Assignment #5: Gradient-Free Optimization — ME 575

due 3/18/2020 before midnight via Learning Suite — **50 possible points**

Objective: Continue progressing on your project, and learn more about gradient-free optimization by using existing tools and developing our own basic implementations.

5.1 *Improve* your project problem from the last homework and optimize it using an existing *gradient-free* algorithm of your choice. You might like to try multiple optimizers, and even compare results using both gradient-based and gradient-free methods (comparing across different optimizers is purely optional, but can be helpful). Discuss the changes/improvements you made to your problem and what you learned from the optimization studies.

5.2 Implement your own gradient-free optimization algorithm (e.g., Nelder-Mead, Genetic Algorithm, Particle Swarm). Apply it to your project problem, or if you project problem is highly complex apply it to a simpler test problem. Compare your results and performance to a similar existing algorithm (presumably what you did in the previous problem unless you needed to create a separate test problem).

5.3 Study the effect of increased problem dimensionality using the n-dimensional Rosenbrock function:

$$f(x) = \sum_{i=1}^{n-1} \left(100(x_{i+1} - {x_i}^2)^2 + (1 - x_i)^2\right)$$

Solve the problem using three different approaches:

(a) Gradient-free

(b) Gradient-based with finite differencing

(c) Gradient-based with exact gradients

You may use existing optimizers or your own implementation if you prefer. In each case repeat the minimization for $n = 2, 4, 8, 16, \ldots$ up to the highest number you can *reasonably* manage. Plot the number of function calls required as a function of dimension size (n) for all three methods on one figure. Discuss any differences in optimal solutions found by the various algorithms and dimensions. Compare and discuss your results.

5.4 Discuss your plans for continued progress towards the final project.

Notes:

- 5.1 can be done as a team (one writeup), but the remaining tasks (5.2–5.4) need to be completed individually.
- The Optimization Toolbox that you have been using has a Nelder-Mead and a mixed-integer linear programming algorithm. The university license does not include the Global Optimization Toolbox (which has more gradient-free solvers like a genetic algorithm, particle swarm, etc.). Look at http://flow.byu.edu/me575/resources/optimizers/ for some options/alternatives if you wish to use one of these solvers.