

AS.2

CS 210

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① a) ^{detailed} line 1a: $t_{\text{fetch}} + t_{\text{store}}$

$$\text{line 1b: } (2t_{\text{fetch}} + t_{\text{L}}) \times (n+2)$$

$$\text{line 1c: } (2t_{\text{fetch}} + t_{\text{r}} + t_{\text{store}}) \times (n+1)$$

$$\text{line 2: } (2t_{\text{fetch}} + t_{\text{r}}) \times (n+1)$$

Simplified

$$\text{line 1a: } 2$$

$$\text{line 1b: } 3 \times (n+2)$$

$$\text{line 1c: } 4 \times (n+1)$$

$$\text{line 2: } 3 \times (n+1)$$

$$T(n) = 2 + 3n + 6 + 4n + 4 + 3n + 3$$

$$T(n) = 10n + 15$$

① b) Detailed

$$\text{line 1a: } t_{\text{fetch}} + t_{\text{store}}$$

$$\text{line 1b: } (2t_{\text{fetch}} + t_{\text{L}}) \times (n+2)$$

$$\text{line 1c: } (2t_{\text{fetch}} + t_{\text{r}} + t_{\text{store}}) \times (n+1)$$

$$\text{line 2: } (2t_{\text{fetch}} + t_{\text{r}} + t_{\text{store}}) \times (n+1)$$

$$\text{line 3: } (2t_{\text{fetch}} + t_{\text{r}} + t_{\text{store}}) \times (n+1)$$

Simplified

$$\text{line 1a: } 2$$

$$\text{line 1b: } 3 \times (n+2)$$

$$\text{line 1c: } 4 \times (n+1)$$

$$\text{line 2: } 4 \times (n+1)$$

$$\text{line 3: } 4 \times (n+1)$$

$$T(n) = 2 + 3n + 6 + 4n + 4 + 4n + 4 + 4n + 4$$

$$T(n) = 15n + 20$$

② ~~Solve~~

Prove by induction

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n=1$

$$(1^2) = \frac{(1)(1+1)(2(1)+1)}{6}$$

$$(1^2) = \frac{6}{6}$$

$$1 = 1$$

True

for $n=k$

$$\sum_{i=1}^k i^2 = \frac{k(k+1)(2k+1)}{6}$$

~~True~~

for $n=k+1$

$$(k+1)^2 + \frac{k(k+1)(2k+1)}{6} = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

$$6(k+1)^2 + k(k+1)(2k+1) = (k+1)(k+2)(2(k+1)+1)$$

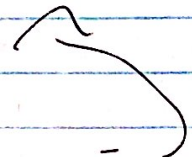
$$2k^3 + 9k^2 + 13k + 6 = 2k^3 + 9k^2 + 13k + 6$$

LHS = RHS

True

③ Solve using repeated substitution

$$T(n) = T(n-1) + 1, \quad T(0) = 1$$

$n = n-1$  substitute back

$$T(n-1) = T(n-2) + 1$$

$$T(n) = T(n-2) + 2$$

$$T(n-2) = T(n-2-1) + 1$$

$$T(n-2) = T(n-3) + 1$$

$$T(n) = T(n-3) + 3$$

$$\therefore T(n) = T(n-k) + k$$

~~Φ~~
when $k = n$

$$T(n) = T(n-n) + n$$

$$T_n = T(0) + n$$

$$\underline{T_n = 1 + n}$$

④ $f(n) = 3n^2 - n + 4$

for $n \geq 4$

$$3n^2 - n + 4 \leq 3n^2 - n + n \leq 3n^2$$

$$f(n) \leq 3n^2$$

$$C = 3 \quad ~~n_0 = 4~~ \quad n_0 = 4$$

$$g(n) = n^2$$

$$\therefore f(n) = O(n^2)$$

$$(5) f(n) = 3n^2 - n + 4$$

$$f(n) = O(g(n))$$

$$f(n) \geq C(g(n))$$

$$3n^2 - n + 4 \geq Cn^2$$

$$n \geq n_0$$

$$3 - \frac{1}{n} + \frac{4}{n^2} \geq C$$

$$n_0 = 1$$

$$3 - 1 + 4 \geq 6$$

$$C = 6$$

$$3n^2 - n + 4 \geq 6n^2$$

$$C = 6, n_0 = 1, n \geq 1$$

$$\underline{3n^2 - n + 4 = O(n^2)}$$