Accumulators

- accumulate information from prior recursive calls
- three categories
    - preserve context from prior recursive calls
    - result so far
    - worklist

lecture 18 (previous)

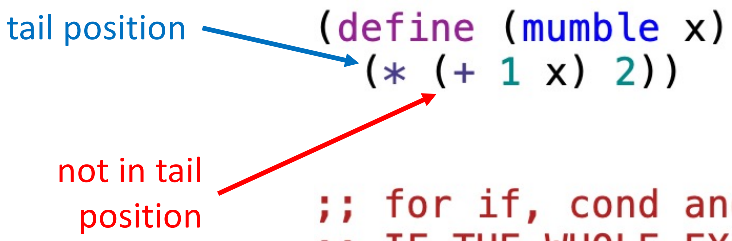
this lecture

Roadmap

- The next four lectures
  - forms of data: trees and graphs
  - recursion: structural non-tail and tail and generative
  - accumulators
    - path in data: previous, upper, lower, pnum, path, visited…
    - rsf (result so far)
    - path in tail recursion: visited, count, …
    - worklist
    - tandem worklist l18 l19 l20 l21

```
(define (sequence? lon0)
  ;; prev is  Natural
  ;; invariant: the element of lon0 immediately before (first lon)
  ;; (sequence? (list 2 3 4 7 5))

  ;; (sequence? (list   3 4 7 5) 2)
  ;; (sequence? (list     4 7 5) 3)
  ;; (sequence? (list       7 5) 4) ==> false
  (local [(define (sequence? lon prev)
            (cond [(empty? lon) true]
                  [else
                   (if (= (first lon) (+ 1 prev)) ;exploit (use)
                       (sequence? (rest lon)
                                  (first lon))    ;preserve
                       false)]))]

    (if (empty? lon0)                ;if original list is empty, we can't
        true                         ;initialize accumulator, so special case
        (sequence? (rest lon0)
                   (first lon0)))))  ;initialize
```

an expression is in tail-position when no surrounding expression in that function is waiting to operate on the value of the expression



tail position

not in tail position

```
(define (mumble x)
  (* (+ 1 x) 2))
```

```
;; for if, cond and local expressions,
;; IF THE WHOLE EXPRESSION IS IN TAIL POSITION,
;; then
;;  questions are never in tail position
;;  answers are in tail position
;;  body is in tail position

(if <question>
    <true answer>
    <false answer>)

(cond [<question> <answer>]
      ...)

(local [<definition> ...]
  <body>)
```

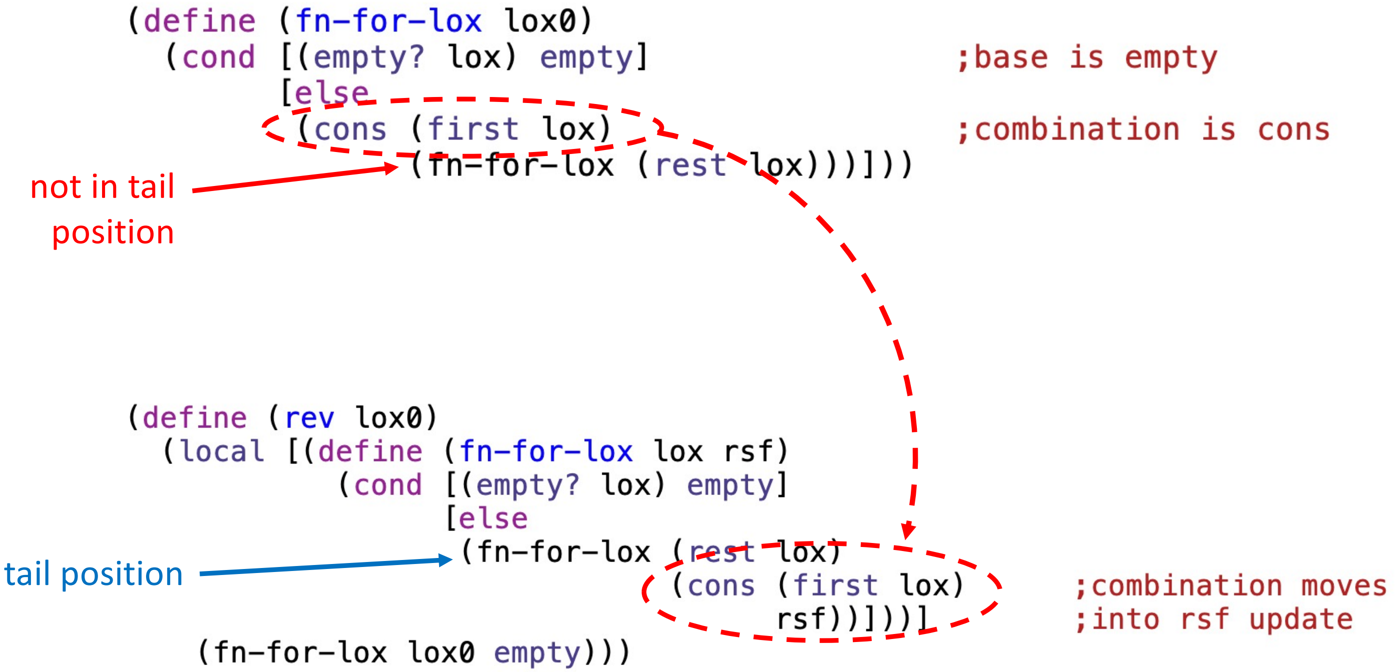
definitions

- an expression is in tail-position when no surrounding expression in that function is waiting to operate on the value of the expression

- a function is tail-recursive when ALL recursive calls are in tail position

- mutually-recursive functions are tail recursive when ALL recursive and mutually recursive calls are in tail position

- in tail recursive functions, base cases produce the final result



```
(define (fn-for-lox lox0)
  (cond [(empty? lox) empty]                    ;base is empty
        [else
         (cons (first lox)                      ;combination is cons
               (fn-for-lox (rest lox)))]))
```

not in tail position

```
(define (rev lox0)
  (local [(define (fn-for-lox lox rsf)
            (cond [(empty? lox) empty]
                  [else
                   (fn-for-lox (rest lox)
                               (cons (first lox)
                                     rsf))]))]          ;combination moves
                                                        ;into rsf update
    (fn-for-lox lox0 empty)))
```

tail position

```
(define (rev lox0)


  (local [(define (fn-for-lox lox)
            (cond [(empty? lox) (... )]
                  [else
                   (... (first lox)
                        (fn-for-lox (rest lox)))]))]

    (fn-for-lox lox0)))
```

produce rsf at end

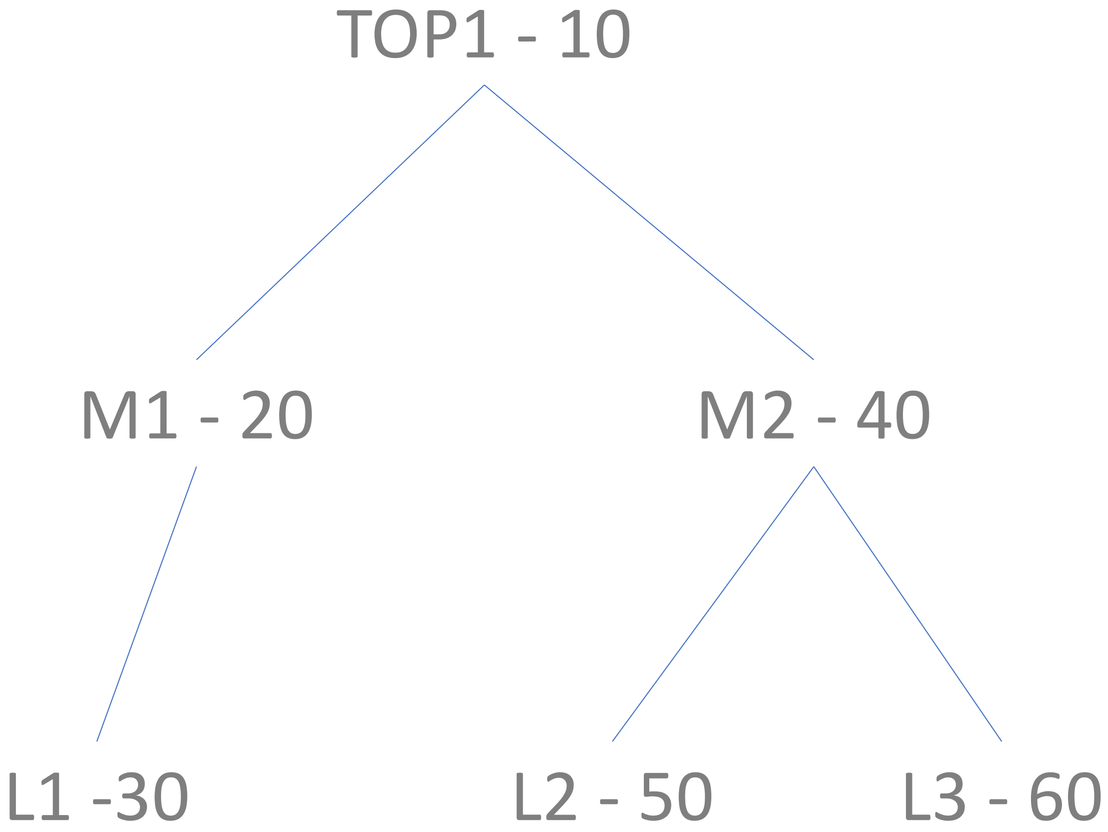
```
(define (rev lox0)
  ;; rsf is (listof X)
  ;; all elements of lox0 before (first lox), in reverse order
  (local [(define (fn-for-lox lox rsf)
            (cond [(empty? lox) rsf]
                  [else
                   (fn-for-lox (rest lox) (cons (first lox) rsf))]))]

    (fn-for-lox lox0 empty)))
```

combination

initialize rsf (base case result)

```
;; QUESTION 1 [45 seconds]
;;
;; Is the call to positive? in tail position?

(define (positive-only lon)
  (cond [(empty? lon) empty]
        [else
         (if (positive? (first lon))
             (cons (first lon)
                   (positive-only (rest lon)))
             (positive-only (rest lon)))]))



;; A. Yes
;; B. No
;; QUESTION 2 [30 seconds]
;;
;; Is the recursive call to positive-only labeled (1) in tail position?

(define (positive-only lon)
  (cond [(empty? lon) empty]
        [else
         (if (positive? (first lon))
             (cons (first lon)
                   (positive-only (rest lon))) ;(1)
             (positive-only (rest lon)))]))    ;(2)


;; A. Yes
;; B. No
;; QUESTION 3 [20 seconds]
;;
;; Is the recursive call to positive-only labeled (2) in tail position?

(define (positive-only lon)
  (cond [(empty? lon) empty]
        [else
         (if (positive? (first lon))
             (cons (first lon)
                   (positive-only (rest lon))) ;(1)
             (positive-only (rest lon)))]))    ;(2)

;; A. Yes
;; B. No
;; QUESTION 4 [40 seconds]
;;
;; Is positive-only tail-recursive?

(define (positive-only lon)
  (cond [(empty? lon) empty]
        [else
         (if (positive? (first lon))
             (cons (first lon)
                   (positive-only (rest lon)))
             (positive-only (rest lon)))]))

;; A. Yes
;; B. No
;; QUESTION 5
;;
;; Is positive-only tail-recursive? [40 seconds]

(define (positive-only lon0)
  (local [(define (positive-only lon rsf)
            (cond [(empty? lon) (reverse rsf)]
                  [else
                   (if (positive? (first lon))
                       (positive-only (rest lon) (cons (first lon) rsf))
                       (positive-only (rest lon) rsf))]))]

    (positive-only lon0 empty)))

;; A. Yes
;; B. No
```

Example Tree



TOP1 - 10

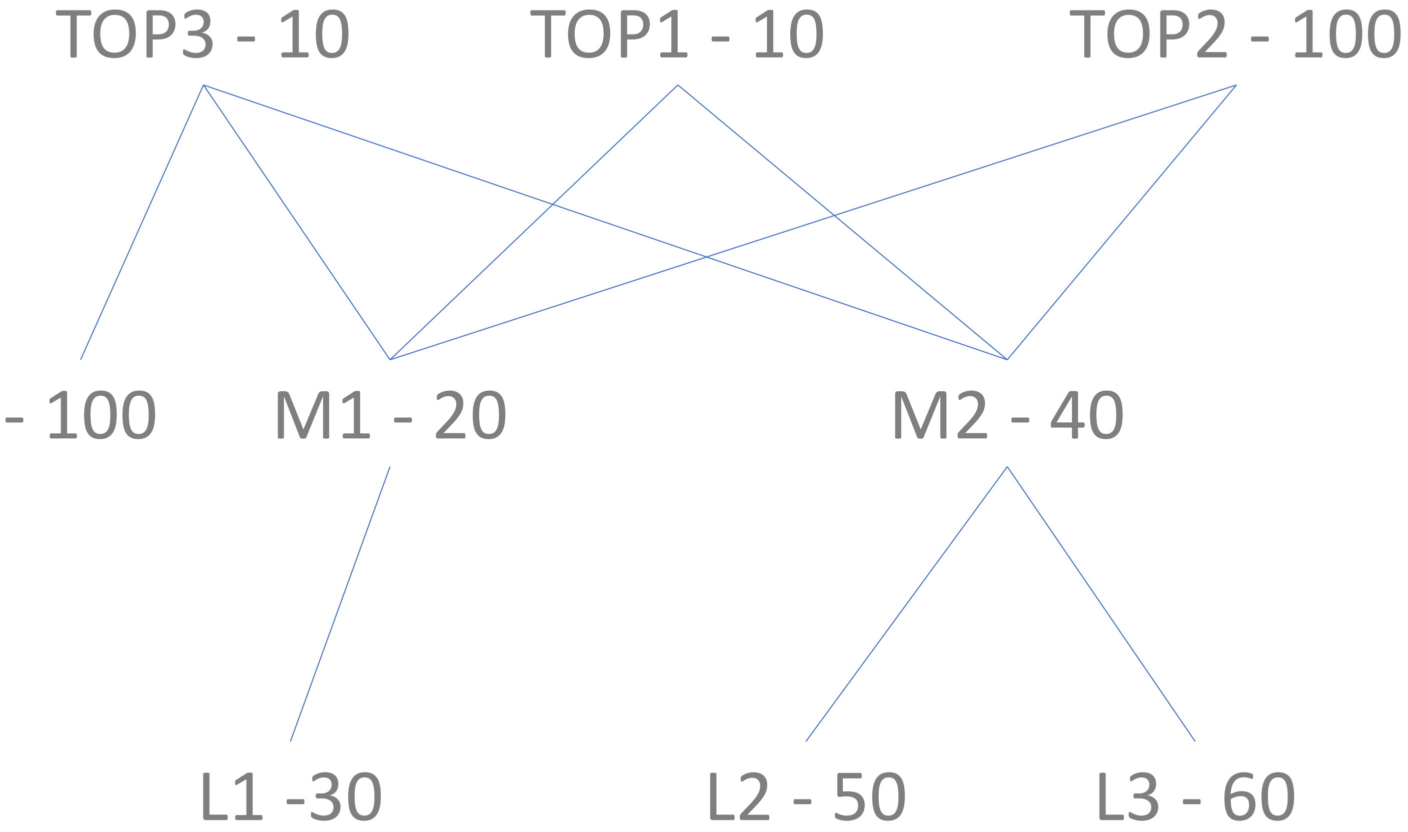
M1 - 20

M2 - 40

L1 -30

L2 - 50

L3 - 60



TOP3 - 10

TOP1 - 10

TOP2 - 100

- 100

M1 - 20

M2 - 40

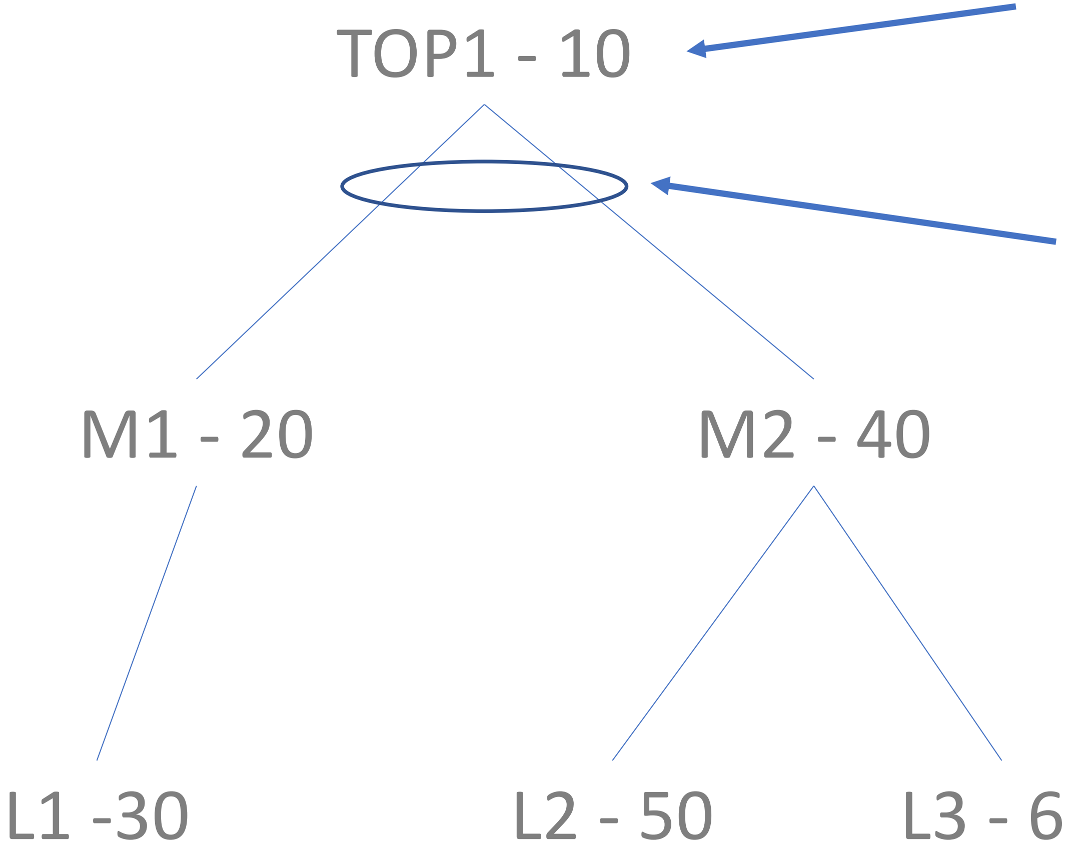
L1 -30

L2 - 50

L3 - 60

2 more
Example Trees

NOTATION



TOP1 - 10

M1 - 20

M2 - 40

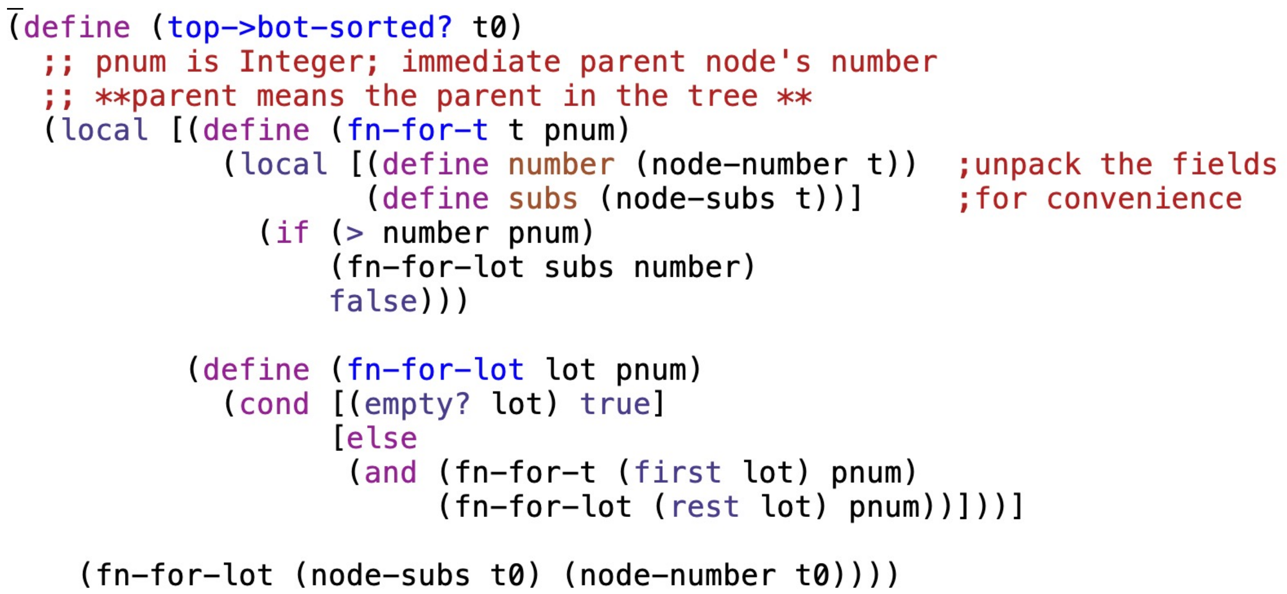
L1 -30

L2 - 50

L3 - 60

fn-for-t is
called with 1 tree

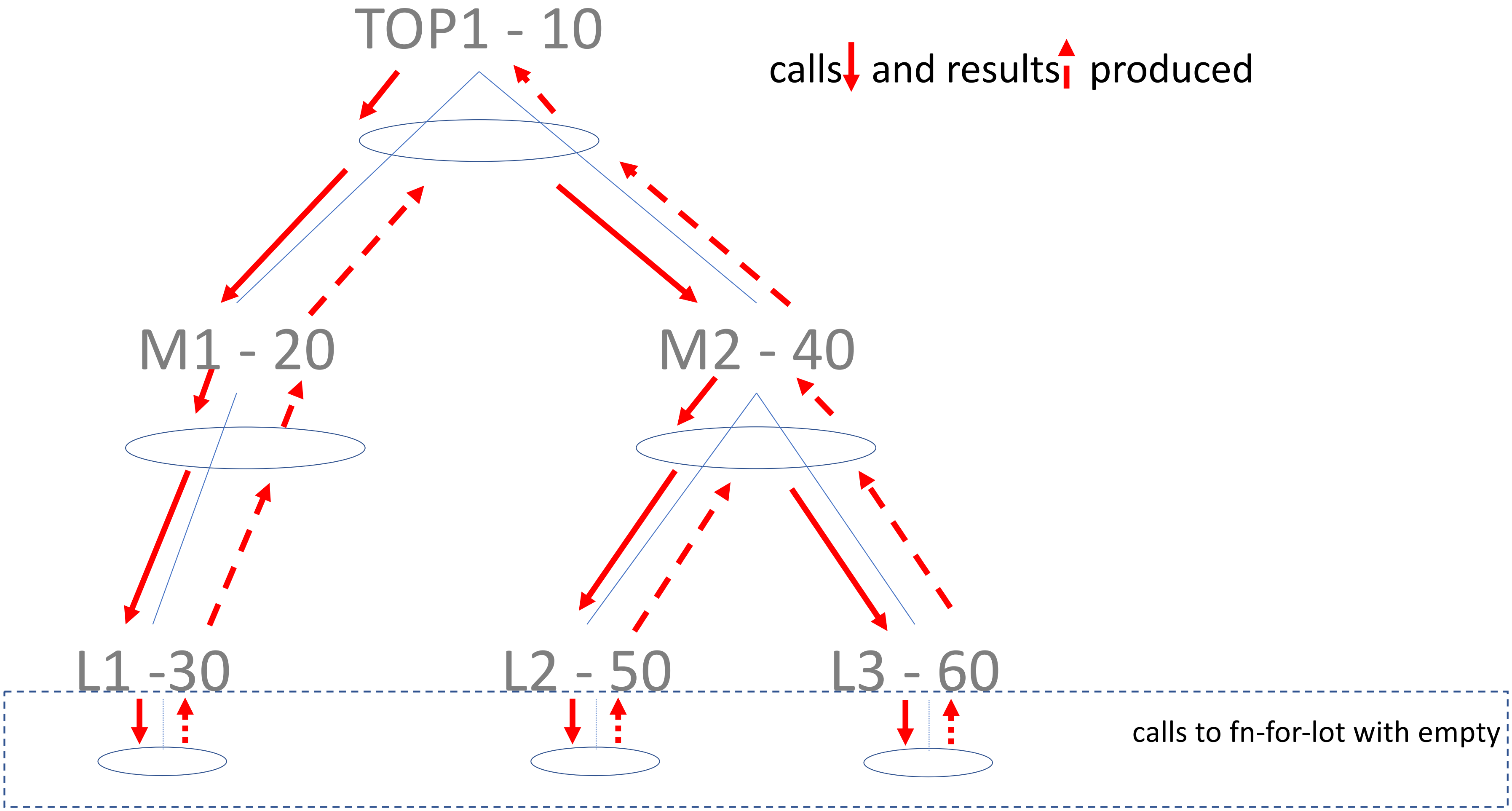
fn-for-lot is
called with list



```
(define (top->bot-sorted? t0)
  ;; pnum is Integer; immediate parent node's number
  ;; **parent means the parent in the tree **
  (local [(define (fn-for-t t pnum)
            (local [(define number (node-number t))  ;unpack the fields
                    (define subs (node-subs t))]     ;for convenience
              (if (> number pnum)
                  (fn-for-lot subs number)
                  false)))

          (define (fn-for-lot lot pnum)
            (cond [(empty? lot) true]
                  [else
                   (and (fn-for-t (first lot) pnum)
                        (fn-for-lot (rest lot) pnum))]))]

    (fn-for-lot (node-subs t0) (node-number t0))))
```

typical structural recursion

calls ↓ and results ↑ produced



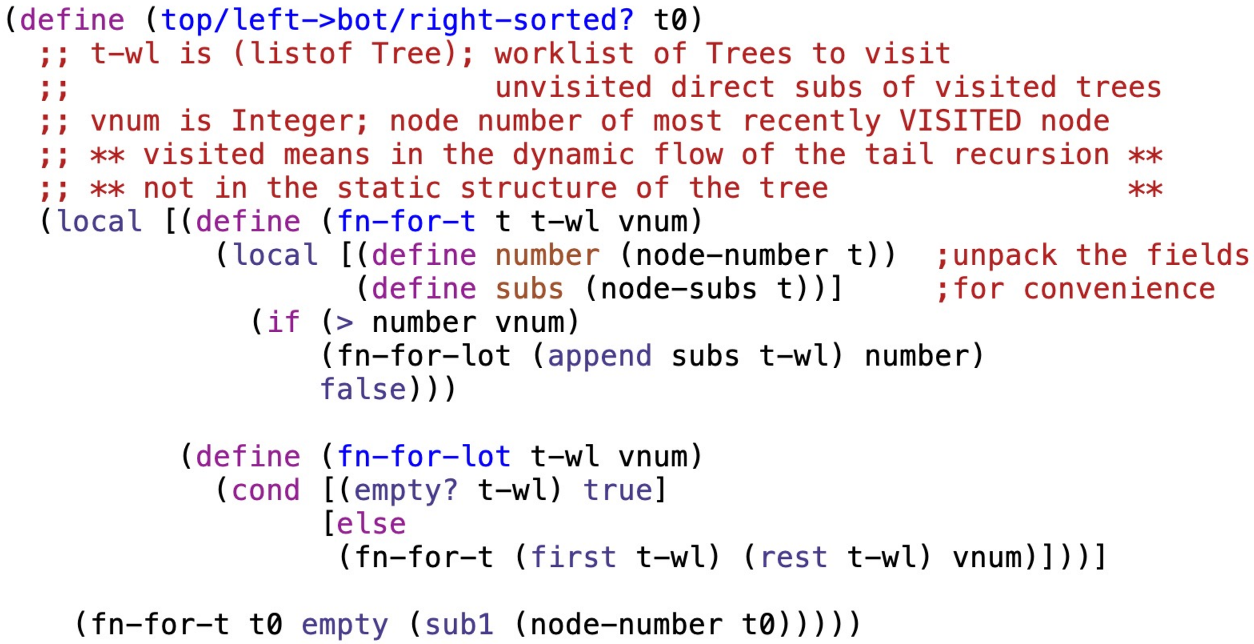
calls to fn-for-lot with empty

What’s in the past depends on the recursion

- tail recursion means current call can have all the preceding context

  → it can produce answer directly

- in a tree
  - ordinary recursion can carry context of what is above current call
  - but tail recursion is required to carry context of what is above and to the LEFT

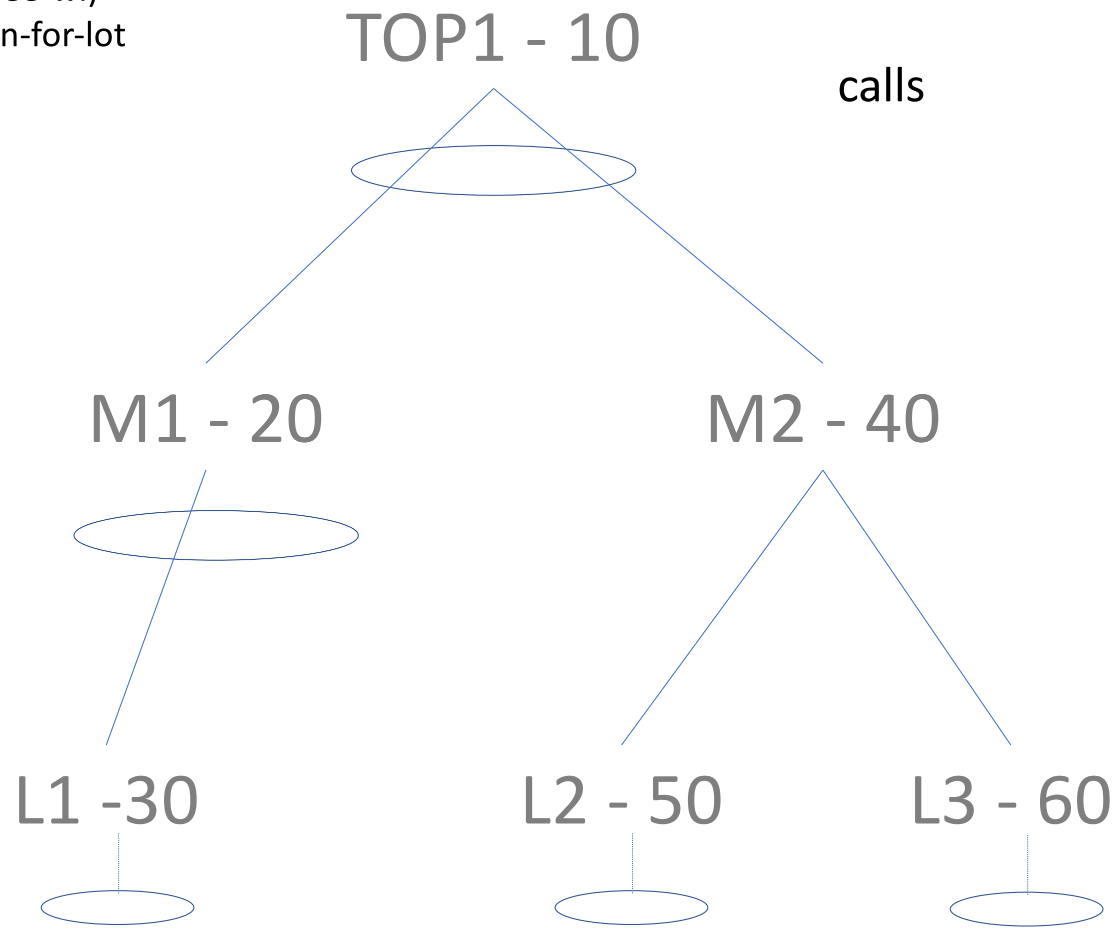


```
(define (top/left->bot/right-sorted? t0)
  ;; t-wl is (listof Tree); worklist of Trees to visit
  ;;                        unvisited direct subs of visited trees
  ;; vnum is Integer; node number of most recently VISITED node
  ;; ** visited means in the dynamic flow of the tail recursion **
  ;; ** not in the static structure of the tree                **
  (local [(define (fn-for-t t t-wl vnum)
            (local [(define number (node-number t))  ;unpack the fields
                    (define subs (node-subs t))]     ;for convenience
              (if (> number vnum)
                  (fn-for-lot (append subs t-wl) number)
                  false)))

          (define (fn-for-lot t-wl vnum)
            (cond [(empty? t-wl) true]
                  [else
                   (fn-for-t (first t-wl) (rest t-wl) vnum)]))]

    (fn-for-t t0 empty (sub1 (node-number t0)))))
```

tree worklist (tree-wl)
at fn-for-t and fn-for-lot

tail recursion with worklist

calls


TOP1 - 10
M1 - 20
M2 - 40
L1 -30
L2 - 50
L3 - 60