Assignment - 2

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Question-1

(a) **First principles model:**

Assuming,

1. The reactor is perfectly mixed.
2. Heat of reaction is constant.

Component balance gives us-

$$V\frac{dc_A}{dt} = qc_{A,i} - qc_A + Vk(T)c_A$$

$$\Rightarrow \frac{dc_A}{dt} = -\left(\frac{q}{V} + k(T)\right)c_A + \frac{qc_{A,i}}{V}$$

The energy balance gives us-

$$V\rho c_p\frac{dT}{dt} = q\rho c_p T_i - q\rho c_p T + (-\Delta H_R)(-k(T)C_A)V$$

$$\Rightarrow \frac{dT}{dt} = (T_i - T)\frac{q}{V} + \frac{\Delta H_R}{\rho c_p}(k(T)C_A)$$

Therefore, the model is-

$$\frac{dc_A}{dt} = -\left(\frac{q}{V} + k(T)\right)c_A + \frac{qc_{A,i}}{V} \qquad --(1)$$

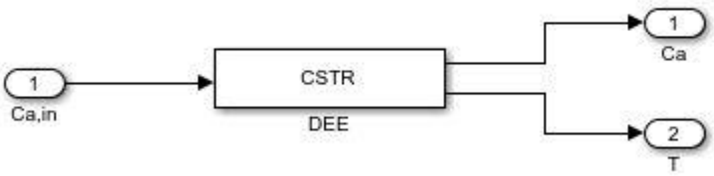
$$\frac{dT}{dt} = (T_i - T)\frac{q}{V} + \frac{\Delta H_R}{\rho c_p}(k(T)C_A) \qquad --(2)$$

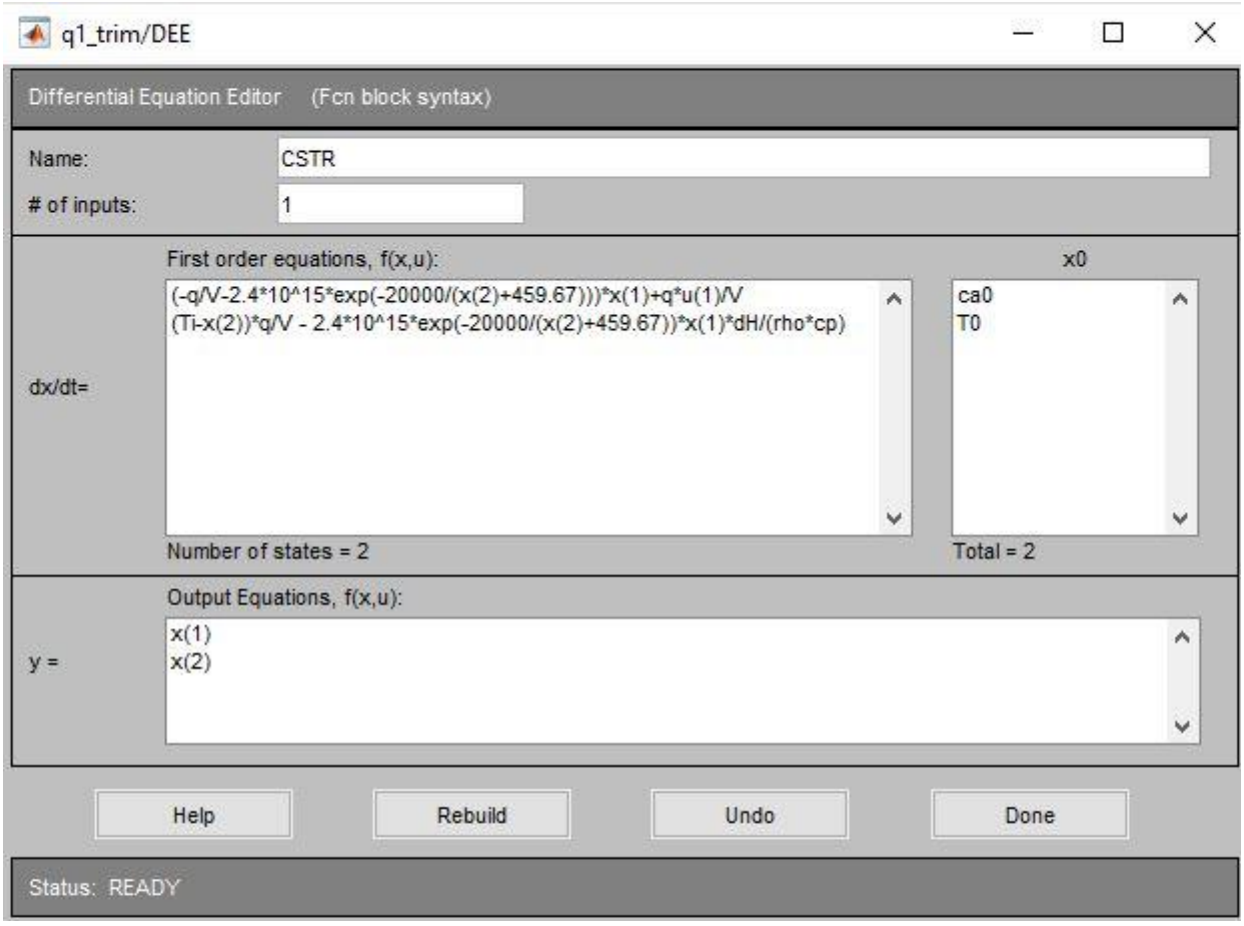
Where,

$$k(T) = -2.4 \times 10^{15} e^{-\frac{20000}{T}}(min^{-1});\ c_p = 0.8\frac{Btu}{lb°\text{F}}$$

$$\rho = 52\frac{lb}{ft^3};\ \Delta\text{H}_\text{R} = -500\frac{kJ}{mol};\ q = q_{ss} = 20\frac{gallons}{minute}$$

(b) **Simulink diagram and steady state:**





- Following code is used:

```
[xs,us,ys] = trim('q1_trim',[1;1],0.8,[1;1],[],1,[])
```

- We get the steady state temperature $\boldsymbol{T_{ss}} = \boldsymbol{xs(2)} = \mathbf{99.391}$°F

(c) **Finding transfer function:**

- Following code is used:

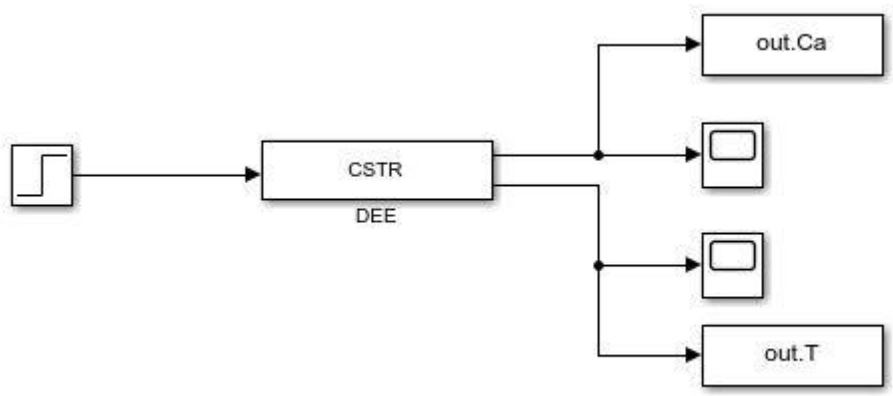
```
[xs,us,ys] = trim('q1_trim',[1;1],0.8,[1;1],[],1,[]) %Finding steady-state
[A,B,C,D]=linmod('q1_trim',xs,0.8) %Linearzing the model
[num,den]=ss2tf(A,B,C,D) %Finding the coefficients of Num and Den of two transfer functions
Gs=tf(num(2,:),den) %Transfer function relating T and Ca,in
```

- Evaluated transfer function is-

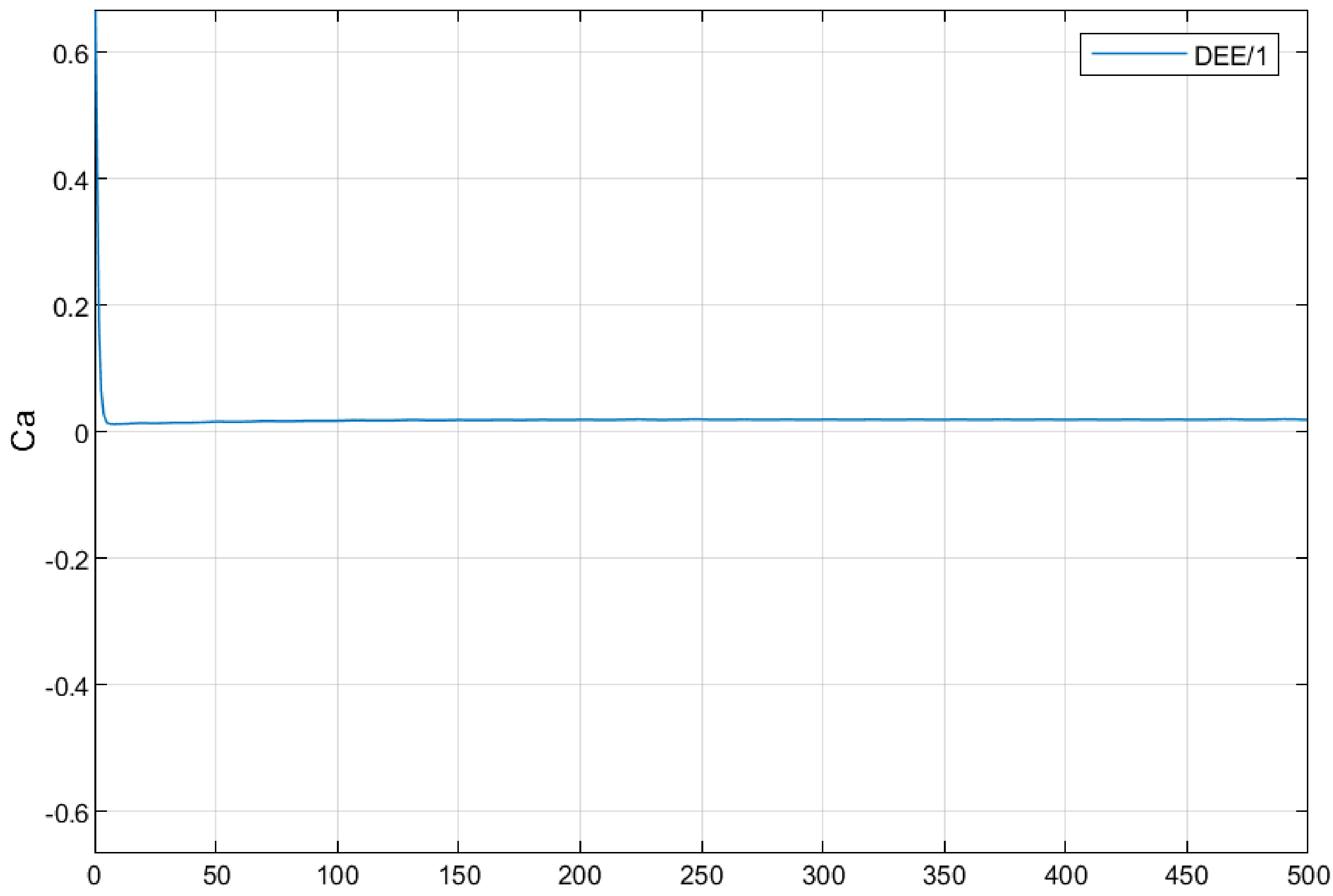
$$G_{21}(s) = \frac{0.1398}{s^2 + 0.721s + 0.01174}$$

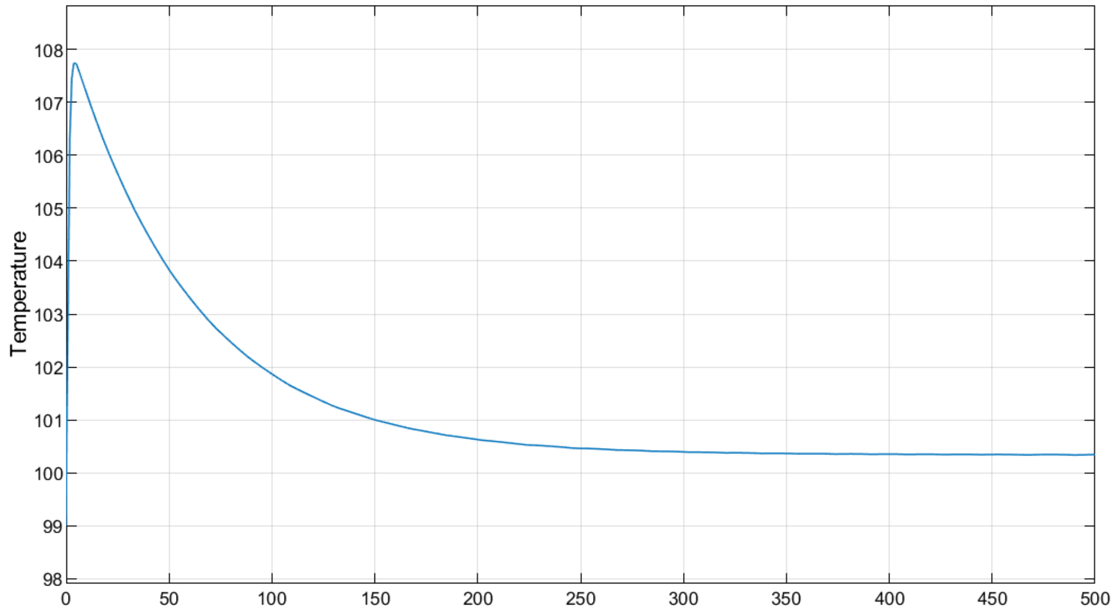
(d) **Step-Responses:**

- Non-linear Simulink model-

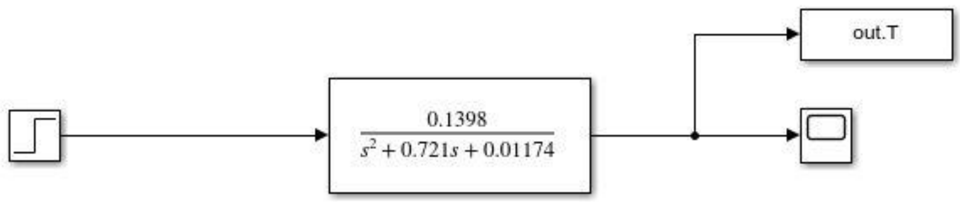


- Step responses for a 10% step in $C_{A,in}$-

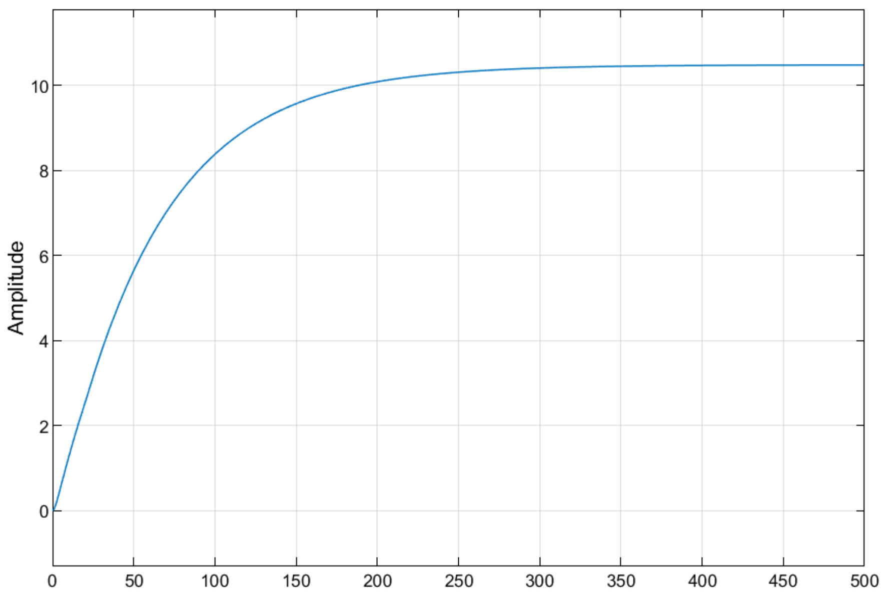




- Linearized Simulink Model in Laplace Domain (between T and $C_{A,in}$)-



- Step response-



Question-2

(a)-(i) **Finding SS description using partial fraction expansion method:**

$$given, G(s) = \frac{s+1}{s^3+10s^2+31s+30} = \frac{s+1}{(s+2)(s+3)(s+5)}$$

$$\Rightarrow \frac{Y(s)}{U(S)} = G(s) = -\frac{1}{3(s+2)} + \frac{1}{s+3} - \frac{1}{6(s+5)}$$

$$Let, X_1(s) = -\frac{U(s)}{3(s+2)};\ X_2(s) = \frac{U(s)}{(s+3)};\ X_3(s) = -\frac{U(s)}{6(s+5)}$$

$$Y(s) = X_1(s) + X_2(s) + X_3(s)$$

- Now, the state equations are:

$$\dot{x_1} = -2x_1 - \frac{1}{3}u$$

$$\dot{x_2} = -3x_2 + u$$

$$\dot{x_3} = -5x_3 - \frac{1}{6}u$$

- Corresponding state-space representation is:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -\frac{1}{3} \\ 1 \\ -\frac{1}{6} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(a)-(ii) **Finding SS description using state-transition method:**

$$\frac{Y(s)}{U(s)} = G(s) = \frac{s+1}{s^3+10s^2+31s+30}$$

$$\Rightarrow Define, X(s) = \frac{Y(s)}{s+1} = \frac{U(s)}{s^3+10s^2+31s+30}$$

$$\Rightarrow Y(s) = sX(s) + X(s)$$

$$And, \quad s^3X(s) + 10s^2X(s) + 31sX(s) + 30X(s) = U(s)$$

We write state equations as-

$$x_1(t) = x(t)$$

$$x_2(t) = \dot{x_1}(t)$$

$$x_3(t) = \dot{x_2}(t)$$

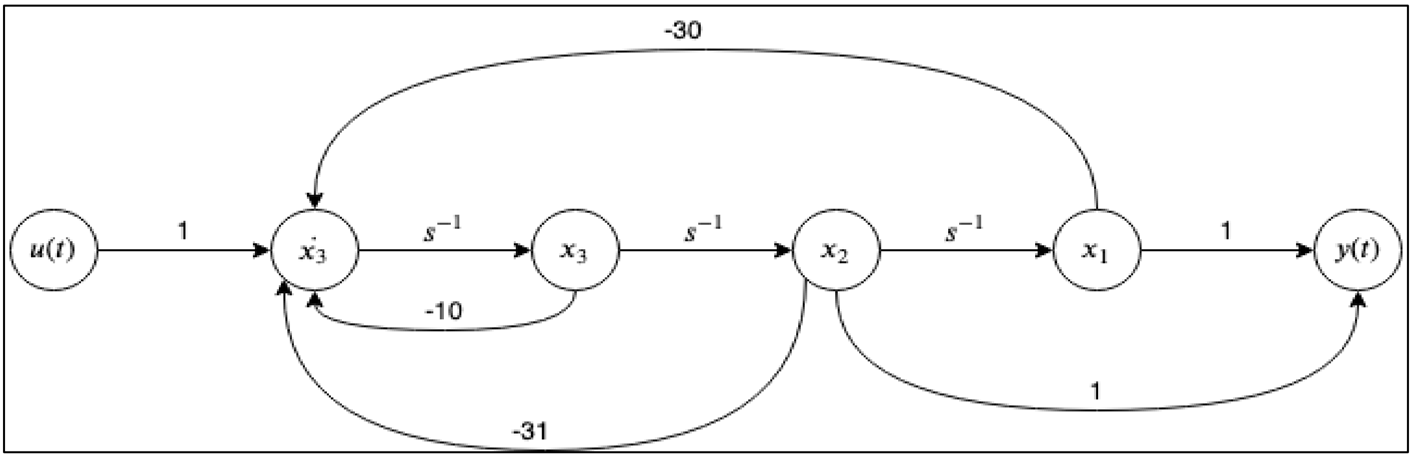
$$\dot{x_3}(t) = u(t) - 30x_1(t) - 31x_2(t) - 10x_3(t)$$

State-space model can be written as-

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -30 & -31 & -10 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

State-transition diagram is-



(b) **SS model for the SITO system:**

$$Given, G_{11}(s) = \frac{4s+1}{(s+1)(s+3)}; \;\; G_{21}(s) = \frac{10s}{(s+2)(s+3)}$$

Now,

$$\frac{Y_1(s)}{U(s)} = \frac{4s+1}{(s+1)(s+3)} = -\frac{1.5}{s+1} + \frac{6.5}{s+3}$$

$$\Rightarrow Y_1(s) = -\underbrace{\frac{1.5U(s)}{s+1}}_{X_1} + \underbrace{\frac{6.5U(s)}{s+3}}_{X_2}$$

$$\Rightarrow Y_1(s) = -X_1(s) + X_2(s)$$

Similarly,

$$\frac{Y_2(s)}{U(s)} = \frac{10s}{(s+2)(s+3)} = -\frac{20}{s+2} + \frac{30}{s+3}$$

$$\Rightarrow Y_2(s) = -\underbrace{\frac{20U(s)}{s+2}}_{X_3} + \frac{30U(s)}{s+3}$$

$$\Rightarrow Y_3(s) = -X_3(s) + \frac{60}{13}X_2(s)$$

From these, state equations can be written as-

$$\dot{x_1} = -x_1(t) + 1.5u(t)$$

$$\dot{x_2} = -3x_2(t) + 6.5u(t)$$

$$\dot{x_3} = -2x_3(t) + 20u(t)$$

And,

$$y_1 = -x_1(t) + x_2(t)$$

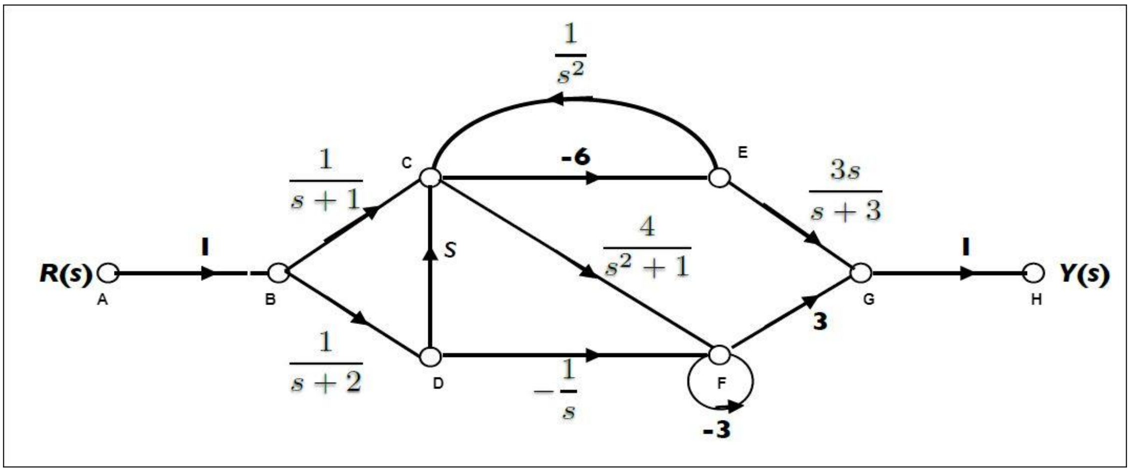
$$y_2 = -x_3(t) + \frac{60}{13}x_2(t)$$

State-space model can be written as-

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1.5 \\ 6.5 \\ 20 \end{bmatrix} u(t)$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & \frac{60}{13} & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

Question-3



(a) **Block diagram of the given process:**

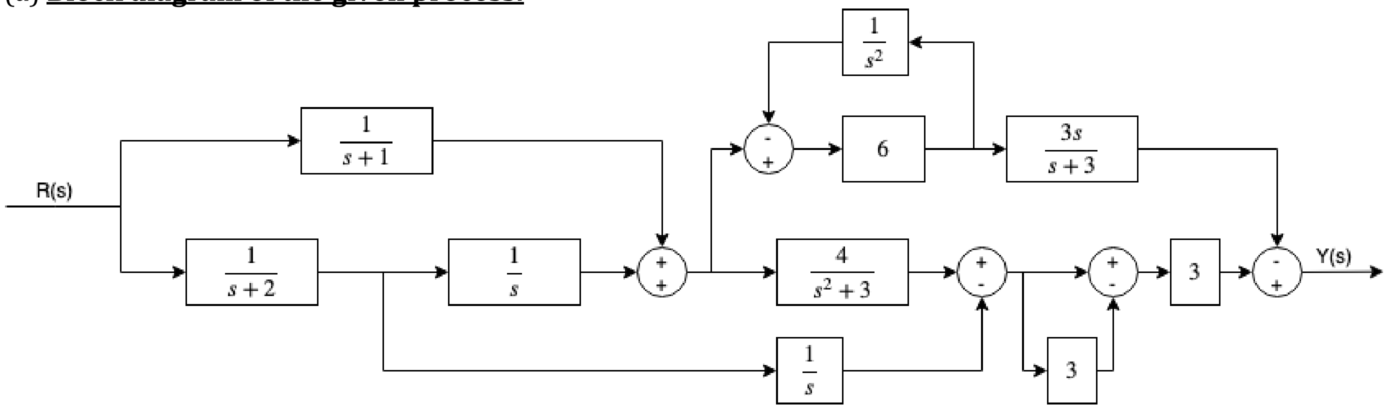


Figure : Block – diagram of the system given in the problem

(b) **For finding transfer function** $\boldsymbol{G(S)} = \frac{\boldsymbol{Y(S)}}{\boldsymbol{R(S)}}$:

- There are five paths connecting Y(S) and R(S). The transmittance of these are given by-

$$P_1(ABCEGH) = \frac{-18s}{(s+1)(s+3)}$$
$$P_2(ABDFGH) = \frac{-3}{s(s+2)}$$
$$P_3(ABDCFGH) = \frac{12s}{(s+2)(s^2+1)}$$
$$P_4(ABDCEGH) = \frac{-18s^2}{(s+2)(s+3)}$$
$$P_5(ABCFGH) = \frac{12}{(s+1)(s^2+1)}$$

- There are two loops in the given diagram. Transmittance of these are given by-

$$L_1(CEC) = \frac{-6}{s^2}$$
$$L_1(FF) = -3$$

- Determinant of the graph is given by-

$$\Delta = 1 - L_1 - L_2 + L_1 L_2 = \frac{4s^2 + 24}{s^2}$$

- Co-factor of each path P_i is given as Δ_i as-

$$\Delta_1 = 4$$
$$\Delta_2 = \frac{s^2+6}{s^2}$$
$$\Delta_3 = 1$$
$$\Delta_4 = 4$$
$$\Delta_5 = 1$$

- With the help of all the above equations, we can get the transfer function-

$$G(s) = \frac{\sum_{i=1}^{5} P_i \Delta_i}{\Delta}$$

$$G(s) = \frac{1}{\Delta}\left(\frac{-72s}{(s+1)(s+3)} + \frac{-3(s^2+6)}{s^3(s+2)} + \frac{12s}{(s+2)(s^2+1)} + \frac{-72s^2}{(s+2)(s+3)} + \frac{12}{(s+1)(s^2+1)}\right)$$

$$Simplifying\ above\ expression\ we\ get -$$

$$\boxed{\frac{Y(s)}{R(s)} = G(s) = \frac{-3(24s^8 + 48s^7 + 69s^6 + 32s^5 + 26s^4 + 4s^3 + 27s^2 + 24s + 18)}{4s(s+1)(s+2)(s+3)(s^2+1)(s^2+6)}}$$