ASSIGNMENT - 4

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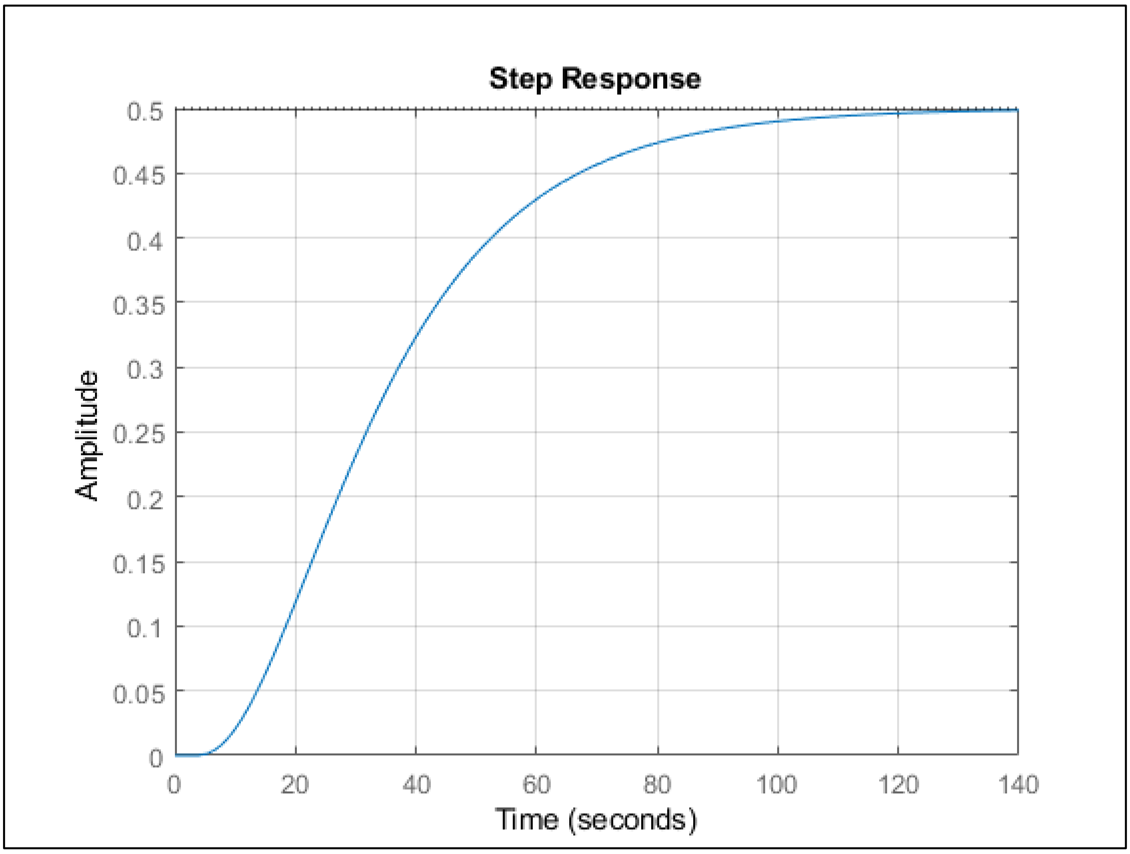
QUESTION 1

- Given,

$$G(s) = \frac{(s + 0.5)e^{-3s}}{(20s + 1)(10s + 1)(5s + 1)(s + 1)}$$



Gain = 0.5; obtained from the step response.

PART (A)

- Gain (K) = 0.5
- Krishnaswamy & Sundaresan model:

$$G_{KS} = \frac{\mathrm{K}e^{-Ds}}{\tau s + 1}$$
$$D = 1.3t_1 - 0.29t_2$$
$$\tau = 0.67(t_2 - t_1)$$

- Here,

$D: delay\ of\ FOPTD\ approximation\ model$
$\tau: Time\ constant\ of\ FOPTD\ approximation\ model$
$t_1: time\ at\ which\ step\ response\ is\ at\ 35.3\%\ of\ steady\ state\ value$
$t_2: time\ at\ which\ step\ response\ is\ at\ 85.3\%\ of\ steady\ state\ value$

- From the step response curve, we get,

$$t_1 = 25s; t_2 = 59s$$

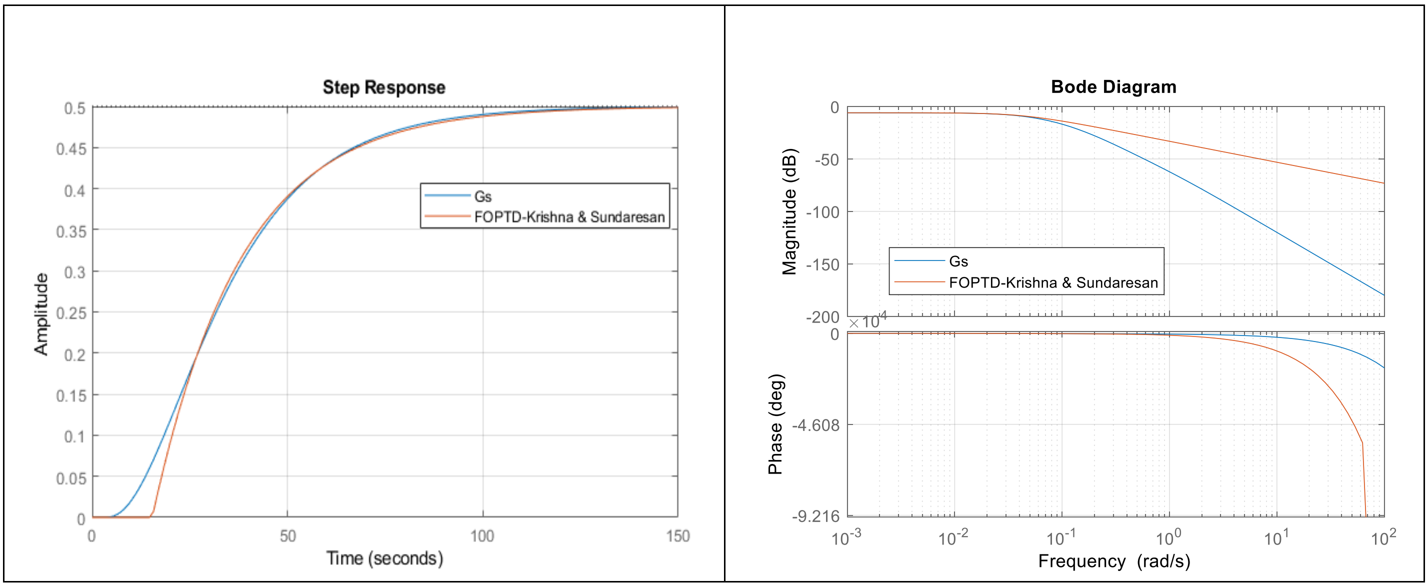
- Using these values, we get,

$$D = 1.3t_1 - 0.29t_2 = 1.3 \times 25 - 0.29 \times 59 = 15.39s$$
$$\tau = 0.67(t_2 - t_1) = 0.67(59 - 25) = 22.78\ s$$

- Hence, the FOPTD model is,

$$\boxed{G_{KS} = \frac{0.5e^{-15.39s}}{22.78s + 1}}$$

- Comparing step response & bode plots of the system and the FOPTD model,



PART (B)

- FOPTD model approximation using Skogestad's Half Rule:

$$\mathrm{K} = 0.5$$
$$\tau = 20 + \frac{10}{2} = 25$$
$$D = 3 + \frac{10}{2} + 5 + 1 - 2 = 12$$

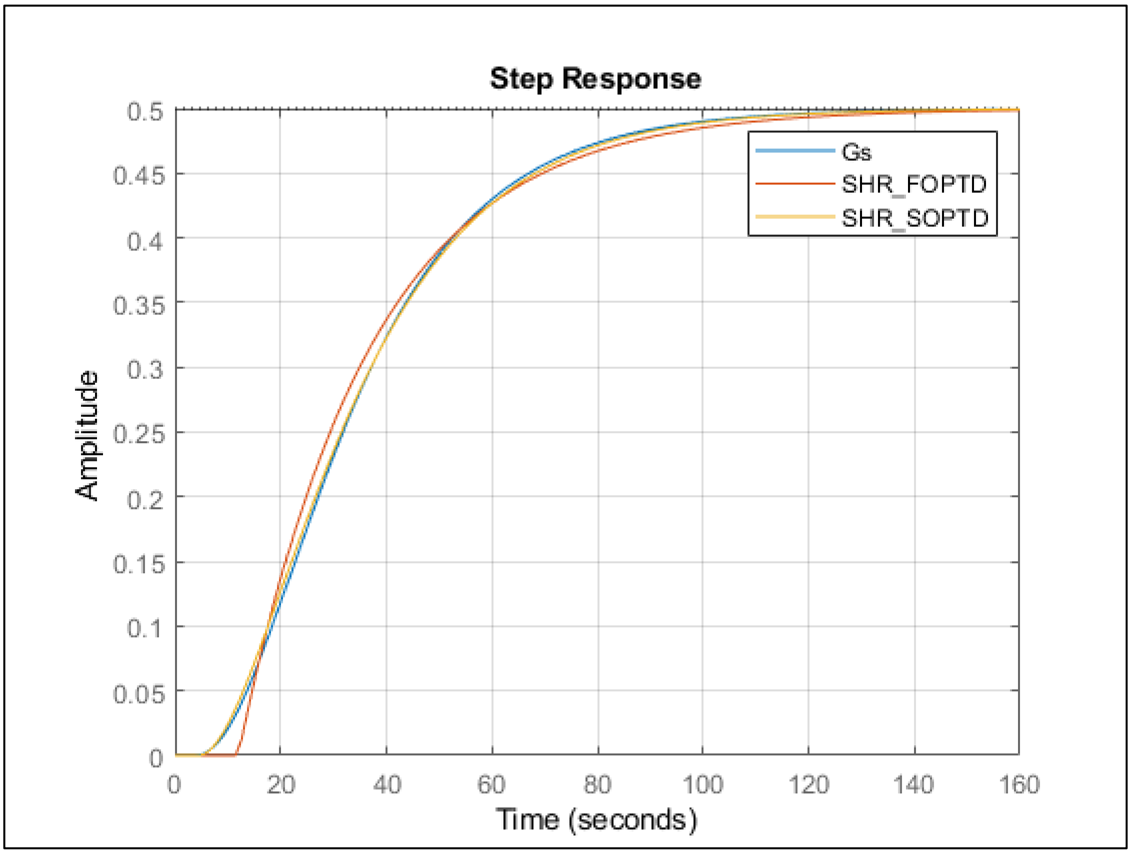
$$\boxed{G_{SHR,FO} = \frac{\mathrm{K}e^{-Ds}}{\tau s + 1} = \frac{0.5e^{-12s}}{25s + 1}}$$

- SOPTD model approximation using Skogestad's Half Rule:

$$\tau_1 = 20; \tau_2 = 10 + \frac{5}{2} = 12.5$$
$$D = \frac{5}{2} + 3 + 1 - 2 = 4.5$$

$$\boxed{G_{SHR,FO} = \frac{\mathrm{K}e^{-Ds}}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{0.5e^{-4.5s}}{(20s + 1)(12.5s + 1)}}$$

- Comparing Step Responses:



PART (C)

- SOPTD approximation using Least Squares fitting on FRF; we use the following code:

```
• %Transfer funcn
• Gs=zpk(-0.5,[-1/20,-1/10,-1/5,-1],1/(10*20*5));
• Gs.iodelay=3;
• Gain=0.5;
• %Least squares apprxn-SOPTD: Gain & Time Constant Estimation using FRF
• [AR,Phi,wout]=bode(Gs);
• options=optimoptions(@lsqcurvefit,'MaxFunctionEvaluations',500);
• mparnew=lsqcurvefit(@(mparnew,wnew)
  AmpRatio(mparnew,wnew),[1,1,1]',wout,reshape(AR,1,size(wout,1))',[],[],op
  tions);
• %Least squares apprxn-SOPTD: Delay Estimation using FRF
• Dnew=lsqcurvefit(@(Dnew,wnew)
  phase(mparnew,wnew,Dnew),[1],wout,(pi/180).*reshape(Phi,1,size(wout,1))',
  [],[],options);
```

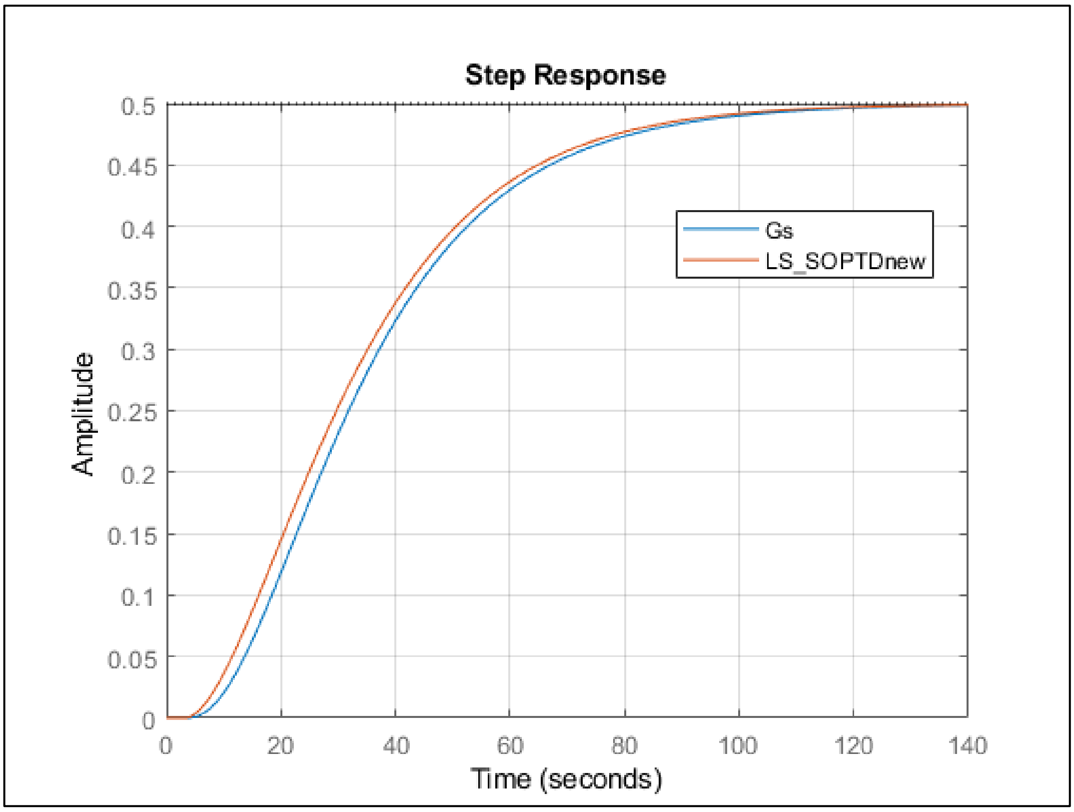
```
• %SOPTD - Least Squares
• LS_SOPTDnew=tf(mparnew(1),conv([mparnew(2),1],[mparnew(3),1]),'iodelay',D
  new);
```

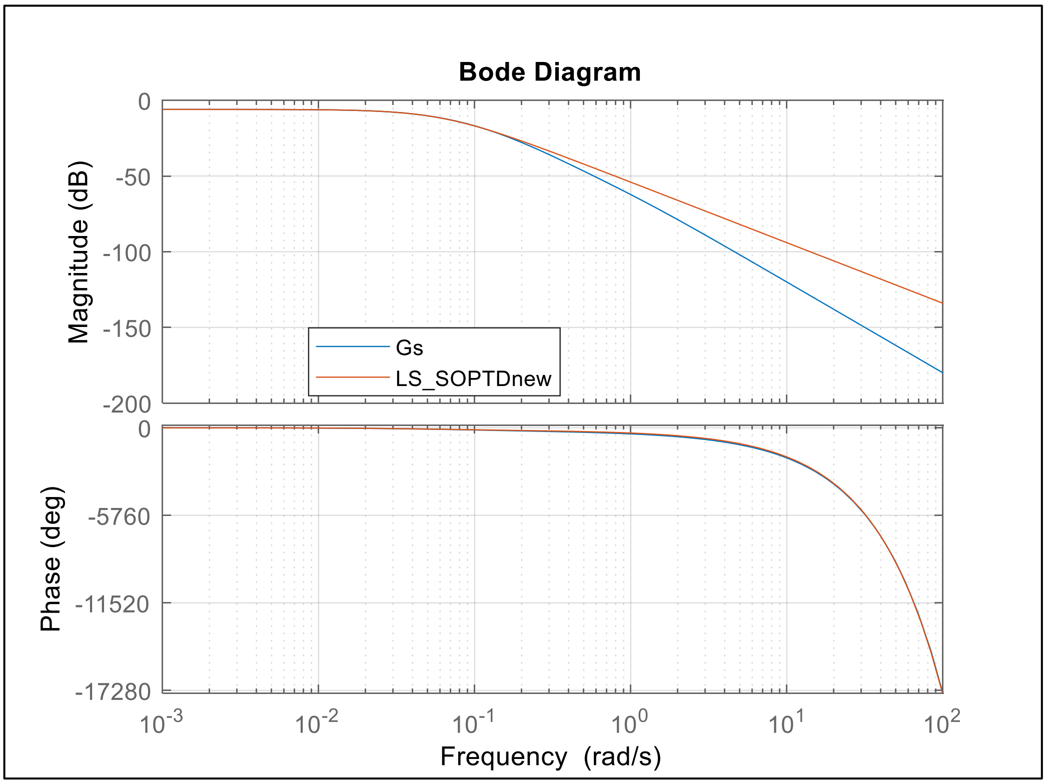
```
%Function
function AR=AmpRatio(mparnew,w)
Kpnew=mparnew(1);
Tau1new=mparnew(2);
Tau2new=mparnew(3);


AR=Kpnew./(sqrt(1+Tau1new^2*w.^2).*sqrt(1+Tau2new^2*w.^2));
end

function Ph=phase(mparnew,w,Dnew)
Kpnew=mparnew(1);
Tau1new=mparnew(2);
Tau2new=mparnew(3);

Ph=-atan(Tau1new.*w)-atan(Tau2new.*w)-Dnew.*w;

end
```

- Estimated model is given by:

$$K = 0.4998$$
$$\tau_1 = 15.8615$$
$$\tau_2 = 15.8615$$
$$D = 3.0358$$

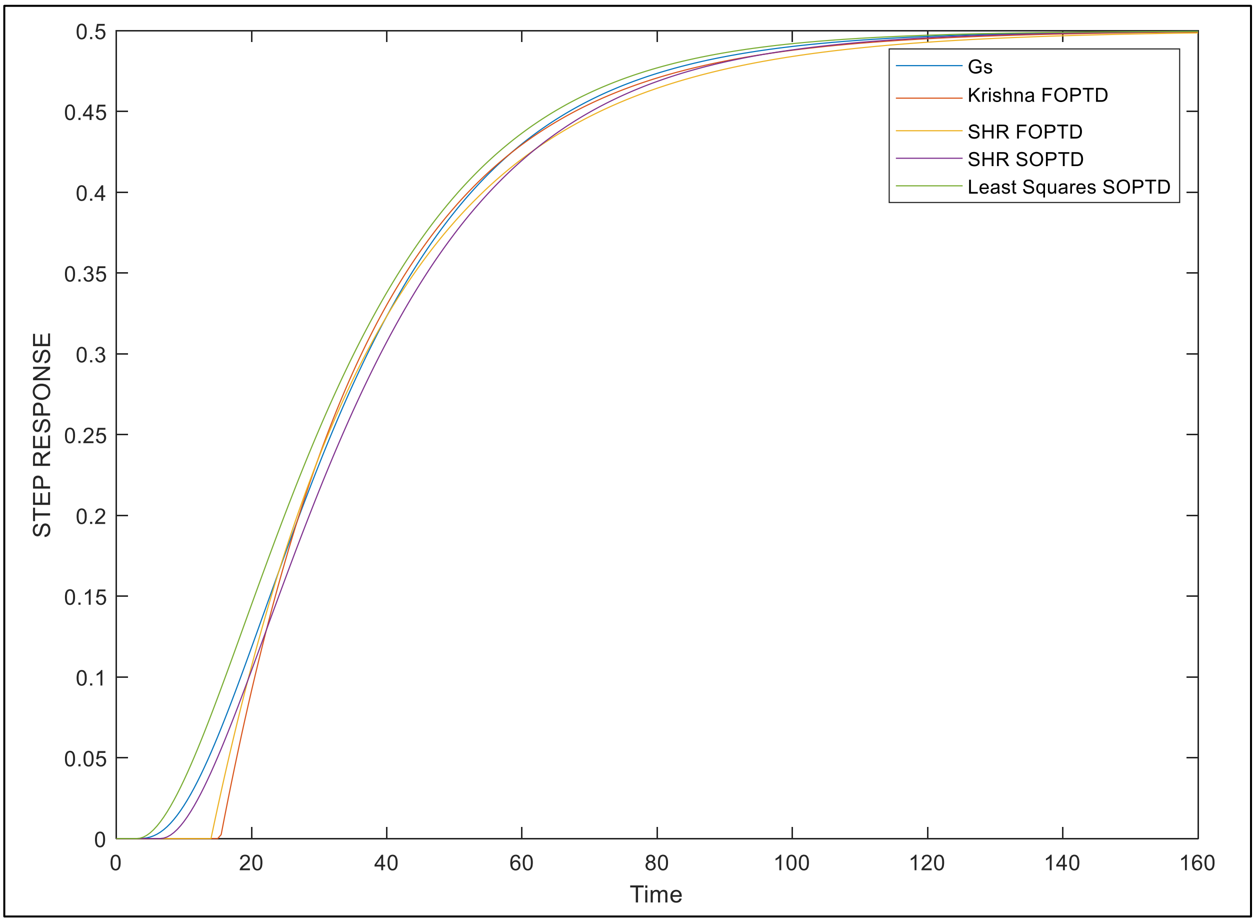
- Step response & Bode plot Comparison:





PART (D)

- Comparing all the step-responses:



- Tabulating Results:

Model	$Gain(\mathrm{K})$	τ_1	τ_2	$Delay(D)$
Krishnaswamy & Sundaresan FOPTD	0.5	22.78	-	15.39
SHR FOPTD	0.5	25	-	12
SHR SOPTD	0.5	20	12.5	4.5
Least Squares SOPTD	0.5	15.86	15.86	3.04

- Step-Responses:

Time	G(s)	Krishnaswamy & Sundaresan FOPTD	SHR FOPTD	SHR SOPTD	Least Squares SOPTD
0	0	0	0	0	0
6	0.0027	0	0	0.0021	0.0077
12	0.0353	0	0	0.041	0.0553
18	0.0954	0.0541	0.1067	0.1041	0.1217
24	0.1649	0.1574	0.1906	0.1722	0.1903
30	0.2317	0.2367	0.2566	0.2358	0.2533
36	0.29	0.2977	0.3086	0.291	0.3072
42	0.3382	0.3445	0.3494	0.337	0.3517
48	0.3767	0.3805	0.3815	0.3742	0.3872
54	0.4068	0.4082	0.4068	0.4037	0.415
60	0.4299	0.4295	0.4267	0.4267	0.4365
66	0.4475	0.4458	0.4423	0.4445	0.4529
72	0.4608	0.4583	0.4546	0.4581	0.4652
78	0.4708	0.468	0.4643	0.4685	0.4744
84	0.4783	0.4754	0.4719	0.4764	0.4812
90	0.4838	0.4811	0.4779	0.4823	0.4863
96	0.488	0.4855	0.4826	0.4868	0.49
102	0.4911	0.4888	0.4863	0.4902	0.4927
108	0.4934	0.4914	0.4893	0.4927	0.4947
114	0.4951	0.4934	0.4915	0.4945	0.4961
120	0.4964	0.4949	0.4934	0.4959	0.4971
126	0.4973	0.4961	0.4948	0.497	0.4979
132	0.498	0.497	0.4959	0.4978	0.4984
138	0.4985	0.4977	0.4968	0.4983	0.4988
144	0.4989	0.4982	0.4975	0.4988	0.4991
150	0.4992	0.4986	0.498	0.4991	0.4993
156	0.4994	0.499	0.4984	0.4993	0.4994

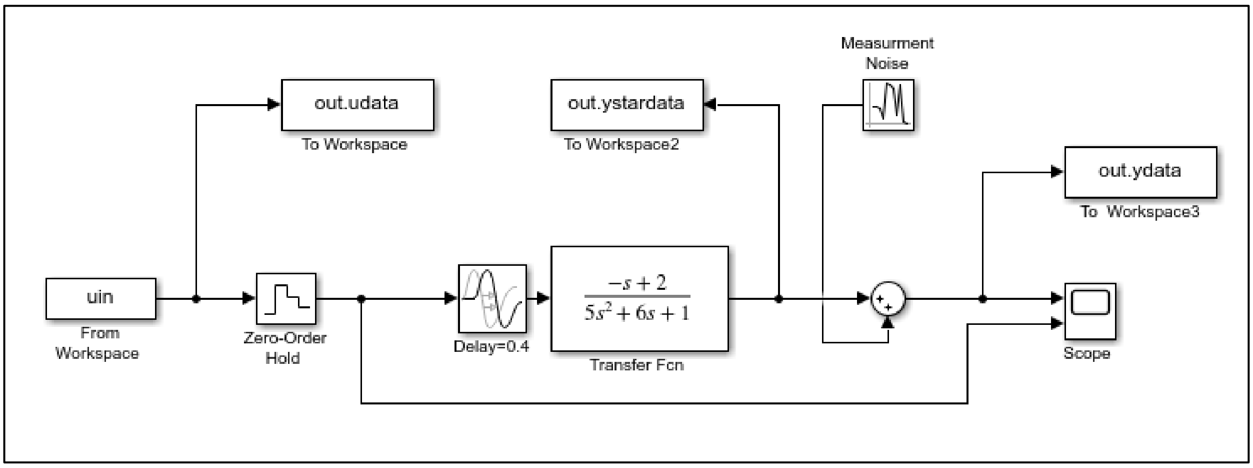
QUESTION-2

- Given,

$$G(s) = \frac{2(-0.5s+1)e^{-0.4s}}{(5s+1)(s+1)}$$

PART (A)

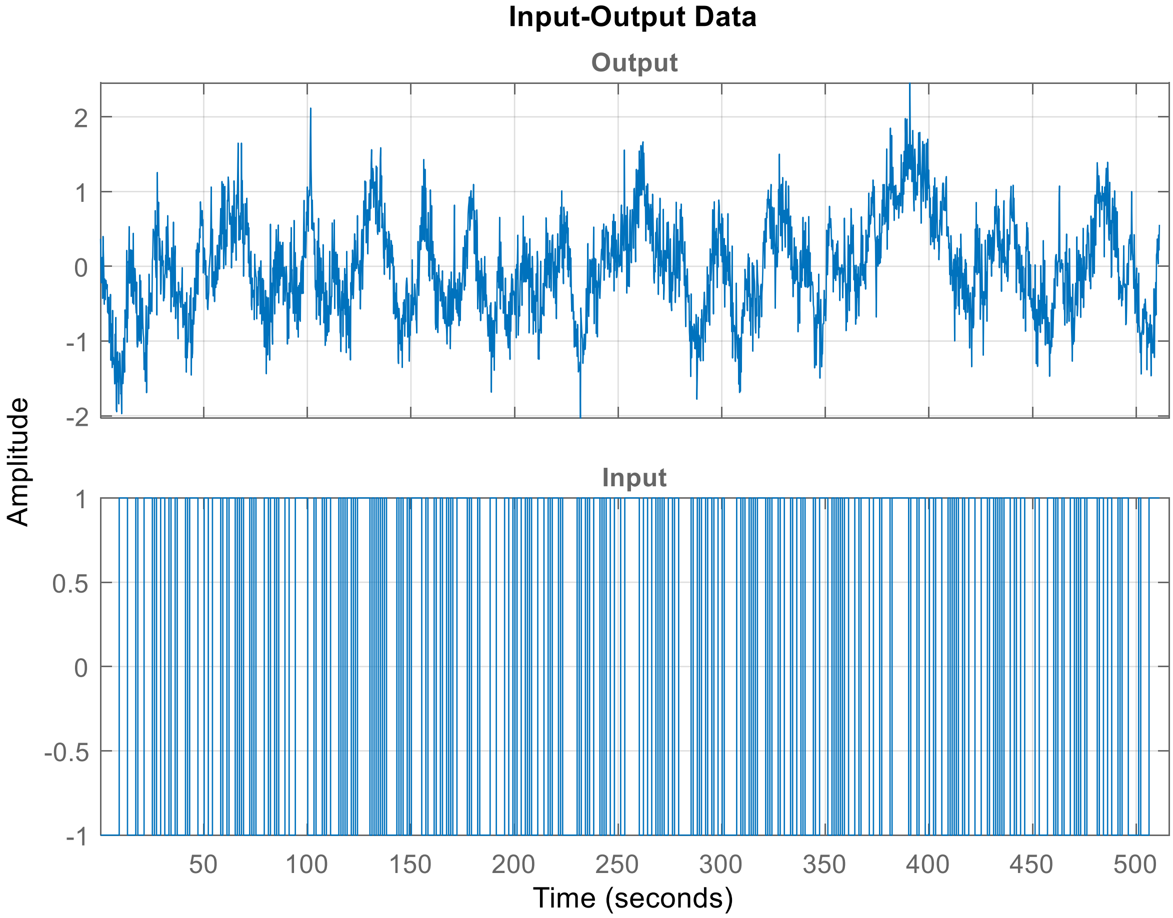
- SIMULINK Diagram of the system:



PART (B)

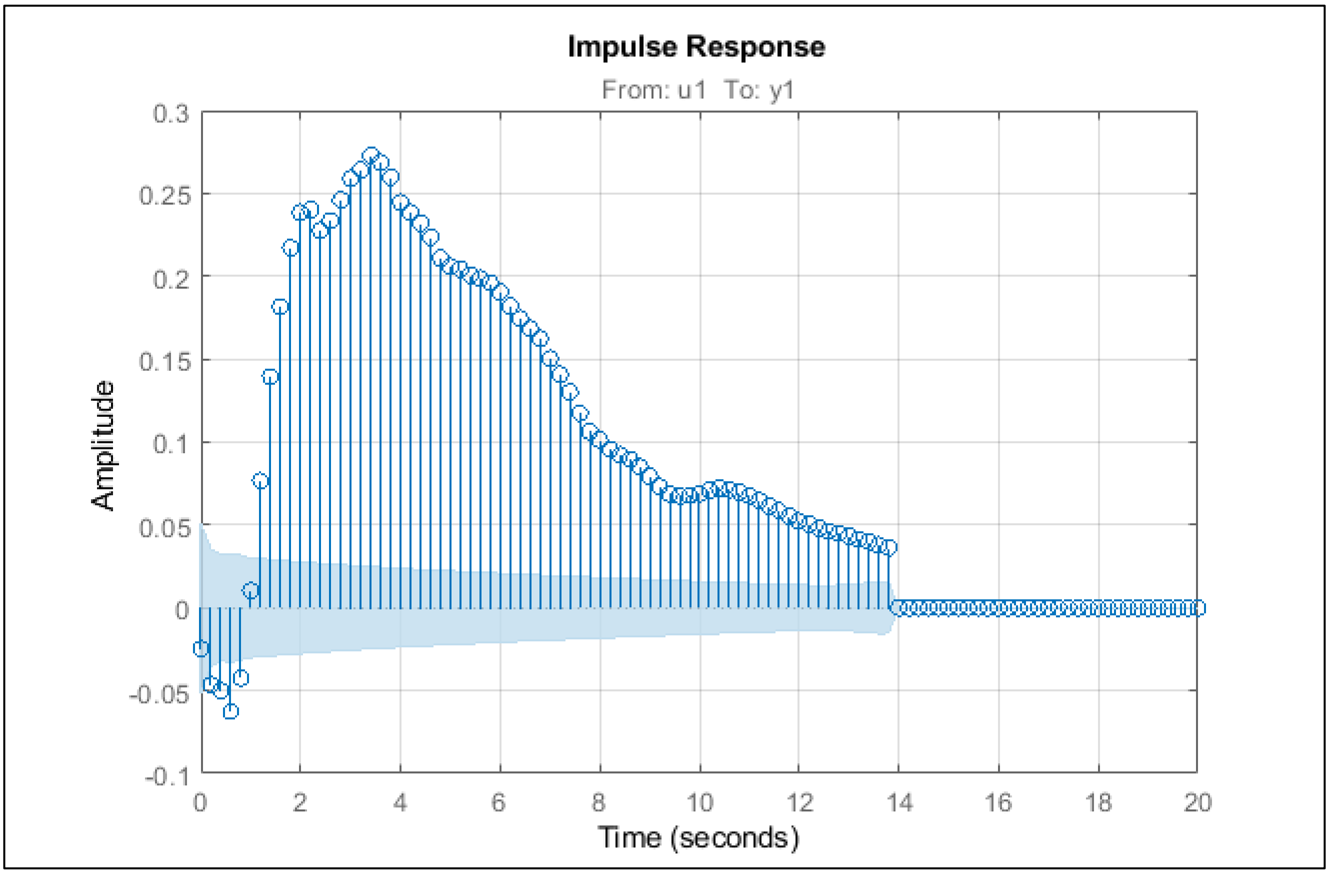
- Using the following code to design input:

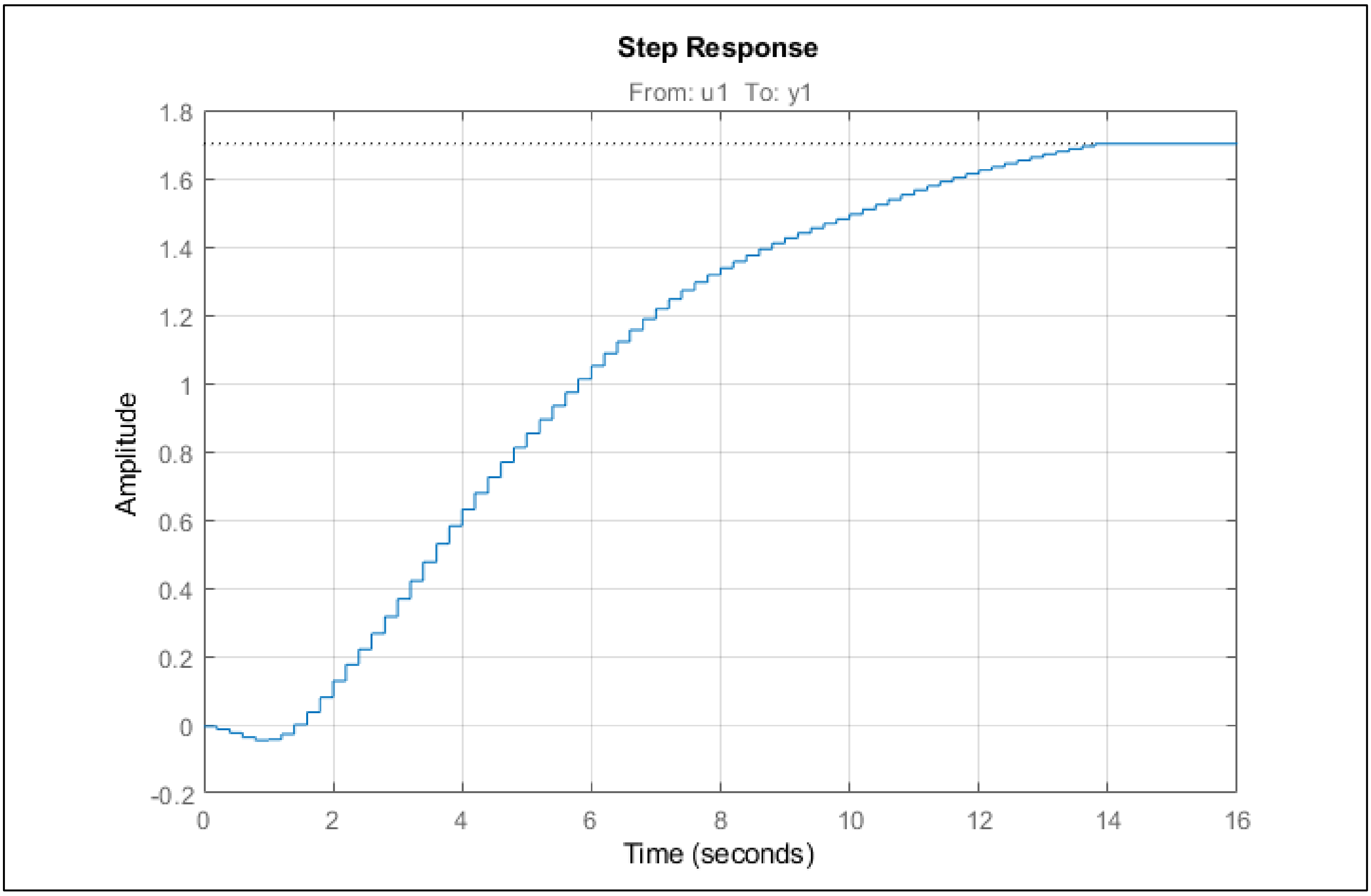
```
%%
%Generating the input signal
Bmax=0.2;
Range=[-1,1];
N=2555;
Ts=0.2;
u1=idinput(N,'prbs',[0,Bmax],Range);
uin = [(0:1:length(u1)-1)'*Ts (u1)];

%%
%I/O data generation
data=iddata(out.ydata.Data,out.udata.Data,0.2);
figure
plot(data)

%%
%Splitting into train and test data
trndata=data(1:1400);
testdata=data(1401:end);
```



PART (C) & (D)

```
%%
%Removing the mean/Mean shifting
[dtrain,Tr]=detrend(trndata,0);
dtest=detrend(testdata,Tr);

%%
figure
plot(dtrain)

%%
%Estimating the FIR
FIR=impulseest(dtrain,[]);
figure
impulse(FIR,'sd',2)
figure
step(FIR)
```





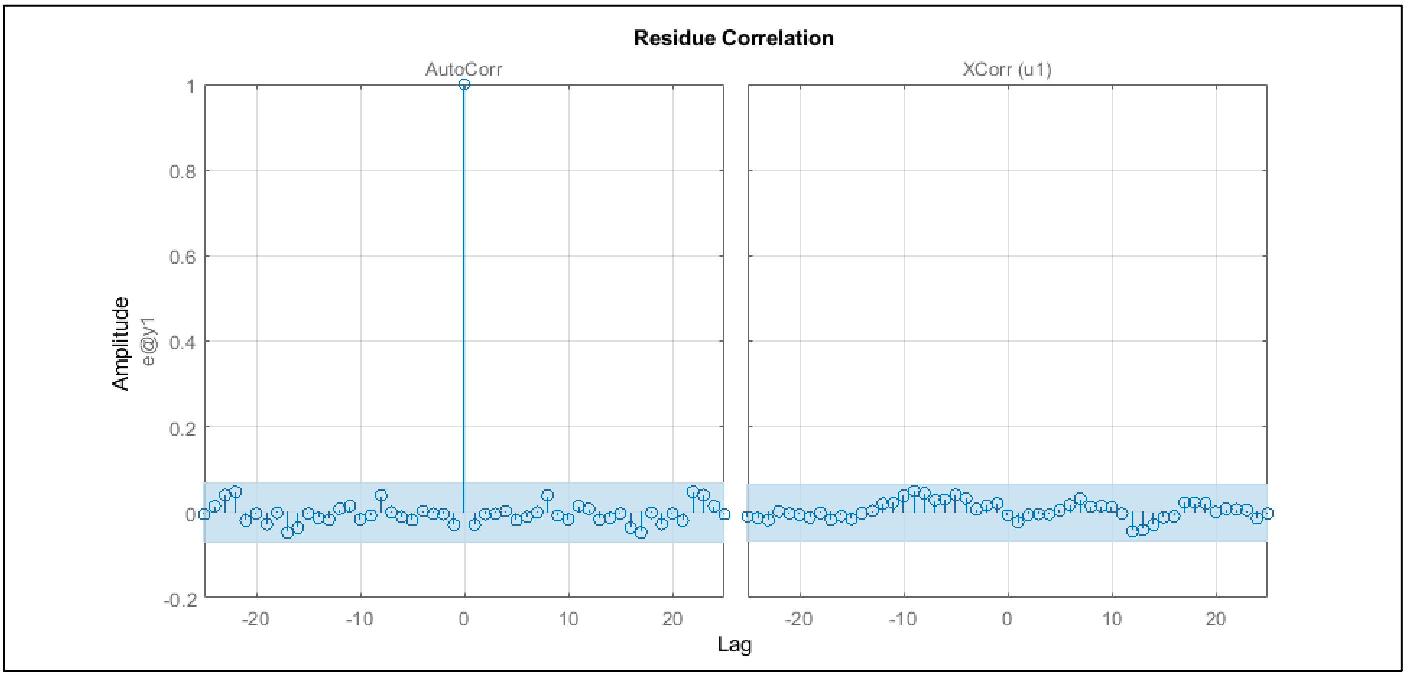
- System is stable with input-ouput delay of n=3 samples, as the first significant non-zero value occurs at the third sample.

- Gain obtained from step-response is 1.7
- As the system has a pole in RHP, the system first responds in the opposite direction of steady state value.

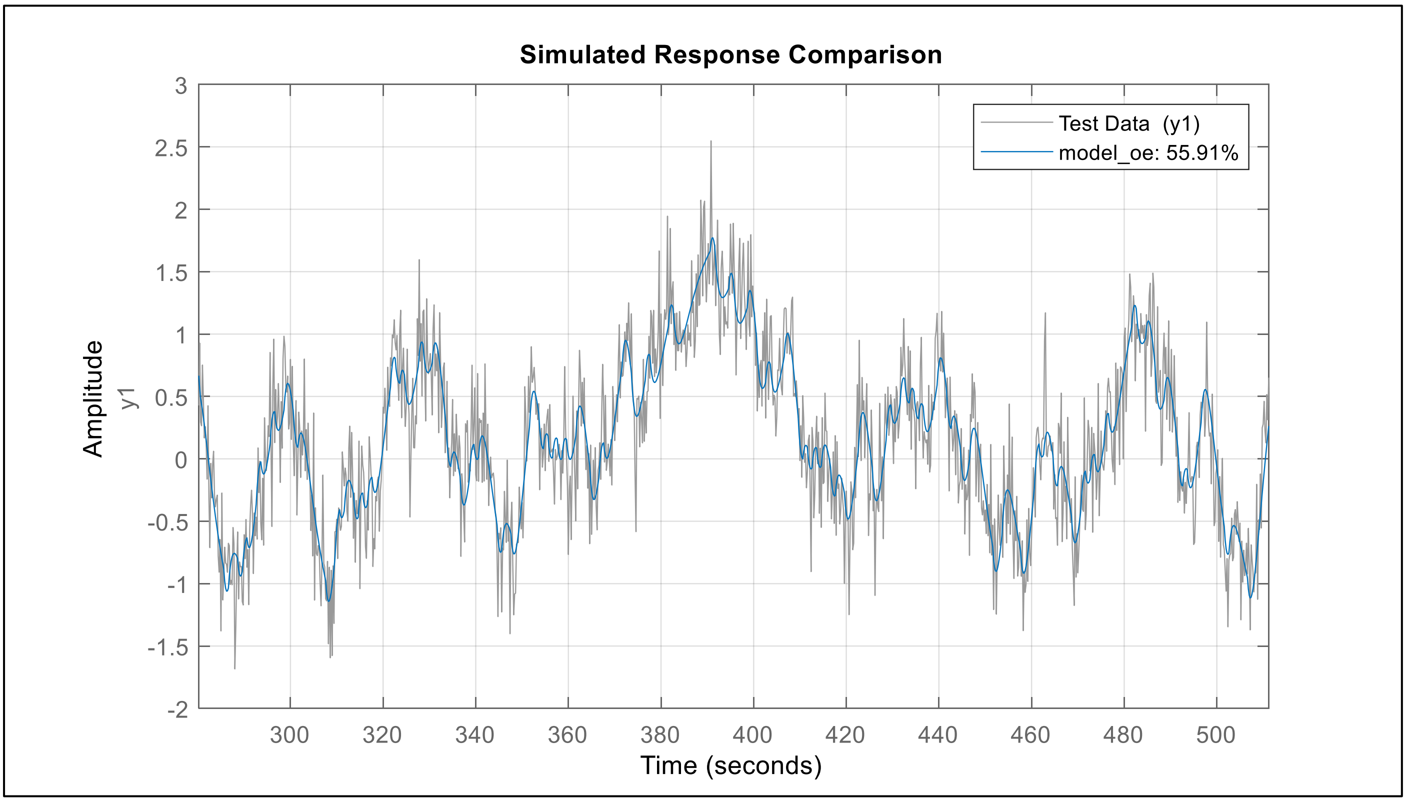
PART (E), (F) & (G)

- We take m=2, n=2 and d=3.
- For a good model:
  - Inputs should not be correlated with the residuals
  - Residuals should not be correlated among themselves
  - Errors in the estimates should be low
  - Model should fit the trend in the test data set well
- We use the following code:

```
%%
%Estimating parametric model using OE
model_oe=oe(dtrain,[2,2,3]);
figure
resid(model_oe,dtrain);
figure
compare(model_oe,dtest);
present(model_oe);

%%
datastar=iddata(out.ystardata.Data,out.udata.Data,0.2);
trnstardata=data(1:1400);
teststardata=data(1401:end);
dteststar=detrend(teststardata,Tr);
figure
compare(model_oe,dteststar);
```



- In the estimated model, we can see that the residuals are not significantly correlated with the input and neither among themselves, hence it is a good estimate.
- Fitness on test set:



- Here, the accuracy of our model on the test set is 55.91%, although it appears to be low numerically, the model is able to capture the trend in the data very well.
- The model is given by:

$$Discrete - time\ OE\ model: y(k) \ = \left[\frac{B(z)}{F(z)}\right] u(k) \ + \ e(k)$$

$$B(z) \ = \ -0.02918\ (+/-\ 0.004051)z^{-3} + \ 0.04249\ (+/-\ 0.004587)z^{-4}$$

$$F(z) \ = \ 1 \ - \ 1.79\ (+/-\ 0.012)z^{-1} \ + \ 0.7968\ (+/-\ 0.01169)z^{-2}$$

$$Sample\ time: 0.2\ seconds$$

END