Operations Research (Master's Degree Course)

8. Discrete Simulation

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Generalities

- Discrete Simulation is used when the system
  - is complex;
  - evolves over time;
  - involves queueing processes;
  - includes a large number of interacting entities.
- **Simulation** is a powerful tool, in many cases the only one available;
- it creates a numerical prototype of the model, used to predict its performance;
- some similarities with video games;
- it is useful both to design a new system, and to improve an existing one.
- We will consider **Discrete event simulation**, in which the system is represented as a sequence of (random) events that modify the status of the system.
- Discrete simulation has anticipated important computer science developments: the **Simula 67** language is regarded as the first **object oriented** language.
- A totally different simulation methodology is **Continuous simulation**, in which the system is modeled by a set of differential equations.

Generalities (cont'd)

- A discrete event simulation model
  - represents the system as sets of independent elements, called **entities**,
  - which have specific characteristics, called **attributes**,
  - and are interconnected by relationships, like belonging to **sets**

    (STATIC structure)

  - The model describes the evolution of the system through actions, called **events**

    (DYNAMIC structure)

- This results in a **computer program**, written in a **simulation language**, whose runs
  - use **random numbers** to generate simulated events over time;
  - give on output **statistical information** on the system performance.
- Through **iterated experiments (runs)** with different system configurations the best one (or a good one) is obtained.
- Computers cannot generate truly random numbers, but any language has software functions to generate sequences of real numbers (usually with values uniformly random in $[0, 1]$), starting from a seed and recursively applying a generating algorithm.

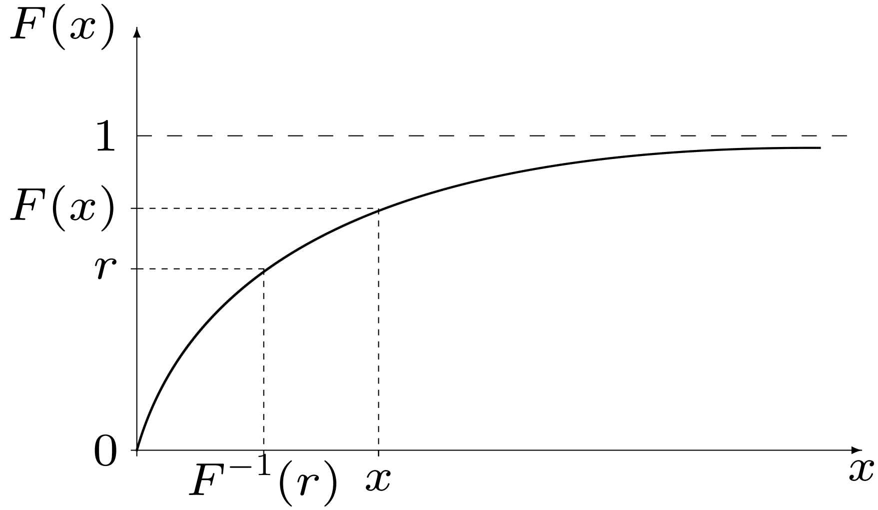
  Such numbers are referred to as **pseudo random**: for all practical applications they can be used as genuine random numbers.

Generating pseudo random numbers from a probability distribution

- X = random variable;
- t = possible value (outcome) of X;
- $f(x)$ = **probability density function** $= \lim_{\Delta x \to 0} \dfrac{\mathbb{P}(x \le t \le x + \Delta x)}{\Delta x}$;
- $f(x) \ge 0$;
- $f(x)dx = \mathbb{P}(x \le t \le x + dx)$;
- $\int_a^b f(x)dx = \mathbb{P}(a \le t \le b)$;
- $\int_{-\infty}^{+\infty} f(x)dx = 1$ (normalization condition).
- E = **expected value** $= \int_{-\infty}^{+\infty} f(x)x\,dx$;
- V = **variance** $= \int_{-\infty}^{+\infty} f(x)(x-E)^2 dx$.
- $F(x)$ = **cumulative distribution function** $= \int_{-\infty}^{x} f(\xi)d\xi = \mathbb{P}(t \le x)$:
  - $0 \le F(x) \le 1$;
  - $F(x)$ monotone non-decreasing.

Generating pseudo random numbers from a probability distribution(cont'd)

Inverse transformation method: *Given a probability density function $f(x)$, and values r uniformly random in $[0,1]$, the values $F^{-1}(r)$ have the probability density function $f(x)$.*



Proof For a random variable Y uniformly distributed in [0,1]: we have

$$f(y) = 1 \Rightarrow \mathbb{P}(r \leq q) = \int_0^q f(\xi)d\xi = q \ (\forall\ q, 0 \leq q \leq 1).$$

Hence $\mathbb{P}(r \leq F(x)) = F(x)$.
From the transformation: $\mathbb{P}(r \leq F(x)) = \mathbb{P}(F^{-1}(r) \leq x)$.

$\Rightarrow F(x) = \mathbb{P}(F^{-1}(r) \leq x)$

i.e., $F^{-1}(r)$ has cumulative distribution function $F(x)$

i.e., $F^{-1}(r)$ has probability density function $f(x)$. $\square$

Generating pseudo random numbers from a probability distribution(cont'd)

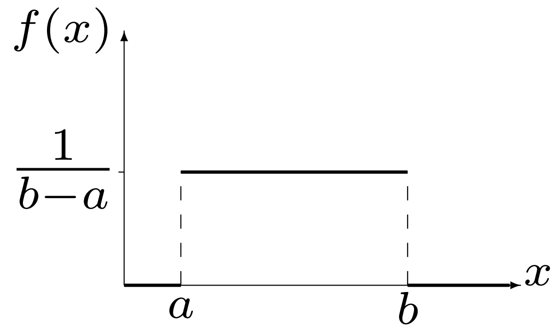
- **Example:** $f(x) = 8x$ for $0 \le x \le \frac{1}{2}$:

$$F(x) = \int_{-\infty}^{x} f(\xi)d\xi = \int_{0}^{x} 8\xi d\xi = 8\left[\frac{\xi^2}{2}\right]_0^x = 4x^2;$$

$$r = 4x^2 \Rightarrow x = \frac{\sqrt{r}}{2}.$$

- **Uniform distribution in $[a, b]$:**

$$f(x) = \frac{1}{b-a} \quad (a \le x \le b);$$



$$F(x) = \int_{a}^{x} \frac{1}{b-a} d\xi = \frac{1}{b-a}\left[\xi\right]_a^x = \frac{x-a}{b-a};$$

$$r = \frac{x-a}{b-a} \Rightarrow x = a + r(b-a);$$

$$E = \int_{a}^{b} \frac{x}{b-a} dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{b-a} \cdot \frac{b^2-a^2}{2} = \frac{a+b}{2}.$$

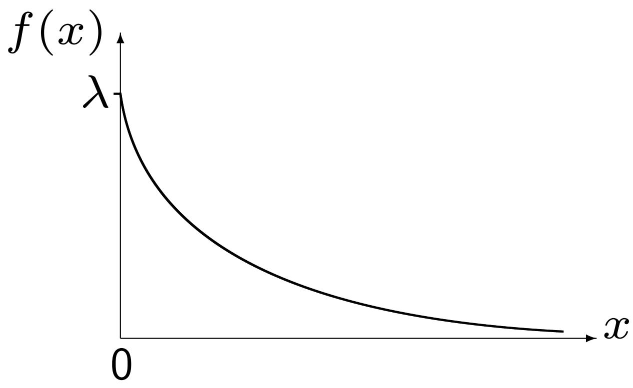
Generating pseudo random numbers from a probability distribution(cont'd)

- **Exponential distribution:**

  $f(x) = \lambda e^{-\lambda x} \quad (\lambda > 0, \; x \geq 0);$

  $E = \dfrac{1}{\lambda};$

  $V = \dfrac{1}{\lambda^2};$



  $$F(x) = \int_0^x \lambda e^{-\lambda \xi} d\xi = \lambda \left[-\frac{e^{-\lambda \xi}}{\lambda} \right]_0^x = 1 - e^{-\lambda x};$$

  $$r = 1 - e^{-\lambda x} \Rightarrow 1 - r = r' = e^{-\lambda x} \Rightarrow \ln r' = -\lambda x$$

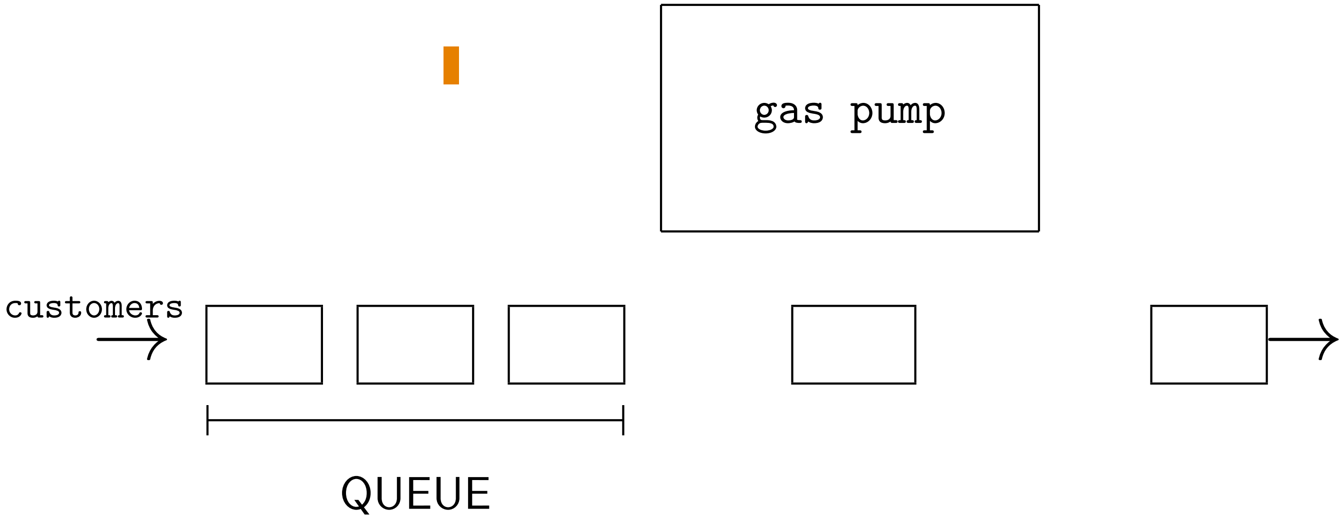
  r' has a uniform distribution in [0,1] $\Rightarrow x = -\dfrac{1}{\lambda} \ln r'$.

- In many cases, **Poisson processes** are good models for random arrivals in which $\lambda =$ expected number of arrivals during a unit time interval.

- In a Poisson process of expected value λ the time between each pair of consecutive events has an exponential distribution of expected value $\dfrac{1}{\lambda}$.

Static and Dynamic description of a simulation model

We will use a first **Example:** gas station (with a single fuel dispenser)



- customers arrival: Poisson distribution of expected value λ;
- refueling time: uniform distribution in $[T1, T2]$;
- if *NMAX* customers are already queueing, the new customer gives up;
- end simulation after having **completely** simulated *NTOT* customers;
- determine the **average time spent in queue**.

Static description

- System status $\leftrightarrow$ data structures.
- Dynamic processes (events) modify the data $\Rightarrow$ they modify the system status.
- **Main objects** in a simulation model:

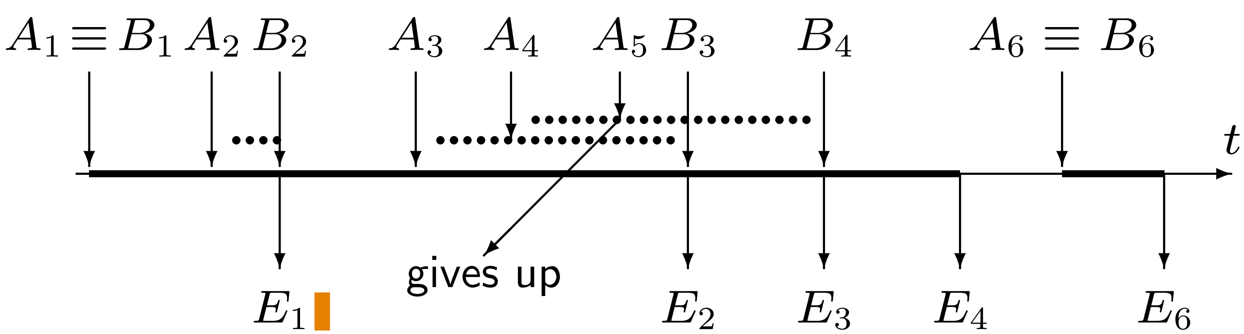
Terminology	Example	SIMSCRIPT
Entity	car	CAR
Attribute	entering time in queue TIC(CAR)	TIC(CAR)
Set	queue	QUEUE
	insert the car in the queue	FILE CAR IN QUEUE
	extract the first car from the queue	REMOVE FIRST CAR FROM QUEUE

- **Creation and destruction:**

Terminology	Example	SIMSCRIPT
Creation (entering the system)	create the car	CREATE CAR
Destruction (leaving the system)	destroy the car	DESTROY CAR

Dynamic description

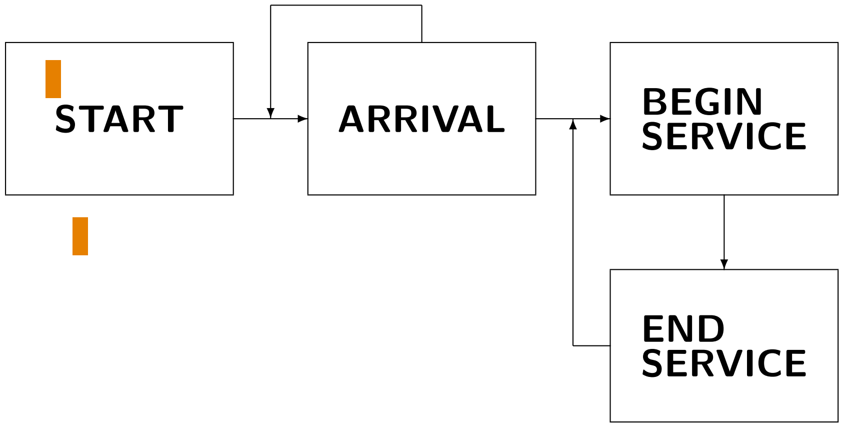
- **Simulated clock (time):**
  - old languages: counter;
  - modern languages: only those time instants in which the system status varies are considered;
- **Example:** gas station with $NMAX = 2$ (**A**rrival, **B**egin service, **E**nd service):

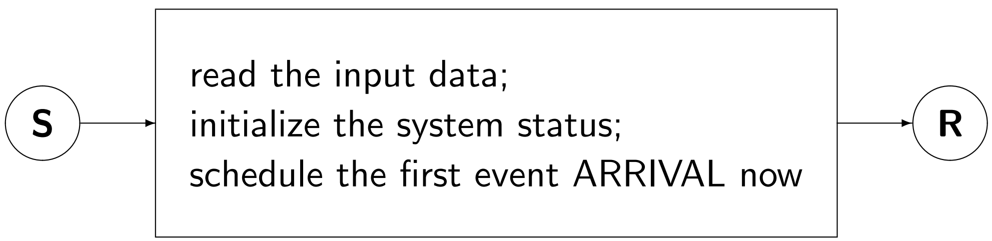


- **Event** = subprogram containing the instructions to execute when the event occurs.
- Each event establishes which future events must occur, and "schedules" them (**Event-scheduling approach**):

Example	**SIMSCRIPT**
schedule an event END SERVICE after T (time units)	SCHEDULE AN END SERVICE AT TIME.V + T

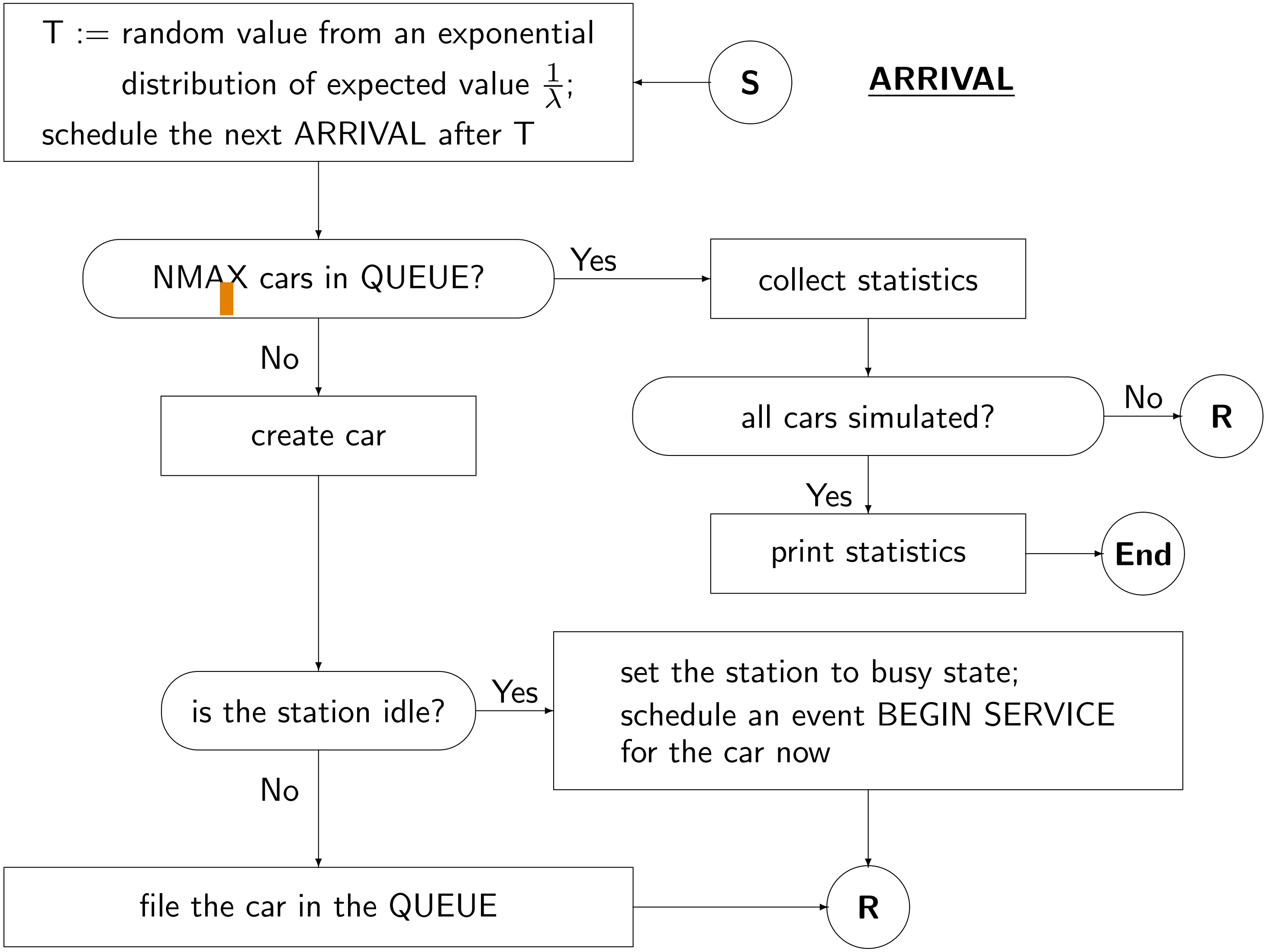
When the execution of an event terminates, the system determines the next scheduled event, and updates the simulated clock.


START
ARRIVAL
BEGIN SERVICE
END SERVICE

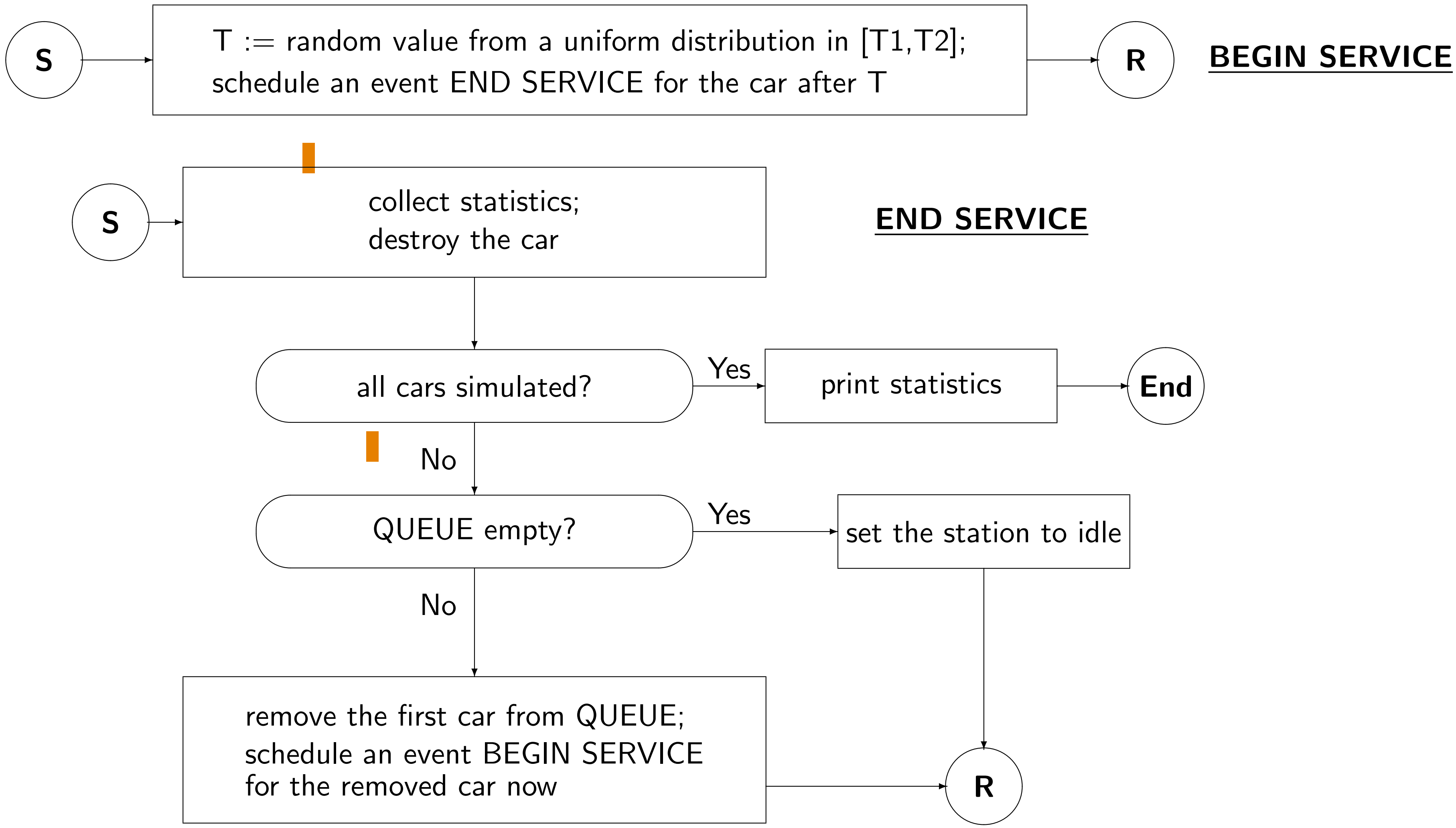

S
read the input data;
initialize the system status;
schedule the first event ARRIVAL now
R

START

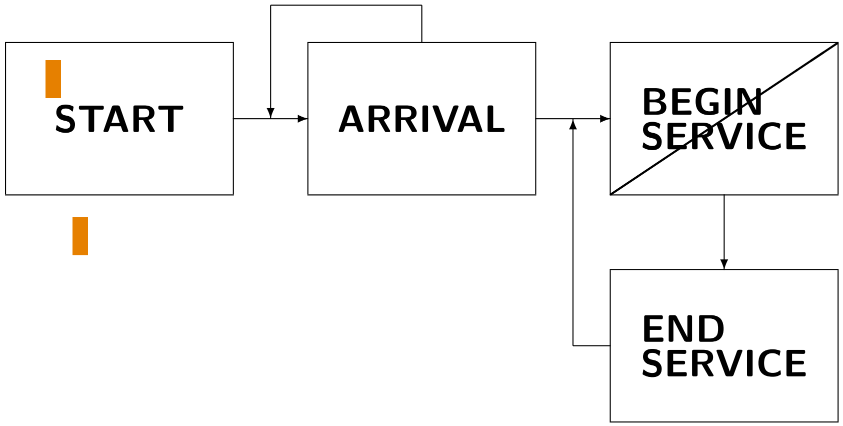
ARRIVAL

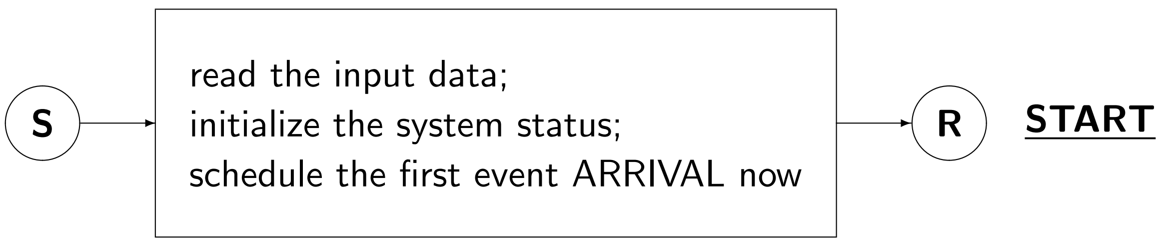


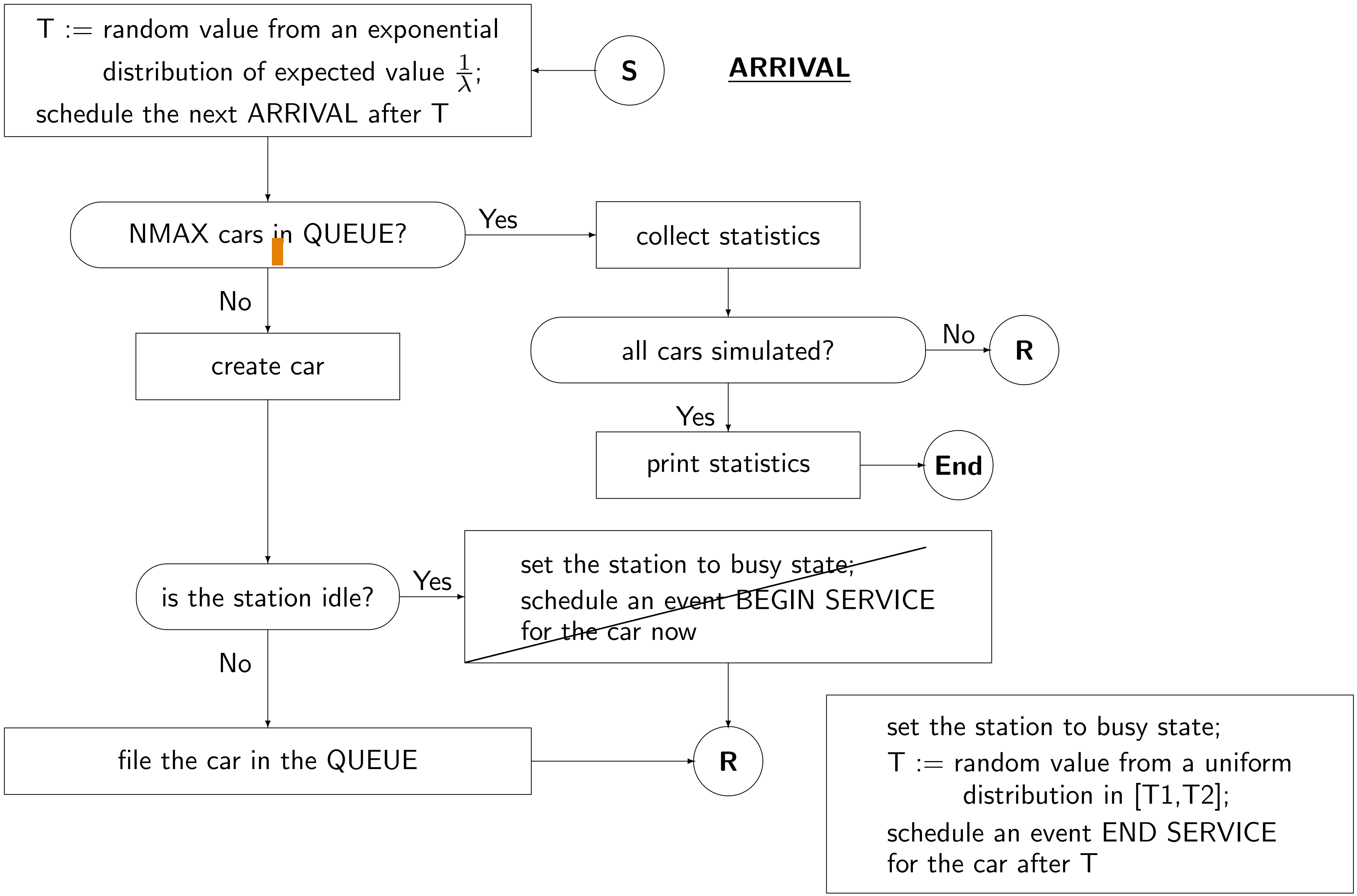
T := random value from an exponential distribution of expected value $\frac{1}{\lambda}$; schedule the next ARRIVAL after T
S
NMAX cars in QUEUE?
Yes
collect statistics
No
create car
all cars simulated?
No
R
Yes
print statistics
End
is the station idle?
Yes
set the station to busy state; schedule an event BEGIN SERVICE for the car now
No
file the car in the QUEUE
R

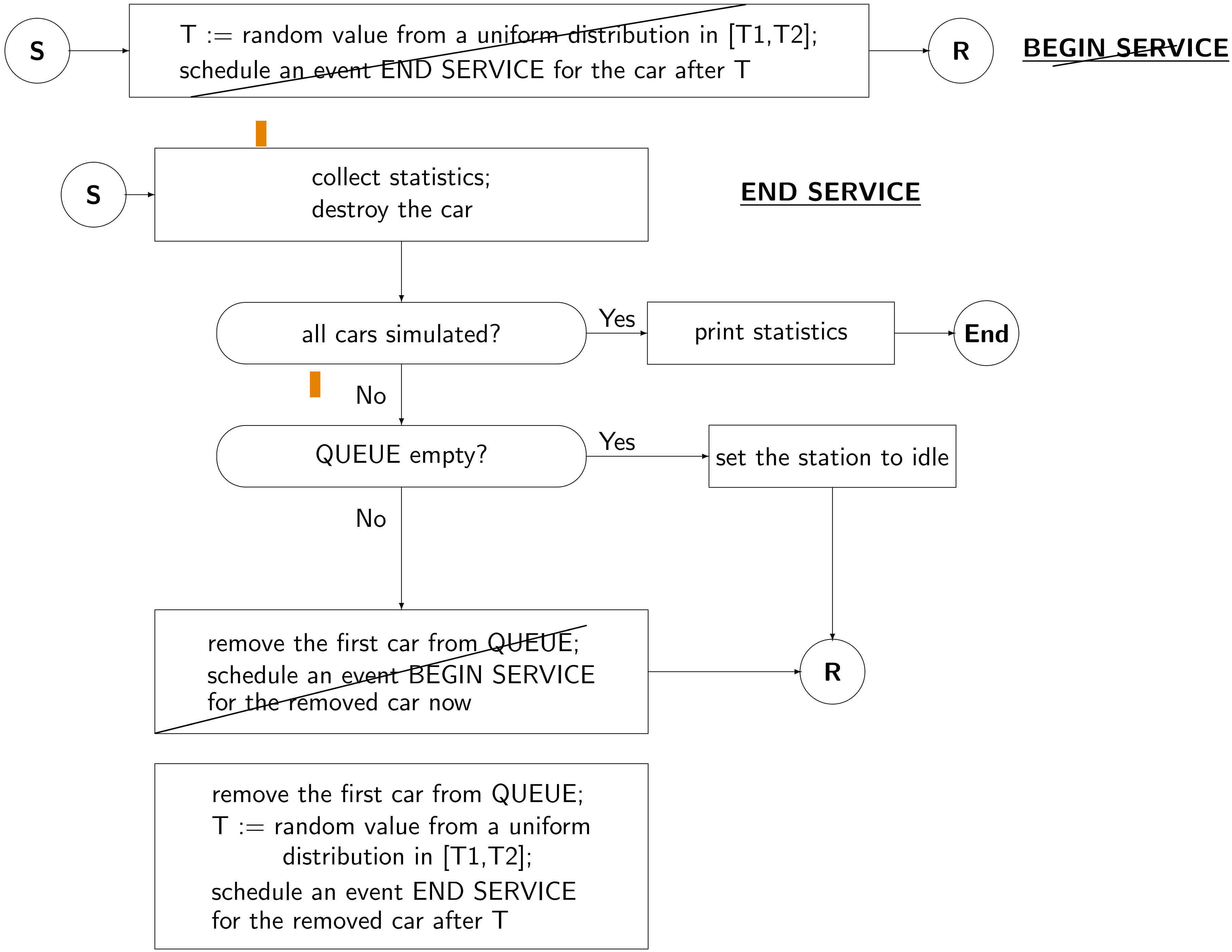


- An event BEGIN SERVICE always coincides with
  - an event ARRIVAL, **or** an event END SERVICE.
- Scheduling events requires computationally heavy inner procedures
- $\Rightarrow$ better to eliminate useless events (further improvement: simpler and more readable model).

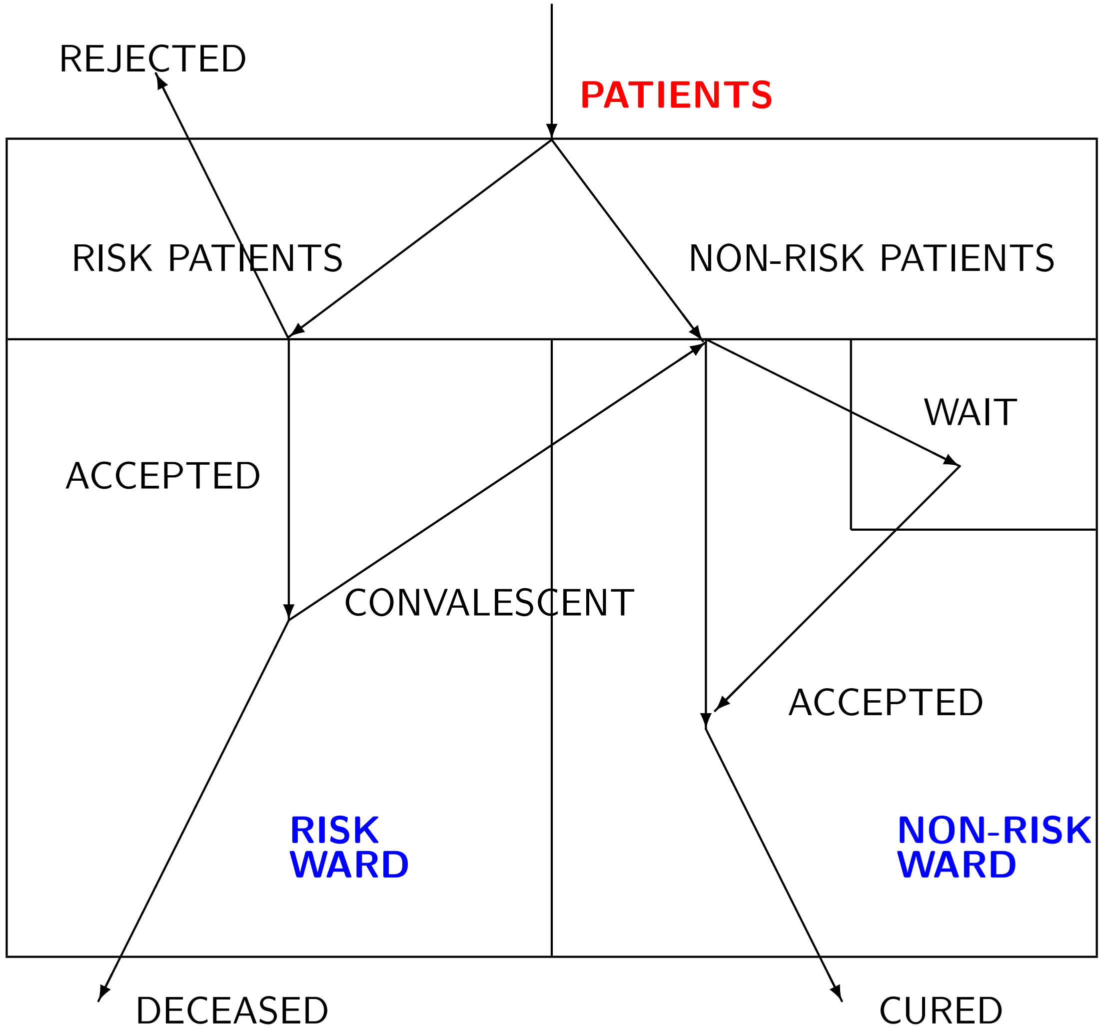

START
ARRIVAL
BEGIN
SERVICE
END
SERVICE


S
read the input data;
initialize the system status;
schedule the first event ARRIVAL now
R
START


T := random value from an exponential distribution of expected value $\frac{1}{\lambda}$;
schedule the next ARRIVAL after T
S
ARRIVAL
NMAX cars in QUEUE?
Yes
collect statistics
No
all cars simulated?
No
R
create car
Yes
print statistics
End
is the station idle?
Yes
set the station to busy state;
schedule an event BEGIN SERVICE
for the car now
No
file the car in the QUEUE
R
set the station to busy state;
T := random value from a uniform distribution in [T1,T2];
schedule an event END SERVICE
for the car after T


S
~~T := random value from a uniform distribution in [T1,T2]; schedule an event END SERVICE for the car after T~~
R
~~BEGIN SERVICE~~
S
collect statistics;
destroy the car
END SERVICE
all cars simulated?
Yes
print statistics
End
No
QUEUE empty?
Yes
set the station to idle
No
~~remove the first car from QUEUE; schedule an event BEGIN SERVICE for the removed car now~~
R
remove the first car from QUEUE;
T := random value from a uniform distribution in [T1,T2];
schedule an event END SERVICE for the removed car after T

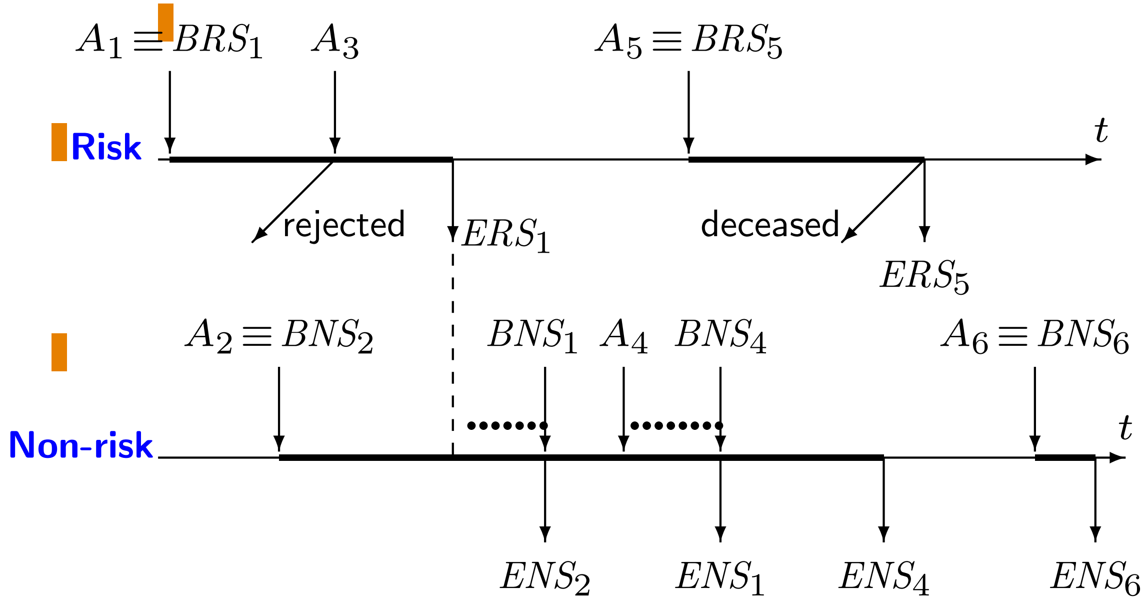
Example: small hospital

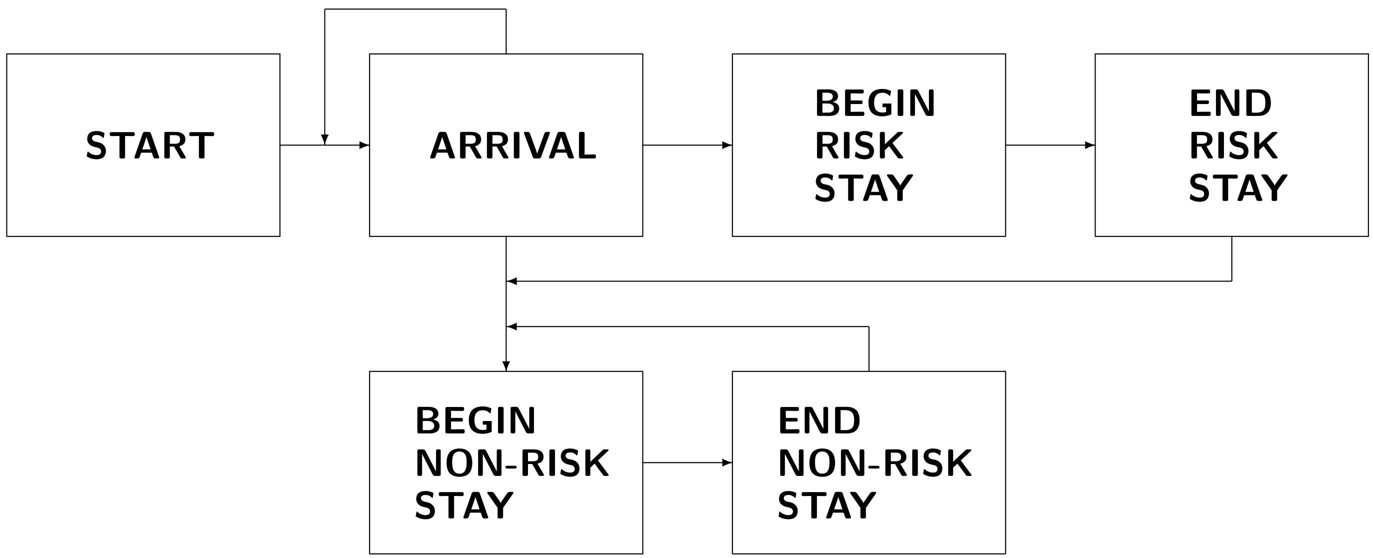


- Numbers of beds: **Risk** ward: NLG; **Non-risk** ward: NLN;
- Patients arrival: Poisson distribution with expected value λ: Risk with probability PG, non-risk with $1 - PG$;
- Risk patients rejected if there is no bed available in the risk ward;
- Stay durations: uniform distribution in $[DMIG, DMAG]$ (Risk ward), in $[DMIN, DMAN]$ (Non-risk ward);
- Outcome of Risk patients: successful with probability PS, unsuccessful with probability $1 - PS$.

***NLG* = *NLN* = 1,**
system initially empty

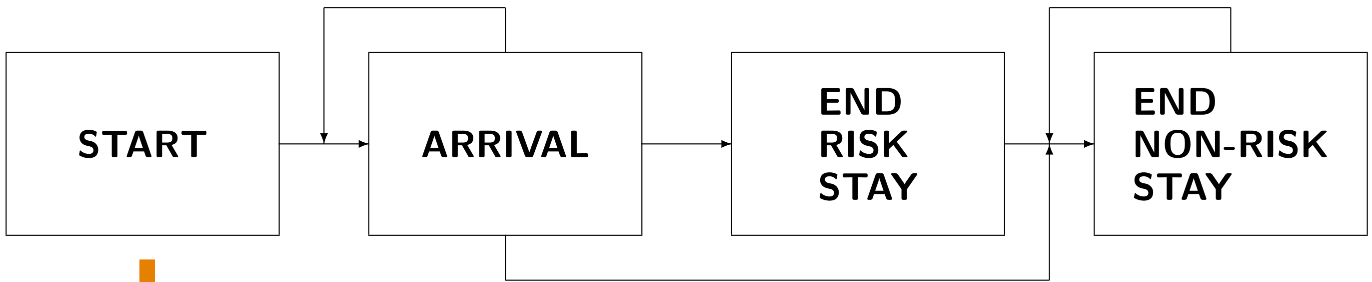
Arrival
Begin ***R***isk ***S***tay
Begin ***N***on-risk ***S***tay
End ***R***isk ***S***tay
End ***N***on-risk ***S***tay

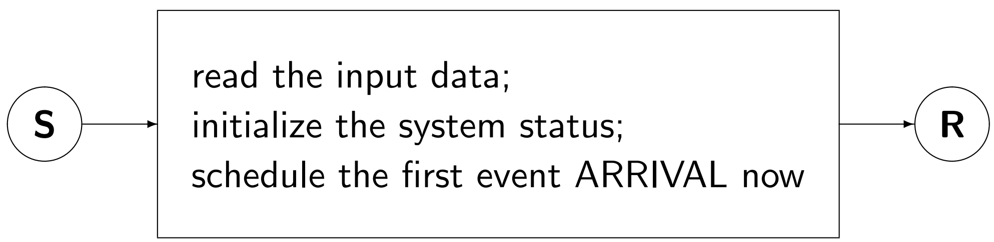




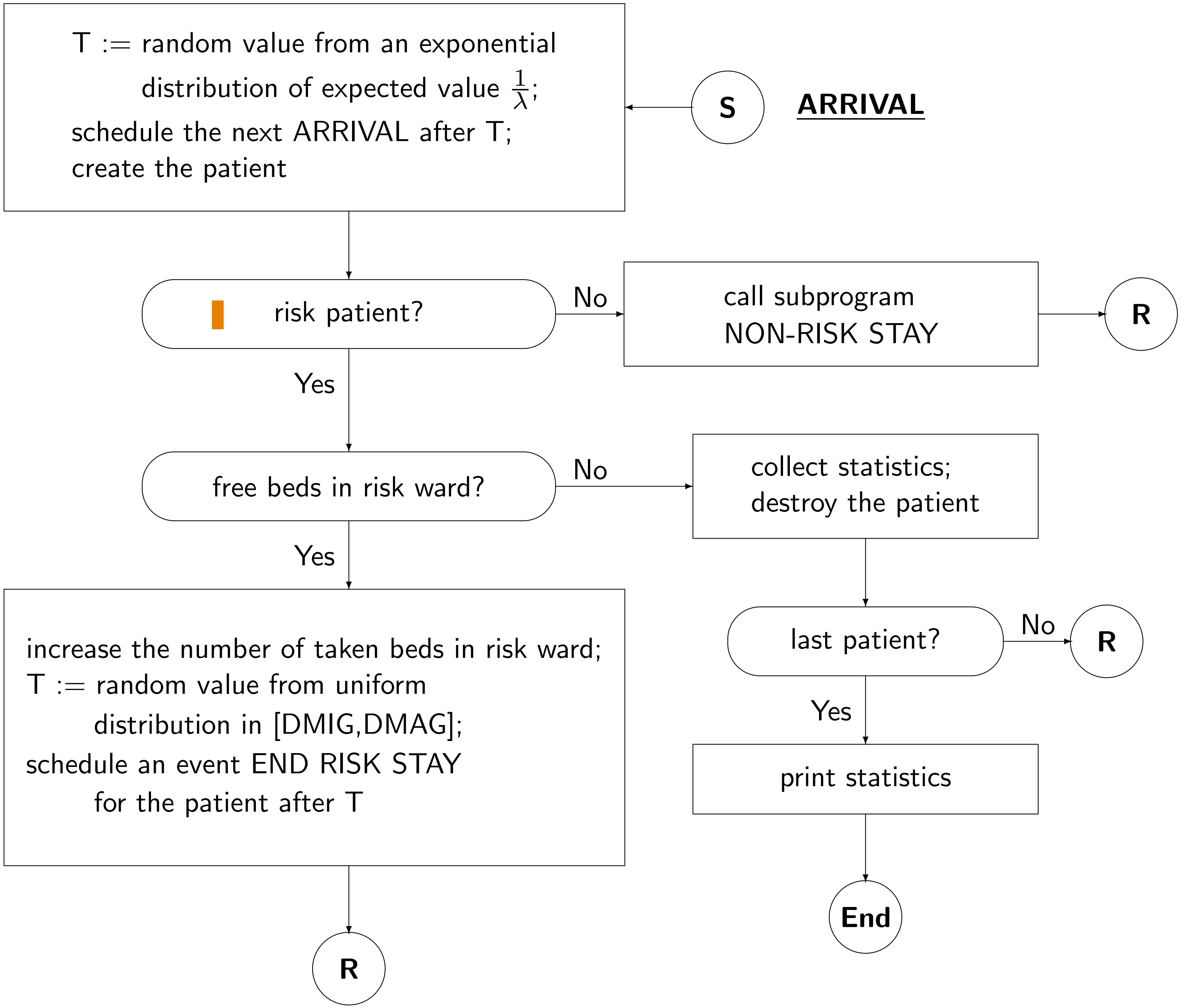
BRS always coincides with A;

BNS always coincides with A, or with ERS, or with ENS.

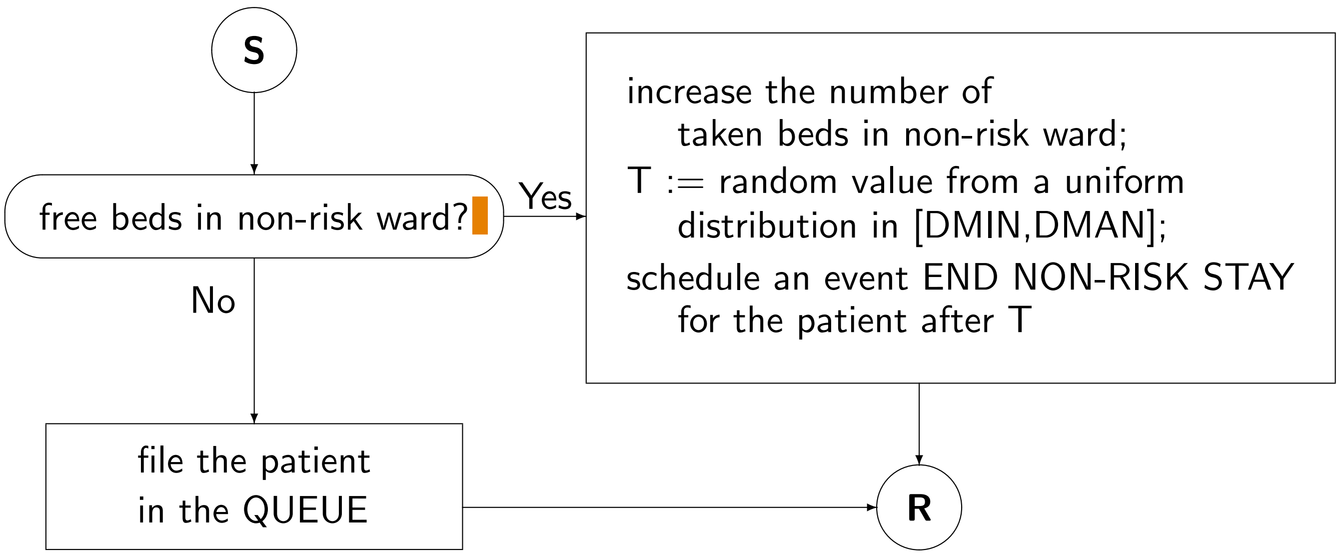

START
ARRIVAL
END RISK STAY
END NON-RISK STAY


S
read the input data;
initialize the system status;
schedule the first event ARRIVAL now
R

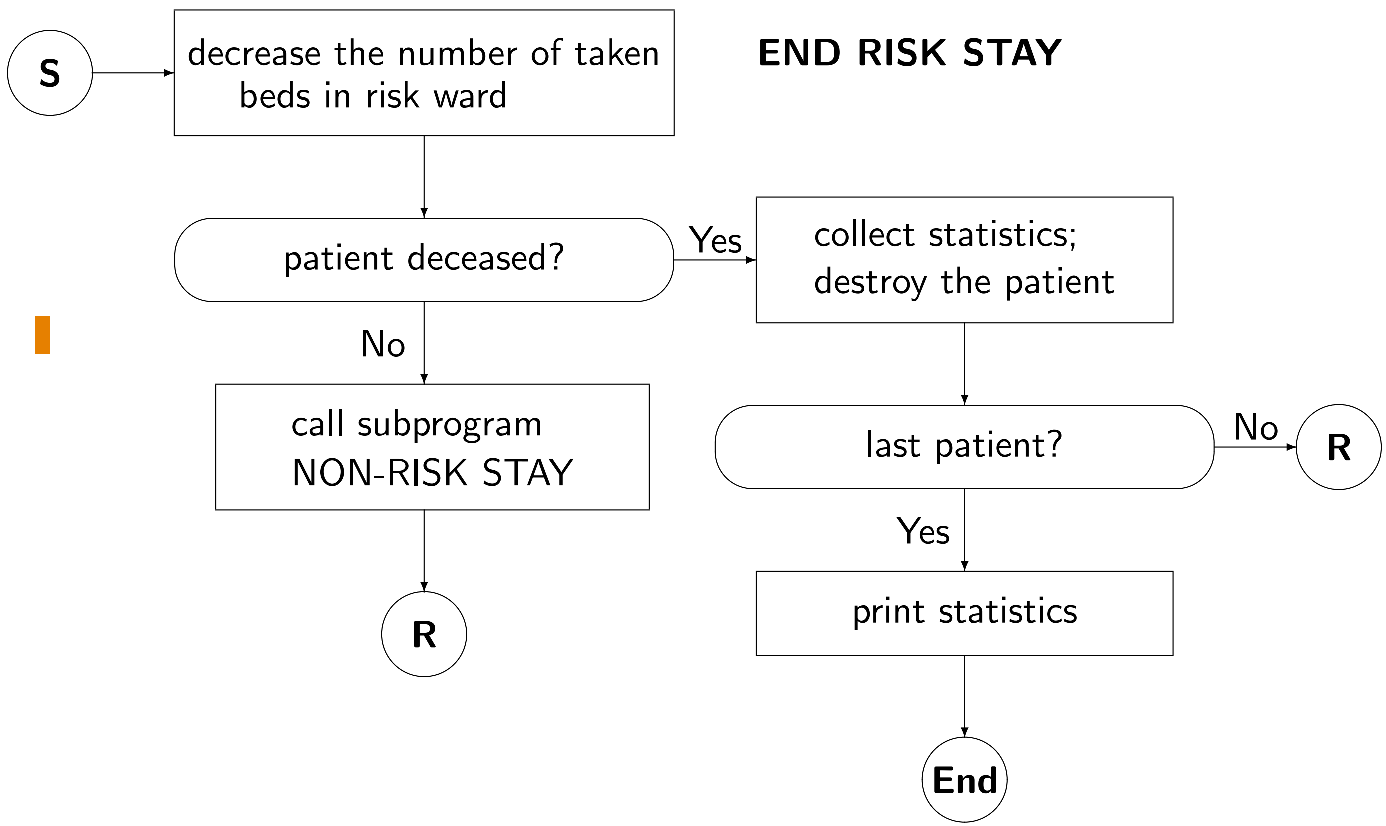
START


ARRIVAL
S
T := random value from an exponential distribution of expected value $\frac{1}{\lambda}$;
schedule the next ARRIVAL after T;
create the patient
risk patient?
No
call subprogram NON-RISK STAY
R
Yes
free beds in risk ward?
No
collect statistics;
destroy the patient
Yes
increase the number of taken beds in risk ward;
T := random value from uniform distribution in [DMIG,DMAG];
schedule an event END RISK STAY for the patient after T
R
last patient?
No
R
Yes
print statistics
End

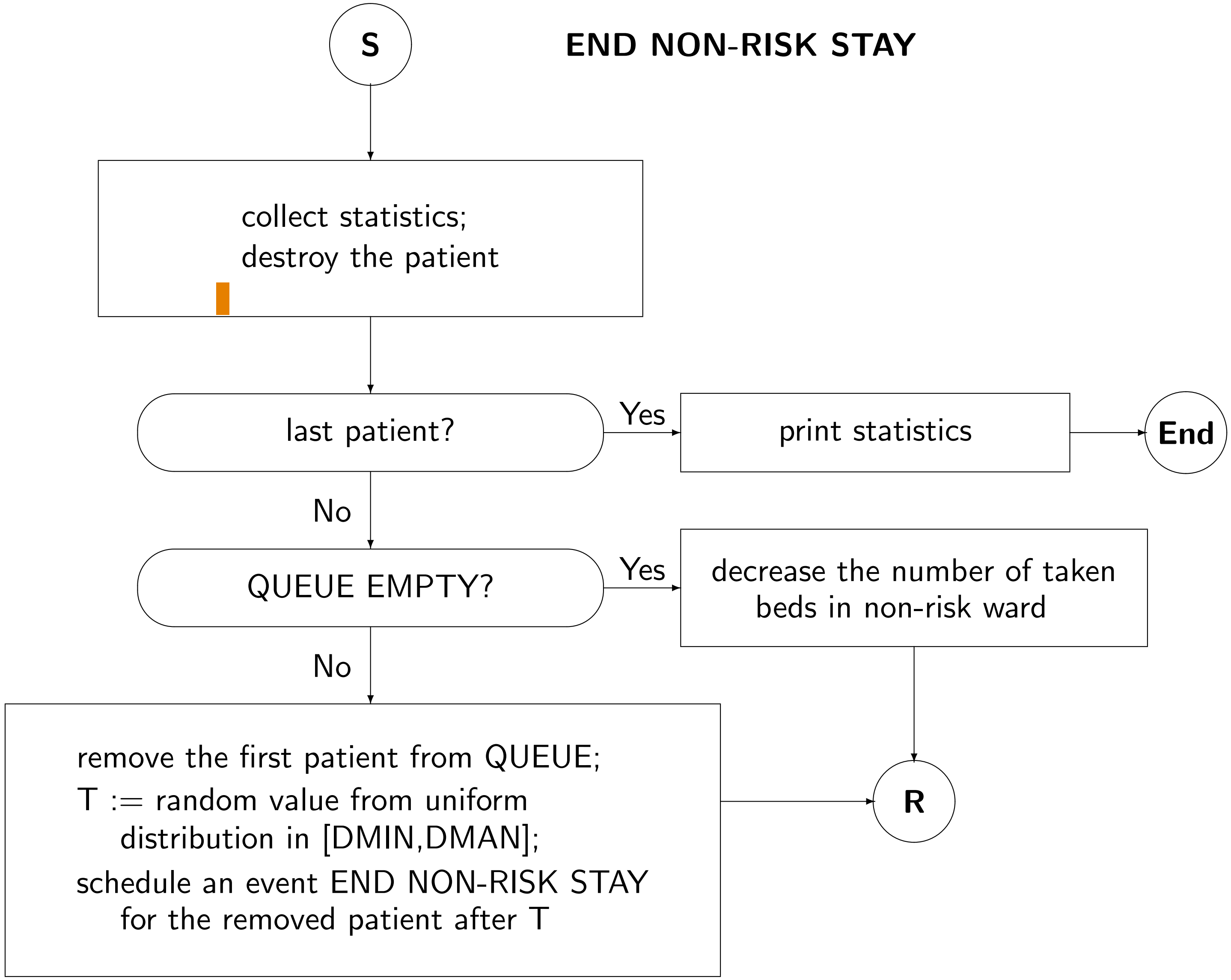
Subprogram NON-RISK STAY



END RISK STAY



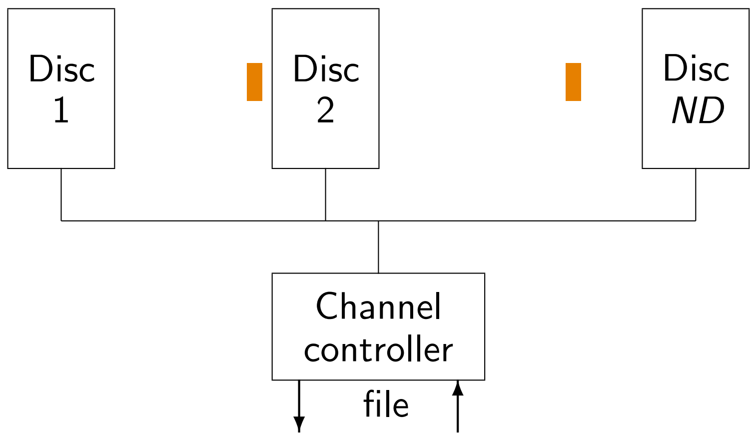
END NON-RISK STAY

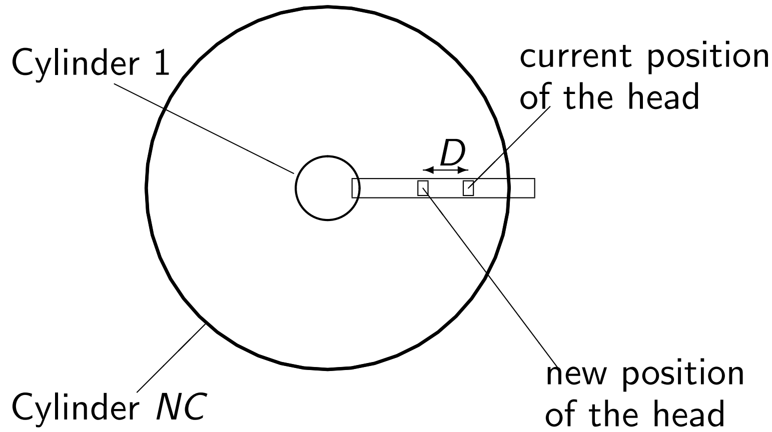


Simulation components

- **Entities:**
  - *temporary entities* are created and destroyed (CAR, PATIENT);
    - $*$ *temporary attributes* (kind of illness: risk, non-risk);
  - *permanent entities*: always present during simulation
    - $*$ fundamental permanent entity: the **System**;
      - · *permanent attributes* (status (idle, busy), T1, T2, NMAX, ...; NLG, NLN, ...)
    - $*$ other permanent entities are represented through **indices** (next example)
- **Events:**
  - *endogenous events*: scheduled by other events (ARRIVAL, END SERVICE, END NON-RISK STAY, ...);
  - *exogenous events*: scheduled from "outside" (START).

Example: simplified version of a multi-disc unit:



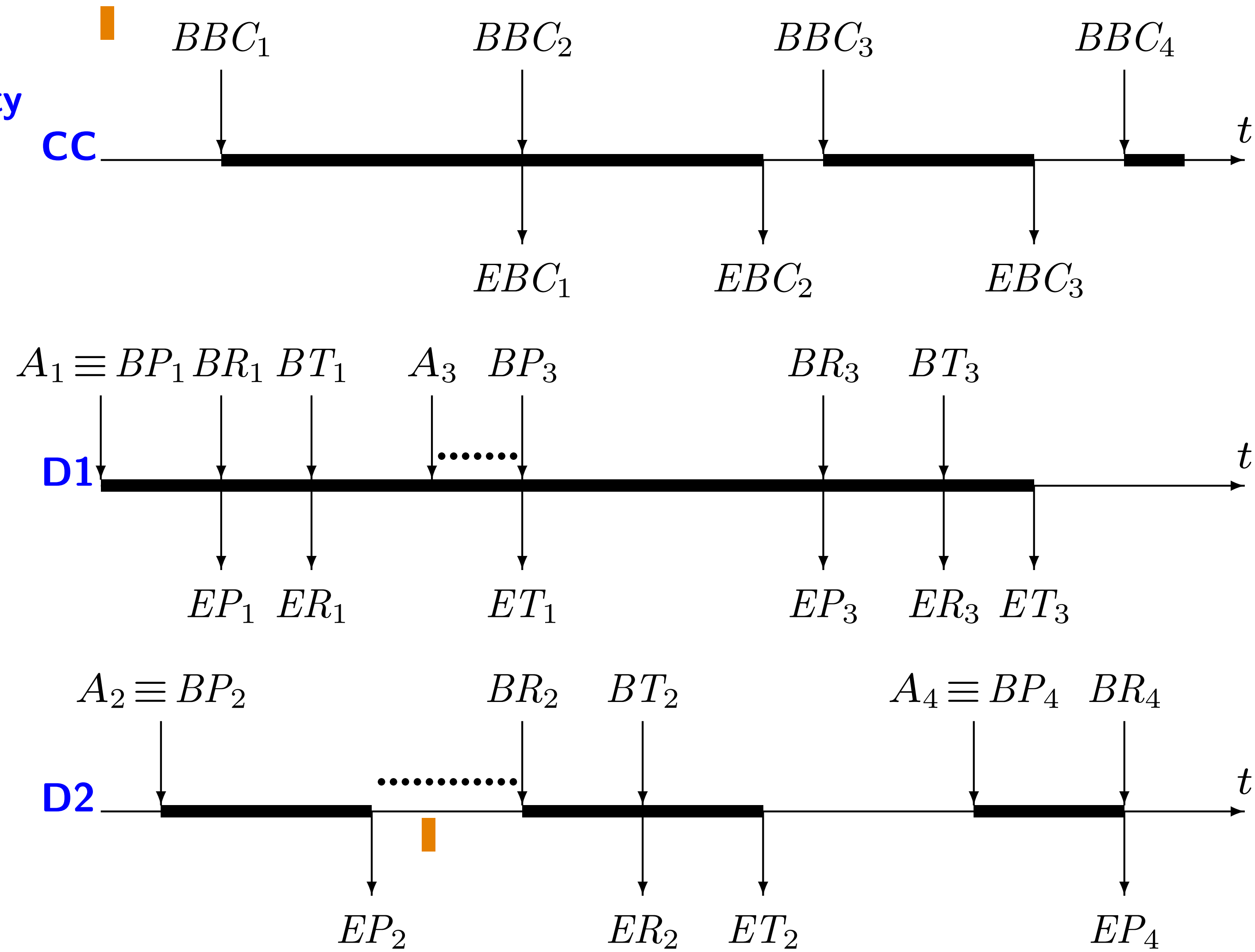


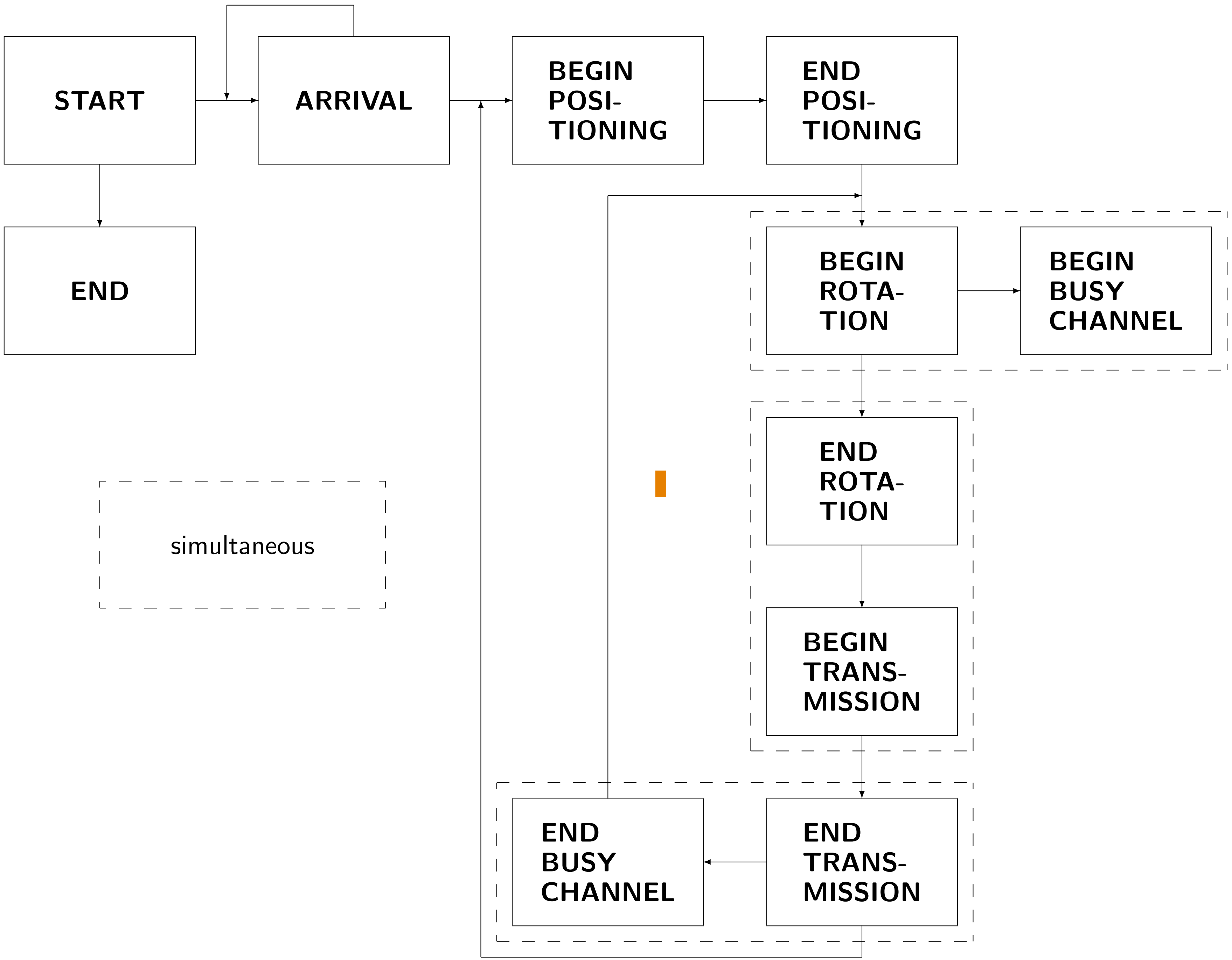
- Arrival of files (indifferently to read or write): Poisson distribution of expected value λ:
  length: value L from uniform distribution in [*MIN*,*MAX*];
  requested disc / cylinder: uniform probability in [1,*ND*] / in [1,*NC*].
- If the disc is idle, the positioning of the head starts:
  positioning time: function $f_1(D)$, with $D =$ distance between current and new position.
- After positioning, if the channel is idle, waiting time for disc rotation (uniform distribution in $[0, TR]$), then transmission (time $f_2(L)$).
- **Queue at channel controller** (precedence to disks with lower number);
  one queue per disk (precedence to files with lower length L).
- Terminate the simulation after TS time units.

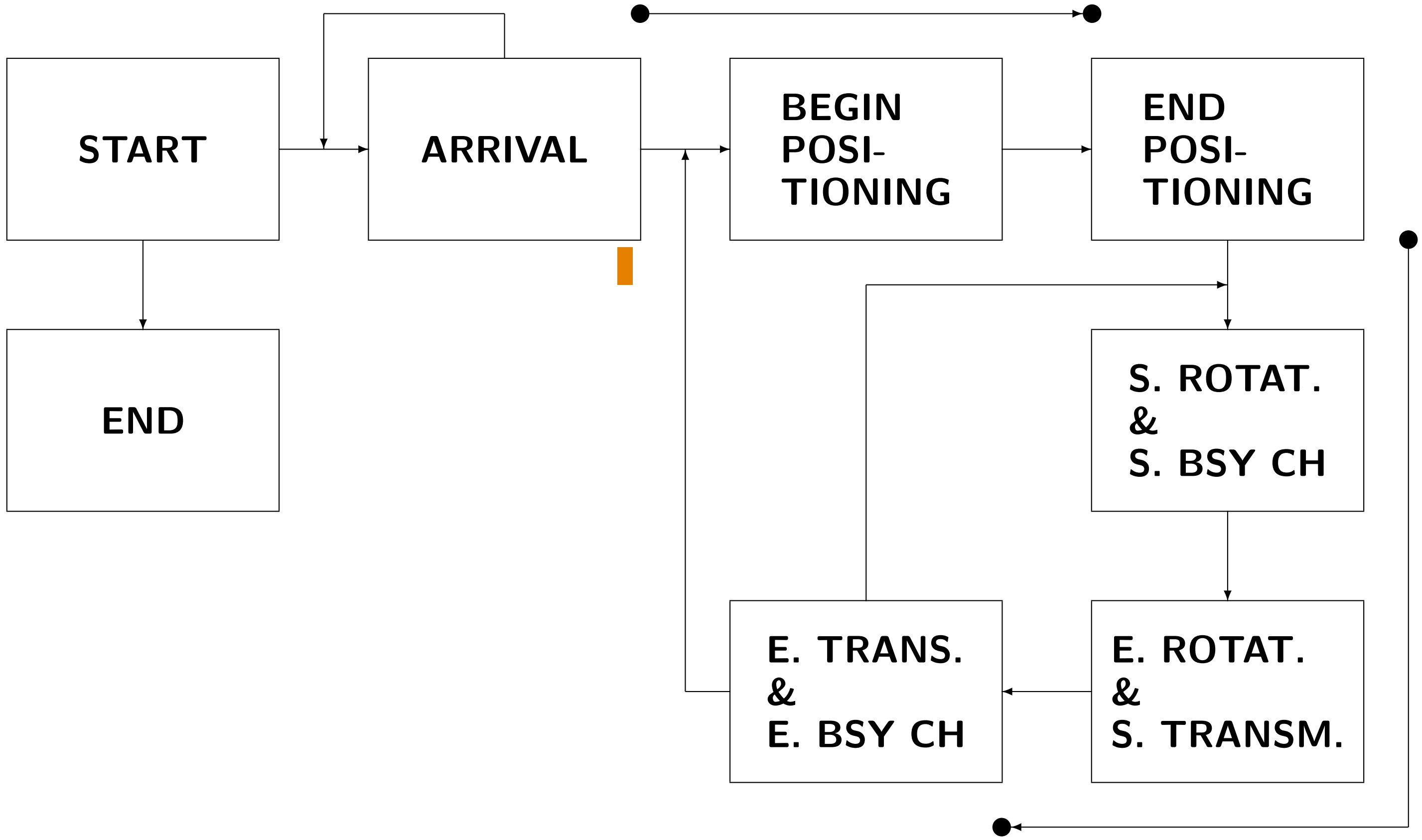
– Temporary entities **file:** Attributes: disc, cylinder, length.
– Permanent entity **System:** Attributes: λ, *ND*, *NC*, *TR*, f_1, f_2, channel status.
– Permanent entities **disks** (integers 1, 2,. . . , *ND*): Attributes: disc status, current head position.

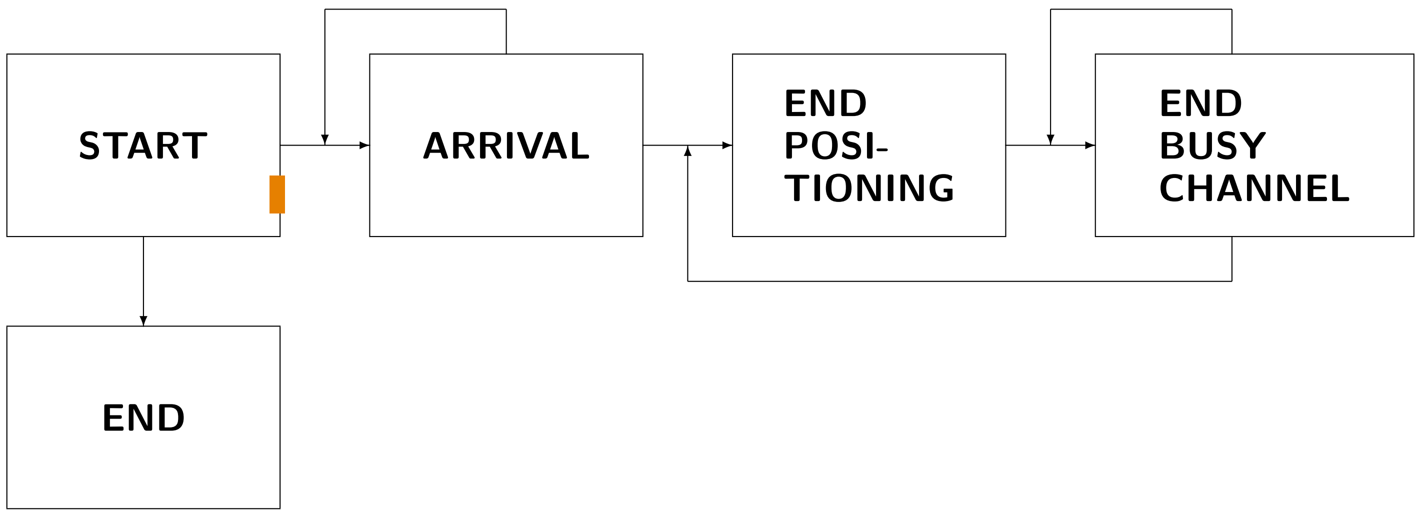
$ND = 2$, system initially empty

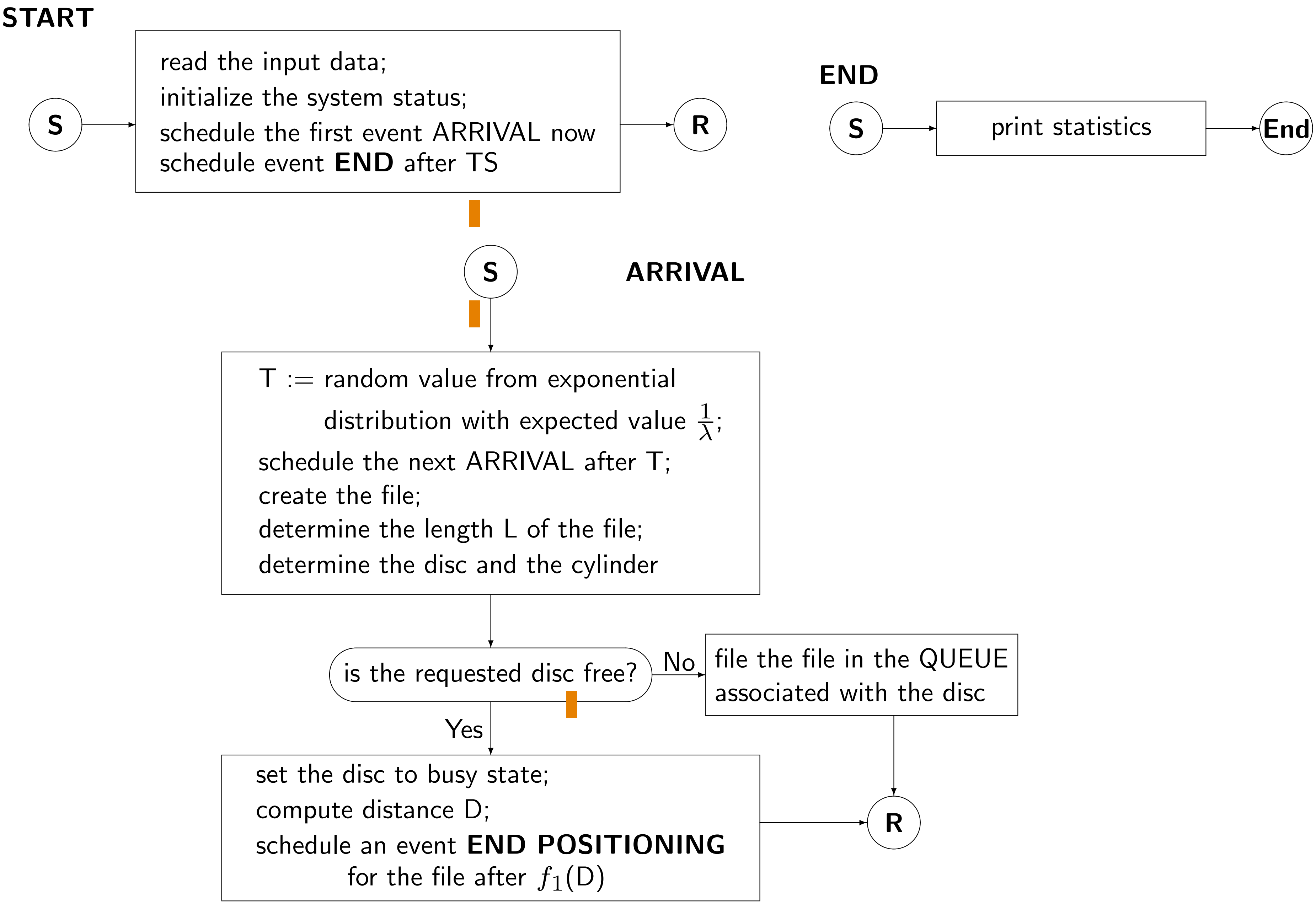
Arrival
Begin ***B***usy ***C***hannel
End ***B***usy ***C***hannel
Begin ***P***ositioning
End ***P***ositioning
Begin ***R***otation
End ***R***otation
Begin ***T***ransmission
End ***T***ransmission



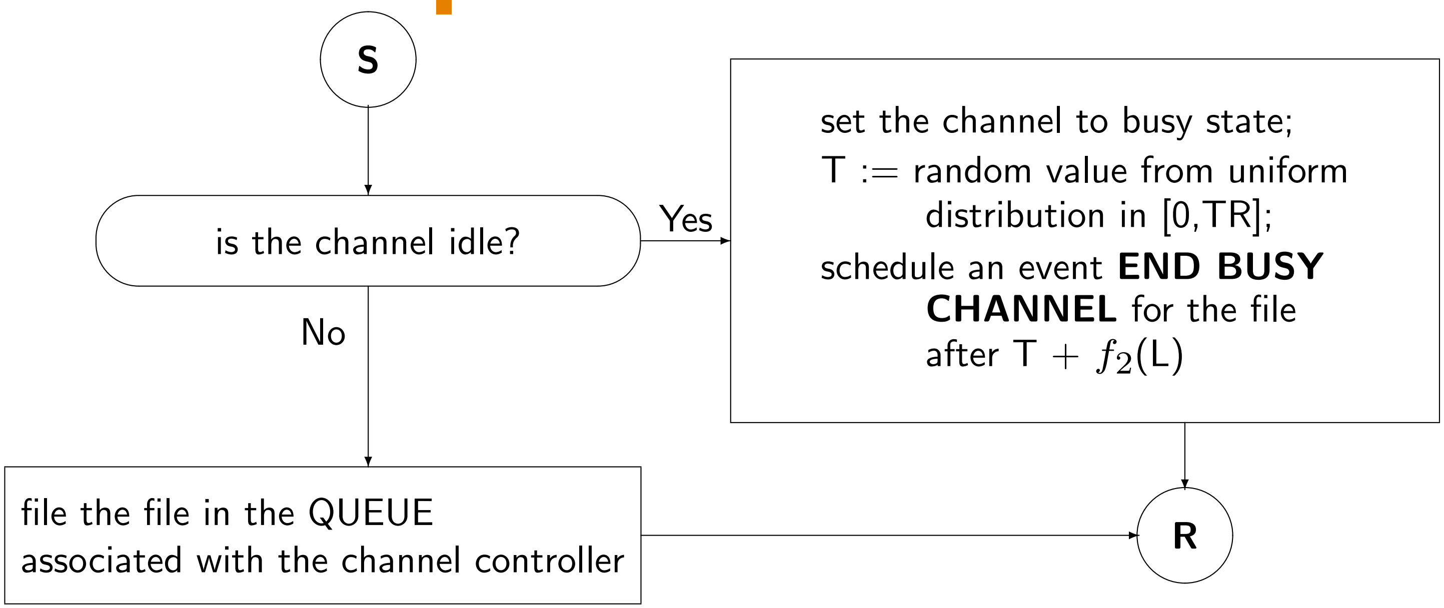

START
ARRIVAL
BEGIN POSI-TIONING
END POSI-TIONING
END
BEGIN ROTA-TION
BEGIN BUSY CHANNEL
END ROTA-TION
simultaneous
BEGIN TRANS-MISSION
END BUSY CHANNEL
END TRANS-MISSION


START
ARRIVAL
BEGIN POSI- TIONING
END POSI- TIONING
END
S. ROTAT. & S. BSY CH
E. TRANS. & E. BSY CH
E. ROTAT. & S. TRANSM.

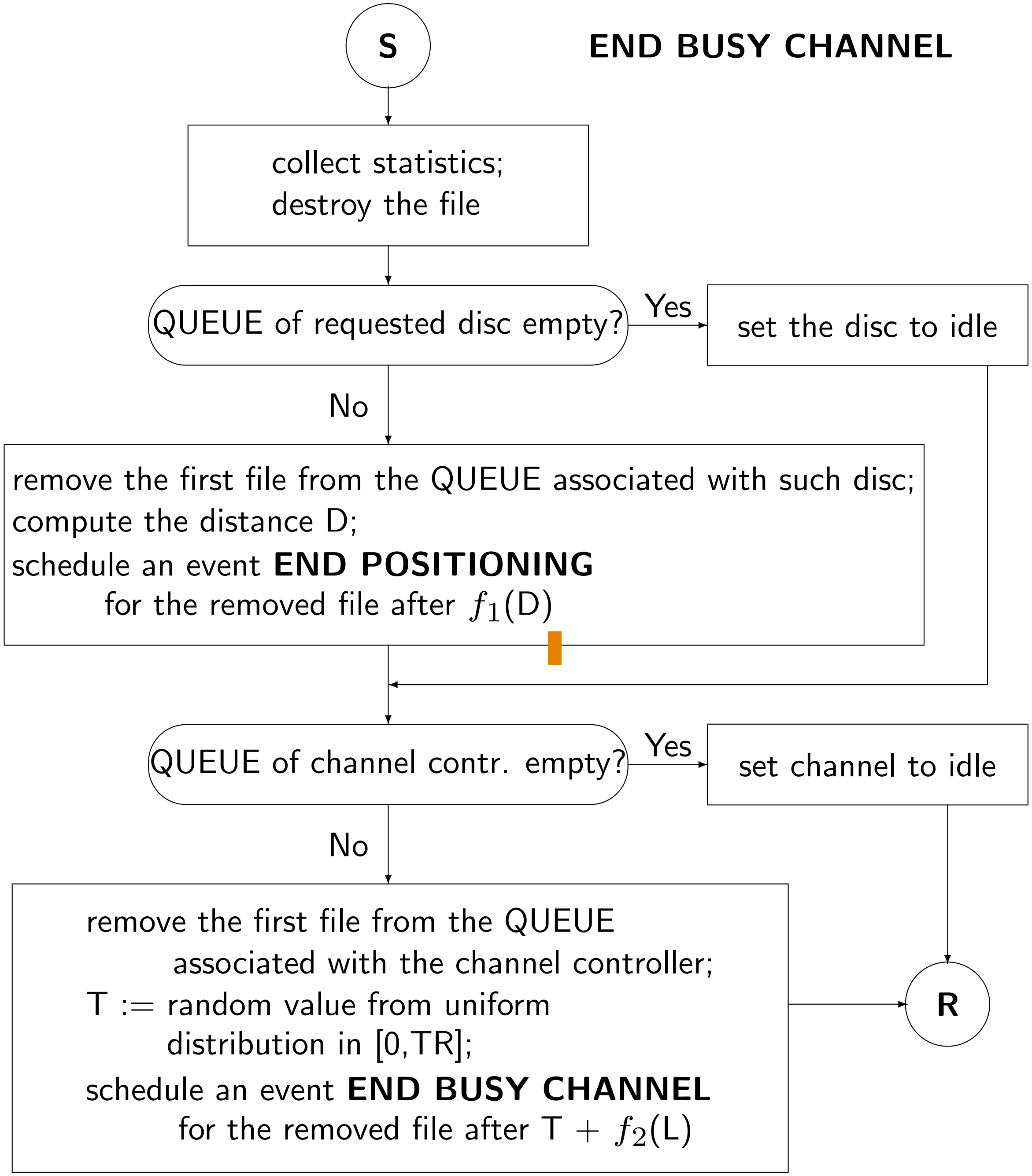

START
ARRIVAL
END POSI- TIONING
END BUSY CHANNEL
END


START
S
read the input data;
initialize the system status;
schedule the first event ARRIVAL now
schedule event **END** after TS
R
END
S
print statistics
End
S
ARRIVAL
T := random value from exponential distribution with expected value $\frac{1}{\lambda}$;
schedule the next ARRIVAL after T;
create the file;
determine the length L of the file;
determine the disc and the cylinder
is the requested disc free?
No
file the file in the QUEUE associated with the disc
Yes
set the disc to busy state;
compute distance D;
schedule an event **END POSITIONING** for the file after f_1(D)
R

END POSITIONING



END BUSY CHANNEL



The main discrete simulation languages

Two logical approaches to discrete simulation:

– **Event scheduling**, preferable from a didactic point of view;

– **Process interaction**: Process = life of an entity = chronological sequence of events.

Imperative specialized languages:

– **Simula I, Simula 67** (process interaction, developed by **Dahl and Nygaard**):

  * important for computer science: it introduced the concepts of **object** and **class**;

  * it influenced the development of **C++**; rarely used today.

– **SIMSCRIPT I, SIMSCRIPT II.5** (event scheduling, developed by **Markowitz and Hausner**):

  * the most powerful and widely used; it also allows process interaction.

  * it influenced Simula, but it is easier to use and more efficient;

Interactive specialized languages (less versatile):

– **Arena** (process interaction):

  * very simple to use; the model is implemented through **boxes** (↔ flow chart);

– **SAS/OR** (event scheduling), integrated software for management.

General purpose languages (more difficult to use):

– Simulation can also be implemented with **general purpose languages** (e.g., **Java**).

Main simulation statements: Temporary entities

- We refer to SIMSCRIPT II.5 (Other languages have logically equivalent statements).
- Temporary entities are created when they enter the system, destroyed when they leave it.
- `CREATE [A, AN, THE]` *`en`* `[CALLED` *`p`*`]`
  1. reserves a new **block of consecutive words** for a new entity of class *en*;
  2. defines a **local variable** *p* containing the corresponding pointer;
     if "`CALLED` *p*" is missing, the variable has name *en* (preferred for single entities).

Example: `CREATE A CAR CALLED A1`
`CREATE A CAR CALLED A2`

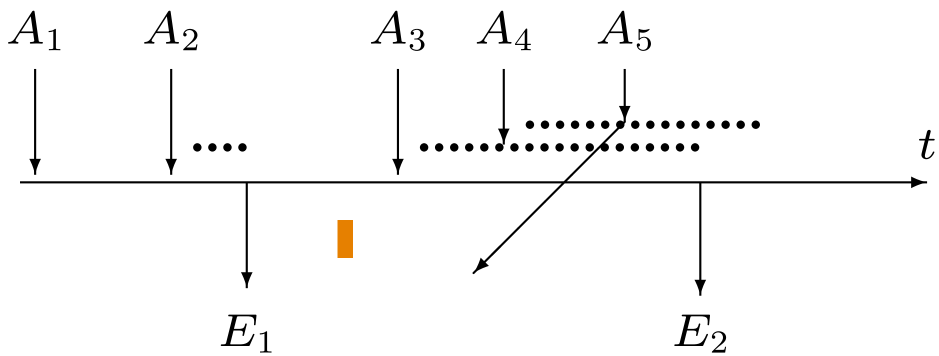
`CREATE A CAR` (equivalent to `CREATE A CAR CALLED CAR`)

- `DESTROY THE` *`en`* `[CALLED` *`p`*`]`
  - releases the reserved block of words pointed by *p* (by *en*, if "`CALLED` *p*" omitted)
- Temporary entities may have **temporary attributes**:

  form: *`attribute`*`(p)`

Example:
```
CREATE A CAR
LET TYPE(CAR) = X
```

Example: gas station, entities `CAR` having two attributes: `TIS` (entry instant time), `TYPE` (X or Y).



In A_1:

`CAR` $\rightarrow$ 550

550	0.0
551	Y

After A_1:

`CAR` $\rightarrow$

550	0.0
551	Y

In A_2:

`CAR` $\rightarrow$ 575

550	0.0
551	Y
575	30.5
576	X

After E_1:

`CAR` $\rightarrow$

550	
$\cdots$ 551	
575	30.5
576	X

Main simulation statements: Permanent entities

- **System:**
  - The System can have **permanent attributes**:

    permanent attributes are *global variables*.

    Example: `NMAX`, `NLG`, `NLN`.
  - The System can own **sets with no index**

    Example: `QUEUE`.

    The system is automatically created, and is never destroyed.
- "True" **Permanent entities:**
  - Permanent entities can have **permanent attributes** (global variables):

    Form: like arrays (index = specific permanent entity).

    Example: `STATUS(J)` = status (idle, busy) of disc `J`.
  - Permanent entities are **created** by a single statement:

    `CREATE EVERY` *en*;

    must be preceded by the definition of `N.`*en* (= number of entities of class *en*).

    Example:
    ```
    READ N.DISC
    CREATE EVERY DISC
    ```

Main simulation statements: Sets

- **Sets** have members and owners:
- **members** are usually temporary entities;
- **owners** are usually permanent entities:
  - sets with no index are owned by the system;
  - sets with index are owned by permanent entities.
- **Sorting policies:**
  - **FIFO**;
  - **LIFO**;
  - ***Ranked***: precedence given by the increasing or decreasing value of an attribute of the member entities.
- `FILE THE` p `IN THE` s inserts the entity pointed by p in set s;
- `REMOVE THE FIRST` q `FROM THE` s
  1. removes the first entity from set s;
  2. stores its pointer in a local variable named q.
- `REMOVE THE` p `FROM THE` s removes the entity pointed by p from set s.
- `IF THE` s `IS EMPTY` I / `IF THE` s `IS NOT EMPTY` I executes *I* if s is / is not empty.

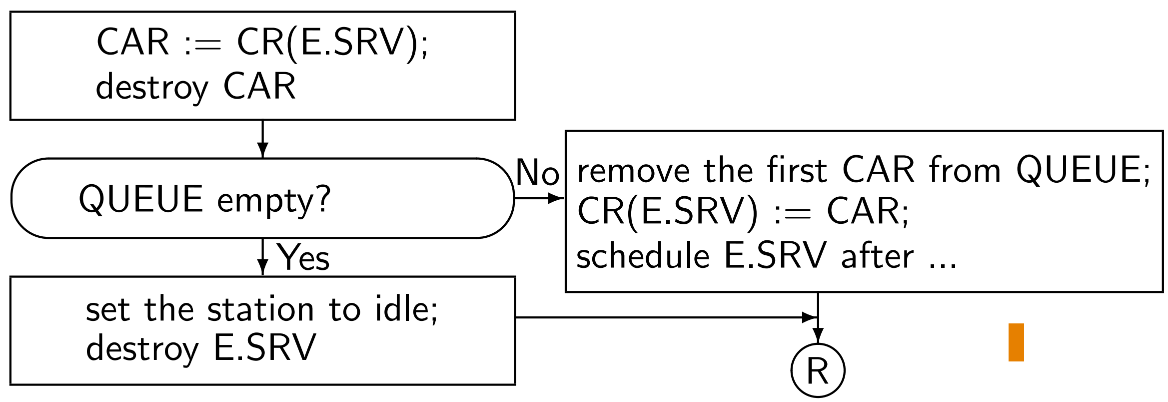
Events and event notices

- **Exogenous events** are scheduled through input data (no need to reserve memory);
- each scheduled **Endogenous event** needs a block of words (time, entity pointer(s), . . .);
- each event has an associated special temporary entity having its name (**event notice**);
- event notices must be created before scheduling the event,

  and destroyed when the event occurs;
- event notices can have attributes (typically used to store pointers to the interested entities);
- when an event is executed, the system stores the pointer to the event notice in a local variable having the event name.
- **Example:** In Arrival:

  set the station to busy state;
  T := random value from a uniform
  distribution in [T1,T2];
  create an E.SRV;
  CR(E.SRV) := CAR
  schedule E.SRV after T

In E.SRV:



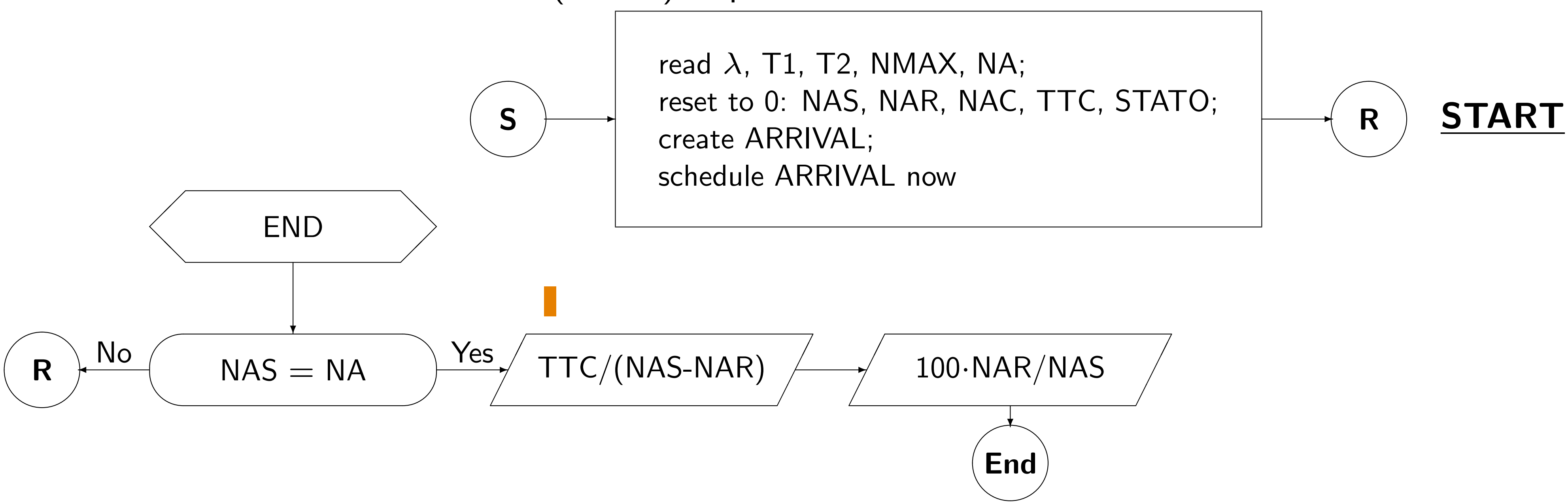
Example: gas station

	In A_1:	After A_1:	In E_1:	after `CAR:=CR(E.SRV)`
`CAR` $\rightarrow$	550			550
`E.SRV` $\rightarrow$	800		800	800
550	0.0	0.0	0.0	0.0
551	Y	Y	Y	Y
800	550	550	550	550

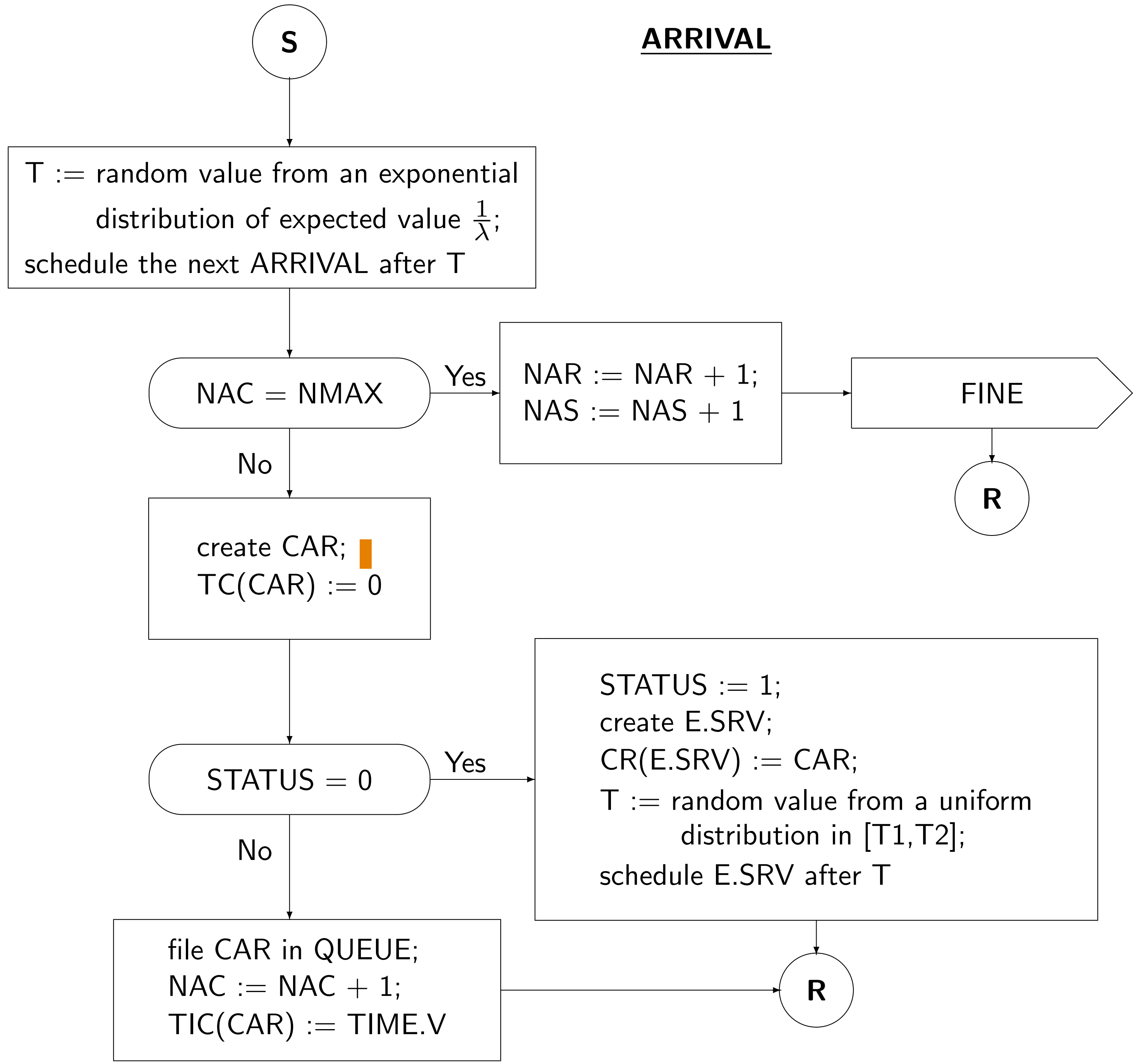
- `CREATE AN` *`ev`* `[CALLED` *`p`*`]`
  1. reserves a new **block of consecutive words** for a new event notice of class *ev*;
  2. defines a **local variable** *p* containing the corresponding pointer;
     if "`CALLED` *p*" is missing, the variable has name *ev*.
- `SCHEDULE THIS` *`ev`* `[CALLED` *`p`*`] AT` *`t`* schedules the event of pointer *p* (or *ev*) at t.
- **Note:** t = absolute time. Implemented as, e.g., `SCHEDULE THIS E.SRV AT TIME.V + T`
- `CANCEL THE` *`ev`* `[CALLED` *`p`*`]` cancels the event of class *ev* having pointer *p* (or *ev*), <u>without</u> destroying the event notice.

Example gas station revisited:

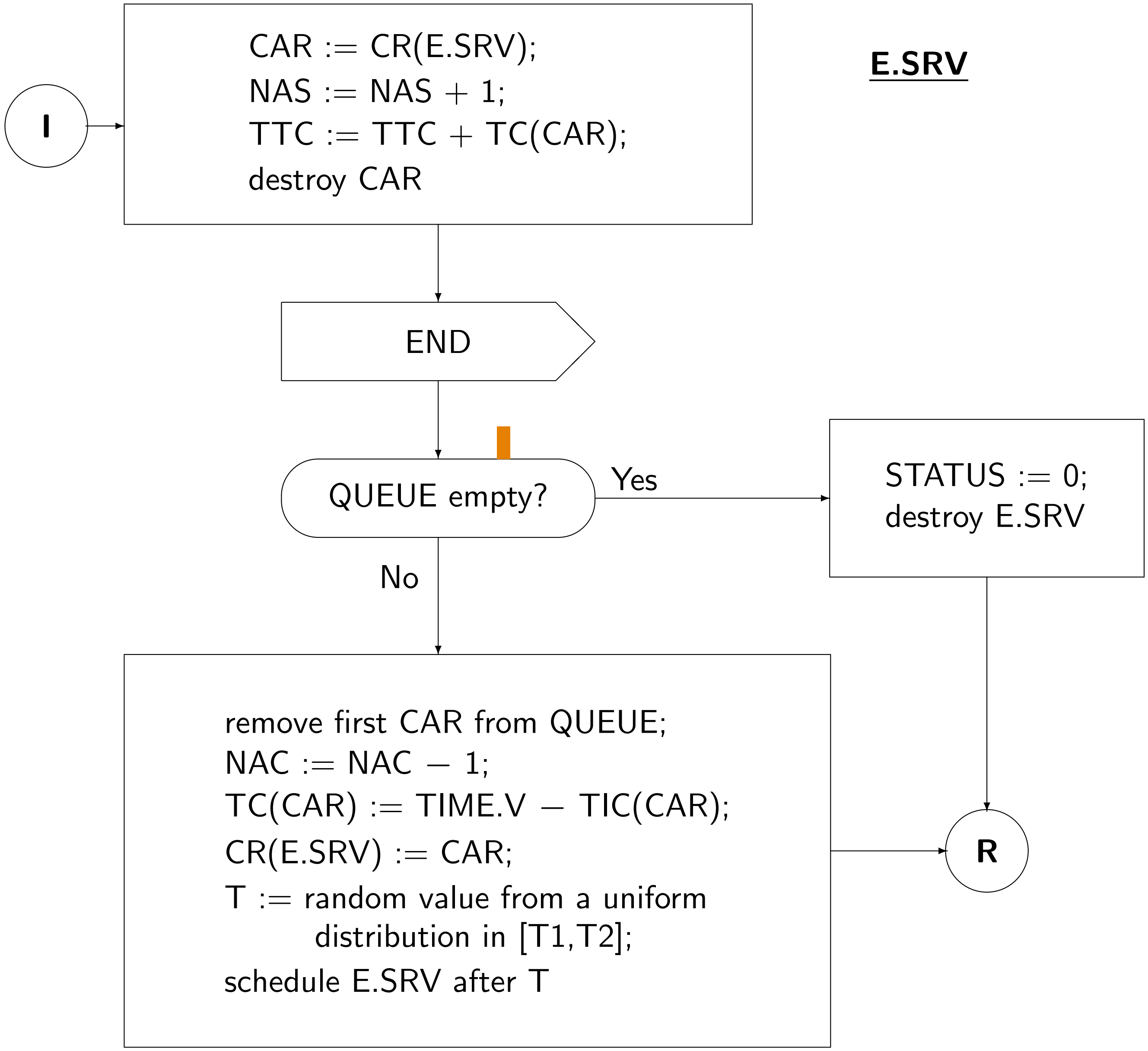
- Determine, over NA simulated cars, the percentage of rejected cars and the average queue time.
- CAR attributes: TIC(CAR) $=$ time instant the car enters the queue, TC(CAR) (time in queue).
- System attributes: input data (λ, T1, T2, NMAX, NA); inner attributes:
  - NAS (number of simulated cars), NAR (number of rejected cars);
  - TTC (total time in queue), NAC (number of cars in queue);
  - STATUS ($=0$ idle, $=1$ busy);
- one FIFO QUEUE, owned by the System, with member entities CAR;
- events START and ARRIVAL have no attribute;
- event E.SRV has attribute CR(E.SER) $=$ pointer to the served car.



ARRIVAL


S
T := random value from an exponential distribution of expected value $\frac{1}{\lambda}$;
schedule the next ARRIVAL after T
NAC = NMAX
Yes
NAR := NAR + 1;
NAS := NAS + 1
FINE
R
No
create CAR;
TC(CAR) := 0
STATUS = 0
Yes
STATUS := 1;
create E.SRV;
CR(E.SRV) := CAR;
T := random value from a uniform distribution in [T1,T2];
schedule E.SRV after T
No
file CAR in QUEUE;
NAC := NAC + 1;
TIC(CAR) := TIME.V
R

E.SRV


I
CAR := CR(E.SRV);
NAS := NAS + 1;
TTC := TTC + TC(CAR);
destroy CAR
END
QUEUE empty?
Yes
STATUS := 0;
destroy E.SRV
No
remove first CAR from QUEUE;
NAC := NAC − 1;
TC(CAR) := TIME.V − TIC(CAR);
CR(E.SRV) := CAR;
T := random value from a uniform
distribution in [T1,T2];
schedule E.SRV after T
R

How do events exchange information?

There are **only three possibilities:**

1. **Permanent attributes**

   Example: TTC is reset to 0 in START, updated in E.SRV, and used in END;

2. **Attributes of event notices**

   Example: CR(E.SRV) is defined in ARRIVAL and used in E.SRV;

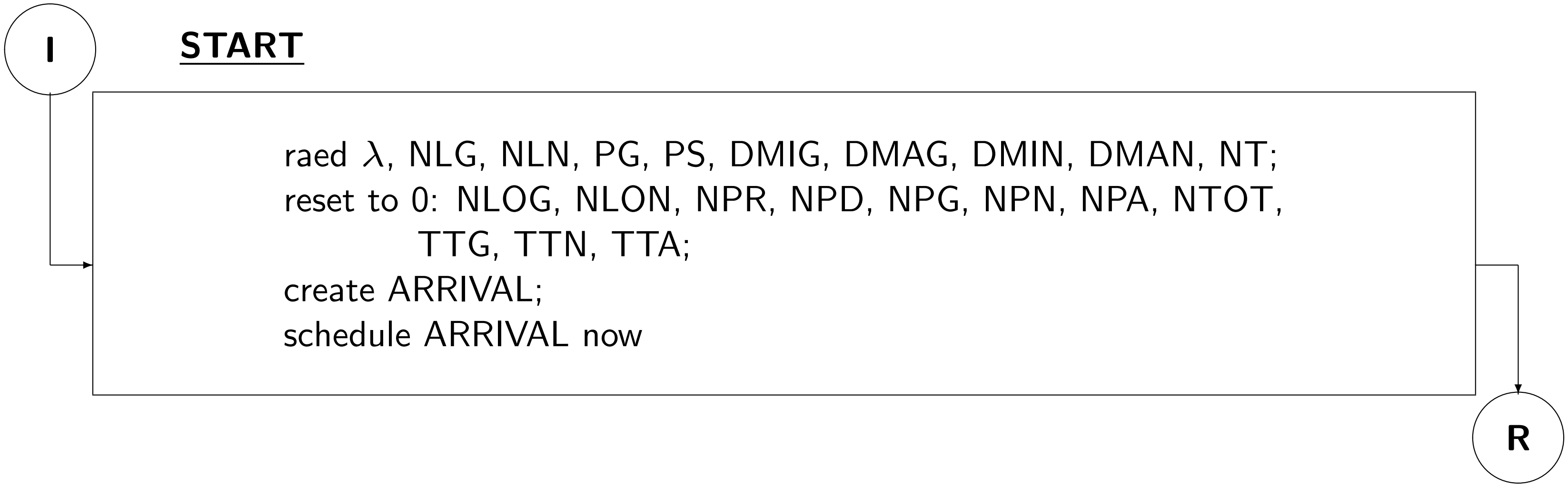
3. **Attributes of temporary entities** inserted and extracted from sets

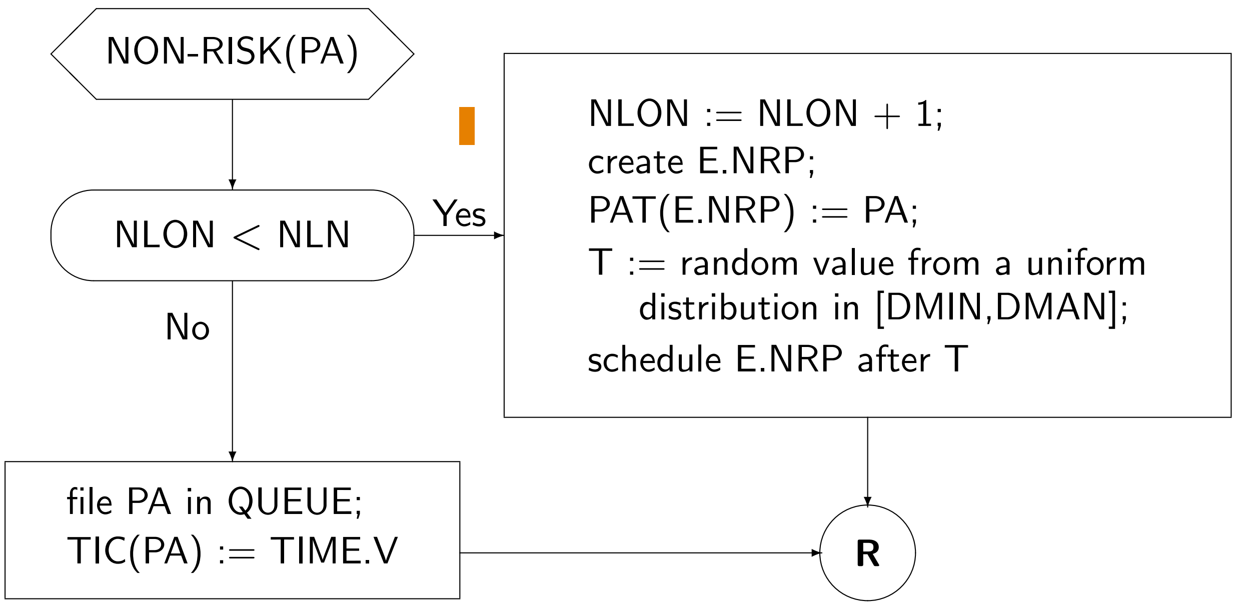
   Example: CAR is inserted in QUEUE in ARRIVAL, and removed from it in E.SRV, where TIC(CAR) is used.

Example small hospital revisited:

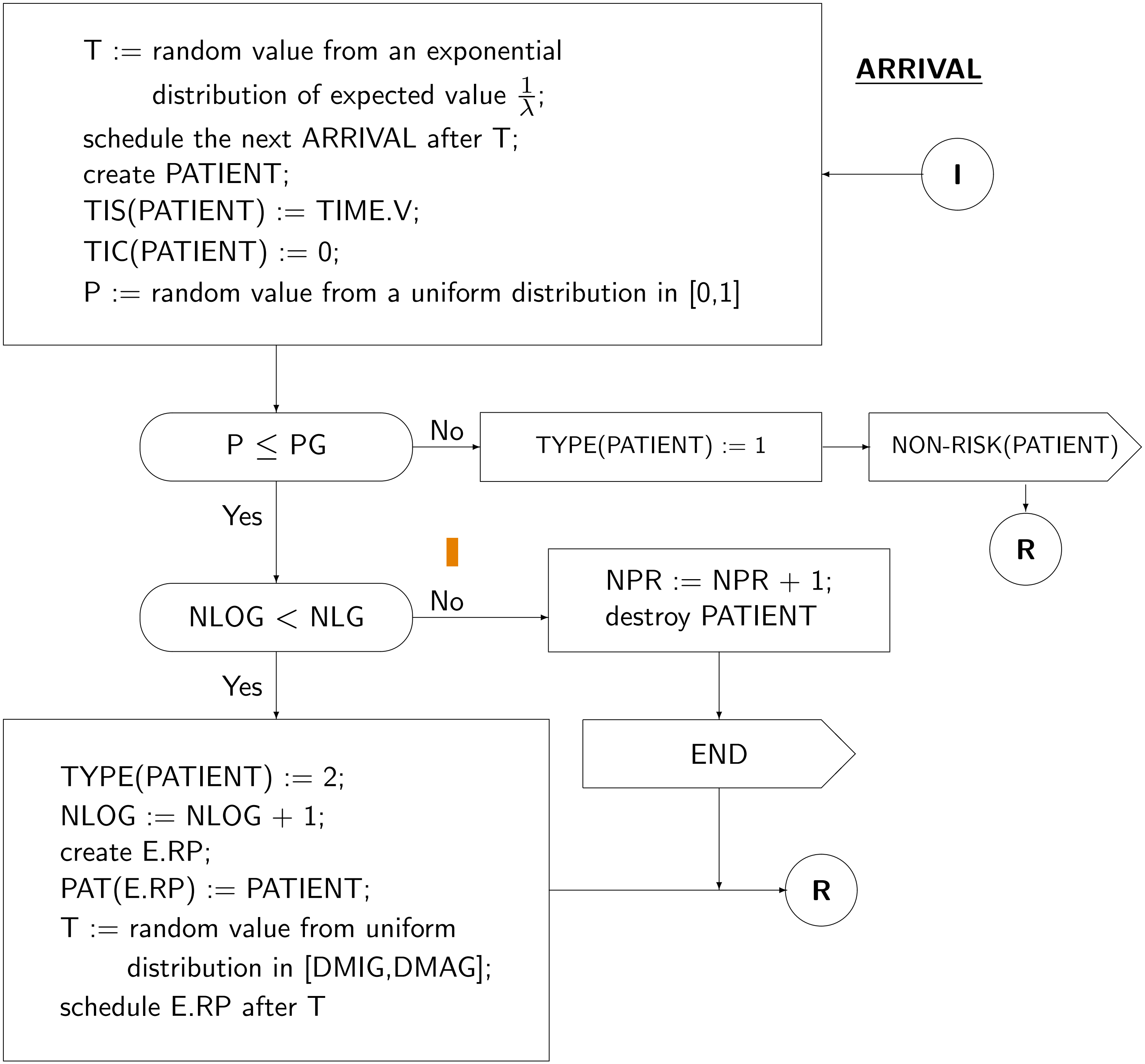
- Determine, over NT simulated patients,
  - average queue time for the patients who queued;
  - average times in the system for risk and non-risk patients, respectively;
  - percentage of rejected risk patients.
- PATIENT attributes:
  - TIS(PATIENT) $=$ time instant the patient enters the system,
  - TIC(PATIENT) $=$ time instant the patient enters the queue,
  - TYPE(PATIENT) ($= 1$ if non-risk, $= 2$ if risk).
- System attributes:
  input data (λ, NLG, NLN, PG, PS, DMIG, DMAG, DMIN, DMAN, NT);

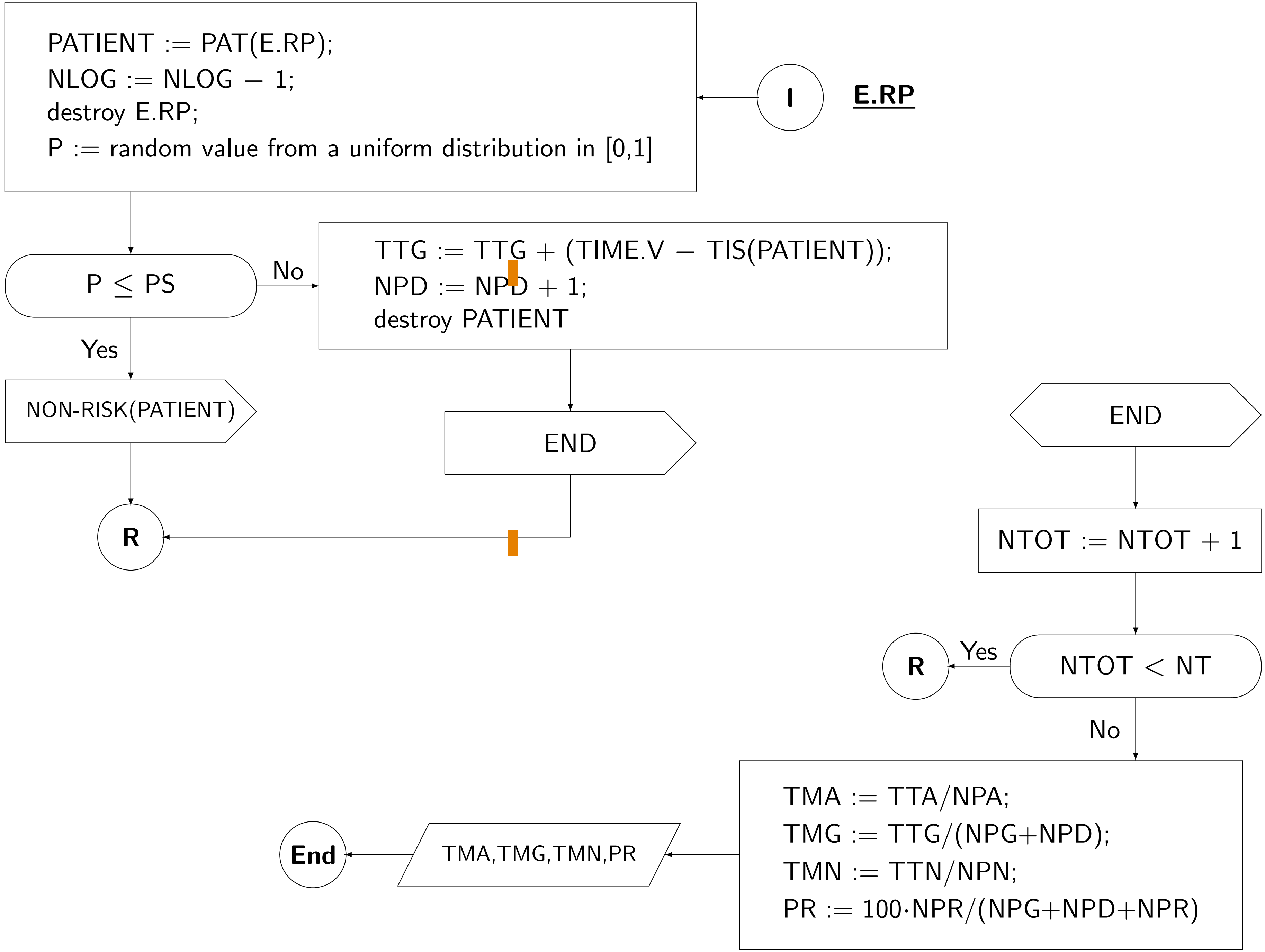
  inner attributes:
  - NPR, NPD and NPG (number of rejected, deceased and survived risk patients);
  - NPN (number of simulated non-risk patients),
  - NPA (number of patients who queued),
  - NTOT (number of simulated patients),
  - TTG and TTN (total time spent in the system by risk and non-risk patients),
  - TTA (total time spent in queue).
- one FIFO QUEUE, owned by the System, with member entities the PATIENTS;
- events START and ARRIVAL have no attribute;
- events E.RP and E.NRP have attribute PAT $=$ pointer to patient ending a risk/non-risk stay.


I
START
raed λ, NLG, NLN, PG, PS, DMIG, DMAG, DMIN, DMAN, NT;
reset to 0: NLOG, NLON, NPR, NPD, NPG, NPN, NPA, NTOT,
TTG, TTN, TTA;
create ARRIVAL;
schedule ARRIVAL now
R

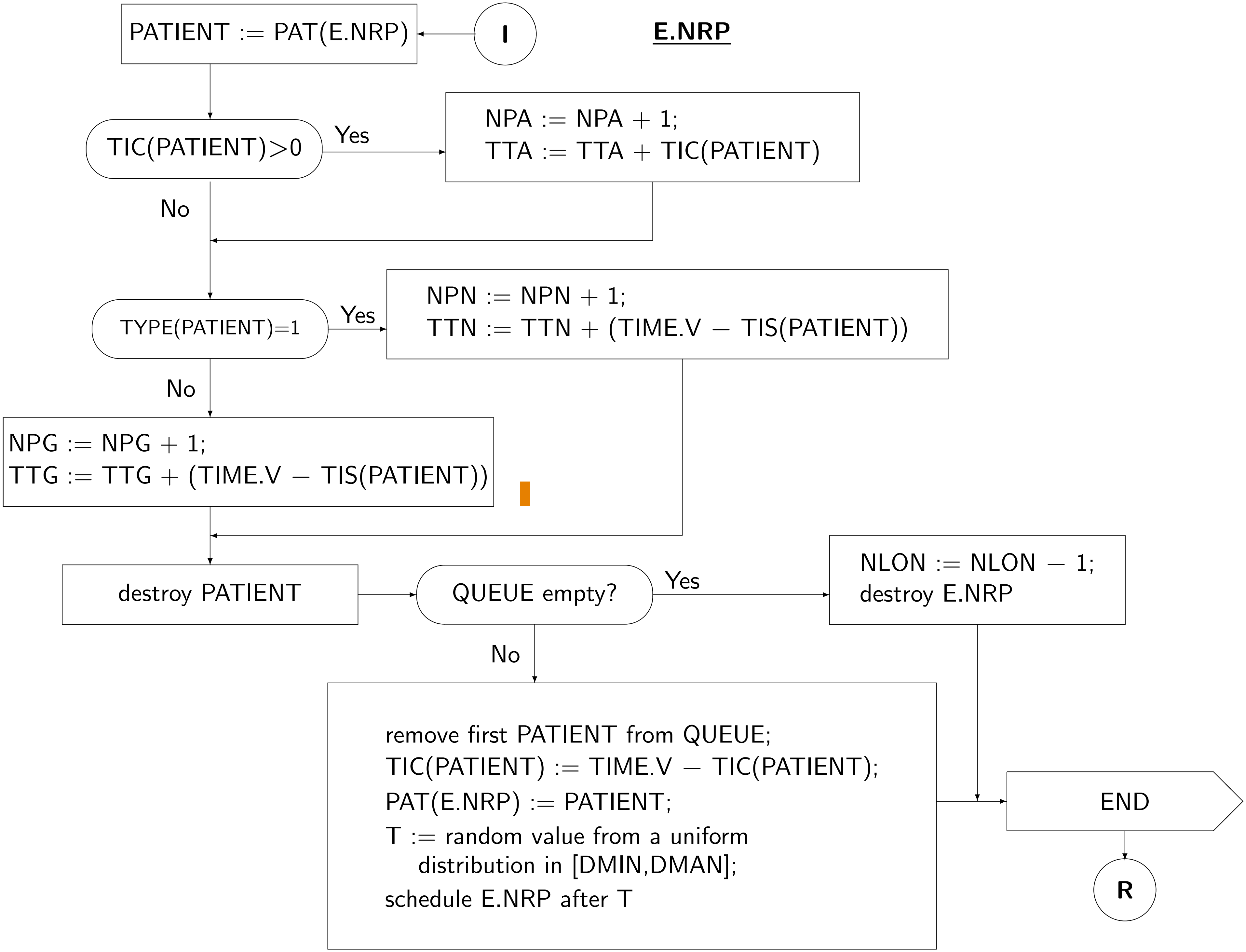

NON-RISK(PA)
NLON < NLN
Yes
No
NLON := NLON + 1;
create E.NRP;
PAT(E.NRP) := PA;
T := random value from a uniform
distribution in [DMIN,DMAN];
schedule E.NRP after T
file PA in QUEUE;
TIC(PA) := TIME.V
R

ARRIVAL


T := random value from an exponential distribution of expected value $\frac{1}{\lambda}$;
schedule the next ARRIVAL after T;
create PATIENT;
TIS(PATIENT) := TIME.V;
TIC(PATIENT) := 0;
P := random value from a uniform distribution in [0,1]
I
$P \leq PG$
No
TYPE(PATIENT) := 1
NON-RISK(PATIENT)
R
Yes
$NLOG < NLG$
No
NPR := NPR + 1;
destroy PATIENT
END
Yes
TYPE(PATIENT) := 2;
NLOG := NLOG + 1;
create E.RP;
PAT(E.RP) := PATIENT;
T := random value from uniform distribution in [DMIG,DMAG];
schedule E.RP after T
R


PATIENT := PAT(E.RP);
NLOG := NLOG − 1;
destroy E.RP;
P := random value from a uniform distribution in [0,1]
I
E.RP
P ≤ PS
No
TTG := TTG + (TIME.V − TIS(PATIENT));
NPD := NPD + 1;
destroy PATIENT
Yes
NON-RISK(PATIENT)
END
R
END
NTOT := NTOT + 1
R
Yes
NTOT < NT
No
TMA := TTA/NPA;
TMG := TTG/(NPG+NPD);
TMN := TTN/NPN;
PR := 100·NPR/(NPG+NPD+NPR)
TMA,TMG,TMN,PR
End

E.NRP



PATIENT := PAT(E.NRP)
I
TIC(PATIENT)>0
Yes
NPA := NPA + 1;
TTA := TTA + TIC(PATIENT)
No
TYPE(PATIENT)=1
Yes
NPN := NPN + 1;
TTN := TTN + (TIME.V − TIS(PATIENT))
No
NPG := NPG + 1;
TTG := TTG + (TIME.V − TIS(PATIENT))
destroy PATIENT
QUEUE empty?
Yes
NLON := NLON − 1;
destroy E.NRP
No
remove first PATIENT from QUEUE;
TIC(PATIENT) := TIME.V − TIC(PATIENT);
PAT(E.NRP) := PATIENT;
T := random value from a uniform distribution in [DMIN,DMAN];
schedule E.NRP after T
END
R

Implementation and Experiments

- **Debugging:**
  - not easy $\Leftarrow$ difficult to spot logical errors $\Leftarrow$ output not known a priori;
  - use of prints in debugging to monitor the program execution: creation/destruction of entities, event scheduling, filing/removals from sets, ...
- **Polarization:**
  - if the system is initially in idle state, the first entities are advantaged: idle resources, (almost) empty queues, ...
  - $\Rightarrow$ expected values depend on the simulation duration. Two methods to overcome this:
    1. create a reliable initial state: increased programming effort, experiments to test reliability; rarely used, unless an initial state is imposed;
    2. start in idle state but, after some time, reset the collected statistical data and perform the actual simulation.
       experiments to decide the duration of the first phase (until final outcomes do not vary)
- **Experiments:**
  - series of experiments with different input configurations to find the best one;
  - preferable to change one (or few) input parameter(s) at a time;
  - for each test, runs with different random sequences and mean of the expected values.

Pros and cons of simulation

Simulation is widely used as a decision support tool for systems whose behavior depends on random elements;

it is very general, in the sense that it is not based on any theoretical hypothesis, and hence it can be applied to very different contexts;

This methodology has many **Pros**. Indeed **Simulation**

- allows the impact of a decision on the system behavior to be evaluated without implementing it in practice;
- allows the analysis of the system evolution over time through time "compression";
- is conceptually simple and flexible;
- can be used to analyze real-world systems that cannot be conveniently modeled through mathematical tools;
- allows the inclusion of every relevant element of the real system;
- allows quick answers to natural questions like "What happens if ...?".

Pros and cons of simulation (cont'd)

The main **cons** of Simulation are:

- the development of good simulation models can be costly and time consuming;
- in order to provide useful evaluations, a good simulation model must include all the relevant system characteristics;
- simulation does not provide an optimal solution (like, e.g., LP or ILP). For a possible decision, it only provides the evaluation of the effect the decision has;
- in order to take an overall decision, managers must separately evaluate every possible choice, and then compare the results of all simulations to reach a final decision;
- a simulation model is generally very specific, and it is unlikely that it can be adapted to different situations.

Simulation it is one of the most widely used Operations Research tools. Together with

- **Linear Programming** and **Integer Linear Programming**, and
- methods based on **Graphs and Networks** (treated in the course *Network Optimization M*)

it is adopted by many industry managers as a decision support tool for many real world systems.

Good luck with your exam! Thank you for your attention