MAM 3040W: Advanced Numerical Methods

PROJECT 1: Shooting Newton

Please email me a single pdf file with your report and the essential parts of your codes. The submission deadline is 16h00 on Friday 24 March 2023.

Self-focusing of light beams in the Kerr medium

The following equation originally appeared in the context of the light propagation in nonlinear dielectrics:

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - v + 2v^3 = 0.$$

The boundary conditions are:

$$\frac{dv}{dr}(0) = 0, \quad v(\infty) = 0.$$

You are required to implement the shooting and the Newton-Kantorovich method to obtain the monotonically decaying and the one-node solutions with the prescribed boundary conditions.

Submit a short scientific report describing the obtained results. Your submission should be coherent, not just a collection of figures with some notes on them. In your report you should compare the implementation of the methods for the given problem from as many points of view as possible, e.g. number of iterations necessary for the convergence, running time, mesh spacing, sensitivity to the initial guess, length of the spatial interval on which it is possible to obtain the solution etc. Your report should contain a detailed description of the implementation of both algorithms.