# Searching for signals in noise

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## I: Motivations

Astrophysics is full of interesting and observed phenomena. The problem we often have is that that our observations are in a low signal to noise regime; this can mean it is difficult to choose between which of several models explains what we have observed.

In this poster we discuss a method for quantitatively assessing how well several models fit some data. The aim being to decide, given the observed data, which astrophysical model is most likely.

## II: Bayesian Data Analysis

We have some data, and several models $\{M_1, \ldots, M_n\}$ which could explain it. An intuitive approach to decide between them be found by writing Bayes rule for the $i^{th}$ model given some data:

$$p(M_i|\text{data}) = p(\text{data}|M_i)\frac{p(M_i)}{p(\text{data})}, \tag{1}$$

where by $P(M_i|\text{data})$ we mean "The probability of the model, given that we have observed some data.

## III: Model comparison

The issue with Bayes rule as written in eqn. (1) is that $p(\text{data})$ is often difficult, or even impossible to define. Instead, we can compare two models, say $M_A$ and $M_B$ by looking at the ratio

$$\frac{p(M_A|\text{data})}{p(M_B|\text{data})} = \frac{p(\text{data}|M_B)}{p(\text{data}|M_A)} \times \frac{p(M_A)}{p(M_B)}. \tag{2}$$

This ratio can directly be interpreted as the 'odds ratio', or how much more we should believe model $A$ over $B$ *given* the data. The last fraction reflects our 'prior' knowledge about the two models. Unless we have a strong reason to believe otherwise this is generally set to unity: $p(M_A) = p(M_B)$.

## IV: Signals in noise

To compute the odds ratio between two models, we need to first define our 'likelihood' function $p(\text{data}|M_i)$: *the probability of the data given the model*. To calculate this, we first calculate the conditional probablity distribution for all the parameters.

For signals in noise we assume that the observed data is the sum of a deterministic signal and a central Gaussian noise process:

$$W(t) = f(t; \vec{\theta}) + n(t; \sigma). \tag{3}$$

Here $f(t;\vec{\theta})$ is the *signal function* with parameters $\vec{\theta}$, while $n(t;\sigma)$ is the noise process with strength $\sigma$. If we subtract the signal model (with the correct parameters) from the observed data we *will* be left with Gaussian noise: $W(t) - f(t;\vec{\theta}) = n(t;\sigma)$. Then the probability of a single observed data point at $t_i$ is given by

$$p(W(t_i)|M_i, \vec{\theta}, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\frac{(W(t_i) - f(t_i)}{2\sigma^2}\right\} \tag{4}$$

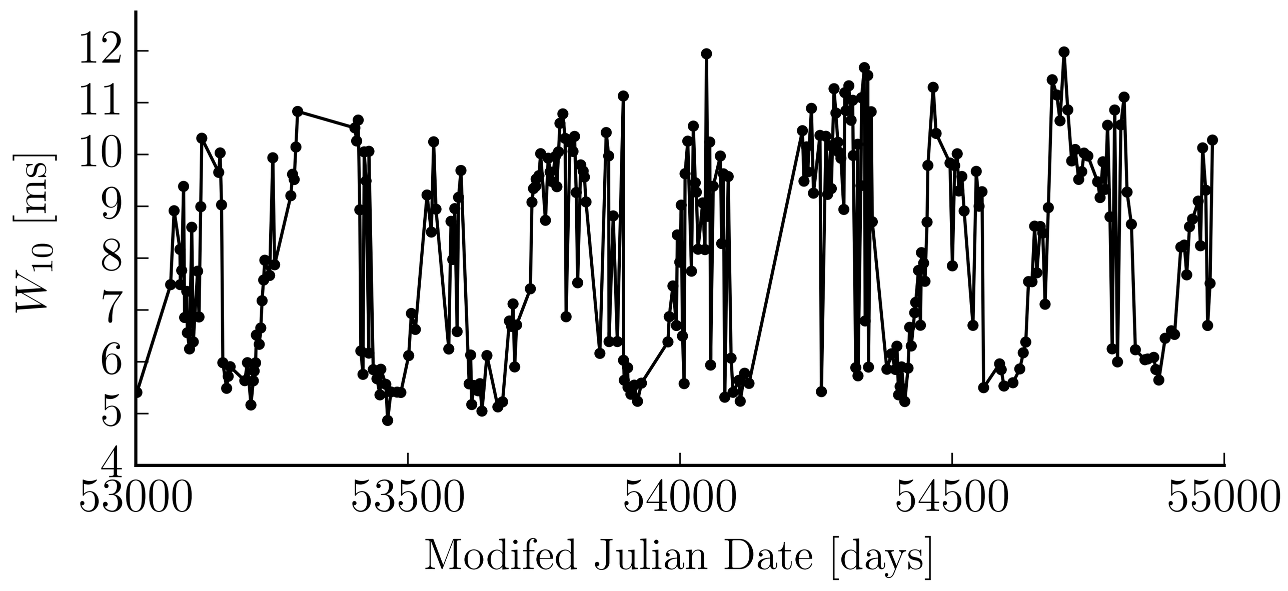
Here the model, and hence the function $f(t)$, are yet to be determined. Given a sequence of $N$ observations (i.e. some data) we can compute the conditional probability distribution of the data:

$$p(\text{data}|M_i, \vec{\theta}, \sigma) = \prod_{i=1}^{N} p(W(t_i)|M_i, \vec{\theta}, \sigma). \tag{5}$$

In box VIII we discuss how to marginalise this and compute the likelihood.

## V: Example

To illustrate how we can apply Bayesian data analysis consider the data shown in the figure below. This is a plot of the measured beam width of pulsar B1828-11 showing a distinct periodic behaviour. This data was originally published in figure 5 of [1] and we are thankful to the authors for allowing us access to this data.




From this data, the authors argue that the beam width $W_{10}$ is periodically *switching* between two distinct values. This is in stark contrast to the standard view that the beam width should be *smoothly* oscillating. The physics which underlies these two ideas are inherently different, so which of these models best explains the data is an important question for neutron star physics.

[1] Lyne, A., Hobbs, G., Kramer, M., Stairs, I., and Stappers, B. (2010). *Switched Magnetospheric Regulation of Pulsar Spin-Down.* Science.

## VI: Model A

The simplest way to model smoothly varying peridic features is with a trig. function. This does not capture all of the physics of alternative models, but nevertheless will test the assumption that $W_{10}$ switches instantaneously between two values. The signal function for this model is

$$f(t; W_0, A, f, \phi_0) = W_0 + A\sin(2\pi f t + \phi_0) \tag{6}$$

## VII: Model B

The model proposed by the authors can be captured by a simple square wave which we give here as a generic function:

$$g(t; W_0, A, f, \phi_0) = W_0 + A\,\text{square}(t\ f, \phi_0) \tag{7}$$

Where the parameters can be directly compared with those in model A. This model can be improved by allowing for a duty cycle as well.

## VIII: Marginalisation and odds ratio

Plugging the model functions (6) and (7) into eqn. (5) we have the conditional likelihood. To compute the likelihood we marginalise over all the parameters:

$$p(\text{data})|M_i) = \iint p(\text{data}|M_i, \vec{\theta}, \sigma) p(\vec{\theta}) p(\sigma) d\vec{\theta} d\sigma \tag{8}$$

Often this integral will be unfeasible analytically, instead we can turn to numerical methods such as MCMC and nested sampling.

## IX: Results

The observed beam width data was tested with both a sinusoidal model and a square-wave model. We computed the marginal likelihoods using MCMC software from *dan.iel.fm/emcee*. This resulted in an odds ratio:

$$\log_{10}\left(\frac{p(M_A|\text{data})}{p(M_B|\text{data})}\right) = -6 \tag{9}$$

This establishes that the square wave model proposed by the authors significantly outperforms a sinusoidal model. We are currently in the process of exploring