Pulse amplitude

Assuming a fixed magnitude of the magnetic dipole, the pulse amplitude will depend on the orientation of the magnetic dipole to the observer and the beam geometry. It will be maximal when pointing directly at the observer and presumably fall off as the angle between the two grows. To model this we take an observers position as Φ_O, Θ_O and then assume the beam geometry follows Gaussian profile with a single conal emmision. This could later be adapated to include a coral emmision.

For such a model of the the beam geometry, the pulse amplitude will vary with the angular separation of the vector from the center of the star to the observer and the magnetic dipole vector. BY considering the intersection of these vectors with the unit sphere, the angular separation can be shown to be

$$\Delta\sigma = \cos^{-1}\left(\sin(\Theta)\sin(\Theta_{\mathrm{obs}}) + \cos(\Theta)\cos(\Theta_{\mathrm{obs}})\cos(\Phi - \Phi_{\mathrm{obs}})\right) \tag{0.0.1}$$

Then the amplitude of the pulse will be given by

$$A(\Theta, \Phi, \Theta_{\mathrm{obs}}, \Phi_{\mathrm{obs}}, \sigma_B) = A_0 \exp\left(-\frac{\Delta\sigma^2(\Theta, \Phi, \Theta_{\mathrm{obs}}, \Phi_{\mathrm{obs}})}{2\sigma_B^2}\right) \tag{0.0.2}$$

where A_0 is some constant amplitude, σ_B is a measure of the angular beam width.

In figure 0.0.1 an illustration is given of variations in amplitude from a torque-free pulsar with some arbitary paramaters. This is intended to demonstrate the fast rotation frequency of individual pulses, along with the longer modulation of the precession.

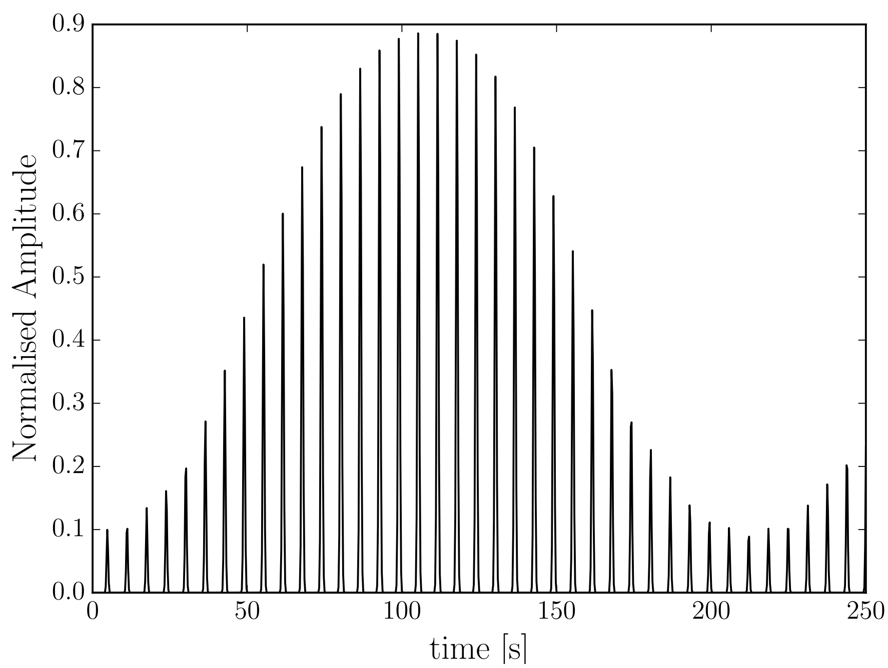

Figure 0.0.1: Amplitude variation using a 2D Gaussian emission.

Pulse width

The amplitude modulations coincide with variations of the pulse width. These have been measured with great accuracy by Lyne et al. (2010) and their correlation with changes in the spindown is key evidence for the two state switching model. We will now extract the pulse width analytically from equation (0.0.2).

To do this we first note that Θ varies on the slow precession timescale, while Φ varies on the rapid spin timescale. We are looking to measure the variations with respect to the slow precession timescale.

The pulse width is measured by the time spent about some fractional amount f of the full pulse amplitude. The condition for when the beam width is greater than this fraction is

$$A(\Phi, \Theta, \Theta_{\text{obs}}, \Phi_{\text{obs}}, \sigma_B) > A_0 \frac{1}{f}. \tag{0.0.3}$$

Substituting equation (0.0.2) and rearranging yeilds

$$\cos(\Phi - \Phi_{\text{obs}}) < \frac{\cos\left(\sqrt{2\sigma_B^2 \ln(f)}\right) - \sin(\Theta)\sin(\Theta)}{\cos(\Theta)\cos(\Theta_{\text{obs}})} \tag{0.0.4}$$

Since we expect Θ to slowly vary, we can over short enough time periods think of it as a constant. Then the whole right hand side of the previous equations is a constant. Lets consider a single rotation with the magnetic dipole starting and ending in the antipodal point to the observers position. Then $\Phi - \Phi_{\text{obs}}$ increase beween $-\pi$ and π during this rotation. The time during which this inequality is true, measures the beam width.

In figure 0.0.2 an illustration is given of this single period showing the constant value on the right hand side of (0.0.4) and the osciallting cosine function. When the cosine is less than this constant this inequality is satisfied. We can calculate the beam width by first measuring the

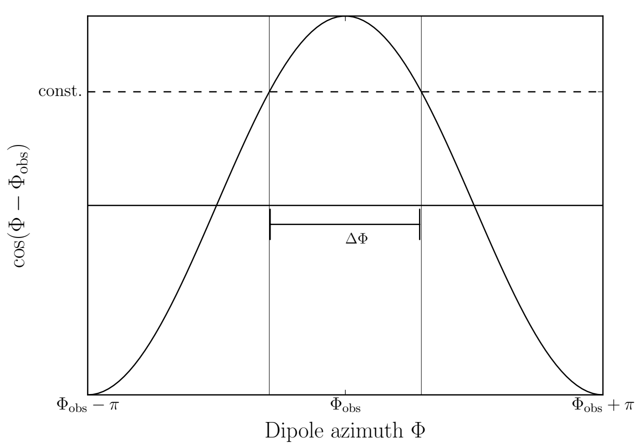

Figure 0.0.2: Illustration of the inequality in equation (0.0.4) the constant value represents the right hand side of this equation. The width $\Delta\Phi$ indicates the angular period during which inequality is satisfied.

angular width $\Delta\Phi$ when the eqeality is not satisified:

$$\Delta\Phi = 2\cos^{-1}\left(\frac{\cos\left(\sqrt{2\sigma_B^2 \ln(f)}\right) - \sin(\Theta)\sin(\Theta)}{\cos(\Theta)\cos(\Theta_{\text{obs}})}\right) \tag{0.0.5}$$

Then the angular fraction at which the inequality *is* satisfied is given by $2\pi - \Delta\Phi$. We can now convert this into a pulse width

$$W_p = T\frac{2\pi - \Delta\Phi}{2\pi} \tag{0.0.6}$$

$$= \frac{1}{\pi\dot{\Phi}}\left(1 - \cos^{-1}\left(\frac{\cos\left(\sqrt{2\sigma_B^2\ln(\frac{100}{p})}\right) - \sin(\Theta)\sin(\Theta)}{\cos(\Theta)\cos(\Theta_{\mathrm{obs}})}\right)\right) \tag{0.0.7}$$

where T is the spin period which we have then written in terms of the spin frequency and p is the percentage of beam width. We should state our notation of W_p is intended to match that of Lyne et al. (2010). This illustrates that the beam width will vary with both the changes in spin-frequency, and with Θ.

In figure 0.0.3 we plot the beam width for some arbitary parameters.

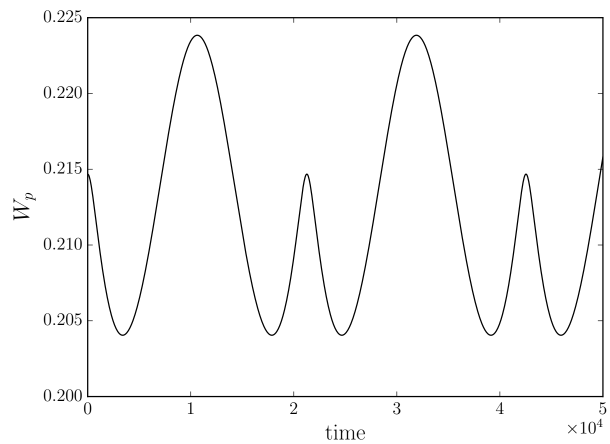

Figure 0.0.3

Bibliography

Lyne, A., Hobbs, G., Kramer, M., Stairs, I., and Stappers, B. (2010). Switched Magnetospheric Regulation of Pulsar Spin-Down. *Science*, 329:408–.