

Y-Intercept-Test

1 Data Processing

In the given data file there is a csv file with values of prices and volumes for different shares. There are 248 different shares. Data for each share are given at different dates, on 2994 different days. The first date 2013-01-04. To simplify we suppose that the volume and the price of a share at a date t_1 are equal to the last volume and last price recorded before t_1 . By this process we get data for all tickers over the 2994 days.

In the file, the function `plotData(int i)` displays the price and volume over the 2994 days of the i th share. The list `last` contains 248 lists containing the different prices for the 248 shares. The same for the list `volume`.

2 Naive Strategy

2.1 Strategy for a Stock Composed by One Share

To start, we try to implement a strategy to decide when buy and sell a unique share according to the evolution of its price and its volume. In this strategy every day we take into account market values over the previous T days, with $T > 0$. Let's consider a single share.

The strategy is the following :

1. If the volume is lower than its average value of the previous T days, we don't do anything. We consider that price moves made on low volume may be said to "lack conviction" and are viewed as being less predictive of future returns.
2. Else
 - (a) If the price is lower than its average value of the previous T days, we sell shares in a proportion p
 - (b) Else we buy new shares

In the file `strategy.py` we set up the model. In the first function `SimpleStrategy(tick, x, dayConsider, buy, propSale, end)` we implement the strategy. According to which share the strategy is applied, results are variable. Two different examples are illustrated in the file `comparaison` : one with positive results, the other with negative results.

2.2 Strategy for a Stock Composed by Several Shares

We assume that the price of shares are decorated (which is clearly not true), and we do not favor any action over another. We apply the previous strategy simultaneously to each share. In the file `strategy.py` we set up the model. In the function `PortFolioSS(initInvest, dayConsider, propSale, end)` we implemented the strategy. We run `PortFolioSS(10000, 3, 1/2, 1000)` in the file `comparaison.py` and the strategy seems to work according to the graph.

3 Black and Scholes Model

3.1 The model

In this strategy we use the Black and Scholes model. Let $(S_t)_{t \in \mathbb{R}_+}$ the price of a share at the time $t \in \mathbb{R}$. Assume that there exist $(\mu, \sigma) \in \mathbb{R} \times \mathbb{R}_+$ such that

$$S_t = s_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t - \sigma W_t\right)$$

with s_0 the initial price at time $t=0$, and (W_t) a brownian motion.

The expected value is $\mathbb{E}(S_t) = s_0 \times e^{\mu t}$. So if μ is non negative, the model predicts that the price will tend to rise. So we should invest.

Then in this strategy we try to obtain the value of μ to decide if we should buy or sell shares.

Let $t_0 = 0 < t_1 < \dots < t_N$ a sequence of time such that for all i $t_{i+1} - t_i = \Delta t$. Let l_i the real price observed at time t_i . We know that

$$\Delta S_i = \log(S_{t_{i+1}}) - \log(S_{t_i}) \stackrel{Law}{=} \mathcal{N}\left(\left(\mu - \frac{\sigma^2}{2}\right) \times \Delta t, \sigma^2 \times \Delta t\right)$$

(Normal law of expected value $(\mu - \frac{\sigma^2}{2}) \times \Delta t$ and variance $\sigma^2 \Delta t$)

At the i -th time t_i we introduce

$$\Delta l_i = \log(l_{i+1}) - \log(l_i)$$

$$\Delta \bar{S}_i = \frac{1}{i} \sum_{j=0}^{i-1} \Delta l_j$$

$$V_i = \frac{1}{i} \sum_{j=0}^{i-1} (\Delta l_j - \Delta \bar{S}_i)^2$$

By the law of large number we know that $\Delta \bar{S}_i$ and V_i are estimators of $(\mu - \frac{\sigma^2}{2}) \times \Delta t$ and $\sigma^2 \times \Delta t$. So we approximating μ by $\frac{\Delta \bar{S}_i + \frac{V_i}{2\Delta t}}{\Delta t}$. Notice that the more time passes the more our model is faithful because we have more and more data.

3.2 Strategy for a Stock Composed by One Share

Now that we know how approximate μ , we have to erect a strategy : when to buy and when to sell ?

Assume that we take a move every time $(t_{T \times i})$ with $T \in \mathbb{R}_+$. Let $\bar{l}_i = \frac{1}{i} \sum_{j=0}^{i-1} l_j$ the empirical mean of the prices observed at time t_i . Let $p \in (0, 1)$ the proportion of sale, $n_s \in \mathbb{R}_+$ an amount of money to invest at each move. We adopt the following strategy at each time $(t_{T \times i})$:

1. If $\Delta \bar{S}_{T \times i} > 0$ we buy shares for n_s dollars
2. Else if $\Delta \bar{S}_{T \times i} < 0$ We sell in p proportion our shares.

In the file BlackScholes.py we set up the model. In the first function BlackScholes(tick,x,begin,end,moveTime,buy,propSell) we implement the strategy. We run two different examples in comparison.py. On these examples the strategy seems to work according to graphs.

One of the possible improvements would be to take volume into account.

3.3 Strategy for a Stock Composed by several shares

We assume that the price of shares are decorated (which is clearly not true), and we do not favor any share over another. We apply the previous strategy simultaneously to each share. In the file `BlackScholes.py` we set up the model. In the function `PortFolioBS(initInvest,begin,end,mooveTime,buy,propSell)` we implement the strategy. We run an example in `comparaison.jl`, and the strategy seems to work according to the graph.

3.4 Defaults of the Strategy

This model presents a lot of defaults. Firstly it doesn't consider the volume. It does not react to the sudden movement of the market. Next it does not take into account the correlation between stocks. So the strategy is risky because we take the risk of having an undiversified portfolio. However this risk is reduced by the fact that we invest uniformly in all stocks.

The comparaison made in the file `comparaison.file` prove that our strategy using the model of Black and Scholes is better than our naive strategy.