11

SENIOR HIGH SCHOOL

DEPARTMENT OF EDUCATION
SCHOOLS DIVISION OF NEGROS ORIENTAL
REGION VII
Kagawasan Ave., Daro, Dumaguete City, Negros Oriental

BASIC CALCULUS

Quarter 3 – Module 2
Limits of Transcendental Functions and Special Limits

ALTERNATIVE DELIVERY MODE
ADM

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Alternative Delivery Mode
Quarter 3 – Module 2: Limits of Transcendental Functions and Special Limits
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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

The module is intended for you compute the limits of exponential, logarithmic, and trigonometric functions using tables of values and graphs of the function; and illustrate limits involving the expressions $\frac{\sin t}{t}$, $\frac{1-\cos t}{t}$ and $\frac{e^t-1}{t}$ using tables of values.

What I Know

Functional relationships are also applied in real world situations, specifically in mathematical models. These mathematical models play an important role in applications of calculus.

There are also other functions, called transcendental, which are very useful both in theory and practice.

By definition, “transcendental function is any function of x which is not algebraic”, (Woods & Bailey, 1917). The basic transcendental functions includes the exponential, logarithmic and trigonometric functions.

PRE-ASSESSMENT

Evaluate the following limits by constructing table of values.

1. $\lim_{x \to 1} 3^x$
2. $\lim_{x \to 2} 5^x$

Lesson	Limits of Transcendental Functions (Exponential, Logarithmic, Trigonometric Functions) & Special Limits

What's In

PRIOR-KNOWLEDGE:

Let us recall some of the limit laws we have learned in the past lesson.

LIMIT LAWS:

Suppose that c is a constant and the limits

$\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then

1. $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

2. $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

3. $\lim_{x\to a} [c\, f(x)] = c \lim_{x\to a} f(x)$

4. $\lim_{x\to a} [f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

5. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, if $\lim_{x\to a} g(x) \neq 0$

6. $\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$, where n is a positive integer

7. $\lim_{x\to a} c = c$

8. $\lim_{x\to a} x = a$

9. $\lim_{x\to a} x^n = a^n$, where n is a positive integer

10. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$, where n is a positive integer

(If n is even, we assume that a > 0.)

11. . $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, where n is a positive integer

(If n is even, we assume that $\lim_{x\to a} f(x) > 0$.)

What's New

Task 1: Remember My Function!

1. If $b > 0$, $b \neq 1$, what is the exponential function with base b?
2. Let $b > 0$, $b \neq 1$. If $b^y = x$. $then\ y\ is\ called\ the\ logarithm\ of\ x\ to\ the\ base\ b$. Then, what denotes the function in symbol?

What is It

I. EVALUATING LIMITS OF EXPONENTIAL FUNCTION

Consider the natural exponential function $f(x) = e^x$, where e is called the Euler number, and has the value of 2.718281….

Illustrative Example 1. Evaluate the $\lim_{x\to 0} e^x$.

Solution:

There are two means to evaluate the limit of the function: through table of values and by graphical approach.

i. through tables of values:

Construct the table of values for $f(x) = e^x$. We will do so by approaching the number 0 from the left or through the values less than but close to 0, and by approaching the number 0 from the right or through the values greater than but close to 0.

Approaching 0 from the left:

x	$f(x) = e^x$
-1	0.36787944117
-0.5	0.60653065971
-0.1	0.90483741803
-0.01	0.99004983374
-0.001	0.99900049983
-0.0001	0.999900049983
-0.00001	0.99999000005

Intuitively, as x approaches to 0 from the left, $f(x) = e^x$

is approaching to 1. Therefore, the limit of $f(x) = e^x$ as x approaches to zero from the left, is 1..

Approaching 0 from the right:

x	$f(x) = e^x$
1	2.71828182846
0.5	1.6487212707
0.1	1.10517091808
0.01	1.01005016708
0.001	1.00100050017
0.0001	1.000100005
0.00001	1.00001000005

Intuitively, as x approaches to 0 from the right, $f(x)$ is approaching to 1.

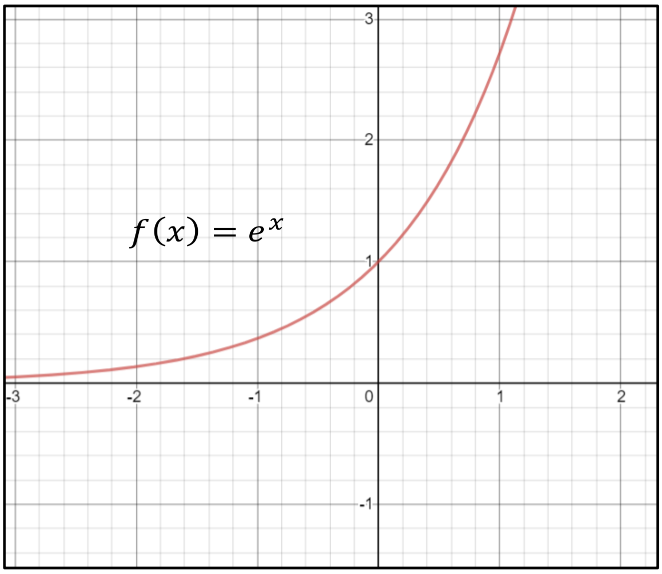
Therefore, the limit of $f(x) = e^x$ is 1 as x approaches to zero from the right.

From the table, as the values of x get closer and closer to 0, the values of $f(x)$ get closer and closer to 1.Combining the two sided limits, we have,

$$\lim_{x \to 0} e^x = 1.$$

ii. through a graph:

To determine the $\lim_{x \to 0} e^x$, use the graph $f(x) = e^x$.



Through inspection of the figure, as the value of x approaches to 0 either from the left or from the right, $f(x)$ gets closer and closer to 1. Therefore, using the graph of $f(x) = e^x$ through a graph, $\lim_{x \to 0} e^x = 1$.

II. EVALUATING LIMITS OF LOGARITHMIC FUNCTION

Consider the natural logarithmic function $f(x) = \ln x$. Recall that

$\ln x = \log_e x$. Moreover, $\ln x$ is the inverse of the natural exponential function $f(x) = e^x$ or $y = e^x$.

Illustrative Example . Evaluate the $\lim_{x \to 1} \ln x$.

Solution

We will evaluate the limit through constructing the tables of values.

Construct the table of values for $f(x) = \ln x$. We start by approaching the number 1 from the left or through the values less than but close to 1, and then approach the number 1 from the right or through the values greater than but close to 1.

Approaching 1 from the left:

x	$f(x) = \ln x$
0.1	-2.30258509299
0.5	-0.69314718056
0.9	-0.10536051565
0.99	-0.01005033585
0.999	-0.00100050033
0.9999	-0.0001000005
0.99999	-0.00001000005

Intuitively, as x approaches to 1 from the left, $f(x)$ is approaching to 0. Therefore limit of $f(x) = \ln x$ is 0 as x approaches to 1 from the left.

Approaching 1 from the right:

x	$f(x) = \ln x$
2	0.69314718056
1.5	0.4054651081
1.1	0.0953101798
1.01	0.00995033085
1.001	0.00099950033
1.0001	0.000099995
1.00001	0.00000999995

Intuitively, as x approaches to 1 from the right, $f(x)$ is approaching to 0. Therefore limit of $f(x) = \ln x$ is 0 as x approaches to 1 from the right.

From the table, as the values of x get closer and closer to 1, the values of $f(x)$ get closer and closer to 0. Combining the two sided limits, hence,

$$\lim_{x \to 1} \ln x = 0.$$

Illustrative Example 3. Evaluate the $\lim_{x \to 1} \log x$.

Solution:

Since, by ***common logarithmic function***

$$f(x) = \log_{10} x.$$

Recall that $f(x) = \log_{10} x = \log x$. Using tables of values, we can determine the $\lim_{x \to 1} \log x$.

Through tables of values:

We shall construct the table of values for $f(x) = \log x$.

Approaching 1 from the left:

x	$f(x) = \log x$
0.1	-1
0.5	-0.30102999566
0.9	-0.04575749056
0.99	-0.0043648054
0.999	-0.00043451177
0.9999	0.00004343161
0.99999	0.00000434296

Intuitively, as x approaches to 1 from the left, $f(x)$ is approaching to 0. Therefore limit of $f(x) = \log x$ is 0 as x approaches to 1 from the left.

Approaching 1 from the right:

x	$f(x) = \log x$
2	0.3010299956
1.5	0.17609125905
1.1	0.04139268515
1.01	0.00432137378
1.001	0.00043407747
1.0001	0.00004342727
1.00001	0.00000434292

Intuitively, as x approaches to 1 from the right, $f(x)$ is approaching to 0. Therefore limit of $f(x) = \log x$ is 0 as x approaches to 1 from the right.

From the table, as the values of x get closer and closer to 1, the values of $f(x)$ get closer and closer to 0. Combining the two sided limits, hence,

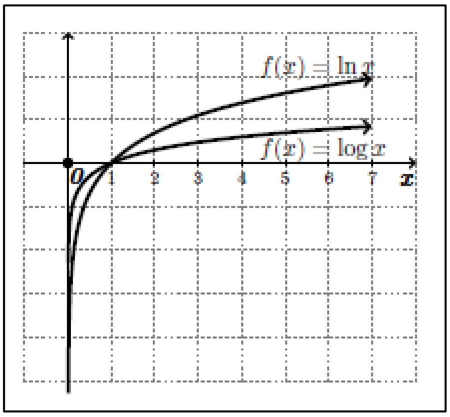
$$\lim_{x \to 1} \log x = 0.$$

Then, we shall verify the limit values in Example 2 and Example 3 by graphical approach.

Illustrative Example 4. Evaluate the $\lim_{x \to 1} \ln x$ and $\lim_{x \to 1} \log x$.

Through a graph:

The graphs of both the natural and common logarithmic functions can be used to determine the limits as x approaches to 1.



By inspecting the graph, we can see that $\lim_{x \to 1} \ln x = 0$ and $\lim_{x \to 1} \log x = 0$

III. EVALUATING LIMITS OF TRIGONOMETRIC FUNCTIONS

Illustrative Example 5. Evaluate the $\lim_{x \to 0} \sin x$.

Solution

We shall evaluate $\lim_{x \to 0} \sin x$ by using tables of values and graphical approach.

Through tables of values:

Construct the table of values for $f(x) = \sin x$. We start by approaching the number 0 from the left or through the values less than but close to 0, and approaching the number 0 from the right or through the values greater than but close to 0.

Approaching 1 from the left:

x	$f(x) = \sin x$
-1	-0.8414709848
-0.5	-0.4794255386
-0.1	-0.09983341664
-0.01	-0.00999983333
-0.001	-0.00099999983
-0.0001	-0.00009999999
-0.00001	-0.00000999999

Intuitively, as x approaches to 0 from the left, $f(x)$ is approaching to 0. Therefore, the limit of $f(x) = \sin x$ is 0 as x approaches to 0 from the left.

Approaching 0 from the right:

x	$f(x) = \sin x$
1	0.8414709848
0.5	0.4794255386
0.1	0.09983341664
0.01	0.00999983333
0.001	0.00099999983
0.0001	0.00009999999
0.00001	0.00000999999

Intuitively, as x approaches to 0 from the right, $f(x)$ is approaching to 0. Therefore limit of $f(x) = \sin x$ is 0 as x approaches to 0 from the right.

From the table, as the values of x get closer and closer to 0, the values of $f(x)$ get closer and closer to 0. Combining the two sided limits, hence,

$$\lim_{x \to 0} \sin x = 0.$$

B. Through a graph:

We can also find $\lim_{x \to 0} \sin x$ by using the graph of the sine function. Consider the graph of $f(x) = \sin x$

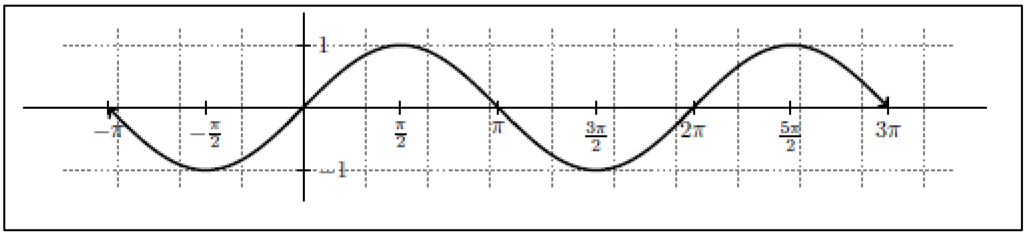


Figure 1: Graph of a sine function

By inspecting, as the value of x approaches to zero from either left or right of it, $f(x)$ approaches to 0. Thus, $\lim_{x \to 0} \sin x = 0$

IV. EVALUATING SPECIAL LIMITS

Illustrative Example 1. Evaluate the $\lim_{t \to 0} \frac{\sin t}{t}$.

Solution

Using tables of values and graphical approach, we can evaluate $\lim_{t \to 0} \frac{\sin t}{t}$.

Through tables of values:

Construct the table of values for $f(t) = \frac{\sin t}{t}$. We start by approaching the number 0 from the left or through the values less than but close to 0, and approaching the number 0 from the right or through the values greater than but close to 0.

Approaching 0 from the left:

t	$f(t)=\frac{\sin t}{t}$
-1	0.8414709848
-0.5	0.9588510772
-0.1	0.9983341665
-0.01	0.9999833334
-0.001	0.99999998333
-0.0001	0.99999999983

Intuitively, as t approaches to 0 from the left, $f(t)$ is approaching to 1. Therefore limit of $f(t) = \frac{\sin t}{t}$ is 1 as t approaches to 0 from the left.

Approaching 0 from the right:

t	$f(t)=\frac{\sin t}{t}$
1	0.8414709848
0.5	0.9588510772
0.1	0.9983341665
0.01	0.9999833334
0.001	0.99999998333
0.0001	0.99999999983

Intuitively, as t approaches to 0 from the right, $f(t)$ is approaching to 1. Therefore limit of $f(t) = \frac{\sin t}{t}$ is 1 as t approaches to 0 from the right.

From the table, as the values of t get closer and closer to 0, the values of $f(t)$ get closer and closer to 1.

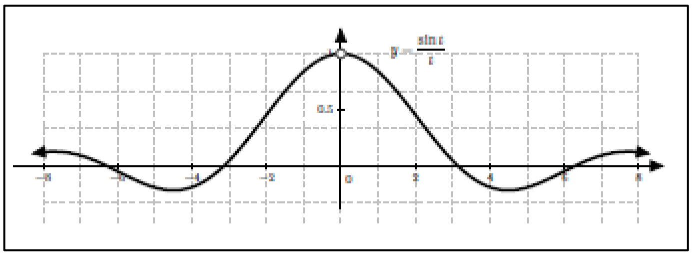
Since, $\lim_{t \to 0^-} \frac{\sin t}{t} = 1$ and $\lim_{t \to 0^+} \frac{\sin t}{t} = 1$ are both equal to 1. Then,

$$\lim_{t \to 0} \frac{\sin t}{t} = 1.$$

Now we shall use the sketch of the graph of $f(t) = \frac{\sin t}{t}$ to inspect the liimit of $\lim_{t \to 0} f(t)$

Through a graph:

The graph of $f(t) = \frac{\sin t}{t}$ below confirms that the $f(t)$ values approach 1 as t approaches to 0.



Illustrative Example 6. Evaluate the $\lim_{t \to 0} \frac{1-\cos t}{t}$.

Solution

Using the tables of values and the graph of the function, we will evaluate $\lim_{t \to 0} \frac{1-\cos t}{t}$.

Through tables of values:

Construct the table of values for $f(t) = \frac{1-\cos t}{t}$. We start by approaching the number 0 from the left or through the values less than but close to 0 and from the right or through the values greater than but close to 0.

Approaching 0 from the left:

t	$f(t) = \frac{1 - \cos t}{t}$
-1	-0.4596976941
-0.5	-0.2448348762
-0.1	-0.04995834722
-0.01	-0.0049999583
-0.001	-0.0004999999
-0.0001	-0.00005

Intuitively, as t approaches to 0 from the left, $f(t)$ is approaching to 0. Therefore limit of $f(t) = \frac{1-\cos t}{t}$ is 0 as t approaches to 0 from the left.

Approaching 0 from the right:

t	$f(t) = \dfrac{1-\cos t}{t}$
1	0.4596976941
0.5	0.2448348762
0.1	0.04995834722
0.01	0.0049999583
0.001	0.0004999999
0.0001	0.00005

Intuitively, as t approaches to 0 from the right, $f(t)$ is approaching to 0. Therefore limit of $f(t) = \frac{1-\cos t}{t}$ is 0 as t approaches to 0 from the right.

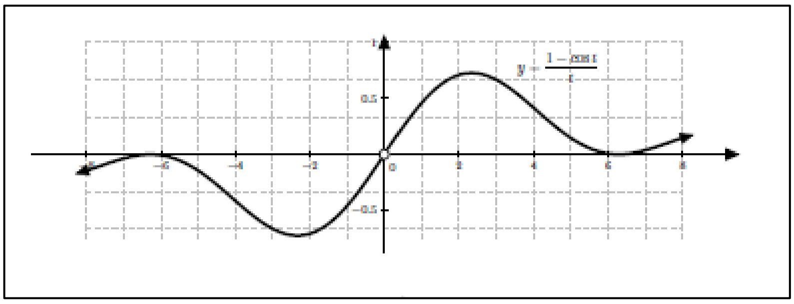
From the table, as the values of t get closer and closer to 0, the values of $f(t)$ get closer and closer to 0.

Since, $\lim_{t\to 0^-} \frac{1-\cos t}{t} = 0$ and $\lim_{t\to 0^+} \frac{1-\cos t}{t} = 0$ are both equal to 0. Then,

$$\lim_{t\to 0} \frac{1-\cos t}{t} = 0.$$

Through a graph:

The graph of $f(t) = \frac{1-\cos t}{t}$ below confirms that the $f(t)$ values approach 0 as t approaches to 0.



Illustrative Example 7. Evaluate the $\lim_{t \to 0} \frac{e^t - 1}{t}$.

Solution

As with the other examples, we shall evaluate Through tables of values:

Construct the table of values for $f(t) = \frac{e^t - 1}{t}$. We start by approaching the number 0 from the left or through the values less than but close to 0, and approaching the number 0 from the right or through the values greater than but close to 0.

Approaching 0 from the left:

t	$f(t) = \dfrac{e^t - 1}{t}$
-1	0.6321205588
-0.5	0.7869386806
-0.1	0.9516258196
-0.01	0.9950166251
-0.001	0.9995001666
-0.0001	0.9999500016

Intuitively, as t approaches to 0 from the left, $f(t)$ is approaching to 1. Therefore limit of $f(t) = \frac{e^t - 1}{t}$ is 1 as t approaches to 0 from the left.

Approaching 0 from the right:

t	$f(t) = \dfrac{e^t - 1}{t}$
1	1.718281828
0.5	1.297442541
0.1	1.051709181
0.01	1.005016708
0.001	1.000500167
0.0001	1.000050002

Intuitively, as t approaches to 0 from the right, $f(t)$ is approaching to 1. Therefore limit of $f(t) = \frac{e^t - 1}{t} = 1$ as t approaches to 0 from the right.

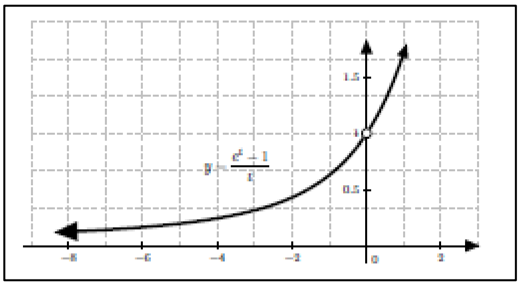
From the table, as the values of t get closer and closer to 0, the values of $f(t)$ get closer and closer to 1.

Since $\lim_{t \to 0^-} \frac{e^t - 1}{t}$ and $\lim_{t \to 0^+} \frac{e^2 - 1}{t}$ are both equal to 1 then,

$$\lim_{t \to 0} \frac{e^t - 1}{t} = 1.$$

Through a graph:

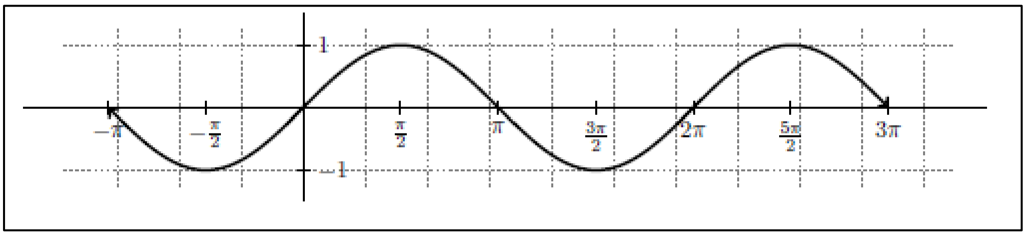
The graph of $f(t) = \frac{e^t - 1}{t}$ below confirms that the y values approach 1 as t approaches to 0.



What's More

Task 2.

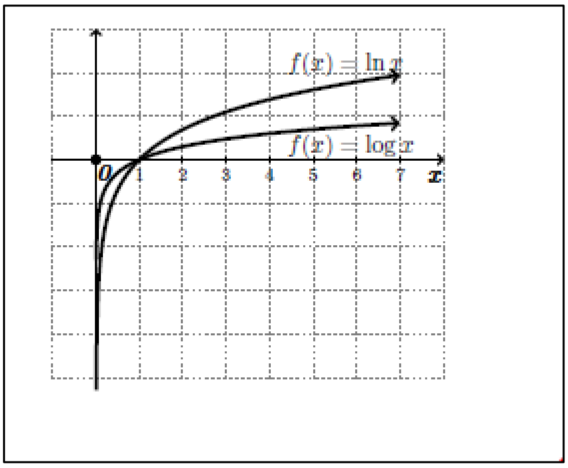
I. Given the graph of $f(x) = sin\ x$. Find the limit of the following functions.



a. $\lim_{x \to \frac{\pi}{2}} \sin x$

b. $\lim_{x \to \pi} \sin x$

II. Given the graph of the function,



Find:

a. $\lim_{x \to e} \ln x$

What I Have Learned

The limit of a function and the functional value at a point is

$$\lim_{x \to c} f(x) = f(c)$$

either f is exponential, logarithmic or trigonometric and c is a real number which is in the domain of f.

Special limits such as,

$$\lim_{t \to 0} \frac{\sin t}{t}, \lim_{t \to 0} \frac{1 - \cos t}{t}, \text{ and } \lim_{t \to 0} \frac{e^t - 1}{t}$$

will result in $\frac{"0"}{"0"}$ upon direct substitution. However, can be found either by using tables of values and graphs of the function.

What I Can Do

Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{x} + 1}$. Write your solutions comprehensively. *(Hint: Direct substitution will result in the expression $\frac{"0"}{"0"}$. To resolve, rationalize the given function first before applying the limit).* Please be guided with the rubric.

RUBRIC

CATEGORY	5	4	3	2
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.

Assessment

Evaluate the following limits through constructing tables of values.

1. $\lim_{x \to -\frac{\pi}{2}} \sin x$
2. $\lim_{x \to -\pi} \sin x$
3. $\lim_{x \to 0^+} \log x$
4. $\lim_{x \to 0^+} \ln x$
5. $\lim_{t \to 0} \frac{1-\cos t}{\sin t}$

Answer Key

What's In

PRIOR-KNOWLEDGE:

1. 3
2. 25

What's New:

1. $f(x) = b^x, x \in \mathbb{R}$
2. $y = \log_b x$

What's More:

1. 1
2. 0
3. 1

Assessment:

1. -1
2. 0
3. $-\infty$
4. $-\infty$
5. 0

References

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