AUTOCORRELATION ANALYSIS

Durbin-Watson Test, Cochrane Orcutt Test, others

Focused on autocorrelation among error terms and how to analyze and solve it

In an OLS model multiple performance problems could happen, we have focused about autocorrelation between errors.

Gregorio Bufalari - Nevro Cerasani

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Preface:

In an OLS model multiple performance problems can happen, we have focused on the problem of autocorrelation between errors.
We started from Durbin-Watson test which confirms the presence of autocorrelation, and then we performed the Cochrane-Orcutt test which eliminates the autocorrelation in residuals. We used also other Econometrics frameworks.

1. Linear Regression Model :

$$y_t = \beta_0 + \beta_t x_t + \ldots .. + \beta_{t-n} x_{t-n} + \varepsilon_t \quad (1.1)$$

- β_0: intercept
- β_t : slope
- ε_t : error term
- X_t : Indipendent Variable
- Y_t: Dependent Variable

One of the most important assumptions in regression is that the error terms are i.i.d (independent and identically distributed) but sometimes some violations could happen when performing the model.

In particular we focused on one kind of performance issue: autocorrelation between errors, which is a relevant problem because the assumption written above is violated. The equation 1.2 shows the linear combination among error terms in a Linear Regression Model.

$$\varepsilon_t = \rho\varepsilon_{t-1} + \omega_t \quad (1.2)$$

- $\varepsilon_t = F(\varepsilon_{t-1})$ the error term at time (t) is function of the previous (t-1)

$\boldsymbol{\rho}$ = correlation coefficient $\quad |\rho| < 1$

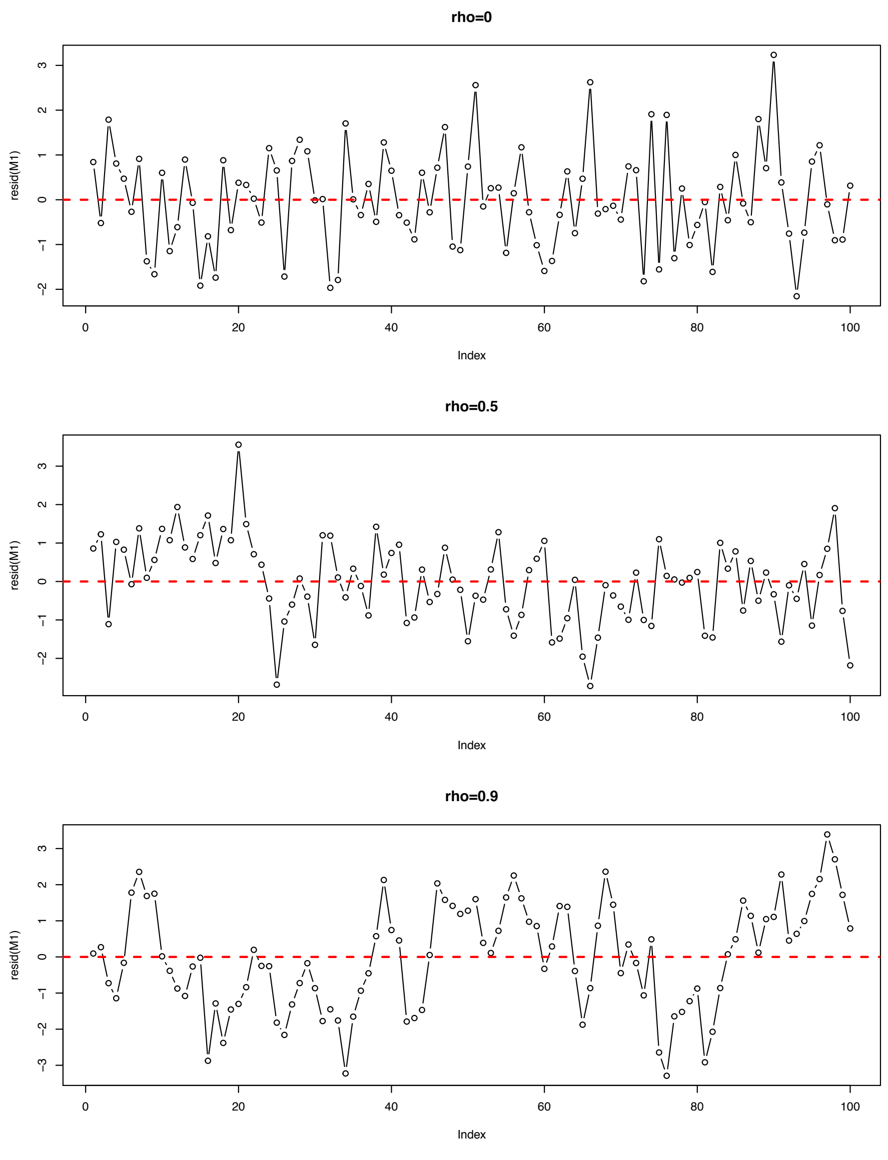
If $\boldsymbol{\rho} = 0$

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\omega}_t \text{ no correlation}$$

If $\boldsymbol{\rho} \neq 0$

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\rho}\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\omega}_t \text{ correlation}$$

As we can see in the graph an increase of rho generates an evident correlation among residuals.



1. Durbin-Watson Test

The Durbin Watson Test (DW) is a test statistic to verify whether there is or not autocorrelation among residuals from a regression analysis.

$$DW = \frac{\sum_{t=2}^{T}((e_t - e_{t-1})^2)}{\sum_{t=1}^{T} e_t^2} \qquad (2.1)$$

The equation 2.1 is the test statistic for performing the Durbin-Watson Test. As we can see it is a ratio: the squared sum of the difference between error at time t and the previous over the sum of the errors at time t squared. The test is based on two hypothesis:

- H_o : $\boldsymbol{\rho} = 0$ (no correlation)
- H_1 : $\boldsymbol{\rho} \neq 0$ (autocorrelation, see equation 1.2)

The result will demonstrate which hypothesis we will accept and so if there is or not autocorrelation in the model. For the analysis there are three important rules: the first is based on p-value assumption, the second confirms the result within the Durbin Watson table and the third compares the result with a value from 0 to 4.

First rule: if the p-value < 5% we reject H_o , in the other cases we don't reject H_o.

Second rule: DW < DW.L result lower than the lower level: we reject H_0.

DW > DW.U result greater than the upper level: we don't reject H_0.

DW.L < DW < DW.U: the test is inconclusive.

Third rule: 2 is no autocorrelation

0 to 2 is positive autocorrelation

2 to 4 is negative autocorrelation.

Durbin Watson Table

Durbin-Watson Statistic: 1 Per Cent Significance Points of dL and dU

	k'*=1		k'=2		k'=3		k'=4		k'=5		k'=6		k'=7		k'=8		k'=9		k'=10	
n	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU
6	0.390	1.142	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----
7	0.435	1.036	0.294	1.676	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----
8	0.497	1.003	0.345	1.489	0.229	2.102	----	----	----	----	----	----	----	----	----	----	----	----	----	----
9	0.554	0.998	0.408	1.389	0.279	1.875	0.183	2.433	----	----	----	----	----	----	----	----	----	----	----	----
10	0.604	1.001	0.466	1.333	0.340	1.733	0.230	2.193	0.150	2.690	----	----	----	----	----	----	----	----	----	----
11	0.653	1.010	0.519	1.297	0.396	1.640	0.286	2.030	0.193	2.453	0.124	2.892	----	----	----	----	----	----	----	----
12	0.697	1.023	0.569	1.274	0.449	1.575	0.339	1.913	0.244	2.280	0.164	2.665	0.105	3.053	----	----	----	----	----	----
13	0.738	1.038	0.616	1.261	0.499	1.526	0.391	1.826	0.294	2.150	0.211	2.490	0.140	2.838	0.090	3.182	----	----	----	----
14	0.776	1.054	0.660	1.254	0.547	1.490	0.441	1.757	0.343	2.049	0.257	2.354	0.183	2.667	0.122	2.981	0.078	3.287	----	----
15	0.811	1.070	0.700	1.252	0.591	1.465	0.487	1.705	0.390	1.967	0.303	2.244	0.226	2.530	0.161	2.817	0.107	3.101	0.068	3.374
16	0.844	1.086	0.738	1.253	0.633	1.447	0.532	1.664	0.437	1.901	0.349	2.153	0.269	2.416	0.200	2.681	0.142	2.944	0.094	3.201
17	0.873	1.102	0.773	1.255	0.672	1.432	0.574	1.631	0.481	1.847	0.393	2.078	0.313	2.319	0.241	2.566	0.179	2.811	0.127	3.053
18	0.902	1.118	0.805	1.259	0.708	1.422	0.614	1.604	0.522	1.803	0.435	2.015	0.355	2.238	0.282	2.467	0.216	2.697	0.160	2.925
19	0.928	1.133	0.835	1.264	0.742	1.416	0.650	1.583	0.561	1.767	0.476	1.963	0.396	2.169	0.322	2.381	0.255	2.597	0.196	2.813
20	0.952	1.147	0.862	1.270	0.774	1.410	0.684	1.567	0.598	1.736	0.515	1.918	0.436	2.110	0.362	2.308	0.294	2.510	0.232	2.174
21	0.975	1.161	0.889	1.276	0.803	1.408	0.718	1.554	0.634	1.712	0.552	1.881	0.474	2.059	0.400	2.244	0.331	2.434	0.268	2.625
22	0.997	1.174	0.915	1.284	0.832	1.407	0.748	1.543	0.666	1.691	0.587	1.849	0.510	2.015	0.437	2.188	0.368	2.367	0.304	2.548
23	1.017	1.186	0.938	1.290	0.858	1.407	0.777	1.535	0.699	1.674	0.620	1.821	0.545	1.977	0.473	2.140	0.404	2.308	0.340	2.479
24	1.037	1.199	0.959	1.298	0.881	1.407	0.805	1.527	0.728	1.659	0.652	1.797	0.578	1.944	0.507	2.097	0.439	2.255	0.375	2.417
25	1.055	1.210	0.981	1.305	0.906	1.408	0.832	1.521	0.756	1.645	0.682	1.776	0.610	1.915	0.540	2.059	0.473	2.209	0.409	2.362
26	1.072	1.222	1.000	1.311	0.928	1.410	0.855	1.517	0.782	1.635	0.711	1.759	0.640	1.889	0.572	2.026	0.505	2.168	0.441	2.313
27	1.088	1.232	1.019	1.318	0.948	1.413	0.878	1.514	0.808	1.625	0.738	1.743	0.669	1.867	0.602	1.997	0.536	2.131	0.473	2.269
28	1.104	1.244	1.036	1.325	0.969	1.414	0.901	1.512	0.832	1.618	0.764	1.729	0.696	1.847	0.630	1.970	0.566	2.098	0.504	2.229
29	1.119	1.254	1.053	1.332	0.988	1.418	0.921	1.511	0.855	1.611	0.788	1.718	0.723	1.830	0.658	1.947	0.595	2.068	0.533	2.193
30	1.134	1.264	1.070	1.339	1.006	1.421	0.941	1.510	0.877	1.606	0.812	1.707	0.748	1.814	0.684	1.925	0.622	2.041	0.562	2.160
31	1.147	1.274	1.085	1.345	1.022	1.425	0.960	1.509	0.897	1.601	0.834	1.698	0.772	1.800	0.710	1.906	0.649	2.017	0.589	2.131
32	1.160	1.283	1.100	1.351	1.039	1.428	0.978	1.509	0.917	1.597	0.856	1.690	0.794	1.788	0.734	1.889	0.674	1.995	0.615	2.104
33	1.171	1.291	1.114	1.358	1.055	1.432	0.995	1.510	0.935	1.594	0.876	1.683	0.816	1.776	0.757	1.874	0.698	1.975	0.641	2.080
34	1.184	1.298	1.128	1.364	1.070	1.436	1.012	1.511	0.954	1.591	0.896	1.677	0.837	1.766	0.779	1.860	0.722	1.957	0.665	2.057
35	1.195	1.307	1.141	1.370	1.085	1.439	1.028	1.512	0.971	1.589	0.914	1.671	0.857	1.757	0.800	1.847	0.744	1.940	0.689	2.037
36	1.205	1.315	1.153	1.376	1.098	1.442	1.043	1.513	0.987	1.587	0.932	1.666	0.877	1.749	0.821	1.836	0.766	1.925	0.711	2.018
37	1.217	1.322	1.164	1.383	1.112	1.446	1.058	1.514	1.004	1.585	0.950	1.662	0.895	1.742	0.841	1.825	0.787	1.911	0.733	2.001
38	1.227	1.330	1.176	1.388	1.124	1.449	1.072	1.515	1.019	1.584	0.966	1.658	0.913	1.735	0.860	1.816	0.807	1.899	0.754	1.985
39	1.237	1.337	1.187	1.392	1.137	1.452	1.085	1.517	1.033	1.583	0.982	1.655	0.930	1.729	0.878	1.807	0.826	1.887	0.774	1.970
40	1.246	1.344	1.197	1.398	1.149	1.456	1.098	1.518	1.047	1.583	0.997	1.652	0.946	1.724	0.895	1.799	0.844	1.876	0.749	1.956
45	1.288	1.376	1.245	1.424	1.201	1.474	1.156	1.528	1.111	1.583	1.065	1.643	1.019	1.704	0.974	1.768	0.927	1.834	0.881	1.902
50	1.324	1.403	1.285	1.445	1.245	1.491	1.206	1.537	1.164	1.587	1.123	1.639	1.081	1.692	1.039	1.748	0.997	1.805	0.955	1.864
55	1.356	1.428	1.320	1.466	1.284	1.505	1.246	1.548	1.209	1.592	1.172	1.638	1.134	1.685	1.095	1.734	1.057	1.785	1.018	1.837
60	1.382	1.449	1.351	1.484	1.317	1.520	1.283	1.559	1.248	1.598	1.214	1.639	1.179	1.682	1.144	1.726	1.108	1.771	1.072	1.817
65	1.407	1.467	1.377	1.500	1.346	1.534	1.314	1.568	1.283	1.604	1.251	1.642	1.218	1.680	1.186	1.720	1.153	1.761	1.120	1.802
70	1.429	1.485	1.400	1.514	1.372	1.546	1.343	1.577	1.313	1.611	1.283	1.645	1.253	1.680	1.223	1.716	1.192	1.754	1.162	1.792
75	1.448	1.501	1.422	1.529	1.395	1.557	1.368	1.586	1.340	1.617	1.313	1.649	1.284	1.682	1.256	1.714	1.227	1.748	1.199	1.783
80	1.465	1.514	1.440	1.541	1.416	1.568	1.390	1.595	1.364	1.624	1.338	1.653	1.312	1.683	1.285	1.714	1.259	1.745	1.232	1.777
85	1.481	1.529	1.458	1.553	1.434	1.577	1.411	1.603	1.386	1.630	1.362	1.657	1.337	1.685	1.312	1.714	1.287	1.743	1.262	1.773
90	1.496	1.541	1.474	1.563	1.452	1.587	1.429	1.611	1.406	1.636	1.383	1.661	1.360	1.687	1.336	1.714	1.312	1.741	1.288	1.769
95	1.510	1.552	1.489	1.573	1.468	1.596	1.446	1.618	1.425	1.641	1.403	1.666	1.381	1.690	1.358	1.715	1.336	1.741	1.313	1.767
100	1.522	1.562	1.502	1.582	1.482	1.604	1.461	1.625	1.441	1.647	1.421	1.670	1.400	1.693	1.378	1.717	1.357	1.741	1.335	1.765
150	1.611	1.637	1.598	1.651	1.584	1.665	1.571	1.679	1.557	1.693	1.543	1.708	1.530	1.722	1.515	1.737	1.501	1.752	1.486	1.767
200	1.664	1.684	1.653	1.693	1.643	1.704	1.633	1.715	1.623	1.725	1.613	1.735	1.603	1.746	1.592	1.757	1.582	1.768	1.571	1.779

*k' is the number of regressors excluding the intercept

(where k is the number of β_i in the linear regression)

Example:

The dataset we worked on is a time series of Tesla stock prices in the last three years, from 01-02-2017 to 01-02-2020. We chose to work on Tesla because of the recent crazy trend in its stock price. We first performed a simple linear regression to study the correlation between the closing price at time t-1 (dependent variable) and the closing price at time t (independent variable). Then we performed a Durbin-Watson test to check whether there is autocorrelation (correlation between residuals) or not. In a first step we imported the dataset in R Studio.

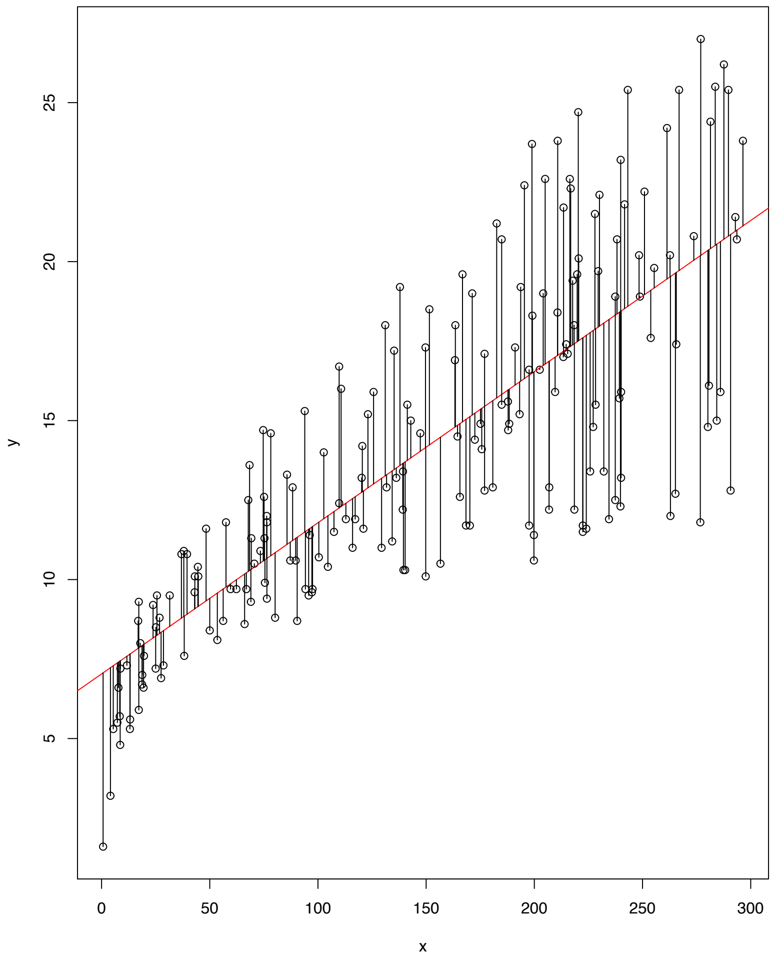
We performed a simple Linear Regression Model. The summary is the following:

```
> summary(slrmodel)

Call:
lm(formula = y ~ x)

Residuals:
    Min      1Q  Median      3Q     Max 
-45.885  -4.785  -0.231   4.786  57.108 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.955627   1.928509  -1.014    0.311    
x            1.008035   0.006132 164.402   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 9.782 on 753 degrees of freedom
Multiple R-squared:  0.9729,	Adjusted R-squared:  0.9729 
F-statistic: 2.703e+04 on 1 and 753 DF,  p-value: < 2.2e-16
```

As we can see from the summary, the resulting model is:

price(t) = -1.955627 + 1.008035 * price(t-1) + eps.

We want to study the error component of the model, therefore we perform the Durbin-Watson test.



(Linear Regression model graph with residuals)

With the command dwtest() we performed the DW-Test and analyzed the result:

Output:

```
> dwmodel

        Durbin-Watson test

data:  slrmodel
DW = 1.9766, p-value = 0.3601
alternative hypothesis: true autocorrelation is greater than 0
```

Now let's verify the three rules: the p-value is 0.3601, so it is greater than 5% and we don't reject the null hypothesis (remember that the null hypothesis confirm there is no correlation); the result of the test is 1.9766 which is close to 2 so no correlation; and finally $1.9766 > 1.684$ is greater than the upper level of the interval confidence for k=1 in the Durbin Watson table, so again it confirms no correlation between error terms.

What happens instead if there is autocorrelation in the model?

2. Cochrane-Orcutt Test

Cochrane–Orcutt estimation is a procedure in econometrics, which adjusts a linear model for serial correlation in the error term. Starting from equations (1.1) and (1.2):

$$\boldsymbol{y_t = \beta_0 + \beta_t x_t + \ldots\ldots + \beta_{t-n} x_{t-n} + \varepsilon_t}$$

$$\boldsymbol{\varepsilon_t = \rho \varepsilon_{t-1} + \omega_t}$$

The Cochrane-Orcutt Test works with variables transformations:

$$\rho Y_{t-1} = \beta_1 \rho + \beta_2 \rho X_{t-1} + \rho u_{t-1}$$

$$Y_t - \rho Y_{t-1} = \beta_1 (1-\rho) + \beta_2 X_t - \beta_2 \rho X_{t-1} + u_t - \rho u_{t-1}$$

$$Y_t = \beta_1 (1-\rho) + \rho Y_{t-1} + \beta_2 X_t - \beta_2 \rho X_{t-1} + \varepsilon_t \tag{3.1}$$

The coefficient β on x_t will be the same β as in the original regression. The intercept is now different: (1−ρ). β could be a little bit different but the estimator is still unbiased and it is effectively the same of making inference on the original β.

Example:

Now we analyze on other database: 'salespred' in which we predict house prices based on four independent variables (prediction; taxi distance; market distance; hospital distance). As usual, first we import the dataset and adjust the dataset.

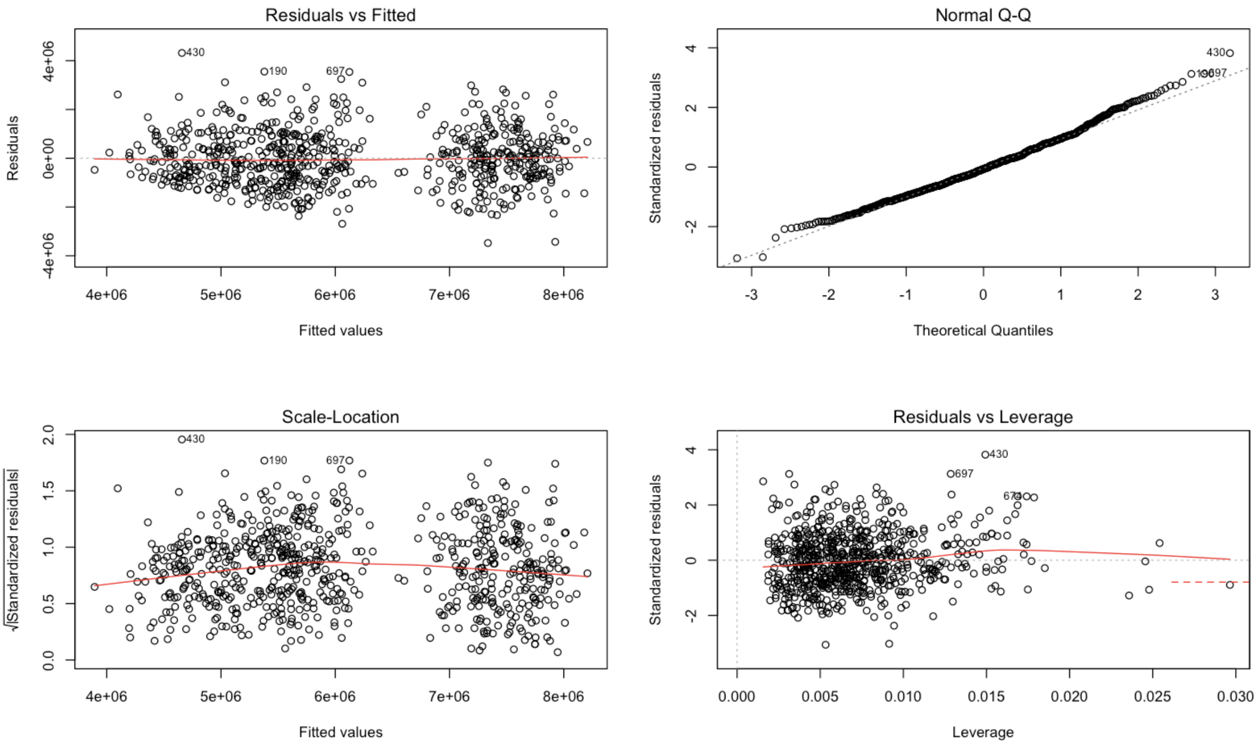
In the second step we split the dataset into training set and test set.

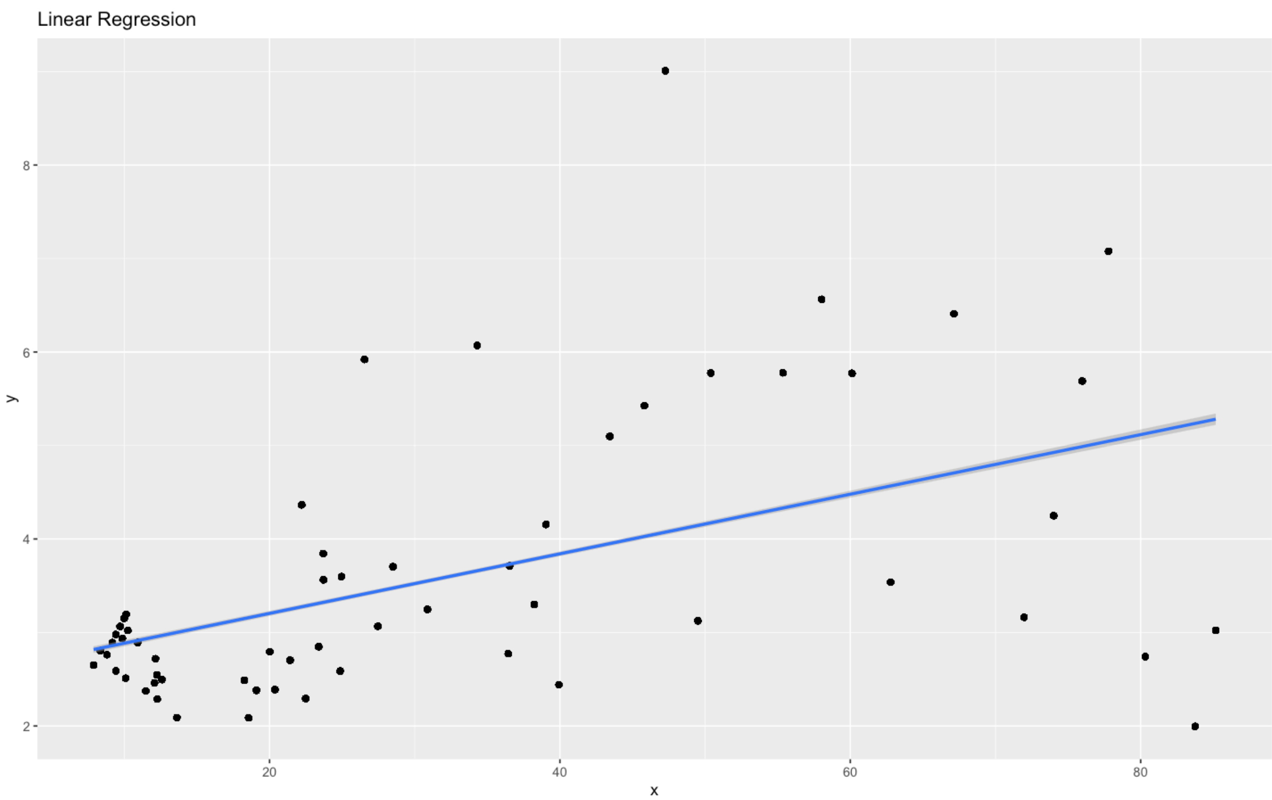
We then determined the dependent and independent variables, plot the points and performed a Linear Regression with the following code:

```
lm(Price_house~prediction+Taxi_dist+Market_dist+Hospital_dist ,tdata)
```

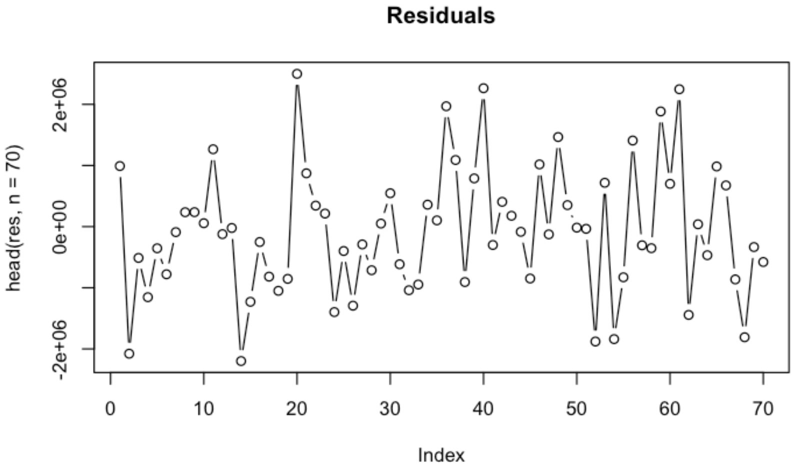
```
> summary(lm.fit)

Call:
lm(formula = Price_house ~ prediction + Taxi_dist + Market_dist +
    Hospital_dist, data = tdata)

Residuals:
     Min       1Q   Median       3Q      Max
-3475909  -784630   -78769   708477  4318118

Coefficients:
                Estimate Std. Error t value
(Intercept)   -1.129e+05  3.099e+05  -0.364
prediction     1.020e+00  3.993e-02  25.543
Taxi_dist      4.868e+01  2.793e+01   1.743
Market_dist    2.682e+01  2.206e+01   1.216
Hospital_dist -5.309e+01  3.129e+01  -1.697
              Pr(>|t|)
(Intercept)     0.7158
prediction      <2e-16 ***
Taxi_dist       0.0817 .
Market_dist     0.2245
Hospital_dist   0.0902 .
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1138000 on 693 degrees of freedom
Multiple R-squared:  0.497,	Adjusted R-squared:  0.4941
F-statistic: 171.2 on 4 and 693 DF,  p-value: < 2.2e-16
```

Prediction is the variable that most influences the model. In particular R-squared is nearly 50% so the variability is explained in a half-portion by the independent variables presented.


Residuals vs Fitted
Residuals
Fitted values
Normal Q-Q
Standardized residuals
Theoretical Quantiles
Scale-Location
√|Standardized residuals|
Fitted values
Residuals vs Leverage
Standardized residuals
Leverage


Linear Regression
y
x

Let's plot the residuals to check if there is correlation among them.



In order for us to be sure, let's run some statistical tests and look carefully at the results.

First Durbin Watson test:

```
> dwtest <- dwtest(lm.fit) #durbin-watson test
> dwtest

	Durbin-Watson test

data:  lm.fit
DW = 1.8974, p-value = 0.08728
alternative hypothesis: true autocorrelation is greater than 0
```

The p-value is greater than 5% so we don't reject the null hypothesis which confirms no correlation among residuals, but the DW value (1.8974) is not very close to 2 but 1.8974 > 1.725 which is the upper lever of the interval confidence in the Durbin-Watson table.

We can approximately say that there is no correlation in the model but the results of the DW-Test is not very precise. To improve and realize a better model in which we can clearly demonstrate no correlation we have to perform the Cochrane-Orcutt Test.

```
> orcutlm.fit <- cochrane.orcutt(lm.fit) #cochrane orcutt test
> summary(orcutlm.fit)
Call:
lm(formula = Price_house ~ prediction + Taxi_dist + Market_dist +
    Hospital_dist, data = tdata)

                 Estimate  Std. Error t value Pr(>|t|)
(Intercept)   -1.4889e+05  3.0983e+05  -0.481   0.6310
prediction     1.0196e+00  3.9826e-02  25.600   <2e-16 ***
Taxi_dist      4.6005e+01  2.7947e+01   1.646   0.1002
Market_dist    2.8708e+01  2.2030e+01   1.303   0.1930
Hospital_dist -5.0122e+01  3.1140e+01  -1.610   0.1080
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1137018 on 692 degrees of freedom
Multiple R-squared:  0.4992 ,  Adjusted R-squared:  0.4964
F-statistic: 172.5 on 4 and 692 DF,  p-value: < 2.04e-102

Durbin-Watson statistic
(original):    1.89735 , p-value: 8.728e-02
(transformed): 1.99667 , p-value: 4.858e-01> dwtest(orcutlm.fit)

        Durbin-Watson test

data:  orcutlm.fit
DW = 1.9967, p-value = 0.4858
alternative hypothesis: true autocorrelation is greater than 0
```

The output clearly says that now we do not reject the null hypothesis and we are sure about no autocorrelation. Why? Because as we saw previously the problem was the result of DW-Test: 1.89735. It was not so enough for affirming our assumption, but now, performing the Cochrane-Orcutt Test, we obtained a new value: 1.9967, which is closer to 2. In our case we had a light autocorrelation, but in other cases the Cochrane-Orcutt is able to correct also heavy autocorrelations in residuals. It's a very powerful tool.

3. Applied Econometrics

Distributed Lag Model is a model for time series data in which a regression equation is used to predict current values of a dependent variable based on both the current values of an explanatory variable and the lagged (past period) values of this explanatory variable. The coefficients β are the lags.

$$y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \ldots + \beta_K X_{t-K} + \mu_t$$

Since autocorrelation is much more common in time series data we have to resort to a time series model. In our model we will use USMacroG dataset from AER package. We performed this kind of regression on R using dynlm (Dynamic Linear Models and Time Series Regression) command with four lags.

We applied as usual the DW-Test to verify autocorrelation:

```
> durbinWatsonTest(consump.1)
 lag Autocorrelation D-W Statistic
   1       0.9244708     0.0866355
 p-value
       0
 Alternative hypothesis: rho != 0
```

```
> durbinWatsonTest(consump1, max.lag = 4)
 lag Autocorrelation D-W Statistic p-value
   1      0.74485565     0.5101068       0
   2      0.57639116     0.8467116       0
   3      0.35261994     1.2936634       0
   4      0.07197978     1.8546563       0
 Alternative hypothesis: rho[lag] != 0
```

The results of both DW-Tests demonstrate that there is autocorrelation in the model but Durbin Watson test fails to capture AC when a lagged term of the dependent variable included and so integrated the model with Breusch Godfrey Test.

a. Breusch-Godfrey Test

The Breusch–Godfrey serial correlation LM test is a test for autocorrelation in the errors in a regression model. It is useful for analyzing residuals from the model being considered in a regression analysis, and a test statistic is derived from these. The null hypothesis is that there is no serial correlation of any order. The test is more general than the Durbin–Watson statistic which is only valid for non-stochastic regressors and for testing the possibility of a first-order regression model for the regression errors. The BG test has none of these restrictions, and it is statistically more powerful than Durbin Watson statistic. It works in three steps: starting from equation (1.1) a general Linear Regression and equation (1.2) which determines correlation among errors, we test the null hypothesis for which rho (correlation coefficient) is equal to zero and so there is no correlation. If the sample size is sufficiently large, then

$$LM = nR^2 \sim \chi^2(p)$$

(apporximatly distributed)

where n is the original sample size and R^2 is the value calculated before. If p-value $< \alpha$, then the null hypothesis is rejected, and so at least one of the p_j is significantly different from zero. The command in R is bgtest(). We realized this equation:

consump.1 <- dynlm(consumption ~ dpi + L(dpi), data = USMacroG)

The result of bgtest()

```
> bgtest(consump.1)

	Breusch-Godfrey test for serial correlation of
	order up to 1

data:  consump.1
LM test = 192.97, df = 1, p-value < 2.2e-16
```

The p-value is very small so we reject the null hypothesis, so there is correlation in the model. Now comparing bgtest with dwtest so we performed another equation called consump2:

consump2 <- dynlm(consumption ~ dpi + L(dpi) + L(consumption, 2)+ L(consumption), data = USMacroG)

And then after performing bgtest and dwtest we look at the results:

```
> durbinWatsonTest(consump2, max.lag = 4)
 lag Autocorrelation D-W Statistic p-value
   1     -0.10692838      2.184723   0.228
   2      0.21865835      1.500568   0.002
   3      0.16513471      1.594958   0.002
   4     -0.05008718      1.987786   0.994
 Alternative hypothesis: rho[lag] != 0
> bgtest(consump2)

	Breusch-Godfrey test for serial correlation of
	order up to 1

data:  consump2
LM test = 11.215, df = 1, p-value = 0.0008116
```

The DW Test it totally inconclusive and chaotic because the results of the test tells us that in case of Lag1 there is not autocorrelation, in Lag2, Lag3 there is correlation but the value of DW for Lag4 is very close to 2 with a high p-value so it tells us that there is no correlation. What is the truth? Let’s analyze the result of bgtest the p-value is very small so we reject the null hp for which rho = 0 and we can confirm autocorrelation in the model.

b. Huber-White HC Robust Standard Error

In regression and time-series modelling, one of the most important assumption is that the errors or disturbances u_i have the same variance across all observation points. When this is not the case, the errors are said to be heteroscedastic, or to have heteroscedasticity, and this behaviour will be reflected in the residuals estimated from a fitted model. Heteroscedasticity-consistent (HC) standard errors are used to allow the fitting of a model that does contain heteroscedastic residuals. Even when the homogeneity of variance assumption is violated the ordinary least squares (OLS) method calculates unbiased, consistent estimates of the population regression coefficients. In this case, these estimates won't be the best linear estimates since the variances of these estimates won't necessarily be the smallest. The Huber-White robust standard errors are equal to the square root of the elements on the diagonal of the covariance matrix.

$$cov(B) = (X^T X)^{-1} X^T S X (X^T X)^{-1}$$

where the elements of S are the squared residuals from the OLS method. We call these standard errors heteroskedasticity-consistent (HC) standard errors. Heteroskedasticity just means non-constant variance. Each estimate is again the square root of the elements of the diagonal of the covariance matrix as described above, except that we use a different version of S.

$$HC0: e_i^2$$

$$HC1: \frac{n}{n-k-1} e_i^2$$

$$HC2: \frac{e_i^2}{1-h_i}$$

$$HC3: \frac{e_i^2}{(1-h_i)^2}$$

$$HC4: \frac{e_i^2}{(1-h_i)^\delta} \text{ where } \delta = min\left\{4, \frac{nh_i}{k+1}\right\}$$

Here, the h_i are the leverage values, $n =$ samples size and $k =$ number of independent variables. HC1 adjusts for degrees of freedom. HC2 reduces the bias due to points of high leverage. HC3 tends to produce superior results than HC2. HC4 is the newest approach that can be superior to HC3. We realize this test in r using the package 'sandwich' and the command coeftest().

```
> summary(consump.1)

Time series regression with "ts" data:
Start = 1950(2), End = 2000(4)

Call:
dynlm(formula = consumption ~ dpi + L(dpi), data = USMacroG)

Residuals:
    Min      1Q  Median      3Q     Max 
-190.02  -56.68    1.58   49.91  323.94 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -81.07959   14.50814  -5.589 7.43e-08 ***
dpi           0.89117    0.20625   4.321 2.45e-05 ***
L(dpi)        0.03091    0.20754   0.149    0.882    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 87.58 on 200 degrees of freedom
Multiple R-squared:  0.9964,	Adjusted R-squared:  0.9964 
F-statistic: 2.785e+04 on 2 and 200 DF,  p-value: < 2.2e-16
```

As we can see R-squared increased to 0.9964, which is a high value and an important result, because the model improved its efficiency. In our case we used HC1 which adjusts for degrees of freedom.

c. Newey West HAC Robust Standard Errors

If the error term u_t in the Lag model is serially correlated, statistical inference isn't clearly and standard errors can be strongly deceptive. Is called HAC for Heteroskedasticity and autocorrelation consistent. This Test was developed by It Whitney K. Newey and Kenneth D. West in 1987. A Newey–West estimator is used in statistics and econometrics to provide an estimate of the covariance matrix of the parameters of a regression-type model when this model is applied in situations where the standard assumptions of regression analysis aren't respected. The problem in autocorrelation, often found in time series data, is that the error terms are correlated over time. Q* is a matrix of sums of squares and cross products that involves $\sigma_{(I,j)}$ and the rows of X. the least squares residuals e_i are "point-wise" consistent estimators of their population counterparts E_i. The general approach, then, will be to use X and e to devise an estimator of Q*. This means that as the time between error terms increases, the correlation between the error terms decreases. The estimator can be used to improve the Linear Regression Model when the residuals are heteroskedastic and/or autocorrelated.

$$Q^* = \frac{1}{T}\sum_{t=1}^{T} e_t^2 x_t x_t' + \frac{1}{T}\sum_{\ell=1}^{L}\sum_{t=\ell+1}^{T} w_\ell e_t e_{t-\ell}(x_t x_{t-\ell}' + x_{t-\ell} x_t')$$

$$w_\ell = 1 - \frac{\ell}{L+1}$$

Where ω_l can be thought of as a weight. It is performed in R using the function NeweyWest() present in the package 'sandwich'.

```
> coeftest(consump.1)

t test of coefficients:

              Estimate Std. Error t value  Pr(>|t|)    
(Intercept) -81.079588  14.508136 -5.5886 7.430e-08 ***
dpi           0.891168   0.206252  4.3208 2.448e-05 ***
L(dpi)        0.030913   0.207542  0.1490    0.8817    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```