INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH3050: Process Dynamics and Control (Jan - May 2019)
Assignment-1 Solutions

Marks distribution

	Question 1	Question 2	Question 3
(a)	5	9	10
(b)	8.5	10	10
(c)	8.5	9	10
(d)	-	10	-
(e)	-	10	-

Question 1

(a)

1. Controlled variable- distillate composition y.
2. Manipulated variables- distillate flow D, bottom flow B, reflux flow R.
3. Disturbance variables- feed flow F, feed composition z

(b)

The schematic diagram of feedback control mechanism for the distillation column is shown in Fig. 1.

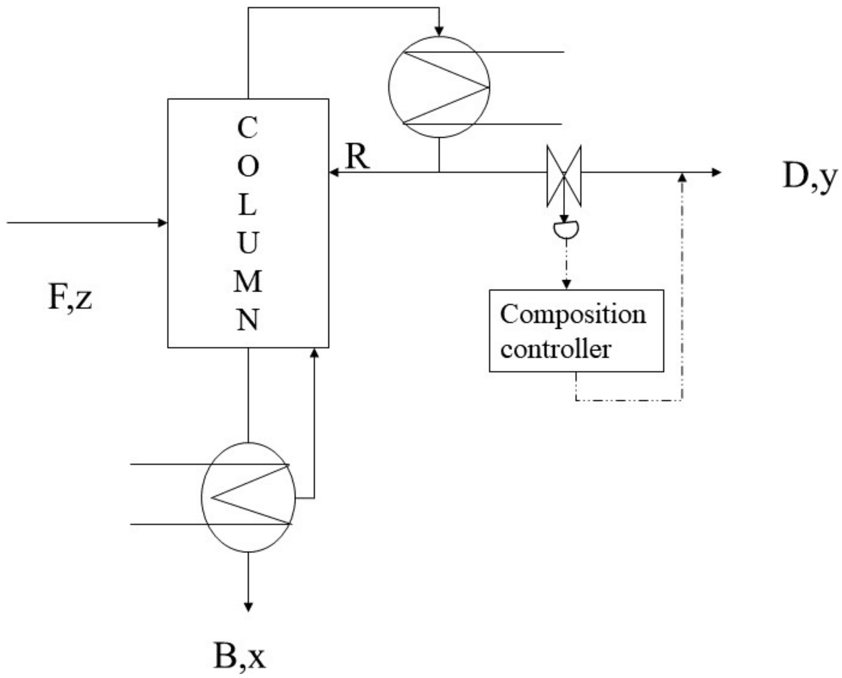


Figure 1: Schematic diagram of feedback control for the distillation column

(c)

The schematic diagram of feed-forward control mechanism for the distillation column is shown in Fig. 2.

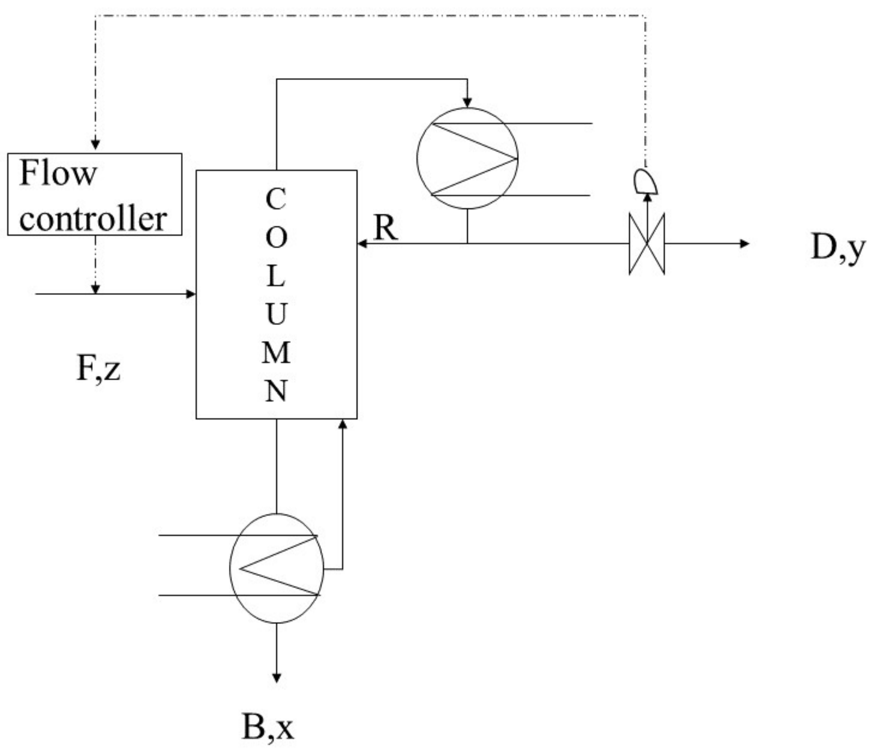


Figure 2: Schematic diagram of feed-forward control for the distillation column

Question 2

Given that

$$\frac{dw}{dt} = \frac{-(L+Va)}{M}w + \frac{Va}{M}z$$
$$\frac{dz}{dt} = \frac{L}{M}w - \frac{(L+Va)}{M}z + \frac{V}{M}z_f$$

where w and z are liquid concentrations on stage 1 and 2, respectively. L and V are the liquid and vapour molar flow rates, z_f si the concentration of the vapour stream entering the column.
The steady-state input values are $L = 80$ gmol inert liquid/min and $V = 100$ gmol inert vapour/min. The parameter values are $M = 20$ gmol inert liquid, $a = 0.5$ and $z_f = 0.1$ gmol solute / gmol inert vapour.

(a)

At steady state, the liquid concentrations are w_0 and z_0

$$0 = \frac{-(L+Va)}{M}w_0 + \frac{Va}{M}z_0 \tag{1}$$
$$z_0 = \frac{L+Va}{Va}w_0 \tag{2}$$

$$0 = \frac{L}{M}w_0 - \frac{(L+Va)}{M}z_0 + \frac{V}{M}z_f \tag{3}$$
$$w_0 = \frac{Va^2Z_f}{(L+Va^2) - LVa} \tag{4}$$

Substituting the given values in the above equations, we get the steady state values as $w_0 = 0.038$ and $z_0 = 0.100$

(b)

The given system is non-linear (non-linearity is caused by the product of state and inputs). The system is linearized around steady-state operations.

$$f_1 = \frac{dw}{dt} = \frac{-L}{M}w - \frac{Va}{M}w + \frac{Va}{M}z$$
$$f_2 = \frac{dz}{dt} = \frac{L(w-z) - Vaz + Vz_f}{M}$$

The state-space representation for the system is

$$\dot{x} = A\bar{x} + B\bar{u}$$
$$\bar{y} = C\bar{x} + D\bar{u} = C\bar{x}$$

State variables: w,z

Input variables : L,V

Output variables : w,z

D = 0

$$x = \begin{bmatrix} w \\ z \end{bmatrix}, \dot{x} = \begin{bmatrix} \frac{dw}{dt} \\ \frac{dz}{dt} \end{bmatrix} \tag{5}$$

$$A = \begin{bmatrix} \dfrac{-(L_0 + V_0 a)}{M} & \dfrac{V_0 a}{M} \\ \dfrac{L_0}{M} & \dfrac{-(L_0 + V_0 a)}{M} \end{bmatrix} \tag{6}$$

$$= \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix} \tag{7}$$

$$B = \begin{bmatrix} \dfrac{-w_0}{M} & \dfrac{-aw_0 + az_0}{M} \\ \dfrac{w_0 - z_0}{M} & \dfrac{-az_0 + zf_0}{M} \end{bmatrix} \tag{8}$$

$$= \begin{bmatrix} 1.9 \times 10^-3 & 1.55 \times 10^-3 \\ -3.1 \times 10^-3 & 2.5 \times 10^-3 \end{bmatrix} \tag{9}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{10}$$

Therefore, the linearized state space model around the steady-state operation is $A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix}$, $B = 10^{-3} \begin{bmatrix} 1.9 & 1.55 \\ -3.1 & 2.5 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $D = 0$.

(c)

Eigenvalues and eigenvectors of the system are

$$|A - \lambda I| = 0$$

The eigenvalues are $\lambda_1 = -3.34$ (slowest) and $\lambda_2 = -9.66$(fastest) and the eigenvectors are

$$V_1 = \begin{bmatrix} 0.79 \\ 1 \end{bmatrix} \tag{11}$$

$$V_2 = \begin{bmatrix} -0.79 \\ 1 \end{bmatrix} \tag{12}$$

Therefore the solution of the system is

$$x(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t} \tag{13}$$

Expected fastest initial condition direction of the system is

$$x(0) = C_2 V_2 e^{\lambda_2 (0)} = C_2 V_2 \tag{14}$$

and the slowest one is

$$x(0) = C_1 V_1 \tag{15}$$

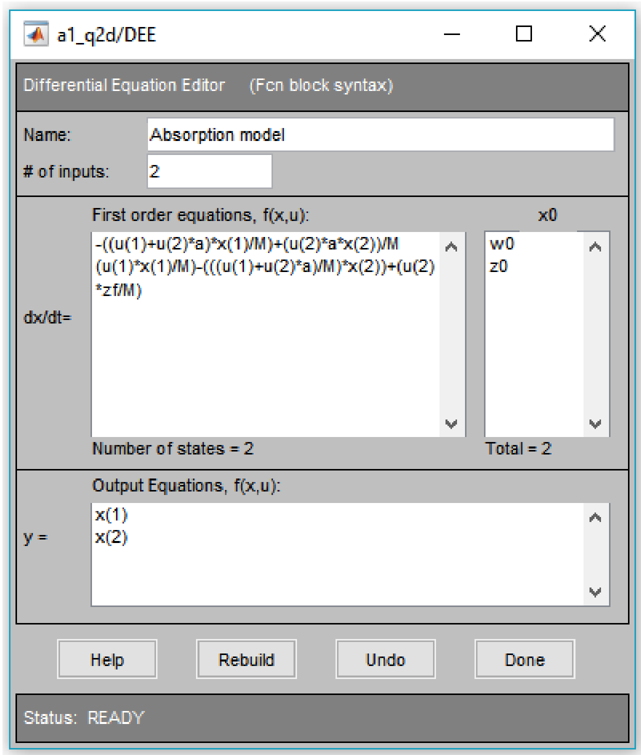
(d)

The Simulink model for the given non-linear system is given Fig. The MATLAB code to to obtain the linearized model for the above non-linear system is given below

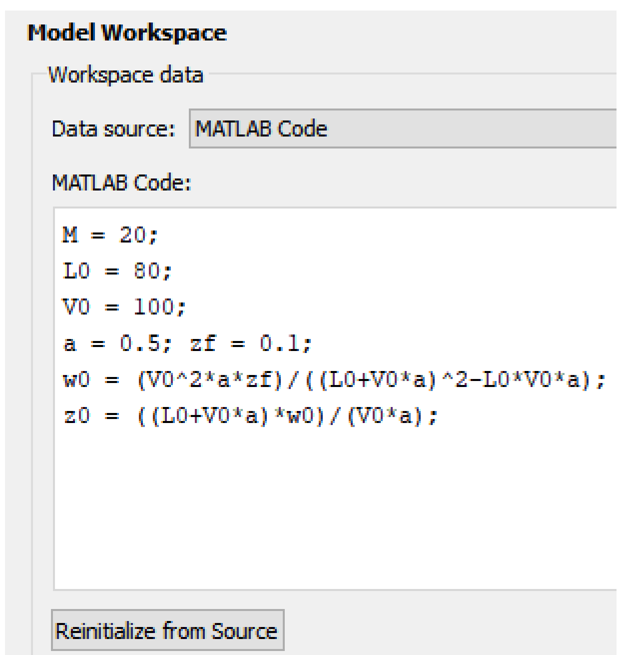
```
%% Question 2(d)

% Finding steady state values
[xs,us,ys] = trim('a1_q2d',[1 1]',[80 100]',[1 1]',[], [1 1]',[]);

% Linearized model
[A,B,C,D] = linmod('a1_q2d',xs,[80 100]');

% Final SS model
modlin_ss = ss(A,B,C,D);
```

The linearized model is

(a) Simulink diagram



(b) DEE block



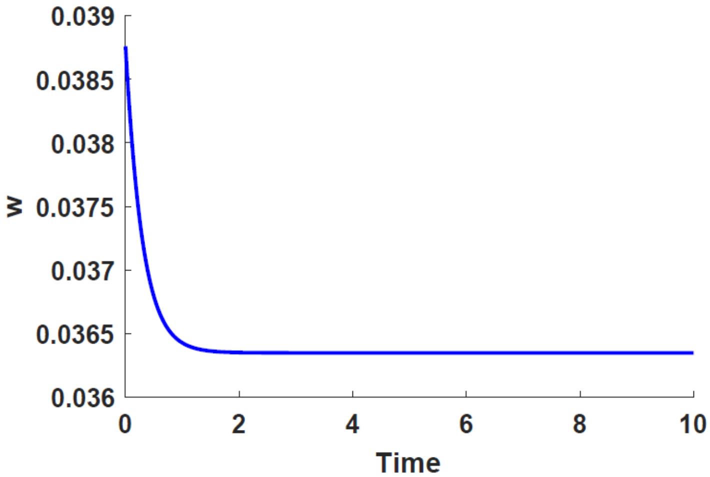
(c) Parameters

Figure 3: Simulink model for the given non-linear system

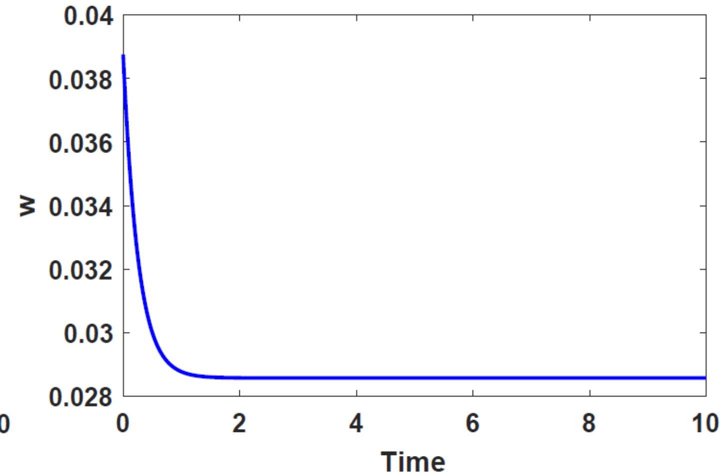
```
A =
x1    x2
x1  -6.5   2.5
x2     4  -6.5

B =
u1          u2
x1  -0.001938    0.00155
x2  -0.003101   0.002481

C =
x1  x2
y1   1   0
y2   0   1

D =
u1  u2
y1   0   0
y2   0   0
```

(e)

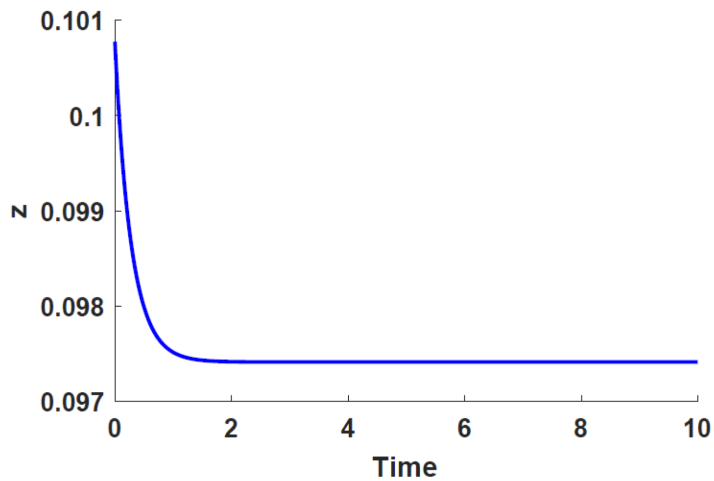
The step responses of the given non-linear system for two different magnitudes of steps (i) 5% and (ii) 20% change in the flow rate are illustrated in Fig. 4. Similarly, the step responses of the linearized system are shown in Fig. 5.



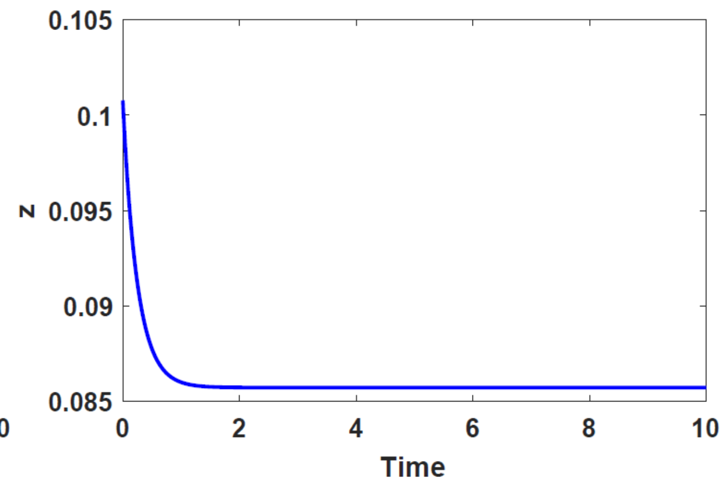
(a) w for 5% change in flow rate



(b) w for 20% change in flow rate

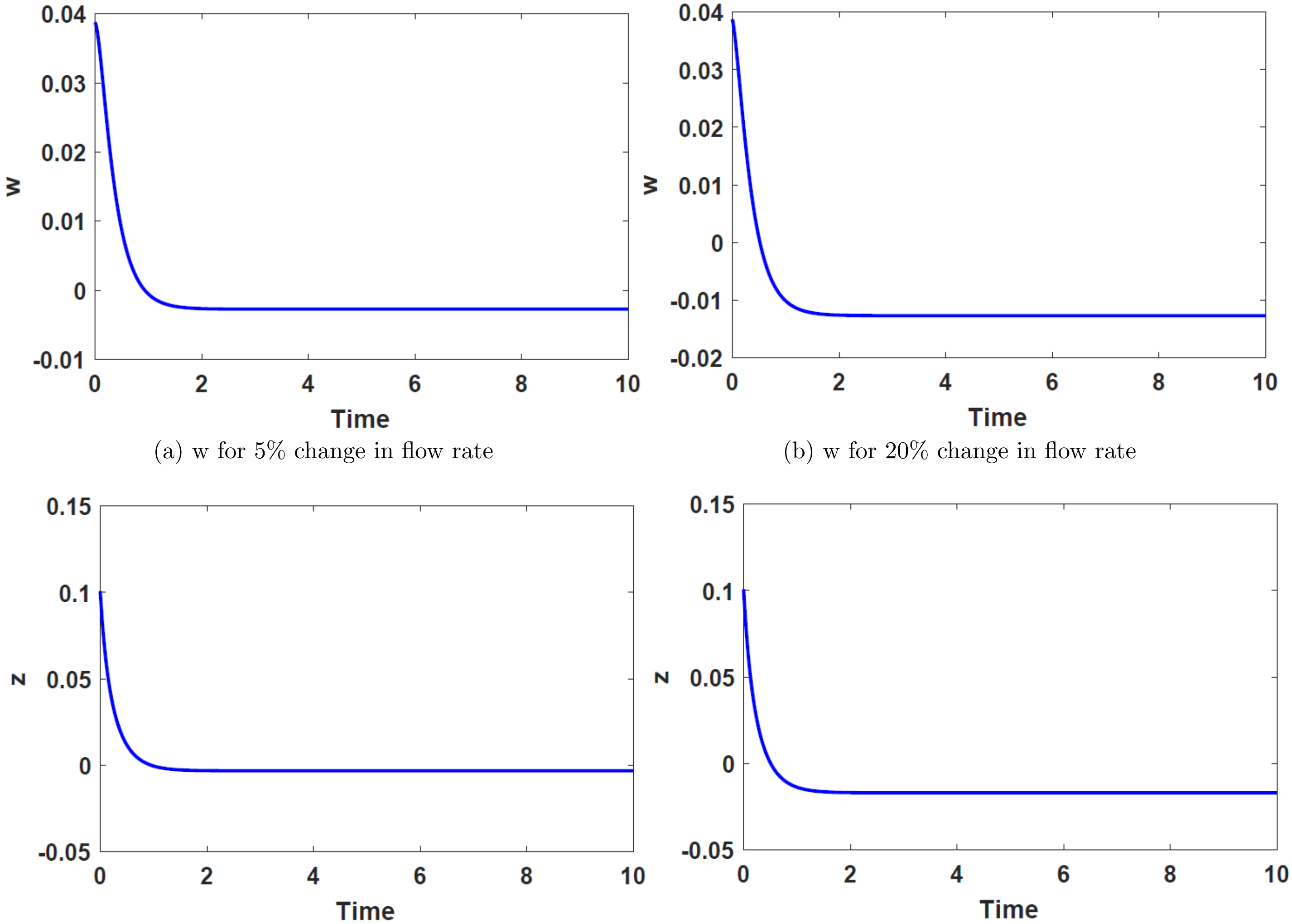


(c) z for 5% change in flow rate



(d) z for 20% change in flow rate

Figure 4: Step responses of non-linear system



(a) w for 5% change in flow rate

(b) w for 20% change in flow rate

(c) z for 5% change in flow rate

(d) z for 20% change in flow rate

Figure 5: Step responses of linearized system

Question 3

(a)

Given signal is

$$x(t) = \begin{cases} t-2 & 0 \leq t < 2 \\ 0 & 2 \leq t < 3 \\ \sin(5\pi(t-3) & 3 \leq t < 5 \\ \exp^{-2t}\sin(5\pi t) & t \geq 5 \end{cases} \tag{16}$$

The Laplace transform of any signal $x(t)$ is

$$\mathcal{L}\{x(t)\} = X(s) = \int_0^\infty x(t)e^{-st}dt \tag{17}$$

Laplace transform of the given signal is

$$X(s) = \int_0^2 (t-2)e^{-st}dt + \int_2^3 0e^{-st}dt + \int_3^5 \sin(5\pi(t-3))e^{-st}dt + \int_5^\infty e^{-2t}\sin(5\pi t)e^{-st}dt \tag{18}$$

Using integration by parts,

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int (f'(x)\int g(x)dx)dx$$

we get

$$X(s) = \frac{-e^{-2s} - 2s + 1}{s^2} + \frac{5\pi e^{-5s}(e^{2s}-1)}{s^2+25\pi^2}\frac{-5\pi e^{-5s-10}}{s^2+4s+25\pi^2+4} \tag{19}$$

(b)

Given that

$$X(s) = \frac{1}{s(\tau^2 s^2 + 2\zeta\tau s + 1)} \tag{20}$$

Case 1: $\zeta > 1$ Roots would be different, negative and real.

$$X(s) = \frac{1}{s(s + \frac{\zeta}{\tau} - \frac{\sqrt{\zeta^2-1}}{\tau})(s + \frac{\zeta}{\tau} + \frac{\sqrt{\zeta^2-1}}{\tau})} \tag{21}$$

The inverse Laplace transform is

$$x(t) = \tau^2 u(t) + \frac{1}{\frac{2}{\tau}\sqrt{\zeta^2-1}}\left[\exp\left(\frac{-(\zeta+\sqrt{\zeta^2-1})t}{\tau}\right) - \exp\left(\frac{-(\zeta+\sqrt{\zeta^2-1})t}{\tau}\right)\right] \tag{22}$$

Case 2: $\zeta = 1$ Roots would be identical, negative and real.

$$x(s) = \frac{1}{s\tau^2(s+\frac{1}{\tau})^2}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

$$x(t) = u(t)\left[1 - (1 + \frac{t}{\tau})e^{-\frac{t}{\tau}}\right]$$

Case 3: $0 \leq \zeta < 1$ Roots would be complex conjugate form.

$$x(s) = \frac{1}{\left(s + \frac{\zeta}{\tau} + j\frac{\sqrt{1-\zeta^2}}{\tau}\right)\left(s + \frac{\zeta}{\tau} - j\frac{\sqrt{1-\zeta^2}}{\tau}\right)}$$

$$x(t) = \left[1 - \frac{1}{\sqrt{1-\zeta^2}} \exp\left(\frac{-\zeta t}{\tau} \sin\left(\frac{\sqrt{1-\zeta^2} t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)\right)\right]$$

(c)

Given differential equation is

$$\ddot{y}(t) + 7\dot{y}(t) + 12y(t) = 2\cos(\omega_0 t) \qquad t > 0 \tag{23}$$

Using Laplace transform on both the sides

$$s^2Y(s) - sY(0) - Y'(0) + 7\left(sY(s) - Y(0)\right) + 12Y(s) = 2\frac{s}{s^2 + \omega_0^2} \tag{24}$$

Assuming zero initial conditions

$$Y(s) = \frac{2s}{(s^2 + 7s + 12)(s^2 + \omega_0^2)} \tag{25}$$

Therefore, the solution of the given differential equation is of the form

$$y(t) = Ae^{-3t} + Be^{-4t} + C\cos(\omega_0 t) + D\sin(\omega_0 t) \tag{26}$$

The solution at steady state does not exist because the denominator of $[sY(s)]$ is a product of complex factors (final value theorem is not valid). Therefore, the response $y(t)$ is oscillatory. Since $y(t)$ is oscillatory, it is not converging either.

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CH3050 Process Dynamics and Control

Assignment 2 Solutions

Marks distribution

	Question 1	Question 2	Question 3
(a)	13	12	15
(b)	10	8	15
(c)	17	-	-
(d)	10	-	-

1

Given that an exothermic reaction $A \longrightarrow 2B$, takes place adiabatically in a stirred-tank reactor. This liquid reaction occurs at constant volume in a 1000-gallon reactor. The reaction can be considered to be first order and irreversible with the rate constant given by $k = 2.4 \times 10^{15} e^{-20000/T}$(min^{-1}) where T is in $^\circ R$. The steady-state conditions are $c_{Ai,ss} = 0.8$ mol/ft^3 and $q_{ss} = 20$ gallons/min. The physical property data for the mixture: $T_i = 90^\circ$F, $C = 0.8$ Btu/(lb $^\circ$F), $\rho = 52$ lb / ft^3 and $\triangle H_R = -500$ kJ/mol

(a)

The first-principles model for the given stirred-tank reactor assuming

1. perfectly mixed reactor
2. constant fluid properties and heat of reaction

is given below.

Component balance is

$$V\frac{dc_A}{dt} = qc_{A_i} - qc_A + Vk(T)c_A$$
$$\frac{dc_A}{dt} = -(\frac{q}{V} + k(T))c_A + \frac{qc_{A_i}}{V}$$

The energy balance is

$$V\rho c_p\frac{dT}{dt} = q\rho c_p T_i - q\rho c_p T + (-\Delta H_R)(-k(T)C_A)V$$
$$\frac{dT}{dt} = (T_i - T)\frac{q}{V} + \frac{\Delta H_R}{\rho c_p}(k(T)C_A)$$

where,

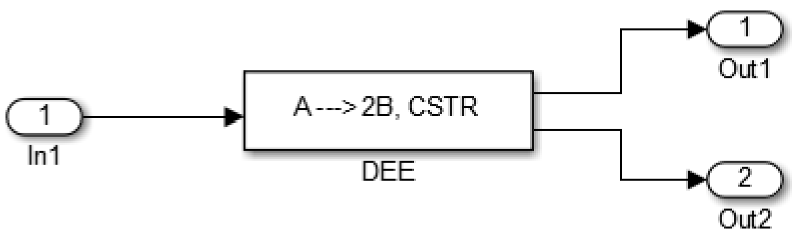
$$k(T) = -2.4 \times 10^{15} e^{\frac{-20000}{T}} (min^{-1}), c_p = 0.8 \frac{Btu}{lb^\circ F},$$
$$\rho = 52 \frac{lb}{ft^3}, \Delta H_R = -500 \frac{KJ}{mol}, q = q_{ss} = 20 \frac{gallons}{min}$$

Therefore the model is

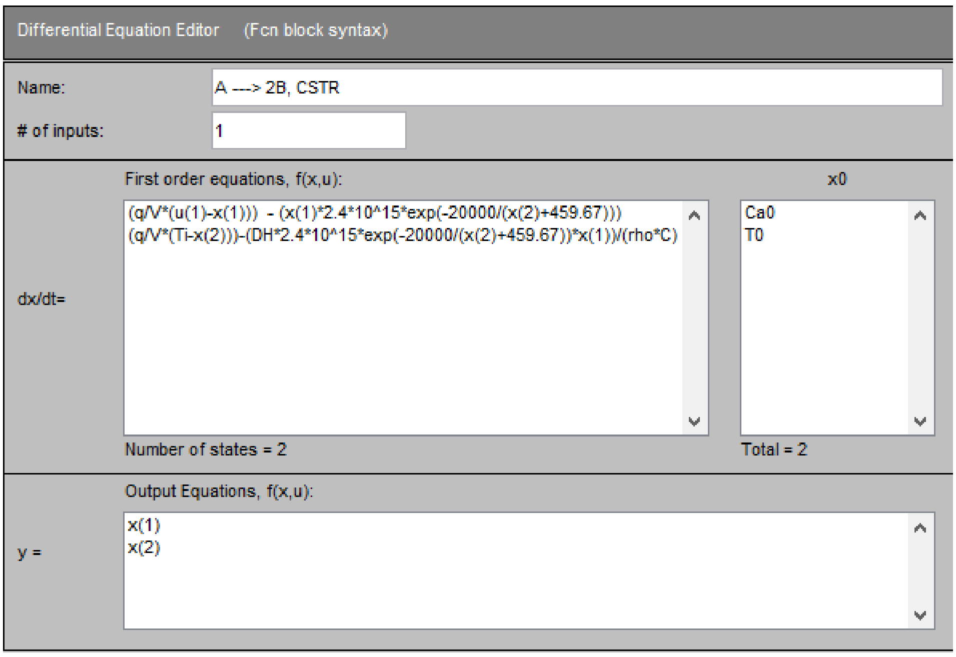
$$\frac{dc_A}{dt} = (-\frac{q}{V} + k(T))c_A + \frac{qc_{A_i}}{V} \tag{1}$$
$$\frac{dT}{dt} = (T_i - T)\frac{q}{V} + \frac{\Delta H_R}{\rho c_p}(k(T)C_A) \tag{2}$$

(b)

The Simulink block diagram to determine the steady-state exit temperature using the `trim` routine of MATLAB is given in Fig.1.



(a) Simulink block diagram



(b) DEE Block

Figure 1: Simulink block diagram

The code for to determine steady state temperature using trim is given below.

```
% Assignment 2, ch3050, 2019
% Question 1 (b)
% Dheeraj Kumar

% Steady-state exit temperature using 'trim', with C_Ai_ss = 0.8
[xs,us,ys] = trim('Temp_ss_trim', [1 1]',0.8,[1 1]',[],1,[]);

% Steady-state exit temperature
Tss = xs(2)
Tss =

99.8705
```

The steady-state exist temperature is $Tss = 99.8705°F$.

(c)

Transfer function using MATLAB

The code in MATLAB to obtain the transfer function relating the exit temperature T to the inlet concentration c_{A_i} is given below.

```
% Assignment 2, ch3050, 2019
% Question 1 (c)
% Dheeraj Kumar

% Steady-state exit temperature using 'trim', with C_Ai_ss = 0.8
[xs,us,ys] = trim('Temp_ss_trim', [1 1]',0.8,[1 1]',[],1,[]);

% Linearized model
[A1,B1,C1,D1] = linmod('Temp_ss_trim',xs,0.8);
% Final SS model
modlin_ss = ss(A1,B1,C1,D1);
% Transfer function
[num, den ] = ss2tf(A1,B1,C1,D1);
Gs = tf(num(2,:),den);

Gs =

        0.1825
------------------------
s^2 + 0.7468 s + 0.01454

Continuous-time transfer function.
```

Therefore, the transfer function obtained using MATLAB is

$$G(s) = \frac{T(s)}{C_{A_i}(s)} = \frac{0.1825}{s^2 + 0.7468s + 0.01454} \tag{3}$$

Transfer function by hand

The transfer function relating the exit temperature T to the inlet concentration c_{A_i} is obtained as follows assuming the other inputs, namely q and T_i, to be constant. Linearizing the above first-principles model, we get

$$\frac{dc_A}{dt} = \frac{dc'_A}{dt}, \quad \frac{dT}{dt} = \frac{dT'}{dt}$$

$$V\frac{dc'_A}{dt} = qc'_{A_i} - (q + Vk(\bar{T})c'_A - V\bar{c_A}k(\bar{T})\frac{20000}{\bar{T}^2}T'$$

Note that $\bar{c_A}$ and $\bar{T}$ represent the steady state values.

$$V\rho c_p\frac{dT'}{dt} = -(q\rho c_p + \Delta H_R V\bar{c_A}k(\bar{T})\frac{20000}{\bar{T}^2})T' + (-\Delta H_R)Vk(\bar{T})c'_A$$

Taking the Laplace transforms and rearranging, we get

$$[Vs + q + Vk(\bar{T})]C'_A(s) = qC'_{A_i}(s) - V\bar{c_A}k(\bar{T})\frac{20000}{\bar{T}^2}T'(s)$$

$$[V\rho c_p s + q\rho c_p - (-\Delta H_R V\bar{c_A}k(\bar{T})\frac{20000}{\bar{T}^2})]T'(s) = (-\Delta H_R)Vk(\bar{T})C'_A(s)$$

Substituting C'_A(s) and rearranging, we get

$$\frac{T'(s)}{C_{A_i}(s)} = \frac{\Delta H_R)Vk(\bar{T})q}{[Vs + q + Vk(\bar{T})][V\rho c_p s + q\rho c_p - (-\Delta H_R V\bar{c_A}k(\bar{T})\frac{20000}{\bar{T}^2})] + -\Delta H_R\bar{c_A}V^2k^2(\bar{T})\frac{20000}{\bar{T}^2}}$$

The value of $\bar{c_A}$ at steady state is (using (1)),

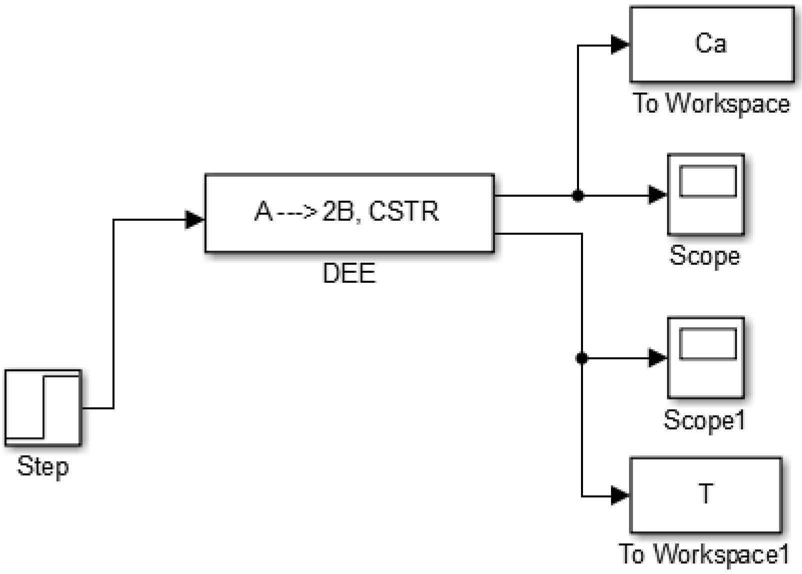
$$\bar{c_A} = \frac{q\bar{c_{A_i}}}{q + Vk(\bar{T})} = 0.0011546\frac{mol}{ft^3}$$

Substituting the numerical values of $\bar{T}, \rho, C, (-\Delta H_R), q, V, \bar{c_A}$, we get the transfer function of the given system as

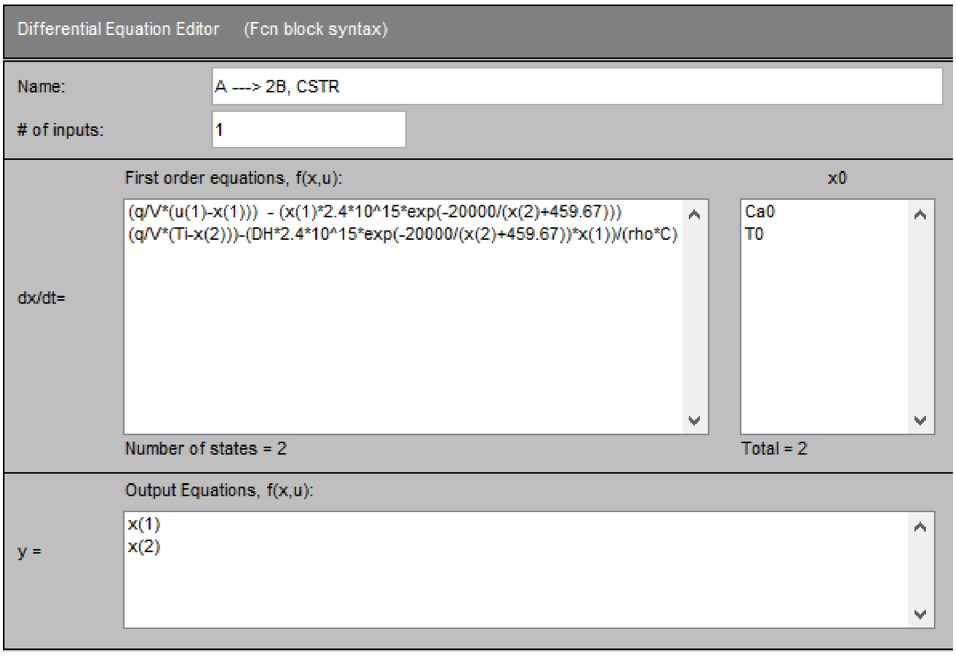
$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{11.38}{(0.0722s + 1)(50s + 1)} \tag{4}$$

(d)

The Simulink block diagram for the non-linear system is given below.



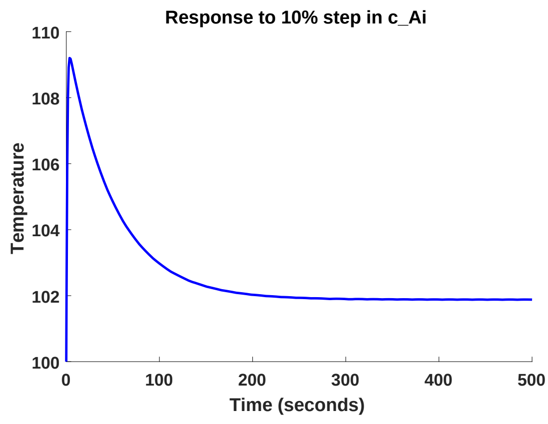
(a) Similink block diagram



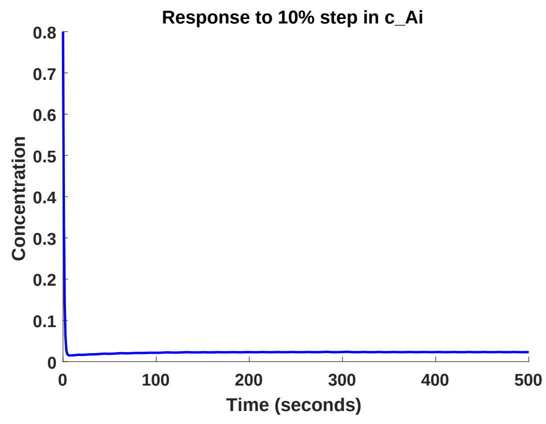
(b) DEE Block

Figure 2: Simulink block diagram for non-linear system

The step response of the non-linear system for 10% step in C_{A_i} is shown in Fig. 3.



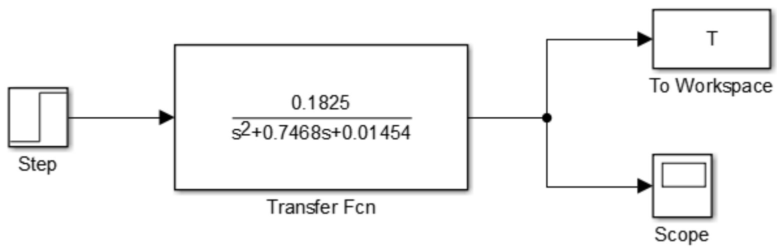
(a) Response of T



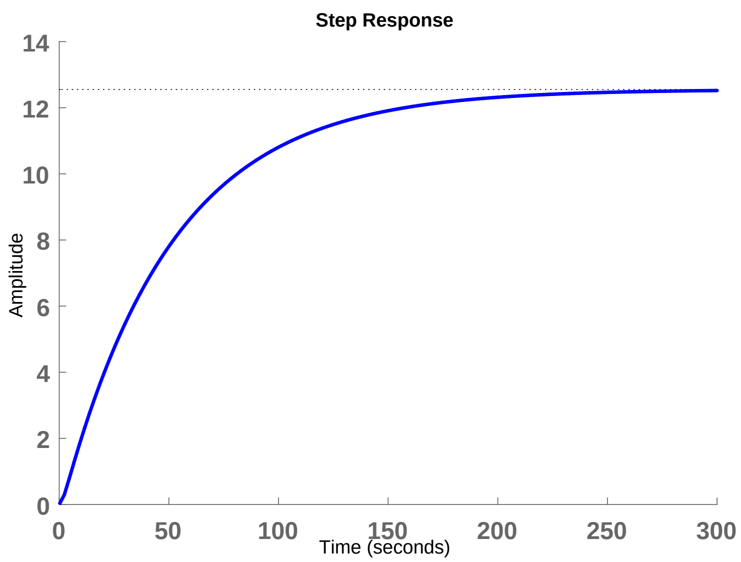
(b) Reponse of C_A

Figure 3: Step responses of non-linear system

The Simulink block diagram for the linearized system and the corresponding step response are shown in Fig. 4(a) and Fig. 4(b), respectively.



(a) Simulink block diagram



(b) Step response

Figure 4: Simulink block diagram for linearized system

2

(a)

Given the transfer function of the system $G(s) = \dfrac{s+1}{s^3+8s^2+19s+12}$, the equivalent SS representation can be written as

(i) Partial fraction expansion method

$$
\begin{aligned}
G(s) &= \frac{s+1}{s^3+8s^2+19s+12} \\
&\implies \frac{s+1}{(s+1)(s+3)(s+4)} \\
\implies \frac{Y(s)}{U(s)} &= \frac{1}{s+3} - \frac{1}{(s+4)}
\end{aligned}
$$

$$
\text{let } X_1(s) = \frac{U(s)}{s+3} \text{ and } X_2(s) = \frac{U(s)}{s+4}
$$

$$
Y(s) = X_1(s) - X_2(s)
$$

Now, the state equations are

$$
\begin{aligned}
\dot{x_1} &= u(t) - 3x_1(t) \\
\dot{x_2} &= u(t) - 4x_2(t)
\end{aligned}
$$

The corresponding state space representation can be written as

$$
\begin{aligned}
\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} &= \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}
\end{aligned}
$$

(ii) State transition diagram method

$$
Y(s) = \underbrace{\frac{U(s)}{s^3+8s^2+19s+12}}_{X(s)}(s+1)
$$

$$
s^3X(s) + 8s^2X(s) + 19sX(s) + 12X(s) = U(s)
$$

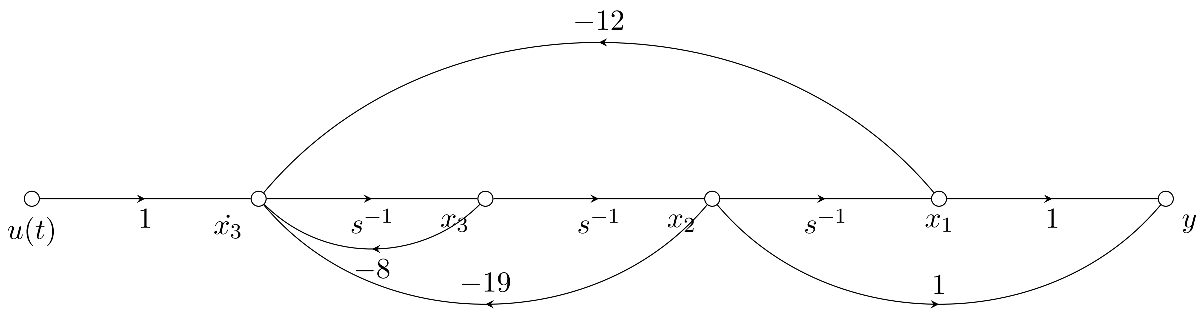
Now the state equations using the state transition diagram are

$$
\begin{aligned}
x_1(t) &= x(t) \\
\dot{x_1} &= x_2(t) \\
\dot{x_2} &= x_3(t) \\
\dot{x_3} &= u(t) - 12x_1(t) - 19x_2(t) - 8x_3(t)
\end{aligned}
$$

Now the corresponding state space representation can also be written as

$$
\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -19 & -8 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)
$$

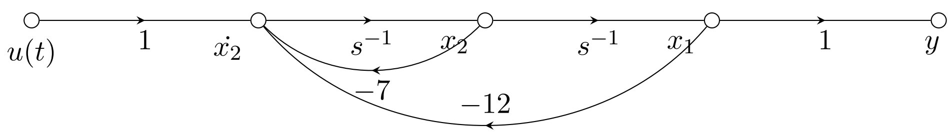
$$
y(t) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}
$$



Now, as it can be seen in state transition method, the system matrix $\mathbf{A} \in \mathbf{R}^{3\times 3}$. Whereas in the case of partial fraction method, due to pole-zero cancellation, the system matrix $\mathbf{A} \in \mathbf{R}^{2\times 2}$ which violates the condition for existence of linear transformation $\mathbf{T}$. For the case of minimal realization the state space representation using state transition diagram can be written as

$$
\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -12 & -7 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{\tilde{\mathbf{A}}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)
$$

$$
y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}
$$



Since, the system is the same for both the representations the eigenvalues of $\mathbf{A}$ and $\tilde{\mathbf{A}}$ should be the same. This ensures the *similarity* relation between $\mathbf{A}$ and $\tilde{\mathbf{A}}$.

So, $\exists\ \mathbf{T}, s.t. \mathbf{T}\tilde{\mathbf{A}} = \mathbf{A}$.

$$\begin{aligned}
\mathbf{T}\tilde{\mathbf{A}} &= \mathbf{A} \\
\mathbf{T} &= \mathbf{A}\tilde{\mathbf{A}}^{-1} \\
\mathbf{T} &= \begin{bmatrix} 1.75 & 0.25 \\ -4 & 0 \end{bmatrix}
\end{aligned}$$

(b)

For the given SITO system

$$\begin{aligned}
\frac{Y_1(s)}{U(s)} &= \frac{4s+1}{(s+2)(s+4)} \\
\implies \frac{Y_1(s)}{U(s)} &= \frac{7.5}{s+4} - \frac{3.5}{s+2} \\
\implies Y_1(s) &= \underbrace{\frac{7.5U(s)}{s+4}}_{X_1} - \underbrace{\frac{3.5U(s)}{s+2}}_{X_2} \\
\implies Y_1(s) &= X_1 - X_2 \\
\frac{Y_2(s)}{U(s)} &= \frac{10s}{(s+1)(s+4)} \\
\implies \frac{Y_2(s)}{U(s)} &= \frac{13.33}{s+4} - \frac{3.33}{s+1} \\
Y_2(s) &= \frac{13.33U(s)}{s+4} - \underbrace{\frac{3.33U(s)}{s+1}}_{X_3} \\
\implies Y_2(s) &= 1.77X_1 - X_3
\end{aligned}$$

Following this,the corresponding state equations can be written as

$$\begin{aligned}
\dot{x_1} &= 7.5u(t) - 4x_1(t) \\
\dot{x_2} &= 3.5u(t) - 2x_2(t) \\
\dot{x_3} &= 3.33u(t) - x_3(t)
\end{aligned}$$

So overall equivalent state space representation of the given system is

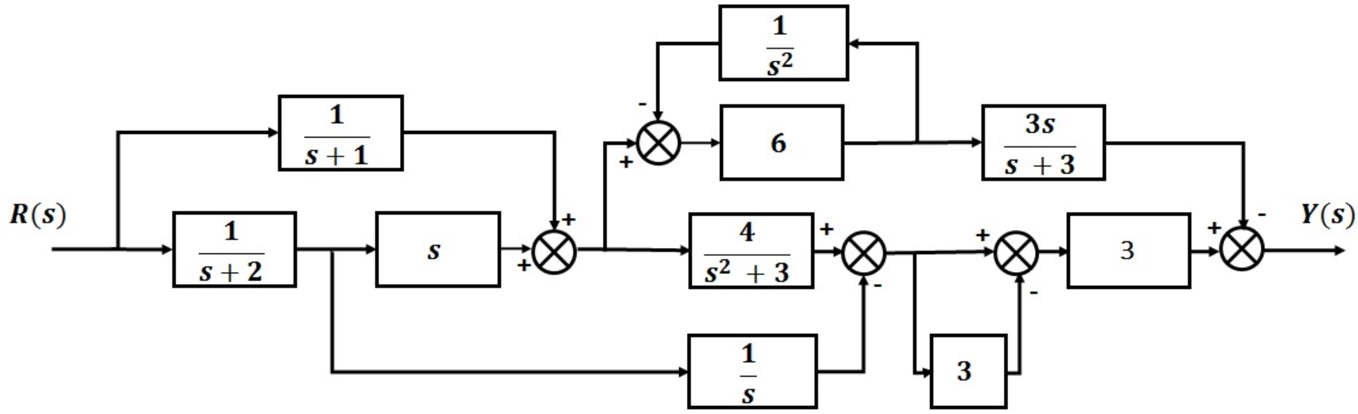
$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 7.5 \\ 3.5 \\ 3.33 \end{bmatrix} u(t)$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1.77 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

3

(a)

The block diagram relating $R(s)$ to $Y(s)$ for the given signal flow graph of the system is given below.



(b)

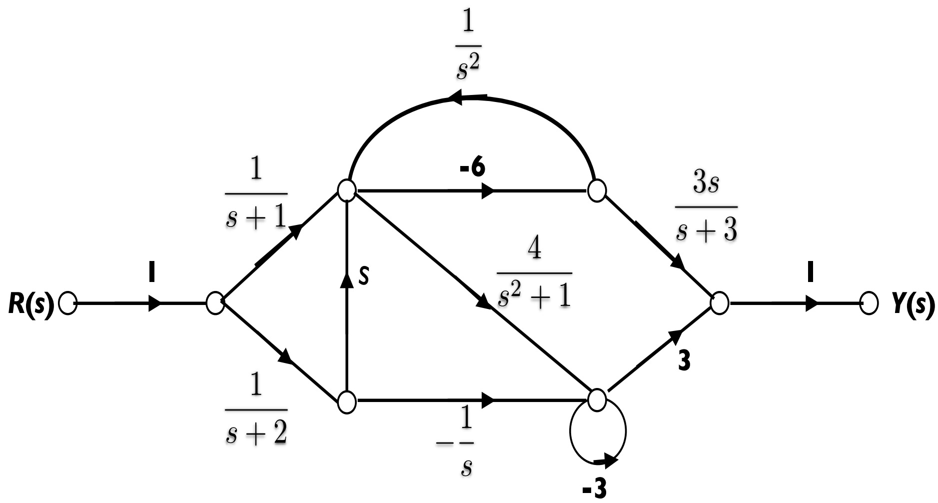


Figure 5: Signal flow graph for Q.4

$1st$ forward path ABCDEF transfer function G_1 is

$$1.\frac{1}{s+1}.(-6).\frac{3s}{s+3}.1 = \frac{-18s}{(s+1)(s+3)} \tag{5}$$

$2nd$ forward path ABCHEF transfer function G_2 is

$$1.\frac{1}{s+1}.\frac{4}{s^2+1}3.1 = \frac{12}{(s^2+1)(s+1)} \tag{6}$$

$3rd$ forward path ABGHEF transfer function G_3 is

$$1.\frac{1}{s+2}.\frac{-1}{s}.3.1 = \frac{-3}{s(s+2)} \tag{7}$$

$4th$ forward path ABGCHEF transfer function G_4 is

$$1.\frac{1}{s+2}.s.(-6).\frac{-3s}{s+3}.1 = \frac{-18s^2}{(s+2)(s+3)} \tag{8}$$

$5th$ forward path ABGCHEF transfer function G_5 is

$$1.\frac{1}{s+2}.s.\frac{4}{s^2+1}.3.1 = \frac{12s}{(s^2+1)(s+2)} \tag{9}$$

$1st$ independent loop CDC transfer function L_1 is

$$-6.\frac{1}{s^2} \tag{10}$$

$2nd$ independent loop HH transfer function L_2 is

$$-3 \tag{11}$$

According to the Mason's gain formula

$$\frac{Y(s)}{R(s)} = \frac{\sum G_k \Delta_k}{\Delta}, \qquad k = 1, 2... \tag{12}$$

where,

$$\Delta = 1 - \sum L_1 + \sum L_1 L_2 + ...$$
$$\sum G_k \Delta_k = G_1 \Delta_1 + G_2 \Delta_2 + ...$$

$$\begin{aligned}\sum G_k \Delta_k &= G_1(1 - L_2) + G_2(1) + G_3 L_1 + G_4 L_2 + G_5(1) \\ &= \frac{-72}{(s+1)(s+3)} + \frac{12}{(s^2+1)(s+1)} - \frac{3(1+\frac{6}{s^2})}{s(s+2)} + \frac{54s^2}{(s+2)(s+3)} + \frac{12s}{(s^2+1)(s+2)}\end{aligned}$$

$$
\begin{aligned}
\Delta &= 1 - (L_1 + L_2) + (L_1 L_2) + ... \\
&= 1 - (\frac{-6}{s^2} - 3) + \frac{18}{s^2} \\
&= \frac{4(s^2+6)}{s^2}
\end{aligned}
$$

Substituting the values of Δ and $\sum G_k \Delta_k$ in (12), we get the transfer function of the system as

$$\frac{Y(s)}{R(s)} = \frac{-72s^3(s^2+1)(s+2) + 12s^3(s+2)(s+3) - 3(s^2+6)(s^2+1)(s+3)(s+1) + 45s^5(s^2+1)(s^2+6)(s+1)(s+2)(s+3)}{4s(s^2+6)(s^{+}1)(s+1)(s+2)(s+3)}$$

INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH3050 Process Dynamics and Control

Jan-May 2019 Assignment 3 Solutions

Marks distribution

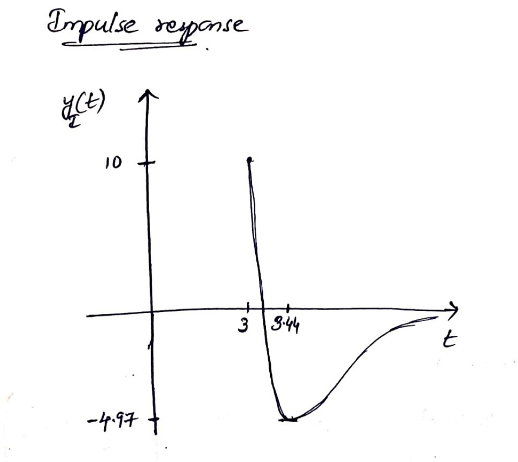
	Question 1	Question 2	Question 3
(a)	15	12	20
(b)	10	8	-
(c)	15	10	-
(d)	5	-	-
(e)	5	-	-

1

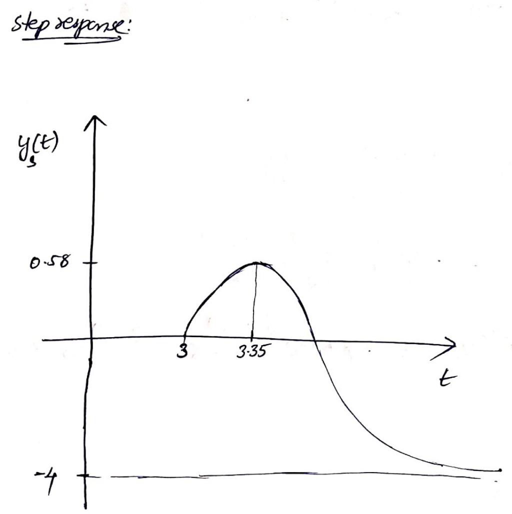
Given the transfer function $G(s) = \frac{10(s-4)}{s^2+7s+10}e^{-3s}$,

(a) Impulse Response

$$
\begin{aligned}
U(s) &= 1 \\
Therefore\ \ Y(s) = G(s)U(s) &= \frac{10(s-4)}{s^2+7s+10}e^{-3s} \\
Using\ \ partial\ \ fractions, & \\
&= 10e^{-3s}\left(\frac{-2}{s+2} + \frac{3}{s+5}\right) \\
Finally & \\
y(t) &= L^{-1}\left\{\frac{-20e^{-3s}}{s+2} + \frac{30e^{-3s}}{s+5}\right\} \\
&= -20e^{-2(t-3)} + 30e^{-5(t-3)}\ \ for\ \ t \geq 3
\end{aligned}
$$



(a) Picture 1



(b) Picture 2

Step Response

$$U(s) = \frac{1}{s}$$

$$Therefore\ \ Y(s) = G(s)U(s) = \frac{10(s-4)}{s^2+7s+10}e^{-3s}\frac{1}{s}$$

$$Using\ \ partial\ \ fractions,$$

$$= \frac{-4e^{-3s}}{s} + \frac{10e^{-3s}}{s+2} - \frac{6e^{-3s}}{s+5}$$

$$Finally$$

$$y(t) = L^{-1}\left\{\frac{-4e^{-3s}}{s} + \frac{10e^{-3s}}{s+2} - \frac{6e^{-3s}}{s+5}\right\}$$

$$= -4 + 10e^{-2(t-3)} - 6e^{-5(t-3)}\ \ for\ \ t \geq 3$$

(b) Large time response of the process for

$u(t) = 2sin(4t) + cos(0.1t)$ Finding the FRF of the given system

$$
\begin{aligned}
G(j\omega) &= \frac{10(j\omega-4)}{-\omega^2+7j\omega+10}e^{-3j\omega} -------- ① \\
G(j\omega) &= \frac{10(j\omega-4)}{10-\omega^2+j7\omega}e^{-3j\omega}\frac{10-\omega^2-j7\omega}{10-\omega^2-j7\omega} \\
G(j\omega) &= \frac{10e^{-3j\omega}}{(10-\omega^2)^2+49\omega^2}\Big((11\omega^2-40)+j(38\omega-\omega^3)\Big) \\
|G(j\omega)| = \frac{B}{A}(\omega) &= \frac{10}{(10-\omega^2)^2+49\omega^2}\sqrt{\Big((11\omega^2-40)^2+(38\omega-\omega^3)^2\Big)}
\end{aligned}
$$

Or we can write equation 1 as

$$
\begin{aligned}
G(j\omega) &= \Big(10(j\omega-4)\Big)\Big(e^{-3j\omega}\Big)\Big(\frac{1}{j\omega+2}\Big)\Big(\frac{1}{j\omega+5}\Big) \\
\frac{B}{A}(\omega) &= \prod_{i=1}^{4}|G_i| = 10\sqrt{\omega^2+16}\times 1\times\frac{1}{\sqrt{\omega^2+4}}\times\frac{1}{\sqrt{\omega^2+25}} \\
\frac{B}{A}(\omega=4) &= 1.975, \frac{B}{A}(\omega=0.1) = 3.995 \\
\phi &= tan^{-1}(\frac{\omega}{-4}) - 3\omega + tan^{-1}(\frac{-\omega}{2}) + tan^{-1}(\frac{-\omega}{5}) \\
\phi(\omega=4) &= -14.565, \phi(\omega=0.1) = -0.395 \\
y_{ss}(t) &= 2\times 1.975\ \ sin(4t-14.565) + 3.995\ \ cos(0.1t-0.395)
\end{aligned}
$$

(c) Bode Plots

$$
\begin{aligned}
dB = 20\log_{10}AR = 20\log_{10}\frac{B}{A}(\omega) = 20\log_{10}\prod_{i=1}^{4}G_i &= 20\log_{10}\Big(10\sqrt{\omega^2+16}\times 1\times\frac{1}{\sqrt{\omega^2+4}}\times\frac{1}{\sqrt{\omega^2+25}}\Big) \\
&= 20\log_{10}\Big(4\sqrt{(\frac{\omega}{4})^2+1}\times\frac{1}{\sqrt{(\frac{\omega}{2})^2+1}}\times\frac{1}{\sqrt{(\frac{\omega}{5})^2+1}}\Big) \\
&= 20log_{10}4 + 10log_{10}\Big((0.25\omega)^2+1\Big) - 10log_{10}\Big((0.5\omega)^2+1\Big) - 10log_{10}\Big((0.2\omega)^2+1\Big)
\end{aligned}
$$

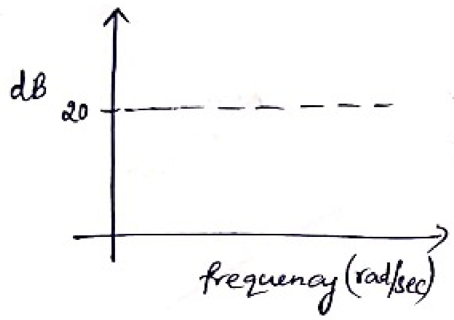
c) i) $G_0 = 10$

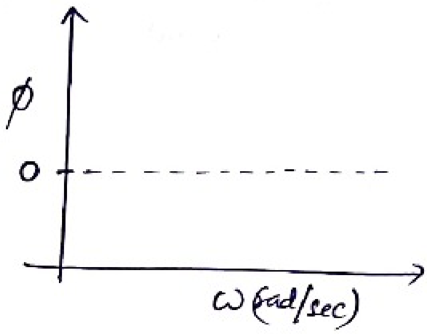
$G_0(j\omega) = 10$

$|G_0(j\omega)| = 10$

$dB = 20 \log_{10} 10 = 20 \quad \forall \omega$

$\phi = 0 \;\forall\, \omega$





ii) $G_1 = s-4$

$G_1(j\omega) = j\omega - 4$

$|G_1(j\omega)| = \sqrt{\omega^2+16}$

$\therefore \; dB = 20 \log_{10} \sqrt{\omega^2+16} = 10 \log_{10} (\omega^2+16)$

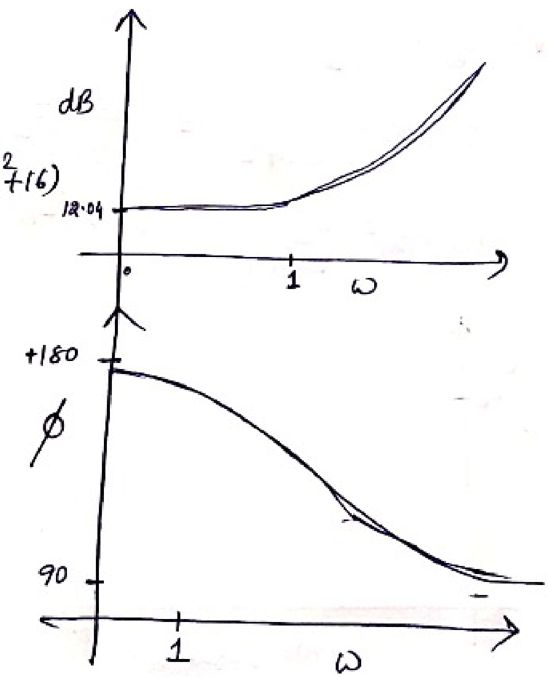
$\phi = -\tan^{-1}\left(\frac{\omega}{4}\right)$

@$\omega = 0$, $dB = 12.04$,

as ω increases initially $(0<\omega<1)$, slow increase in dB as well,

But $(\omega > 1)$, there is square effect

Therefore, dB increases at faster rate



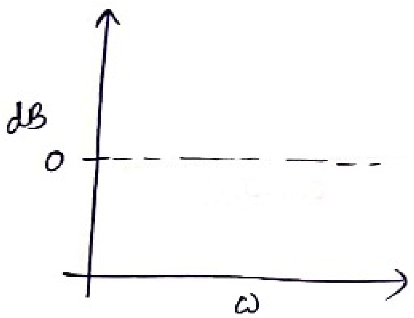
iii) $G_2 = e^{-3s}$

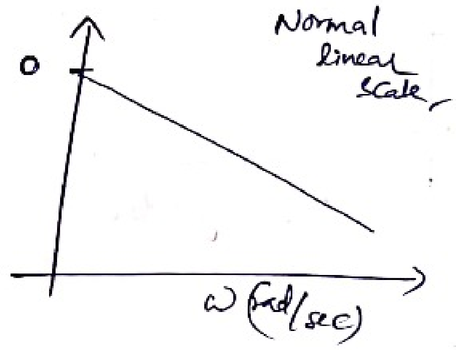
$$G_2(j\omega) = e^{-3j\omega} = \cos(-3\omega) + j\sin(-3\omega)$$
$$= \cos 3\omega - j\sin(3\omega)$$

$$|G_2(j\omega)| = 1$$

$\therefore dB = 0 \ \forall \omega,$

$$\phi = \tan^{-1}\left(\frac{-\sin 3\omega}{\cos 3\omega}\right) = -\tan^{-1}\tan(3\omega) = -3\omega$$

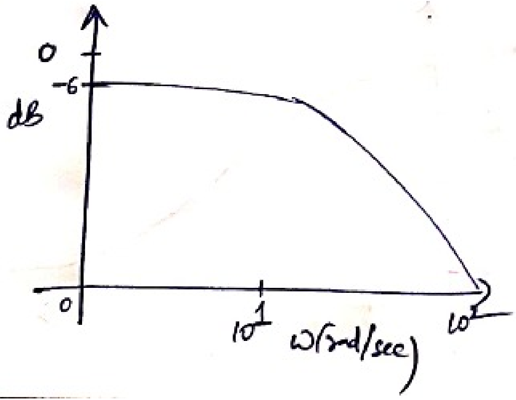


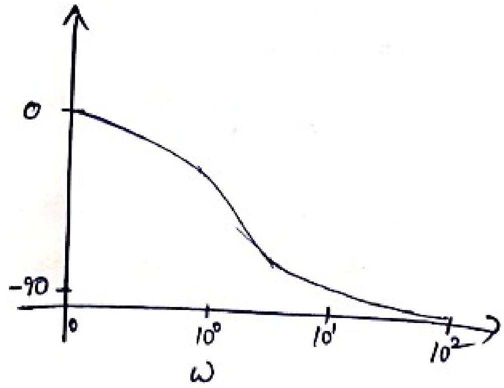


iv) $G_3 = \frac{1}{s+2}$

$$G_3(j\omega) = \frac{1}{j\omega+2} = \frac{2-j\omega}{4+\omega^2}$$

$$|G_3(j\omega)| = \frac{1}{\sqrt{\omega^2+4}}, \quad dB = -10\log(\omega^2+4), \quad \phi = \tan^{-1}\left(\frac{-\omega}{2}\right)$$

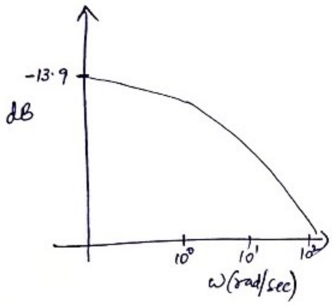


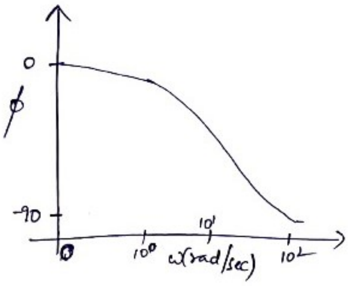


v) $G_4 = \frac{1}{s+5}$

$G_4(j\omega) = \frac{1}{j\omega+5} = \frac{5-j\omega}{25+\omega^2}$

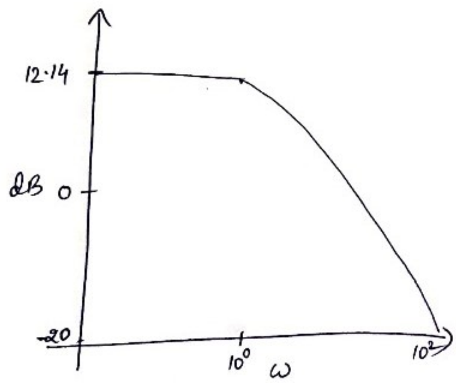
$\therefore |G_4(j\omega)| = \frac{1}{\sqrt{\omega^2+25}}$, $dB = -10\log(\omega^2+25)$, $\phi = \tan^{-1}\left(-\frac{\omega}{5}\right)$

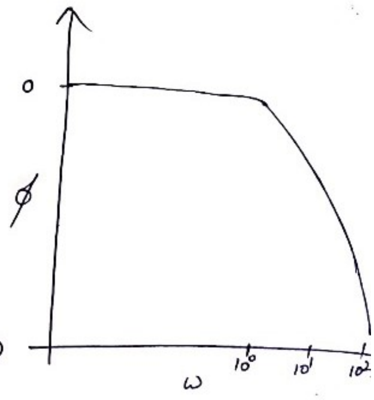




vi) Finally,

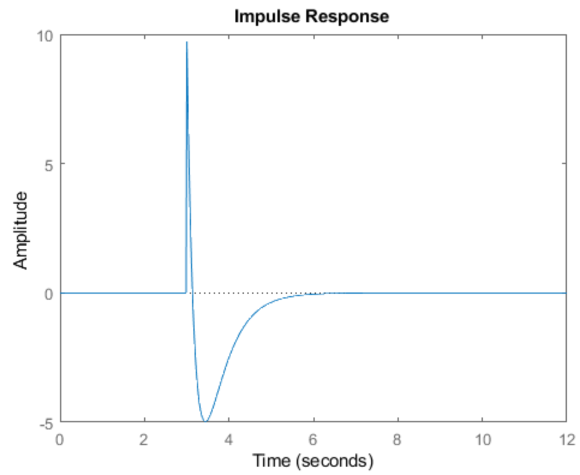
$$dB\Big|_{overall} = \sum_{i=0}^{4} dB_i$$

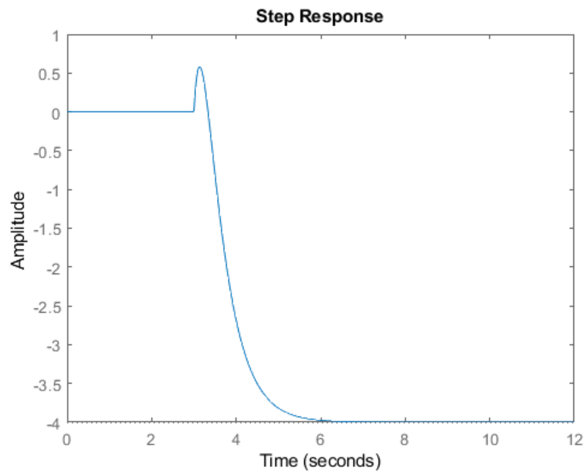


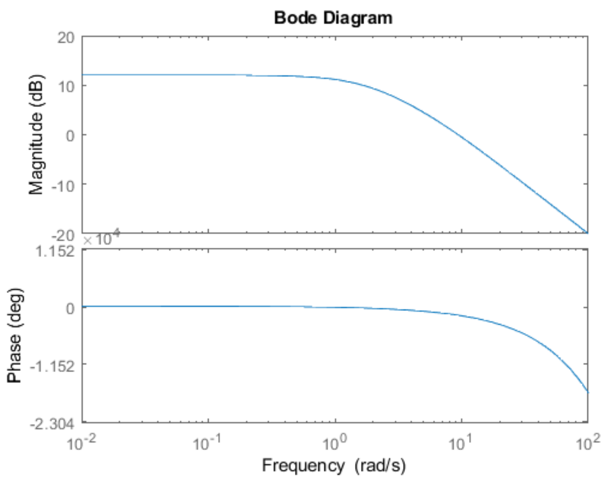


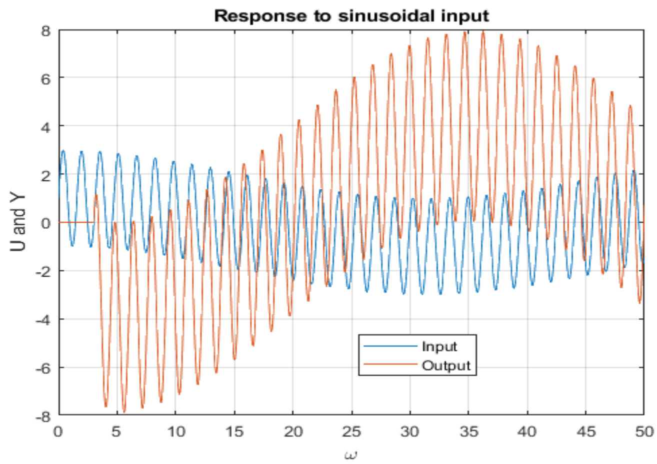
(d) LTI

The LTI stable system that has the same magnitude at all frequencies as that of given system but with the lowest phase is the same system with negative zero i.e., $G(s) = \frac{10(s+4)}{s^2+7s+10}e^{-3s}$









(e) Verification using matlab

2

Given the governing equation is

(a) Rearranging the equation

$$\frac{d^2h}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh}{dt} + \frac{3}{2}\frac{g}{L}h = \frac{3}{4\rho L}p(t)$$

Applying Laplace transform

$$s^2H(s) + \frac{6\mu}{R^2\rho}sH(s) + \frac{3}{2}\frac{g}{L}H(s) = \frac{3}{4\rho L}P(s)$$

$$G(s) = \frac{H(s)}{P(s)} = \frac{\frac{3}{4\rho L}}{s^2 + \frac{6\mu}{R^2\rho}s + \frac{3}{2}\frac{g}{L}}$$

$$\textit{Multiplying numerator and denominator with } \frac{2L}{3g}$$

$$G(s) = \frac{\frac{1}{2\rho g}}{\frac{2L}{3g}s^2 + \frac{4\mu L}{R^2\rho g}s + 1}$$

Comparing above expression with standard second order transfer function

$$G(s) = \frac{K_p}{\tau_n^2 s^2 + 2\tau_n\zeta s + 1}$$

$$K_p = \frac{1}{2\rho g}, \tau_n = \sqrt{\frac{2L}{3g}}, \zeta = \frac{\mu}{\rho R^2}\sqrt{\frac{6L}{g}}$$

(b) For oscillation,

System must be underdamped one
$0 < \zeta < 1 \Longrightarrow 0 < \frac{\mu}{\rho R^2}\sqrt{\frac{6L}{g}} < 1$

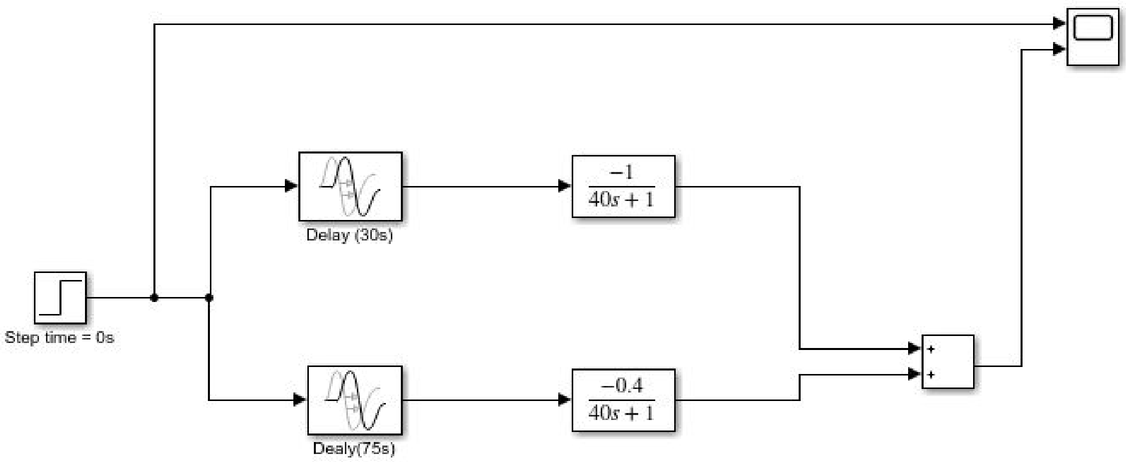
(C) Analysis,

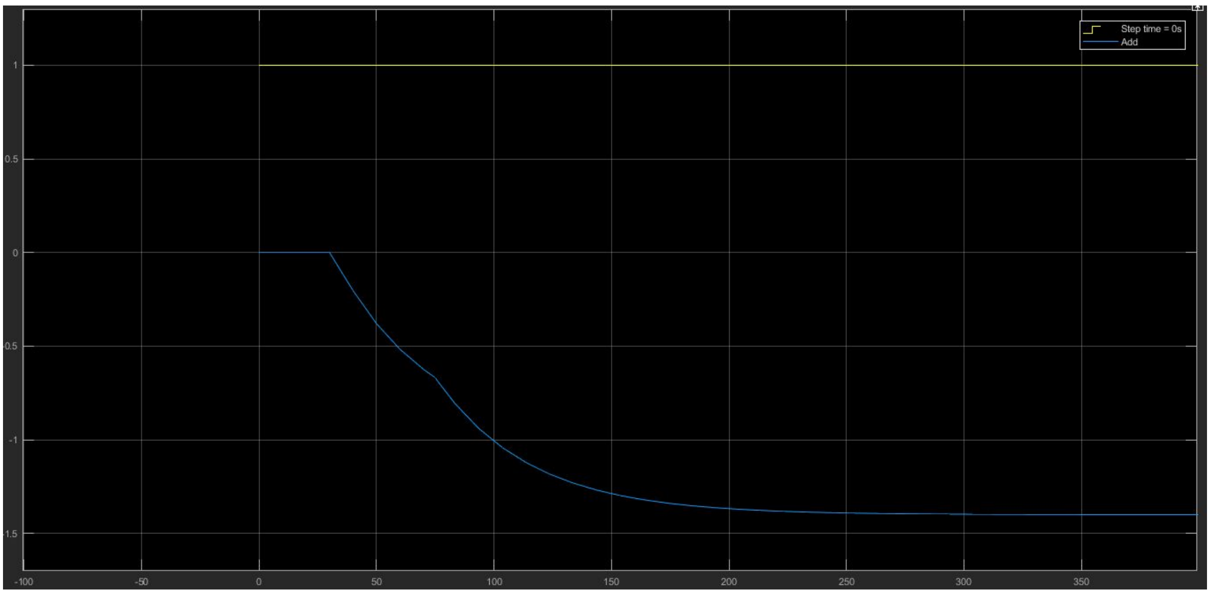
ζ gives the influence on oscillatory system that has the effect of reducing the oscillations ζ close to 1 implies system dampens the oscillations and ζ close to 0 implies high oscillations. Thus increase in manometer Leg(L) or viscosity (μ) restricts the oscillations and decrease in same results in more oscillatory response

3

Given

$$G_p(s) = \frac{Ke^{-\theta_1 s}(1+\alpha e^{-\theta_2 s})}{\tau s+1}$$
$$G_p(s) = \frac{Ke^{-\theta_1 s}}{\tau s+1} + \frac{\alpha K e^{-(\theta_1+\theta_2)s}}{\tau s+1}$$
$$G_p(s) = G_1(s) + G_2(s)$$





The system is excited with a unit step at t=0sec shown as yellow line. The corresponding response for the above system is shown in blue color. We observe that there is a discontinuity in slope at two points. One is at t = 30sec and other is at 75 sec
1) Transfer functions of parallel filters sum together 2) Therefore current system can be viewed as first order system with two different delays, One with 30sec and other at 30+45=75sec whereas usual first order system with a delay will have only one discontinuity

INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering

CH3050 Process Dynamics and Control
Assignment 4 Solutions

Marks distribution

	Question 1	Question 2
(a)	15	10
(b)	10	15
(c)	10	10
(d)	5	7
(e)	-	8
(f)	-	5
(g)	-	5

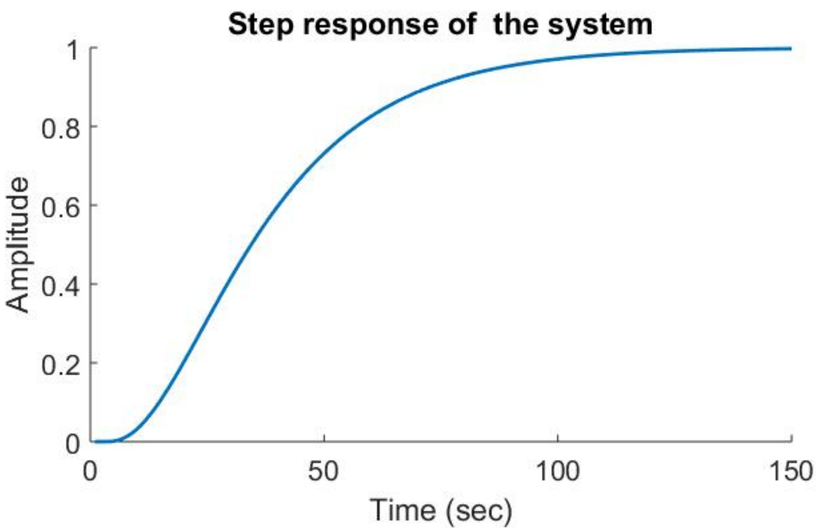
1

(a)

Given transfer function is

$$G(s) = \frac{(2s+1)e^{-2s}}{(20s+1)(10s+1)(5s+1)(s+1)}$$

Step response of the system is



t_1 is the value at 35.3% of the steady state magnitude of step response.
t_2 is the value at 85.3% of the steady state magnitude of step response.

$$t_1 = 23.95$$
$$t_2 = 58.02$$

By using Krishnaswamy and Sundaresans method (of two points) fit a FOPTD model, therefore need to find new delay and time constant.

$$\tau = 0.67(t_2 - t_1) = 22.83$$
$$D = 1.3t_1 - 0.29t_2 = 14.30$$

Estimated gain value $K = 0.9971$.
Therefore,

$$G_{FOPTD} = \frac{0.9971e^{-14.3s}}{22.83s + 1}$$

MATLAB codes are given below for fitting an approximate FOPTD and SOPTD model by using Skogestad's method.

```
Gs = tf([2 1],[1000 1350 385 36 1],'InputDelay',2); % Given transfer function
[y t] = step(Gs); % Step resonse of the Gs
Kp = y(end);
[M2, t1prime] = min(abs(y-(0.353*Kp)));
[M2, t2prime] = min(abs(y-(0.853*Kp)));
t1 = t(t1prime);
t2 = t(t2prime);
D  = 1.3*t1 - 0.29*t2;
tau = 0.67*(t2 - t1);
G_FOPTD = tf([Kp],[tau 1],'InputDelay',D); % Final FOPTD model
step(Gs,Gapx); % Comparision between true model and approximated model.
```

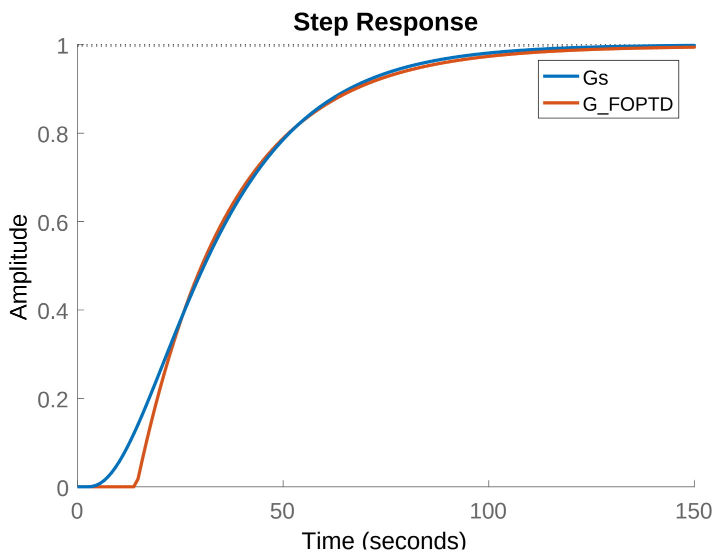


Figure 1: Comparison between true model and approximated model

(b)

Skogestads half-rule method
FOPTD approximated model structure

$$G_{FOPTD} = \frac{Ke^{-\left(\frac{\tau_2}{2}+\tau_3+\tau_4\right)s}}{(\tau_1 + \frac{\tau_2}{2})s + 1}$$

where, $\tau_1 = 20$, $\tau_2 = 10$, $\tau_3 = 5$, $\tau_4 = 1$

$$\tau = \tau_1 + \frac{\tau_2}{2} = 25$$

$$D = \frac{\tau_2}{2} + \tau_3 + \tau_4 = 11$$

$$G_{FOPTD} = \frac{e^{-11s}}{25s + 1}$$

SOPTD approximated model structure

$$G_{SOPTD} = \frac{Ke^{-(\frac{\tau_3}{2}+\tau_4)s}}{(\tau_1 s + 1)((\tau_2 + \frac{\tau_3}{2})s + 1)}$$

SOPTD approximated model is

$$G_{SOPTD} = \frac{e^{-3.5s}}{(20s + 1)(12.5s + 1)}$$

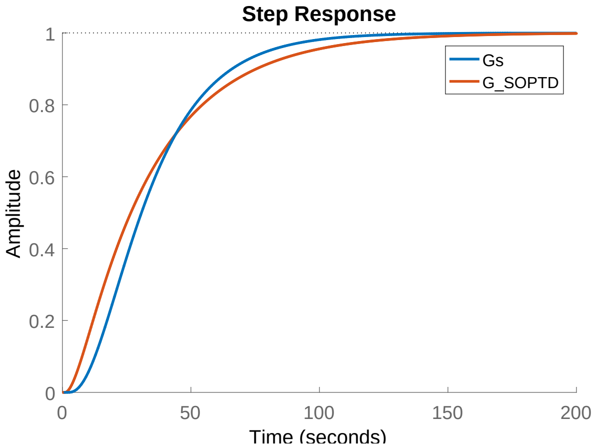
(c)

MATLAB code for estimating SOPTD model by using least square method

```
clear all;close all;clc;
Gs = tf([2 1],[1000 1350 385 36 1],'InputDelay',2);
[Gmag,Gphase,wvec] = bode(Gs,[0,0.1,100]');
% For gain, time cnstant estimation fit magnitude plot
param = lsqcurvefit(@(param,wval) modpred(param,wval),[1 1 1]',wvec,squeeze(Gmag));
Kp = param(1);
taup = param(2);
zeta = param(3);
% For delay estimatin fit phase plot
z = cos(squeeze(Gphase))
G_phase = lsqcurvefit(@(param,wval) phasepred(param,wval,Kp,taup,zeta),1,wvec,z);
D = G_phase;
s = tf('s');
G_SOPTD_ls = Kp*exp(-D*s)/(taup^2*s^2 + 2*zeta*taup*s + 1);
G_SOPTD_ls % approximated SOPTD model
```

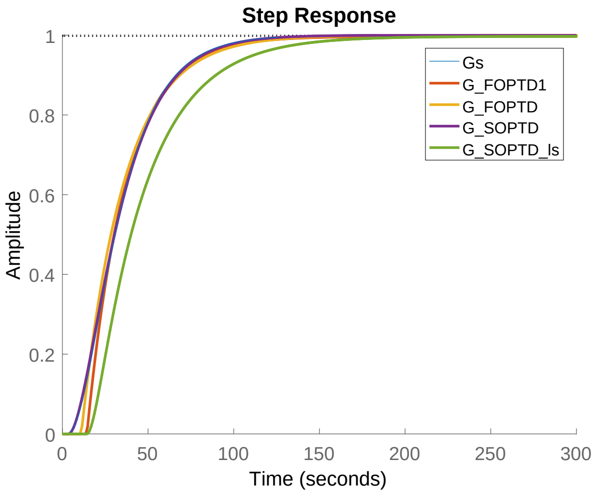
```
                                           1
G_SOPTD_ls = exp(-0.994*s) * -----------------------
                              130.8 s^2 + 34.65 s + 1
```

Step response comparison of true and approximated transfer function are as shown in figure.



(d)

Comparison between true model and different approximated model obtained in (a)-(c) has been plotted in below figure.



Observation table for approximated FOPTD and SOPTD:

Method	K	D	τ
K-S	0.9971	14.30	22.83
Skogestad's	1	11	25

Method	K	D	τ	ζ
Skogestad's	1	3.5	15.81	1.028
LSQ	1	0.9943	11.43	1.52

2

(a)

The SIMULINK block diagram of the process with delay and ZOH is shown below.

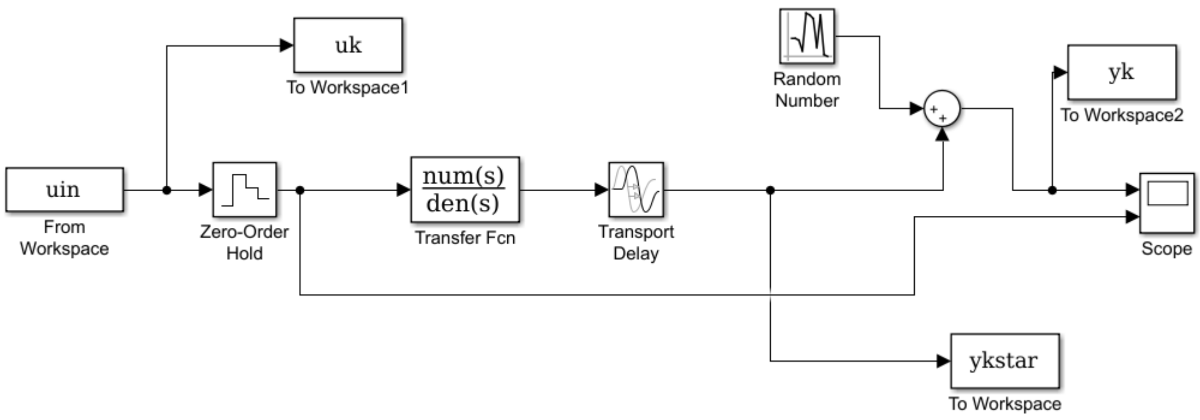


Figure 2: Simulink Block with measurement noise

(b)

MATLAB code for input design and measurement data.

```
% Input design
B_max = 1/5;
Ts= 0.2;
usig = idinput(2555,'prbs',[0 B_max],[-1 1]);
uin = [(0:1:length(usig)-1)'*Ts (usig)];

% Input-output data profile
data=iddata(yk,uk,1);
plot(data)

% Partitioning data into train and test data
data_train=data(1:1300);
data_test=data(1300:end);

% Removing the mean
[ztrain,Tr]=detrend(data_train,0);
ztest=detrend(data_test,Tr);
```

Snapshot of input-output data is shown below. From it is observed that no visible trends and non-stationarities in the data.

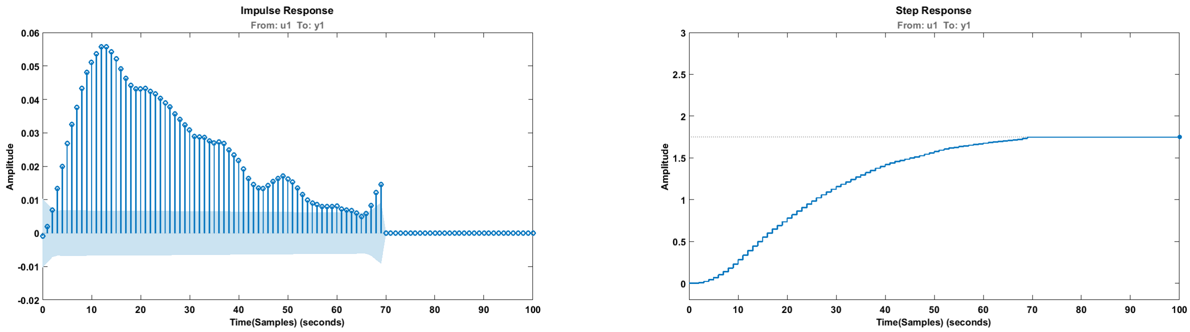


Figure 4: Impulse and step response plots.

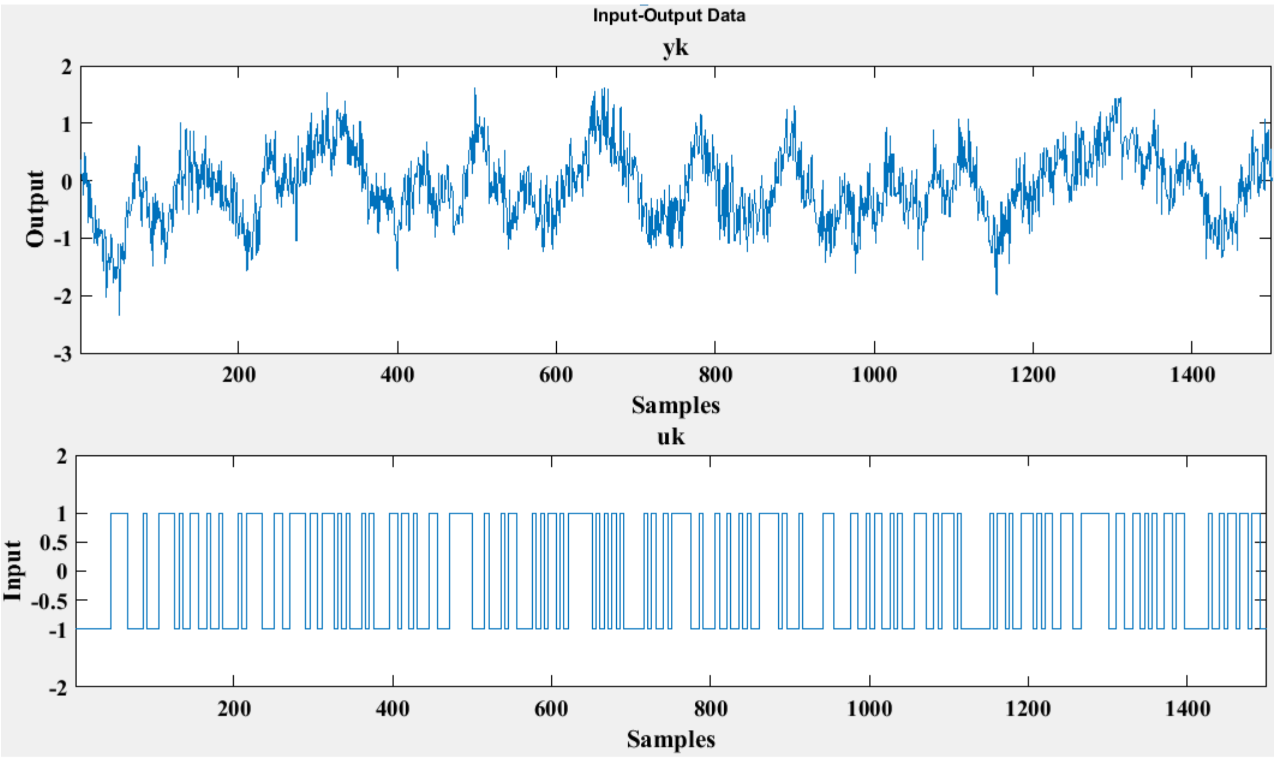


Figure 3: Snapshot of input-output profile

(c & d)

MATLAB code for estimation of delay and step response.

```
% Non-parametric(impulse ansd step responses)model estimation
 impulse_est= impulseest(ztrain);
 figure
 impulse(impulse_est,'sd',3);
 figure
 step(impulse_est)
```

The estimated impulse ands step responses for the given system is shown in respective figures.
From the estimated non-parametric analysis, we can observe that:

- System is stable with input-output delay of 3 samples. Hence n=3.

- When the damping is large the frictional force is so great that the system cant oscillate and the process is sluggish. Hence it is an overdamped system.

- The steady state gain of the system is 1.8 units

With the initial idea obtained from the non-parametric analysis, we use the following conditions to identify which model best fits the given data. The general conditions are:

- The inputs should uncorrelated with the residuals.

- The residuals should be uncorrelated amongst themselves.

- Error in the estimates should be low, which implies our estimated model is parsimonious.

(e , f & g)

MATLAB Code for parametric estimation.

```
%OE model
model_oe=oe(ztrain,[1,2,3]);
figure
resid(model_oe,ztrain);
figure
compare(model_oe,ztest);
present(model_oe);
```

The output error model, assumes that the white noise directly affects the output.The model is denoted as OE(m,n) with a delay of d samples.
From the analysis of impulse and step responses, m=1,n=2 and d=3. A total of (m+n) parameters have to be estimated.
Goodness of the model
A good model should not leave behind residuals from training. The given figure below displays the correlation between any two samples separated in time by lag l. This is auto-correlation function which explains the random effects in stochastic part of model. ACF is unity at lag zero.
The figure also displays the correlation between the residuals and lagged inputs.It is clear from the figure that there exists no significant correlation between residuals and inputs.This is known as cross-correlation.

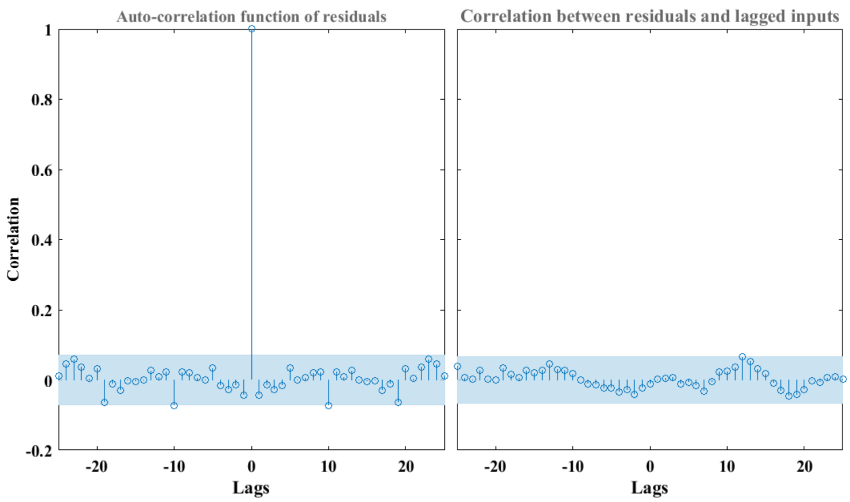


Figure 5: Correlation analysis

Thus the estimated OE model has satisfactorily captured the dynamics of the deterministic process.

Final model

```
%Results
model_oe =
Discrete-time OE model:  y(t) = [B(z)/F(z)]u(t) + e(t)
B(z) = 0.01262 (+/- 0.0007002) z^-3

F(z) = 1 - 1.802 (+/- 0.01158) z^-1 + 0.8085 (+/- 0.01133) z^-2

Sample time: 1 seconds

Parameterization:
Polynomial orders:   nb=1  (m),nf=2 (n),k=3 (d)
Number of free coefficients: 3
```

A good model yields good predictions on fresh data set. Snapshot of predictions of OE model on the fresh data is shown below:

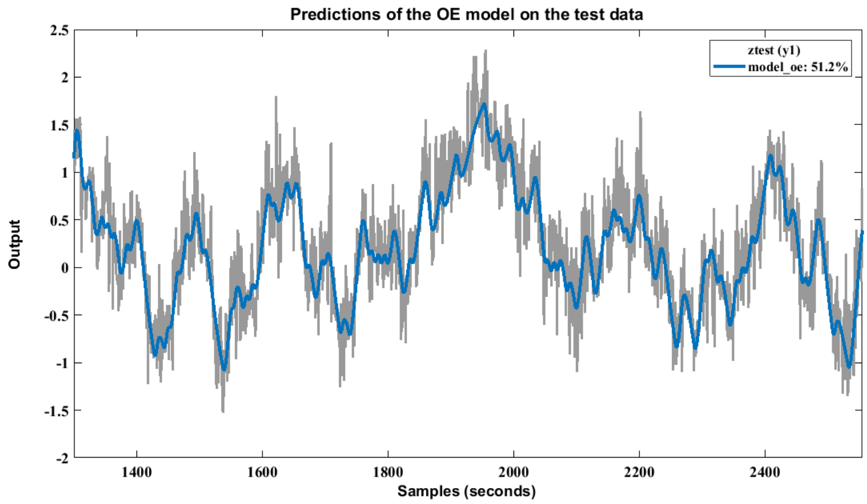


Figure 6: Cross-validation

Based on the results of the model assessment tests, the OE(1,2,3) in the transfer function form is written as:

$$y[k] = \frac{0.01262(+/-0.0007002)q^{-3}}{1-1.802(+/-0.01158)q^{-1}+0.8085(+/-0.01133)q^{-2}}u[k]+e[k]$$

INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering

CH3050 Process Dynamics and Control
Jan-May 2019 Assignment 5 Solutions

Marks distribution

	Question 1	Question 2	Question 3
(a)	10	5	10
(b)	10	10	10
(c)	10	12	10
(d)	5	4	–
(e)	–	4	–

1

(a & b)

Given the tranfer function of the system

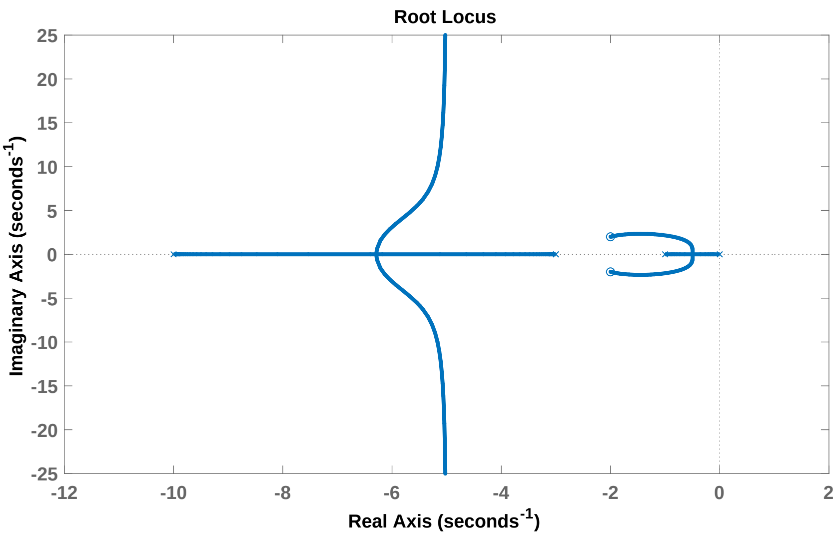


Figure 1: Root locus plot of the system

$$G(s) = \frac{s^2 + 4s + 8}{s(s+1)(s+3)}$$

with the controller with a transfer function of $H(s) = \dfrac{K_c}{s+10}$ the open loop transfer function can be written as

$$G(s)H(s) = \frac{K_c(s^2 + 4s + 8)}{s(s+1)(s+3)(s+10)}$$

Assymptotic angles

$$\begin{aligned}\phi_k &= \frac{(2k+1)\pi}{\#poles - \#zeros}, \forall k = 0(i)\#poles - \#zeros - 1\\ &\implies \frac{\pi}{2}, \frac{3\pi}{2}\end{aligned}$$

Centroid:

$$\begin{aligned}\text{centroid} &= \frac{\sum poles - \sum zeros}{\#poles - \#zeros}\\ \implies \text{centroid} &= -5\end{aligned}$$

Breakin points:
At break in points the gain attain it's local maximum as the system changes from being overdanped to critically damped.

$$\begin{aligned}|K_c| &= \frac{1}{G(s)}\\ |K_c| &= \frac{s(s+1)(s+3)(s+10)}{s^2+4s+8}\\ \left.\frac{dK_c}{ds}\right|_{s=s*} &= 0\\ s* = -0.479, -6.3\end{aligned}$$

Angle of arrival:
The contribution by the pole at origin is= $\tan^{-1}(-1) = 135$
The contribution by the pole at $s = -1$ is= $\tan^{-1}(\frac{2}{-1}) = 116.56$
The contribution by the pole at $s = -3$ is= $\tan^{1}(\frac{2}{1}) = 63.44$
The contribution by the pole at $s = -10$ is= $\tan^{1}(\frac{2}{8}) = 14.036$
The contribution by the zero at $s = -2 - 2j$ is= $\tan^{1}(\frac{2}{8}) = 90$
So, the angle of arrival at the complex zero is $180 + 90 - (135 + 116.56 + 63.44 + 14.036) = 59.036$

(c)

Form the root locus editor in MATLAB the associated value of gain with $\zeta = 0.4$ is found to be 27 dB. The corresponding complex poles are situated at $s = -0.896 \pm 2.05j$. The corresponding relative stability can be obtained as -0.896.

(d)

For a PI controller the canonical form can be written as $G_c(s) = K_c(1 + \tau_i/s)$.

$$\begin{aligned}1 + G_p(s)G_c(s)H(s) &= 0\\ \implies s^2(s+10)(s+3)(s+1) + K_c(s^2+4s+8)(s+\tau_i) &= 0\\ \implies 1 + \tau_i\frac{K_c(s^2+4s+8)}{s(K_c(s^2+4s+8) + s(s+1)(s+3)(s+10))} &= 0\end{aligned}$$

So the effective open loop transfer with τ_i as tuning parameter is $G'(s) = \frac{K_c(s^2+4s+8)}{s(K_c(s^2+4s+8)+s(s+1)(s+3)(s+10))}$

2

(a)

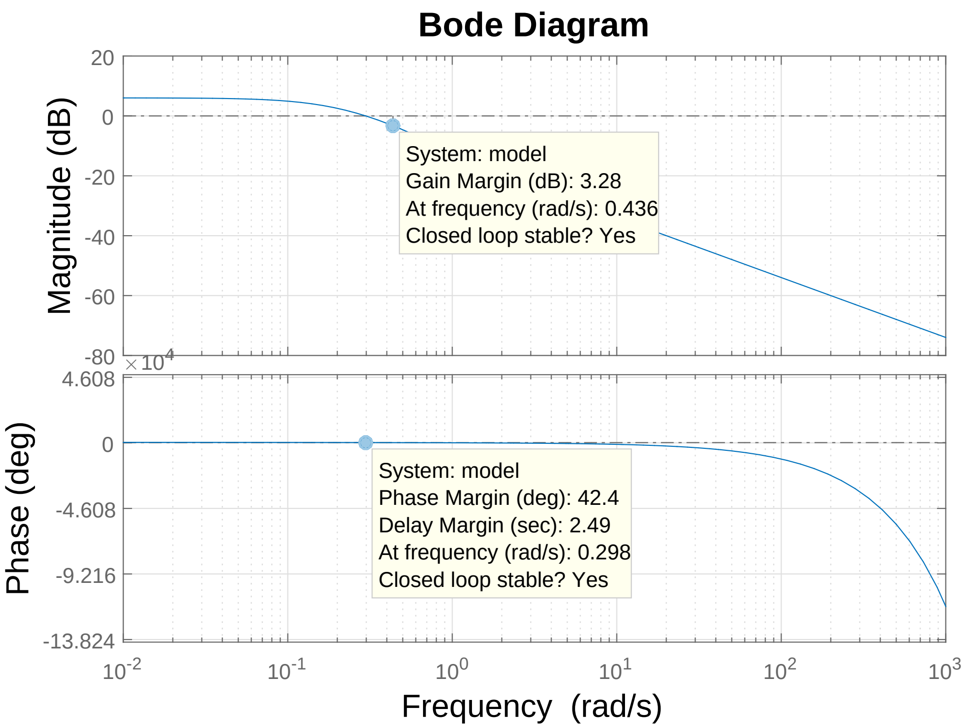
Given process transfer function is

$$G(s) = \frac{2(-s+1)e^{-2s}}{10s^2+7s+1}$$

When the controller is P type, have K_c gain, then the close loop transfer function is

$$G_{CL}(s) = 1 + K_c\frac{2(-s+1)e^{-2s}}{10s^2+7s+1}$$

Bode diagram for the open loop process is as shown in figure. The gain and phase margins are indicated in the plot.



(b)

New gain margin of the process is given as 1.7 dB $\implies 10^{\frac{1.7}{20}} = 1.216(abs)$

Controller transfer function for proportional controller in this case will be, $G_c(s) = K_c$
Open loop transfer function,

$$G_p(s) = \frac{2(-s+1)}{(2s+1)(5s+1)}e^{-2s}$$

$$G_{OL}(s) = K_c \times \frac{2(-s+1)}{(2s+1)(5s+1)}e^{-2s}$$

$$G_{OL}(j\omega) = K_c \times \frac{2(-j\omega+1)}{(2j\omega+1)(5j\omega+1)}e^{-2j\omega}$$

$$\underline{/G_{OL}(j\omega)} = \sum_{i=1}^{4} \underline{/G_i(j\omega)} = tan^{-1}(-\omega) + tan^{-1}(-2\omega) + tan^{-1}(-5\omega) - 2\omega$$

For, $\omega = \omega_c, \underline{/G_{OL}(j\omega)} = -\pi$

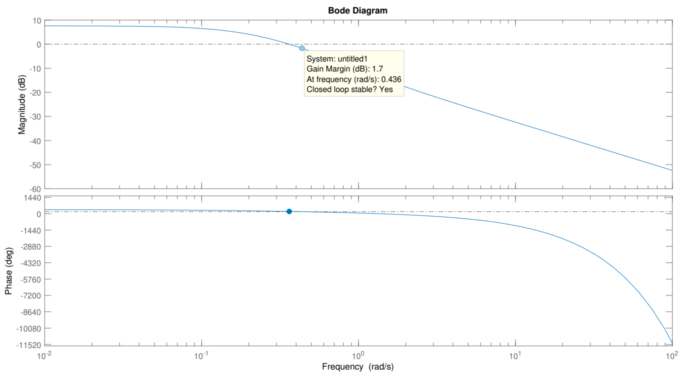
Therefore, $tan^{-1}(-\omega_c) + tan^{-1}(-2\omega_c) + tan^{-1}(-5\omega_c) - 2\omega_c = -\pi$

$$\implies \omega_c = 0.4361\frac{rad}{sec}$$

$$|G_{OL}(j\omega)| = \prod_{i=1}^{4}|G_i(j\omega)| = 2 \times K_{cu} \times \sqrt{1+\omega_c^2} \times \frac{1}{\sqrt{1+4\omega_c^2}} \times \frac{1}{\sqrt{1+25\omega_c^2}} = 1$$

$$K_{cu} = \frac{1}{0.6854} = 1.4588$$

Therefore, $K_c = \dfrac{K_{cu}}{GM} = 1.1996$



(c)

PI Controller transfer function, $G_C = (K_p + \frac{K_I}{s})$

Open loop transfer function,

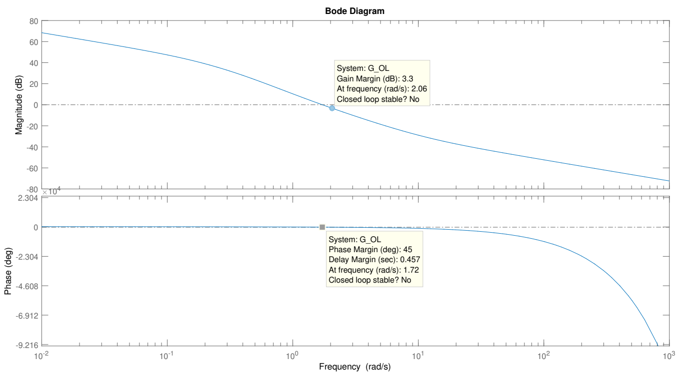
$$G_p(s) = (K_c + \frac{K_I}{s})\frac{2(-s+1)}{(2s+1)(5s+1)}e^{-2s} \tag{1}$$

Given phase margin for new system is 45 deg.

```
% MATLAB Code:
s=tf('s');
Kp = 1.1996;
Ki = 13.22;
G_OL=(Kp+(Ki/s))*((2*(-s+1)*exp(-2*s))/((5*s+1)*(2*s+1))) ;
figure
bode(G_OL)
figure
step(G_OL)



x=fsolve(@q2c,[1.5 13])


function f=q2c(x)
Kp = 1.1996;
w = x(1);
```

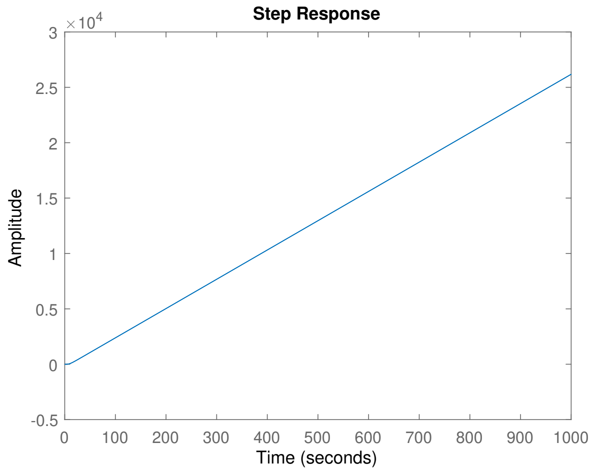
```
k = x(2);
f(1) = ((2*sqrt((w^2+1))*sqrt((Kp^2+(k/w)^2))) - (sqrt((25*w^2+1))*sqrt((4*w^2+1))));
f(2) = -atan(w)-2*w*57.29 + atan((w*Kp)/k) - atan(5*w) - atan(2*w) + 135;
end
```

Integral controller gain, $K_I = 13.22$



(d)

Step response of final system:



As it can be seen the system consists a zero on the RHP which makes it non-minimum phase which in turn implies instability for a positive phase margin as the case here.

(e)

$$S = \frac{1}{1 + G_p}$$

```
% Matlab Code
S = 1/(1 + GpGc);
w = linspace(0.001,50, 1000000);
[magnitude, phase, l] = bode(S, w);
trapz(l, log(abs(magnitude(:))))
```

Since the difference between poles and zeros is one and system is unstable, the Bode sensitivity integral does not hold.

3

Given the transfer function

$$G_p(s) = \frac{2}{s^2+3s-10}e^{-s}$$

3.1

Ignoring the delay

$$G(s) = \frac{2}{s^2+3s-10} = \frac{2}{(s+5)(s-2)}$$

The characteristic equation is,

$$1 + G_{c1}G(s) = 0 \implies 1 + G_{c1}\frac{2}{(s+5)(s-2)} = 0$$

$s = -1$ is the root of this equation

$$1 + G_{c1}\frac{2}{(-1+5)(-1-2)} = 0$$

Therefore, $G_{c1} = 6$

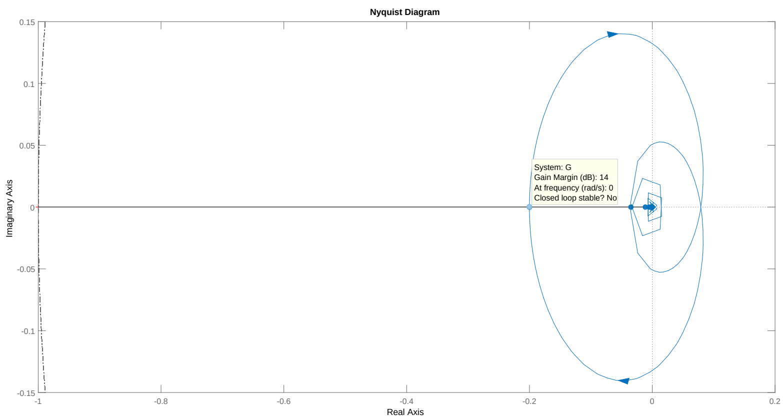
3.2

$$G_p(s) = \frac{2}{(s+5)(s-2)}e^{-s}$$

substitute, $s = j\omega$

$$G_p(j\omega) = \frac{2}{(j\omega+5)(j\omega-2)}e^{-j\omega}$$

$$|G_p(j\omega)| = \frac{2}{\sqrt{(\omega^2+25)(\omega^2+4)}}$$



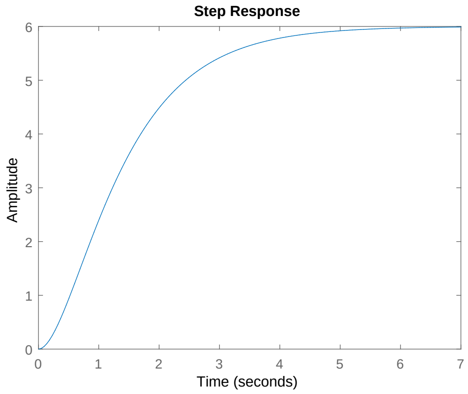
$\omega = 0$ is the cross over frequency,

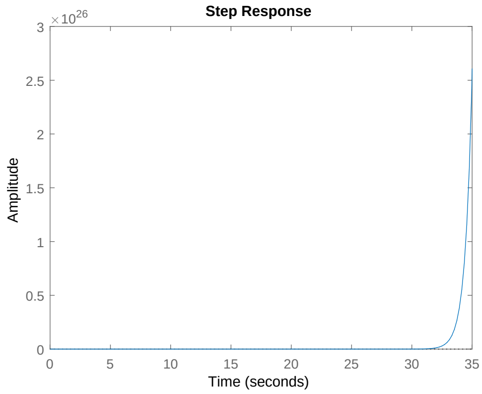
Therefore, $|G_p| = 0.2$Hence, $20\log\frac{1}{|G_p|} = 1.5$

$G_{c2} = 4.217$

3.3

$$T_1 = \frac{G_{c1}G}{1+G_{c1}G}$$
$$T_2 = \frac{G_{c2}G_p}{1+G_{c2}G_p}$$





Step response of T_1 is stable with a settling time of 7sec whereas step response of T_2 is unstable for the given gain margin
Now we take into account the delay using a Pade's second-order approximation

$$e^{-s} = \frac{1-\frac{s}{2}+\frac{s^2}{12}}{1+\frac{s}{2}+\frac{s^2}{12}}.$$

$$\text{Therefore, } G_f(s) = \frac{2}{(s+5)(s-2)}\frac{1-\frac{s}{2}+\frac{s^2}{12}}{1+\frac{s}{2}+\frac{s^2}{12}}$$

The characteristic equation is,

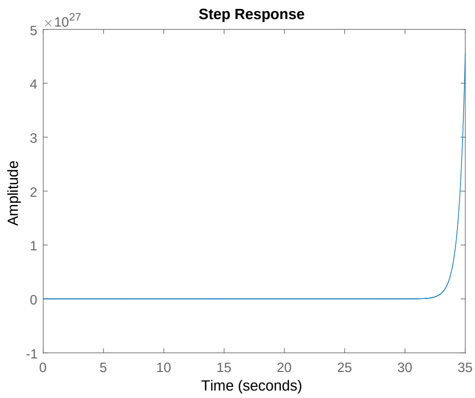
$$1+G_{c3}G_f(s)=0 \implies 1+G_{c3}\frac{2}{(s+5)(s-2)}\frac{1-\frac{s}{2}+\frac{s^2}{12}}{1+\frac{s}{2}+\frac{s^2}{12}}=0$$

$s=-1$ is the root of this equation

$$1+G_{c3}\frac{2}{(-1+5)(-1-2)}\frac{1-\frac{(-1)}{2}+\frac{(-1)^2}{12}}{1+\frac{(-1)}{2}+\frac{(-1)^2}{12}}=0$$

Therefore, $G_{c3} = 2.2$

$$T_3 = \frac{G_{c3}G_f}{1+G_{c3}G_f}$$



Step response of T_3 is becoming unstable by Pade's approximation
Matlab code for above graphs is shown below

```
close all
clc
clear

s = tf('s');
Gp = 2 * exp(-s) / (s^2 + 3*s - 10);
G = 2  / (s^2 + 3*s - 10);
Gd = (1-s/2+s^2/12)/(1+s/2+s^2/12); % pades approximation
Gf = (2*Gd )/ (s^2 + 3*s - 10);

figure(1)
nyquist(Gp);

Gc1 = 6;
T1 = (Gc1*G)/(1+Gc1*G);
figure(2)
step(T1)

Gc2 = 4.217;
T2 = (Gc2*Gp)/(1+Gc2*Gp);
figure(3)
step(T2)

Gc3 = 2.2;
T3 = (Gc3*Gf)/(1+Gc3*Gf);
figure(4)
step(T3)
```