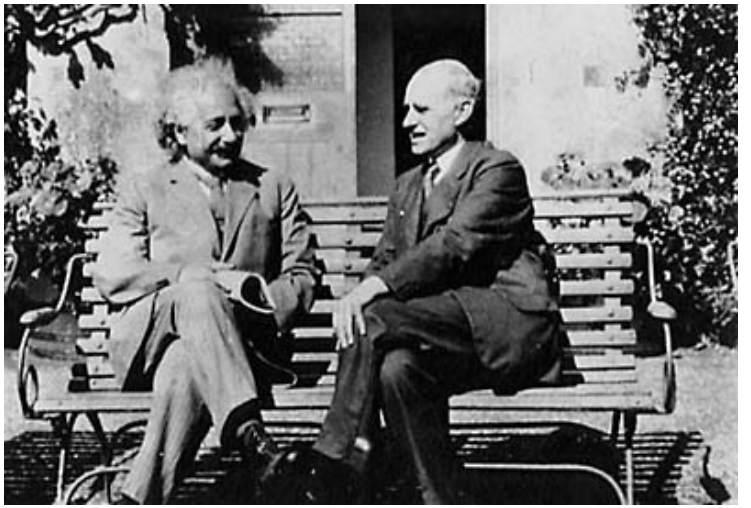
Teleparallel gravity

“You may be amused to hear that one of our great Department Stores (Selfridges) has pasted up in its window your paper... so that passers by can read it all through.

Large crowds gather round to read it!”

Eddington to Einstein, 1929



Overview

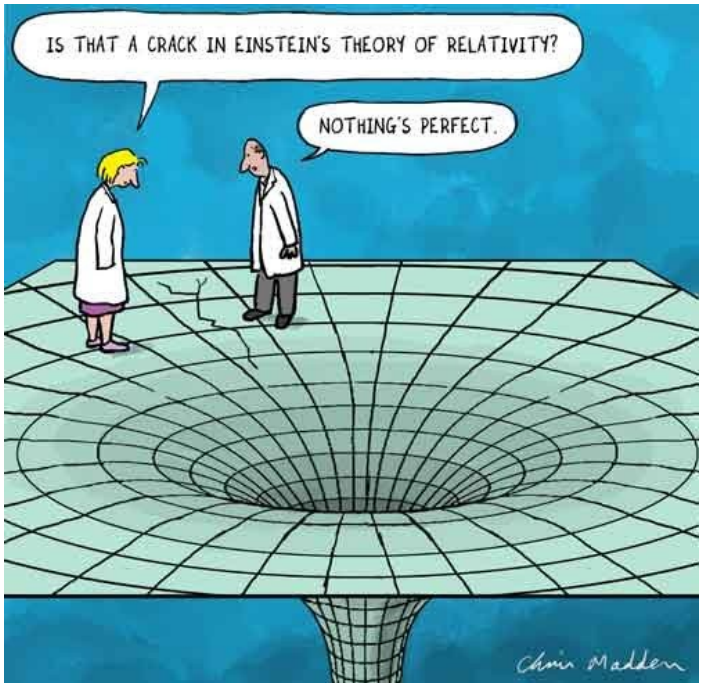
- The history of teleparallel gravity
- “What is gravity?”
- An analogy: defects in a crystal
- Back to gravity...

Unifying the forces

- 1916: general relativity

  But still need to unify gravity with EM

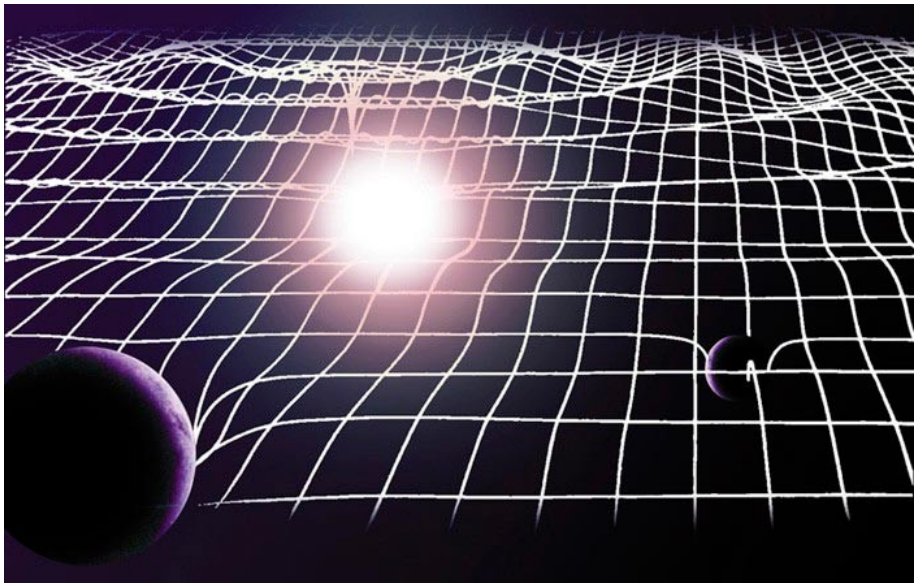
- ~1925: Einstein gets interested in unification. Kaluza-Klein.
- 1928 – 1931: teleparallel gravity
- 1931 – 1955: more attempts



2013: **still** need to unify gravity with EM (and the strong force, and the weak force...)

Rough idea

- General relativity describes gravity in terms of a **curvature** of spacetime
- Teleparallel gravity has zero curvature. Instead it uses a different geometric quantity: **torsion**
- “Distant parallelism” - parallel lines stay parallel



“For the moment, this theory seems to me to be like a starved ape who after a long search, has found an amazing coconut, but cannot open it; so he doesn't even know whether there is anything inside”

Einstein to Cartan, 1930

So, is there anything inside?

Well, probably not a unified field theory. But:

- good for describing electron spin (Dirac equation) and maybe neutrinos
- nice links to elasticity theory / defects in crystals
- cosmology?

Teleparallel equivalent of GR

Same field equations, so why bother?

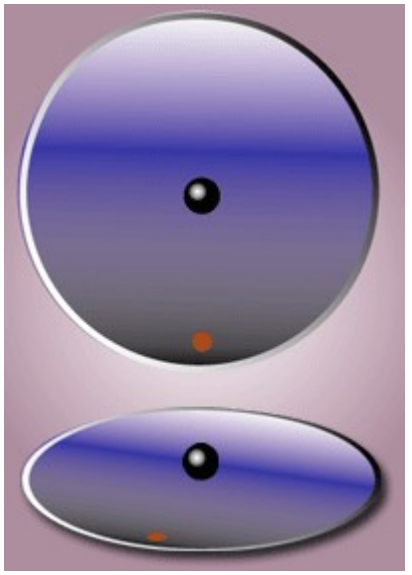
"The formulation is mathematically equivalent. There are, therefore, no fundamentally new results.

However, there is a pleasure in recognising old things from a new point of view"

Feynman, 1948, on his new path-integral formulation of quantum physics

Inertial forces

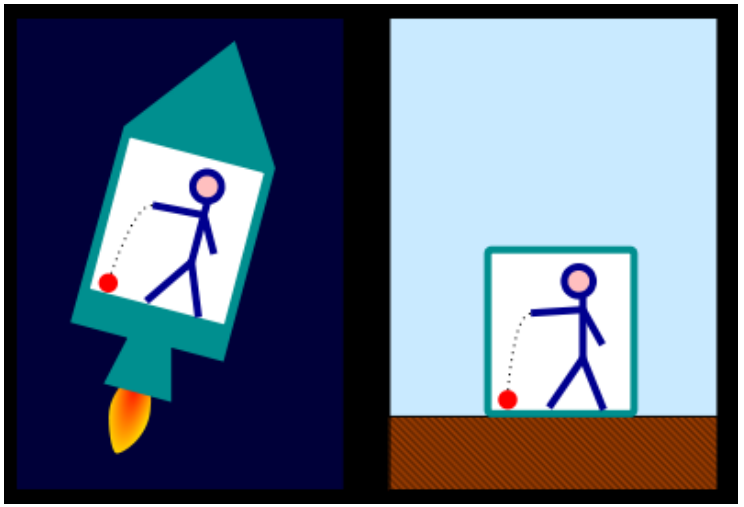
Coriolis effect:



- affects all matter equally
- can be transformed away by a coordinate change

Gravity

Can transform away locally...

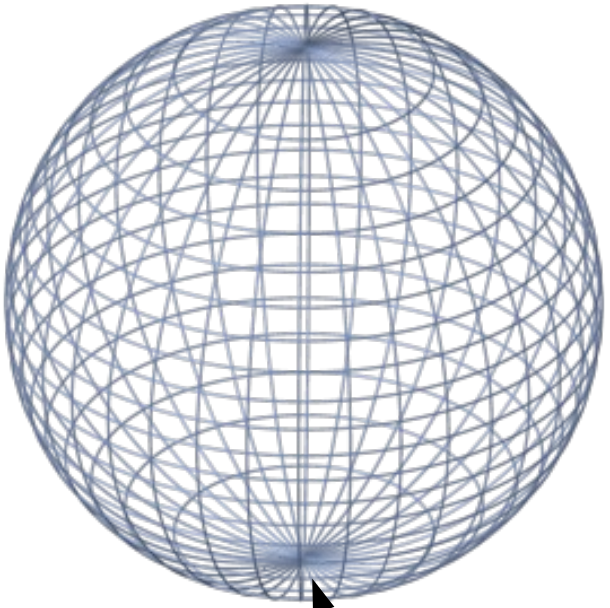


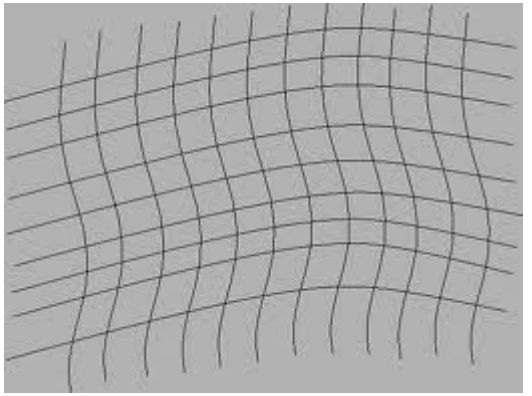
...this is the
equivalence principle

BUT...

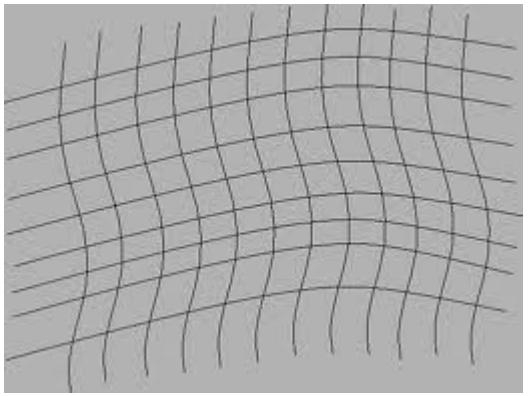
… no single transformation will work everywhere

Coordinates on a sphere





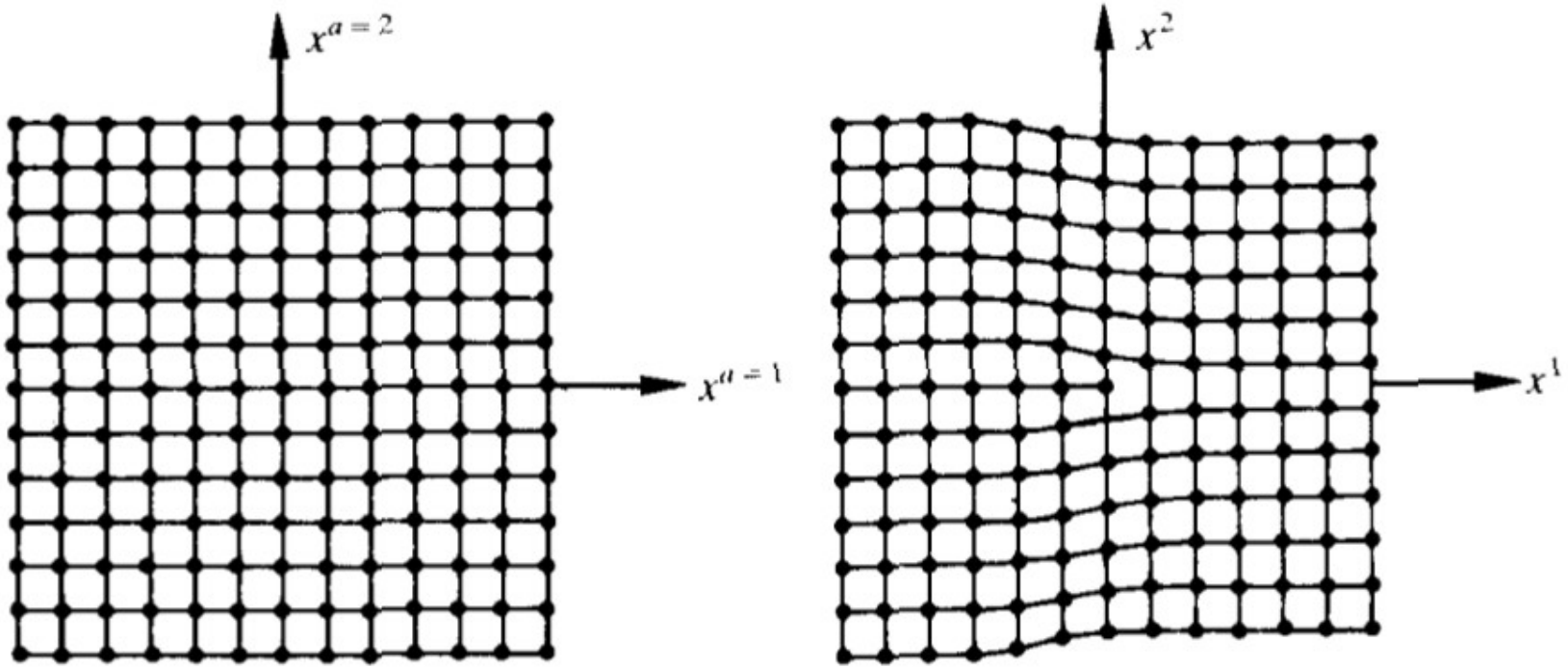
can find coordinates locally...



… but any one choice is singular somewhere

Edge defects

FIG. 2.4. Atomic positions in a crystal with and without a dislocation of the edge type.

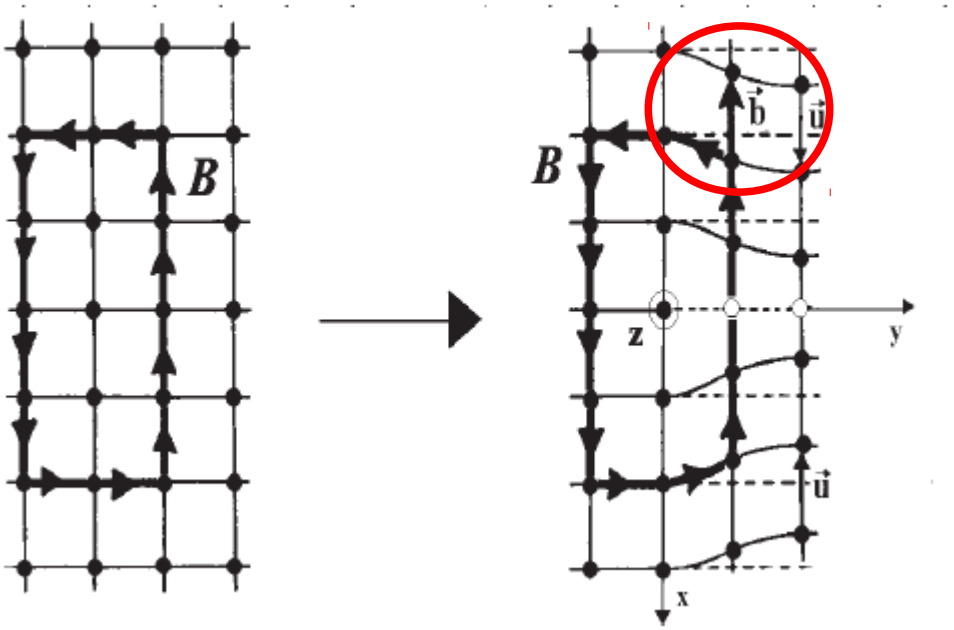


Break in translational invariance

Edge defects – parallel transport

"ideal" crystal

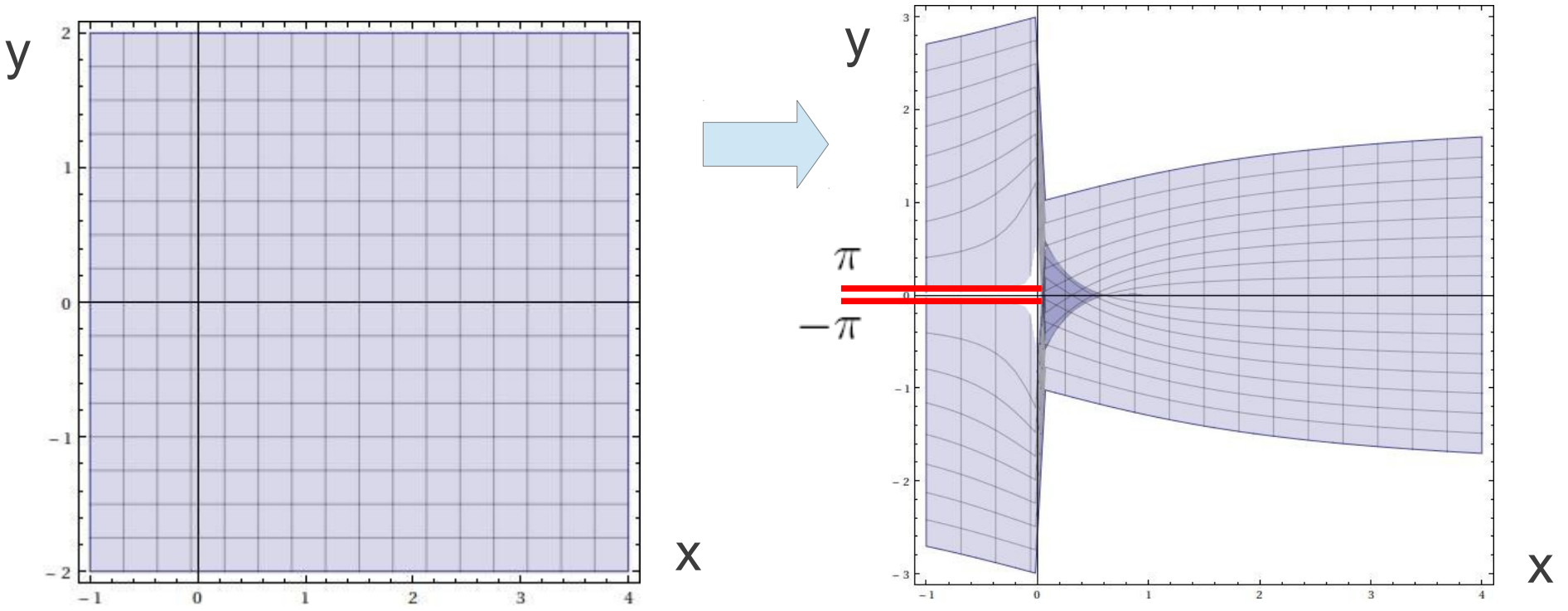
actual crystal



Burgers vector: measures extra translation of vector round circuit

This is a measure of **torsion**

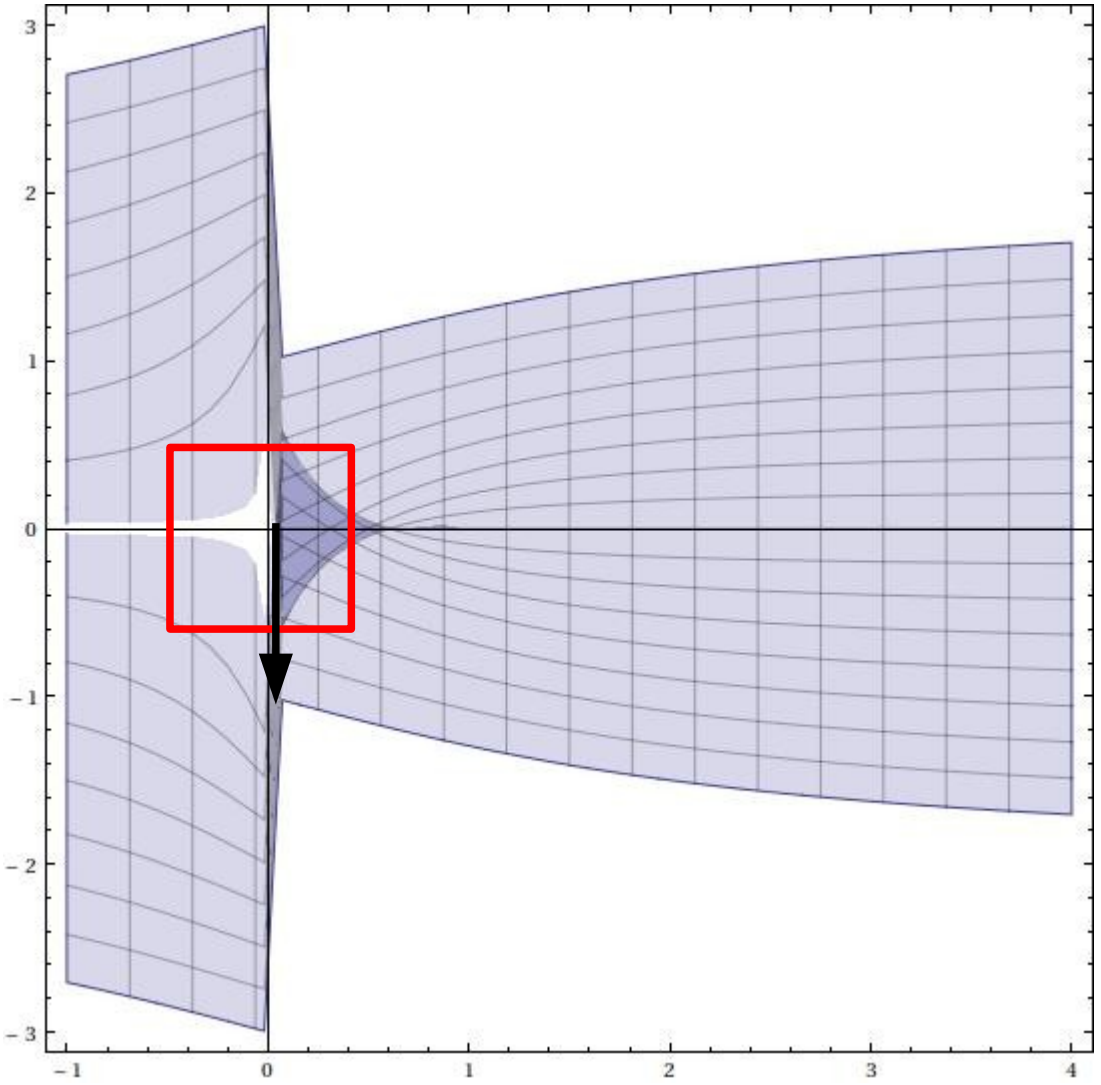
Edge defect example



$$u = x$$

$$v = y - \frac{b}{2\pi} \arctan\left(\frac{y}{x}\right)$$

Edge defect example



$$u = x$$

$$v = y - \frac{b}{2\pi} \arctan\left(\frac{y}{x}\right)$$

$$du = dx$$

$$dv = dy + \frac{b}{2\pi}\left(\frac{y\,dx - x\,dy}{x^2 + y^2}\right)$$

Integrate round red contour C:

$$b^1 = \oint_C du = 0$$

$$b^2 = \oint_C dv = -b$$

Wedge defects

FIG. 2.2. Lattice planes in a crystal in which two types of disclinations of $-90°$ and $-180°$ have been formed by Volterra process.

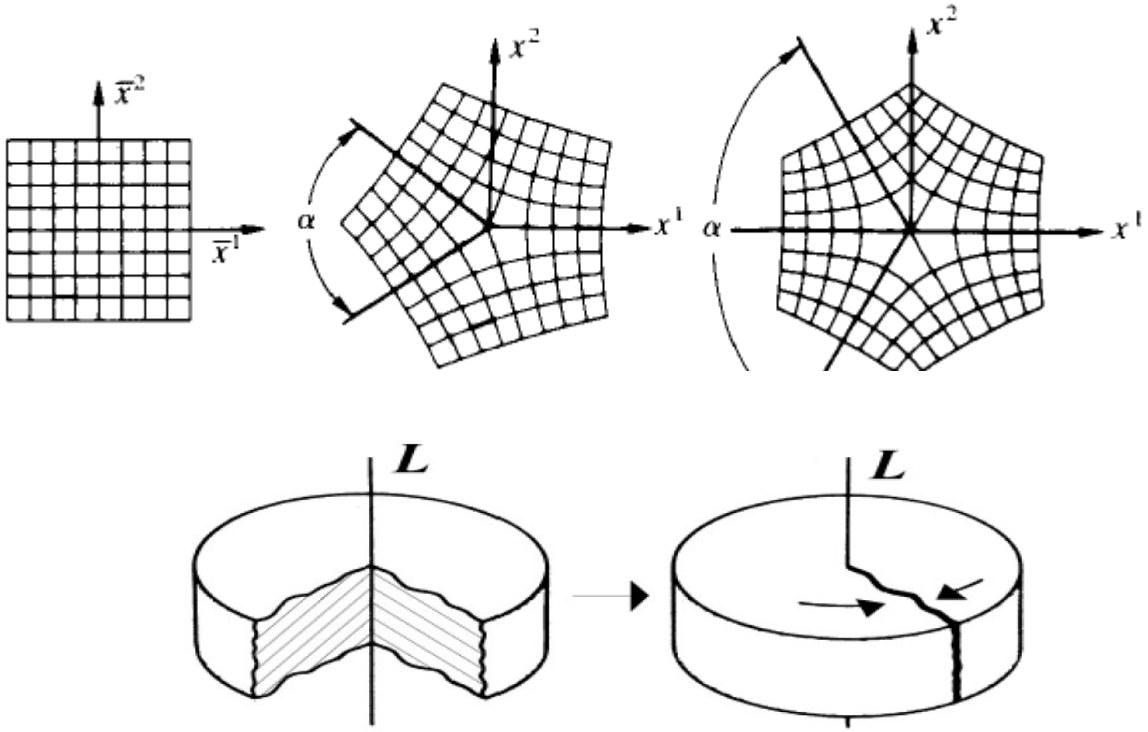
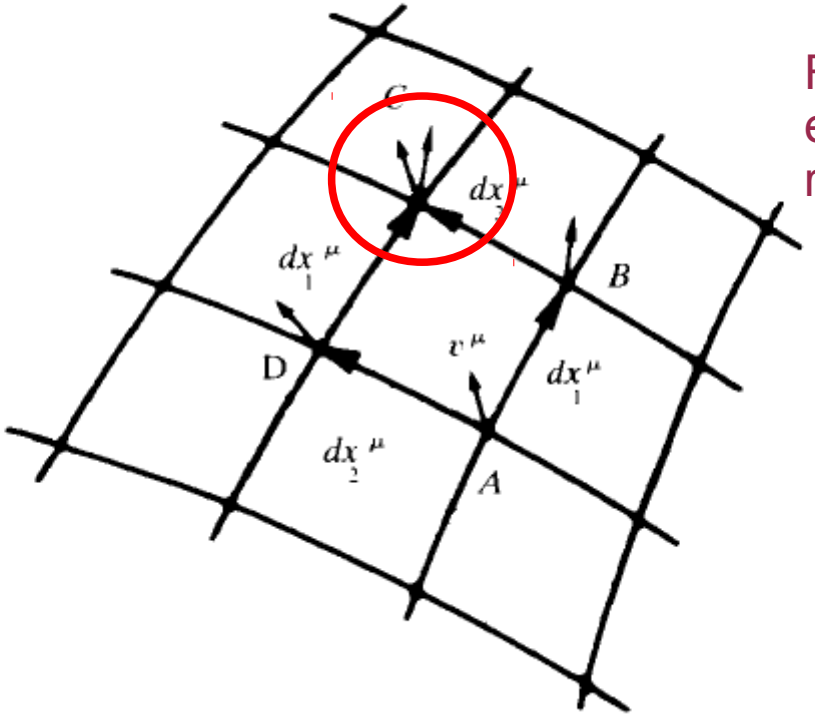


FIG. 6: We obtain a wedge disclination removing a slice from the disk-like piece of medium, and then uniting the opposite faces along the cut. L is the disclination line.

Break in rotational invariance

Wedge defects – parallel transport

FIG. 2.3. Illustration of parallel transport of a vector around a closed circuit ABCD.

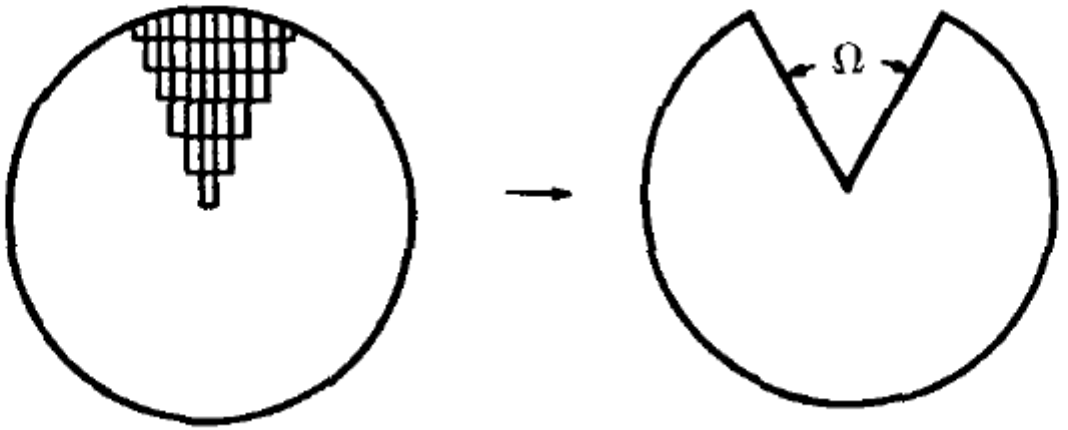


Frank angle: measures extra rotation of vector round circuit

This is a measure of **curvature**

Wedge defects from edge defects

FIG. 2.5. Formation of a disclination from a stack of layers of missing atoms (cf. Fig. 2.2). Equivalently, one may cut out an entire section of the crystal. In a real crystal, the section has to conform with the symmetry angles. In the continuum approximation, the angle Ω is meant to be very small.

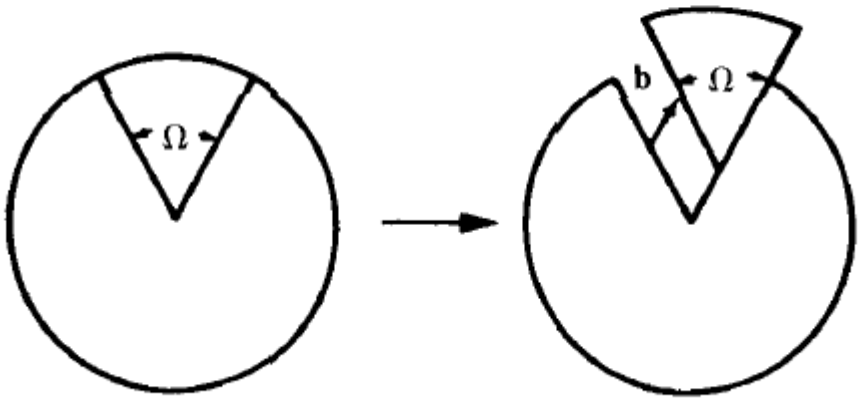


Torsion

Curvature

Edge defects from wedge defects

FIG. 2.13. The generation of a dislocation line from a pair of disclination lines running in opposite directions at a fixed distance b. The Volterra process amounts to cutting out a section and reinserting it, but shifted by the amount **b**.



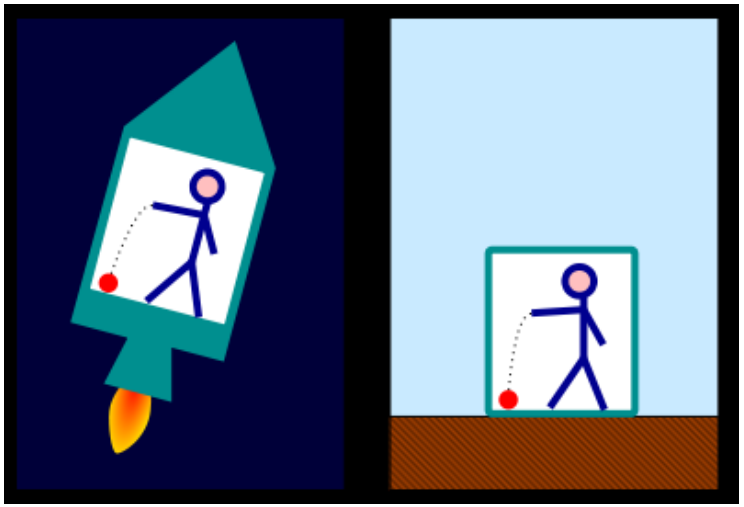
Curvature Torsion

Parallel transport in spacetime

No crystal axes in spacetime: we pick what counts as parallel. The rule for parallel transport is called a connection.

- GR uses the Levi-Civita connection: curvature but no torsion
- Teleparallel (Weitzenböck) connection: torsion but no curvature

Returning to the lift...



Could have a mixture of inertial forces and gravity

GR attributes all local effects to gravity

Teleparallel gravity – somehow! – isolates the inertial bit.

This could be really useful for defining energy in GR...

“More research is needed…”

Things I want to understand better:

- How does the equivalence between dislocations and disclinations work mathematically?

- How does the teleparallel connection separate out inertial forces from gravitation?

Anything else I should think about?

Any questions?

Stupid ones are fine...