Coding Assignment 2

Consider our model boundary value problem,

$$\begin{aligned} u_{,xx} + f &= 0 \\ u(1) &= 0 \\ u_{,x}(0) &= 0 \end{aligned}$$

Employing piecewise quadratic and cubic ($p = 2, 3$) B-spline (C^{p-1}) finite element spaces implement a finite element code that solves the Galerkin finite element equations for $n = 1, 10, 100, 1000$ and $f = x^2$. For the B-spline basis place nodes at the Greville abscissae. Do not approximate f but instead evaluate f exactly at the Gauss points.

1. Employing the data for $n = 1, 10, 100, 1000$, $p = 2, 3$ for the B-spline basis compute the error between the exact and approximate solutions and generate a log-log plot of e versus h. Determine and report the slope of each convergence curve. How can the slopes be explained through the *a priori* error estimate $||e||_0 \leq Ch^{p+1}||u||_{p+1}$?

2. Employing the data for $n = 1, 10, 100, 1000$, $p = 2, 3$ for the B-spline basis compute the error between the exact and approximate solutions and generate a log-log plot of e versus number of nodes.

Now consider the Bernoulli-Euler beam theory for a cantilever beam under a distributed load f (force per unit length)

$$\begin{aligned} EIu_{,xxxx} &= f \\ u(1) &= 0 \\ u_{,x}(1) &= 0 \end{aligned}$$

where E is Young's modulus and I is the moment of inertia of the beam cross-section, both of which are assumed to be constant, and u describes the beam deflection. Let $E = 1000000$. For the B-spline basis place nodes at the Greville abscissae.

1. Let the beam width $b = 0.005$ and height $h = 0.005$. Assume a uniform distributed load $f = 10h^3$. For $n = 1, 10, 100$ use C^1 quadratic B-splines and C^2 cubic B-splines to solve for the maximum tip deflection of the beam. Generate a plot of the computed deflection versus number of nodes. Include on your plot a line representing the exact deflection computed from the theory (Hint: Look in the back of you mechanics of materials textbook). In other words your plot will have 2 computed curves and 1 exact curve. Reasoning with basic beam theory why is your finite element code unable to compute the exact tip deflection? What could you do to the basis so that the exact tip deflection could be computed?

2. Set the beam width $b = 0.005$ and the let the height vary as $h = 0.1, 0.01, 0.005, 0.002, 0.001$. Normalize the uniform distributed load as $f = 10h^3$ for each case. For $n = 10$, use C^1 quadratic B-splines and C^2 cubic B-splines to solve for the maximum tip deflection of the beam. Generate a plot of the computed deflection versus the

beam slenderness L/h. Include on your plot a line representing the exact deflection computed from the theory. In other words your plot with have 2 computed curves and 1 exact curve. What can you conclude about the sensitivity of your method to increasing beam slenderness?

For this assignment you will turn in your code for the *beam bending problems* only. I will grade you on whether or not you correctly implemented and used the following:

1. The Bézier extraction of B-splines of degree $p = 2, 3$.
2. Greville abscissae to compute nodal locations for B-splines.
3. The IEN, ID, and LM arrays.
4. Local and global assembly routines. A loop over elements must be employed where element stiffness and force matrices and vectors are constructed and subsequently assembled into the global system $\mathbf{Kd} = \mathbf{F}$.
5. All integrals must be evaluated using Gaussian quadrature.
6. A local shape function subroutine which returns shape function values and parametric derivatives at a given Gauss point.
7. A global shape function subroutine which returns the global derivatives of the shape functions and the Jacobian determinant at a given Gauss point.