0.1 Timing residuals

The principle observational quantifty reported on for a pulsar is the timing residual. This is measured by the difference between the TOA of a pulse and a timing model of the pulsar. It is the quasi-periodic structure in the timing residual which we term timing-noise and hence we naturally want to be able to calculate the residual from our analytic models.

Equation (**??**) gives the exact phase evolution of the star, we will label this as Φ_{exact}. We then take a Taylor expansion to the phase about some fixed time t_0

$$\Phi(t; t_0, \phi, f, \dot{f}, \ddot{f}) = \phi + 2\pi \left(f(t - t_0) + \frac{\dot{f}}{2!}(t - t_0)^2 + \frac{\ddot{f}}{3!}(t - t_0)^3 \right), \tag{0.1.1}$$

and use a least-squares fitting method to minimise the squared error between the Taylor expansion and Φ_{exact}. The resulting coefficients $\{\phi_0, f_0, \dot{f}_0, \ddot{f}_0\}$ are the quantities best describing the NS under a power law spindown; in pulsar astronomy these are refered to as the timing model. We will refer to the best-fit phase as described by these coeffients as

$$\Phi_{\mathrm{fit}}(t) = \Phi(t; t_0, \phi_0, f_0, \dot{f}_0, \ddot{f}_0) \tag{0.1.2}$$

The phase-residual is then defined as the difference between the exact phase and the fitted phase

$$\Delta\Phi(t) = \Phi_{\mathrm{exact}}(t) - \Phi_{\mathrm{fit}}(t). \tag{0.1.3}$$

It is worth noting that a phase-residual depends on the fit to the entire length of date provided in Φ_{exact}. The phase residual can be rescaled to give the timing residual by calculating the residual as a fraction of a cycle then multiplying by the period

$$\Delta T = \frac{\Delta\Phi(t)}{2\pi} P. \tag{0.1.4}$$

Over a typical observation periods it is possible for the period to fractionally change due to the spindown. For this work we will report only on phase-residuals although to make contact with observational results we will require this rescaling.

The effect of free-precession and the inclusion of a spin-down torque was considered analytically by **?**. This provides a set of useful results with which to verify our simulations and the subsequent processing required to calculate the timing residual.

Before continuing we need to define the wobble angle $\tilde{\theta}$, this is the angle between the spinvector and the axis about which it precesses. Without the EM torque, this is the body-frame $\hat{\boldsymbol{z}}$ axis defined as one of the principle axis of the moment of inertia. In such a situation the wobble angle is simply given by

$$\tilde{\theta} = \theta, \tag{0.1.5}$$

as used in **?**. Including the EM torque introduces two effects. The first and the largest is the transformation induced by the anomalous torque which causes the spin-vector to precess about the principle axis of the effective body-frame. This has already been discussed in chapter **??** and results in a wobble angle

$$\tilde{\theta} = \theta - \beta \tag{0.1.6}$$

where β is the rotation from the body-frame to the effective body-frame. The second effect is that, even without the anomalous torque the spin-down torque rotates the axis about which the spin-vector precesses. This is a smaller effect and can be shown to produce a wobble angle of the order P/τ_s. This will always be significantly smaller than the other angles and so can be neglected for now.

0.1.1 Effect of free precession on the phase residual: geometric effect

? analysed the geometric effect that precession will have on the timing residual. This was done by considering the motion of the magnetic dipole in the inertial frame as the superposition of motions due to the fast rotation period and the slow precession. The results must be separated into two cases when $\theta > \chi$ and $\theta < \chi$. Of these two the authors argue that 'the wobble angle (θ) of rapidly rotating stars are to small values by the finite crustal breaking strain'. Therefore, the second case $\theta < \chi$ holds greater physical relevance and so we focus on this region of parameter space. The phase residual was found to be given by

$$\Delta\Phi^{49}(t) = -\tilde{\theta}\frac{\cos\chi}{\sin\chi}\cos(\dot{\psi}t), \tag{0.1.7}$$

where the superscript here refers to the equation number from **?**. For a freely precession star $\dot{\psi} = \epsilon_{\mathrm{I}} f$ is the constant free precession frequency. Therefore, the magnitude of variations is given by $|Delta\Phi^{49}| = \tilde{\theta}\cot(\chi)$.

Equation (0.1.7) is calculated in the absence of any EM torque. Nevertheless, it is still appropriate when such a torque is applied provided that the geometric effect is stronger that any others(these are discussed in the next few sections). As such we begin by simulating a NS with the properties listed in table 0.1.1. Unphysical values of the rotation frequency and magnetic field have been used to aid the computational speed. The resutlting phase residual, in cycles, is given in figure **??**.

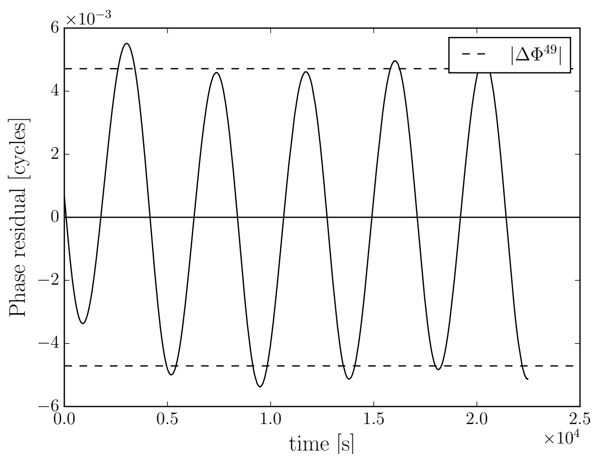

Figure 0.1.1: The phase residual in cycles for a simulated NS with the properties described in table 0.1.1. This is used to illustrate the agreement with the magnitude of variations from equation (0.1.7) taken from **?**.

Simulation parameters		
ω_0	$=$	155.0 rad/s
B_0	$=$	1.581×10^{14} G
χ	$=$	49.99°
a_0	$=$	2.00°
$\tilde{\theta}$	$=$	2.02°

Table 0.1.1

0.1.2 Effect of free precession on the phase residual: effect of the electromagnetic torque

Considering a vacuum point-dipole spin-down torque (like the one introduced in **??**) **?** found that the EM torque can amplify the geometric effect of equation (0.1.7). The magnitude of variation due to EM torque is given by

$$|\Delta\Phi^{63}| = \frac{1}{\pi}\left(\frac{\tau_P}{P}\right)\left(\frac{\tau_P}{\tau_S}\right)|\Delta\Phi^{49}| \tag{0.1.8}$$

The two ratios of time-scales define an 'amplification factor'. This effect is important for young pulsars with short periods.

We simulate such a star using the properties in table 0.1.2 noting that the amplification factor is ~ 3 when including the factor of π. The resulting phase residual is plotted in figure 0.1.2 along with the magnitude of variations due to free precession along and the amplification due to the EM torque.

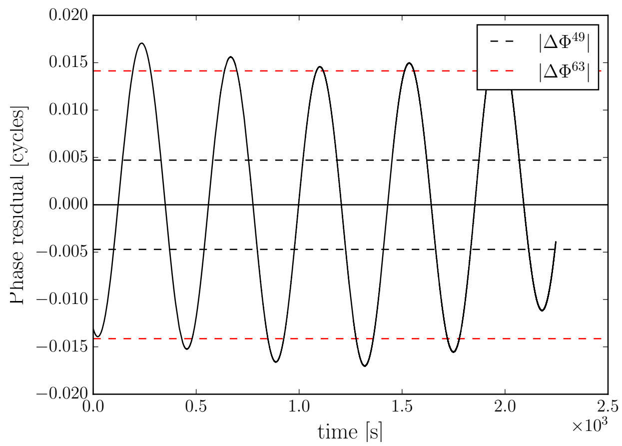

Figure 0.1.2: The phase residual in cycles for a simulated NS with the properties described in table 0.1.2. This is used to illustrate the agreement with the magnitude of variations from equation (0.1.8) taken from **?**.

Simulation parameters		
ω_0	$=$	1550.0 rad/s
B_0	$=$	1.581×10^{14} G
χ	$=$	$70.00°$
a_0	$=$	$2.00°$
$\tilde{\theta}$	$=$	$2.01°$

Table 0.1.2

0.1.3 Effect of free precession on phase residuals: oblique rotator with electromagnetic torque

0.1.4 Variations in the phase residual due to magnetic dipole inclination