First we write down the spin-down torque in component form

$$\boldsymbol{T}_{\mathrm{s}} = \begin{pmatrix} -\frac{\omega_{\mathrm{x}}}{2}\left(\cos(2\chi)+1\right) + \frac{\omega_{\mathrm{z}}}{2}\sin(2\chi) \\ -\omega_{\mathrm{y}} \\ \frac{\omega_{\mathrm{x}}}{2}\sin(2\chi) + \frac{\omega_{\mathrm{z}}}{2}\left(\cos(2\chi)-1\right) \end{pmatrix} \tag{0.0.1}$$

First we will assume that the wobble-angle is small, such that $\omega = \omega_{\mathrm{z}} + \eta$ where $\eta \sim \omega_{\mathrm{x}}, \omega_{\mathrm{y}}$ and is small. Second, we assume that the magnetic dipole is close to the equator such that $\chi \approx \pi/2$. Expanding the trig. functions about this point we have

$$\sin(2\chi) \approx -2\left(\chi - \pi/2\right) + \mathcal{O}\left((\chi-\pi/2)^2\right) \tag{0.0.2}$$

$$\cos(2\chi) \approx -1 + \mathcal{O}\left((\chi-\pi/2)^2\right) \tag{0.0.3}$$

Under these assumptions, the spin-down torque is given by

$$\boldsymbol{T}_{\mathrm{s}} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{\mathrm{z}} \end{pmatrix} + \mathcal{O}(\eta) + \mathcal{O}(\chi - \pi/2) \tag{0.0.4}$$

Considering the three coupled ODEs for the components of $\boldsymbol{\omega}$, unlike in the free-precession case where $\dot{\omega_{\mathrm{z}}} = 0$, we now have:

$$\dot{\omega_{\mathrm{z}}} = -\frac{2R}{3c}\epsilon_{\mathrm{A}}\omega_{\mathrm{z}}^3 \tag{0.0.5}$$

This has a physical solution (where $\omega_{\mathrm{z}} > 0$) given by

$$\omega_{\mathrm{z}}(t) = \left(\frac{4R}{3c}\epsilon_{\mathrm{A}}t + C_0\right)^{-1/2} \tag{0.0.6}$$

Solving for C using the initial condition

$$\omega_{\mathrm{z}}(t) = \left(\frac{4R}{3c}\epsilon_{\mathrm{A}}t + \frac{1}{\omega_{\mathrm{z}}(0)^2}\right)^{-1/2} \tag{0.0.7}$$

Without the EM torque, the solution to Eulers rigid body equations are given by

$$\omega_{\mathrm{x}} = C_2\cos(\epsilon_{\mathrm{I}}C_1 t) - C_3\sin(\epsilon_{\mathrm{I}}C_1 t) \tag{0.0.8}$$

$$\omega_{\mathrm{y}} = C_3\cos(\epsilon_{\mathrm{I}}C_1 t) + C_2\sin(\epsilon_{\mathrm{I}}C_1 t) \tag{0.0.9}$$

$$\omega_{\mathrm{z}} = C_1 \tag{0.0.10}$$

We fix the integration constants by specifying the initial coniditions: as in the simulation results we begin with the spin vector $\boldsymbol{\omega}$ in the $x-z$ plane at an angle a_0 to the z axis such that $\omega_{\mathrm{z}}(0) = \omega_0\cos(a_0)$. Under the assumptions made above then we obtain a set of solutions for the spin-vector in the body frame:

$$\omega_{\mathrm{x}} = \omega_0\sin(a_0)\cos(\epsilon_{\mathrm{I}}\omega_{\mathrm{z}}t) \tag{0.0.11}$$

$$\omega_{\mathrm{y}} = \omega_0\sin(a_0)\sin(\epsilon_{\mathrm{I}}\omega_{\mathrm{z}}t) \tag{0.0.12}$$

$$\omega_{\mathrm{z}} = \left(\frac{4R}{3c}\epsilon_{\mathrm{A}}t + \left(\omega_0\cos(a_0)\right)^{-2}\right)^{-1/2} \tag{0.0.13}$$

0.0.1 Euler angles

Having obtained approximate solutions for the components of the spin-vector in the body frame we now need to use these to find approximate solutions for the Euler angles θ, ϕ, ψ. This can be done by solving the differential equations with these approximate solutions

$$\dot{\psi} = -\omega_{\mathrm{z}}\epsilon_{\mathrm{I}} \tag{0.0.14}$$

$$= -\epsilon_{\mathrm{I}}\left(\frac{4R}{3c}\epsilon_{\mathrm{A}}t + (\omega_0\cos(a_0))^{-2}\right)^{-1/2} \tag{0.0.15}$$

Integrating and using the initial condition $\psi(t=0) = \pi/2$ we get

$$\psi(t) = \frac{-3\epsilon_{\mathrm{I}}c}{2R\epsilon_{\mathrm{A}}}\left(\frac{4R}{3c}\epsilon_{\mathrm{A}}t + (\omega_0\cos(a_0))^{-2}\right)^{-1/2} + \frac{\pi}{2} + \frac{3\epsilon_{\mathrm{I}}c}{2R\epsilon_{\mathrm{A}}\omega_0\cos(a_0)} \tag{0.0.16}$$