Modifications for the Differential Evolution algorithm

Vasileios Charilogis, Ioannis G. Tsoulos, Alexandros Tzallas, Evangelos Karvounis

Department of Informatics and Telecommunications, University of Ioannina, Greece

Abstract

Differential Evolution (DE) is a global optimization method used in various continuous problems but it can be applied also in problems that are not even continuous, are noisy and changed over time. The DE method maintains a set of candidate solutions that repeatedly replaced by new solutions produced with combinations of the previous solutions. This article proposes two major modifications for the DE method designed to improve the speed of the algorithm without sacrificing the efficiency of the method. The proposed method is tested on a series of well known optimization problems from the relevant literature and the results are compared against the original method.

1 Introduction

The location of the global minimum of a continuous and differentiable function $f : S \to R, S \subset R^n$ is formulated as

$$x^* = \arg\min_{x \in S} f(x) \tag{1}$$

where the set S is defined as

$$S = [a_1, b_1] \otimes [a_2, b_2] \otimes \ldots [a_n, b_n]$$

In the recent literature there are a plethora of real world problems that can formulated as global optimization problems such as problems from the physics area [1, 2, 3, 4], chemistry [5, 6, 7], economics [8, 9] etc. There are a variety of proposed methods to handle the global minimum problem such as Adaptive Random Search [10], Competitive Evolution [11], Controlled Random Search [12], Simulated Annealing [13, 14, 16], Genetic Algorithms [17, 18], Differential Evolution [21], Particle Swarm Optimization [23], Bee optimization [19, 20], Ant Colony Optimization [21] etc. Recently many works have been appeared that

take advantage of the GPU processing units to implement parallel global optimization methods [24, 25, 26]. This work introduces two major modifications for the Differential Evolution (DE) method, that aim to speed up the algorithm and to reduce the total number of function evaluations required by the method. The DE method initially creates a population of candidate solutions and through a series of iterations creates new solutions by combining the previous ones. The method does not require any prior knowledge of the derivative and is therefore quite fast and has low memory requirements.

After a literature review it was found that differential evolution is used in several areas and many modifications of the original algorithm have been introduced in the recent literature. More specifically, in the research of Zongjun et al [27], genetic and differential calculus algorithms were used to optimize the parameters of two models aimed at estimating evapotranspiration in three regions and it was found that the performance of evolution algorithms was better than the genetic algorithm. Another research focuses on a case study of a cellular neural network aimed at generating fractional classes of neurons. The best solutions provided by differential calculus and the use of accelerated particle swarm optimization (APSO) are presented concretely in the work of Tlelo-Cuautle et al [28]. Another article [29] proposes a regeneration framework based on space search adaptation (ARSA), which can be integrated into different variants of different evolution to address the problems of early convergence and population stability faced by different calculus. Also another interesting variation of the method is the Bernstain Search Differential Evolution Algorithm [30] for optimizing numerical functions. Another interesting algorithm is a new design of differential evolution to solve the travel salesman problem [31].

The differential evolution method was applied also to energy science. Specifically, the article of Liang et al [32] evaluates the parameters of solar photovoltaic models through a self-adjusting differential evolution. Similarly, in the study of Peng et al [33], differential evolution is used for the prediction of electricity prices. Also, differential evolution was also incorporated in neural architecture search [34].

The rest of this article is organized as follows: in section 2 the base DE algorithm as well as the proposed modifications are presented clearly, in section 3 the test functions used in the experiments are presented accompanied with the experimental results and finally in section 4 some conclusions are presented.

2 Modifications

This section starts with the detailed description of the DE method and continues with the modifications suggested in this article. The first modification is a new stopping rule which measures the difference of the mean of function values between the iterations of the algorithm and the second modification suggests a new scheme for a critical parameter of the DE algorithm called Differential Weight.

2.1 The base algorithm

The base DE algorithm has the steps described below

1. **Set** the population size NP $\geq$4, usually NP $= 10n$, where n is the dimension of the input problem.
2. **Set** the crossover probability CR $\in [0, 1]$. A typical value for this parameter is 0.9
3. **Set** the differential weight $F \in [0, 2]$. A typical value for this parameter is 0.8
4. **Initialize** all members of the population in the search space. The members of the population are called agents.
5. **Until** some stopping criterion is met repeat
   (a) **For** $i = 1 \ldots \text{NP}$ **do**
      i. **Set** x as the agent i.
      ii. **Pick** randomly three agents a, b, c
      iii. **Pick** a random index $R \in \{1, \ldots, n\}$
      iv. **Compute** the trial vector $y = [y_1, y_2, \ldots, y_n]$ as follows
      v. **For** $j = 1, \ldots, n$ **do**
         A. **Set** $r_i \in [0, 1]$ a random number.
         B. **If** $r_j <$ CR **or** $j = R$ **then** $y_j = a_j + F \times (b_j - c_j)$ **else** $y_j = x_j$.
      vi. If $f(y) \leq f(x)$ then $x = y$.
      vii. **EndFor**
   (b) **EndFor**
6. **Return** the agent x_{best} in the population with the lower function value $f(x_{\text{best}})$.

2.2 The new termination rule

Typically the DE method is terminated when a predefined number of iterations is reached. This can be extremely inefficient in some problems and in others it can lead to premature termination, ie termination before the total minimum is found. In the work of Ali et al [35] a different termination rule is proposed ie. terminate when

$$f_{\max} - f_{\min} \leq \epsilon \tag{2}$$

where $f_{\max}$ is the function value of the worst agent in the population, $f_{\min}$ is the function value of the best agent and $\epsilon$is a small positive number.

In the proposed termination rule, the average function value of the population is calculated in each iteration. If this value does not change significantly

for a repetitive number of iterations, then it is very likely that the method may not discover a new global minimum and should therefore be terminated. Hence in every generation t we measure the quantity:

$$\delta^{(t)} = \left| \sum_{i=1}^{\mathrm{NP}} \left| f_i^{(t)} \right| - \sum_{i=1}^{\mathrm{NP}} \left| f_i^{(t-1)} \right| \right| \tag{3}$$

and the termination rule is defined as: **terminate** if $\delta^{(t)} \leq \epsilon$ for a predefined number of M generations.

2.3 The new differential weight

The differential weight initially proposed in the DE algorithm was a static value, which means that some tuning is required in order to discover the global minimum in every optimization function. Ali et al [35] proposed an adaptation mechanism for this parameter in order that the algorithm should search in larger spaces at the the first generations and to become more focused at latter generations. The mechanism proposed is expressed as:

$$F = \begin{cases} \max\left(l_{\min}, 1 - \left|\frac{f\max}{f\min}\right|\right) & , \quad \text{if} \quad \left|\frac{f\max}{f\min}\right| \leq 1 \\ \max\left(l_{\min}, 1 - \left|\frac{f\min}{f\max}\right|\right) & , \quad \text{otherwise} \end{cases} \tag{4}$$

The current work proposes a stochastic mechanism similar to the crossover operation of the Genetic algorithms. The proposed scheme is expressed as:

$$F = -\frac{1}{2} + 2 \times R \tag{5}$$

where $R \in [0, 1]$ is a random number.

3 Experiments

In order to determine the effectiveness of the proposed modifications, a series of experiments were performed on known functions from the relevant literature [36, 37]. The experiments were divided into two major categories. In the first category all the schemes for the Differential Weight were tested using the termination rule of Equation 2 and in the second category the same schemes were tested using the proposed termination criterion. Also, after every successful termination the local optimization method BFGS [38] was applied in order to get even closer to the global minimum.

3.1 Test functions

The description of the test functions used in the experiments has as follows:

- **Bf1** function. The function Bohachevsky 1 is given by the equation

$$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1) - \frac{4}{10}\cos(4\pi x_2) + \frac{7}{10}$$

with $x \in [-100, 100]^2$. The value of global minimum is 0.0.

- **Bf2** function. The function Bohachevsky 2 is given by the equation

$$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$$

with $x \in [-50, 50]^2$. The value of the global minimum is 0.0.

- **Branin** function. The function is defined by $f(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$ with $-5 \leq x_1 \leq 10,\ 0 \leq x_2 \leq 15$. The value of global minimum is 0.397887.with $x \in [-10, 10]^2$. The value of global minimum is -0.352386.

- **CM** function. The Cosine Mixture function is given by the equation

$$f(x) = \sum_{i=1}^{n} x_i^2 - \frac{1}{10}\sum_{i=1}^{n}\cos(5\pi x_i)$$

with $x \in [-1, 1]^n$. The value of the global minimum is -0.4 and in our experiments we have used $n = 4$.

- **Camel** function. The function is given by

$$f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4, \quad x \in [-5, 5]^2$$

The global minimum has the value of $f(x^*) = -1.0316$

- **Easom** function. The function is given by the equation

$$f(x) = -\cos(x_1)\cos(x_2)\exp\left(\left(x_2 - \pi\right)^2 - \left(x_1 - \pi\right)^2\right)$$

with $x \in [-100, 100]^2$. The value of the global minimum is -1.0

- **Exponential** function. The function is given by

$$f(x) = -\exp\left(-0.5\sum_{i=1}^{n} x_i^2\right), \quad -1 \leq x_i \leq 1$$

The global minimum is located at $x^* = (0, 0, ..., 0)$ with value -1. In our experiments we used this function with $n = 2, 4, 8, 16, 32$.

- **Goldstein and Price function**
  The function is given by the equation

$$\begin{aligned} f(x) &= \Big[1+(x_1+x_2+1)^2 \\ &\quad \left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)] \times \\ &\quad [30+(2x_1-3x_2)^2 \\ &\quad \left(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2\right)] \end{aligned}$$

  With $x \in [-2,2]^2$. The global minimum is located at $x^* = (0,-1)$ with value 3.0

- **Griewank2** function. The function is given by

$$f(x) = 1 + \frac{1}{200}\sum_{i=1}^{2} x_i^2 - \prod_{i=1}^{2} \frac{\cos(x_i)}{\sqrt{(i)}}, \quad x \in [-100,100]^2$$

  The global minimum is located at the $x^* = (0,0,...,0)$ with value 0.

- **Gkls** function. $f(x) = \text{Gkls}(x,n,w)$, is a function with w local minima, described in [39] with $x \in [-1,1]^n$ and n a positive integer between 2 and 100. The value of the global minimum is -1 and in our experiments we have used $n = 2,3$ and $w = 50,\ 100$.

- **Hansen** function. $f(x) = \sum_{i=1}^{5} i\cos\left[(i-1)x_1+i\right]\sum_{j=1}^{5} j\cos\left[(j+1)x_2+j\right]$, $x \in [-10,10]^2$. The global minimum of the function is -176.541793.

- **Hartman 3** function. The function is given by

$$f(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right)$$

  with $x \in [0,1]^3$ and $a = \begin{pmatrix} 3 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3 & 10 & 30 \\ 0.1 & 10 & 35 \end{pmatrix}$, $c = \begin{pmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{pmatrix}$ and

$$p = \begin{pmatrix} 0.3689 & 0.117 & 0.2673 \\ 0.4699 & 0.4387 & 0.747 \\ 0.1091 & 0.8732 & 0.5547 \\ 0.03815 & 0.5743 & 0.8828 \end{pmatrix}$$

  The value of global minimum is -3.862782.

- **Hartman 6** function.

$$f(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right)$$

with $x \in [0,1]^6$ and $a = \begin{pmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{pmatrix}$, $c = \begin{pmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{pmatrix}$

and

$$p = \begin{pmatrix} 0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5886 \\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991 \\ 0.2348 & 0.1451 & 0.3522 & 0.2883 & 0.3047 & 0.6650 \\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381 \end{pmatrix}$$

the value of global minimum is -3.322368.

- **Potential** function. The molecular conformation corresponding to the global minimum of the energy of N atoms interacting via the Lennard-Jones potential[40] is used as a test case here. The function to be minimized is given by:

$$V_{LJ}(r) = 4\epsilon \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6} \right] \tag{6}$$

  In the current experiments three different cases were studied: $N = 3,\ 4,\ 5$

- **Rastrigin** function. The function is given by

$$f(x) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2), \quad x \in [-1,1]^2$$

  The global minimum is located at $x^* = (0,0)$ with value -2.0.

- ***Rosenbrock*** *function.*
  This function is given by

$$f(x) = \sum_{i=1}^{n-1} \left(100 \left(x_{i+1} - x_i^2 \right)^2 + (x_i - 1)^2 \right), \quad -30 \leq x_i \leq 30.$$

  The global minimum is located at the $x^* = (0,0,...,0)$ with $f(x^*) = 0$. In our experiments we used this function with $n = 4,\ 8,\ 16$.

- **Shekel 7** function.

$$f(x) = -\sum_{i=1}^{7} \frac{1}{(x - a_i)(x - a_i)^T + c_i}$$

with $x \in [0,10]^4$ and $a = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \\ 2 & 9 & 2 & 9 \\ 5 & 3 & 5 & 3 \end{pmatrix}$, $c = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \\ 0.6 \\ 0.3 \end{pmatrix}$. The value of global minimum is -10.342378.

- **Shekel 5** function.

$$f(x) = -\sum_{i=1}^{5} \frac{1}{(x-a_i)(x-a_i)^T + c_i}$$

with $x \in [0,10]^4$ and $a = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \end{pmatrix}$, $c = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \end{pmatrix}$. The value of global minimum is -10.107749.

- **Shekel 10** function.

$$f(x) = -\sum_{i=1}^{10} \frac{1}{(x-a_i)(x-a_i)^T + c_i}$$

with $x \in [0,10]^4$ and $a = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \\ 2 & 9 & 2 & 9 \\ 5 & 5 & 3 & 3 \\ 8 & 1 & 8 & 1 \\ 6 & 2 & 6 & 2 \\ 7 & 3.6 & 7 & 3.6 \end{pmatrix}$, $c = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \\ 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \\ 0.6 \end{pmatrix}$. The value of global minimum is -10.536410.

- **Sinusoidal** function. The function is given by

$$f(x) = -\left(2.5 \prod_{i=1}^{n} \sin(x_i - z) + \prod_{i=1}^{n} \sin(5(x_i - z))\right), \quad 0 \le x_i \le \pi.$$

The global minimum is located at $x^* = (2.09435, 2.09435, ..., 2.09435)$ with $f(x^*) = -3.5$. In our experiments we used $n = 4, 8, 16, 32$ and $z = \frac{\pi}{6}$ and the corresponding functions are denoted by the labels SINU4, SINU8, SINU16 and SINU32 respectively.

- **Test2N** function. This function is given by the equation

$$f(x) = \frac{1}{2} \sum_{i=1}^{n} x_i^4 - 16x_i^2 + 5x_i, \quad x_i \in [-5, 5].$$

The function has 2^n in the specified range and in our experiments we used $n = 4, 5, 6, 7$. The corresponding values of global minimum is -156.664663 for $n = 4$, -195.830829 for $n = 5$, -234.996994 for $n = 6$ and -274.163160 for $n = 7$.

- **Test30N** function. This function is given by

$$f(x) = \frac{1}{10}\sin^2(3\pi x_1)\sum_{i=2}^{n-1}\left((x_i-1)^2\left(1+\sin^2(3\pi x_{i+1})\right)\right)+(x_n-1)^2\left(1+\sin^2(2\pi x_n)\right)$$

  with $x \in [-10, 10]$. The function has 30^n local minima in the specified range and we used $n = 3, 4$ in our experiments. The value of global minimum for this function is 0.0

3.2 Experimental results

All the experiments were conducted using different seed for the random generator each time and averages were taken. The code has been implemented in ANSI C++ and the random generator used was the function drand48() of the C programming languages. The execution environment was an Intel Xeon E5-2630 multi core machine. The parameters used in the experiments are shown in Table 1. The experiments where the stopping rule of Equation 2 was used are outlined in Table 2 and the experiments with the proposed stopping rule are listed in Table 3. The numbers in cells represent average function calls. The fraction in parentheses stands for the fraction of runs where the global optimum was found. If this number is missing then the global minimum was discovered in every independent run (100% success). The column STATIC represents the static value for the differential weight ($F = 0.8$), the column ALI stands for the mechanism given in the Equation 4 and lastly the column PROPOSED stands for the proposed scheme given in the Equation 5.

From the experiments performed we observe that the two proposed variations drastically reduce the required number of function calls. Also, the proposed changes do not seem to affect the average performance of the method, as it remains high in all cases. The effect of the proposed scheme for the differential weight is presented graphically in Figure 1, where we plot the average function calls for the functions ROSENBROCK4, ROSENBROCK8 and ROSENBROCK16 using the three schemes of the differential weight. Also, in the plot of Figure 2, the average calls for the same functions are plotted using both the proposed scheme for the differential weight and the proposed termination rule. It is evident the the combination of both modifications reduced even more the average number of function calls required to locate the global minimum of the test functions.

4 Conclusions

In this text, two main additions to the DE method are presented. In the first an asymptotic termination rule was introduced and used successfully even in multidimensional problems. This rule is based on the observation that from one point onwards the average of the functional values of the agents does not change. This means that either the algorithm has already found the global minimum or its further continuation will have no meaning.

Table 1: Experimental parameters.

PARAMETER	VALUE
NP	$10n$
F	0.8
CR	0.9
M	20
ϵ	10^{-4}

Figure 1: The effect of the usage of the new scheme for the differential weight.

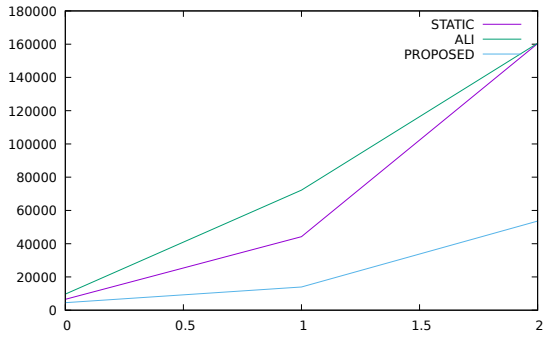

Figure 2: Plot of function calls using the two modifications.

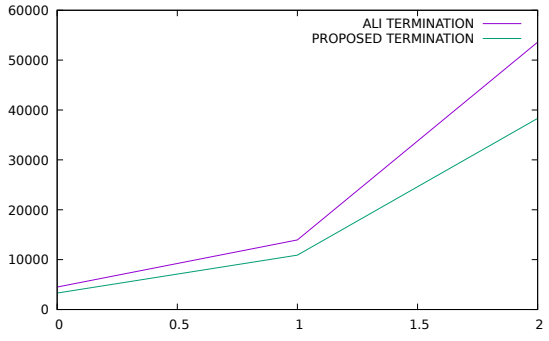

Table 2: Experiments with the termination rule of Ali.

FUNCTION	**STATIC**	**ALI**	**PROPOSED**
BF1	1142	1431	847
BF2	1164	1379	896
BRANIN	984	816	707
CM4	3590	7572	2079
CAMEL	1094	18849	685
EASOM	1707	2014	1327
EXP2	532	323	449
EXP4	2421	1019	1494
EXP8	15750	3670	5632
EXP16	160031	15150	21416
EXP32	320039	152548	77936
GKLS250	784	944	614
GKLS2100	772	1531	599(0.97)
GKLS350	1906(0.93)	3263	1275(0.93)
GKLS3100	1883	3539	1373
GOLDSTEIN	988	818	769
GRIEWANK2	1299(0.97)	1403	883(0.93)
HANSEN	2398	2968	1400
HARTMAN3	1448	836	1050
HARTMAN6	9489(0.97)	4015(0.97)	4667(0.80)
POTENTIAL3	90027	89776	21824
POTENTIAL4	120387(0.97)	120405(0.33)	45705(0.97)
POTENTIAL5	150073	150104	83342
RASTRIGIN	1246	1098(0.93)	871
ROSENBROCK4	6564	9695	4499
ROSENBROCK8	44240	72228	13959
ROSENBCROK16	160349(0.90)	160538(0.60)	53594
SHEKEL5	5524	3810	3057(0.83)
SHEKEL7	5266	3558	2992(0.87)
SHEKEL10	5319	3379	3076
TEST2N4	4200	1980	2592
TEST2N5	7357	2957	4055
TEST2N6	12074	4159	5836
TEST2N7	18872	5490	7904
SINU4	3270	1855	2216
SINU8	23108	6995	8135
SINU16	160092	36044	30943
SINU32	213757(0.70)	160536(0.53)	83369(0.80)
TEST30N3	1452	1732	959
TEST30N4	1917	2287	1378
TOTAL	1564515(0.97)	1062714(0.96)	506404(0.98)

Table 3: Experiments with the proposed termination rule.

FUNCTION	**STATIC**	**ALI**	**PROPOSED**
BF1	996	1124	889
BF2	926	1026	816
BRANIN	878	900	730
CM4	1148(0.70)	1991	1103
CAMEL	1049	904(0.93)	846
EASOM	447	448	446
EXP2	470	461	467
EXP4	915	903	892
EXP8	1797	3558	1796
EXP16	3578	7082	3521
EXP32	7082	14125	7022
GKLS250	498	576	493
GKLS2100	533	884(0.97)	515
GKLS350	823	1130(0.93)	814(0.97)
GKLS3100	858	1495(0.97)	829(0.93)
GOLDSTEIN	945	993	915
GRIEWANK2	947	921	826
HANSEN	2104	1949	1479
HARTMAN3	1017	1005	952
HARTMAN6	4679(0.90)	3744(0.97)	3128(0.87)
POTENTIAL3	21473	2284	8197
POTENTIAL4	44191(0.43)	3098(0.33)	24659(0.97)
POTENTIAL5	75910	3443	52664
RASTRIGIN	841	994	777
ROSENBROCK4	4934	7192	3300
ROSENBROCK8	29583	49696	10907
ROSENBCROK16	160349	160538(0.60)	38315
SHEKEL5	4389(0.97)	4266	2839(0.83)
SHEKEL7	3905	3685	2668
SHEKEL10	4049	3548	2629
TEST2N4	2785	2275	2221
TEST2N5	4481	3170	3122
TEST2N6	6852	4286	4296
TEST2N7	11971	5701	6267
SINU4	2322	1987	1755
SINU8	9990	6156	5113
SINU16	6892	3628(0.97)	16905
SINU32	7235(0.80)	7438(0.83)	7218
TEST30N3	1033	1098	951
TEST30N4	1355	1444	1285
TOTAL	432610(0.98)	321166(0.96)	224567(0.99)

In the second case, a stochastic scheme was used to produce the differential weight. This scheme helped the algorithm to better explore the search space of the objective function without the need for more calls to the objective function.

Compliance with Ethical Standards

All authors declare that they have no has no conflict of interest.

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