11

SENIOR HIGH SCHOOL

DEPARTMENT OF EDUCATION
SCHOOLS DIVISION OF NEGROS ORIENTAL
REGION VII
Kagawasan Ave., Daro, Dumaguete City, Negros Oriental

BASIC CALCULUS

Quarter 3 – Module 5
Derivative of a Function and Rules for Differentiation

ALTERNATIVE DELIVERY MODE
ADM

Basic Calculus – Grade 11
Alternative Delivery Mode
Quarter 3 – Module 5: Derivative of a Function and Rules for Differentiation
Second Edition, 2021

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer: Rickleoben V. Bayking

Editors: Ronald G. Tolentino & Gil S. Dael

Reviewer: Littie Beth S. Bernadez

Layout Artist: Radhiya A. Ababon

Management Team: Senen Priscillo P. Paulin CESO V, Joelyza M. Arcilla EdD, CESE, Marcelo K. Palispis JD, EdD, Nilita L. Ragay EdD, Elisa L. Bagiuo EdD, Rosela R. Abiera, Maricel S. Rasid, Elmar L. Cabrera

Printed in the Philippines by ________________________

Department of Education –Region VII Schools Division of Negros Oriental

Office Address: Kagawasan, Ave., Daro, Dumaguete City, Negros Oriental
Tel #: (035) 225 2376 / 541 1117
E-mail Address: negros.oriental@deped.gov.ph

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

The module is intended for you to determine the relationship between differentiability and continuity of a function; and apply the differentiation rules in computing the derivative of an algebraic, exponential, logarithmic, trigonometric functions and inverse trigonometric functions.

What I Know

Test yourself!

Task 1: Find the derivatives of the following functions. Simplify.

1. $y = x^2 - x + 1$
2. $f(x) = 5x^3 - 3x^5$
3. $y = (3x-1)(2x+5)$
4. $f(x) = 25$
5. $g(x) = \frac{2x}{3x^2+1}$
6. $h(x) = 2x^{\frac{1}{3}}$
7. $y = \frac{2}{3x^{\frac{1}{2}}}$
8. $f(x) = 5\sqrt[3]{x^2}$
9. $y = x^2 \sec x$
10. $y = 2x^2e^x + 5e^x$

What's In

The difference between continuity and differentiability is a critical issue. Most, but not all, of the functions we encounter in calculus will be differentiable over their entire domain. Before we can confidently apply the rules regarding derivatives, we need to be able to recognize the exceptions to the rule.

Recall the following definitions:

Definition 1 (Continuity at a Number). A function f is continuous at a number c if all of the following conditions are satisfied:

a. *f(c)* is defined;
b. $\lim_{x \to c} f(x)$ exists: and
c. $\lim_{x \to c} f(x) = f(c)$.

If at least one of these conditions is not satisfied, the function is said to be **discontinuous** at c.

Definition 2 (Continuity on $\mathcal{R}$). A function f is said to be **continuous everywhere** if *f* is continuous at every real number.

Definition 3. A function *f* is **differentiable at the number c** if $f'(c) = \lim_{h \to \infty} \frac{f(c+h)-f(c)}{h}$ or $f'(x) = \lim_{\triangle x \to \infty} \frac{f(x+\triangle x)-f(x)}{\triangle x}$ exists.

Note: The variable h is equivalent to Δx in some books.

What's New

Task 2: Deal with me.

Determine the limits of the following function if it exist and identify whether it is continuous or not.

1. The piecewise function defined by

$$f(x) = \begin{cases} \dfrac{x^2 + 2x - 3}{x - 1} & if\ x \neq 1 \\ 4 & if\ x = 1 \end{cases}$$

2. The function defined by

$$f(x) = \begin{cases} -x^2 & if\ x < 2 \\ 3 - x & if\ x \geq 2 \end{cases}$$

What is It

For $y = f(x)$, we define the derivative of f at x, denoted by $f'(x)$, to be $\boldsymbol{f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}$ if the limit exists.

The existence of a derivative at $x = a$ depends on the existence of a limit at $x = a$, that is, on the existence of

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}.$$

If the limit does not exist at $x = a$, we say that the function f is nondifferentiable at $x = a$, or $f'(a)$ does not exist.

There are functions which are continuous at a point but is not differentiable at that point. The next theorem however says that the converse is always **TRUE**.

> **Theorem.** If a function f is differentiable at a, then f is continuous at a.

Proof. That function f is differentiable at a implies that $f'(a)$ exists. To prove that f is continuous at a, we must show that

$$\lim_{h \to a} f(x) = f(a)$$

or equivalently,

$$\lim_{h \to 0} f(a+h) = f(a)$$

If $\boldsymbol{h \neq 0}$, then

$$f(a+h) = f(a) + f(a+h) - f(a)$$

$$= f(a) + \frac{f(a+h) - f(a)}{h} \cdot h$$

Taking the limit as $h \to 0$, we get

$$\lim_{h \to 0} f(a+h) = \lim_{h \to 0} f(a) + \lim \left[\frac{f(a+h) - f(a)}{h} \cdot h \right]$$

$$= f(a) + f'(a) \cdot 0$$

$$= f(a)$$

Remarks 1:

a. If f is continuous at $x = a$, it does not mean that f is differentiable at $x = a$.
b. If f is not continuous at $x = a$, then f is not differentiable at $x = a$.
c. If f is not differentiable at $x = a$, it does not mean that f is not continuous at $x = a$.
d. A function f is not differentiable at $x = a$ if one of the following is true:
   i. f is not continuous at $x = a$.
   ii. the graph of f has a vertical tangent line at $x = a$.
   iii. the graph of f has a corner or cusp at $x = a$.
   iv. If the graph of f has a hole or break at $x = a$.
   v. If the graph of f has a sharp corner at $x = a$ (the left and the right hand derivative exist but are not equal)

(Cespedes 2020)

DERIVATIVE RULES/DIFFERENTIATION RULES

1. The Constant Rule

The graph of a constant function is a horizontal line and a horizontal line has zero slope. The derivative measures the slope of the tangent, and so the derivative is zero.

If $f(x) = c$ where c is a constant, then $f'(x) = 0$. The derivative of a constant is equal to zero.

Proof

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{c-c}{h}$$
$$= \lim_{h\to 0} 0$$
$$= 0$$

Examples:

a. If $f(x) = 10$, then $f'(x) = 0$
b. If $h(x) = -\sqrt{3}$, then $h'(x) = 0$
c. If $y = 2$, then $\frac{dy}{dx} = 0$
d. If $g(t) = -\frac{3}{2}$, then $g'(t) = 0$
e. $\frac{d}{dx}[7] = 0$

2. The (Simple) Power Rule

A function of the form $f(x) = x^k$, where k is a real number, is called a power function. Below are some examples of power functions.

a. $f(x) = x$
b. $g(x) = x^2$
c. $h(x) = \sqrt{x}$
d) $p(x) = \sqrt[4]{x}$
e) $l(x) = x^{-5}$
f) $s(x) = \frac{1}{x^8}$

Power Rule: If $f(x) = x^n$ where $n \in \mathbb{N}$, then $f'(x) = nx^{n-1}$.

Proof: The cases $n = 1$. Using the limit definition.

$$f(x) = x$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)-(x)}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1$$

or $f(x) = x$

$$f'(x) = 1x^{1-1} = 1x^0 = 1(1) = 1$$

The cases $n = 2$. Using the limit definition.

$$f(x) = x^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2-(x^2)}{h}$$ Substitution

$$= \lim_{h\to 0} \frac{x^2+2hx+h^2-x^2}{h}$$ square of binomial

$$= \lim_{h\to 0} \frac{2hx+h^2}{h}$$ combined like terms

$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$ factor out

$$= \lim_{h\to 0} 2x + h$$ cancelation

$$= 2x + 0 = 2x$$

or $f(x) = x^2$

$$f'(x) = 2x^{2-1} = 2x^1 = 2x$$

Examples: Find the derivatives of the following functions:

a. $f(x) = x^3$ b. $y = \frac{1}{x^2}$

Solution. a. By the Power Rule we have

$f(x) = x^3$ function
$f'(x) = 3x^{3-1}$ apply the power rule
$f'(x) = 3x^2$ derivative

b. By the Power Rule we have

$y = \frac{1}{x^2}$ function
$y = x^{-2}$ rewrite (apply law of exponent)
$\frac{dy}{dx} = (-2)x^{-2-1}$ apply power rule
$\frac{dy}{dx} = -2x^{-3}$ derivative
$\frac{dy}{dx} = -\frac{2}{x^3}$ simplify

Note: In example b before differentiating, we rewrite $\frac{1}{x^2}$ as x^{-2}. Rewriting is the first step in many differentiation problems.

3. The Constant Multiple Rule

If $f(x) = cf(x)$ where c is a constant, then $f'(x) = c\,f'(x)$.

Note: Rule 3 states that the derivative of a constant times a differentiable function is the constant times the derivative of the function. Its proof is a direct consequence of the constant multiple theorem for limits

Examples: Differentiate the following functions:

a. $y = 2x^{\frac{1}{2}}$

b. $f(x) = \frac{4x^2}{5}$

c. $g(x) = \frac{1}{3}\sqrt[3]{x}$

Solution

a.

$y = 2x^{\frac{1}{2}}$	function
$\frac{dy}{dx} = \frac{d}{dx}\left[2x^{\frac{1}{2}}\right]$	
$= 2\frac{d}{dx}\left[x^{\frac{1}{2}}\right]$	constant multiple rule
$= 2\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)$	power rule
$= 2\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$	
$= x^{-\frac{1}{2}}$	derivative
$\frac{dy}{dx} = \frac{1}{x^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{x}}$	simplify

b.

$f(x) = \frac{4x^2}{5}$	function
$f(x) = \frac{4}{5}x^2$	rewrite
$f'(x) = \frac{4}{5}(2x^{2-1})$	constant multiple rule
$f'(x) = \frac{4}{5}(2x^1)$	
$f'^{(x)} = \frac{8}{5}x \quad or \quad \frac{8x}{5}$	derivative

c.

$g(x) = \frac{1}{3}\sqrt[3]{x}$	function
$g(x) = \frac{1}{3}x^{\frac{1}{3}}$	rewrite from radical form to exponential
$g'^{(x)} = \frac{1}{3}\left(\frac{1}{3}x^{\frac{1}{3}-1}\right)$	constant multiple rule
$g'^{(x)} = \frac{1}{3}\left(\frac{1}{3}x^{-\frac{2}{3}}\right)$	
$g'^{(x)} = \frac{1}{9}x^{-\frac{2}{3}}$	derivative
$g'^{(x)} = \frac{1}{9x^{\frac{2}{3}}}$	simplify

4. The Sum Rule

If $f(x) = g(x) + h(x)$ where g and h are differentiable functions, then $f'(x) = g'(x) + h'(x)$.

Rule 4 states that the derivative of the sum of two differentiable functions is the sum of the derivatives of the functions. Its proof relies on the Addition Theorem for limits.

By similar procedure, the derivative of a difference is the difference of the derivatives. The derivative of the function

$f(x) = g(x) - h(x)$ is $f'(x) = g'(x) - h'(x)$.

Example: Consider the given sample functions in rule 3 $f(x) = \frac{4x^2}{5}$ and $g(x) = \frac{1}{3}\sqrt[3]{x}$. Differentiate the following:

a. $f(x) + g(x)$

b. $f(x) - g(x)$

Solution:

a. Copying the derivatives in the solution of Example (3), and substituting them into the formula of the Sum Rule, we obtain

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$
$$= \frac{8}{5}x + \frac{1}{9x^{\frac{2}{3}}}$$

b. Using Rules 3 and 4, we deduce that the derivative of $f(x) - g(x)$ is equal to the difference of their derivatives: $f'(x) - g'(x)$. Therefore, we obtain

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$
$$= \frac{8}{5}x - \frac{1}{9x^{\frac{2}{3}}}$$

Remarks 2

a. The Sum Rule can also be extended to a sum of a finite number of functions. If $f(x) = f_1(x) + f_2(x) + \cdots + f_n(x)$ where $f_1, f_2, \ldots, f_n$ are differentiable functions, then $f'(x) = f_1'(x) + f_2'(x) + \cdots + f_n'(x)$.

b. The same is true for the difference of a finite number of functions. That is,

$$f'(x) = f_1'(x) - f_2'(x) \cdots f_n'(x)$$

Take Note

With the four differentiation rules given so far, we can now differentiate any polynomial function. This is illustrated in the next two examples.

Example: Find the derivatives of the two polynomial functions.

a. $f(x) = x^3 - 4x + 2$

b. $g(x) = -\frac{x^4}{2} + 3x^3 - 2x$

Solution:

a. The derivative of $f(x)$ is

$$f'(x) = \frac{d}{dx}x^3 - \frac{d}{dx}4x + \frac{d}{dx}2$$

$f'(x) = 3x^2 - 4 + 0$ apply 4 rules

$f'(x) = 3x^2 - 4$ derivative

b. The derivative of $g(x)$ is

$$g'(x) = \frac{d}{dx}\left(-\frac{x^4}{2}\right) + \frac{d}{dx}3x^3 - \frac{d}{dx}2x$$

$g'^{(x)} = -\frac{4x^{4-1}}{2} + 3(3)x^{3-1} - 2x^{1-1}$ apply the 4 rules

$g'^{(x)} = -2x^3 + 9x^2 - 2$ derivative

5. The Product Rule

The derivative of the product of two differentiable functions is equal to the first function times the derivative of the second function, plus the second function times the derivative of the first function.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Example: Find $f'(x)$ if $f(x) = (3x^2 - 4)(x^2 - 3x)$

Solution.

By the product rule,

$$f'(x) = \overbrace{(3x^2-4)}^{first}\overbrace{\frac{d}{dx}(x^2-3x)}^{\substack{derivative\\ of\ second}} + \overbrace{(x^2-3x)}^{second}\overbrace{\frac{d}{dx}(3x^2-4)}^{\substack{derivative\\ of\ first}}$$

$$f'(x) = (3x^2 - 4)(2x - 3) + (x^2 - 3x)(6x)$$

$$= 6x^3 - 9x^2 - 8x + 12 + 6x^3 - 18x^2$$

$$= 12x^3 - 27x^2 - 8x + 12$$

Remark 3: In the above example, we could have also multiplied the two factors and get

$$f(x) = 3x^4 - 9x^3 - 4x^2 + 12x.$$

Then by the rule 2,3, and 4, the derivative of $f(x)$ is

$$f'(x) = 12x^3 - 27x^2 - 8x + 12$$

which is consistent with the one derived using the product rule.

Example: Find $f'(x)$ if $f(x) = \sqrt{x}(6x^3 + 2x - 4)$

Solution: By the Product Rule,

$$f'(x) = \overbrace{\left(x^{\frac{1}{2}}\right)}^{first}\overbrace{\frac{d}{dx}(6x^3 + 2x - 4)}^{\substack{derivative \\ of\ second}} + \overbrace{(6x^3 + 2x - 4)}^{second}\ \overbrace{\frac{d}{dx}x^{\frac{1}{2}}}^{\substack{derivative \\ of\ first}}$$

$$= (x^{\frac{1}{2}})(18x^2 + 2) + (6x^3 + 2x - 4)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= 18x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + 3x^{\frac{5}{2}} + x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$$

$$= 21x^{\frac{5}{2}} + 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$$

6. The Quotient Rule

The derivative of the quotient of two differentiable functions is equal to the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}\ , g(x) \neq 0$$

The following form may assist you in memorizing the rule:

$$\frac{d}{dx}[quotient] = \frac{(denominator)\frac{d}{dx}[numerator] - (numerator)\frac{d}{dx}[denominator]}{(denominator)^2}$$

A very common mnemonic for the quotient rule is

$$D_x\frac{high}{low} = \frac{low\ D(high) - high\ D(low)}{low\ squared}$$

Example: Let $h(x) = \frac{(3x+5)}{x^2+4}$. Find the derivative of the function $h(x)$.

Solution: If $h(x) = \frac{(3x+5)}{x^2+4}$, then $f(x) = 3x + 5$ and $g(x) = x^2 + 4$ and therefore $f'(x) = 3$ and $g'(x) = 2x$. Thus,

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(x) = \frac{(x^2+4)(3) - (3x+5)(2x)}{[x^2+4]^2}$$

$$= \frac{3x^2 + 12 - 6x^2 - 10x}{[x^2+4]^2}$$

$$= \frac{12 - 10x - 3x^2}{[x^2+4]^2}$$

7. Derivative of Trigonometric Functions

Derivative of Trigonometric Functions	
a) $D_x(\sin x) = \cos x$	d) $D_x(\cot x) = -csc^2\, x$
b) $D_x(\cos x) = -\sin x$	e)$D_x(\sec x) = \sec x \tan x$
c) $D_x(\tan x) = sec^2\, x$	f) $D_x(\csc x) = -\csc x \cot x$

Example: Differentiate the following functions:

a. $f(x) = \sec x + 3\csc x$
b. $g(x) = x^2 \sin x - 3x\cos x + 5\sin x$

Solution: Apply the formula above.

a. If $f(x) = sec\, x + 3\, csc\, x$, then

$$f'(x) = D_x \sec x + d_x\, 3\csc x$$
$$= \sec x \tan x - 3\csc x \cot x$$

b. If $g(x) = x^2 \sin x - 3x\cos x + 5\sin x$, then

$$g'^{(x)} = [x^2 D_x \sin x + \sin x\, D_x x^2] - [3x D_x \cos x + \cos x\;\; D_x 3x + D_x 5\sin x$$
$$= \; x^2 \cos x + 2x \sin\; x - [-3xsin\, x + 3\cos x] + 5\cos x$$
$$= \; x^2 \cos x + 2x \sin x + 3xsin\, x - 3\cos x + 5\cos x$$
$$= \; x^2 \cos x + 5x \sin x + 2\cos x$$

Remarks 4:

a. Whenever Rule 7 is applied to problems where the trigonometric functions are viewed as functions of angles, the unit measure must be in radians.
b. Every trigonometric function is differentiable on its domain. In particular, the sine and cosine functions are everywhere differentiable.

8. Derivative of an exponential function

Derivative of Exponential Function
If $f(x) = e^x$, then $f'(x) = \; e^x$.

Example:

a. Find $f'(x)$ if $f(x) = 3e^x$

Solution: Applying rules 3 and 7, we have

$$f'(x) = 3D_x[e^x]$$
$$= 3`e^x$$

b. Find $g'(x)$ if $g(x) = -4x^2e^x + 5xe^x - 10e^x$

Solution: Applying Rule 5 to the first two terms and rule 3 to the third term, we have

$$g'^{(x)} = -[4x^2 D_x e^x + e^x D_x 4x^2] + [5x D_x e^x + e^x D_x 5x] - D_x 10e^x$$
$$= -4x^2e^x - 8xe^x + 5xe^x + 5e^x - 10e^x$$
$$= -4x^2e^x - 3xe^x - 5e^x$$

What's More

Task 3: Challenge yourself!

Find the derivatives of the following functions.

1. $y = x^3 - 4x^2 - 3x$
2. $y = (2x^2 + 2)(x^2 + 3)$
3. $y = 2x \cos x - 2 \sin x$
4. $y = 5x^3 e^x$

What I Have Learned

Task 4: Do your best!

Apply what you have learned by giving your own examples of differentiable functions then solve for its derivative. Give example/s in each rule.

CRITERIA

OUTSTANDING (20 pts) – demonstrate/show superior knowledge of basic mathematical concepts and operations on differentiation.

EFFECTIVE (17 pts) – demonstrate/show appropriate use of basic mathematical concepts and operations on differentiation.

ADEQUATE (13 pts) – understands the basic mathematical concepts and operations on differentiation.

INEFFECTIVE (8 pts) – cannot demonstrate/show knowledge of mathematical concepts and operation on differentiation.

http://www.csu.edu/CTRE/pdf/rubricexamples-all.pdf

What I Can Do

Task 5: You can do it!

Find an equation of the tangent line to the graph of the given function at the indicated point.

1. Function: $f(x) = (x-1)(x^2-2)$
   Point: (0, 2)
2. Function: $f(x) = \frac{x}{x-1}$
   Point: (2, 2)

RUBRIC

CATEGORY	5	4	3	2
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.

Assessment

Task 6:

I. Use the Rules of Differentiation/derivatives to differentiate the following functions:

1. $y = 2x^3 + 6x$
2. $f(x) = 4x^3 - 18x^2 + 6x$
3. $y = (4x+5)(7x^3 - 2x)$
4. $f(x) = 100$
5. $g(x) = \frac{2x-1}{3x^2+1}$
6. $h(x) = 3x^{\frac{2}{3}}$
7. $y = \frac{4}{2x^{\frac{2}{3}}}$
8. $f(x) = 3\sqrt[3]{x^2}$
9. $y = 2x^2 \tan x$
10. $y = x^2 e^x - 2e^x$

II. Find the derivative of $f(x) = x^2 - 3x$. Use the result to find the slope of the tangent line to the curve $f(x) = x^2 - 3x$ at the point where $x = 2$.

Answer Key

Task 1.

1. $\frac{dy}{dx} = 2x - 1$
2. $f'(x) = 15x^2 - 15x^4$
3. $\frac{dy}{dx} = 12x + 13$
4. $f'(x) = 0$
5. $g'(x) = \frac{2-6x^2}{(3x^2+1)^2}$
6. $h'(x) = \frac{2}{3x^{\frac{2}{3}}}$
7. $\frac{dy}{dx} = -\frac{1}{3x^{\frac{3}{2}}}$
8. $f'(x) = \frac{10}{3x^{\frac{1}{3}}}$ *or* $\frac{10}{3\sqrt[3]{x}}$
9. $\frac{dy}{dx} = x^2 \sec x \tan x + 2x \sec x$ *or* $xsec\,x(xtan\,x + 2)$
10. $\frac{dy}{dx} = 2x^2e^x + 4xe^x + 5e^x$

Task 2.

1. Limit exist
   $\lim_{z\to 1} f(x) = 4$
   Continuous at c = 1
2. Limit does not exist
   Not continuous

Task 3.

1. $\frac{dy}{dx} = 3x^2 - 8x - 3$
2. $\frac{dy}{dx} = 8x^3 + 16x$
3. $\frac{dy}{dx} = -2x \sin x$
4. $\frac{dy}{dx} = 5x^3e^x + 15x^2e^x$

Task 6.

1. $\frac{dy}{dx} = 6x^2 + 6$
2. $f'(x) = 12x^2 - 36x + 6$
3. $\frac{dy}{dx} = 112x^3 + 105x^2 - 16x - 10$
4. $f'(x) = 0$
5. $g'(x) = \frac{2+6x-6x^2}{(3x^2+1)^2}$
6. $h'(x) = \frac{2}{x^{\frac{1}{3}}}$ *or* $\frac{2}{\sqrt[3]{x}}$
7. $\frac{dy}{dx} = -\frac{4}{3x^{\frac{5}{3}}}$
8. $f'(x) = \frac{2}{x^{\frac{1}{3}}}$ *or* $\frac{2}{\sqrt[3]{x}}$
9. $\frac{dy}{dx} = 4x \tan x + 2x^2 sec^2 x$
10. $\frac{dy}{dx} = x^2e^x + 2xe^x - 2e^x$

References

(Cespedes 2020)

Balmaceda, J. 2016. *Teaching Guide for Senior High School Basic Calculus.* Quezon City, Philippines: Department of Education - Bureau of Learning Resources.

2022. *Chicago State University.* Accessed 2021. https://bit.ly/3f4xfiC.

Larson, R. 1987. *Brief Calculus with application.* Canada: Canada: D.C. health and Company.

For inquiries or feedback, please write or call:

Department of Education – Schools Division of Negros Oriental
Kagawasan, Avenue, Daro, Dumaguete City, Negros Oriental

Tel #: (035) 225 2376 / 541 1117
Email Address: negros.oriental@deped.gov.ph
Website: lrmds.depednodis.net