# Assignment - 1

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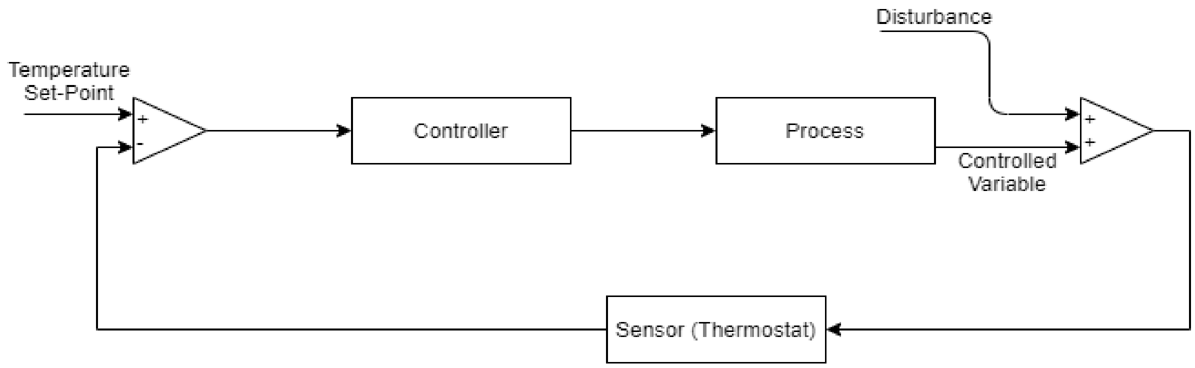
Question-1

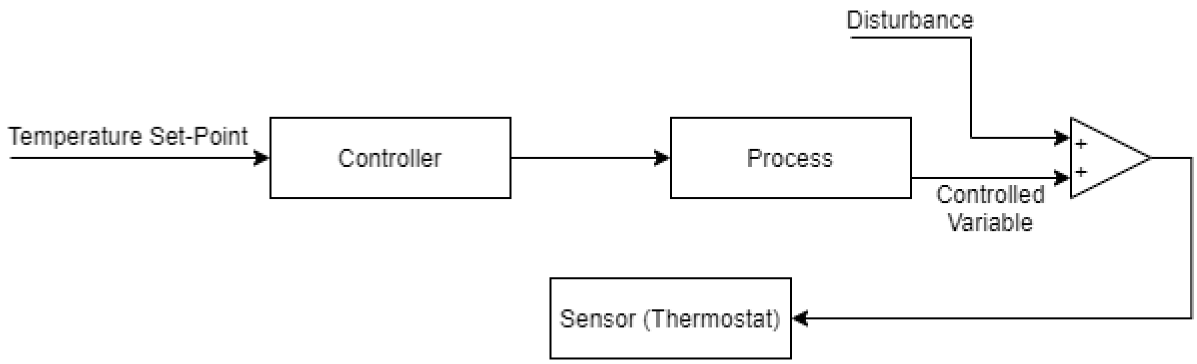
(a)

1. <u>Controlled Variable:</u> Temperature of the room.
2. <u>Manipulated Variable:</u> Switch Position (On/Off).
3. <u>Disturbance Variable:</u> Ambient Air temperature

(b) <u>Feedback control diagram:</u>




(c) <u>Feedforward control diagram:</u>

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**Question – 2**

The process given in this problem is a continuous process.

1. Controlled variable: Flow rate of the blood.
2. Manipulated Variable: Temperature of the unit, concentration of waste substances in blood.
3. Disturbance Variable: Changes in waste concentration due to metabolic activities in the body.

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**Question – 3**

**<u>For finding steady state values-</u>**

At steady state $\frac{dw}{dt} = 0 \; and \; \frac{dz}{dt} = 0$

$$\frac{dw}{dt} = -\frac{L + Va}{M}w + \frac{Va}{M}z = 0$$

$$\frac{dz}{dt} = \frac{L}{M}w - \frac{L + Va}{M}z + \frac{V}{M}z_f = 0$$

Solving these two equations, we get:

$$w_{ss} = \frac{V^2 a Z_f}{V^2 a^2 + L^2 + VaL}$$

$$z_{ss} = \left(\frac{L}{Va} + 1\right) w_{ss}$$

L = 80 gmol/min; V = 100 gmol/min; M = 20 gmol/min; a = 0.5; $z_f$ = 0.1 gmol solute/gmol vapour

From these, we get, $w_{ss} = 0.0387; \; z_{ss} = 0.1006$

**<u>For linearizing the system of ODEs:</u>**

The model has two states, w and z.

And two outputs, again w and z.

We define-

$$\text{State vector: } \boldsymbol{x} = \begin{bmatrix} w \\ z \end{bmatrix}$$

$$\text{Output vector: } g(x) = \boldsymbol{y} = \begin{bmatrix} w \\ z \end{bmatrix}$$

$$\text{Input vector: } \boldsymbol{u} = \begin{bmatrix} L \\ V \end{bmatrix}$$

Given system of equations:

$$\text{f(x)} = \dot{\boldsymbol{x}} = \begin{bmatrix} \dot{w} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -\frac{L + Va}{M}w + \frac{Va}{M}z \\ \frac{L}{M}w - \frac{L + Va}{M}z + \frac{V}{M}z_f \end{bmatrix}$$

Linearized system is written as:

$$\dot{\overline{\boldsymbol{x}}} = A\overline{\boldsymbol{x}} + B\overline{\boldsymbol{u}}$$
$$\overline{\boldsymbol{y}} = C\overline{\boldsymbol{x}} + D\overline{\boldsymbol{u}}$$

Where, $\overline{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_{ss}; \; \overline{\boldsymbol{y}} = \boldsymbol{y} - \boldsymbol{y}_{ss}; \; \overline{\boldsymbol{u}} = \boldsymbol{u} - \boldsymbol{u}_{ss}$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial w} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial w} & \frac{\partial f_2}{\partial z} \end{bmatrix}_{ss} = \begin{bmatrix} -\frac{L+Va}{M} & \frac{Va}{M} \\ \frac{L}{M} & -\frac{L+Va}{M} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial L} & \frac{\partial f_1}{\partial V} \\ \frac{\partial f_2}{\partial L} & \frac{\partial f_2}{\partial V} \end{bmatrix}_{ss} = \begin{bmatrix} -\frac{w_{ss}}{M} & \frac{a}{M}(z_{ss} - w_{ss}) \\ \frac{w_{ss} - z_{ss}}{M} & -\frac{z_f - az_{ss}}{M} \end{bmatrix}$$

Putting all the input, steady state and parameter values to get-

$$A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix} \; B = \begin{bmatrix} -0.002 & 0.0015 \\ -0.003 & -0.0025 \end{bmatrix} \; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \; D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**<u>Eigen-Values & direction of slowest and fastest change:</u>**

Eigen values of the matrix A are -3.33 and -9.66.

The eigen value -9.66 corresponds to that eigen vector which is the direction of fastest change at the initial steady state.

$\begin{bmatrix} -0.620 \\ 0.784 \end{bmatrix}$ is the direction of fastest change, and $\begin{bmatrix} 0.620 \\ 0.784 \end{bmatrix}$ is the direction of smallest change.

**<u>SIMULINK Modelling, steady state using 'trim', and linearized model using 'linmod':</u>**

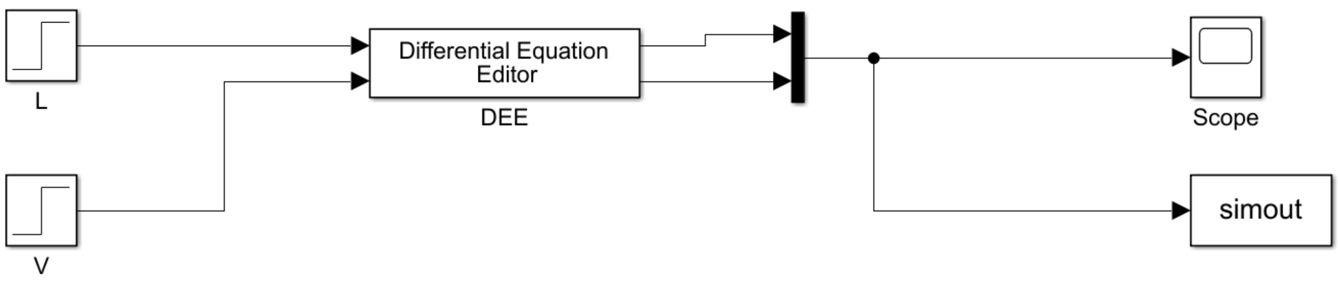



Figure: Block diagram of non-linear system of ODEs

Figure: Output of step input (in L) response

Using 'trim' we get, $w_{ss} = 0.0386$; $z_{ss} = 0.1006$.

Using 'linmod' we get the linearized model with

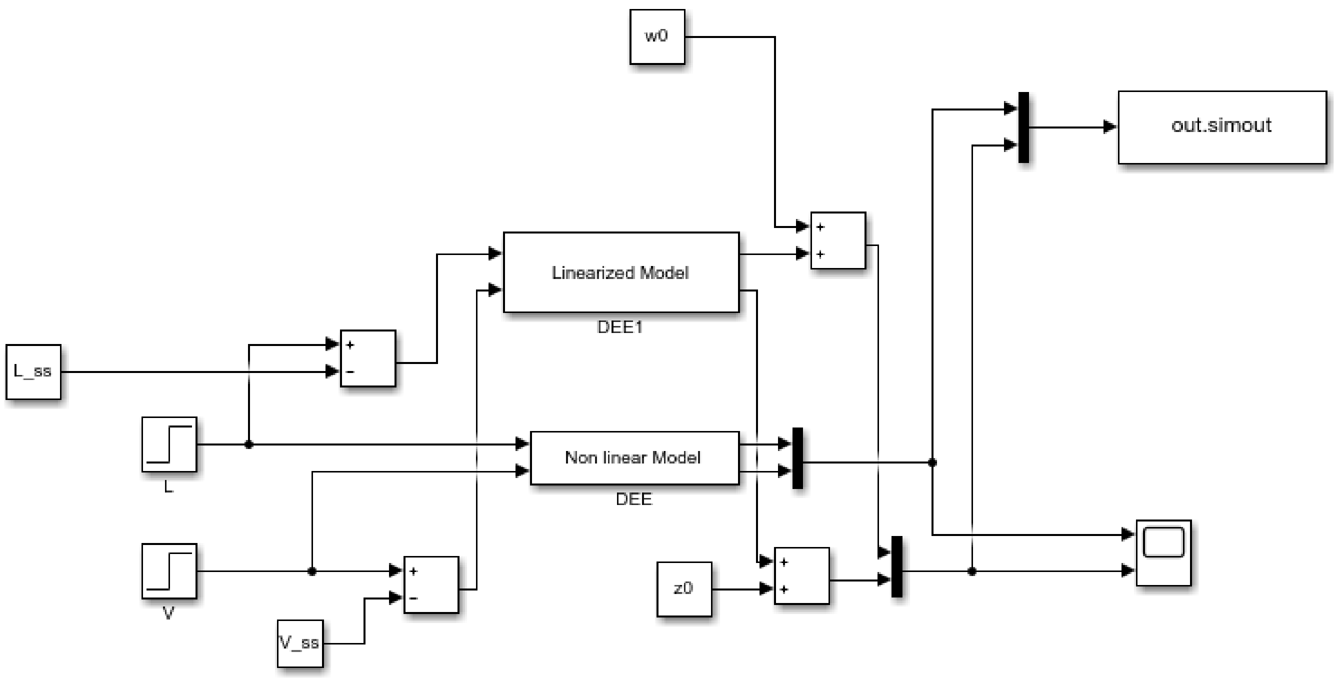
$$A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix} \quad B = \begin{bmatrix} -0.002 & 0.0015 \\ -0.003 & -0.0025 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The following code is used:

```
x0 = [0;0]
y0 = [0;0]
u0 = [80;100]
[X,U,Y] = trim('A1trim', x0, u0)

[A,B,C,D] = linmod('A1trim',X,u0)
```

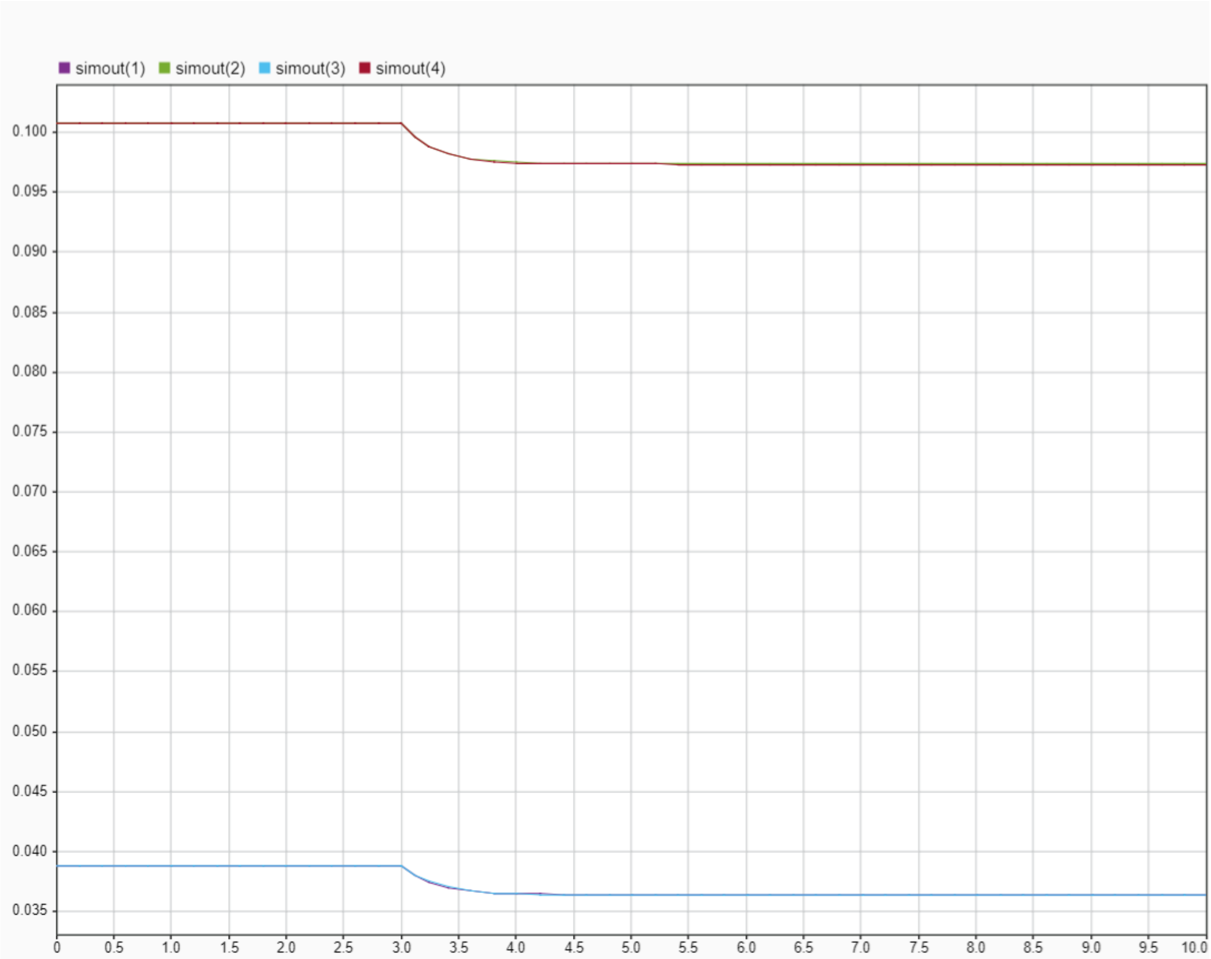
## Responses to step input changes in linear and non-linear models:




The plots obtained by a 5% and 15% increase in L are as shown below.

For 5% change in flow rate L:

Simout(1) – w as calculated by non linear model
Simout(1) – z as calculated by non linear model
Simout(3) – w as calculated by linear model
Simout(4) – z as calculated by linear model

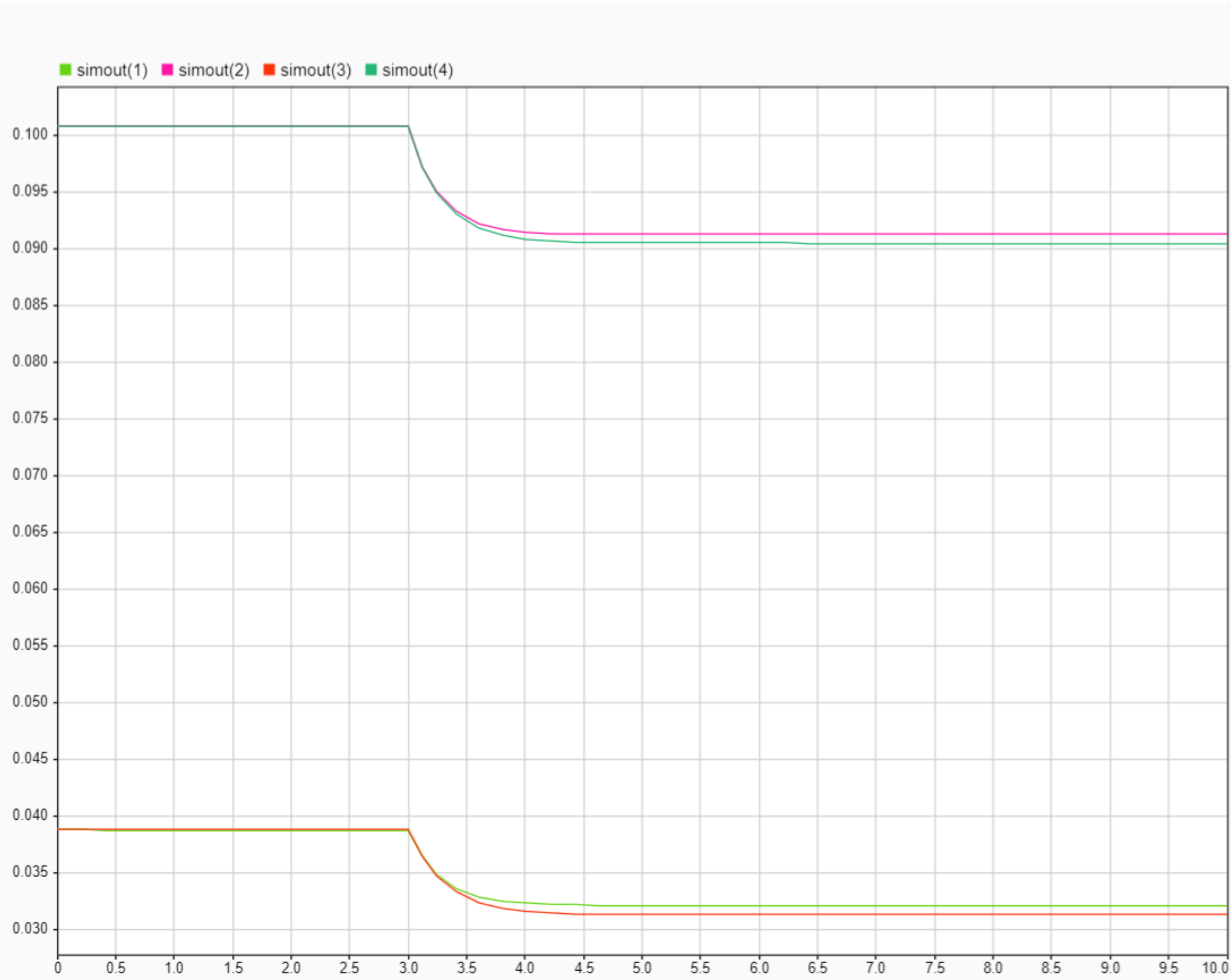



Plot showing the response in w and z as calculated by the linear and non-linear models for a 5% change in L.

The linear model is sufficiently capable of capturing the transience and the final steady state in this case.

<u>For 15% change in flow rate L:</u>

Simout(1) – w as calculated by non-linear model
Simout(1) – z as calculated by non-linear model
Simout(3) – w as calculated by linear model
Simout(4) – z as calculated by linear model




Plot showing the response in w and z as calculated by the linear and non-linear models for a 15% change in L.

In this case the linear model is not sufficient in estimating the steady state but however is capable of modelling the transience to a fair degree as seen from the plots.

**Question-4**

## (a) Laplace transform of function:

$$x = \begin{cases} t-3; & 0 \le t < 3 \\ 0; & 3 \le t < 4 \\ \sin(3\pi(t-4)); & 4 \le t < 5 \\ e^{-2t}\sin(5\pi t); & 5 \le t < \infty \end{cases}$$

$$X(s) = \mathcal{L}(x(t)) = \int_0^3 (t-3)e^{-st}dt + \int_3^4 (0)e^{-st}dt + \int_4^5 \sin(3\pi(t-4))e^{-st}dt + \int_5^\infty e^{-2t}\sin(5\pi t)\, e^{-st}dt$$

$$I_1 = \int_0^3 (t-3)e^{-st}dt = \frac{-3s + e^{-3s} - 1}{s^2}$$

$$I_2 = \int_3^4 (0)e^{-st}dt = 0$$

$$I_3 = \int_4^5 \sin(3\pi(t-4))e^{-st}dt = \frac{3\pi e^{-5s}(e^s+1)}{s^2+9\pi^2}$$

$$I_4 = \int_5^\infty e^{-2t}\sin(5\pi t)\, e^{-st}dt = \frac{-5e^{-10}\pi e^{-5s}}{s^2+4s+25\pi^2+4}$$

$$\boldsymbol{X(s) = I_1 + I_2 + I_3 + I_4 = \frac{-3s + e^{-3s} - 1}{s^2} + \frac{3\pi e^{-5s}(e^s+1)}{s^2+9\pi^2} + \frac{-5e^{-10}\pi e^{-5s}}{s^2+4s+25\pi^2+4}}$$

## (b) Inverse Laplace:

We break up the expression into the constitutive partial fractions.

$$\frac{s-1}{s(T^2s^2+2\epsilon Ts+1)} = -\frac{1}{s} + \frac{s}{2\epsilon\frac{s}{T}+s^2+\frac{1}{T^2}} + \frac{1}{T^2}\frac{(1+2\epsilon T)}{2\epsilon\frac{s}{T}+s^2+\frac{1}{T^2}}$$

Rewriting the denominator in terms of a square, we can write:

$$= -\frac{1}{s} + \frac{s}{\left(s+\frac{\epsilon}{T}\right)^2 + \frac{1-\epsilon^2}{T^2}} + \frac{1}{T^2}\frac{(1+2\epsilon T)}{\left(s+\frac{\epsilon}{T}\right)^2 + \frac{1-\epsilon^2}{T^2}}$$

Manipulating the second term:

$$= -\frac{1}{s} + \frac{\left(s+\frac{\epsilon}{T}\right)}{\left(s+\frac{\epsilon}{T}\right)^2 + \frac{1-\epsilon^2}{T^2}} + \frac{1}{T^2}\frac{(1+\epsilon T)}{\left(s+\frac{\epsilon}{T}\right)^2 + \frac{1-\epsilon^2}{T^2}}$$

For the case where $\epsilon > 1$ we have:

$$-L(t) + e^{\left(\frac{-\epsilon}{T}\right)}\cosh(wt) + \frac{1+\epsilon T}{T}\frac{1}{(1-\epsilon^2)^{0.5}}e^{-\frac{\epsilon}{T}}\sinh(wt)\ where\ w = \frac{(\epsilon^2-1)^{0.5}}{T}$$

For the case where $0 < \in < 1$ we have:

$$-L(t) + e^{\left(\frac{-\in}{T}\right)} \cos{(wt)} + \frac{1+\in T}{T} \frac{1}{(1-\in^2)^{0.5}} e^{-\frac{\in}{T}} \sin(wt)\ where\ w = \frac{(1-\in^2)^{0.5}}{T}$$

For the case where $\in = 1$ we have:

$$-L(t) + e^{\left(\frac{-\in}{T}\right)} + \frac{1+\in T}{T} e^{-\frac{\in}{T}}\ where\ w = \frac{(1-\in^2)^{0.5}}{T}$$