String Theory:
an introduction

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Disclaimer:

I must have been crazy to suggest I'd do this!

I'll try and give an unbiased view.

"The moment you encounter string theory and realize that almost all of the major developments in physics over the last hundred years emerge -- and emerge with such elegance -- from such a simple starting point, you realize that this incredibly compelling theory is in a class of its own" M. Green

"What makes string theory so difficult to assess dispassionately is that it gains its support and chooses its directions of development almost entirely from aesthetic judgements guided by mathematical desiderata. I believe that it is important to record each of the turnings that the theory has undergone, and to point out that almost every turn has taken us further from observationally established facts." R. Penrose

Contents

- Why do we need string theory
  (or at least new physics...)
- Overview of the properties of string theory
- Classical string theory as a toy model

Why bother with string theory?

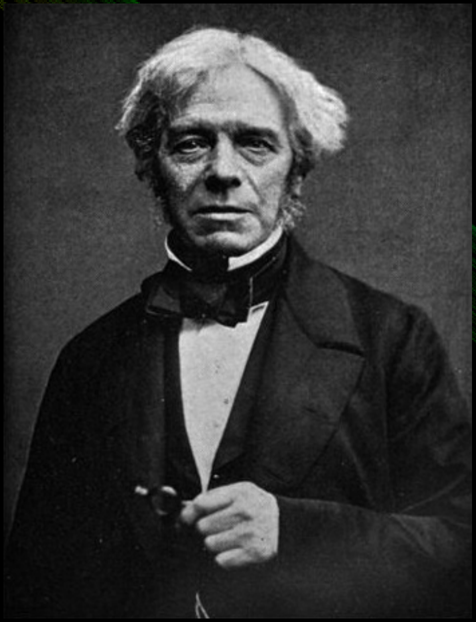
UNIFICATION OF PHYSICAL LAWS!

- In 1820 Biot, Savart and Ampère find electric currents produce B-fields

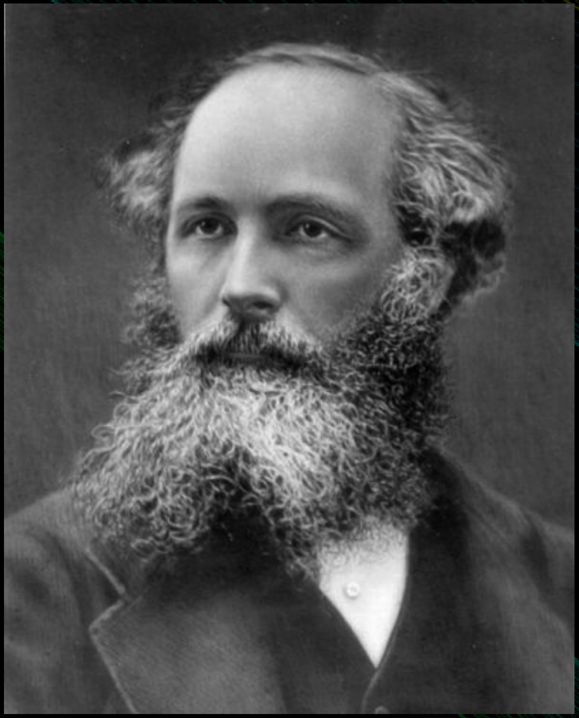
- In 1831 Faraday shows that a changing magnetic flux induces a current

- In 1865 James Clerk Maxwell derives his famous Maxwell equations, formally uniting Electricity and Magnetism into Electromagnetism.

JB Biot



M Faraday



JCM

O Heaviside

Why bother with string theory?

UNIFICATION OF PHYSICS!

- Next big unification of fundamental forces: 1960s (Nobel prize 1979)
  Glashow, Weinberg and Salam unite EM with the Weak interaction

- Electroweak theory defines the electroweak scale, (~250 GeV) above which EM and the weak interaction unite.

- W^+, W^-, B^0, A^0, become W^+, W^-, Z^0 and γ via EWSB (~ masses via Higgs mechanism)

- Success of QFT = SR + QM

- Birth of the Standard Model

Weinberg

Glashow

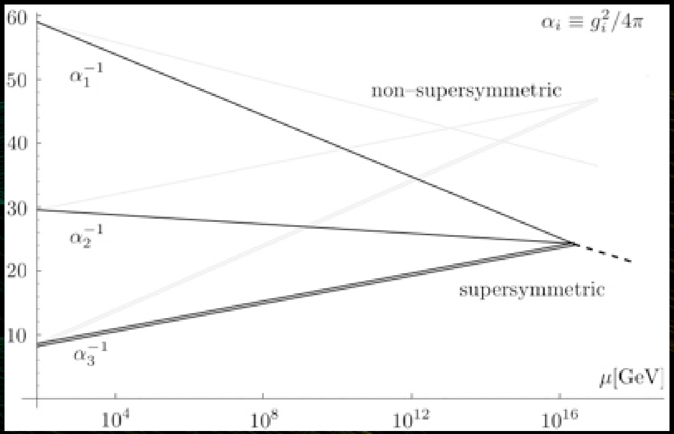
Salam

Why bother with string theory?

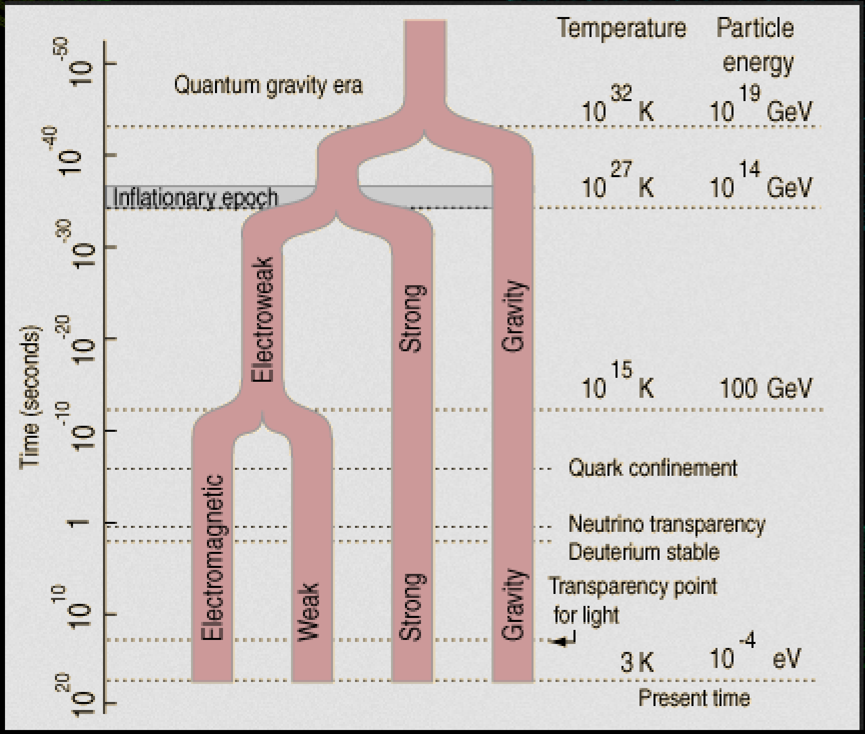
UNIFICATION OF PHYSICS!

- What's Next?

GUTs = QCD + EW



- Bigger picture: SM ⊃ QED + QCD + EW has problems at high energies



Why bother with string theory?

STANDARD MODEL PROBLEMS

- Standard Model Lagrangian... Large and Arbitrary?

- 18(25) Free Parameters
  - 12 Fermion Masses
  - 3 CKM mixing angles + 1 phase
  - 3 PMNS mixing angles + 1 phase
  - 3 coupling constants
  - Higgs mass
  - Z^0 mass

Plus more...

- Heirarchy Problems
- Neutrino Masses
- Dark Matter/ 'Energy'
- Imbalance of Particle to Antiparticle

$$
\begin{gathered}
-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g^2_s f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\frac{1}{2}ig^2_s(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c^2_w}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
\frac{1}{2}m^2_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c^2_w}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} + \\
\frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c^2_w(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s^2_w(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - \\
gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c^2_w}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
\phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s^2_w}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c^2_w}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \\
\frac{1}{4}g^2\frac{1}{c^2_w}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s^2_w}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s^2_w}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c^2_w - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^1 s^2_w A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + \\
m^\lambda_d)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma d^\lambda_j)] + \frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \\
\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s^2_w - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s^2_w - 1 - \gamma^5)u^\lambda_j) + \\
(\bar{d}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s^2_w - \gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1 + \\
\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 + \gamma^5)u^\lambda_j)] + \\
\frac{ig}{2\sqrt{2}}\frac{m^\lambda_e}{M}[-\phi^+(\bar{\nu}^\lambda(1 - \gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1 + \gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m^\lambda_e}{M}[H(\bar{e}^\lambda e^\lambda) + \\
i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1 - \gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1 + \\
\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 + \gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 - \gamma^5)u^\kappa_j] - \\
\frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \frac{M^2}{c^2_w})X^0 + \bar{Y}\partial^2 Y + \\
igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + \\
igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + \\
igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) - \\
\frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c^2_w}\bar{X}^0 X^0 H] + \frac{1-2c^2_w}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \\
\bar{X}^- X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + igMs_w[\bar{X}^0 X^-\phi^+ - \\
\bar{X}^0 X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]
\end{gathered}
$$

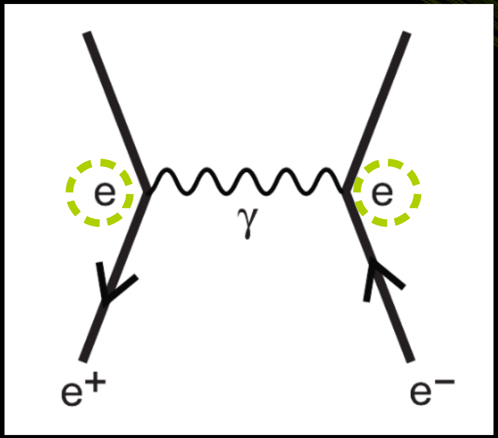
Why bother with string theory?

STANDARD MODEL PROBLEMS: GRAVITY

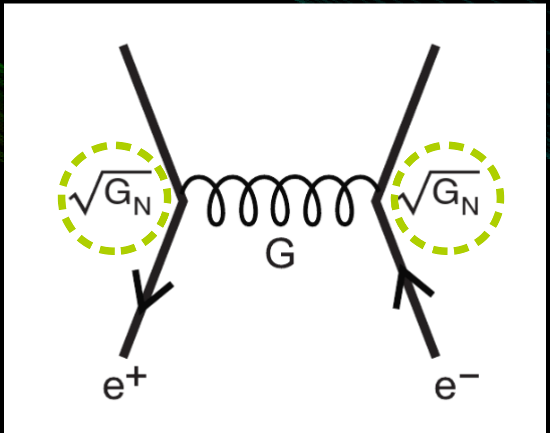
- Adapt (NR)QM for gravity : Schrödinger-Newton equation

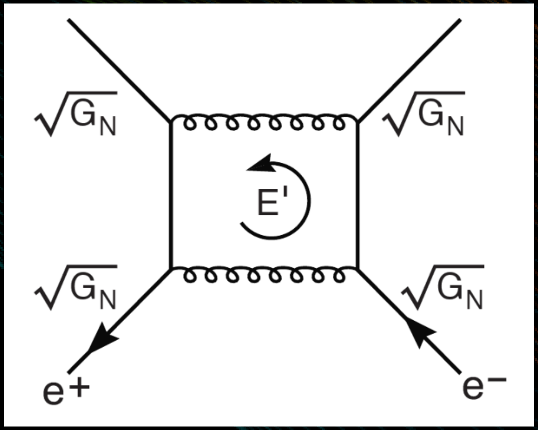
$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2m}\nabla^2\Psi + V\Psi + m\Phi\Psi$$

- OK for low energies... how about QFT and Gravity? Non-renormalisable!



$\Rightarrow$





$$\mathcal{A} \propto e^2 = 4\pi\alpha_{\mathrm{EM}}$$

$$\mathcal{A}^{(0)}_{\mathrm{grav}} \propto G_N E^2$$

$$\mathcal{A}^{(1)}_{\mathrm{grav}} \propto G_N^2 E^2 \int_0^\infty E' dE'$$

"Strongly Divergent"

$$G_N \approx 7 \times 10^{-33} \mathrm{TeV}^{-2}$$

Problem when $E \sim M_P$

And now for some string theory...

Answer to all the problems? (Maybe) String Theory

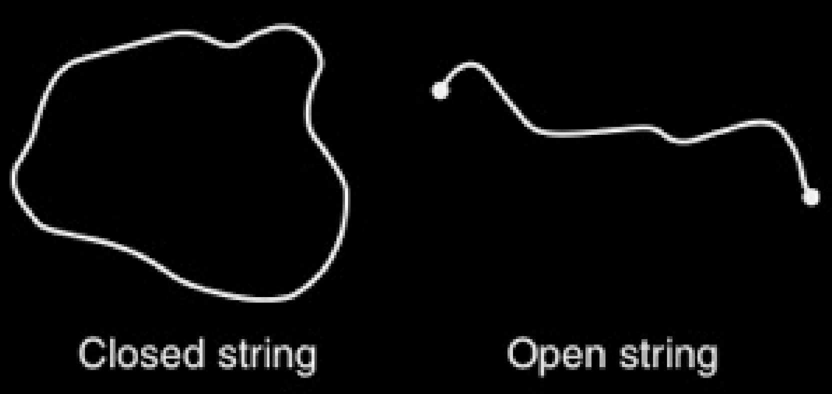
WHAT EXACTLY IS STRING THEORY?

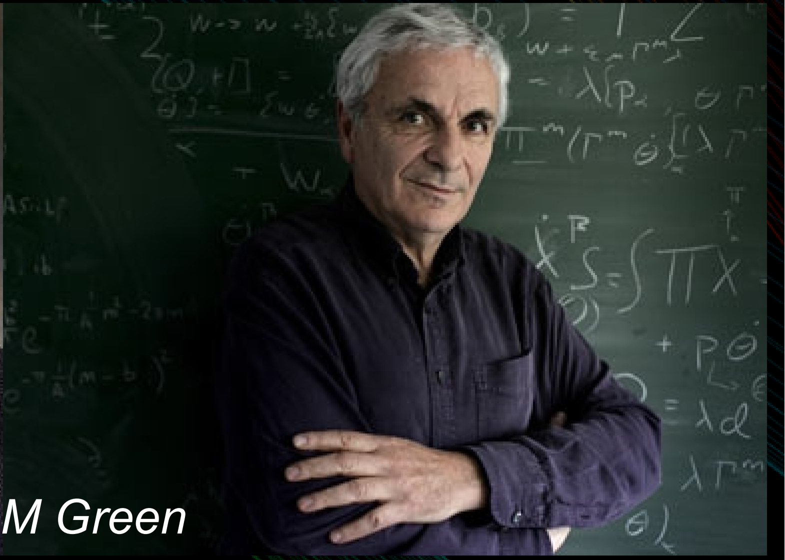
1 dimensional extension to the 'normal' point-particle (0-D) representation of physics (QFT)

Admits open and closed strings

WHY IS THIS BETTER THAN QFT AND THE SM?

String theory combines all of the ideas of QFT and the SM into a more general, tighter, unified structure.





String Theory Ideas

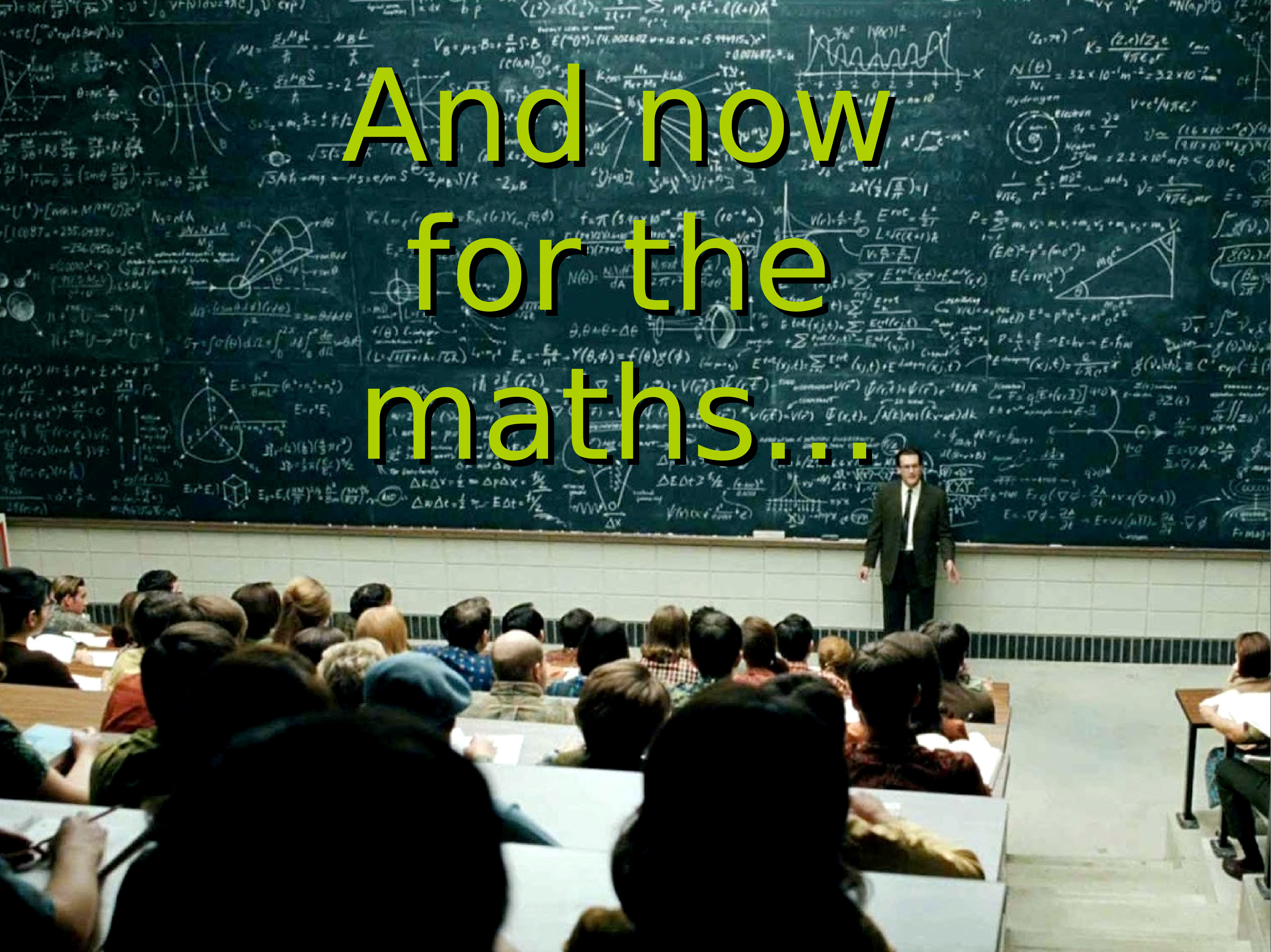
STRING THEORY PROVIDES (for better or for worse)

- Gravity – All consistent STs contain massless spin-2 graviton whose interactions → GR in low energy
- Quantum Gravity – In contradiction to QFT. No infinities here!
- GUTs – ST gauge groups large enough to swallow SM. Reduces to SM in low energy limit

- Extra Dimensions – ST requires definite # of EDs. 10 (26)
- Supersymmetry – All consistent STs require SUSY
- Many Vacua – Quantized ST → many possible ground states

- Free Parameters? - ST has at most 1 free parameter
- Uniqueness – Lack of adjustable free parameters. Spacetime dimension # inherent.

- Testing – No decisive tests (?)


And now for the maths...

- General theory: p-brane theory

(this is an example of the humour you can expect from string theorists)

- Point Particle = 0-brane
- String = 1-brane
- Membrane = 2-brane

And so on...

The 0-brane

Action of a point particle

$$S_0 = -m \int ds \; = -m \int d\tau \sqrt{-g_{\mu\nu}(x)\dot{x}^\mu \dot{x}^\nu}$$

- Difficult to quantise → remove square root
- How about massless particles?

Introduce the Einbein auxiliary field... can reform into

$$\tilde{S}_0 = \frac{1}{2} \int d\tau \left(e(\tau)^{-1} \dot{x}^2 - m^2 e(\tau) \right)$$

Plus EoM in e(τ)

Generalising the 0-brane action

Can generalise $\tilde{S}_0$ to any p-brane

$$S_p = -T_p \int d\mu_p$$

Our (p+1)-dim volume element $d\mu_p$ generalises to

$$d\mu_p = \sqrt{-\det(G_{\alpha\beta})}d^{p+1}\sigma$$

With the pullback,

$$G_{\alpha\beta} = \frac{\partial x^\mu}{\partial \sigma^\alpha}\frac{\partial x^\nu}{\partial \sigma^\beta} g_{\mu\nu}(x)$$

$$\alpha, \beta \in \{0, 1, ..., p\}$$

$$\mu, \nu \in \{0, 1, ..., D \geq p\}$$

Comparison

$$S_0 = -m \int ds$$

For p=0, α,β=0:

$$d\mu_p = \sqrt{-\det(G_{00})}d\tau$$

With

$$G_{00} = \dot{x}^\mu \dot{x}^\nu g_{\mu\nu}(x)$$

Regain

$$d\mu_0 = ds$$

1-brane or string action

- Worldline → Worldsheet (parameterised by two coördinates)
- $\alpha,\beta = 0,1$

- Assume background is Minkowski: $\eta_{\mu\nu}$

$$G_{\alpha\beta} = \frac{\partial x^\mu}{\partial \sigma^\alpha}\frac{\partial x^\nu}{\partial \sigma^\beta}\eta_{\mu\nu}(x) \quad \rightarrow \quad G_{\alpha\beta} = \begin{pmatrix} \dot{x}^2 & \dot{x}x' \\ \dot{x}x' & x'^2 \end{pmatrix}$$

Leads to Nambu-Goto action:

$$S_{NG} = -T\int d\tau d\sigma \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}$$

Add in auxillary field (analogue to e(τ)) to get POLYAKOV ACTION

$$S_\sigma = -\frac{T}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\frac{\partial x^\mu}{\partial \sigma^\alpha}\frac{\partial x^\nu}{\partial \sigma^\beta}g_{\mu\nu}$$

Finding the field equations

$$S_\sigma = -\frac{T}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\frac{\partial x^\mu}{\partial \sigma^\alpha}\frac{\partial x^\nu}{\partial \sigma^\beta}g_{\mu\nu}$$

Polyakov action

- Can use two gauge symmetries of the action

- Reparameterisation Invariance $\sigma \to \sigma' = f(\sigma)$

- Weyl Invariance $h_{\alpha\beta}(\tau,\sigma) \to h'_{\alpha\beta}(\tau,\sigma) = e^{2\omega(\sigma)}h_{\alpha\beta}(\tau,\sigma)$

to gauge auxiliary field into Minkowski metric

- Weyl Inv is local → cannot extend globally to worldsheet...
UNLESS... Worldsheet manifold has Euler characteristic = 0 (Topologically unobstructed)

A Polyakov

$$S_\sigma = \frac{T}{2}\int d\tau d\sigma (\dot{x}^2 - x'^2)$$

Finding the field equations

- Remarkably simple field equations result:

$$(\partial_\tau^2 - \partial_\sigma^2)x^\mu = 0$$

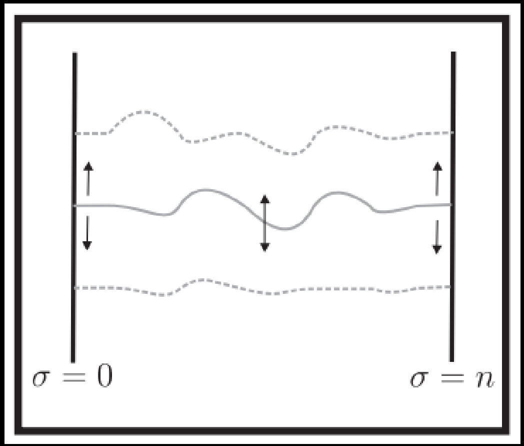
- Plus boundary conditions:

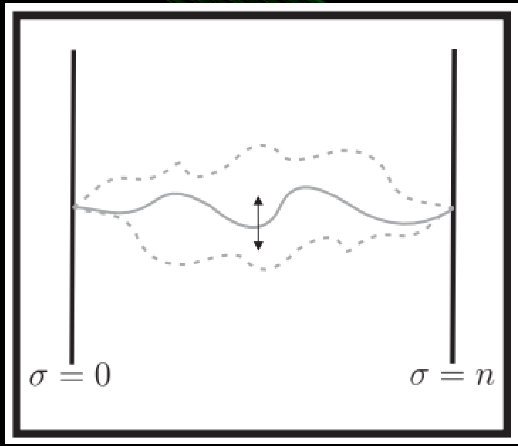
$$x^\mu(\tau, \sigma + n) = x^\mu(\tau, \sigma)$$ CLOSED STRING

$$\partial_\sigma x^\mu(\tau, \sigma) = \partial_\sigma x^\mu(\tau, \sigma + n) = 0$$ OPEN NEUMANN

$$x^\mu(\tau, \sigma = 0) = x_0^\mu \text{ AND } x^\mu(\tau, \sigma = n) = x_n^\mu$$

OPEN DIRICHLET





Finding the field equations

- Most general solution can be expanded in Fourier modes,

CLOSED STRING

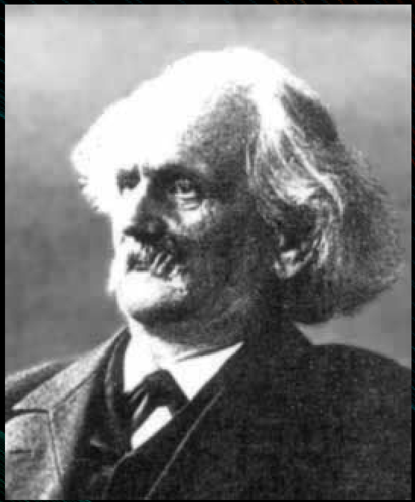
$$x^\mu = x_0^\mu + \tau l_s^2 p_0^\mu + \frac{i}{2} l_s \sum_{n\neq 0} \frac{1}{n}(\alpha_n^\mu e^{2in\sigma} + \tilde{\alpha}_n^\mu e^{-2in\sigma}) e^{-2in\tau}$$

OPEN STRING (NEUMANN)

$$x^\mu = x_0^\mu + l_s^2 \tau p_0^\mu + i l_s \sum_{m\neq 0} \frac{1}{m} \alpha_m^\mu e^{-im\tau} \cos(m\sigma)$$

J Fourier

C Neumann



$$\ddot{T}\hbar\tilde{\alpha}_N k \ \ \Upsilon\mathcal{O}(u)$$