As **?** described, the motion of $\mathbf{M}$ in the rotating frame can be understood from by conservation laws. For the time being let us consider, in the rotating frame, a torque free body with moments of inertia I_1, I_2, I_3. If the spin-vector components are given by $M_i = I_i\Omega_i$ then in momentum space, we can write the conservation of energy and angular momentum as

$$\frac{M_1^2}{I_1} + \frac{M_2^2}{I_2} + \frac{M_3^2}{I_3} = 2E \tag{0.0.1}$$

$$M_1^2 + M_2^2 + M_3^2 = M^2 \tag{0.0.2}$$

The conservation of energy describes an ellipsoid with semi-axis $\sqrt{2EI_1}, \sqrt{2EI_2}$ and $\sqrt{2EI_3}$. The conservation of angular momentum describes a sphere of radius M. The intersection of the sphere and ellipse at fixed E and M describe the precession of the angular momentum and hence the spin-vector.

0.1 Torque free biaxial body

For a biaxial body free from torques we can parameterise the principle components of the moment of inertia by

$$I_1 = I_2 = I_0, \qquad I_3 = I_0(1 + \epsilon_{\mathrm{I}}) \tag{0.1.1}$$

For such a system, the Euler rigid body equations have an exact solution with $\Omega_3 = \text{const}$ and the other components are given by

$$\Omega_1 = \left(\Omega^2 - \Omega_3^2\right)^{1/2} \cos\left(\epsilon_{\mathrm{I}}\Omega_3 t\right), \tag{0.1.2}$$

$$\Omega_2 = \left(\Omega^2 - \Omega_3^2\right)^{1/2} \sin\left(\epsilon_{\mathrm{I}}\Omega_3 t\right), \tag{0.1.3}$$

where Ω^2 is also a constant since there is no torque. In the rotating body frame this solution demonstrates that the spin-vector will precess in a circle about the symmetry axis. The circle is precisely the intersection of the ellipsoid and sphere. Since the cone is aligned with the 3 axis, we can define a polar angle θ made by the spin-vector with the 3 axis. This will be constant during a precessional period and can be calculated as follows:

$$\sin\theta = \frac{M_3}{M} \tag{0.1.4}$$

From equation (0.0.1) we can rearrange

$$M_3^2 = (1 + \epsilon_{\mathrm{I}})\left(2EI_0 - M_1^2 - M_2^2\right) \tag{0.1.5}$$

$$= I_0(1 + \epsilon_{\mathrm{I}})\left(2E - I_0\left(\Omega^2 - \Omega_3^2\right)\right) \tag{0.1.6}$$

and similarly

$$M^2 = M_1^2 + M_2^2 + M_3^2 \tag{0.1.7}$$

$$= I_0^2\left(\Omega_1^2 + \Omega_2^2\right) + I_0(1 + \epsilon_{\mathrm{I}})\left(2E - I_0\left(\Omega^2 - \Omega_3^2\right)\right) \tag{0.1.8}$$

$$= I_0^2\left(\Omega^2 - \Omega^3\right) + I_0(1 + \epsilon_{\mathrm{I}})\left(2E - I_0\left(\Omega^2 - \Omega_3^2\right)\right) \tag{0.1.9}$$

$$= I_0\left(2E + \epsilon_{\mathrm{I}}\left(2E - I_0\left(\Omega^2 - \Omega_3^2\right)\right)\right) \tag{0.1.10}$$

Then we can write the polar angle as

$$\sin\theta = \left(\frac{(1 + \epsilon_{\mathrm{I}})\left(2E - I_0\left(\Omega^2 - \Omega_3^2\right)\right)}{2E + \epsilon_{\mathrm{I}}\left(2E - I_0(\Omega^2 - \Omega_3^2)\right)}\right) \tag{0.1.11}$$

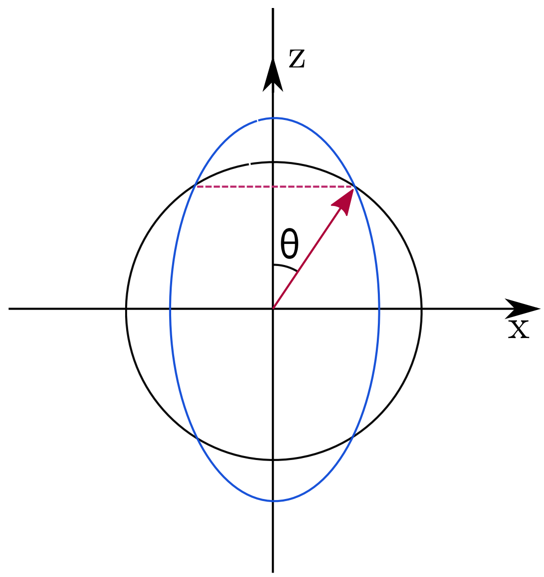

Figure 0.1.1