ASSIGNMENT - 3

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QUESTION-1

- Given, the transfer function-

$$G(s) = \frac{10(s-4)e^{-3s}}{(s+2)(s+5)}$$

PART (A)

- The above transfer function can be written after partial fraction expansion as:

$$G(s) = \frac{-20e^{-3s}}{(s+2)} + \frac{30e^{-3s}}{(s+5)}$$

- **Impulse response:**

$$Y_{IR}(s) = G(s)U_{impulse}(s) = \left(\frac{-20e^{-3s}}{(s+2)} + \frac{30e^{-3s}}{(s+5)}\right) \times 1 = \frac{-20e^{-3s}}{(s+2)} + \frac{30e^{-3s}}{(s+5)}$$

$$Taking\ inverse\ laplace: y_{IR}(t) = \mathcal{L}^{-1}\left(\frac{-20e^{-3s}}{(s+2)} + \frac{30e^{-3s}}{(s+5)}\right)$$

$$\boldsymbol{y_{IR}(t) = -20e^{-2(t-3)} + 30e^{-5(t-3)}} \qquad t \geq 3$$

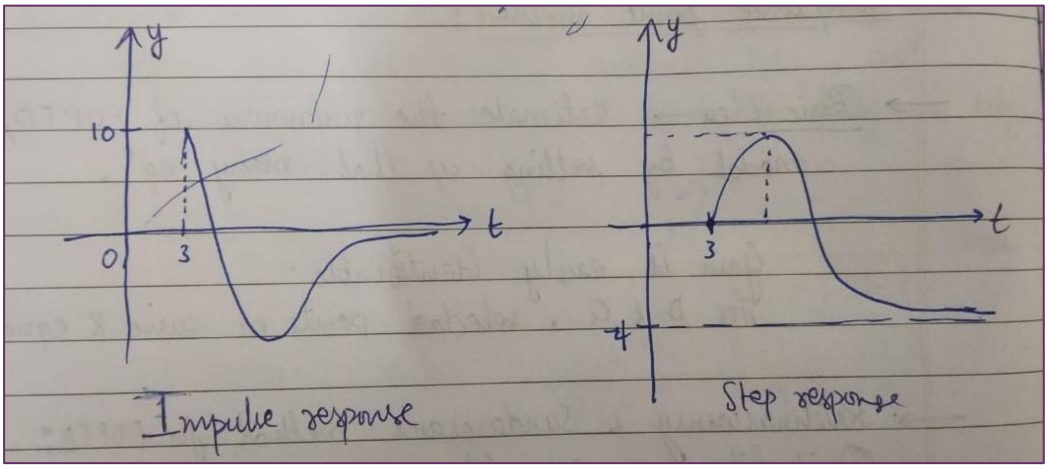
- **Step response:** Similarly,

$$Y_{SR}(s) = G(s)U_{step}(s) = \left(\frac{10(s-4)e^{-3s}}{(s+2)(s+5)}\right) \times \frac{1}{s} = \frac{-4e^{-3s}}{s} + \frac{10e^{-3s}}{s+2} - \frac{6e^{-3s}}{s+5}$$

$$Taking\ inverse\ laplace: y_{SR}(t) = \mathcal{L}^{-1}\left(\frac{-4e^{-3s}}{s} + \frac{10e^{-3s}}{s+2} - \frac{6e^{-3s}}{s+5}\right)$$

$$\boldsymbol{y_{SR}(t) = 10e^{-2(t-3)} - 6e^{-5(t-3)} - 4} \qquad t \geq 3$$

- Sketched Impulse Response (Left) & Step-Response (Right):



PART (B)

- Given, input-

$$u(t) = 2\sin 4t + \cos 0.1t = A_1 \sin\omega_1 t \ + A_2 \cos\omega_2 t$$

$$where, A_1 = 2; A_2 = 1; \omega_1 = 4;\ \omega_2 = 0.1$$

- First, we will find out the expressions for amplitude ratio (AR) and phase:

$$G(j\omega) = \frac{10(\omega j - 4)e^{-3\omega j}}{(\omega j + 2)(\omega j + 5)}$$

$$AR = |G(j\omega)| = \frac{10\sqrt{16+\omega^2}}{\sqrt{(4+\omega^2)(25+\omega^2)}}$$

$$\phi = \angle G(j\omega) = -3\omega + \tan^{-1}\left(\frac{\omega}{-4}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

- Evaluating the expressions of AR and Phase at $\omega_1 = 4$ and $\omega_2 = 0.1$:

$$AR_1 = |G(j\omega_1)| = \frac{10\sqrt{16+16}}{\sqrt{(4+16)(25+16)}} = 1.975$$

$$AR_2 = |G(j\omega_2)| = \frac{10\sqrt{16+0.01}}{\sqrt{(4+0.01)(25+0.01)}} = 3.995$$

$$\phi_1 = -14.565; and\ \phi_2 = -0.395$$

- Finally, long time response is given by-

$$y_{ss}(t) = (AR_1 \times A_1)\sin(\omega_1 t + \phi_1) \ + (AR_2 \times A_2)\cos(\omega_2 t + \phi_2)$$

$$y_{ss}(t) = (1.975 \times 2)\sin(4t - 14.565) + \ (3.995 \times 1)\cos(0.1t - 0.395)$$

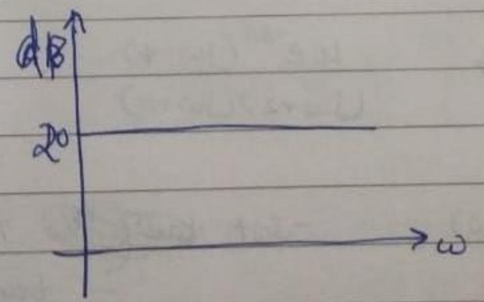
$$\boxed{\boldsymbol{y_{ss}(t) = 3.95\sin(4t - 14.565) + \ 3.995\cos(0.1t - 0.395)}}$$

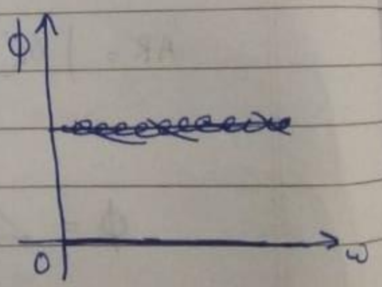
PART (C)

- Bode-Plots sketched by hand are shown below:

$$G(s) = \frac{10(s-4)e^{-3s}}{(s+2)(s+5)}$$

→ (1) Plotting the constant : $G_1(s) = 10$ $dB = 20$



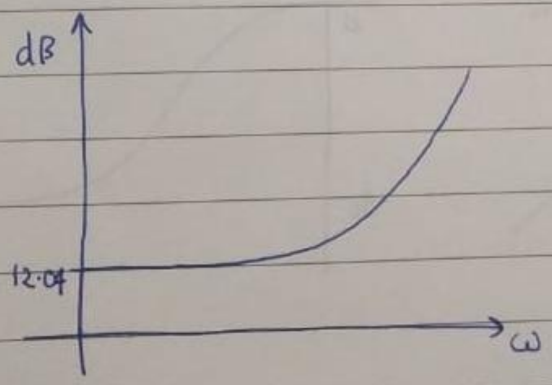


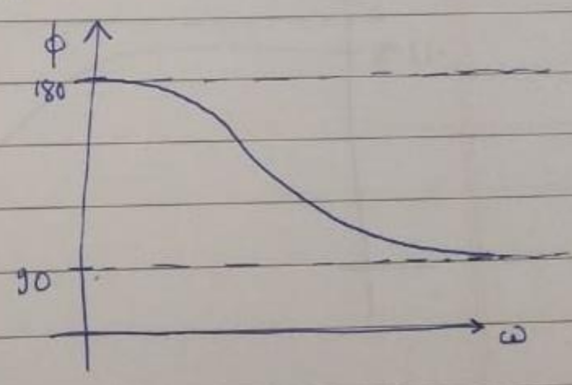
→ (2) Plotting the zero : $G_2(s) = s-4$

$$G_2(j\omega) = j\omega - 4$$

$$dB = 10\log_{10}(\omega^2+16)$$

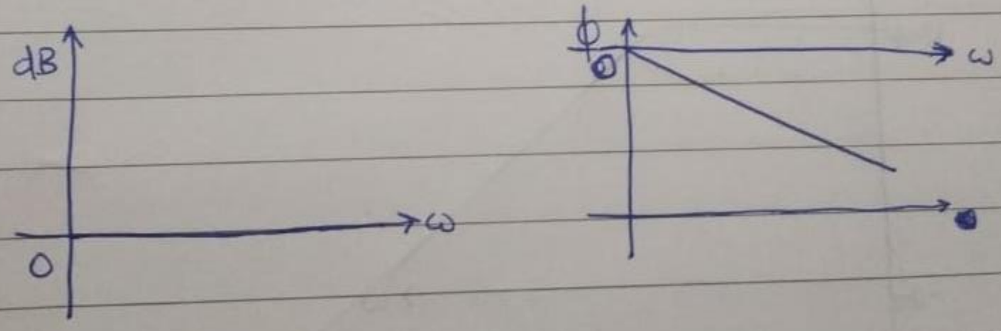
$$\phi = \tan^{-1}\left(\frac{\omega}{-4}\right)$$





→ ③ Plotting the delay: $G_3(s) = e^{-3s}$

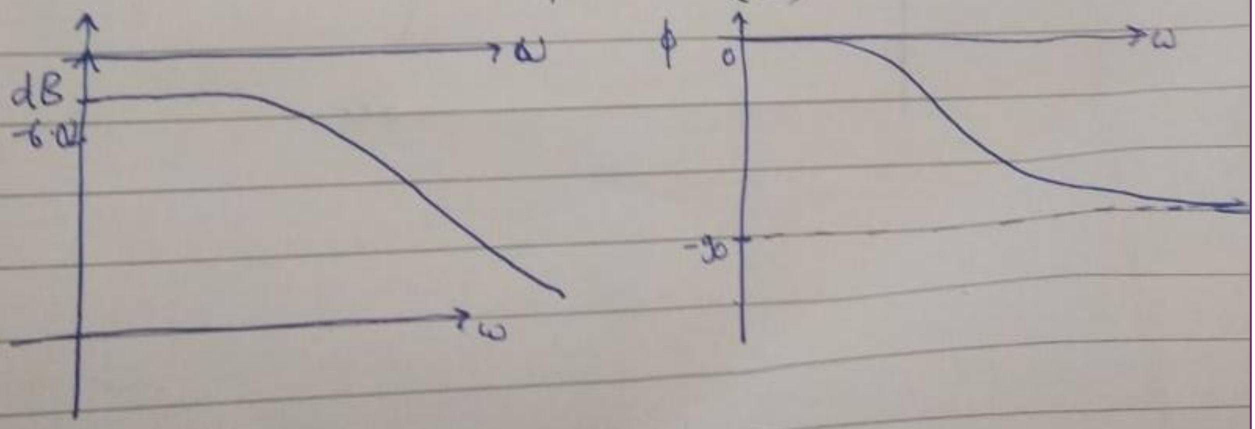
$G_3(j\omega) = e^{-3\omega j}$ | $dB = 0$ | $\phi = -3\omega$

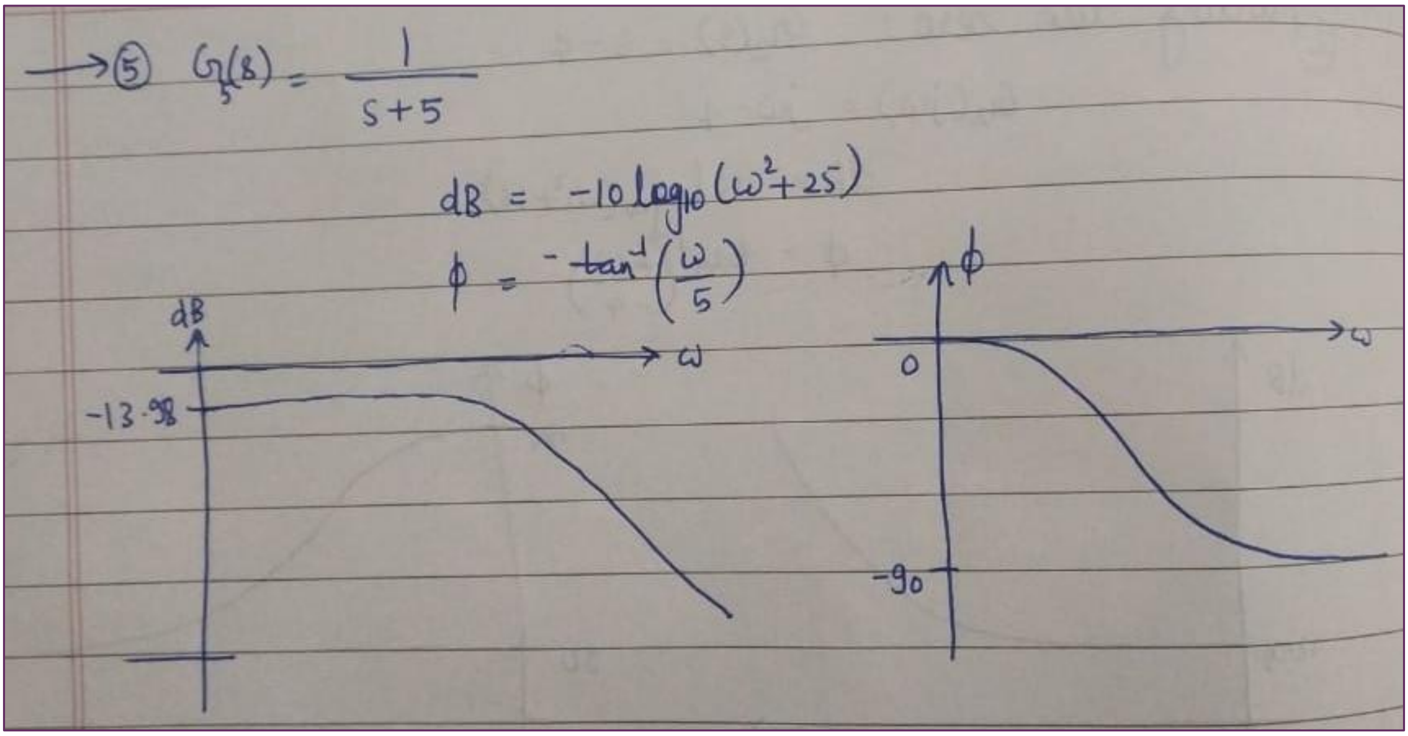


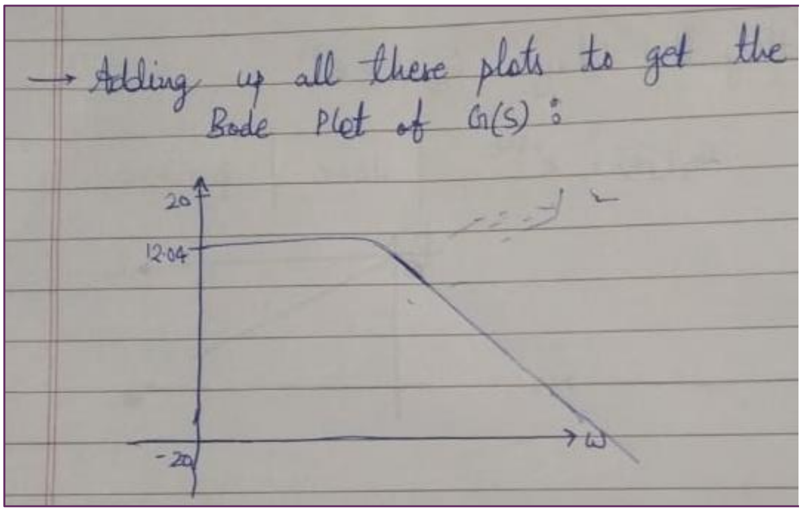
→ ④ Plotting $G_4(s) = \frac{1}{s+2}$

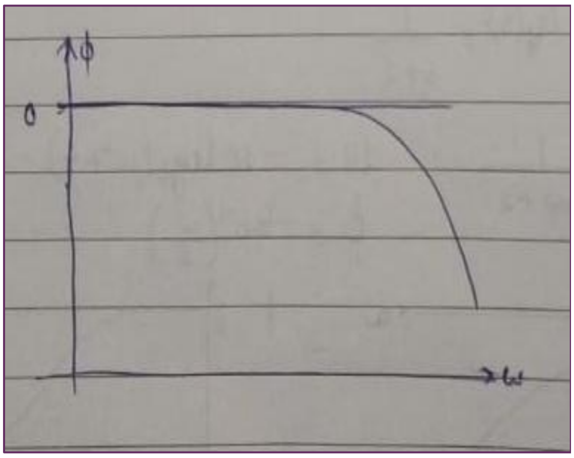
$G_4(j\omega) = \frac{1}{j\omega + 2}$ $dB = -10\log_{10}(\omega^2 + 4)$

$\phi = -\tan^{-1}\left(\frac{\omega}{2}\right)$









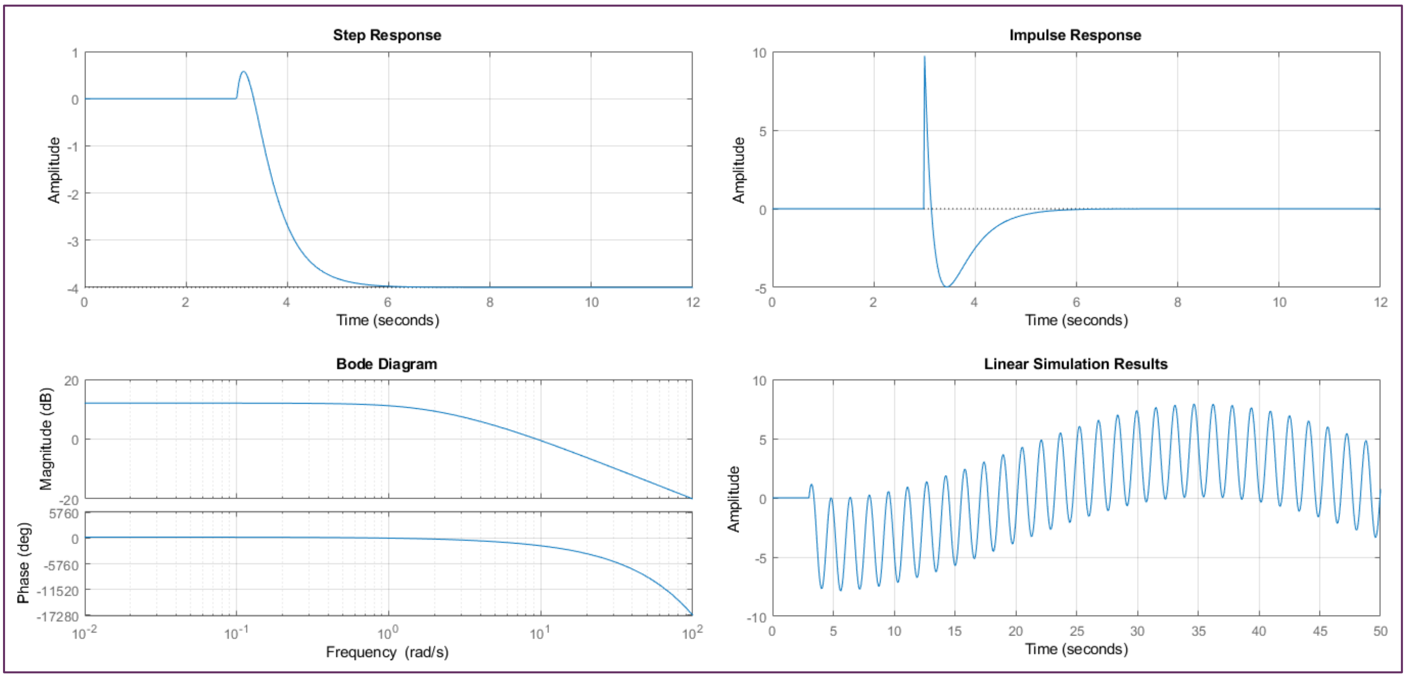
PART (D)

- LTI system having same magnitude as that of the given system at any given frequency but with minimum phase, is given by the same system but with all its zeros in LHP:

$$G_{min-phase}(s) = \frac{\mathbf{10}(s+\mathbf{4})e^{-\mathbf{3}s}}{(s+\mathbf{2})(s+\mathbf{5})}$$

PART (E)

- Verifying parts (a) to (c) using MATLAB (using *ltiview*):



QUESTION-2

Given,

$$\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t) \tag{1}$$

Where,

h(t): level of fluid measured with respect to initial steady state value

p(t): pressure change

PART (A)

- Taking Laplace transform of (1):

$$\left(s^2 + \frac{6\mu}{R^2\rho} + \frac{3g}{2L}\right)H(s) = \frac{3}{4\rho L}P(s)$$

$$\Rightarrow G(s) = \frac{H(s)}{P(s)} = \frac{\frac{3}{4\rho L}}{\left(s^2 + \frac{6\mu}{R^2\rho} + \frac{3g}{2L}\right)} \tag{2}$$

- Rearranging G(s) in the standard second-order transfer function form-

$$\frac{\mathrm{K}}{\tau s^2 + 2\tau\zeta s + 1} \tag{3}$$

- We get,

$$\Rightarrow G(s) = \frac{\frac{1}{2g\rho}}{\frac{2L}{3g}s^2 + \frac{4\mu L}{gR^2\rho}s + 1} \quad (4)$$

- Comparing the expressions in (3) and (4), we get-

$\mathbf{K = \frac{1}{2\rho g}}$
$\boldsymbol{\tau = \sqrt{\frac{2L}{3g}}}$
$\boldsymbol{\zeta = \frac{\mu}{\rho R^2}\sqrt{\frac{6L}{g}}}$

PART (B)

- For the system response to oscillate, it should be underdamped, i.e.,

$$0 < \zeta < 1$$

$$\boldsymbol{\Rightarrow 0 < \frac{\mu}{\rho R^2}\sqrt{\frac{6L}{g}} < 1}$$

PART(C)

- Damping factor, ζ, characterizes the effect of damping on the oscillations on a system. $\zeta = 0$ implies no damping or high oscillations, whereas ζ close to 1 means high damping.
- For more oscillatory response, we need to reduce ζ to bring it closer to zero.
- For less oscillatory and more damped response, we need to increase ζ.
- **Increasing μ or L will result in the increase in value of damping factor ζ, hence reducing oscillations, whereas decreasing these parameters would mean more oscillatory response.**

QUESTION-3

- Given transfer function:

$$G_p(s) = \frac{\mathrm{K}e^{-\theta_1 s}\left(1 + \alpha e^{-\theta_2 s}\right)}{\tau s + 1}$$

$$where, \mathrm{K} = -1\frac{mm\ Hg}{ml/h}; \alpha = 0.4; \theta_1 = 30s;\ \theta_2 = 45s; \tau = 40s$$

- The above transfer function can be written as a sum of two transfer functions with same time constant and different delays and gains:

$$\Rightarrow G_p(s) = \frac{\mathrm{K}e^{-\theta_1 s}}{\tau s + 1} + \frac{\alpha\mathrm{K}e^{-(\theta_1+\theta_2)s}}{\tau s + 1}$$

- SIMULINK block diagram for the system:

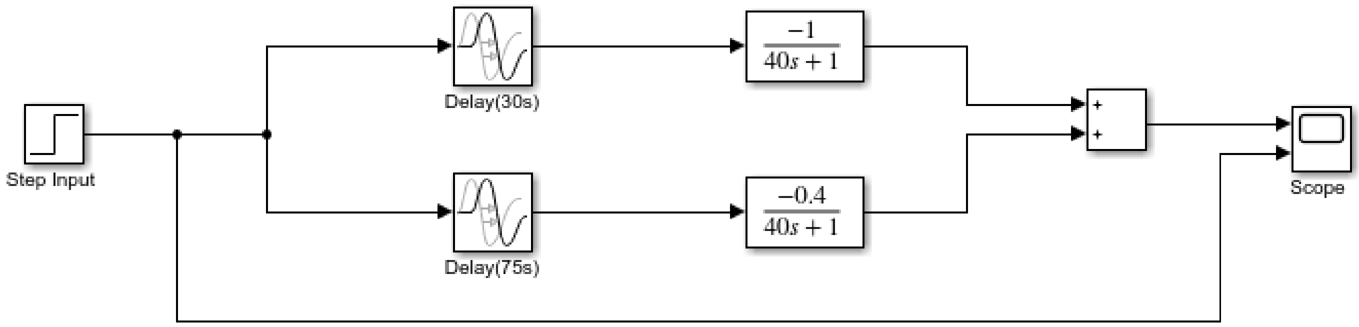
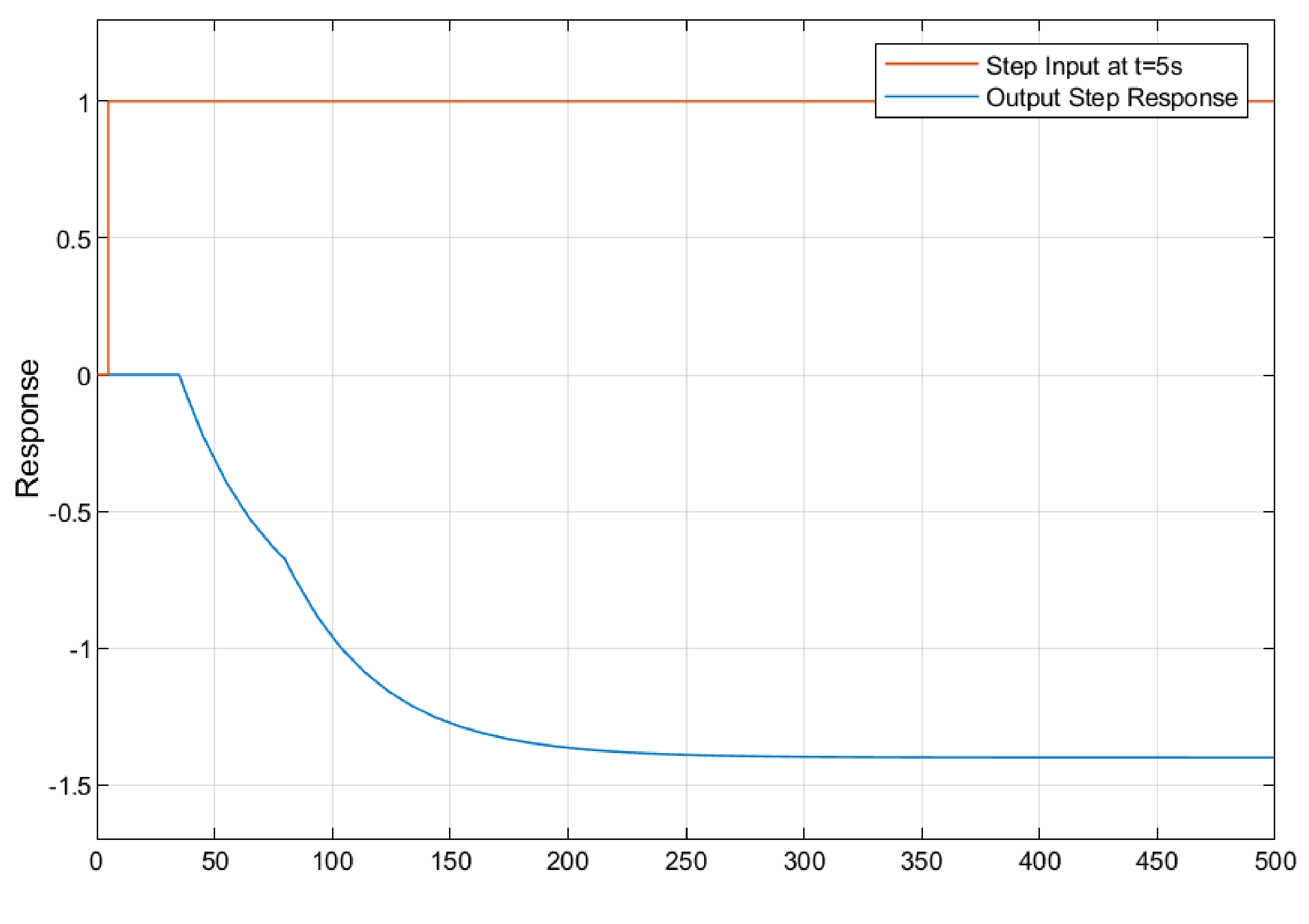


Fig. SIMULINK block diagram of the given transfer function, as sum of two first order transfer functions with different delays

- Step response of the system of a unit step input given (at t=5s) is given below:



- Transfer functions of parallel systems sum together. Hence, we can see our transfer function as a sum of two first order transfer functions, with delays $D_1 = \theta_1 = 30$, and $D_2 = \theta_1 + \theta_2 = 75s$.
- This can also be verified by the two non-differentiable points in the step response (at t=30s and 75s); whereas normal first-order with delay systems have only one non-differentiable point.

END