Department of Mathematics and Applied Mathematics
MAM 3040W: Advanced numerical methods

PROJECT 2: Head-on collisions

Please email me a single pdf file containing your project. The submission deadline is **Friday 12 May**.

Coupled system with saturated nonlinearity

The interaction of optical pulses in nonlinear media can be studied in a system of two coupled Schrödinger equation with the following nonlinearity:

$$iu_t + u_{xx} + \left(|u|^2 + \frac{|v|^2}{1+\alpha|v|^2}\right) u = 0,$$
$$iv_t + v_{xx} + \left(|v|^2 + \frac{|u|^2}{1+\alpha|u|^2}\right) v = 0.$$

Here α is a parameter. We impose periodic boundary conditions.

An important practical question that arises in optical communications is how two or more solitons interact with each other. Such collisions can be studied by choosing your initial configuration as a sum of two solitons separated by a sufficiently large distance

$$u(x,0) = \sum_{j=1,2} \mathrm{sech}[(x-x_j)]\mathrm{e}^{ic_j(x-x_j)},$$
$$v(x,0) = \sum_{j=1,2} \mathrm{sech}[(x-x_j)]\mathrm{e}^{ic_j(x-x_j)},$$

where $|c_j| < 2$ are velocities of the colliding solitons.

Explore the soliton collisions for different values of the parameter α, the velocity c_j and different values of the initial inter-soliton distance $|x_2 - x_1|$. In particular, please examine the effect of positive and negative values of α as well as small and large velocities v and initial inter-soliton distances.

Requirements

Write a short scientific report. You should verify your observations using two different numerical algorithms: a finite difference method and a method based of Fourier expansions (the split-step or the pseudospectral method).

To make the comparison between the methods consistent, choose some small ε suitable for the computational power of your computer and "calibrate" the parameters of both

numerical routines (e.g. space and time steps) such that both codes compute the conserved quantity

$$N = \int_{-\infty}^{+\infty} |u|^2 + |v|^2 dx$$

to the chosen accuracy ε for one soliton for short time. Use these parameters for the simulations of the soliton collisions.

Include a comparison between the methods from as many points of view as possible in your report. Especially interesting is to compare of the running times required by the two schemes. (You can time your code in MATLAB using the built-in functions `tic` and `toc`.) Illustrate you report with several representative figures, choose to depict only one of the variables u or v. Note that for the 3D figures it is important to choose the viewing angle in such a way that both the initial configuration and the evolution after the collision are clearly seen.