Eg(1), say ki = 3 {say real GDP (y_t), inflation (π_t), interest rate (i_t)}

$$\underset{3\times1}{\begin{pmatrix} x_{1it} \\ x_{2it} \\ x_{3it} \end{pmatrix}} = \underset{3\times1}{\begin{pmatrix} a_{1i0} \\ a_{2i0} \\ a_{3i0} \end{pmatrix}} \oplus \underset{3\times1}{\begin{pmatrix} a_{1ii} \\ \vdots \\ a_{3ii} \end{pmatrix}} t \oplus \underset{3\times3}{\begin{pmatrix} \wedge_{11} & \wedge_{12} & \\ \wedge & \wedge_{22} & \\ \wedge & \wedge & \wedge_{33} \end{pmatrix}} \underset{3\times1}{\begin{pmatrix} x_{1it-1} \\ x_{2it-1} \\ x_{3it-1} \end{pmatrix}}$$

$$\oplus \underset{3\times3}{\begin{pmatrix} \wedge & & \\ & \wedge & \\ & & \wedge \end{pmatrix}} \underset{3\times1}{\begin{pmatrix} x^*_{1it} \\ x^*_{2it} \\ x^*_{3it} \end{pmatrix}} \oplus \cdots \oplus \underset{3\times1}{\begin{pmatrix} \varepsilon_{1it} \\ \varepsilon_{2it} \\ \varepsilon_{3it} \end{pmatrix}}$$

Eg(2), say we've countries (i) & (j) & (l) (3 countries) $N = 3$.

$$\underset{(3\times1)}{\begin{pmatrix} x^*_{1it} \\ x^*_{2it} \\ x^*_{3it} \end{pmatrix}} = \begin{pmatrix} \sum_{j\neq i}^{N} w_{ij} x_{jjt} \equiv w_{ij} x_{ijt} + w_{il} x_{ilj} \\ \sum_{j=1} w_{ij} x_{ijt} \neq \cdots \\ \cdots = \cdots \end{pmatrix}$$

- (foreign variable)
- % of trade flow from (j) to (i) (the weight)
- Say, real GDP of country j
- Also the base for Input out put leontief matrix.

(2)

eg 3

Say $k_i = 3$

$$A_i z_{i,t} = \begin{pmatrix} I & 0 & 0 & -\Lambda & -\Lambda & -\Lambda \\ 0 & I & 0 & -\Lambda & -\Lambda & -\Lambda \\ 0 & 0 & I & -\Lambda & -\Lambda & -\Lambda \end{pmatrix} \begin{pmatrix} x_{1it} \\ x_{2it} \\ x_{3it} \\ x^*_{1it} \\ x^*_{2it} \\ x^*_{3it} \end{pmatrix}$$

3×6 $\qquad$ 6×1

$(k_i \cdot \ast (k + k^*))$

$n \leftarrow$ # of countries.

$K = \sum_{i=1}^{n} k_i$

$= n3$

$= 3n$

Define $W_i = \begin{pmatrix} x_1 & - & - \\ x_2 & & \\ x_3 & & \\ x^*_{1i} & & \\ x_2 & & \\ x_3 & & \end{pmatrix}$

$6 \times 3n$

$x_t = \begin{pmatrix} x & x_k \\ x & \vdots \\ x & \vdots \\ x & x \end{pmatrix}$

Eg 4

Def of $N=3$ (i, j, l) $k=3$,

$$z_{it} = W x_t \qquad 6 \times 1 \quad (3\times 3) \times 1$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_{1ij} & 0 & 0 & w_{1il} & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{2ij} & 0 & 0 & w_{2il} & 0 \\ 0 & 0 & 0 & 0 & 0 & w_{3ij} & 0 & 0 & w_{3il} \end{pmatrix} \begin{pmatrix} x_{1it} \\ x_{2it} \\ x_{3it} \\ x_{1jt} \\ x_{2jt} \\ x_{3jt} \\ x_{1kt} \\ x_{2kt} \\ x_{3kt} \end{pmatrix}$$

6×9 $\qquad$ 9×1.

Eg 5

$N=3,\ K=3$ for i

(3)

Rewrite :. $A_i W_i x_t = a_{i0} + a_{i1} t + B_i W_i x_{t-1} + \varepsilon_t$

$$\begin{pmatrix} I & 0 & 0 & -1 & -1 & -1 \\ 0 & I & 0 & -1 & -1 & -1 \\ 0 & 0 & I & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \sim & \sim & \sim & \sim & \sim & 0 \\ 0 & 0 & 1 & \sim & \sim & \sim & \sim & \sim & 0 \\ 0 & 0 & 0 & w_{1ij} & 0 & 0 & w_{1il} & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{2ij} & 0 & 0 & w_{2il} & 0 \\ 0 & 0 & 0 & 0 & 0 & w_{3ij} & 0 & 0 & w_{3il} \end{pmatrix} \begin{pmatrix} x_{1ic} \\ x_{2ic} \\ x_{3ic} \\ x_{1jc} \\ x_{2jc} \\ x_{3jc} \\ x_{1lc} \\ x_{2lc} \\ x_{3lc} \end{pmatrix}$$

A_i (3×6) — W_i (6×9) — x_t (9×1).

eg 6

G = Stacking i, j, l country.

$$\begin{pmatrix} 3\times 1 \\ \oplus\ 3\times 1 \\ \oplus\ 3\times 1 \\ \hline 9\times 1 \end{pmatrix} = G$$

Say $\Rightarrow$ $a_{i0} = \begin{pmatrix} a_{i0} \\ a_{i0} \\ a_{i0} \\ a_{j0} \\ a_{j0} \\ a_{j0} \\ a_{l0} \\ a_{l0} \\ a_{l} \end{pmatrix}$ $\begin{matrix} \} i \\ \\ \\ \} j \\ \\ \\ \} l \\ \\ \end{matrix}$ 9×1