.29**	**7.19**	9.47	**6.61**	**5.84**	**6.83**
3.45	**2.93**	**6.38**	**4.38**	**6.76**	**9.01**	**8.47**	**6.8**	**6.4**
8.6	**3.99**	**7.68**	2.24	**5.32**	**6.24**	**6.19**	**5.63**	**5.37**
5.26	**7.35**	**6.11**	**7.34**	**5.87**	**6.56**	**6.18**	**7.35**	**4.21**

Therefore, there are 34 babies out of 36 babies or about 94% of the babies are within 2 standard deviation from the mean.

c) To find the frequencies and relative frequency within/under 3standard deviation, we need to add 3 standard deviation from the mean and subtract 3 standard deviation from the mean to identify the range/area under 3 standard deviation.

Range: $\mu - 3\sigma < x < \mu + 3\sigma$

$$6.11 - 3(1.61) < x < 6.11 + 3(1.61)$$
$$6.11 - 4.83 < x < 6.11 + 4.83$$
$$1.28 < x < 10.94$$



These are the weights of babies in bold face that are within/under 3 standard deviation from 1.28 lbs. to 10.94 lbs.

4.94	**4.69**	**5.16**	**7.29**	**7.19**	**9.47**	**6.61**	**5.84**	**6.83**
3.45	**2.93**	**6.38**	**4.38**	**6.76**	**9.01**	**8.47**	**6.8**	**6.4**
8.6	**3.99**	**7.68**	**2.24**	**5.32**	**6.24**	**6.19**	**5.63**	**5.37**
5.26	**7.35**	**6.11**	**7.34**	**5.87**	**6.56**	**6.18**	**7.35**	**4.21**

Therefore, there are 36 babies out of 36 babies or about 100% of the babies are within 3 standard deviation from the mean. (Albacca, Zita, Albert, Jose Ramon G., Ayaay, Mark John V., et al. 2016)

What's More

TASK 2.1 Validating the Empirical Rule

Fifty students were asked to run a 100-meter dash. The data below represents the time it took to finish the dash, and the histogram. The mean time for the 50 students is 15.8 seconds, and the standard deviation s is approximately 3.29 seconds.

16	14	14	16	21	14	17	15	16	21
14	10	9	20	12	12	19	11	15	14
18	18	13	18	23	8	20	13	16	23
16	17	15	18	17	16	13	15	18	19
12	12	15	17	14	16	17	16	16	21

Draw the normal curve, find the probability/relative frequency and the number of students under/within:

a) 1 standard deviation
b) 2 standard deviation
c) 3 standard deviation

What I Have Learned

Generalization

Task 3

A. Now, that we are finished with our lesson, let us review the concepts we have learned by filling in the blanks with appropriate word or phrase to make meaningful statements.

1. The curve of a probability distribution is formed by ________.
2. The area under a normal curve is ________.
3. The important values that best describe a normal curve are _________.
4. There are ______ standard units at the baseline of a normal curve.
5. The curve of a normal distribution extends indefinitely at the tails but does not ________.
6. The mean, the median, and the mode of a normal curve are ________.
7. A normal curve is used in ___________.

B. Directions: Reflect the learning that you gained after taking up the two lessons in this module by completing the given statements below. Do this on your activity notebook. Do not write anything on this module.

What were your thoughts or ideas about the topic before taking up the lesson?

I thought that

____

____

____.

What new or additional ideas have you had after taking up this lesson?

I learned that (write as many as you can)

____

_____

____

____

____

____

____

_____________.

How are you going to apply your learning from this lesson?

I will apply

____

____

____.

What I Can Do

TASK 4. Application

Direction: In your activity notebook/activity sheet, perform the task given below.

A. The number of inquiries received per day by the Office Admissions in a certain university is shown below. Find the variance and standard deviation of the distribution. Draw the graph of the probability distribution.

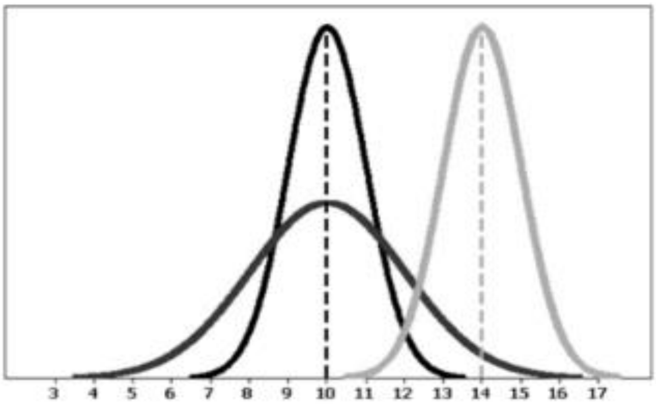
No. of inquiries, X	22	23	24	25	26	27
Probability, P(X)	0.08	0.19	0.36	0.25	0.07	0.05

Rubric for Performance Task 1. This performance task is worth 45 points.

	Unsatisfactory (0 point)	**Needs Improvement (5points)**	**Satisfactory (10 points)**	**Exemplary (15 points)**
Completeness of solution	0-25% solution complete	26-50% solution complete	51-75% solution complete	76-100% solution complete
Answer accuracy	0-25% answer accuracy	26-50% answer accuracy	51-75% answer accuracy	76-100% answer accuracy
On the graph/ normal curve	0-25% Correct direction of shading and region are shown	26-50% Correct direction of shading and region are shown	51-75% Correct direction of shading and region are shown	76-100% Correct direction of shading and region are shown

B. There are many normal curves. Even though all normal curves have the same bell shape, they vary in their center and spread. What observation can you make as you look into the figure? Write you observation in your notebook.
[https://courses.lumenlearning.com

Observations of Normal Distributions



[https://www.ck12.org/c/probability/August 2012]

Rubrics

	Excellent (5pts)	Competent (3pts)	Not acceptable (1pt)
Analysis/evaluation/interpretation are effective and consistent			
Claims and ideas are supported and elaborated			
Information and evidence are accurate, appropriate, and integrated effectively			
Sentence form and word choice are varied and appropriate.			

(Note to teacher: The final say as to the number of credit points in each column still depends on you).

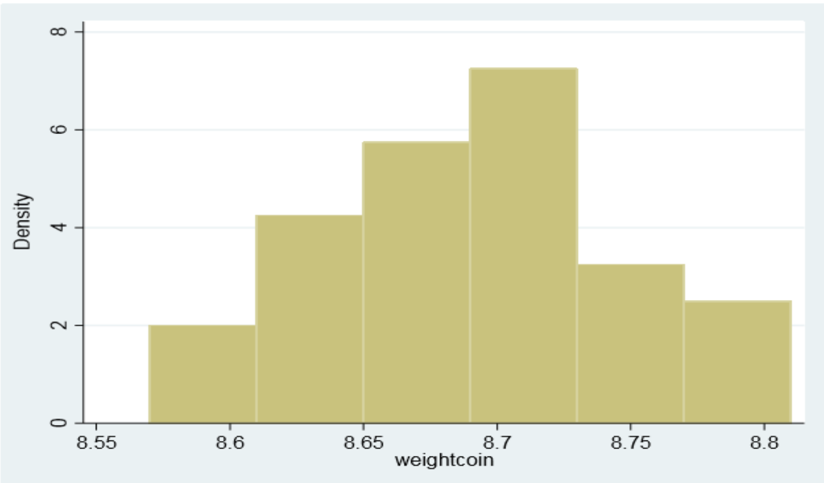
Assessment

A. Suppose that in a casino, a certain slot machine pays out an average of Php15, with a standard deviation of Php5000. Every play of the game costs a gambler Php20.

1) Why is the standard deviation so large?
2) If your parent decides to play with this slot machine 5 times, what are the mean and standard deviation of the casino's profit?
3) If gamblers play with this slot machine 1000 times in a day, what are the mean and standard deviation of the casino's profits?

B. The data below and the accompanying histogram give the weights, to the nearest hundredth of a gram, of a sample of 100 coins (each with a value of P10). The mean weight is 8.69 grams and the standard deviation s is approximately 0.055 gram.

8.57	8.62	8.65	8.67	8.68	8.7	8.71	8.73	8.74	8.77
8.57	8.62	8.65	8.67	8.68	8.7	8.71	8.73	8.74	8.77
8.58	8.63	8.65	8.67	8.69	8.7	8.71	8.73	8.74	8.77
8.59	8.63	8.65	8.67	8.69	8.7	8.72	8.73	8.75	8.78
8.6	8.63	8.65	8.67	8.69	8.7	8.72	8.73	8.75	8.78
8.6	8.63	8.66	8.67	8.69	8.71	8.72	8.73	8.75	8.79
8.61	8.64	8.66	8.68	8.69	8.71	8.72	8.73	8.76	8.79
8.61	8.64	8.66	8.68	8.69	8.71	8.72	8.74	8.76	8.8
8.62	8.64	8.66	8.68	8.7	8.71	8.72	8.74	8.76	8.81
8.62	8.64	8.66	8.68	8.7	8.71	8.72	8.74	8.76	8.81



https://www.teacherph.com/statistics-&-probability-senior-high-school-teaching-guide/

a. Compare the mean and median.

b. What percentage of the data is within one standard deviation of the mean? Within two standard deviations? Within three standard deviations?

c. Suppose you were to randomly select a coin from this collection. What is the chance that its weight would be within one standard from the mean? Two standard deviations? Three standard deviations?

d. What percentage of the data is below the mean?

e. Suppose you were to randomly select a coin from this collection. What is the chance that its weight would be below the mean?

Additional Activities

A. Conduct a survey on the number of sports related activities your classmates are involved in. construct a probability distribution and compute the mean, variance, and standard deviation.

B. History regarding the Normal Curve

Historical Notes on the Normal Curve:

(i) The French-English mathematician Abraham de Moivre first described the use of the normal distribution in 1733 when he was developing the mathematics of chance, particularly for approximating the binomial distribution. (Pungtilan, 2017)

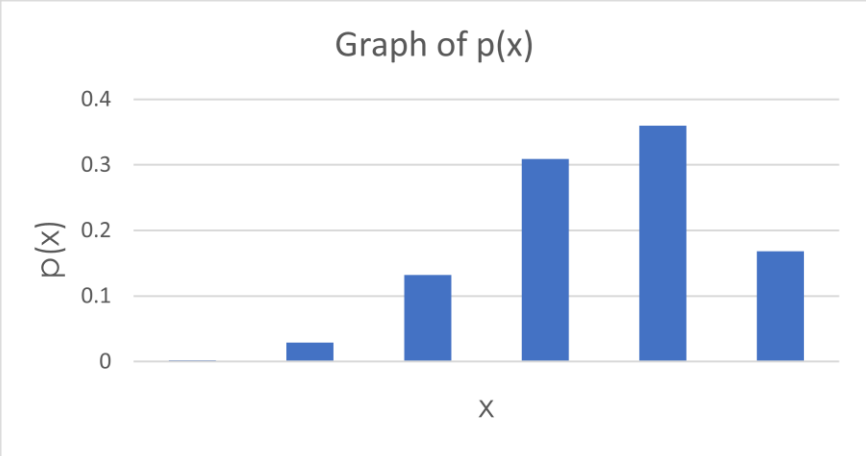
Marquis de Laplace used the normal distribution as a model of measuring errors. Adolphe Quetelet and Carl Friedrich Gauss popularized its use. Quetelet used the normal curve to discuss “the average man” with the idea of using the curve as some sort of an ideal histogram while Gauss used the normal curve to analyze astronomical data in 1809.

(ii) In some disciplines, such as engineering, the normal distribution is also called the Gaussian distribution (in honor of Gauss who did not first propose it!). The first unambiguous use of the term “normal” distribution is attributed to Sir Francis Galton in 1889 although Karl Pearson's consistent and exclusive use of this term in his prolific writings led to its eventual adoption throughout the statistical community.

Answer Key

Pre Assessment

A. 1. $\mu = 3.5$ 2. $\sigma^2 = 1.05$ 3. $\sigma = 1.02$



4.

We can use the mean, or μ, and the standard deviation, or σ, to describe p(x) in the same way we used $\bar{x}$ ans s to describe the relative frequency distribution. Notice that μ = 3.5 is the center of probability distribution. In other words, if the five cancer patients receive chemotherapy treatment, we expect the number of them who are cured to be near 3.5. The standard deviation which is σ = 1.02 in this case, measures the spread of the probability distribution p(x).

B.

1. True	3. False, Mean	5. True	7. True	9. True
2. True	4. True	6. True	8. True	10. True

What's In

Task 1.1

1) C 2) A 3) E 4) B 5) D

What's More

Task 2.1

a) There are 35 students or 70% of the students are within 1 standard deviation from the mean.
b) There are 46 students or 92% of the students are within 2 standard deviation from the mean
c) There are 50 students or 100% of the students are within 3 standard deviation from the mean

What I have learned

Task 3

1. the mean and standard deviation
2. 1
3. Mean and standard deviation
4. 6 (3 below the mean, 3 above the mean)
5. touch the x-axis
6. equal
7. inferential statistics

What I can Do
Task 4
A

Solve for the mean;

$\mu = E(X) = X1\ P1 + X2\ P2 + \cdots + Xk$

$= \Sigma\ Xi\ Pi$

$\mu = (22)(0.08) + (23)(0.19) + (24)(0.36) + (25)(0.25) + (26)(0.07) + (27)(0.05)$

$\mu = 1.76 + 4.37 + 8.64 + 6.25 + 1.82 + 1.35$

$\mu = 24.19$

Now solve for variance;

$\sigma^2 = Var(X) = \Sigma\ (Xi - \mu)^2\ Pi$

$\sigma^2 = (22 - 24.19)^2 (0.08) + (23 - 24.19)^2 (0.19) + (24 - 24.19)^2 (0.36) + (25 - 24.19)^2 (0.25) + (26 - 24.19)^2 (0.07) + (27 - 24.19)^2 (0.05)$

$\sigma^2 = 1.45$

The standard deviation is $\sigma = \sqrt{1.45} = 1.20$

Assessment:

A.

1. Gamblers lose a small amount of money most of the time, but there are a few large payouts by the slot machine

In one play of the game, the slot machine loses X pesos to the gambler, with a mean E(X) = 15, while SD(X) = 5000, or Var(X) = 25,000,000. Note that every play of the game, gamblers are charged Php20. Thus, the casino actually loses only an average of E(X-20) = Php15 - Php20 = - Php5 (i.e. on average, the casino wins Php5 per game), with a variability of Var(X-20) = 25,000,000

2. For 5 plays of the game, expected value of "losses" would be 5 times (-Php5) = -Php25, i.e. (Php25 earned by a casino per game), with a variability of Var(5 (X-20)) =
   25 * 25,000,000 = 625,000,000

And thus, a standard deviation of 25,000

3. For 1000 plays of the game, the expected losses of the casino would be E (1000 (X-20)) = 1000 x (-Php5) = -Php5000 with a variance of Var (1000 (X - 20)) = 1000^2 x (25,000,000)

B.

a. very close, the median is 8.7 grams.
b. 67%, 95%, 100%
c. According to the empirical rule, the chances are 68%, 95%, and 99.7%.
d. 48%
e. 50%

References

Book

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