AS.2 CS 210

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① a) Detailed

line 1a: $t_{fetch} + t_{store}$

line 1b: $(2t_{fetch} + t_{<}) \times (n+2)$

line 1c: $(2t_{fetch} + t_{+} + t_{store}) \times (n+1)$

line 2: $(2t_{fetch} + t_{+}) \times (n+1)$

Simplified

line 1a: 2

line 1b: $3 \times (n+2)$

line 1c: $4 \times (n+1)$

line 2: $3 \times (n+1)$

$T(n) = 2 + 3n + 6 + 4n + 4 + 3n + 3$

$t(n) = 10n + 15$

① b) Detailed

line 1a: $t_{fetch} + t_{store}$

line 1b: $(2t_{fetch} + t_{<}) \times (n+2)$

line 1c: $(2t_{fetch} + t_{+} + t_{store}) \times (n+1)$

line 2: $(2t_{fetch} + t_{\%} + t_{store}) \times (n+1)$

line 3: $(2t_{fetch} + t_{?} + t_{store}) \times (n+1)$

simplified

line 1a: 2

line 1b: $3 \times (n+2)$

line 1c: $4 \times (n+1)$

line 2: $4 \times (n+1)$

line 3: $4 \times (n+1)$

$t(n) = 2 + 3n + 6 + 4n + 4 + 4n + 4 + 4n + 4$

$T(n) = 15n + 20$

② ~~Soln.~~

Prove by induction

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n=1$

$$(1^2) = \frac{(1)(1+1)(2(1)+1)}{6}$$

$$(1^2) = \frac{6}{6}$$

$$1 = 1$$

True

for $n=k$

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

~~True~~

for $n=k+1$

$$(k+1)^2 + \frac{k(k+1)(2k+1)}{6} = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

$$6(k+1)^2 + k(k+1)(2k+1) = (k+1)(k+2)(2(k+1)+1)$$

$$2k^3+9k^2+13k+6 = 2k^3+9k^2+13k+6$$

LHS = RHS

True

③ Solve using repeated substitution

$T(n) = T(n-1) + 1 \quad , \; T(0) = 1$

$n = n-1$ substitute back

$T(n-1) = T(n-2) + 1$

$T(n) = T(n-2) + 2$

$T(n-2) = T((n-2)-1) + 1$

$T(n-2) = T(n-3) + 1$

$T(n) = T(n-3) + 3$

$\therefore T(n) = T(n-k) + k$

when $k = n$

$T(n) = T(n-n) + n$

$T_n = T(0) + n$

$T_n = 1 + n$

4 $f(n) = 3n^2 - n + 4$

<u>for $n >= 4$</u>

$3n^2 - n + 4 <= 3n^2 - n + n <= 3n^2$

$f n \le= 3n^2$

$C = 3$ $\quad n_0 = 4$

$g(n) = n^2$

$\therefore \; f(n) = O(n^2)$

5 $f(n) = 3n^2 - n + 4$

$f(n) = \Omega(g(n))$

$f(n) >= C(g(n))$

$3n^2 - n + 4 >= Cn^2$

$n >= n_0$

$3 - \frac{1}{n} + \frac{4}{n^2} >= C$

$n_0 = 1$

$3 - 1 + 4 >= 6$

$C = 6$

$3n^2 - n + 4 >= 6n^2$

$C = 6, n_0 = 1, n >= 1$

$3n^2 - n + 4 = \Omega(n^2)$