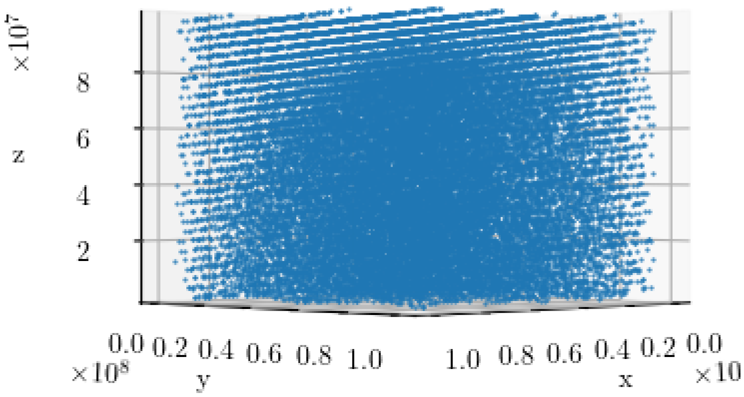
In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt

plt.rc('text', usetex=True)
plt.rc('font', family='serif')
plt.rcParams['figure.dpi'] = 100
```

Question 1

I could not figure out how to use the C pseudo-random number generator given by Jon. So I'll just use the txt file provided. Also, I'll check if the Python (`numpy.random.randint`) PRNG is flawed or not.

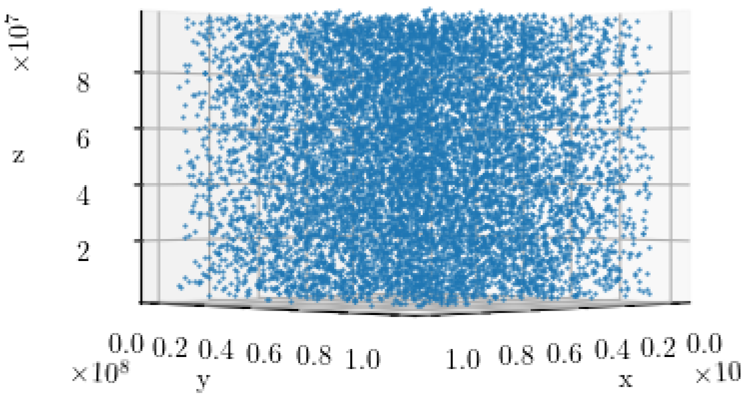
In [ ]:
```
# flawed C-PRNG
# Loading the txt file provided
points=np.loadtxt('rand_points.txt')
x=points[:,0]
y=points[:,1]
z=points[:,2]

# plotting stuff
fig=plt.figure()
ax=fig.add_subplot(projection='3d')
ax.plot(x,y,z,'.',markersize='1')
ax.view_init(0, 45)
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
plt.show()
```



We can see lines; it really does look like the 'randomly' generated points lie on a set of planes! Not so random.

In [ ]:

```
# Numpy PRNG
# getting (x,y,z) coordinates between 0 and 1e8, using numpy random number generator
x=[];y=[];z=[]
for i in range(10000):
    x.append(np.random.randint(0,1e8))
    y.append(np.random.randint(0,1e8))
    z.append(np.random.randint(0,1e8))

# plotting stuff
fig=plt.figure()
ax=fig.add_subplot(projection='3d')
ax.plot(x,y,z,'.',markersize=1)
ax.view_init(0, 45)
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
plt.show()
```



It does not look like the points lie on a set of planes; completely randomly generated points!

Question 2

Theoretically, I could use any of the Lorentzians, Gaussians or power laws to generate the exponential deviates, so long as the bounding distribution is greater than the exponential for all values of x. The Gaussian function will be a little harder to use, because the CDF does not really have simple form (it is the ERF, error function), but I dont doubt that it is possible to work with it in Python. However, in this question I will just use the Lorentzian because why not.

A Lorentzian has the form: $\text{PDF}(x) = \frac{1}{1+x^2}$. So the CDF is:

$$\text{CDF}(x) = \int_0^x \frac{1}{1+x^2} dx = \arctan(x)$$

Set the CDF equal to some random number between 0 and 1:

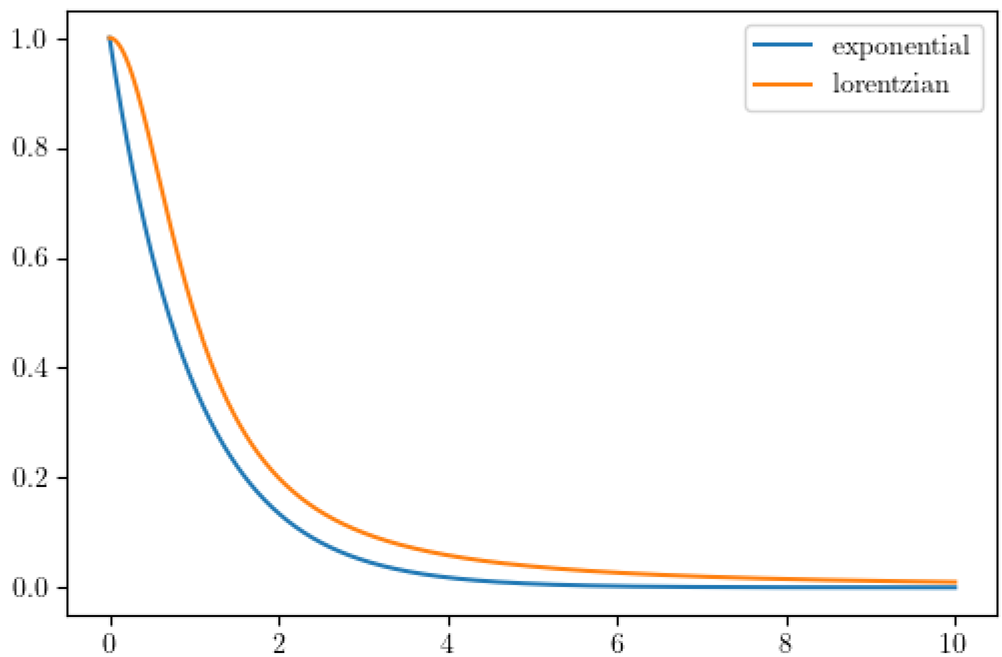
$$\text{CDF}(x) = \arctan(x) = r.$$

Solve for x:

$$x = \tan(r).$$

In [ ]:
```
# relation we derived above: x=tan(r)
def lorentzian(n):
    r=np.pi*(np.random.rand(n)-0.5) # tan only takes numbers between -pi and pi
    return np.tan(r)
```

```python
# first let's check if the Lorentzian is always above the exponential we
want to generate
x=np.linspace(0,10,1001)

def model(x):
    return np.exp(-x)

def lorentz(x):
    return 1/(1+x**2)

plt.plot(x,model(x),label='exponential')
plt.plot(x,lorentz(x),label='lorentzian')
plt.legend()
plt.show()
print('all good')
```



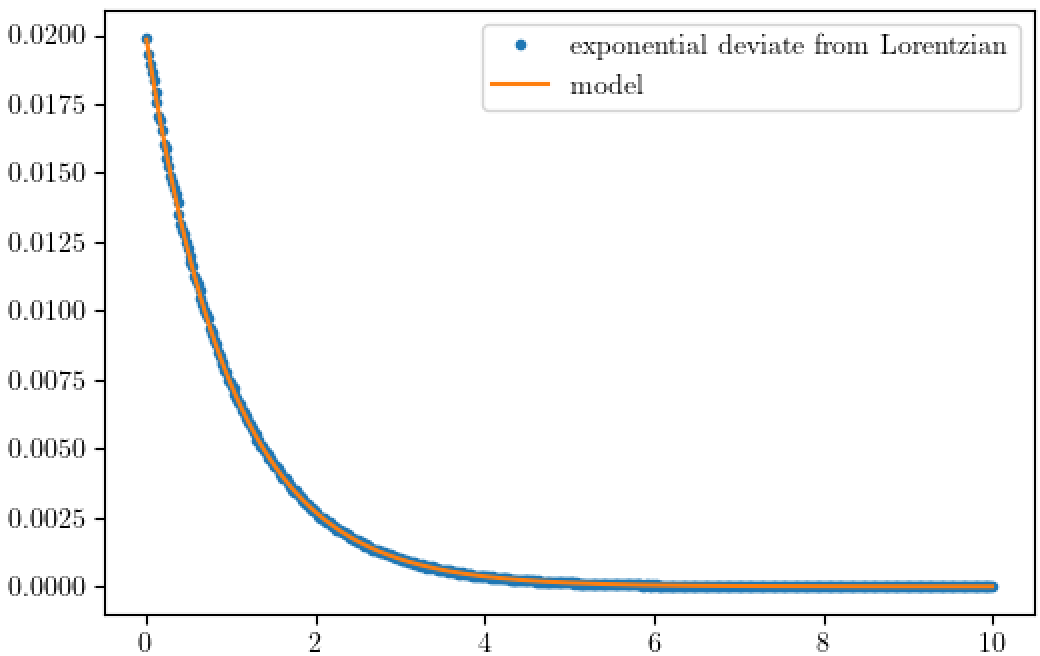

```
all good
```

In [ ]:

```python
n=10000000 # number of random numbers

# generating the non-uniform random numbers
l=lorentzian(n)
u=lorentz(l)*np.random.rand(n)

# seletcing only those that lie *below* the exponential
accept=u<model(l)
```

```
use=l[accept]

# effeciency (how many samples we actually use)
eff=np.size(use)/np.size(accept)*100

# plotting stuff
a,b=np.histogram(use,np.linspace(0,10,500))
a=a/a.sum()
bb=0.5*(b[1:]+b[:-1]) # get center of bins
pred=model(bb)
pred=pred/pred.sum()

plt.plot(bb,a,'.', label='exponential deviate from Lorentzian')
plt.plot(bb,pred,label='model')
plt.legend()
plt.show()
print('Effeciency of generator is: ',eff)
```

```
C:\Users\Greg\AppData\Local\Temp\ipykernel_3820\1798579393.py:10: RuntimeWarning: ove
rflow encountered in exp
  return np.exp(-x)
```




```
Effeciency of generator is:  81.80398
```

Looks good!

Question 3

Same as problem 2, but now I use a ratio-of-uniforms generator.

The math is that we take a (u, v) plane, where

$$0 < u < \sqrt{\mathrm{PDF}(v/u)}.$$

In our case, $\mathrm{PDF}(x) = e^{-x}$ and u runs from 0 to 1

So we have

$$0 < u < \sqrt{e^{-v/u}},$$

$$0 < u^2 < e^{-v/u},$$

$$\ln{(0)} < \ln{(u^2)} < -v/u,$$

$$\ln{(0)} > -2u\ln{(u)} > v.$$

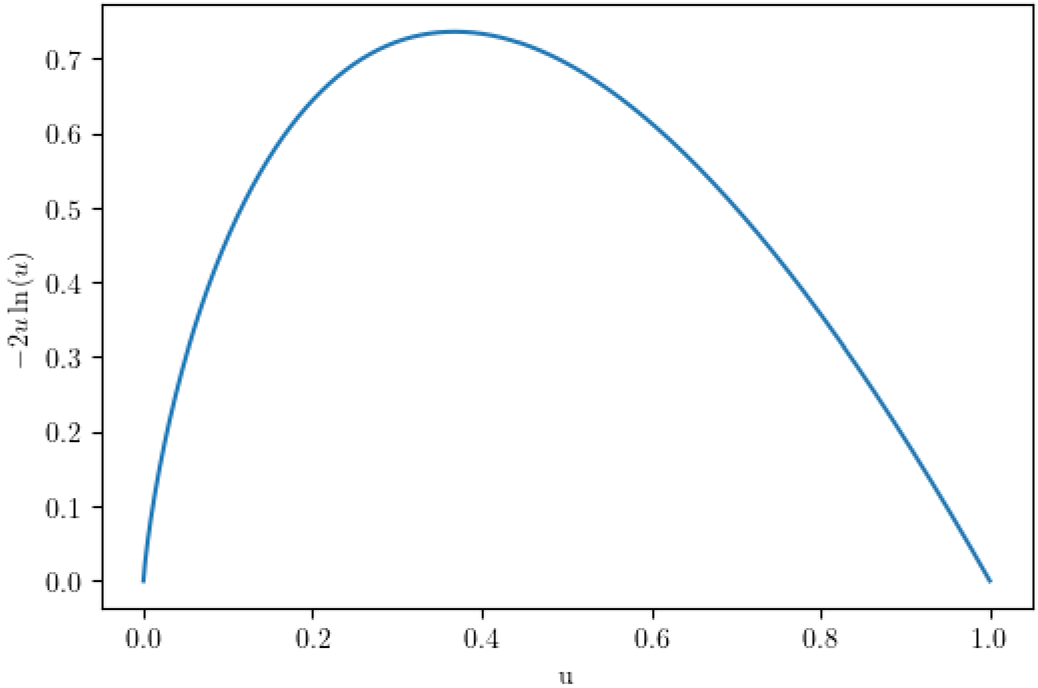
$\ln 0 = \infty$ so we can discard that bound (of course v will always be smaller than ∞). So we are left with

$$-2u\ln{(u)} > v.$$

To find the upper bound on v (the lower bound is 0) let's pick the u, between 0 and 1, that will maximize $-2u\ln{(u)}$

In [ ]:

```
x=np.linspace(0.0000001,1,1001)
plt.plot(x,-2*x*np.log(x))
plt.xlabel('u')
plt.ylabel('$-2u\ln{(u)}$')
plt.show()
```



There is one extremum (it is a maximum) betweem 0 and 1 so let's take the derivative and set it equal to zero:

$$\frac{d}{du}(-2u\ln(u)) = -2 - 2\ln(u) = 0,$$

$$\ln u = -1,$$

$$u = e^{-1}.$$

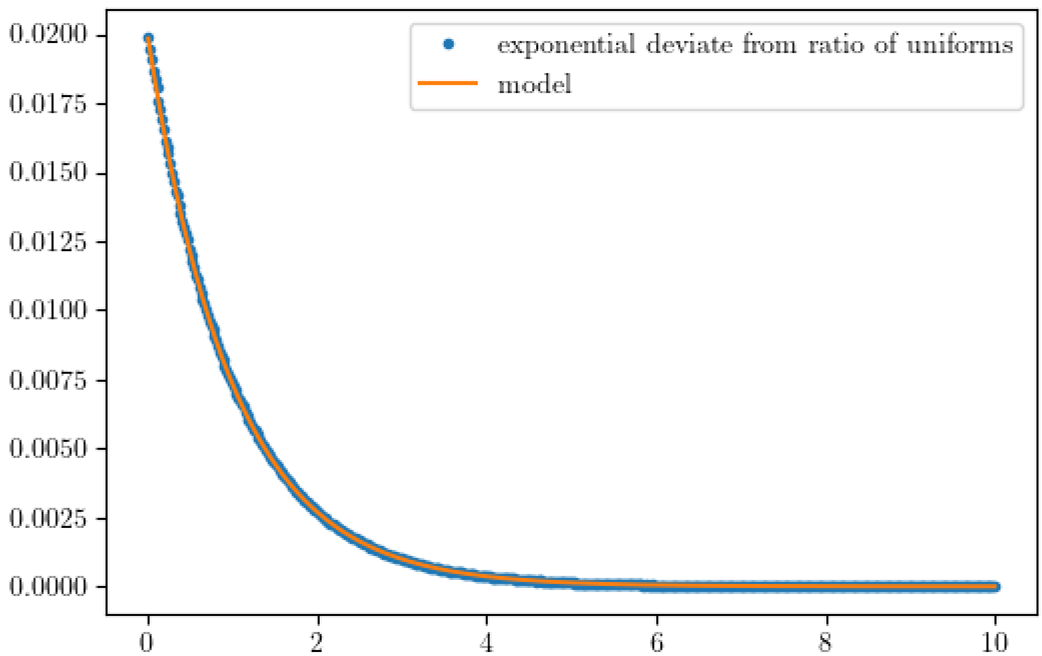
So the upper bound on v is $-2(e^{-1})\ln(e^{-1}) = 2e^{-1}$.

Since the lower bound is 0:

$$0 < v < 2e^{-1}.$$

In [ ]:
```
n=10000000 # number of random numbers

u=np.random.rand(n) # random numbers from 0 to 1
v=(np.random.rand(n))*2*np.exp(-1) # random numbers from 0 to e^-1

# ratio of uniforms
a=u<np.sqrt(np.exp(-v/u))
use=v[a]/u[a]

# effeciency (how many ratios we actually use)
eff=np.size(use)/np.size(a)*100
```

```python
# plotting stuff
a,b=np.histogram(use,np.linspace(0,10,500))
a=a/a.sum()
bb=0.5*(b[1:]+b[:-1]) # get center of bins
pred=model(bb)
pred=pred/pred.sum()

plt.plot(bb,a,'.', label='exponential deviate from ratio of uniforms')
plt.plot(bb,pred,label='model')
plt.legend()
plt.show()

print('Effeciency of generator is: ',eff)
```




```
Effeciency of generator is:  67.96546
```

Not bad!!