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# STATISTICS and PROBABILITY

Quarter 3 - Module 8  
Confidence Interval



**Statistics and Probability – Grade 11**  
**Alternative Delivery Mode**  
**Quarter 3 – Module 8: Confidence Interval**  
**Second Edition, 2021**

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## **Introductory Message**

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



# What I Need to Know

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

The module is intended for you to identify the length of a confidence interval, compute for the length of the confidence interval and compute for an appropriate sample size using the length of the interval.

After going through this module, you are expected to solve problems involving sample size.



## What I Know

## **PRE-ASSESSMENT**

Choose the letter that corresponds to the correct answer:

1. A/an \_\_\_\_\_ of a population parameter is a single value of a statistic.  
A. Interval estimate      C. Good estimate  
B. Point estimate      D. Accurate estimate
  2. A/an \_\_\_\_\_ is defined by two numbers, between which a population parameter is said to lie  
A. Interval estimate      C. Good estimate  
B. Point estimate      D. Accurate estimate
  3. A confidence interval consists of three parts except  
A. Parameter      C. Statistic  
B. Confidence level      D. Margin of error
  4. In a confidence interval, the range of values above and below the sample statistics is called  
A. Outlier      C. Margin of error  
B. Extreme values      D. Minimum and maximum values
  5. The interval estimate of a confidence interval is defined by the sample statistic + margin of error.  
A. True      B. False      C. Sometimes true      D. It depends
  6. The critical value is the same as the z-value. The value is found by first be given the confidence level.  
A. True      B. False      C. Sometimes true      D. It depends

# Lesson

## Identifying and Computing the Length of the Confidence Interval



### What's In

In the previous discussion, it talks about how to use the t-distribution table and find the degrees of freedom. In order for us to understand our new lesson, we need to recall those things with this activity.

**Instruction:** Fill in the blank/s to complete the sentences below.

1. The t-distribution is \_\_\_\_\_ and symmetric about the \_\_\_\_\_.
2. The t-distribution is a family of curves, each determined by a parameter called the \_\_\_\_\_. When you use a t-distribution to estimate a population mean, the degrees of freedom are equal to one less than the sample size, in symbol, \_\_\_\_\_.
3. The total area under a t-curve is \_\_\_\_\_.
4. The mean, median, and mode of the t-distribution are equal to\_\_\_\_\_.
5. As the degrees of freedom increase, the t-distribution approaches the \_\_\_\_\_. After 30 df the t-distribution is very close to the standard normal z-distribution.
6. A \_\_\_\_\_ is a number on a statistical distribution who is less-than the probability in the given percentage.

Let us also recall these important points.

#### Point estimates and Interval estimates

**Point estimate.** A point estimate of a population parameter is a single value of a statistic. For example, the sample mean is a point estimate of the population mean  $\mu$ . Similarly, the sample proportion  $p$  is a point estimate of the population proportion  $P$ . (Malate J.S., 2017)

Like, the average height of STEM 11 learners in a certain senior high school is 4 feet and 8 inches, we are giving point estimate.

**Interval estimate.** An interval estimate is defined by two numbers, between which a population parameter is said to lie. For example,

$a < \mu < b$  is an interval estimate of the population mean  $\mu$ . It indicates that the population mean is greater than  $a$  but less than  $b$ . (Malate J.S., 2017)

Like, the area of the classrooms in a certain senior high school is between 40 square meters and 50 square meters, we are giving an interval estimate.

### Confidence Interval

A confidence interval consists of three parts namely *A statistic*, *A margin of error*, and *A confidence level*. But if we consider the confidence level as part of the margin of error, then confidence interval consists only of two parts namely *A statistic* and *A margin of error*. These parts help us compute the ends or confidence limit of an interval.

*Example:*

1. 4ft. 8in. to 5ft. 1in., the left end of the confidence interval is 4ft. 8in.  
and 5ft. 1in is the right end.
2. (5ft. 2in, 40kgs), 5ft. 2in is the left end while 40kgs is the right end.

### Confidence Level

A confidence level is part of all possible samples (in percent) taken from the same population that can be expected to include the true population parameter.

*Example:*

1. 95% confidence level means 95% of the intervals contain the true population parameter.
2. 99% confidence level means 99% of the intervals contain the true population parameter.

### Margin of Error

The margin of error is the maximum likely difference (in percent) between sample mean  $\bar{x}$  and the real population value  $\mu$ .

Example. A teacher conducts a survey from a group of senior high school learners whether they love mathematics and found out that 60% love mathematics. The teacher states that his/her findings had a margin of error of 5% and a confidence level of 95%. This means that if we ask group of learners from the same group of senior high school learners we are 95% confident that between 55% and 65% will respond they '*Love mathematics*'.

Let  $\sigma$  be the population standard deviation,  $n$  is the sample size and  $Z_{\alpha/2}$  is the suitable  $z$ -value for the preferred level of confidence. So, the general formula for the margin of error for the sample mean  $\bar{x}$  is

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

However, when the population standard deviation is unknown the sample standard deviation  $s$  is used to approximate  $\sigma$ . So,

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \approx z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

**Steps for computing the margin of error for a sample mean:**

Note: Must meet the condition that the population is normally distributed or  $n \geq 30$  (large)

1. Determine the population standard deviation  $\sigma$ , the sample size  $n$  and the suitable  $z$  – value.
2. Multiply the  $z$  – value to the quotient of the population standard deviation  $\sigma$  and the square root of the sample size  $n$ .

**Example:** Consider the following information:

$\bar{x} = 98$ ,  $\sigma = 5$  and  $n = 190$ . Find the maximum error  $E$  for 95% Confidence Interval

**Solution:**

$$\begin{aligned} E &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} && \text{Formula} \\ E &= 1.96 \left( \frac{5}{\sqrt{190}} \right) && \text{by substitution} \\ E &= 0.71 \end{aligned}$$



## What's New

Before going to the meat of the discussion, let's have this epic poem, an excerpt from the E-book of Daan Van Schalkwijk entitled "Confidence Interval".

### Confidence Interval

"If one has a single sample,  
And wants with confidence to say,  
Oh, where the people's mean is lying:  
It is surprising that one may."

"Say one wants confidence at level,  
Of even ninety-five percent,  
Then one finds all thy people's values,  
One may, from sample's worth, defend."

The high and lowest of these values,  
Give us a bell-curve that lies so:  
The sample mean does mark the outskirts,  
With two-and-a-half percent to go.  
(Schalkwijk, 2012)

Schalkwijk's explanation to his poem is that a squire teaches the High King statistics to overcome a crisis of confidence, at risk to his own life. Just when the High King is at loss in whom to confide, a squire offers to solve his problems by teaching him about confidence intervals.

This lesson teaches us how to identify and compute the length of the confidence interval.



## What Is It

### Discussion

#### Identifying the length of a confidence interval

#### *Confidence Interval Estimates of Population Parameters*

Let  $\mu_s$  be the mean and  $\sigma_s$  be the standard deviation of the sampling distribution of a statistics S. The sampling distribution S is approximately normal or  $n \geq 30$  (large). The sample statistics S is expected to lie in the interval as shown in table 1.0 below.

TABLE 1.0

Confidence level	80%	90%	95%	98%	99%
Z <sub>α/2</sub> (Confidence Coefficients)	1.28	1.64	1.96	2.33	2.58
Confidence interval formula for estimating the sample mean $\mu_s$ .	$S \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $S \pm 1.28 \frac{\sigma}{\sqrt{n}}$	$S \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $S \pm 1.645 \frac{\sigma}{\sqrt{n}}$	$S \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $S \pm 1.96 \frac{\sigma}{\sqrt{n}}$	$S \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $S \pm 2.33 \frac{\sigma}{\sqrt{n}}$	$S \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $S \pm 2.58 \frac{\sigma}{\sqrt{n}}$

*Example* 1. Identify the confidence interval in which the sample mean  $\mu_s$  lies at 98% confidence level.

*Solution*

$$S \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = \quad \text{Formula}$$

$$= S \pm 2.33 \frac{\sigma}{\sqrt{n}} \quad \begin{aligned} &2.33 \text{ is the confidence coefficient at 98\% confidence} \\ &\text{level} \end{aligned}$$

So, at 98% confidence level the sample mean  $\mu_s$  lies in the confidence level  $S \pm 2.33 \frac{\sigma}{\sqrt{n}}$ .

## Computing the length of the Confidence Interval

### A. Large Sample or $n \geq 30$ Confidence Interval for a Population Mean

- If  $\sigma$  is known and the sample mean  $\bar{x}$  is specified, the formula for Confidence Interval is  $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
- If  $\sigma$  is unknown and the sample mean  $\bar{x}$  is specified, the formula for Confidence Interval is  $\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

#### Steps

1. Determine the confidence coefficient.
2. Find the maximum error E.
3. Find the lower and the upper confidence limits.
4. Describe the results.

(Belecina R.R., et.al., 2016)

**Example 1.** A sample of size 49 has sample mean 35 and sample standard deviation 14. Construct a 98% confidence interval for the population mean using this information. Interpret its meaning.

**Solution:**

Step 1. The confidence coefficient is 2.33 (see table 1.0)

Step 2. Find the maximum error E

$$E = z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{because } s \text{ is the best estimate}$$

for  $\sigma$  if unknown.

$$E = 2.33 \cdot \frac{14}{\sqrt{49}} \quad \text{by substitution}$$
$$E = 4.66$$

Step 3. Find the lower and the upper confidence limits.

$$\bar{x} - z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$35 - E < \mu < 35 + E$$

$$35 - 4.66 < \mu < 35 + 4.66$$

$$30.34 < \mu < 39.66$$

#### Step 4. Describe the results.

We are 98% confident that the population mean  $\mu$  lies in the interval [30.34 , 39.66] in the sense that in repeated sampling 98% of all intervals constructed from the sample data in this manner will contain  $\mu$ .(Malate J.S., 2017)

### **B. Large Sample Estimation of a Population Proportion**

### Definition:

- i. Let  $p$  denote the proportion of the population that lean towards a specific proportion.
  - ii. To estimate  $p$ , a random sample should be chosen, then  $p$  should be estimated by the proportion of the sample.
  - iii. We call this estimator  $\hat{p}$ , we can express it by
$$\hat{p} = \frac{X}{n}, \text{ where } X \text{ is the number of members in the sample who lean towards a specific proposition and } n \text{ is the sample size.}$$
  - iv. The complement of  $\hat{p}$  is  $\hat{q}$

$$\hat{q} = 1 - \hat{p}$$

**Example:** In a survey of 150 households, 60 have air conditioner. Find the estimator  $\hat{p}$  and  $\hat{q}$ .

## Solutions:

$$n = 150 \quad X = 60$$

$$\hat{p} = \frac{X}{n} = \frac{60}{150} = 0.4$$

$$\hat{q} = 1 - \hat{p} = 1 - 0.4 = 0.6$$

(Malate J.S., 2017)

## *Confidence Interval Estimate for Proportions*

- The error of an estimate is  $E = z \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$
  - The confidence interval is:  $\hat{p} - E < p < \hat{p} + E$

## Example:

A sample poll of 100 voters chosen at random from all voters in a given district indicated that 55% of them were in favor of a particular candidate. Find 98% confidence limits for the proportion of all the voters in favor of this candidate.

**Solution:**

$$\text{Given: } n = 100 \quad \hat{p} = 0.55 \quad \hat{q} = 1 - 0.55 = 0.45$$

**Step 1.** The confidence coefficient  $z = 2.33$  at 98% level of confidence (see table 1.0).

**Step 2.**

$$E = z \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} = (2.33) \cdot \sqrt{\frac{(0.55) \cdot (0.45)}{100}} = (2.33) \sqrt{0.002475} = (2.33)(0.05) = 0.12$$

**Step 3.** To find the lower and the upper confidence limits

$$\hat{p} - E < p < \hat{p} + E$$

$$0.55 - 0.12 < p < 0.55 + 0.12$$

$$0.43 < p < 0.67 \quad , 0.43 \text{ and } 0.67 \text{ are the 98\% confidence limits.}$$

**Step 4.** We are 98% confident that the interval 43% to 67% contains the true proportion of all the voters in favor of this candidate.

### C. Small Sample Estimation of a Population Mean

The confidence interval formulas in the previous examples are based on the Central Limit Theorem, the statement that for large samples  $\bar{x}$  is normally distributed with mean  $\mu$  and standard deviation  $\frac{\sigma}{\sqrt{n}}$ . According to the Central Limit Theorem, the sampling distribution of a statistic (like a sample mean) will follow a normal distribution as long as the sample size is sufficiently large. Therefore, when we know the standard deviation of the population, we can compute a z-score, and use the normal distribution to evaluate probabilities with the sample mean.

But sample sizes are sometimes small, and often we do not know the standard deviation of the population. When either of these problems occur, we use  $t$  score whose formula is given as:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where  $\bar{x}$  is the sample mean,  $\mu$  is the population mean,  $s$  is the standard deviation of the sample, and  $n$  is the sample size. The distribution of the  $t$  statistic is called the  $t$  distribution or the Student  $t$  distribution (Malate J.S., 2017)

The maximum error of the estimate is given by the formula for  $E$  as:

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$$

The limits for the confidence interval is given as:

$$\bar{x} - E < \mu < \bar{x} + E$$

**Example:**

A sample of size 15 drawn from a normally distributed population has a sample mean of 35 and a sample standard deviation of 14. Construct a 95% confidence interval for the population mean and interpret its meaning.

**Solution:**

Given:  $n = 15$      $\bar{x} = 35$      $s = 14$

Step 1: The confidence level 95% means that  $\alpha = 1 - 0.95 = 0.05$  so  
 $\alpha/2 = 0.05/2 = 0.025$

Since the sample size is  $n = 15$ , there are  $n - 1 = 15 - 1 = 14$  degrees of freedom. By the student's t distribution table below the confidence coefficient  $t \alpha/2 = t_{0.025} = 2.145$

Step 2: To find E,

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}} = (2.145) \frac{14}{\sqrt{15}} = (2.145)(3.61478) = 7.75 \approx 7.8$$

Step 3: To find the lower and the upper confidence limits,

$$\begin{aligned} \bar{x} - E &< \mu < \bar{x} + E \\ 35 - 7.8 &< \mu < 35 + 7.8 \\ 27.2 &< \mu < 42.8 \quad \text{Confidence Interval} \end{aligned}$$

Step 4. We are 95% confident that the interval 27.2 to 42.8 contains the true population mean.

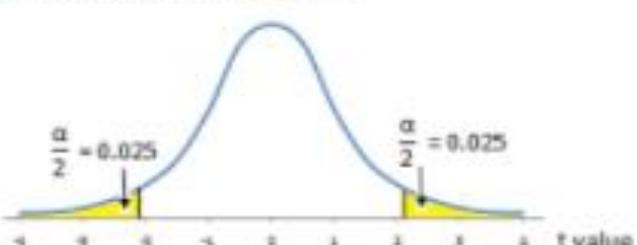
## Student's t Distribution Table

For example, the t value for

18 degrees of freedom

is 2.101 for 95% confidence

interval (2-Tail  $\alpha = 0.05$ ).



	90%	95%	97.5%	99%	99.5%	99.95%	1-Tail Confidence Level
	80%	90%	95%	98%	99%	99.9%	2-Tail Confidence Level
<i>df</i>	0.100	0.050	0.025	0.010	0.005	0.0005	1-Tail Alpha
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9290	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3049	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4379	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6469	

<https://breitheroqw.blogspot.com/2021/08/t-statistic-table.html>

## Sample Size Considerations

### Minimum Sample Size for Estimating a Population Mean

The estimated minimum sample size  $n$  needed to estimate a population mean  $\mu$  to within  $E$  units at  $100(1 - \alpha)\%$  confidence is.

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$$

**Example** Find the minimum sample size necessary to construct a 99% confidence interval for  $\mu$  with a margin of error  $E = 0.2$ . Assume that the population standard deviation is  $\sigma = 1.3$

#### Solution.

*Step 1. Determine the confidence level:*

The confidence level is 99%.

*Step 2. Determine the confidence coefficient:*

With a 99% confidence level,  $\alpha = 0.01$ , so  $z_{\alpha/2} = 2.58$  (see table 1.0)

*Step 3. Determine the margin of error E.*

$E = 0.2$

*Step 4. Compute n using the formula:*

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{(2.58)(1.3)}{0.2}\right)^2 = \left(\frac{3.354}{0.2}\right)^2 = (16.77)^2 = 281.2329 \approx 282 \text{ (Round up the resulting)}$$

So, the required minimum sample size is 282.

### Minimum Sample Size for Estimating a Population Proportion

The estimated minimum sample size  $n$  needed to estimate a population proportion  $p$  to within  $E$  units at  $100(1 - \alpha)\%$  confidence is.

$$n = \frac{(z_{\alpha/2})^2 \cdot \hat{p}(1-\hat{p})}{E^2}$$

#### Steps in Solving the Sample Size Involving Proportions

- Step 1. Determine the confidence level.
- Step 2. Determine the confidence coefficient.
- Step 3. Determine the margin of error E.
- Step 4. Determine  $\hat{p}$ .
- Step 5. Solve  $n$  by substituting the values in the formula.
- Step 6. Round up the resulting value.

**Example:** Find the necessary minimum sample size to construct a 98% confidence interval for  $p$  with a margin of error  $E = 0.05$ ,

- a. Assuming that no prior knowledge about  $p$  is available; and
- b. Assuming that prior studies suggest that  $p$  is about 0.1

**Solutions:**

Step 1. The confidence level is 98%.

Step 2. The confidence coefficient  $z_{\alpha/2} = 2.33$ . (see table 1.0)

Step 3. The margin of error  $E = 0.05$ .

Step 4. Since  $p \approx 0.1$  we estimate  $\hat{p} = 0.1$

Step 5. Solving for  $n$ :

$$n = \frac{(z_{\alpha/2})^2 \cdot \hat{p}(1 - \hat{p})}{E^2} = \frac{(2.33)^2 \cdot (0.1)(1 - 0.1)}{0.05^2} = \frac{5.4289 \cdot (0.1)(0.9)}{0.0025} = 195.44$$

Step 6. Rounding up the result  $195.44 \approx 196$

So, the necessary minimum sample size is 196.



## What I Have Learned

### Generalization

Directions: Reflect the learning that you gained after taking up this lesson on “Confidence Interval” by completing the given statements below. Do this in your activity notebook. Do not write anything on this module.

*What were your thoughts or ideas about the topic before taking up the lesson?*

I thought that \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.

*What new or additional ideas have you had after taking up this lesson?*

I learned that \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.

*How are you going to apply your learning from this lesson?*

I will apply \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.



## What I Can Do

Research on real-life applications of confidence intervals. You are provided with 2 examples. Give another 3.

### 1. Biology

For example, a biologist may be interested in measuring the mean weight of a certain species of frogs in the Visayas region. Since it would take too long to go around and weigh thousands of individual frogs, the biologist may instead collect simple random sample of 50 frogs and measure the mean and standard deviation of the frogs in the sample. *She could then use the sample mean and sample standard deviation to construct an interval for the true mean of the frogs in the entire population.*

### 2. Clinical trials

For example, a doctor may believe that a new drug is able to reduce blood pressure in patients. To test this, he may recruit 20 patients to participate in a trial in which they used the new drug for one month. At the end of the month, the doctor may record the mean decrease in blood pressure and the standard deviation of the decrease in each in the sample. *He could then use the sample mean and sample standard deviation to construct an interval for the true mean change in blood pressure that patients are likely to experience in the population.*

(Zach, 2021)



## Assessment

Solve the problems as indicated:

1. A random sample of size  $n = 36$  is drawn from a population with a standard deviation of  $s = 11.3$  and sample mean  $\bar{x} = 105.2$ . Construct a 90% confidence interval for the population mean  $\mu$ .
2. A sample of 250 workers aged 16 and older produced an average length of time with the current employer ('job tenure') of 4.4 years with a standard deviation of 3.8 years. Construct a 99% confidence interval for the mean job tenure of all workers aged 16 and older.
3. A random sample of size  $n = 16$  is drawn from a normally distributed population. The sample mean is 98 with known standard deviation of 5. Construct a 98% confidence interval for the population mean.
4. Estimate the interval for the population proportion from  $X = 500$ ,  $n = 812$  and 95% confidence. Then interpret the results.

5. In a survey, 1000 Grade 7 students were asked if they read storybooks. There were 318 who said *Yes*. Using 90% confidence level determine the proportion  $p$  of all Grade 7 students who read storybooks.



# Answer Key

## What I know

- |      |      |
|------|------|
| 1. B | 4. C |
| 2. A | 5. A |
| 3. A | 6. A |

## What's In

1. Bell-shaped, mean
2. Degrees of freedom,  $df=n-1$
3. 1 or 100%
4. Zero
5. Normal distribution
6. percentile

## What I have learned

(Answers may vary)

## Assessment

1.  
Step 1. Confidence Coefficient is 1.64  
Step 2.  $E = 3.09$   
Step 3.  $102.1 < \mu < 108.3$
2.  
Step 1. Confidence Coefficient is 2.58  
Step 2.  $E = 0.62$   
Step 3.  $3.78 < \mu < 5.02$
3.  
Step 1. Confidence Level is 98%,  $\alpha = 0.02$  and  $\alpha/2 = 0.01$   
Step 2.  $t_{\alpha/2} = 2.603$ ,  $E = 3.25$   
Step 3.  $94.75 < \mu < 101.25$
4.  
Step 1. Confidence Coefficient  $z = 1.96$   
Step 2.  $E = 0.03$   
Step 3.  $0.586 < p < 0.646$   
Step 4. We are 95% confident that the interval 0.586 to 0.646 contains the true proportion.
5.  
Step 1. Confidence Coefficient  $z = 1.64$   
Step 2.  $E = 0.024$   
Step 3.  $0.294 < p < 0.342$   
Step 4. We are 90% confident that the proportion of Grade 7 students read storybooks lie within 0.294 to 0.342

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