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Local Features (Part 2)

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Correspondences are Key !

- A great variety of Computer Vision problems can be dealt with by finding

corresponding points

between two (or more) images of a scene.

- Corresponding (aka homologous) points: image points which are the projection of the same 3D point in different views of the scene.

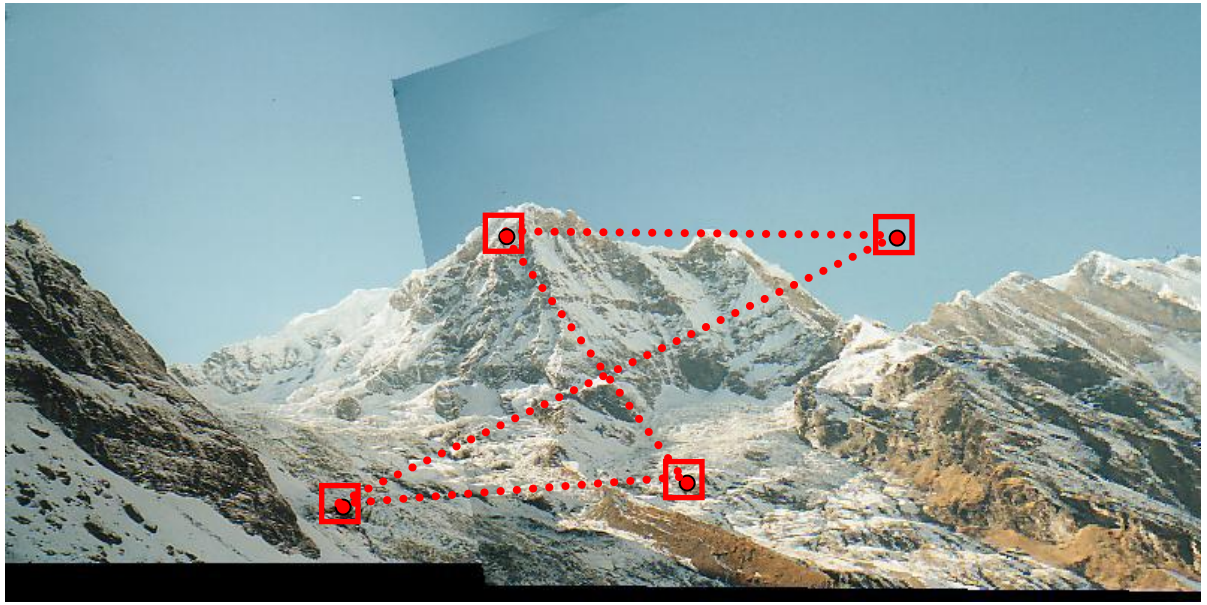
- Establishing correspondences may be difficult as homologous points may look different in the different views, e.g. due to viewpoint variations and/or light changes.

Example: Image Mosaicing

Create a larger image by aligning two images of the same scene.

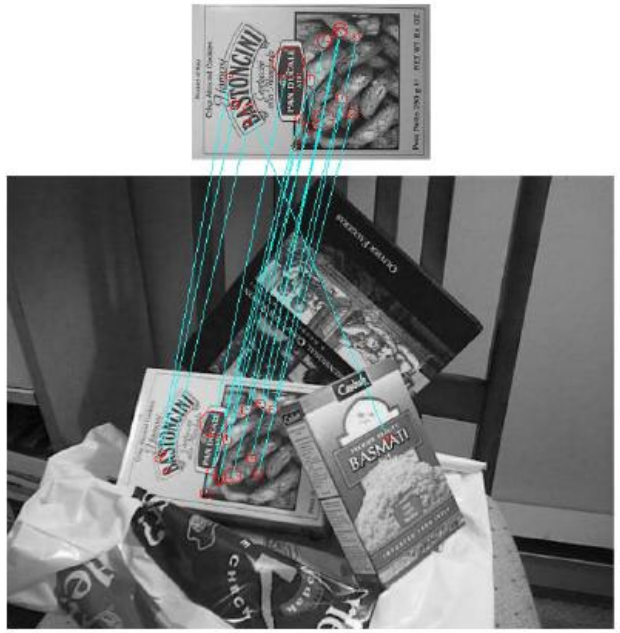
The two images may be aligned by estimating a *homography*, which requires at least 4 correspondences (the more the better).

1. Find salient points independently in the two images.
2. Compute a local "description" to recognize salient points.
3. Compare descriptors to tell apart salient points.



Other examples

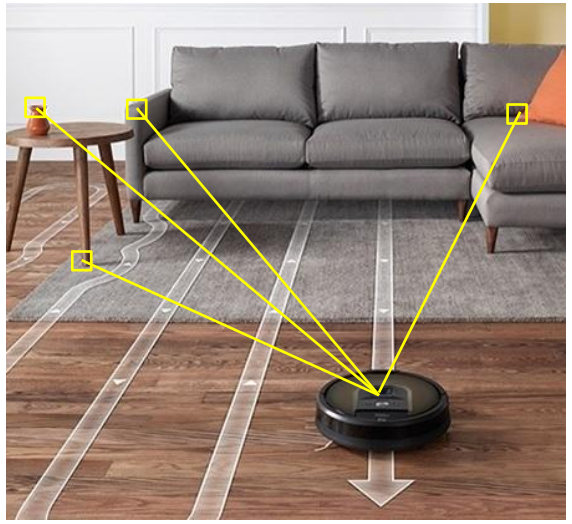
Instance-level Object Detection



3D Reconstruction from images, e.g. by *Structure-from-Motion [18]*

https://demuc.de/colmap/

SLAM (Simultaneous Localizazion and Mapping), e.g, for Robot Navigation



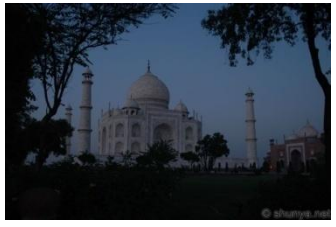
Camera Tracking, e.g. for Markerless Augmented Reality

The Local Invariant Features Paradigm

The task of establishing correspondences is split into 3 successive steps:

1. Detection of salient points (aka keypoints, interest points, features ...).
2. Description - Computation of a suitable descriptor based on a neighbourhood around a keypoint.
3. Matching descriptors between images.

and should be invariant (robust) to the tranformations that may relate images.



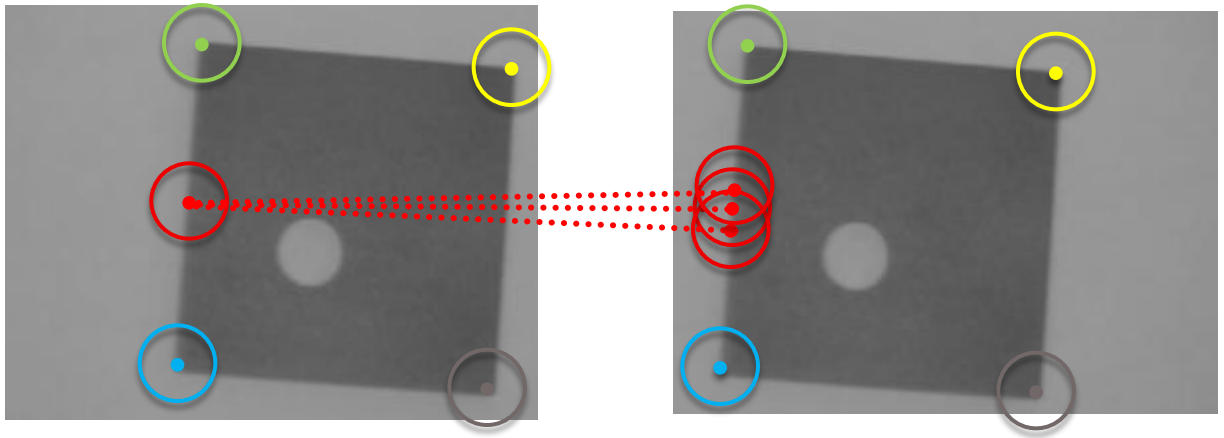
Properties of good detectors/descriptors

- **Detector**
  - **Repeatability**: it should find the same keypoints in different views of the scene despite the tranformations undergone by the images.
  - **Interestingness/Saliency:** it should find keypoints surrounded by informative patterns of intensities, which would render them amenable to be told apart by the matching process.

- **Descriptor**
  - **Distinctiveness vs. Robustness Trade-off**: the description algorithm should capture the salient information around a keypoint, so to capture the distinctive pattern and disregard changes due to nuisances (e,g. light changes) and noise.
  - **Compactness**: the description should be as concise as possibile, to minimize memory occupancy and allow for efficient matching.

Speed is desirable for both, and in particular for detectors, which need to be run on the whole image whereas decriptors are computed at keypoints only.

Keypoints vs. Edges



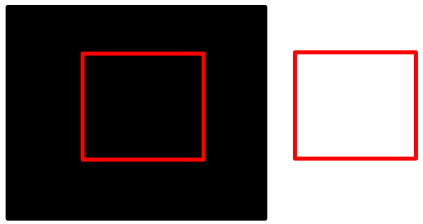
- Edge pixels can be hardly told apart as, locally, they look very similar along the direction perpendicular to the gradient.
- Conversely, pixels exhibiting a large variation along both the gradient direction and the direction perpendicular to the gradient are more amenable to establish reliable correspondences.

Moravec Corner Detector [1]

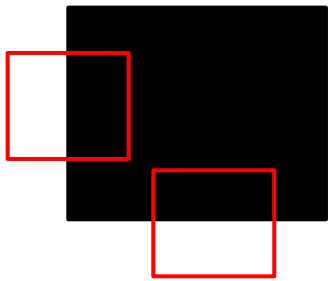
$$C(\mathbf{p}) = \min_{\mathbf{q} \in n_8(\mathbf{p})} \|N(\mathbf{p}) - N(\mathbf{q})\|^2$$ →

the *cornerness* at p is given by the minimum squared difference between the patch (e.g. 7x7) centerd at **p** and those centered at its 8 neighbours.

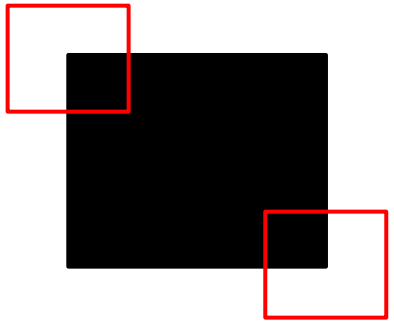
uniform region: no change in all directions



edge: no change along the edge direction

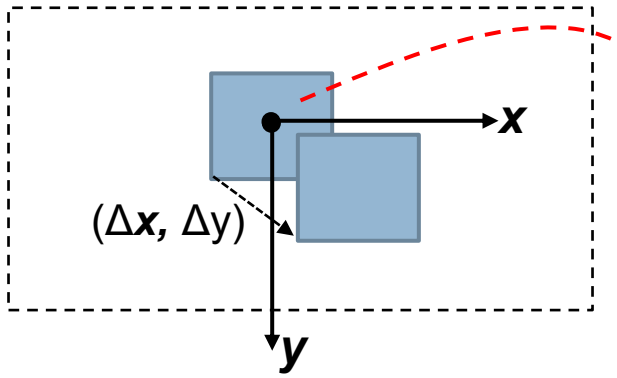


corner: significant change in all directions.



Harris Corner Detector (1)

Harris&Stephens [2] proposed to rely on a continuous formulation of Moravec's "error" function. Denoted as (Δx, Δy) a generic infinitesimal shift, such a function may be written as :



$$w(x,y) = \begin{cases} 1 & \rightarrow \blacksquare \\ 0 & \rightarrow \square \end{cases}$$

$$E(\Delta x, \Delta y) = \sum_{x,y} w(x,y)\big(I(x+\Delta x, y+\Delta y) - I(x,y)\big)^2$$

Due to the shift being infinitesimal, we can deploy Taylor's expansion of the intensilty function at (x,y):

$$I(x+\Delta x, y+\Delta y) - I(x,y) \cong \frac{\partial I(x,y)}{\partial x}\Delta x + \frac{\partial I(x,y)}{\partial y}\Delta y = I_x(x,y)\Delta x + I_y(x,y)\Delta y$$

Harris Corner Detector (2)

$$E(\Delta x,\Delta y)=\sum_{x,y} w(x,y)\left(I_x(x,y)\Delta x+I_y(x,y)\Delta y\right)^2=$$

$$=\sum_{x,y} w(x,y)\left(I_x(x,y)^2\,\Delta x^2+I_y(x,y)^2\,\Delta y^2+2\,I_x(x,y)I_y(x,y)\Delta x\Delta y\right)$$

$$=\sum_{x,y} w(x,y)\left([\Delta x\ \ \Delta y]\begin{pmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y)\\ I_x(x,y)I_y(x,y) & I_y(x,y)^2\end{pmatrix}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}\right)$$

$$=[\Delta x\ \ \Delta y]\begin{pmatrix} \sum_{x,y} w(x,y)I_x(x,y)^2 & \sum_{x,y} w(x,y)I_x(x,y)I_y(x,y)\\ \sum_{x,y} w(x,y)I_x(x,y)I_y(x,y) & \sum_{x,y} w(x,y)I_y(x,y)^2\end{pmatrix}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}$$

$$E(\Delta x,\Delta y)=[\Delta x\ \ \Delta y]\,M\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}$$

Harris Corner Detector (3)

M encodes the local image structure around the considered pixel.
To understand why, let us hypothesize that M is a diagonal matrix:

$$M = \begin{pmatrix} \sum_{x,y} w(x,y) I_x(x,y)^2 & \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) \\ \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) & \sum_{x,y} w(x,y) I_y(x,y)^2 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \rightarrow$$

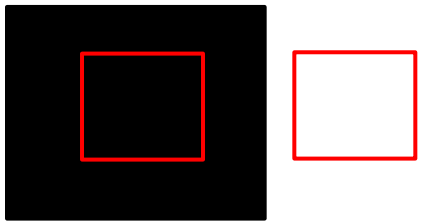
$$E(\Delta x, \Delta y) = [\Delta x \;\; \Delta y] \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

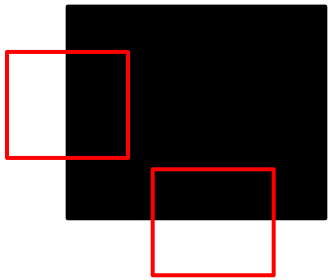
$$= \lambda_1 \Delta x^2 + \lambda_2 \Delta y^2$$

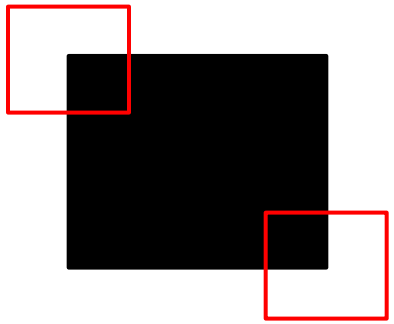
$\lambda_1, \lambda_2 \cong 0$: **Flat**

$\lambda_1 \gg \lambda_2$: **Edge**

$\lambda_1, \lambda_2 \uparrow$: **Corner**





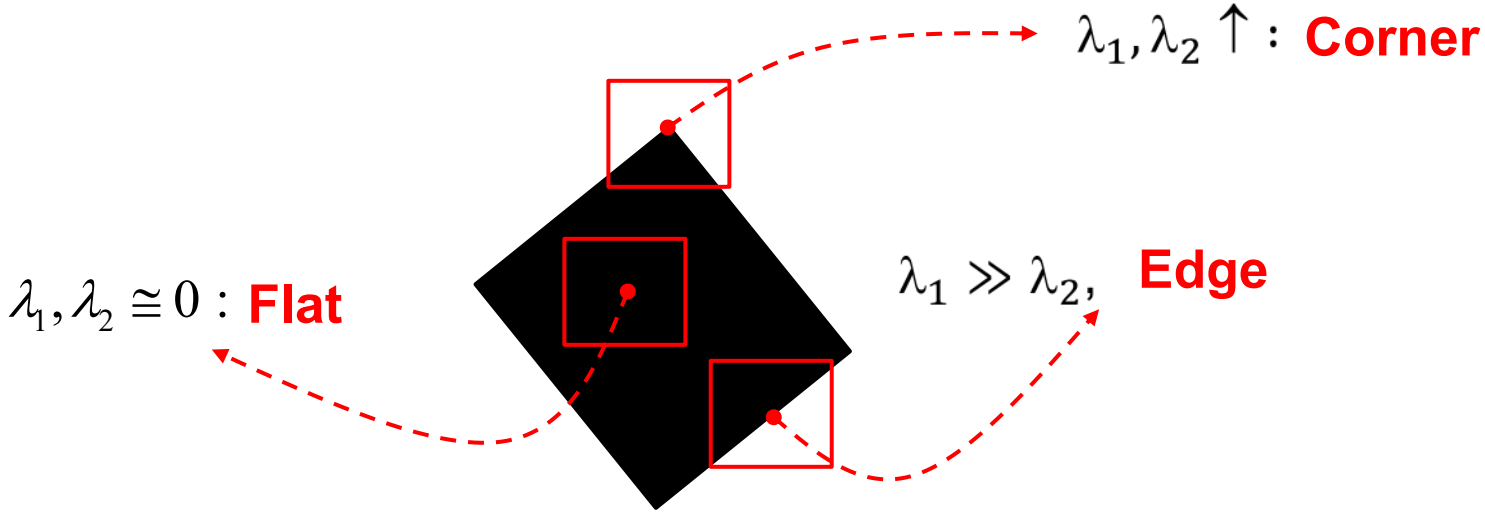


Harris Corner Detector (4)

Indeed, the previous considerations have general validity as M is real and symmetric, and thus can always be diagonalized by a rotation of the image coordinate system:

$$M = R \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R^T$$

the columns of *R* are the ortogonal unit eigenvectors of M, λ_i the corresponding eigenvalues. R^T is the rotation matrix that aligns the eigenvectors of M to the image axes.



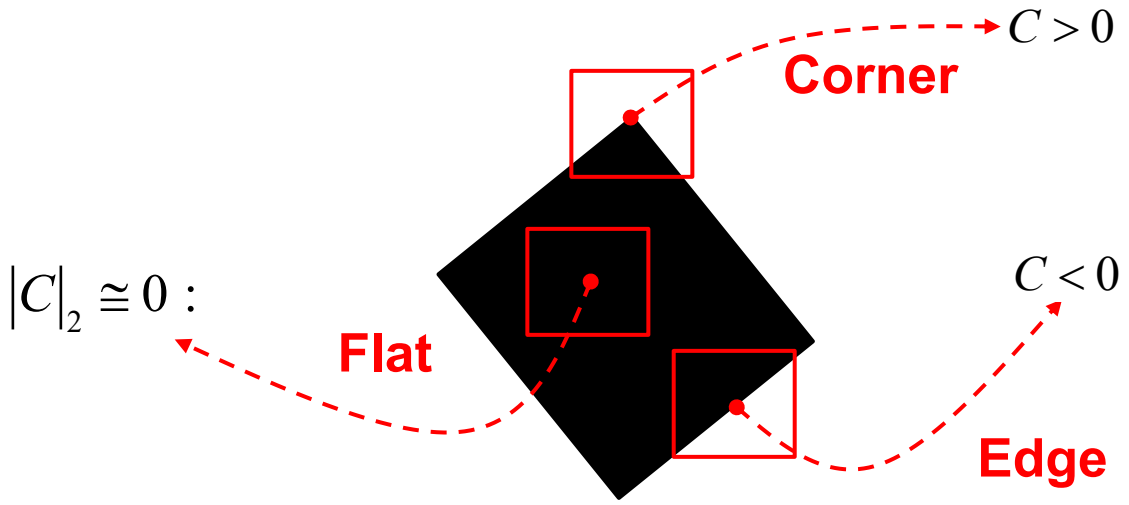
Harris Corner Detector (5)

Computing the eigenvalues of M may be slow. Thus, Harris&Stephens propose to compute a more efficient "cornerness" function:

$$C = \det(M) - k \cdot \mathrm{tr}(M)^2 = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2$$

$k \in [0.04; 0.15]$

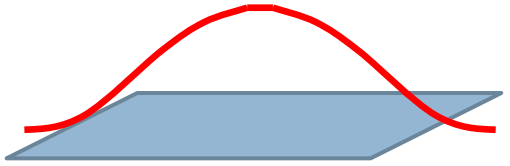
Analyis of the above function would show that:



Harris Corner Detector (6)

- **The Harris corner detection algorithm can thus be summarized as follows:**
  1. **Compute *C* at each pixel.**
  2. **Select all pixels where C is higher than a chosen positive threshold (*T*).**
  3. **Within the previous set, detect as corners only those pixels that are local maxima of C (NMS).**
- **It is worth highlighting that the weighting function *w(x,y)* used by the Harris corner detector is *Gaussian* rather than Box-shaped, so to assign more weight to closer pixels and less weight to those farther away.**

1 in window, 0 outside

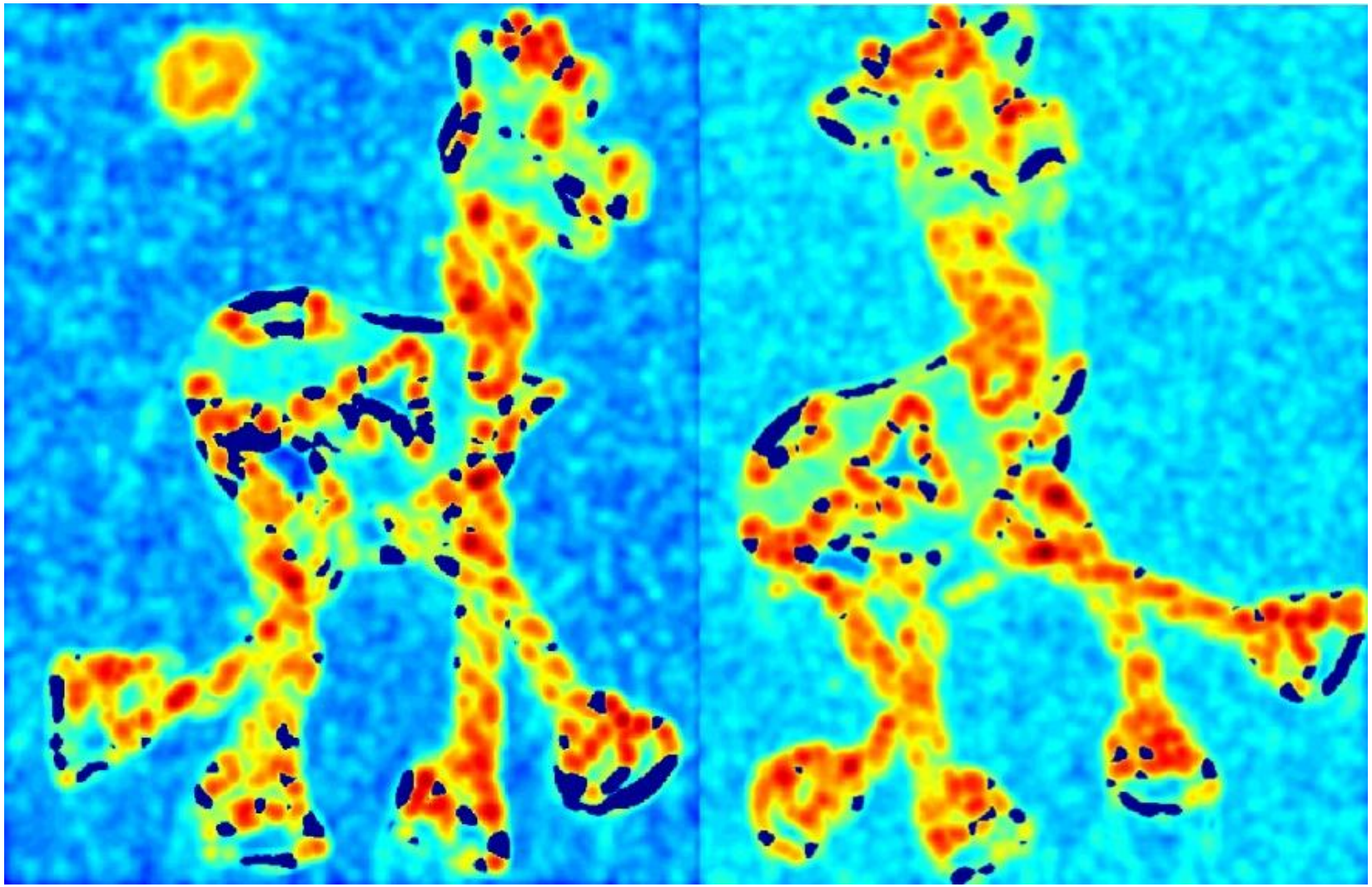


Gaussian

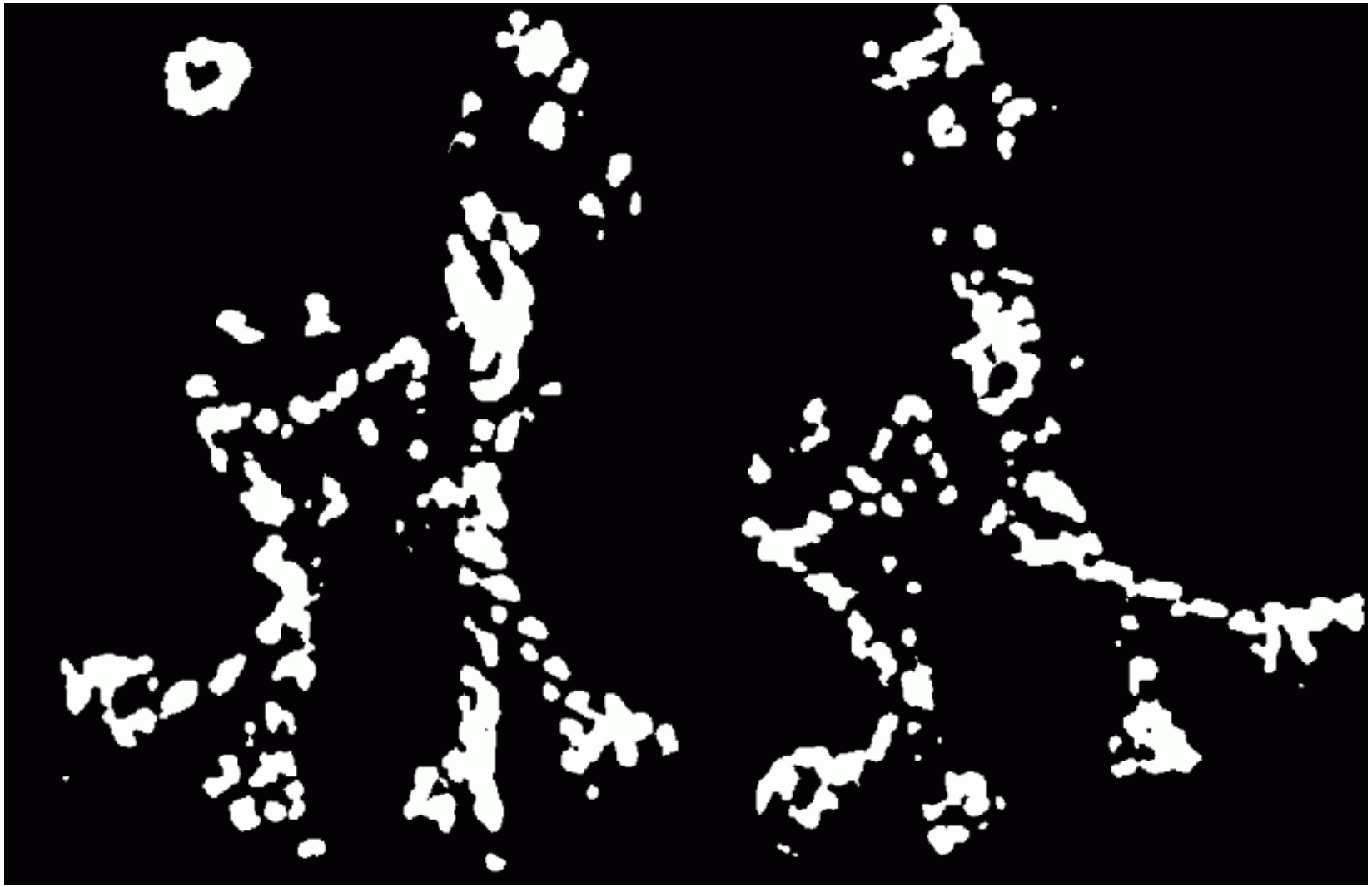
OpenCV: cv2.cornerHarris

Example: input images

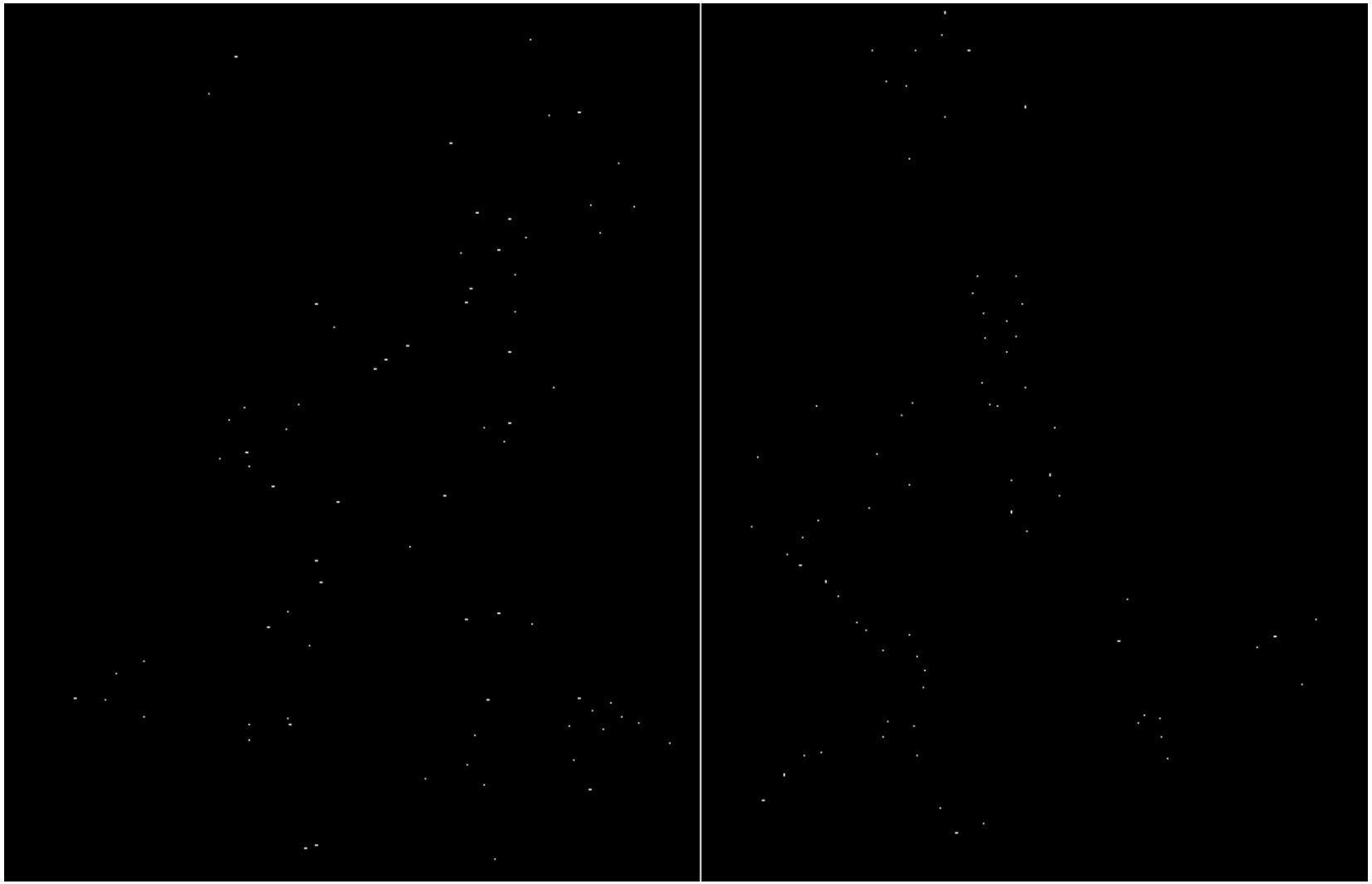
Example: *C* displayed as a heatmap



Example: $C > T$



Example: *NMS*

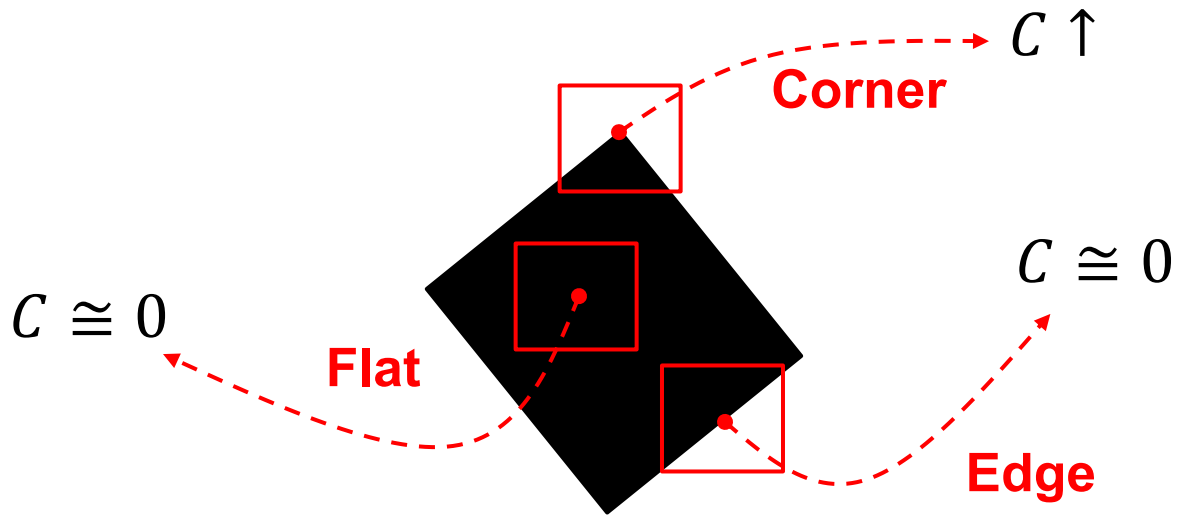


Example: corners displayed on input images

The Shi-Tomasi Corner Detector

A popular variant of the Harris Corner Detector is due to Shi and Tomasi [3] who showed that a different *cornerness* function provides better results when tracking corner points across video frames (*good features to track*):

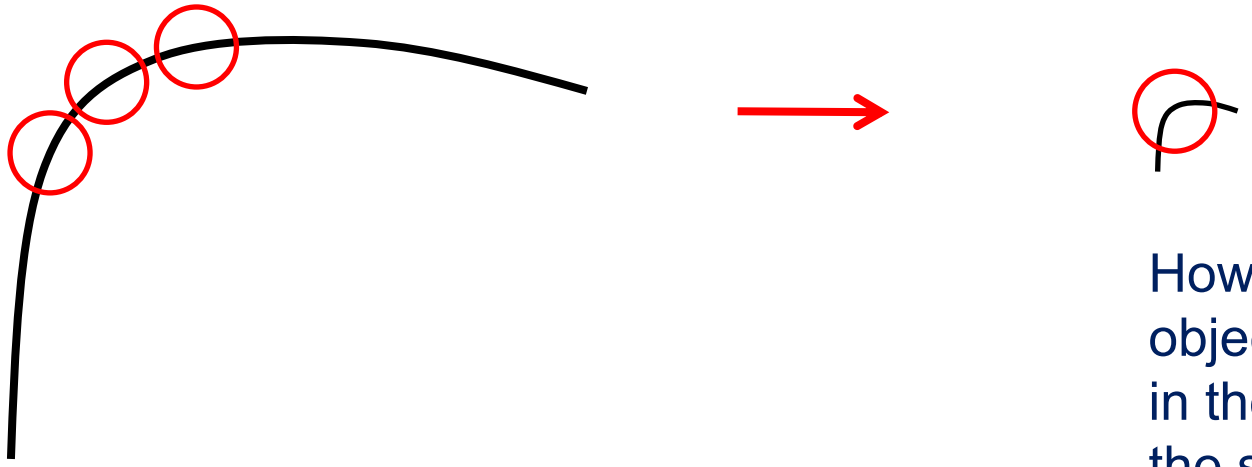
$$C = \min(\lambda_1, \lambda_2)$$



OpenCV: cv2.goodFeaturesToTrack

Scale Invariance (1)

- **Is the Harris Corner Detector invariant to scale changes ?**



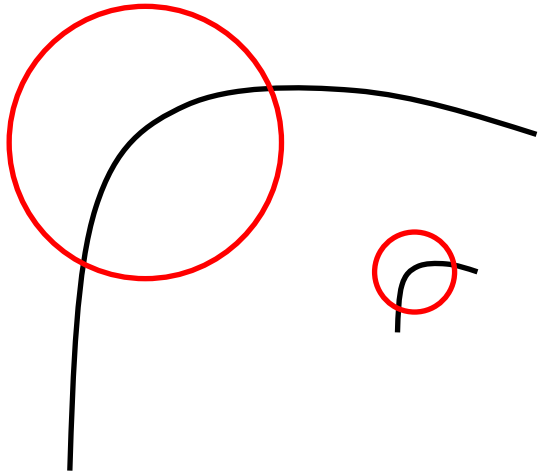
Given the chosen size of the detection window, all the points along the border are likely to be classified as **edges**.

However, should the object appear smaller in the image, use of the same window size would lead to detect a **corner**.

The use of a <u>fixed-size detection window</u> makes it impossible to repeatably detect homologous keypoints when they appear at different scales in images.

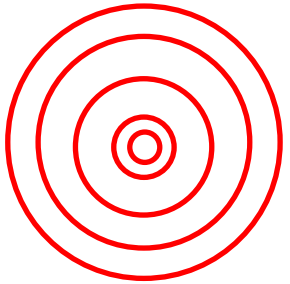
How to achieve scale invariance ?

- Images contain **keypoints at different scales**, i.e. points that stand-out as salient as long as a proper neighnourhood size is chosen to evaluate the salinecy criterion.



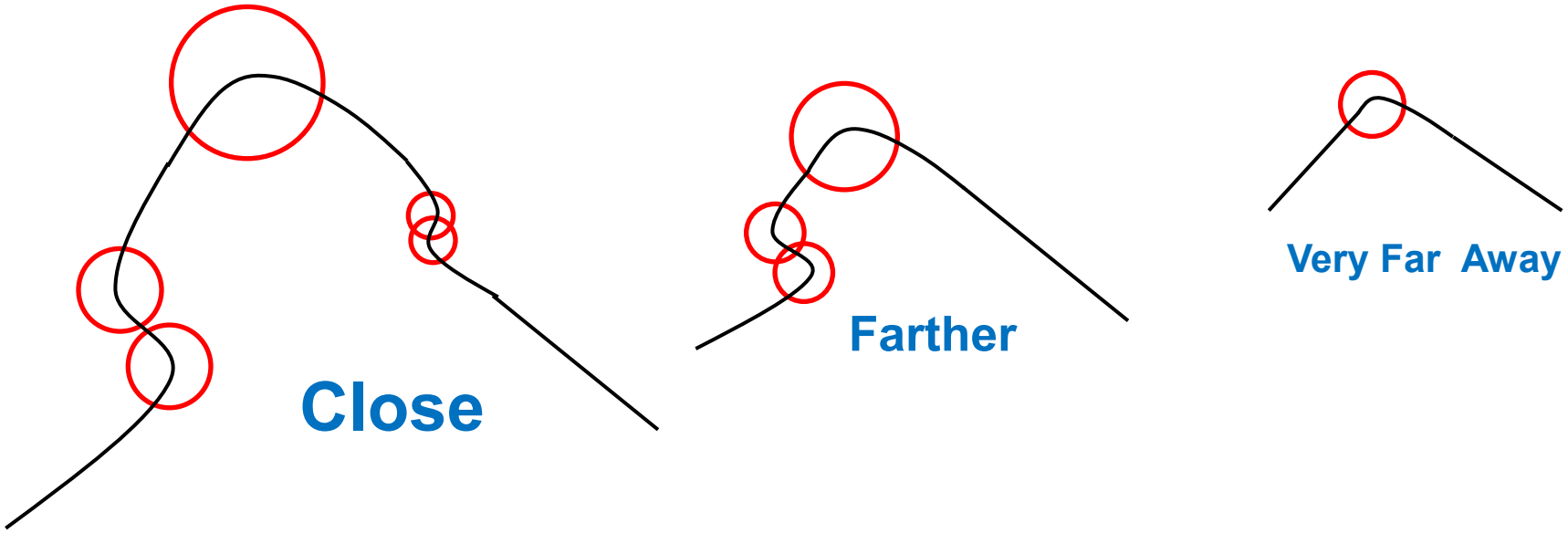
- The more keypoints we detect the higher the chances to establish correspondences between images.

- Thus, detecting <u>all</u> keypoints calls for a “tool” capable of analyzing the image across the whole range of the scales deemed as relevant.



Scale Invariance (2)

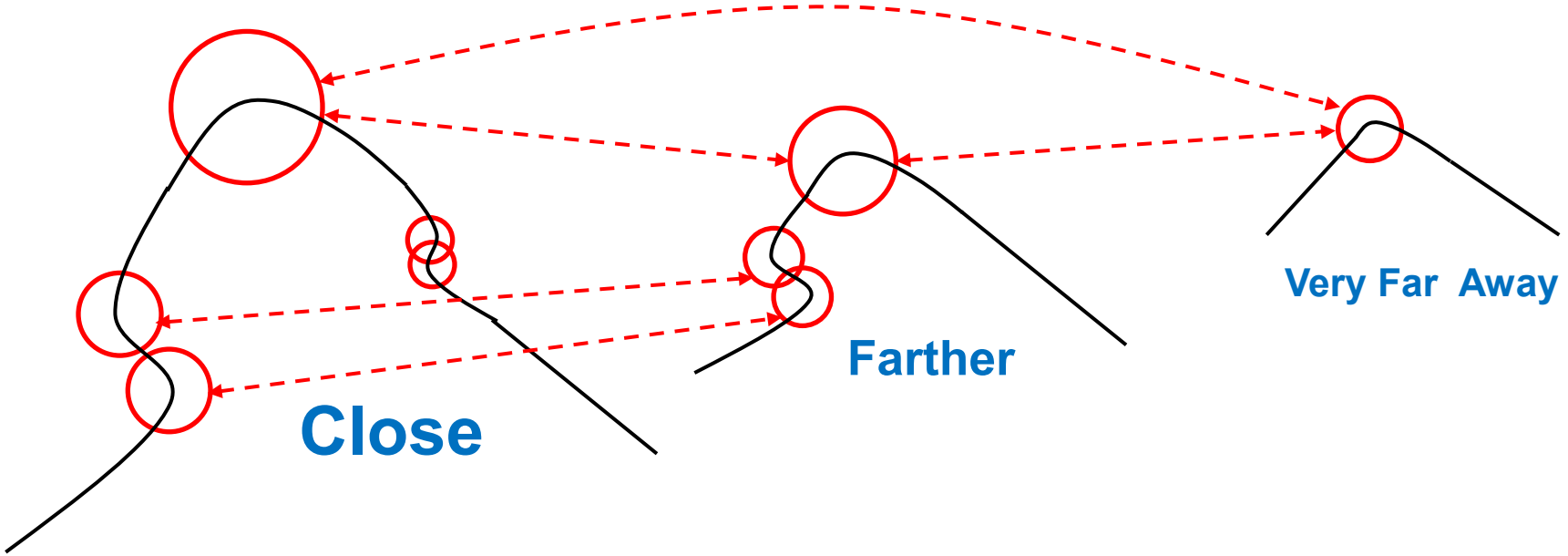
- Depending on the acquisition settings (distance and focal length) an object my look differently in the image, in particular it may exhibit more/less details (i.e. features).



- Hence, features do exist within a certain range of scales. The previously mentioned multi-scale feature detection tool would thus also allow for finding homologous features despite they may show-up at different scales in different images: the same feature would simply be detected at different steps within a multi-scale detection process.

Scale Invariance (3)

- We detect features in order to match their descriptors.



- Yet, the neighbourhoods surrounding large scale features are far richer of details than those around small scale ones. The higher the scale the finer the details present in the neighbourhood of a feature.
- To make it possible to compute similar descriptors – and so match them- the details that do not appear across the range of scales should be canceled-out by means of image smoothing.

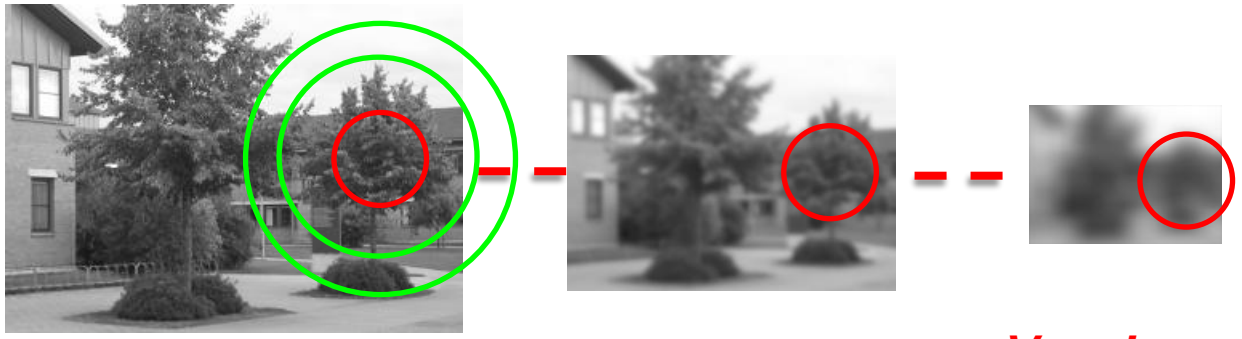
Multi-scale feature detection (1)

- Scale invariance has been the main issue addressed by more advanced local invariant features.
- One key finding concerns applying a ***fixed-size detection window*** on increasingly ***downsampled*** versions of the input image. Indeed, a fixed window size has a larger ***receptive field*** in a downsampled version of the input image.

Small Features ***Larger Features*** ***Very Large Features***

Multi-scale feature detection (2)

- Scale invariance has been the main issue addressed by more advanced local invariant features.
- One key finding concerns applying a ***fixed-size detection window*** on increasingly ***downsampled*** and ***smoothed*** versions of the input image. Indeed, a fixed window size has a larger ***receptive field*** in a downsampled version of the input image.



Small Features *Larger Features* *Very Large Features*

- As features exist across a range of scales, we need criteria for both *feature detection* and *optimal* (aka *characteristic*) *scale selection*.

Gaussian Scale-Space

- A ***Scale-Space*** is a one-parameter (i.e. ***scale***) family of images created from the original one so that the structures at smaller scales are successively suppressed by smoothing operations. Morever, one would not wish to create new structures while smoothing the images. In other words, a scale-space should continuosly simplify the image without introducing artifacts.

- Several researchers, such as, prominently, Witkin [4] and Koenderink [5], have studied the problem and shown that a *Scale-Space* must be realized by ***Gaussian Smoothing***.

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

- Thus, a *Scale-Space* is created by repeatedly smoothing the original image with larger and larger Gaussian kernels or, equivalently, by solving the 2D diffusion PDE over time starting from the original image:

$$\frac{\partial I(x, y, t)}{\partial t} = k\nabla^2 I(x, y, t) \rightarrow \sigma = \sqrt{2kt}$$

Feature Detection & Scale Selection

- **The Gaussian Scale-Space is the right tool to smooth an image in order to progressively simplify its content. However, it neither includes any criterion to detect features nor to select their characteristic scale. Indeed, as features exist across a range of scales within the Gussian Scale-Space, the notion of characteristic scale deals with establishing at which scale a feature turns out maximally interesting and should therefore be described.**
- **The fundamental research work on multi-scale feature detection and automatic scale selection is due to Lindberg [6], who proposed to find the extrema of certain combinations of *scale-normalized derivatives* of the *Gaussian Scale-Space* (*normalized Gaussian derivatives*).**

  The method consists in stacking the chosen Gaussian derivatives and seeking for the extrema in space (pixel position) and along scale (i.e. σ). As we smooth more (larger σ) derivatives become weaker and struggle to stand-out as extrema along scale. Hence, Gaussian derivatives are multiplied by σ^2 such that weaker/stronger ones are normalized by larger/smaller values.

Scale-Normalized LOG

Between Lindberg's proposed combination of derivatives is the scale-normalized Laplacian of Gaussian (LOG):

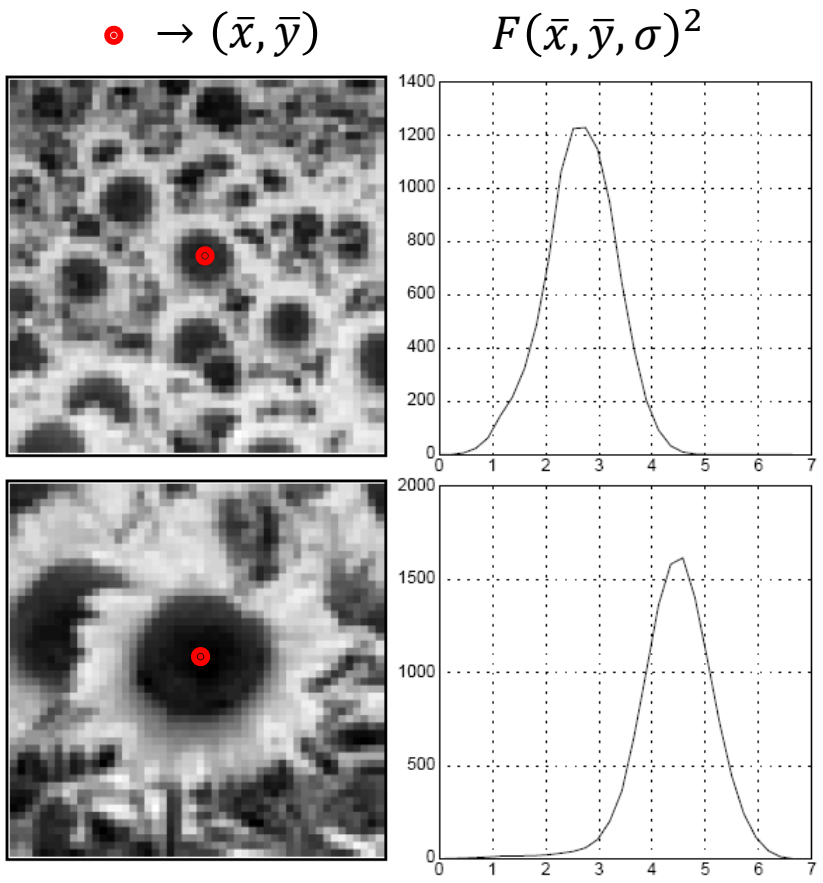
$$F(x, y, \sigma) = \sigma^2 \nabla^2 L(x, y, \sigma) = \sigma^2 (\nabla^2 G(x, y, \sigma) * I(x, y))$$

normalization

Considering the centers (red points) of the two dark blobs, it can be observed.that the extremum of the scale-normalized LOG is found at a larger scale for the larger blob.

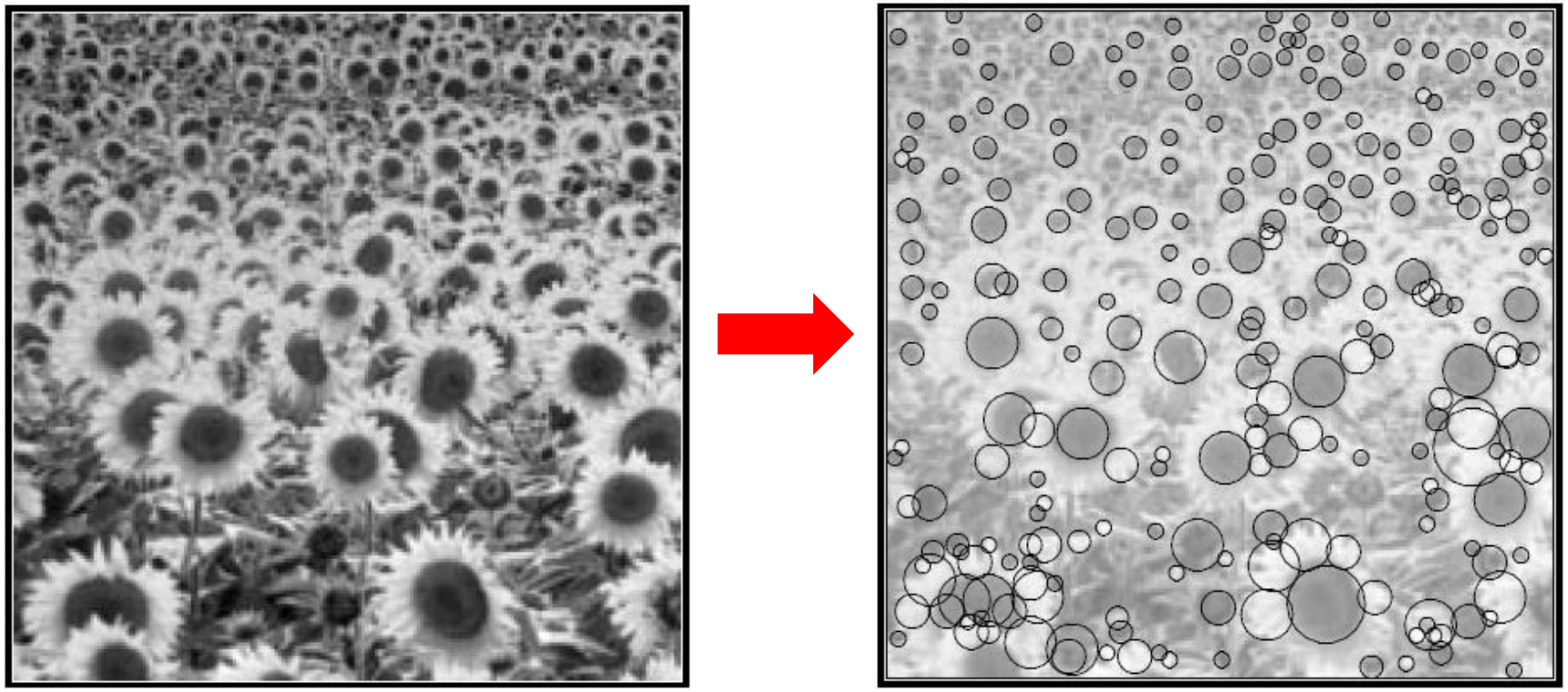
The ratio between the two characteristic scales (is roughly the same as the ratio between the sizes (diameters) of the two blobs.

Thus, with the scale-normalized LOG, an extremum at $(\bar{x}, \bar{y}, \bar{\sigma})$ identifies a feature at pixel position $(\bar{x}, \bar{y})$ having scale equal to $\bar{\sigma}$



Multi-Scale Feature Detection

Features (blob-like) and scales detected as extrema of the scale-normalized LOG.



DoG Detector

- Lowe [7] proposes to detect *keypoints* by seeking for the extrema of the DoG (*Difference of Gaussian*) function across the *(x,y,σ)* domain:

Gaussian Scale-Space

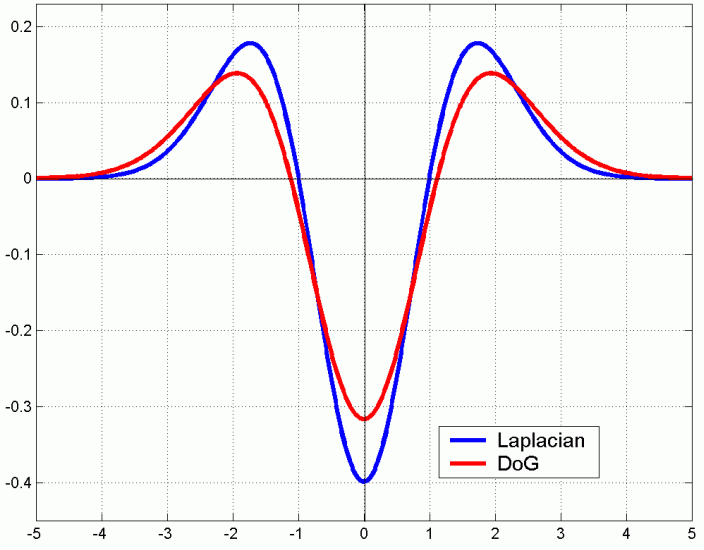
$$DoG(x,y,\sigma) = (G(x,y,k\sigma) - G(x,y,\sigma)) * I(x,y) = \underbrace{L(x,y,k\sigma)}_{\text{Gaussian Scale-Space}} - \underbrace{L(x,y,\sigma)}_{\text{Gaussian Scale-Space}}$$

- This approach provides a ***computationally efficient*** approximation of Lindeberg's scale-normalized LOG:

$$G(x,y,k\sigma) - G(x,y,\sigma) \approx (k-1)\sigma^2\Delta^2 G(x,y,\sigma)$$

- As (*k* -1) is a constant factor, it does not influence extrema location. As such, the choice of k is not critical.

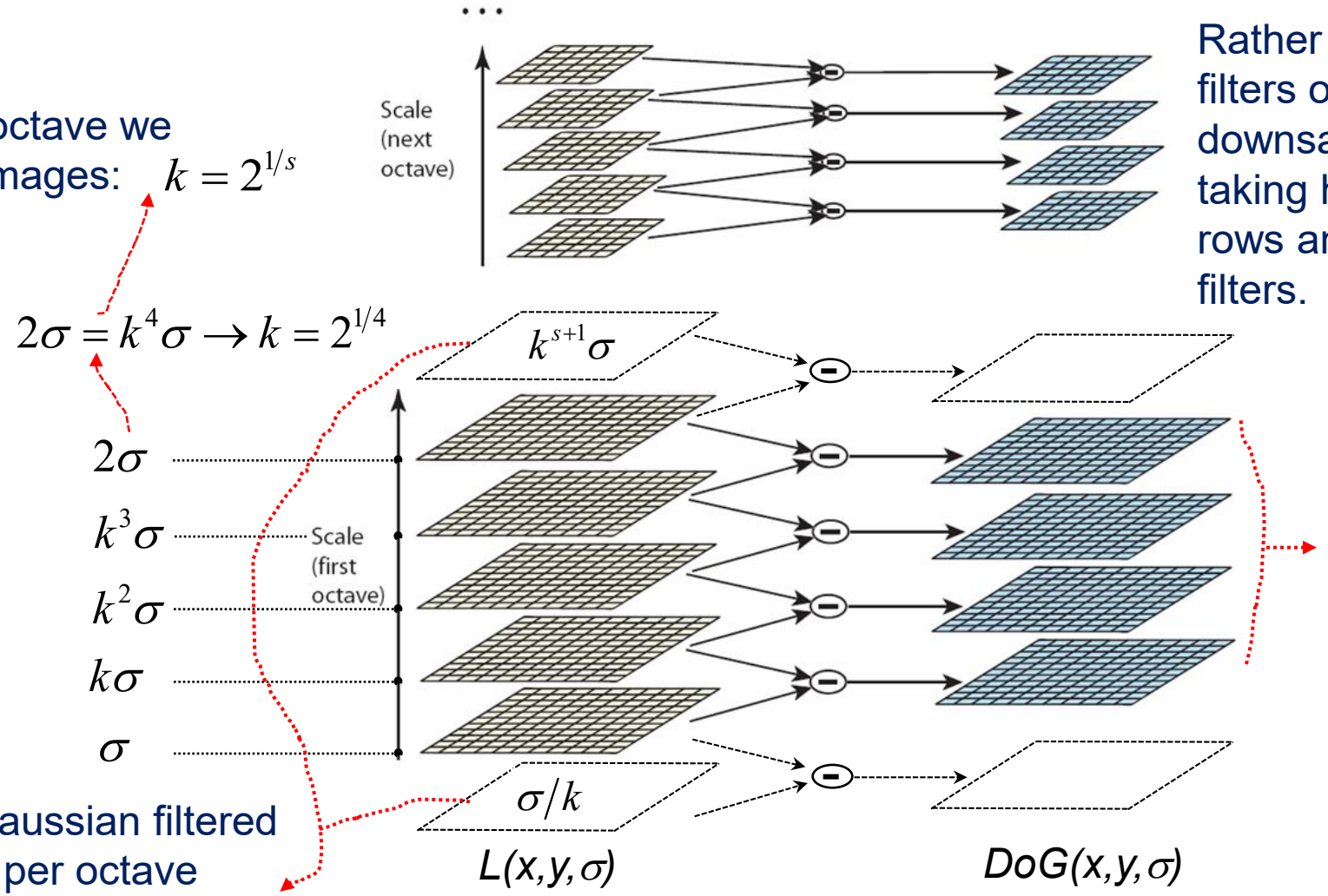
- Both detectors are rotation invariant (circularly symmetric filters) and find blob-like features.



Computation of *DoG(x,y,σ)*

In general, in an octave we compute *s* DoG images: $k = 2^{1/s}$

$2\sigma = k^4\sigma \rightarrow k = 2^{1/4}$



(s+3) Gaussian filtered images per octave

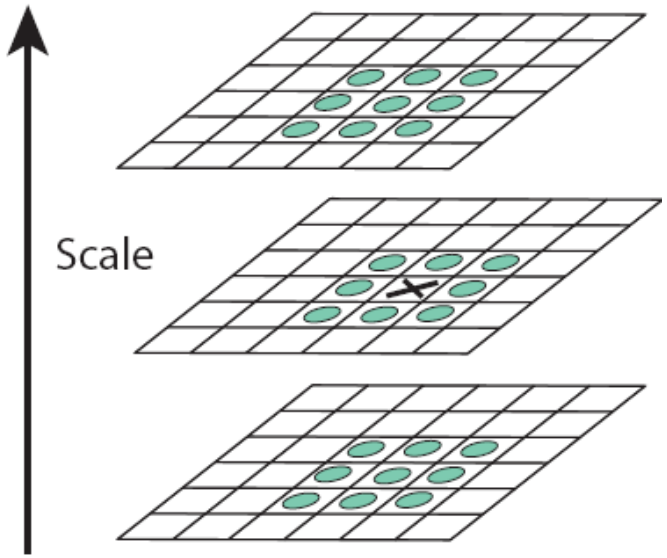
Rather than doubling σ for the filters of the next octave, we downsample *L(x,y,2σ)* by taking half of the columns and rows and then use the same filters.

Extrema detection on *s DoG* images per octave

Keypoint Detection and Tuning

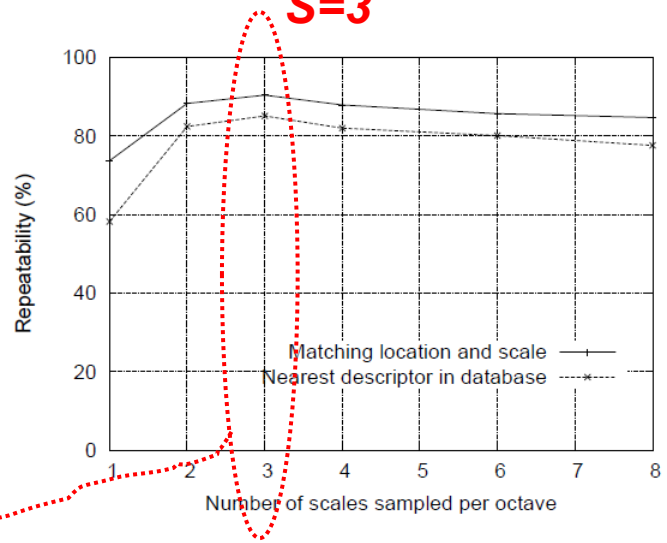
- **Keypoint Detection**

Extrema detection: a point *(x,y,σ)* is detected as a *keypoint* iff its *DoG* is *higher* (*lower*) than that of the 26 neighbours (8 at the same scale and 18=9+9 at the two nearby scales) in the *(x,y,σ)* space.



- **Parameter Tuning:**

Best repeatability with S=3 DoG images per octave



$\sigma = 1.6$ ***(initial σ at each new octave)***

The input image is enlarged by a factor of 2 in both dimensions.

Keypoint Localizazion and Pruning

- **Accurate Keypoint Localzation**

  To localize keypoints more accurately, the DoG function can be approximated around each extrema by its second degree Taylor expension. This procedure yields sub-pixel estimation of position and scale for each detected keypoint.

  (*Details in Appendix 1*)

- **Pruning Unstable Keypoints**

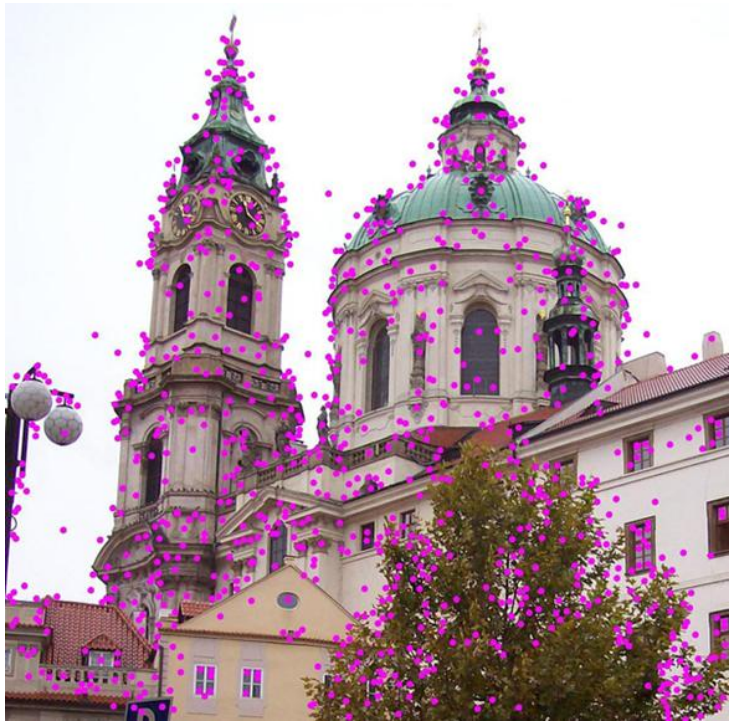
  Extrema featuring weak DoG response turn out scarcely repeatable. They can be pruned by thresholding the absolute value of the DoG. Lowe also notices that unstable keypoints featuring a sufficiently strong DoG may be found along edges and devises a further pruning step aimed at dealing with them.

  (*Details in Appendix 1*)

Exemplar DoG keypoints (1)

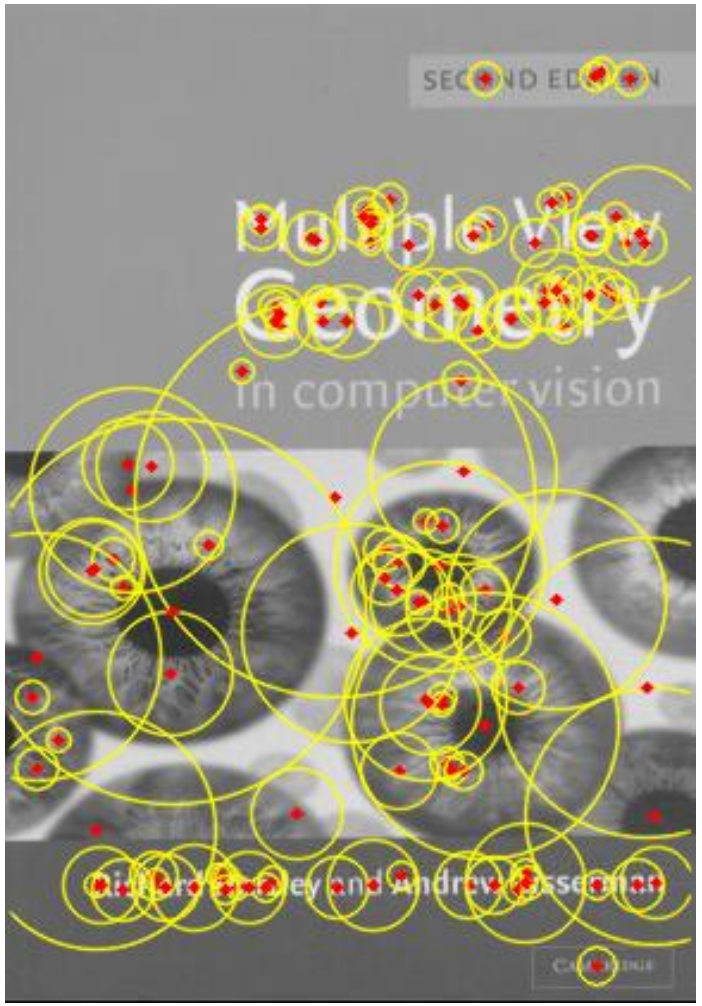
DoG extrema

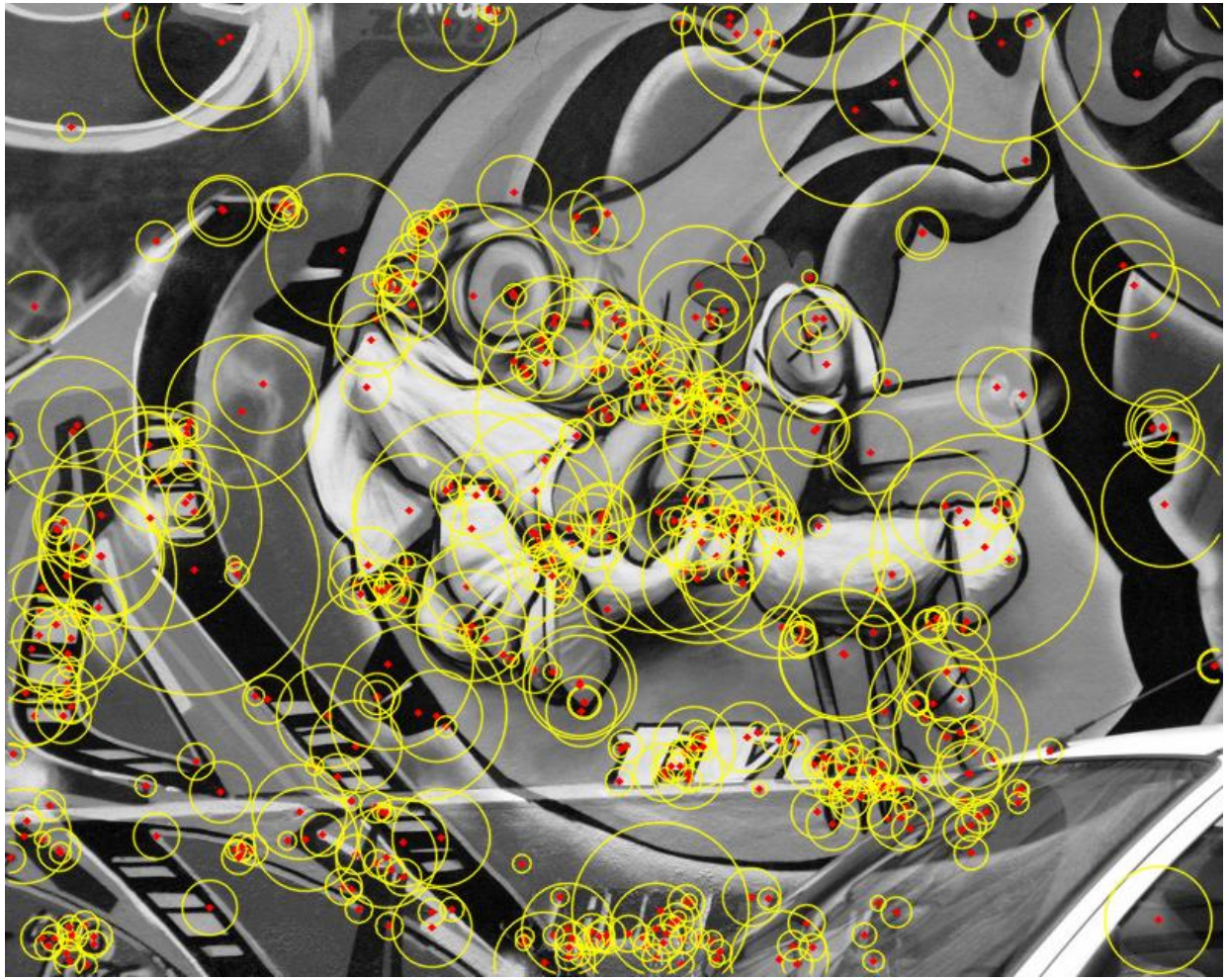
Keypoints after pruning weak responses



Final keypoints after pruning those located along edges.

Exemplar DoG keypoints (2)

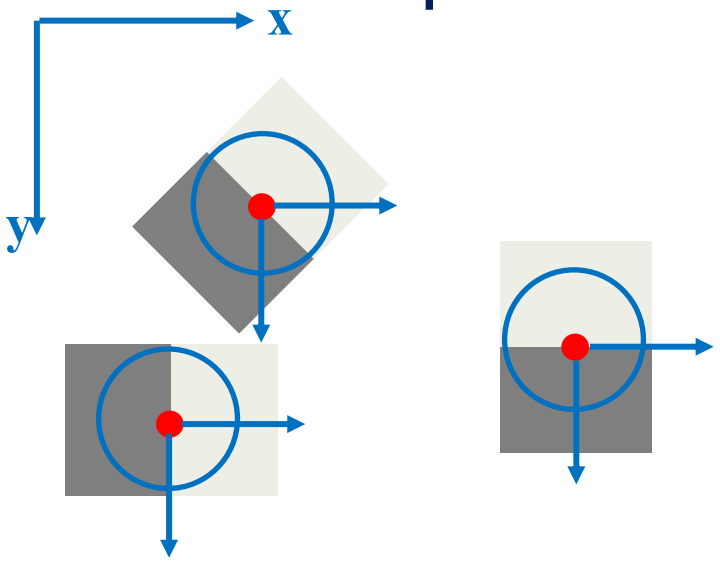




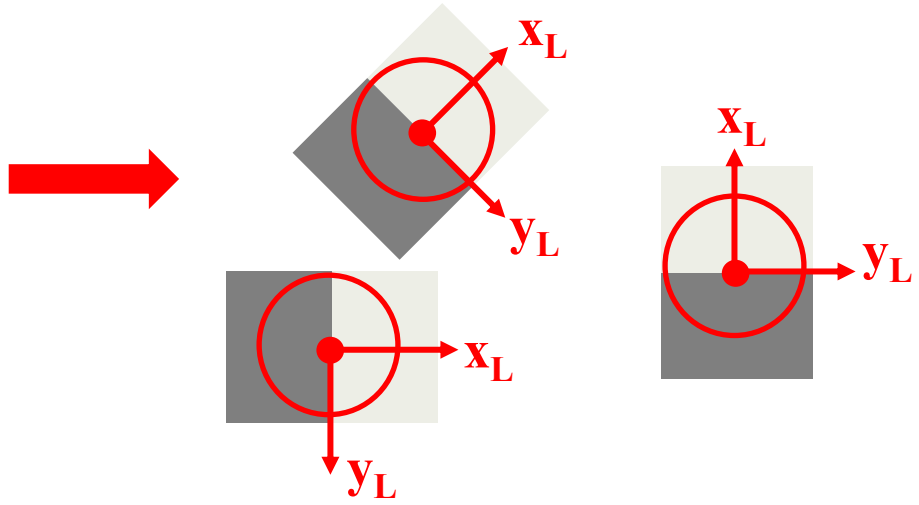
The size of the circle is proportional to σ, i.e. the scale (size) of the feature.

Scale and Rotation Invariant Description

- Then, once each keypoint has been extracted, the surrounding patch is considered to compute its descriptor. A desirable property concerns such descriptor being scale and rotation invariant.
- To attain scale invariance, the patch is taken from the stack image ($L(x,y,\sigma)$ in Lowe's) that correspond to its characteristic scale. As such, the descriptor is computed on a smooth patch that does not contain fine details that are not preserved across the range of scales.
- To attain rotation invariance, a canonical (aka characteristic) patch orientation is computed, so that the descriptor can then be computed on a canonically-oriented patch.



Find a prominent direction inherent to the patch (canonical orientation) and, accordingly, define a local reference frame.



Canonical Orientation with DoG (1)

- Lowe proposes to compute the canonical orientation of DoG keypoints as follows [7]:
  - Given the keypoint, the magnitude and orientation of the gradient are computed at each pixel within a surrounding patch of the associated Gaussian-smoothed image, $L(x,y,\sigma_s)$:

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

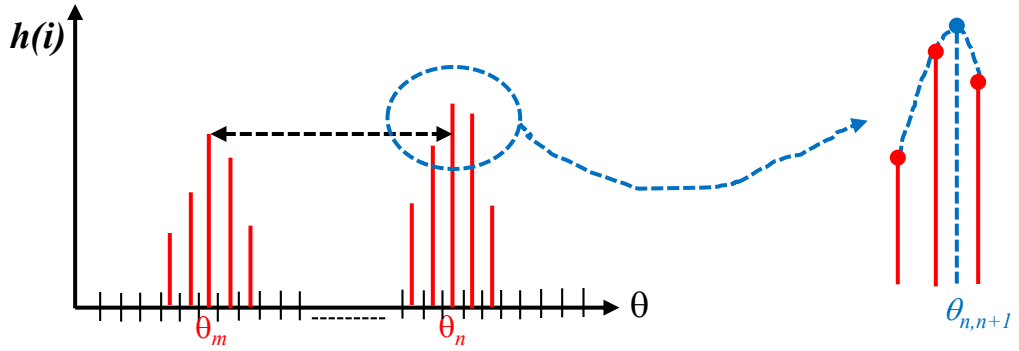
$$\theta(x,y) = \tan^{-1}((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$

  - Then, an orientation histogram (with bin size equal to 10°) is created by accumulating the contributions of the pixels belonging to a neighborhood of the keypoint location.
  - The contribution of each such pixel to its designated orientation bin is given by the gradient magnitude weighted by a Gaussian with $\sigma=1.5\cdot\sigma_s$, σ_s denoting the scale of the keypoint.

Canonical Orientation with DoG (2)

- The canonical orientation of the keypoint is given by the highest peak of the orientation histogram.
- Moreover, other peaks higher than 0.8 of the main one would be kept as well. Accordingly, a keypoint may have multiple canonical orientations and, in turn, multiple descriptors sharing the same location/scale with diverse orientations. This has been found to occur quite rarely (≈ 15% of the keypoints), though.

$$\theta_i = i \cdot \Delta\theta = i \cdot (\pi/18)$$

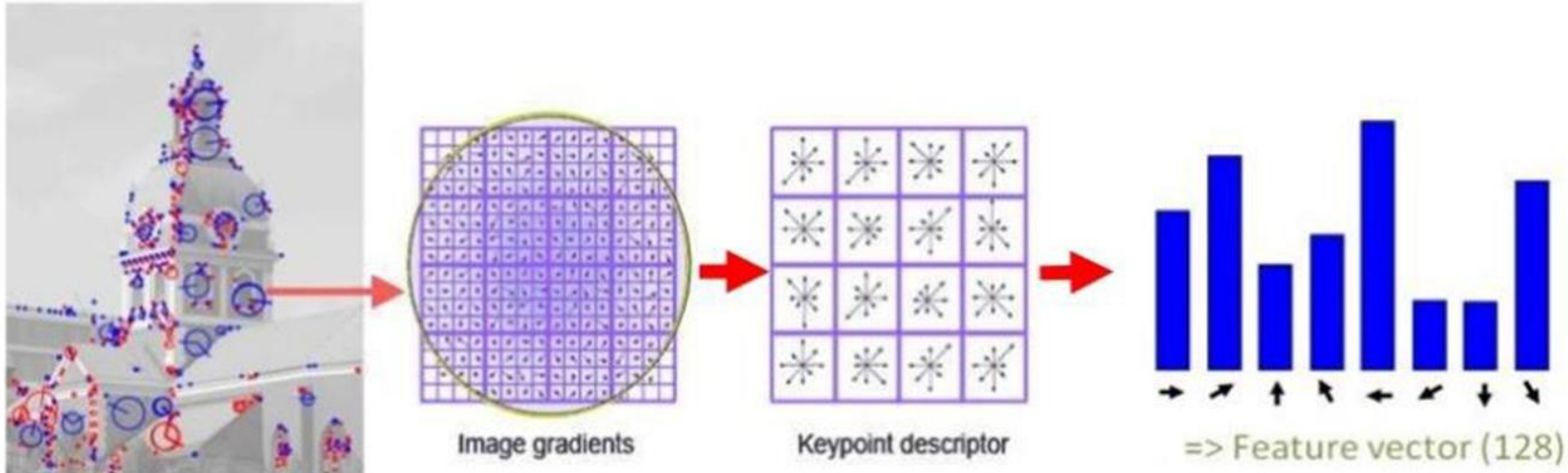


- Finally, a parabola is fitted to the neighborhood of each peak to achieve a more accurate estimation of the canonical orientation. Purposely, the two adjacent bins to the found peak are considered.

SIFT Descriptor (1)

The SIFT (Scale Invariant Feature Transform) descriptor is computed as follows [7]. A 16x16 oriented pixel grid around each keypoint is considered. This is further divided into 4x4 regions (each of size 4x4 pixels) and a gradient orientation histogram is created for each region. Each histogram has 8 bins (i.e. bin size 45°).

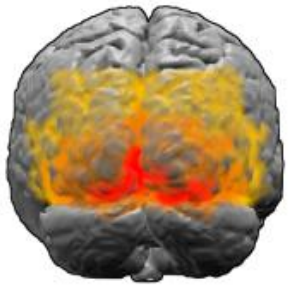
Gradients are rotated according to the canonical orientation of the keypoint. Each pixel in the region contributes to its designated bin according to gradient magnitude as well as to a Gaussian weighting function centred at the keypoint (with σ equal to half the grid size).



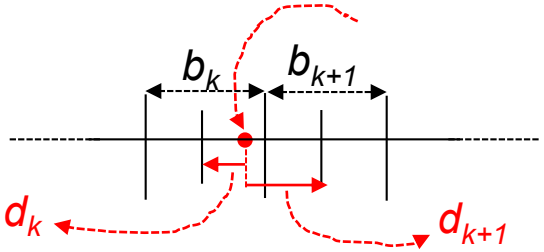
The descriptor size is given by the number of regions times the number of histogram bins per region, i.e. 4x4x8 = 128.

SIFT Descriptor (2)

- SIFT is biologically inspired. There exist studies suggesting that neurons in the primary visual cortex (V1) do match gradient orientations robustly with respect to a certain degree of shift of the input pattern for recognition purposes.



- To avoid boundary effects, a *soft* rather than *hard* assignment is employed in SIFT, whereby the contribution to two adjacent bins is weighted by the distance to the bin center:



$$\begin{cases} w_k = 1 - d_k \\ w_{k+1} = 1 - d_{k+1} \end{cases}$$

This is done within an histogram as well as between regions (the contribution is spread bilinearly between 4 adjacent regions). Hence, the overall scheme is referred to as *trilinear interpolation*.

- The descriptor is normalized to unit length to gain invariance wrt affine intensity changes. Then, to increase robustness to non-linear changes, all elements larger than 0.2 are saturated and the descriptor normalized again.

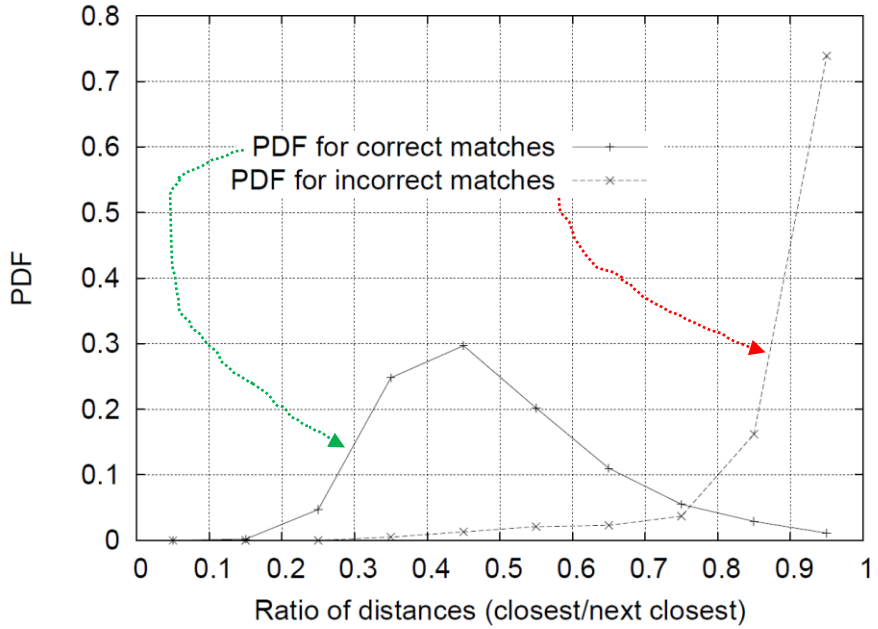
Matching Process: NN Search

Given two images, *T* (target) and *M* (model), for each keypoint in *T* we find the keypoint in *M* such that the distance between their SIFT descriptors is the smallest (Nearest Neighbour Search). As some keypoints in *T* may not have a corresponding one in *M*, we may wish to enforce criteria to accept/reject a match found by the NN search. Two such criteria are as follows;

1) $d_{NN} \leq T$ (NN distance)

2) $\frac{d_{NN}}{d_{2-NN}}$ (ratio of distances)

Lowe [7]

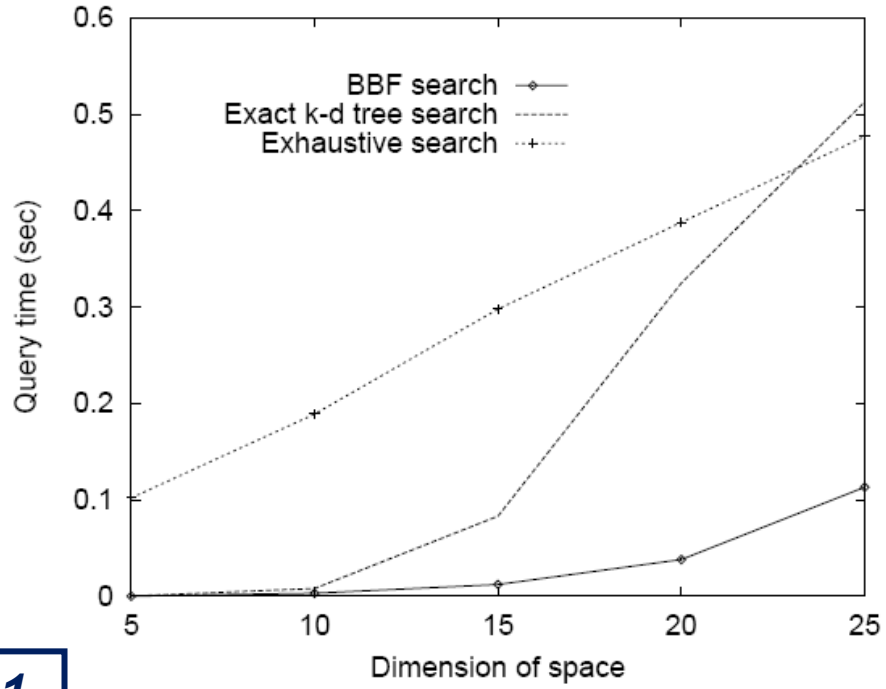


Lowe shows that T=0.8 may allow for rejecting 90% of wrong matches while missing only 5% of correct ones.

Efficient NN-Search

- Exhaustively searching for the NN of the *query* feature, *q*, has linear complexity in the size of *S*, which, oftentimes, turns out exceedingly slow.
- Thus, efficient ***indexing techniques*** borrowed from database management and information retrieval are deployed to speed-up the NN-search process.

- The main *indexing* technique deployed for feature matching is known as ***k-d tree*** [8]. In particular, the preferred approach is the ***approximate*** variant referred to as BBF (***Best Bin First***) proposed in [9]. Indeed, unlike the basic *k-d tree*, the BBF formulation is efficient also in high-dimensional spaces such as descriptor spaces (e.g., R^{128} for SIFT).



More information on k-d tree and BBF can be found in Appendix 1

Libraries for Efficient NN-Search

- **FLANN - Fast Library for Approximate Nearest Neighbors**

FLANN is a library for performing fast approximate nearest neighbor searches in high dimensional spaces. It contains a collection of algorithms found to work best for nearest neighbor search and a system for automatically choosing the best algorithm and optimum parameters depending on the dataset. FLANN is written in C++ and contains bindings for the following languages: C, MATLAB, Python, and Ruby (available in OpenCV).

https://github.com/flann-lib/flann

- **Faiss - Facebook AI Similarity Search**

Faiss is a library for efficient similarity search and clustering of dense vectors. It contains algorithms that search in sets of vectors of any size, up to ones that possibly do not fit in RAM. It also contains supporting code for evaluation and parameter tuning. Faiss is written in C++ with complete wrappers for Python/numpy. Some of the most useful algorithms are implemented on the GPU. It is developed primarily at *Facebook AI Research*.

https://github.com/facebookresearch/faiss

A few other relevant proposals

- A fast alternative to SIFT is the **SURF (Speeded-Up Robust Features)** algorithm. Blob-like features are detected through efficiently computable filters. The stack of filtered images is computed more efficiently by mean filtering.
- **MSER (Maximally Stable Extremal Regions)** detects interest regions of arbitrary shapes, in particular approximately uniform areas either brighter or darker than their surroundings. Description of MESR regions may be carried out by SIFT.
- **FAST (Features from Accelerated Segment Test)** is a very efficient detector of corner-like features.
- A variety of ***binary descriptors***, such as **BRIEF, ORB and BRISK**, have been proposed to minimize memory occupancy and accelerate the matching step thanks to the use of the **Hamming distance**.
- Finally, **BOLD (Bunch Of Lines Descriptor)** allows for deploying the local features paradigm with texture-less objects based on line-segments extracted along edges.

OpenCV: SIFT, SURF, MSER, FAST, BRIEF, ORB
OpenCV: Feature Detection and Description

Appendix 1 - Accurate Keypoint Localization

- To localize *keypoints* more accurately, the *DoG* function can be approximated around each extrema by its second degree Taylor expension:

$$D(\mathbf{x}) = D + \frac{\partial D}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x}$$

Displacement wrt the (x,y,σ) position of the found extremum

DoG at the found extremum

Gradient of the DoG function at the found extremum

Hessian matrix of the DoG function at the found extremum

- The new extremum (either maximum or minimum) can be localized by imposing the gradient of the approximated function to be zero:

$$\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$$

- If the displacement $\hat{\mathbf{x}}$ turns out large (>0.5 in at least one dimension), the extremum is re-assigned to the closest discrete position and the approximation computed again. Once the displacement is small, the procedure is stopped and the found displacement provides sub-pixel interpolation wrt to the last discrete position.

Appendix 1 - Pruning Unstable Keypoints

- Extrema featuring weak DoG response turn out scarcely repeatable. They can be pruned by estimating the DoG at a found extremum

$$D(\hat{\mathbf{x}}) = D + \frac{1}{2}\frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}}.$$

  and thresholding its magnitude: $| D(\hat{\mathbf{x}}) | < T_{DoG}$.

- Lowe also notices that unstable keypoints featuring a sufficiently strong DoG may be found along edges and devises a further pruning step based on the eigenvalues of the Hessian, which are proportional to the principal curvatures of the intensity surface
- Thus, the higher is the ratio between the larger and smaller eigenvalues, *r*, the more similar to an edge rather than a blob is the considered pixel
- Similarly to Harris, explicit computation explicit computation of the eigenvalues may be avoided

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}$$

This increasing function of *r* can be thresholded to prune edges

Appendix 2 – k-d Tree and Best Bin First (BBF)

- Extrema featuring weak DoG response turn out scarcely repeatable. They can be pruned by estimating the DoG at a found extremum

$$D(\hat{\mathbf{x}}) = D + \frac{1}{2}\frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}}.$$

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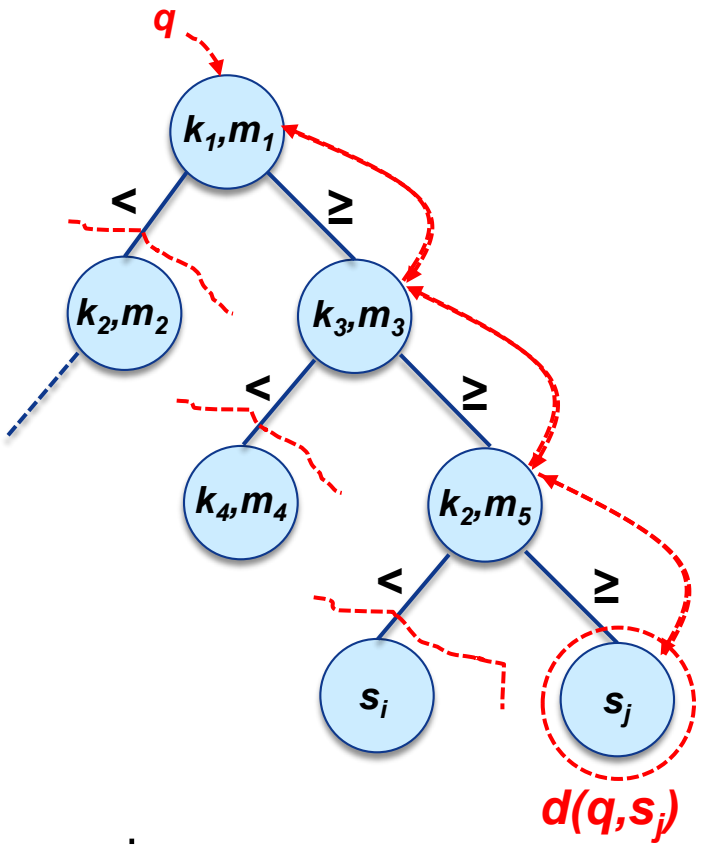
This increasing function of *r* can be thresholded to prune edges

k-d Tree

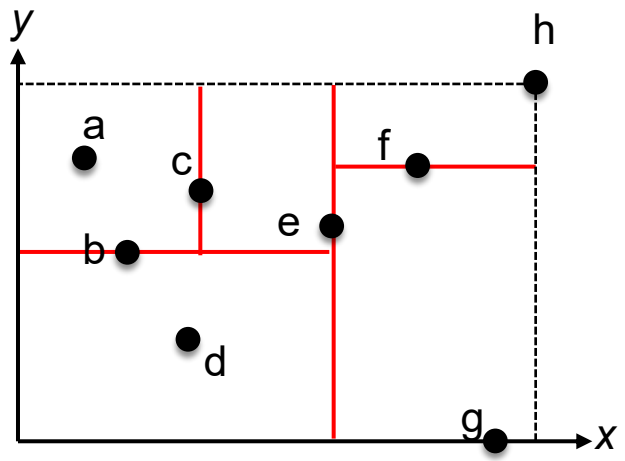
- The *k-d* tree generalizes the *binary search tree* to the case of *k*-dimensional data.
- Each node defines a *split* of the data according to one of the *k* dimensions.
- During the search, the query point, *q*, traverses the tree from the root down to a *leaf* according to the the splits defined by nodes. The datum stored in the *leaf* is close to *q* though not guaranteed to be the NN.

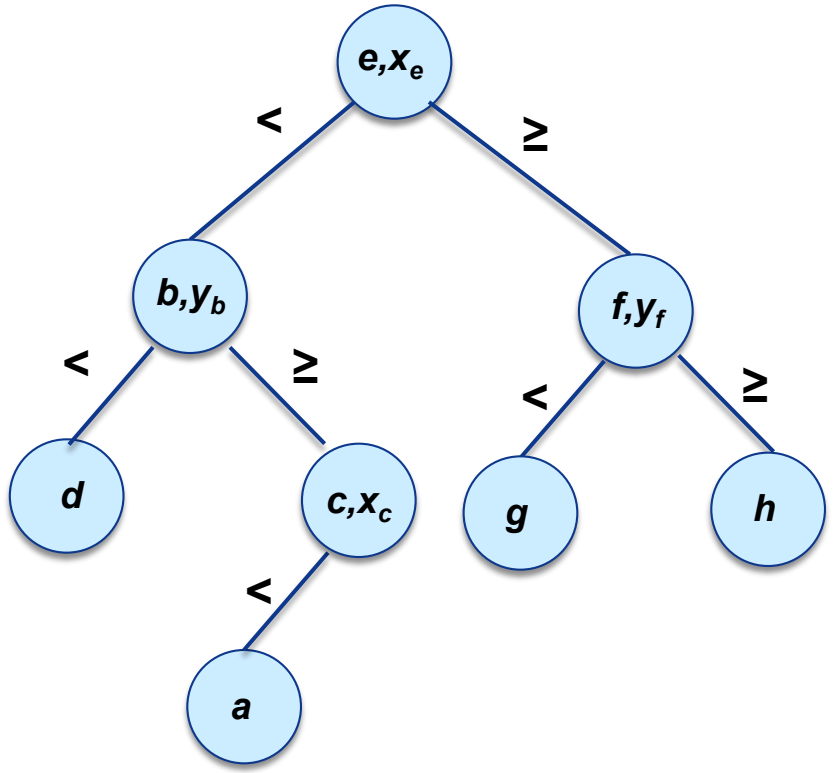


- Therefore, the *tree* is traversed back from the *leaf* to the *root* to refine the search. During *backtracking,* only the *sub-trees* that may contain closer data are explored.
- If the tree is *balanced*, the initial search requires $\boldsymbol{log_2(size(S))}$ comparisons, that is **logatithmic complexity**, which yields a large speed-up as long as *backtracking* involves visiting relatively few nodes.

Building the Tree

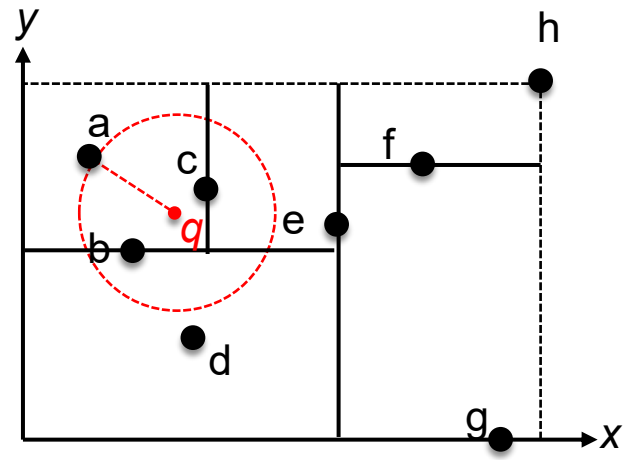
- The data are always split by choosing the dimension showing the highest variance. The datum taking the median value of such dimension defines the split.
- Thus, the root is created first. This splits the initial set of data, S, into two sub-sets, S_1 and S_2, of, roughly, the same sizes.
- The process is then applied recursively.

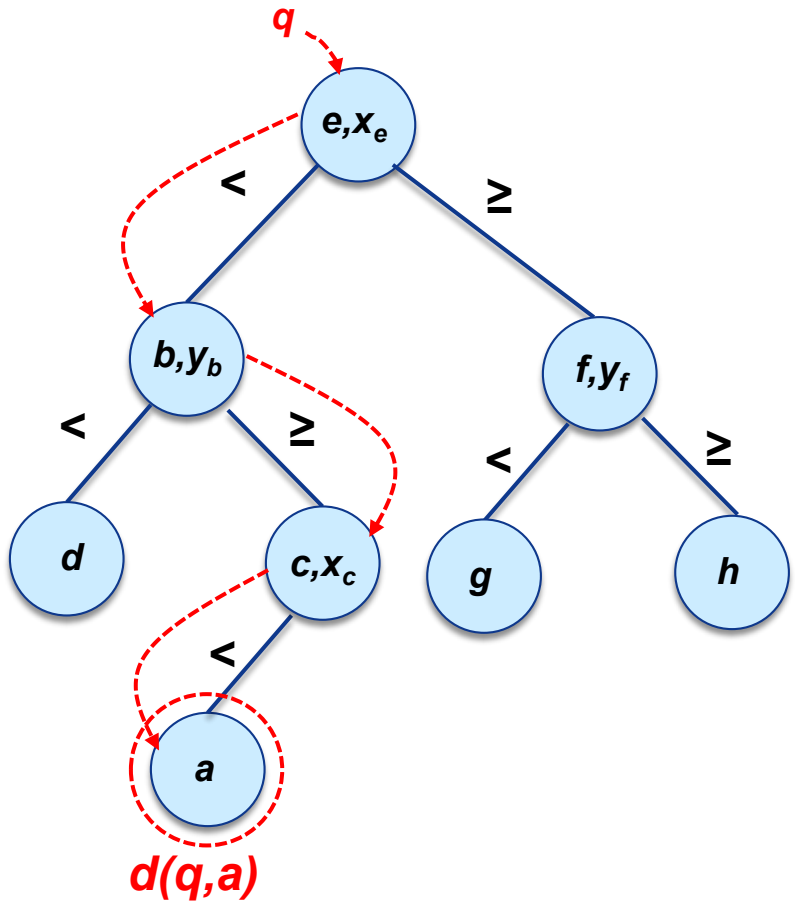




The k-d tree partitions the data space into bins adaptively: bins are smaller in higher density regions, larger in regions of lower data density.

NN Search – Traversing the tree





As the (hyper)sphere centred at *q* with radius *d(q,a)* does intersect the (hyper)planes separating the partition of the space associated with leaf *a* from its adjacent partitions, it may be necessary to explore these partitions (together with their parent nodes) to find the NN of *q*.

NN Search - Backtracking

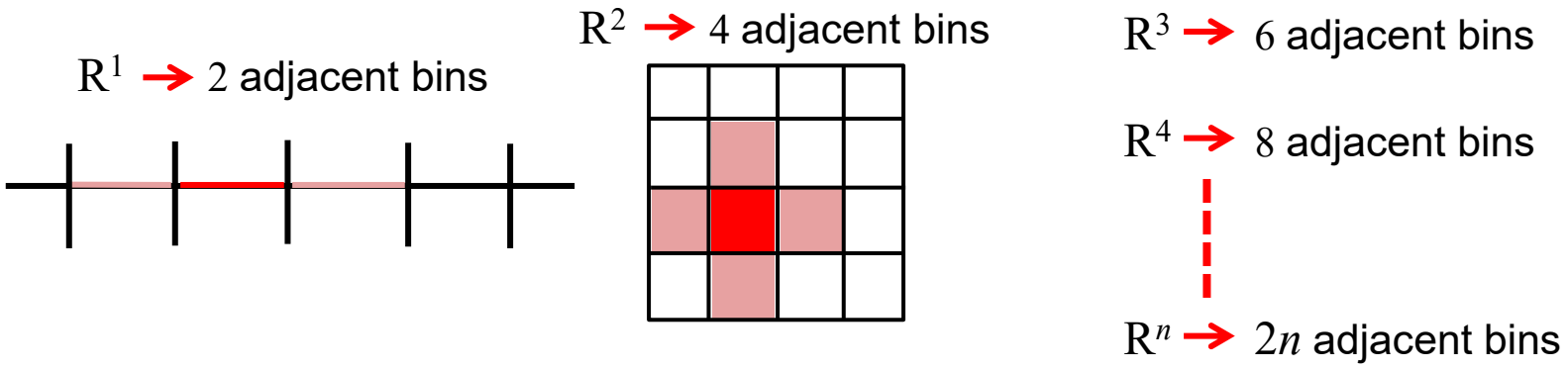
1. Given the query, q, the tree is traversed from the root to the closest leaf, denoted here as NN_{curr}, the associated distance given by $d(q, NN_{curr})$.

2. Backtracking:
   - The parent node to the found leaf is considered. If its distance to q is less than $d(q, NN_{curr})$, the parent node becomes the new NN_{curr} and $d(q, NN_{curr})$ gets updated accordingly.
   - A check is carried out to determine whether the other half-space defined by the current parent node may contain or not a closer point. This depends on whether or not the hypersphere of radius $d(q, NN_{curr})$ centred at q does intersect the splitting hyperplane.
     - Should the former be the case, the sub-tree whose root is the parent node is traversed down to the next closest leaf.
     - Conversely, the parent node to the current one is considered.
   - The process ends upon getting to the root of the tree, with NN_{curr} turning out the NN to q.

The curse of dimensionality

- **A *k-d tree* may be thought of as partitioning the space into "bins". During *backtracking*, the bins adjacent to that containing the found leaf might be examined to find the NN.**

- **However, as the dimensionality of data increases, so does the number of bins adjacent to a given one, which renders *k-d trees* inefficient (perhaps even slower than exhaustive search) with high-dimensional data.**



- **The issue comes from the inherent structure of the R^n space, thus it is hardly addressable.**

Approximate Search (Best Bin First)

Hence, Beis & Lowe in [9] have proposed an approximate search algorithm, referred to as *Best Bin First* (BBF), which deploys a *priority queue* and limits the maximum number of reachable leaves (E_{max}).

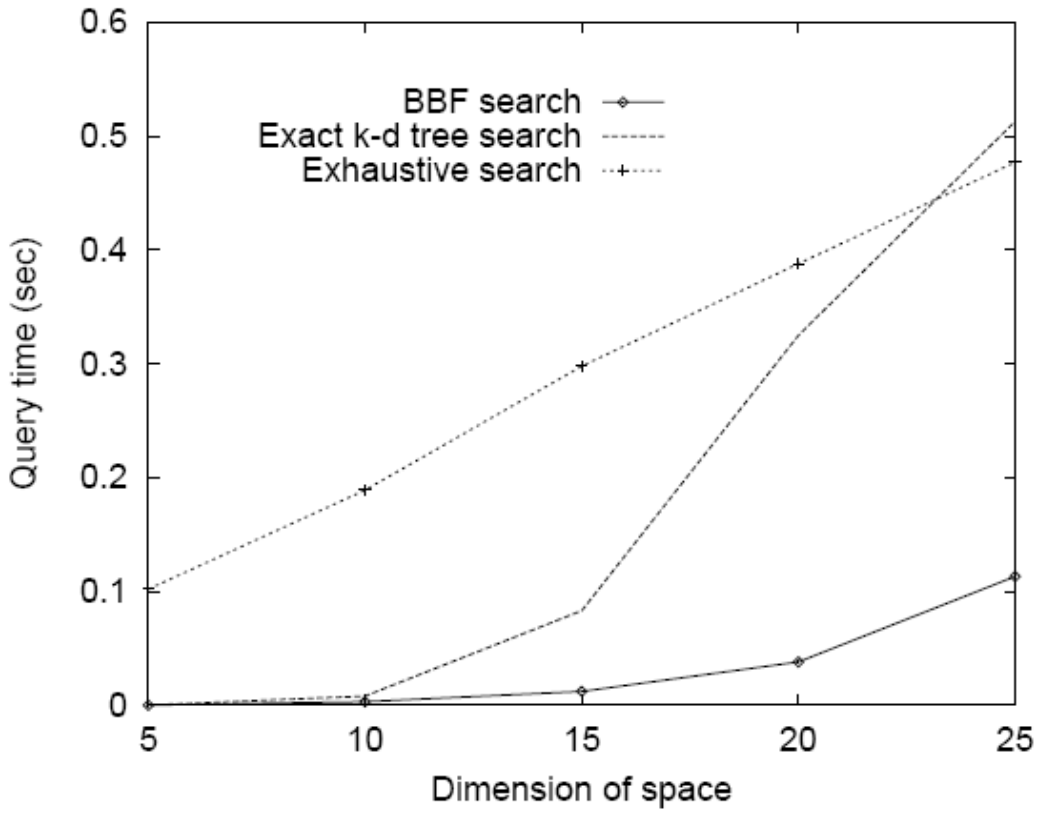
1. Akin to the standard algorithm, given the query, *q*, the tree is traversed down to the closest *leaf*, so to find the initial NN_{curr} and associated distance ($d(q, NN_{curr})$). Moreover, though, the visited nodes are ordered in a *priority queue* according to their distance to *q*.
2. Backtracking:
   - The first element in the *priority queue* is extracted so to traverse its unexplored branch down to a *leaf*. During the process, the *priority queue* is always kept updated.
   - The previous step is iterated until a fixed number of leaves (E_{max}) has been reached.

The found *approximate* NN is the closest data point (NN_{curr}) after E_{max} leaves have been visited.

BBF vs. *k-d* tree

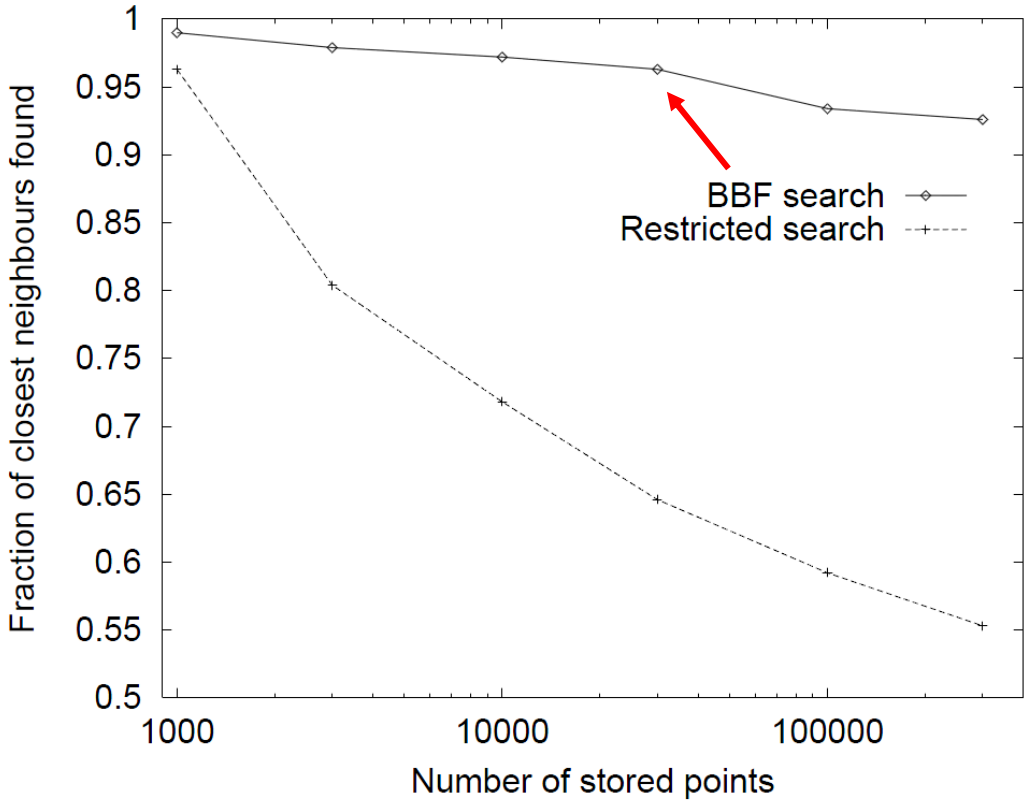
Efficiency

(30.000 uniformly distributed data points)



Accuracy

(E_{max}=200, *n*=12)



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