

The effect of timing noise on targeted and narrow-band coherent searches for continuous gravitational waves

G. Ashton,^{1,*} D.I. Jones,¹ and R. Prix²

¹*School of Mathematics, University of Southampton, Southampton SO17 1BJ*

²*Max Planck Institut für Gravitationsphysik (Albert Einstein Institut), 30161 Hannover, Germany*

(Dated: October 3, 2014)

Most continuous gravitational-wave searches use Taylor expansions in the phase to model the spin-down of neutron stars. Studies of pulsars demonstrate that their electromagnetic (EM) emissions suffer from *timing noise*, small deviations in the phase from Taylor expansion models. How the mechanism producing EM emission is related to any continuous gravitational-wave (CW) emission is unknown; if they either interact or are locked in phase then the CW will also experience timing noise. Any disparity between the signal and the search template used in matched filtering methods will result in a loss of signal-to-noise ratio (SNR), referred to as “mismatch”. In this work we assume the CW suffers a similar level of timing noise to its EM counterpart. We inject and recover fake CW signals, which include timing noise generated from observational data on the Crab pulsar. Measuring the mismatch over durations of order ~ 10 months, the effect is for the most part found to be small. This suggests recent so-called ‘narrow-band’ searches which placed upper limits on the signals from the Crab and Vela pulsars will not be significantly affected. At a fixed observation time, we find the mismatch depends upon the observation epoch. Considering the averaged mismatch as a function of observation time, we find that it increases as a power law with time, and so may become relevant in long baseline searches.

I. INTRODUCTION

Rotating neutron stars capable of supporting non-axisymmetric mass distributions will emit continuous gravitational waves (CWs) due to their time-varying quadrupole moments. The emitted signals will persist longer than typical CW searches, but are weak in amplitude making them difficult to detect in the noise of the detector. To find a signal CW searches use matched filtering techniques such as the \mathcal{F} -statistic [1] which compare the output of the detector with a template. These techniques are powerful provided that the signal and template remain coherent for the duration of the observation. If the signal can be perfectly matched by a template then the signal to noise ratio, used to quantify the detection likelihood, scales as $\rho^2 \propto T_{\text{obs}}$ (e.g. see [2]). This suggests searching over longer observations increases the chances of making a detection.

CW templates model the monotonic spin-down of the source due to the electromagnetic (EM) and gravitational torque; this is done by Taylor expanding the phase:

$$\phi(t) = \phi_0 + 2\pi \left(f_0(t - t_0) + \frac{\dot{f}_0}{2!}(t - t_0)^2 \right) + \dots, \quad (1)$$

where t_0 is the reference time at which the pulsar frequency, and spin-down parameters $[\phi_0, f_0, \dot{f}_0, \dots]$ are defined. Pulsar astronomers fit this model to observed time of arrivals (TOAs). If the best fit model is accurate enough to track the pulsar to within a single rotation the resulting timing solution is described as *phase-connected*.

Often such solutions are capable of tracking the pulsar over durations greater than a year. For gravitational-wave searches, this level of accuracy motivates the use of the same Taylor expansion phase models to account for the spin-down. Pulsar observers measure the frequency f_0^{EM} and higher order coefficients describing the rotation the pulsar itself. In CW searches we primarily look for the emission from non-axisymmetric neutron stars at $f_0^{CW} = 2f_0^{EM}$ [3]. For this work all frequencies and spin-downs refer to the CW emission.

While Taylor expansion models are on average reliable enough to track the spin-down, pulsars do show deviations known as *timing noise*. This is often represented by structure in the timing residual, which is the difference between the best fit Taylor expansion, typically up to second order in spindown \dot{f}_0 , and the observed phase. Timing noise refers specifically to deviations from Taylor expansions that are intrinsic to the pulsar and not to systematic errors such as dispersion in the interstellar medium. Hobbs et al. [4] conducted a wide ranging study on timing noise across the pulsar population. They concluded, amongst other things, that timing noise is ubiquitous and inversely correlated to the age of the pulsar. There already exists measures used to quantify the strength of timing noise such as the Δ_8 value introduced by Arzoumanian et al. [5] and the generalisation of the Allan invariance [6]. These do not convert directly into the effect that timing noise may have on CW searches. To quantify this, we need to measure the *mismatch* due to timing noise. This is closely related to the loss of signal to noise ratio due to the imperfect matching between the template and signal (which we define explicitly in section III).

Although a variety of models exist to interpret timing noise, there is currently no consensus on a single mech-

*E-mail: G.Ashton@soton.ac.uk

anism. However, for the issue of timing noise and CW searches, we only need to consider the relation between the components of the neutron star which produce the EM and CW signals. This was investigated by Jones [7] who identified three possible scenarios. First, the two signals are strongly coupled: the same timing noise will be observed in both. Second, the two signals are loosely coupled: a similar, but different level of timing noise will be observed in both. Third, timing noise exists only in the EM signal, there is no corresponding variations in the CW signal. Of course these are really three cases from a full spectrum of possibilities which could also include the CW being significantly more noisy.

The significance of timing noise will vary between different types of CW searches; these can be divided into targeted, narrow-band, directed, and all-sky searches. *Targeted* searches involve a single known pulsar where an estimate of the spin parameters has been obtained from the EM signal. If we assume that the EM and CW signals are strongly coupled, then we can use a *single-template* targeted search. Under this assumption, when the level of timing noise in the EM signal is small, then a single Taylor expansion is sufficient. If instead the level of timing noise is large, then the EM data can be used to account for it; this is done by applying an adapted matched-filtering phase-model that closely follows the observed EM phase model [8]. If instead we assume such that the EM and CW signals are loosely coupled, then we should perform a *narrow-band* search in a small area of parameter space. These narrow-band searches aim to allow for small frequency offsets between the EM and CW signals, such as could be caused by free precession, or a finite coupling time between the two components of the neutron star [9]. *Directed* searches look for non-pulsing neutron stars predicted by other means such as at the centre of the super-nova remnant Cassiopeia A. An *all-sky* search involves searching over the entire sky for unknown pulsars. For both directed and all-sky searches the lack of EM data necessitates wide bands in the frequency and its derivatives. For fully coherent matched filtering methods these searches can rapidly become computationally prohibitive. To circumvent this, semi-coherent search techniques are used that incoherently combine short fully-coherent sections of data [10]; these will be less sensitive to timing noise. Nevertheless, semi-coherent searches ultimately need to be followed up by targeted fully coherent searches, for which timing noise may be an issue.

For the properties of the CW signal, the most general case is that it will exhibit some timing noise, but it could be different to the timing noise observed in the EM signal. Until a detection is made, we can only make assumptions about how the two are correlated. To probe these assumptions, we will define two special cases corresponding to different sorts of errors in a CW search:

- **Special Case 1:** Timing noise, exactly like that in the EM signal, exists in the CW signal but is not included in the template. This will result in a loss of signal to noise ratio for searches which

assumed that timing noise was negligible. The error potentially affects the narrow-band, directed, and all-sky searches since the level of timing noise is unknown. The single template targeted searches will not be effected since they either check that the level of timing noise is negligible, or correct for it using an adaptive phase model.

- **Special Case 2:** Timing noise is included in the template but does not exist in the signal. This will result in a loss of signal to noise ratio for single-template targeted searches that account for timing noise using an adapted phase model (for example Abbott et al. [9]). Instead, these searches will now erroneously introduce timing noise into the template while the signal will be a smooth Taylor expansion.

In this work we will mimic narrow-band and single-template searches to directly simulate special case 1. Specifically, we will inject a fake CW signal exactly matched to the Crab's rotation and recover it using templates based on a single global Taylor series. This tests the scenarios in which the timing noise in the CW signal is either exactly coupled to the EM signal, or they are at least similar. However, this also quantifies special case 2 since the signal and template are interchangeable in matched filtering methods. That is, timing noise in the signal but not in the template is equivalent to timing noise in the template but not the signal.

We have chosen to study the Crab pulsar because it produces exceptional levels of timing noise. Jones [7] estimates that of all the pulsars targeted by CW searches this is one of the few for which timing noise may be an issue. Because it is so noisy, the Crab has been regularly observed by the Jodrell bank observatory. These observations are made available through the Crab ephemeris, see section II for details. Also, the Crab is a good candidate for CW emission and so has been the subject of several targeted searches. A single-template search was performed on data collected during the LIGO S5 science run [9]. This search used the Crab ephemeris and an adapted phase model to account for timing noise. In addition to this single-template search, a narrow-band search for signals from the Crab was also performed by Abbott et al. [9] on the S5 data. Another narrow-band search for the Crab is being carried out using data from the VIRGO VSR4 science run along with a search for the Vela pulsar [11].

The structure of this paper is as follows. In section II we describe the observational data available from the Crab ephemeris and discuss its relation to CW searches. In section III we describe the signal injection and recovery method. Results from this method are presented in section IV: we begin by considering the effect timing noise has on narrow-band searches, then we consider the mismatch on stretches of data for which narrow-band searches have been performed; we further investigate how the mismatch depends upon epoch; and finally examine

how the mismatch depends on the duration of observation. We summarise our results in section V.

II. TIMING NOISE AS DESCRIBED BY THE CRAB EPHEMERIS

The monthly Crab ephemeris [12] provides the phase evolution of the EM signal between 1982 and the present and can be found at <http://www.jb.man.ac.uk/pulsar/crab.html>. Each monthly update consists of the frequency and spin-down coefficients along with a reference time coinciding with the TOA of a pulse at the solar system barycentre. The coefficients are calculated by least-squares fitting of a Taylor expansion to the measured TOAs. The reference time for each month is chosen as the TOA of the pulse closest to the mid-point; this is done to minimise the average phase error of the local Taylor expansion. The period of a month is short enough such that these coefficients and equation (1) track the rotational phase during the month.

The Crab ephemeris gives a distinct picture of the variations due to timing noise superimposed on the monotonic spin-down. To illustrate how this manifests itself, figure 1 depicts the frequency evolution in two adjacent months. Notice that a discontinuity occurs at the interface between months. Such discontinuities will occur in the spin-down, frequency, and phase; timing noise can then be described by the magnitude of these jumps. From the Crab ephemeris it can be shown that the distribution of jumps in phase, frequency and spin-down appear to follow standard normal distributions. This is consistent with timing noise models consisting of a large number of small unresolved events accumulating over a month (e.g. the models considered by Cordes and Greenstein [13]).

Timing noise is usually depicted by structure in the phase residuals calculated by removing the best fit Taylor expansion to the phase from the real phase evolution. A best fit Taylor expansion consists of a single set of coefficients f_0 , \dot{f}_0 , and \ddot{f}_0 valid over the *entire* observation period. To make this distinct from the *local* Taylor expansions describing the evolution in each month this will be referred to as the *global template*. In figure 1 we see that if the discontinuity is non-zero, then it is impossible for any global Taylor expansion template to exactly match the local templates in both months. The phase residual, and hence timing noise, results from the inability to match a single global template to all the local ones.

In this work we aim to quantify the significance of timing noise in CW searches by generating signals from the Crab ephemeris. This is an empirical description of timing noise and so we make no assumptions on the underlying astrophysical model.

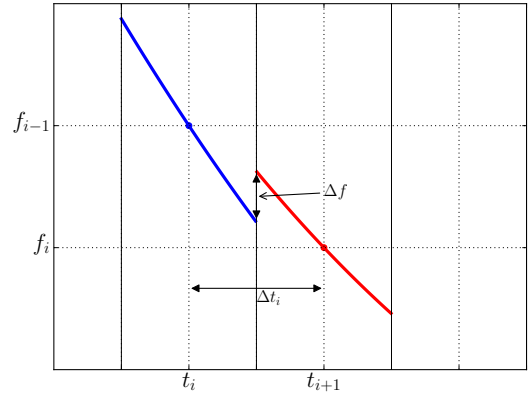


FIG. 1: Illustration of the jumps between 'local' per-month templates in frequency space, defining the frequency jump Δf .

III. METHOD

We now describe the method to quantify the effect of timing noise on CW searches. As much as possible we have attempted to mimic the real process of CW detection. The results can be interpreted as measuring the consequence of special case 1 on narrow-band and single-template searches; that is we assume the CW signal has a similar level of timing noise as the EM signal and search using global Taylor expansion templates. For this study, a single-template search refers to a single Taylor expansion template and not the adapted phase model proposed by Pitkin and Woan [8].

We begin by generating a CW signal emulating timing noise using the Crab ephemeris:

1. From the ephemeris select a period of data consisting of the reference times t_i , frequency f_i , and spin-down \dot{f}_i for each month i .
2. Generate the phase as a function of time from the data and then fit a global Taylor expansion up to \ddot{f} for the whole observation time. The fit results in set of interpolated coefficients $[f_0, \dot{f}_0, \ddot{f}_0]$ at a global reference time halfway through the data. These coefficients are used to centre the narrow-band search parameters.
3. We supplement the local monthly data $\{t_i, f_i, \dot{f}_i\}$ with the fixed value of \ddot{f}_0 calculated in the previous step. The phase of the CW signal is always zero at each monthly reference time t_i of the ephemeris, which by construction coincides with a pulse arrival time.

In this process we have assumed that a fixed value of \ddot{f}_0 is sufficient. This can be justified by considering the next term in the Taylor expansion (1) and typical values of $\ddot{f} \sim 10^{-30}$ Hz/s³. Over typical search durations ~ 1 yr

this term contributes less than a radian to the phase, and it can therefore be safely neglected.

We use the **LALSuite** [14] gravitational-wave analysis routines to generate a fake CW signal; for these experiments we work without any simulated detector noise. The standard tool to generate fake CW signals uses single Taylor expansion models. Therefore, to include timing noise in the signal we do the following.

4. We inject each month-long segment of data from the Crab ephemeris as a rectangular transient window lasting for only the duration of that month. Ensuring that all windows are adjacent, so that the signal is continuous, this method creates fake signals with timing noise corresponding to the monthly ephemeris.

Once we have produced data, we then use **LALSuite** tools to recover the signal using the \mathcal{F} -statistic [1]. This is a matched filtering method in which the output of the detector is compared to a signal template (see Prix [2] for more details).

Two types of searches are performed: a single template search at the interpolated coefficients $[f_0, \dot{f}_0, \ddot{f}_0]$ and a narrow-band search in f and \dot{f} centred on the interpolated coefficients. The narrow-band consists of a grid of points in f and \dot{f} ; we keep \ddot{f}_0 fixed for this experiment. The grid spacing is parameterised by m , the one-dimensional maximal mismatch between two adjacent Taylor expansion templates. From Aasi et al. [15] the corresponding grid spacing is given by

$$df = \frac{\sqrt{12m}}{\pi T_{\text{obs}}}, \quad d\dot{f} = \frac{\sqrt{720m}}{\pi T_{\text{obs}}^2}, \quad (2)$$

For the single-template and at each grid point in the narrow-band search we measure the squared SNR value ρ^2 . In order to quantify the relative loss compared to the perfectly phase-matched squared SNR ρ_s^2 , we define the mismatch in the usual way (e.g. see Prix [16]) as

$$\mu = \frac{\rho_s^2 - \rho^2}{\rho_s^2}, \quad (3)$$

It is well known (e.g. see Prix [2]) that the SNR for a perfectly phase-matched signal is independent of the signal phase evolution. Therefore, in the absence of timing noise the measured value of ρ^2 can reach the maximum value of ρ_s^2 , and the mismatch therefore vanishes in that template. In the presence of timing noise, even the best-matching template will suffer some mismatch, and this effect will increase with the level of timing noise.

In the single-template search, we measure a single mismatch value. The single-template search can also be interpreted to quantify the error made in special case 2, when the template is adapted to account for EM timing noise but none exists in the CW signal. We can think of the narrow-band search as repeating the single-template search over a grid of points; this allows us two degrees of

freedom, corresponding to the frequency and spin-down parameters, over which to minimise the mismatch. The grid point with the minimum mismatch, which we denote by μ_{min} , is the best candidate and will be used to quantify the success of the search. Because the narrow-band can minimise the mismatch, μ_{min} must always be equal or smaller than the mismatch in the single-template search.

IV. RESULTS

A. The effect of timing noise on narrow-band searches

We begin by describing how timing noise degrades a narrow-band search. This is done by comparing the result for a signal containing no timing noise with a signal generated from the Crab ephemeris between MJD 45150 and 56668. This period holds no special significance and is used simply to demonstrate the essential features of a signal containing timing noise.

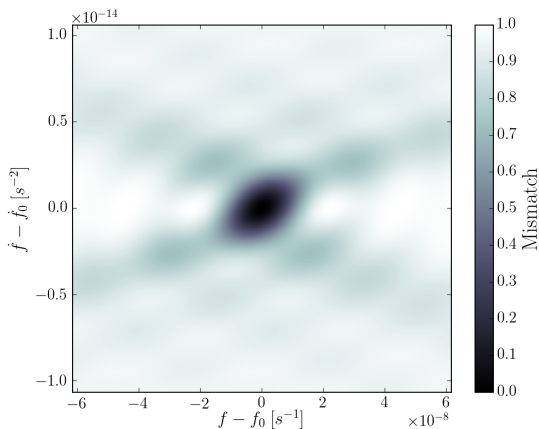
In figure 2 we show the mismatch as a function of parameter space offset for (a) a signal without timing noise, and (b) a signal containing timing noise. The signal without timing noise is injected at the interpolated coefficients $[f_0, \dot{f}_0, \ddot{f}_0]$. Therefore, we find the minimum mismatch with $\mu_{\text{min}} = 0$ at exactly the centre of the grid and the iso-mismatch contours in the local neighbourhood around the origin are well described by ellipses (e.g. see Prix [16]). For the signal with timing noise (b) we notice two distinctive effects: the minimum achievable mismatch μ_{min} is non-zero, and the iso-mismatch contours around the best-match template are more irregular and less well described by ellipses. In the following we will quantify the effect of timing noise by considering only the location and value of the minimum mismatch grid point in the narrow-band search.

B. Results relevant to recent narrow-band searches

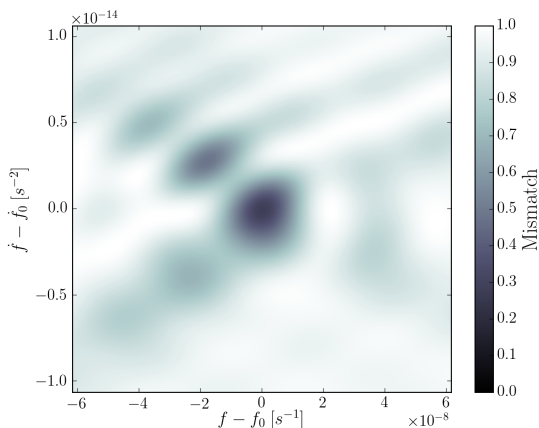
First we consider two particular periods of the Crab ephemeris corresponding to recent narrow-band searches for the Crab: the LIGO S5 period [9] and the VIRGO VSR4 period [11]. The mismatch in the single-template and the minimum mismatch for the narrow-band searches during both periods are listed in table I. For these periods timing noise is found to produce a mismatch of $\approx 1\%$. As expected, the narrow-band mismatch is smaller than the single-template search. The fractional difference between the two searches is relatively small.

Provided that the timing noise observed in the CW signal is at the same level (or less) as that observed in the EM signal, this result signifies that the recent LIGO and VIRGO narrow-band searches would not suffer significantly from the effects of timing noise.

In addition to producing a mismatch, timing noise may result in the best candidate being found at some distance



(a) Signal without timing noise



(b) Signal with timing noise

FIG. 2: In figure (a) we show the mismatch as a function of parameter space for a signal without timing noise. The injected signal has parameters $[f_0, \dot{f}_0, \ddot{f}_0]$, as a result the mismatch has a minimum at this point.

This can be compared with figure (b) showing the mismatch from a signal including timing noise. The signal is generated from the Crab ephemeris between MJD 45150 and 45668.

from the centre of the narrow-band search. However, we find that the distance from the centre of the grid is small when compared to the grid spacing used in actual narrow-band searches such as the S5 and VSR4. For the S5 period narrow-band search, we find that the minimum mismatch was a fraction ~ 0.01 of the grid spacing used in the Abbott et al. [9] search. At the resolutions used in real narrow-band searches, the effects of timing noise on the location of the minimum mismatch will not be evident.

Figure 3 shows the convergence of the measured best mismatch μ_{\min} for the narrow-band search over the S5 period with the value of m . This demonstrates that the non-zero values of μ_{\min} given in table I are not the re-

	Dates	Single template	Narrow band
	MJD	μ	μ_{\min}
S5	53673 - 53977	0.00968	0.00933
VSR4	55681 - 55839	0.00659	0.00584

TABLE I: Measurements of the mismatch during the S5 and VSR4 narrow-band search periods.

sult of grid coarseness. For signals without timing noise, the measured best mismatch μ_{\min} will have a minimum of $\sim m$ when the putative signal is located halfway between grid points. In the limit of $m \rightarrow 0$ we then expect the measured mismatch to tend to zero. Instead, for a signal with timing noise we observe a plateau after some initial reduction. This indicates that the grid is now *fully* resolving the variations due to timing noise.

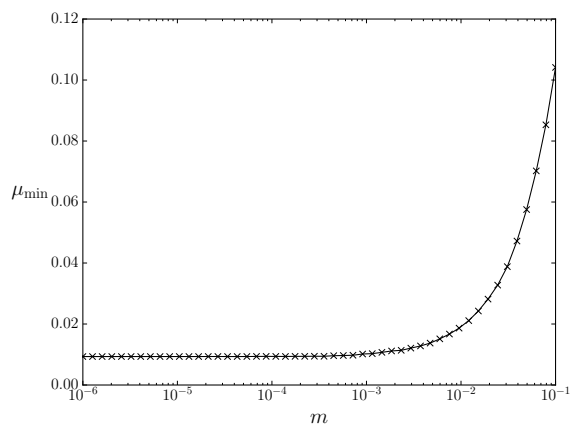


FIG. 3: Measured best mismatch μ_{\min} as a function of grid spacing m , for the Crab pulsar over the S5 period. This demonstrates that at infinite grid resolution the best mismatch μ_{\min} is nonzero and hence the result of timing noise rather than a coarse grid.

C. Averaged mismatch as a function of the observation epoch

We will now investigate how the best mismatch μ_{\min} varies as a function of the observation epoch. We only show the narrow-band search, as the results were found to be very similar for the single-template search. The method consists of measuring the mismatch μ_{\min} in a 6-month window, which is shifted in 1 month intervals over all the available ephemeris data. The observation time of 6 months is chosen to be similar to typical CW search durations. We are restricted to multiples of 1 month by the frequency of updates to the Crab ephemeris.

Timing noise is not the only variability in the spin-down of pulsars - they can also undergo sudden increases in rotation frequency known as *glitches*. The Crab fre-

quently glitches and these are catalogued by Espinoza et al. [17]. The mechanism which causes a glitch is not well understood and may involve unpredictable variations in the CW signal. As a result, targeted CW searches such as Abbott et al. [9] avoid periods with known glitches. For this reason, we are interested only in the effect of timing noise and not glitches; we therefore omit windows including glitches from the search.

This method can be computationally intensive for large numbers of grid points. To reduce the amount of computation, we begin by searching in a small 40×40 grid in frequency and spindown; we work with a fixed grid space mismatch of $m = 1 \times 10^{-5}$ and the grid spacing is defined in equation (2). It is possible that the minimum mismatch is found at the edge of the narrow-band grid; such candidates are not true local minima in the mismatch. If this is the case, the search is repeated with an increasingly larger grid size, but the same fixed grid spacing. This process continues until we find a minimum mismatch which is not at the edge of the grid; the largest grids required to find a suitable minimum mismatch where 2200×2200 .

Figure 4 shows the measured minimum mismatch in the narrow-band search for a sliding 6-month window at the centre of the observation time. The mismatch due to timing noise is the low level noise occurring in between glitches. Greater mismatches are observed in the post-glitch periods; this is expected as the relaxation time after glitches for the Crab is of the order 1 month [18]. We note the presence of an anomalous period of large mismatch for all windows that include the ephemeris time MJD 55362. The cause for this is unclear from the available data, but it may be caused either by a measurement error or a small undetected glitch. In general, we find that the level of mismatch due to timing noise is between $\mu_{\min} \sim 10^{-3} - 10^{-2}$ for these 6-month searches.

D. Averaged mismatch as a function of the observation duration

We can study the averaged behaviour of the mismatch μ_{\min} as a function of time by varying the size of the sliding window in the previous section. This was done for both the narrow-band and single-template searches; the mismatch from the narrow-band search was found to be a fraction $\lesssim 0.1$ smaller on average than the single-template search. We therefore will only present results from the narrow band search. The shortest possible window ~ 6 months is restricted by the number of points needed to generate a fit to the phase. Setting the upper limit at ~ 17 months retains a statistically meaningful number of points to average over. Having obtained the data from all sliding window sizes in this range we want to analyse the average behaviour as a function of the observation time. Before doing this we filter results in the following ways:

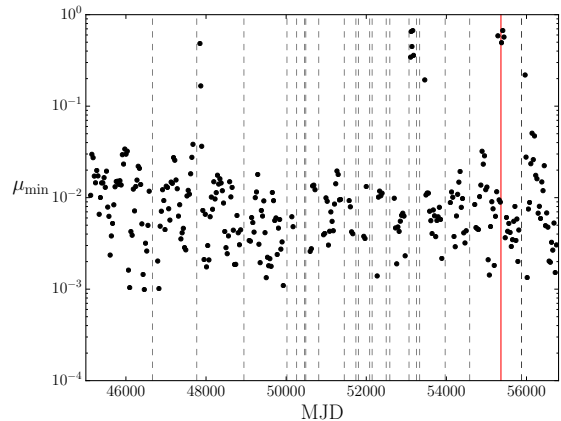


FIG. 4: Minimum mismatch μ_{\min} found in 6-month sliding window searches as a function of epoch at the centre of window. Vertical dashed lines indicate glitch events as described by Espinoza et al. [17] available at <http://www.jb.man.ac.uk/pulsar/glitches.html>. The vertical red line indicates the date MJD 55362, a period of anomalously large mismatch.

- We do not consider any windows that include or are bounded by glitch events
- Windows including the anomalous epoch MJD 55362 are omitted. We wish to study the fluctuations due to timing noise, and this period is either an unidentified glitch, or another highly unusual and unrepresentative form of timing noise
- While each entry of the ephemeris is on average valid over a whole month, some months were truncated due to glitches. The sliding window, which works on a fixed number of entries of the ephemeris will occasionally be shorter than average. To ensure we are averaging over windows of a similar length we omit windows for which the observation time differs from the average by 2 weeks.

In figure 5 we plot the averaged minimum mismatch $\langle \mu_{\min} \rangle$ as a function of observation time. This indicates a growth of $\langle \mu_{\min} \rangle$ with observation time resembling a power law. We perform a least-squares fitting to the power law to quantify the non-linear growth of the average mismatch. Fitting the expression

$$\langle \mu_{\min} \rangle_{\text{fit}} = \kappa \left(\frac{T_{\text{obs}}}{1 \text{ sec}} \right)^n, \quad (4)$$

we find the best fit parameters

$$\kappa = 1.5 \pm 0.8 \times 10^{-23} \quad (5)$$

$$n = 2.88 \pm 0.030. \quad (6)$$

For perfectly matched signals the squared SNR increases linearly [2] with observation time. This suggests

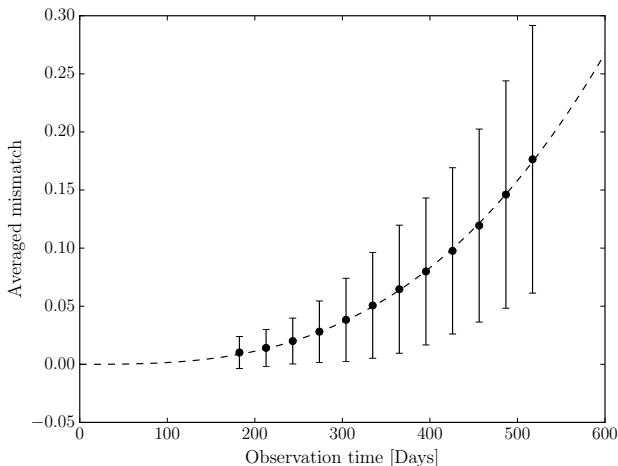


FIG. 5: Averaging the mismatch for sliding window searches and varying the observation times. The points give the mean while the bars correspond to one standard deviation.

that longer observation times yield a greater likelihood of detection. The power law fit with $n > 1$ implies that the average mismatch from Crab timing noise grows faster than the squared SNR. Gains in SNR from longer observation time will therefore eventually be outweighed by the increasing mismatch from timing noise. To estimate when this may occur, we can rearrange equation (3) to give

$$\rho^2 = \rho_s^2 (1 - \mu_{\min}). \quad (7)$$

Substituting the time dependencies for the perfectly matched SNR and the averaged mismatch we have

$$\rho^2 \propto T_{\text{obs}} - \kappa T_{\text{obs}}^{n+1}. \quad (8)$$

Differentiating and solving for T_{obs} yields an expression for the observation time (in seconds) beyond which the ρ^2 value of a signal containing timing noise starts to decrease

$$T_{\text{obs}} = \left(\frac{1}{\kappa(1+n)} \right)^{1/n}. \quad (9)$$

For the fit values from equation (6), this yields a critical observation time of $T_{\text{obs}} \approx 600$ days after which the mismatch exceeds $\langle \mu_{\min} \rangle \approx 0.25$. In this case it is no longer true that further increases in observation time will yield greater detectability.

Jones [7] estimated the maximum time the signal and template would remain coherent given a random walk in frequency. A crude method used a phase residual of 1 rad for the decoherence criteria. For the Crab, this estimates the decoherence time at 200 days. We can improve upon this result by setting a mismatch of 0.1 as the decoherence criteria; using the fit to the averaged mismatch this gives us a decoherence time of $T_{\text{obs}} \approx 400$ days.

It would be useful to relate the growth in mismatch to various timing noise interpretations such as a random walk [13]. This has been done for other measures of timing noise, but not the mismatch. This approach will be presented in future work.

V. CONCLUSIONS

We have used observational data on the Crab pulsar to characterise the possible effects of timing noise on coherent targeted single-template and narrow-band continuous gravitational-wave searches. This was done by generating fake signals based on the Crab ephemeris data and searching for them using templates without timing noise. This work has resulted in three findings.

Firstly, for the S5 and VSR4 narrow-band searches, if the timing noise in the CW signal from the Crab is at a similar level (or lower) to that in the EM signal, then we find it will only have a small ($\approx 1\%$) effect on the measured squared SNR of the putative signal. We found the mismatch in single-template searches to be only fractionally larger than the narrow-band searches. This suggests phase-adapted searches would not be significantly effected if the signal does not contain timing noise.

Secondly, searching over all available Crab data with a 6-month window, we looked at the mismatch as a function of observation epoch. Post glitch periods tend to admit significant levels of mismatch; this is expected due to the exponential recovery from the glitch. (We also discovered a period around MJD 55362 which has a large mismatch and is not connected to a known glitch). The narrow-band and single-template searches performed similarly in this and subsequent tests. Typically the mismatch due to timing noise for 6-month searches was found to be between 10^{-3} and 10^{-2} .

Thirdly, we considered the averaged mismatch as a function of observation time in an effort to extrapolate to future searches. Here we find that the averaged mismatch grows as a power law with observation time and may become problematic for Crab searches in excess of a 1 year.

The scope of this work can be extended to directed and all-sky searches, which target young rapidly spinning down stars which may emit the strongest CWs. These stars are also known to exhibit the highest levels of timing noise and glitch frequently. Crucially the lack of EM data means we cannot be certain a glitch does not occur during the observation and we cannot account for timing noise in the signal. In future work we would like to quantify both these effects and estimate safe upper limits for the search durations.

VI. ACKNOWLEDGEMENTS

GA acknowledges financial support from the University of Southampton and the Albert Einstein Institute

(Hannover). DIJ acknowledge support from STFC via grant number ST/H002359/1, and also travel support from CompStar (a COST-funded Research Networking Programme). All authors are grateful for useful feed-

back from members of the Continuous Waves group of the LIGO Scientific Collaboration and the Virgo Scientific Collaboration. This paper has been assigned document numbers AEI-2014-054 and LIGO-P1400207-v1.

-
- [1] P. Jaranowski, A. Królak, and B. F. Schutz, Phys. Rev. D **58**, 063001 (1998), gr-qc/9804014.
 - [2] R. Prix, in *Astrophysics and Space Science Library*, edited by W. Becker (2009), vol. 357 of *Astrophysics and Space Science Library*, p. 651, <https://dcc.ligo.org/LIGO-P060039/public>.
 - [3] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars* (John Wiley & Sons, Inc, 1983).
 - [4] G. Hobbs, a. G. Lyne, and M. Kramer, MNRAS **402**, 1027 (2010), ISSN 00358711.
 - [5] Z. Arzoumanian, D. J. Nice, J. H. Taylor, and S. E. Thorsett, Astrophys. J. **422**, 671 (1994).
 - [6] D. N. Matsakis, J. H. Taylor, and T. M. Eubanks, Astronomy & Astrophysics **326**, 924 (1997).
 - [7] D. Jones, Phys. Rev. D **70**, 1 (2004), ISSN 1550-7998.
 - [8] M. Pitkin and G. Woan, Classical and Quantum Gravity **21**, S843 (2004).
 - [9] B. Abbott, R. Abbott, R. Adhikari, P. Ajith, B. Allen, G. Allen, R. Amin, S. B. Anderson, W. G. Anderson, M. A. Arain, et al., Astrophys. J. **683**, L45 (2008), 0805.4758.
 - [10] J. Abadie, B. P. Abbott, R. Abbott, T. D. Abbott, M. Abernathy, T. Accadia, F. Acernese, C. Adams, R. Adhikari, C. Affeldt, et al., Phys. Rev. D **85**, 022001 (2012), 1110.0208.
 - [11] C. Palomba, in preparation (2014).
 - [12] A. G. Lyne, R. S. Pritchard, and F. Graham-Smith, MNRAS **265**, 1003 (1993).
 - [13] J. M. Cordes and G. Greenstein, Astrophys. J. **245**, 1060 (1981).
 - [14] LIGO Scientific Collaboration, *LALSuite: FreeSoftware (GPL) Tools for Data-Analysis* (2014), <https://www.lsc-group.phys.uwm.edu/daswg/projects/lalsuite.html>.
 - [15] J. Aasi, J. Abadie, B. P. Abbott, R. Abbott, T. D. Abbott, M. Abernathy, T. Accadia, F. Acernese, C. Adams, T. Adams, et al., Physical Review D **87**, 042001 (2013), 1207.7176.
 - [16] R. Prix, Phys. Rev. D **75**, 023004 (2007), gr-qc/0606088.
 - [17] C. M. Espinoza, A. G. Lyne, B. W. Stappers, and M. Kramer, MNRAS **414**, 1679 (2011), 1102.1743.
 - [18] A. Lyne and F. Graham-Smith, *Pulsar Astronomy* (Cambridge University Press, 2012).