11

SENIOR HIGH SCHOOL

DEPARTMENT OF EDUCATION
SCHOOLS DIVISION OF NEGROS ORIENTAL
REGION VII
Kagawasan Ave., Daro, Dumaguete City, Negros Oriental

STATISTICS and PROBABILITY

Quarter 3 - Module 7
T-Distribution and Percentiles Using the T-Table

ALTERNATIVE DELIVERY MODE
ADM

NegOr_Q3_Stat_and_Prob11_Module7_v2

Statistics and Probability– Grade 11
Alternative Delivery Mode
Quarter 3 – Module 7: T-Distribution and Percentiles Using the T-Table
Second Edition, 2021

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writers: Evangeline K. Ije and Mae Sherida O. Magalso

Editors: Didith T. Yap & Rickleoben V. Bayking

Reviewer: Mercyditha D. Enolpe

Layout Artist: Jerry Mar B. Vadil

Management Team: Senen Priscillo P. Paulin, CESO V — Elisa L. Baguio, EdD
Joelyza M. Arcilla EdD, CESE — Rosela R. Abiera
Marcelo K. Palispis JD, EdD — Maricel S. Rasid
Nilita L. Ragay, EdD — Elmar L. Cabrera

Printed in the Philippines by _____

Department of Education –Region VII Schools Division of Negros Oriental

Office Address: Kagawasan, Ave., Daro, Dumaguete City, Negros Oriental
Telephone #: (035) 225 2376 / 541 1117
E-mail Address: negros.oriental@deped.gov.ph

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you

What I Need to Know

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

The module is intended for you to illustrate the T-distribution table and identify the percentiles using the t-table.

What I Know

PRE-ASSESSMENT

A. Multiple Choice. Direction: Read each item carefully. Write only the letter of the correct answer in your notebook.

1. The following statements are true about the properties of t-distribution, except
   A. The t-distribution is bell shaped. C. The total area under the curve is 100%.
   B. It is symmetrical about the mean. D. The curve touches or intersects the x-axis.
2. In the t-distribution, the mean, median and mode are located at the center of the distribution.
   A. True B. False C. maybe D. sometimes
3. In the t-distribution, the confidence interval for the Mean when the standard deviation σ is unknown and n < 30.
   A. True B. False C. cannot be determined D. maybe
4. The t-distribution is a family of curves based on the degrees of freedom, which is a number related to the sample size.
   A. True B. False C. cannot be determined D. maybe
5. Given the t-distribution with 9 degrees of freedom. What is the t-value to the left of the t-distribution at 0.90 or 90th percentile, +90?
   A 3.498 B. 2.8214 C. 2.2622 D. 1.3830

B. Find the t-values of the t distribution with a critical value α=0.05 given the following degrees of freedom:

A.16 B. 27 C. 14 D. 25

Lesson 1

Illustrating the T-Distribution and Identifying the Percentiles Using the T-Table

What's In

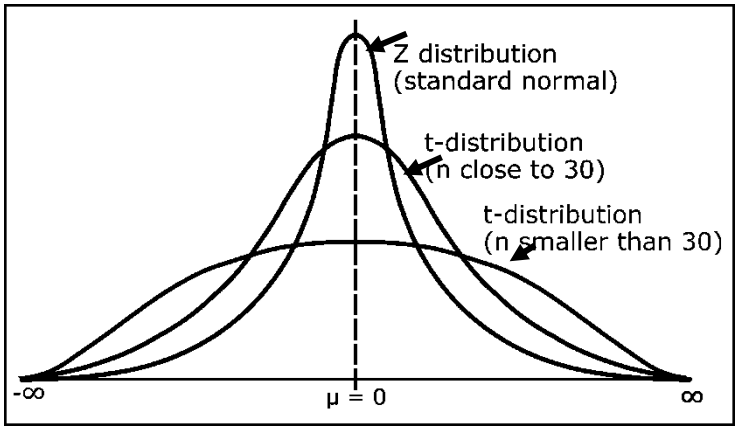
In previous lesson, we encountered the use of Central Limit theorem to estimate the population mean. Theoretically, the population parameters are never known and can only be discerned through a sample. For instance, it is never possible for anyone to determine the average height of all male Filipino teenagers from Batanes to Sulu. But if one is asked to find it out, the best he/she can do is to obtain a sample of some Filipino male teenagers. Then from the sample, he/she can estimate with a certain degree accuracy what the average height of all male Filipino male students is. How accurate his or her estimate is can be best computed using the Central Limit theorem.

To do this is to use the z-table and the area under the standard normal distribution. We call this interval 95% confidence interval for the true mean. Therefore, when we know the standard deviation of the population mean, we can compute a z-score.

But sample sizes are sometimes small, and often we don't know the standard deviation of the population. When either of these problems occur, statisticians rely on the distribution of the t -statistic (also known as the t score) whose formula is given as:

Type	T-Statistic	Degrees of Freedom
One-sample t-test	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$d_{f\,=\,n-1}$

where $\bar{x}$ is the sample mean, μ is the population mean, s is the standard deviation of the sample, and n is the sample size? The distribution of the t statistic is called the t distribution.



Standard Normal Curve

https://www.geeksforgeeks.org/students-t-distribution-in-statisticsc/

What's New

How to Use the t-table

The t-table is a table of t-values of the t-distribution. The entries of the top-most row are levels of significance denoted by α while the entries of the left-most column are degrees of freedom, denoted by v, (*Greek-letter nu*) or simply df and inside the body of the table are the t-values. The student's t-distribution was created by William T. Gosset, an Irish Brewery worker.

TRY THIS

Look at the t-value of $\alpha = 0.01$ having 14 degrees of freedom.

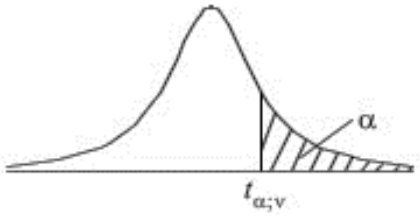
Procedure:

Run your finger to the column under $\alpha = 0.01$ until you come across the row where 14 degrees of freedom is located. The table entry of **2.624** is the t-value when $\alpha = 0.01$ and df = 14, that is, $t_{(0.01,\ 14)} = 2.624$.

Critical Values of the t-Distribution

Table of the Student's *t*-distribution

The table gives the values of $t_{\alpha;\nu}$ where
$\Pr(T_\nu > t_{\alpha;\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

What is It

The t-distribution

The t-distribution is a probability distribution that arises when estimating the mean of a normally distributed population in situations where the sample size is small, and population standard deviation is unknown.

The concept of the degrees of freedom is used in the t-distribution. The degrees of freedom, denoted by df, are the numbers of values that are free to vary after a sample statistic has been computed.

Properties of the t-distribution are as follows:

1. The t-distribution is bell shaped and symmetric about the mean.
2. The t-distribution is a family of curves, each determined by a parameter called the degrees of freedom. The degrees of freedom are the number of free choices left after a sample statistic is calculated. When you use a t-distribution to estimate a population mean, the degrees of freedom are equal to one less than the sample size, df = n-1.
3. The total area under a t-curve is 1 (or 100%).
4. The mean, median, and mode of the t-distribution are equal to zero.
5. As the degrees of freedom increase, the t-distribution approaches the normal distribution. After 30 df the t-distribution is very close to the standard normal z-distribution.

Degrees of Freedom

A degree of freedom occurs for every data value which is allowed to vary once a statistic has been fixed. For a single mean, there are n – 1 degrees of freedom. This value will change depending on the statistic being used.

If the population standard deviation, sigma is unknown, then the mean has a student t distribution and the sample standard deviation and the sample standard deviation is used instead of the population standard deviation.

What is $t_{a/2}$, and why must we use it when $n < 30$ and σ is unknown?

When n < 30, the quantity does not have an approximately standard normal distribution, even though we assume here that the population is normal. Instead, t has a "Student's t distribution with n- 1 degrees of freedom".

There is a different t distribution for each value for the degrees of freedom (df). These distributions are not normal, although they are symmetrical around zero, and mound shaped.

The quantity $t_{\alpha/2}$ denotes the t-value such that the area to its right under the Student's t distribution (with df = n-1) is $\alpha/2$.

Example No. 1

1. Find the $t_{\alpha/2}$ for a 99% confidence interval when the sample size is 20.

Solution:

$$df = n - 1$$

$$= 20\text{-}1 = 19$$

Find 19 in the left column and 99% in the row labeled confidence intervals. The intersection where the two meets gives the value for $t_{\alpha/2}$ which is 2.861.

	Confidence interval	0.800	0.900	0.950	0.980	0.990	0.998	0.999
d.f.		0.100	0.050	0.025	0.010	0.005	0.001	0.0005
1								
2								
3								
19								**2.861**

How to find the Percentiles of the t-Distribution

When you want to find percentiles for a t-distribution, you can use the t-table. A percentile is a number on a statistical distribution who is less-than the probability in the given percentage; for example, the 95th percentile of the t-distribution with n – 1 degrees of freedom is that value of whose left-tail (less than) probability is 0.05.

The t-table shows right-tail probabilities for selected t-distributions. You can use it to solve the following problems.

Example No. 2

Suppose you have a sample size 10 and you want to find the 95th percentile of its corresponding t-distribution. You have n – 1 = 9. The 95th percentile is the number where 95% of the values lie below it and 5% lie above it, so you want the right-tail area to be 0.05. Move across the row, find the column for 0.05, and you get 1.8331. This is the 95th percentile of the t-distribution with 9 degrees of freedom.

Now, if you increase the sample size to n = 20, the value of the 95th percentile decreases; look at the row for 20 – 1 =19 degrees of freedom, and in the column for 0.05 (a right-tail probability of 0.05) you find 1.7291.

What's More

Activity 1. Supply the Missing Piece!

Find the missing value. Use the t-distribution Table.

1. Given: $90^{th}\ percentile, sample\ size\ n = 10$

   Find the following:

   a) α
   b) t – value
   c) df

2. Given: t-value = 2.492

   Find the following:

   a) sample size n
   b) df
   c) α
   d) Percentile

What I Have Learned

Activity 2

A. Find the values for each of the following degree of freedom and t-values with the given percentile. Write your answer on the space provided.

1. $t_{\alpha/2}$ and n = 16 for the 99% confidence interval
   df = _____ $t_{(.005,15)}$ = _____
2. $t_{\frac{\alpha}{2}}$ and n = 25 for the 98% confidence interval
   df = _____ $t_{(.01,24)}$ = _____
3. $t_{\alpha/2}$ and n = 8 for the 95% confidence interval
   df = _____ $t_{(.025,7)}$ = _____
4. $t_{\alpha/2}$ and n = 12 for the 90% confidence interval
   df = _____ $t_{(.05,11)}$ = _____
5. $t_{\alpha/2}$ and n = 20 for the 99% confidence interval
   df = _____ $t_{(.005,19)}$ = _____

B. Summarize the lesson by completing the sentences below:

1. When asked of an estimation of population mean but population standard deviation is unknown, the _____can be used.
2. The t-distribution is similar to the standard normal distribution in the following ways:

   2.1 _____
   2.2 _____
   2.3 _____
   2.4 _____

What I Can Do

Activity 3

(NOTE: For Students who can access Microsoft excel)

Use of Microsoft Excel in finding the t-value given degrees of freedom and level of significance.

A. Finding the t-value given level significance (alpha) and degrees of freedom *v*(nu)

(Note: The computer t-value is for two-tailed, thus α must be multiplied by 2.

Example $t_{(0.01,14)} = 2.624494$

1. Highlight A1;
2. On A1, Type [=TINV(0.01*2,14)];→ 14-degrees of freedom;
3. Press ENTER
4. You will find the value 2.624494 on A1;
5. Good Luck!

Assessment

I. Multiple Choice:

Direction: Read each item carefully. Write only the letter of the correct answer on your activity notebook/activity sheets.

	Confidence interval	0.800	0.900	0.950	0.980	0.990	0.998	0.999
d.f.		0.100	0.050	0.025	0.010	0.005	0.001	0.0005
1								
2								
3								
19								**2.861**

For numbers 1 to 3, use the t table to answer the questions below.

1. Given 5 degrees of freedom at 5% level of significance, the t-value is equal to _____.

A. 1.476 C. 2.015
B. 2.571 D. 3.365

2. At 10% level of significance with 10 degrees of freedom, the t-value is _____.

A. 2.764 B.4.144 C. 1.372 D. 4.587

3. When n = 21 at α = 0.005, the t-value is _____.

A. 1.323 B. 2.845 C. 2.831 D. 3.527

4. When the original variable is normally distributed, the distribution of the sample means will be normally distributed for any sample size *n*.

A. True B. False C. maybe D. cannot be determined

5. As the degrees of freedom increase, the t-distribution approaches the normal distribution. After 30 df the t-distribution is very far to the standard normal z-distribution.

A. True B. False C. maybe D. cannot be determined

II. Find Find the t-values of the t distribution with a critical value α=0.05 given the following degrees of freedom:

A. 16 B. 27 C. 26 D. 15

Additional Activities

Note: Only for students who can access internet.

Use the link on Stattrek.com and statistics fun (video on How to calculate t-distributions) for enrichment activity and write your learning insights in your math journal.

Answer Key

PRE-ASSESSMENT:

A. 1. D 2. A 3. A 4. A 5. A

B. a. At df=16 $t_{0.05} = 1.7459$

b. At df=7 $t_{0.05} = 1.7033$

WHAT'S MORE

Activity 1

1. **a) 0.10**
b) 1.383
c) 9

2. **a) n = 25**
b) df = 24
c) α = 0.01
d) 99^{th} *percentile*

ASSESSMENT

I.

1. C 2. A 3. B 4. A 5.B

II.

a) 1.746
b) 1.703
c) 1.706
d) 1.753

References

Books

Malate, Jose S. 2017. "Confidence Interval." In *Statistics and Probability*, by Jose S. Malate, 102-110. Purelybooks Trading, Inc.

Millard R. Mamhot, et al. n.d. "Statistics for General Education." 79-97. Purelybooks Trading & Publishing Corporation.

Rene Belecina, et al. n.d. "T-Distribution." In *Statistics and Probability*, by Rene Belecina. Rex Bookstore.

Websites

Agnes Ogee, Bruno Scibilia. n.d. *surveymonkey.com/mp/t-tests-explained.* Accessed January 2021. https://tinyurl.com/28h8mpdt

Rumsey, Deborah J. "How to Find Percentiles for a t-Distribution." *dummies.com.* March 26, 2016. https://www.dummies.com/article/academics-the-arts/math/statistics/how-to-find-percentiles-for-a-t-distribution-169844 (accessed January 4, 2022).

https://www.statisticshowto.com/probability-and-statistics/t-distribution/t-score-formula/ **(January, 10, 2022).**

2020. *www.geeksforgeeks.org/students-t-distribution-in-statistics/.* Jul 09. Accessed January 2021.

https://tinyurl.com/zamv3wj3. (January, 10, 2022).

For inquiries or feedback, please write or call:

Department of Education – Schools Division of Negros Oriental
Kagawasan, Avenue, Daro, Dumaguete City, Negros Oriental

Tel #: (035) 225 2376 / 541 1117
Email Address: negros.oriental@deped.gov.ph
Website: lrmds.depednodis.net