11

SENIOR HIGH SCHOOL

DEPARTMENT OF EDUCATION
SCHOOLS DIVISION OF NEGROS ORIENTAL
REGION VII
Kagawasan Ave., Daro, Dumaguete City, Negros Oriental

STATISTICS and PROBABILITY

Quarter 3 - Module 2

Probabilities, Mean and Variance of Discrete Variable

ALTERNATIVE DELIVERY MODE

ADM

NegOr_Q3_Stat_and_Prob11_Module2_v2

Statistics and Probability – Grade 11
Alternative Delivery Mode
Quarter 3 – Module 2: Probabilities, Mean and Variance of a Discrete Variable
Second Edition, 2021

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer: Carmelita I. Carlos

Editors: Didith T. Yap & Rickleoben V. Bayking

Reviewer: Mercyditha D. Enolpe

Layout Artist: Jerry Mar B. Vadil

Management Team: Senen Priscillo P. Paulin, CESO V; Elisa L. Baguio, EdD; Joelyza M. Arcilla EdD, CESE; Rosela R. Abiera; Marcelo K. Palispis JD, EdD; Maricel S. Rasid; Nilita L. Ragay, EdD; Elmar L. Cabrera

Printed in the Philippines by ________________________

Department of Education –Region VII Schools Division of Negros Oriental

Office Address: Kagawasan, Ave., Daro, Dumaguete City, Negros Oriental
Telephone #: (035) 225 2376 / 541 1117
E-mail Address: negros.oriental@deped.gov.ph

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

The module is intended for you to compute probabilities corresponding to a given random variable, illustrate the mean and variance of a discrete random variable.

After going through this module, you are expected to calculate the mean and the variance of a discrete random variable.

What I Know

Task 1. PRE-ASSESSMENT

Answer the following in your activity sheets/activity notebook.

1. A discrete random variable X has the following probability distribution:

x	77	78	79	80	81
$P(x)$	0.15	0.15	0.20	0.40	0.10

Compute each of the following quantities:

a) $P(X = 80)$
b) $P(X > 80)$
c) $P(X \leq 80)$
d) The mean μ of X.
e) The variance σ^2 of X.
f) The standard deviation σ of X.

Lesson 1

Computation on Probabilities Corresponding to a Given Random Variable, Illustration and Calculation for the Mean and Variance of a Discrete Variable

What's In

In the previous module, you learned that a **discrete random variable** has a countable number of possible values. The probability of each value of a **discrete random variable** is between 0 and 1, and the sum of all the probabilities is equal to 1 and a continuous **random variable** takes on all the values in some interval of numbers.

What's New

In this module, you will be learning on how to solve probabilities on a given discrete random variables and be able to solve the mean and variance on it.

The idea of a random variable builds on the fundamental ideas of probability. You need to understand that random variables are conceptually different from the mathematical variables that you have met before. A random variable is linked to observations in the real world, where uncertainty is involved. An informal — but important — understanding of a random variable is that it is a variable whose numerical value is determined by the outcome of a random procedure.

In this module, you will also see the more formal understanding, which is that a random variable is a function from the event space of a random procedure to the set of real numbers. Random variables are central to the use of probability in practice. They are used to model random phenomena, which means that they are relevant to a wide range of human activity. In particular, they are used extensively in many branches of research, including agriculture, biology, ecology, economics, medicine, meteorology, physics, psychology and others. They provide a structure for making inferences about the world when it is impossible to measure things comprehensively. They are used to model outcomes of processes that cannot be predicted in advance. Random variables have distributions. (Michael Evans, Peter Brown, Sue Finch, et al n.d.)

Furthermore, we describe the essential properties of distributions of discrete random variables. Distributions can have many forms, but there are some special types of distributions that arise in many different practical uses.

This module covers the mean of a discrete random variable, which is a measure of central location, and the variance and standard deviation, which are measures of spread. (Mayang n.d.)

Task 2.

Experiment: Tossing two coins.

Random variable X = number of heads

a) List all possible sample space.
b) Find the random variable values.
c) Find the probabilities for the random variable values.

What is It

Probability Distribution of a Discrete Random Variable

The probability distribution of a discrete random variable X is a list of each possible value of X together with the probability that X takes that value in one trial of the experiment. The probabilities in the probability distribution of a random variable X must satisfy the following two conditions: Each probability $P(x)$ must be between 0 and 1 and the sum of all the probabilities is 1. (Probability Distributions for Discrete Random Variables 2021)

<u>Definition: Probability Distribution</u>

The probability distribution of a discrete random variable X is a list of each possible value of X together with the probability that X takes that value in one trial of the experiment.

The probabilities in the probability distribution of a random variable X must satisfy the following two conditions:

- Each probability $P(x)P(x)$ must be between 0 and 1:

$$0 \leq P(x) \leq 1$$

- The sum of all the possible probabilities is 1:

$$\sum P(x) = 1$$

Example:

A fair coin is tossed twice. Let X be the number of heads that are observed.

a. Construct the probability distribution of X.

b. Find the probability that at least one head is observed.

Solution:

a. The possible values that X can take are 0, 1, and 2. Each of these numbers corresponds to an event in the sample space $S = \{hh, ht, th, tt\}\ or\ S = \{hh, ht, th, tt\}$ of equally likely outcomes for this experiment:
$X = 0\ to\ \{tt\}$ – no heads (1 possible outcome)
$X = 1\ to \{ht, th\}$ – at least 1 head (two possible outcomes that has at least 1 head)
$X = 2\ to\ hh$ - two heads (1 possible outcome)

The probability of each of these events, hence of the corresponding value of X, can be found simply by counting, to give

X	0	1	2
Frequency (f)	1	2	1
$P(X)$	$\frac{1}{4}$ 0r 0.25	$\frac{2}{4}$ or 0.50	$\frac{1}{4}$ or 0.25

Table 1.0 Probability Distribution of Random Variable X.

You can have a table this way.

Random variable X	Frequency (f)	Probability of X or $P(X)$
0	1	$\frac{1}{4}$ 0r 0.25
1	2	$\frac{2}{4}$ or 0.50
2	1	$\frac{1}{4}$ or 0.25

Table 1.0 Probability Distribution of Random Variable X.

b. “At least one head” is the event $X \geq 1$, which is the union of the mutually exclusive events $X = 1$ and $X = 2$. Thus

$$P(X \geq 1) = P(1) + P(2)$$
$$= 0.50 + 0.25$$
$$= 0.75$$

A histogram that graphically illustrates the probability distribution is given in Figure 1.0

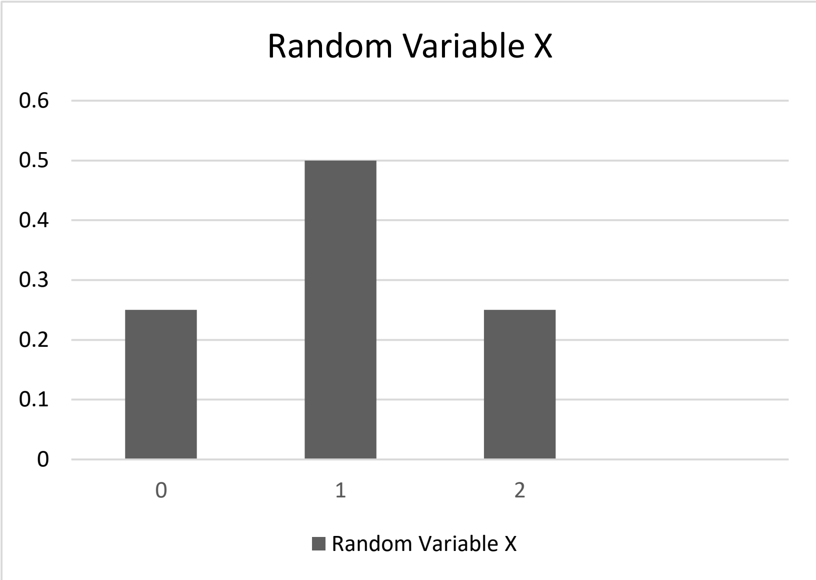

Figure 1.0 Probability Distribution for tossing a fair coin twice.

Example.

Give the probability distribution on the random variable X defined as the number of good cellphones drawn from 5 cellphones of which 3 are good ones and 2 are defectives when 3 cellphones are drawn at random.

Random variable X- number of good cellphones

X= number of good cellphones	Frequency f	Probability of x or $P(x)$
1	3	$\frac{3}{10}$
2	6	$\frac{6}{10}$
3	1	$\frac{1}{10}$
total	10	1

Let us now find the probability corresponding to a given random variable.

1. What is the probability of getting exactly 2 good cellphones?
2. What is the probability of getting at most 2 good cellphones?

3. What is the probability of getting at least 2 good cellphones

Solution:

1. $P(x = 2) = P(2)$
$= \frac{6}{10}$

2. $P(x \leq 2) = P(2) + P(1)$
$= \frac{6}{10} + \frac{3}{10}$
$= \frac{9}{10}$

3. $P(x \geq 2) = P(2) + P(3)$
$= \frac{6}{10} + \frac{1}{10}$
$= \frac{7}{10}$

Mean of a discrete random variable

The mean of the discrete random variable X is also called the *expected value of* X. The expected value of X is denoted by $E(X)$ or μ_x. Given a random variable X and its corresponding probability distribution, the expected value (EV) of X is the sum of the product of all the possible values of X and their relative frequencies. Use the formula to compute the mean of a discrete random variable.

$$\mu_x \text{ or } EV = \sum [x \cdot P(x)]$$

Example:

1. Find the mean in Figure 1.0 Probability Distribution for tossing a fair coin twice.

$\mu_x \text{ or } EV = \sum [x \cdot P(x)]$

$= 0(0.25) + 1(0.5) + 2(0.25)$

$= 0 + 0.5 + 0.5$

$= 1$

2. Find the mean of the discrete random variable X whose probability distribution is as follow:

Suppose we toss a coin three times.

Random variable X- the number of heads.

x	3	2	1	0

$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

μ_x *or* $EV = \sum[x \cdot P(x)]$

$$= 3\left(\frac{1}{8}\right) + 2\left(\frac{3}{8}\right) + 1\left(\frac{3}{8}\right) + 0\left(\frac{1}{8}\right)$$

$$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} + 0$$

$$= \frac{12}{8} \text{ or } \frac{3}{2} \text{ or } 1.5$$

The mean can be regarded as a measure of `central location' of a random variable. It is the weighted average of the values that X can take, with weights provided by the probability distribution. (saylordotorg.github.io 2012)

Variance and Standard Deviation of a discrete random variable

The variance and standard deviation of a discrete random variable X may be interpreted as measures of variability of the values assumed by the random variable in repeated trials of the experiment. The units on the standard deviation match those of X.
The equation for computing the variance of a discrete random variable is shown below.

$$\sigma^2 = \sum (x - \mu)^2 \cdot P(x)$$

Example:

3. Find the variance in Figure 1.0 Probability Distribution for tossing a fair coin twice.

$\sigma^2 = \sum(x - \mu)^2 \cdot P(x)$
$= (0 - 1)^2 \cdot 0.25 + (1 - 1)^2 \cdot 0.5 + (2 - 1)^2 \cdot 0.25$
$= 0.25 + 0 + 0.25$
$= 0.5$ or $\frac{1}{2}$

The standard deviation σ of a discrete random variable X is the square root of its variance, hence the formula is:

$$\sigma = \sqrt{\sigma^2}$$

Example:

4. Find the standard deviation in Figure 1.0 Probability Distribution for tossing a fair coin twice.

$\sigma = \sqrt{\sigma^2}$
$\sigma = \sqrt{0.5}$ *or* 0.7071

- a small standard deviation (or variance) means that the distribution of the random variable is narrowly concentrated around the mean
- a large standard deviation (or variance) means that the distribution is spread out, with some chance of observing values at some distance from the mean.

Note: The mean of a discrete random variable is a measure of central location, and the variance and standard deviation are measures of spread of the variable.

What's More

Task 3
A discrete random variable X has the following probability distribution:

x	-1	0	1	4
$P(x)$	0.2	0.5	a	0.1

Compute each of the following quantities.

a. a
b. $P(0)$
c. $P(X > 0)$
d. $P(X \geq 0)$
e. $P(X \leq -2)$
f. The mean μ of X
g. The variance σ^2 of X
h. The standard deviation σ of X

Solution:

a. Since all probabilities must add up to 1,

$$\begin{aligned} a &= 1 - (0.2 + 0.5 + 0.1) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

x	-1	0	1	4
$P(x)$	0.2	0.5	0.2	0.1

b. Directly from the table, $P(0) = 0.5$

c. From Table

$$\begin{aligned}(X > 0) &= P(1) + P(4) \\ &= 0.2 + 0.1 \\ &= 0.3\end{aligned}$$

d. From Table

$$\begin{aligned}P(X \geq 0) &= P(0) + P(1) + P(4) \\ &= 0.5 + 0.2 + 0.1 \\ &= 0.8\end{aligned}$$

e. Since none of the numbers listed as possible values for X is less than or equal to -2, the event $X \leq -2$ is impossible, so

$$P(X \leq -2) = 0$$

f. Using the formula in the definition of μ

$$\begin{aligned}\mu_x &= \sum[x \cdot P(x)] \\ &= (-1) \cdot (0.2) + (0) \cdot (0.5) + (1) \cdot (0.2) + (4) \cdot (0.1) \\ &= 0.4\end{aligned}$$

g. Using the formula in the definition of σ^2 and the value of μ that was just computed,

$$\begin{aligned}\sigma^2 &= \sum(x - \mu)^2 \cdot P(x) \\ &= (-1 - 0.4)^2 \cdot (0.2) + (0 - 0.4)^2 \cdot (0.5) + (1 - 0.4)^2 \cdot (0.2) \\ &\quad + (4 - 0.4)^2 \cdot (0.1) \\ &= 0.392 + 0.08 + 0.072 + 1.296 \\ &= 1.84\end{aligned}$$

h. Using the result of part (g),

$$\sigma = \sqrt{\sigma^2}$$

$$\begin{aligned}\sigma &= \sqrt{1.84} \\ &= 1.356\end{aligned}$$

What I Have Learned

- The probability distribution of a discrete random variable X is a listing of each possible value x taken by X along with the probability $P(x)$ that X takes that value in one trial of the experiment.
- The mean μ of a discrete random variable X is a number that indicates the average value of X over numerous trials of the experiment. It is computed using the formula $\mu = \sum xP \cdot (x)$.
- The variance σ^2 and standard deviation σ of a discrete random variable X are numbers that indicate the variability of X over numerous trials of the experiment. They may be computed using the formula

  $$\sigma^2 = \sum(x-\mu)^2 \cdot P(x)$$

  $$\sigma = \sqrt{\sigma^2}$$

Task 4

Make your own experiment and trial. Choose your own random variable X and compute for its mean, variance and standard deviation.

<u>CRITERIA</u>

OUTSTANDING (20 pts) – demonstrate/show superior knowledge of basic mathematical concepts and operations on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

EFFECTIVE (17 pts) – demonstrate/show appropriate use of basic mathematical concepts and operations on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

ADEQUATE (13 pts) – understands the basic mathematical concepts and operations on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

INEFFECTIVE (8 pts) – cannot demonstrate/show knowledge of mathematical concepts and operation on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

http://www.csu.edu/CTRE/pdf/rubricexamples-all.pdf

What I Can Do

Task 5
Try answering the following:

Sales versus Profit The monthly sales, X, of a company have a mean of ₱25,000 and a standard deviation of ₱4,000. Profits, Y, are calculated by multiplying sales by 0.3 and subtracting fixed costs of ₱6,000. What are the mean profit and the standard deviation of profit?

<u>CRITERIA</u>
OUTSTANDING (20 pts) – demonstrate/show superior knowledge of basic mathematical concepts and operations on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

EFFECTIVE (17 pts) – demonstrate/show appropriate use of basic mathematical concepts and operations on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

ADEQUATE (13 pts) – understands the basic mathematical concepts and operations on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

INEFFECTIVE (8 pts) – cannot demonstrate/show knowledge of mathematical concepts and operation on Probability of Random Variable, Mean, Variance and Standard Deviation of Random Variable.

Assessment

Task 5

1. Julia and Tony play the hand game *rock-paper-scissors.* Assume that, at each play, they make their choices with equal probability ($\frac{1}{3}$) for each of the three moves, independently of any previous play.

a) On any single play, what is the chance of a tie?
b) What is the chance that Julia wins at the first play?

Let Random variable X – rock
The probability distribution is shown below

X	0	1	2
$f$requency f	4	4	1
$P(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

c) Find $P(X = 1)$
d) Find $P(X \geq 1)$

2. Consider again the example of the number of languages spoken by Filipino school children. Define X to be the number of languages in which a randomly chosen Filipino child attending school can hold an everyday conversation. Assume that the probability function of X, $P(x)$, is as shown in the following table.

x	1	2	3	4	5	6
$P(x)$	0.663	0.226	0.066	0.022	0.019	0.004

a. What is the mean of X?
b. Find the variance of X
c. Find the standard deviation of X.

Answer Key

What I know Task 1

a) 0.40 c) 0.90 e) $\sigma^2 = 1.5275$

b) 0.10 d) $\mu_x or\ EV = 79.15$ f) $\sigma = 1.2359$

What's New task 2

a) (Head, Head), (Head, Tail), (Tail, Head), (Tail, Tail)

b) Random Variable value of X: $\{0, 1, 2\}$

c)

X	0	1	2
$P(X)$	$\frac{1}{4}$ or 0.25	$\frac{1}{4}$ or 0.25	$\frac{1}{4}$ or 0.25

What I Have Learned? Task 4

Answer may vary.

What I can Do Task 5

We know that: $E(X) = 25000$

$V(X) = 4000\ squared = 16000000$

$and\ Y = 0.3X - 6000$

$Therefore,$

$E(Y) = 0.3E(X) - 6000$

$= 0.3 \cdot 25000 - 6000$

$= 1500$

Assessment Task 6

1.

a) $\frac{1}{3}$ c) $\frac{4}{9}$

b) $\frac{1}{2}$ d) $\frac{5}{9}$

2.

a) $\mu_x = 1.52$

b) $\mu^2 = 0.8216$

c) $\sigma = 0.9064$

References

Books

Malate, S. 2017. *Statistics and Probability for Senior High School.* Manila: Vicarish Publicationsand Trading, Inc.

Millard Mamhot, Alice A. Mamhot. 2016. *Statistics and Probability.* Intramuros Manila: Unlimited Books Library Services & Prublishing Inc. .

Websites

Mayang, Manilyn Mugatar. n.d. *Discrete & Continuous Random Variables 12SPIII - A1.* Accessed December 14, 2021. https://www.scribd.com/document/415341293/Chapter-2-01-Discrete-continuous-random-variables-12SPIII-a1-docx.

Michael Evans, Peter Brown, Sue Finch, et al. n.d. *amsi.org.au.* Accessed December 14, 2021. https://amsi.org.au/ESA_Senior_Years/SeniorTopic4/4c/4c_1intro.html.

2021. *Probability Distributions for Discrete Random Variables.* Accessed December 14, 2021. https://stats.libretexts.org/Bookshelves/Introductory_Statistics

2012. *saylordotorg.github.io.* Accessed December 14, 2021. https://saylordotorg.github.io/text_introductory-statistics/s08-02-probability-distributions-for-.html.

For inquiries or feedback, please write or call:

Department of Education – Schools Division of Negros Oriental
Kagawasan, Avenue, Daro, Dumaguete City, Negros Oriental

Tel #: (035) 225 2376 / 541 1117
Email Address: negros.oriental@deped.gov.ph
Website: lrmds.depednodis.net