Course 5: Reinforcement Learning

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Summary

Last session

1. Combinatorial game theory
2. Definition of a game
3. Proof of determined games

Today's session

1. Reinforcement Learning
2. Value and Policy Functions
3. Q-Learning

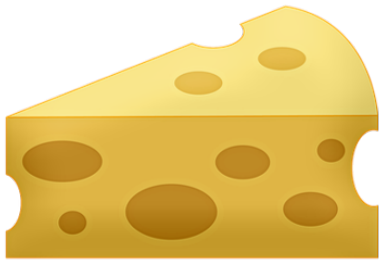
Note: reinforcement learning and combinatorial game theory share a common mathematical framework. But to ease access to online resources, we will adopt a new vocabulary.

Learning Approaches

Supervised L

1. **Data:** (x=data,y=labels)
2. **Goal:** Learn a mapping $x \rightarrow y$

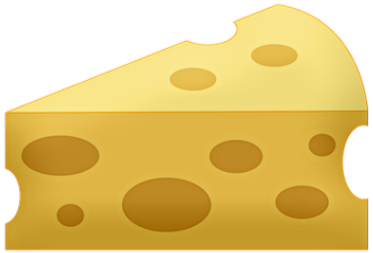
This thing is a cheese



Reinforcement L

1. **Data:** (state, action)
2. **Goal:** Maximize future rewards

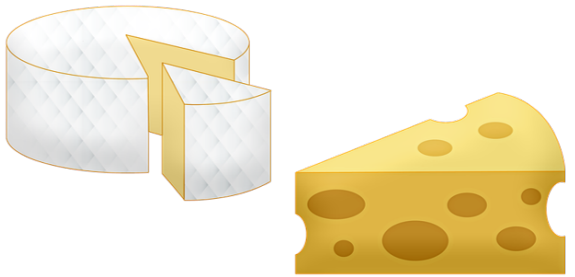
Eat this thing for reward



Unsupervised L

1. **Data:** x=data, no labels
2. **Goal:** Learn structure in x

This thing is like the other thing



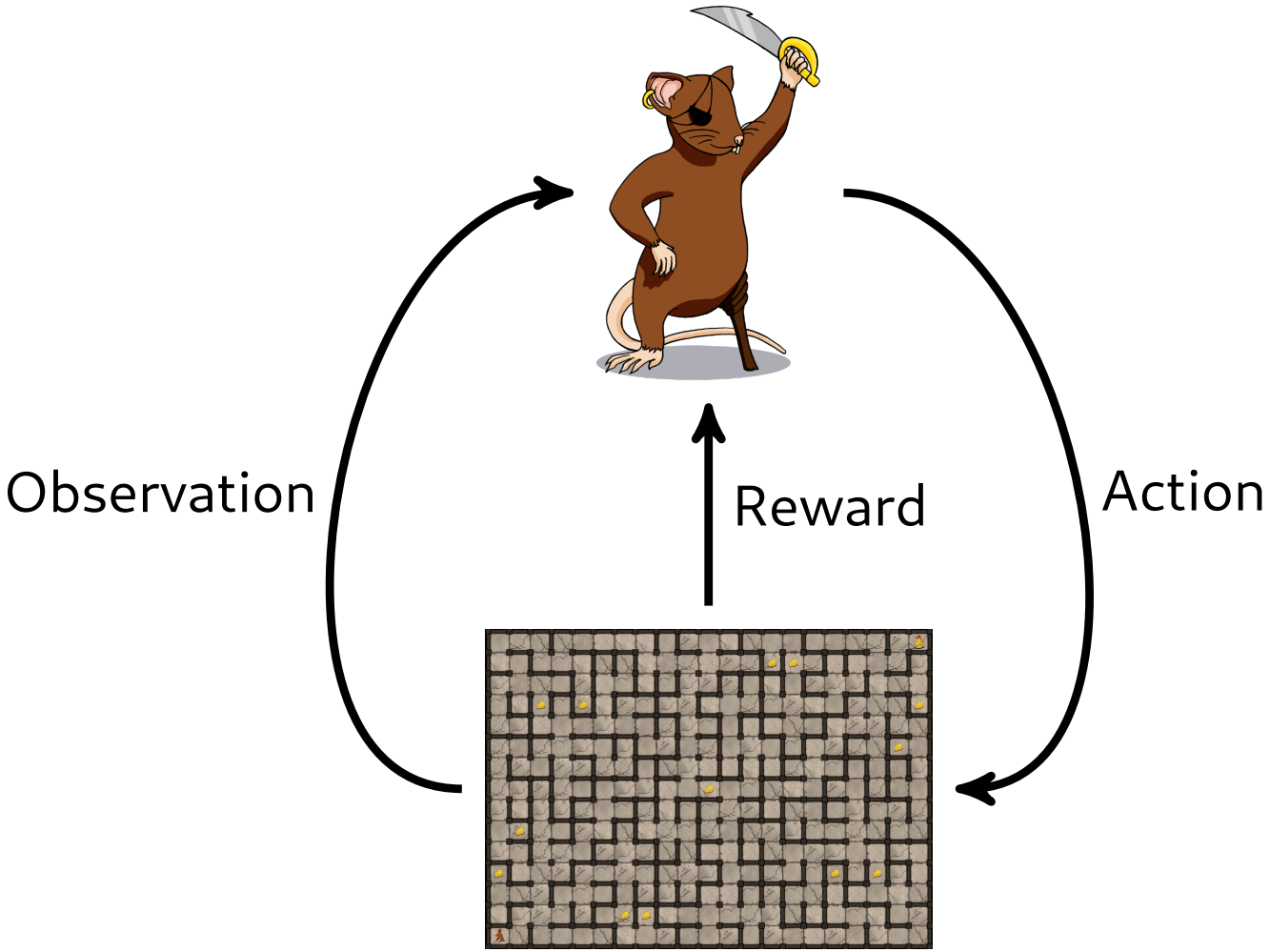
Outline of the course

1. Definitions of Reinforcement Learning (RL)
   - Fundamentals
   - Example: PyRat
   - Policy and values
2. Q-learning
   - Q-learning definitions
   - Example
   - Approximate Q-learning
   - Exploration/Exploitation

Plan

Agent and environment

Our objective is to train an agent to maximize its reward through actions that affect an environment.



Fundamentals

Reward hypothesis

- *All goals can be described by the maximization of expected cumulated reward over time.*

Specificities of reinforcement learning

- No supervision, only a reward signal,
- Delayed feedback, the reward can come (much) later,
- Importance of the temporal dimension,
- Agent's actions affect the subsequent data it receives.

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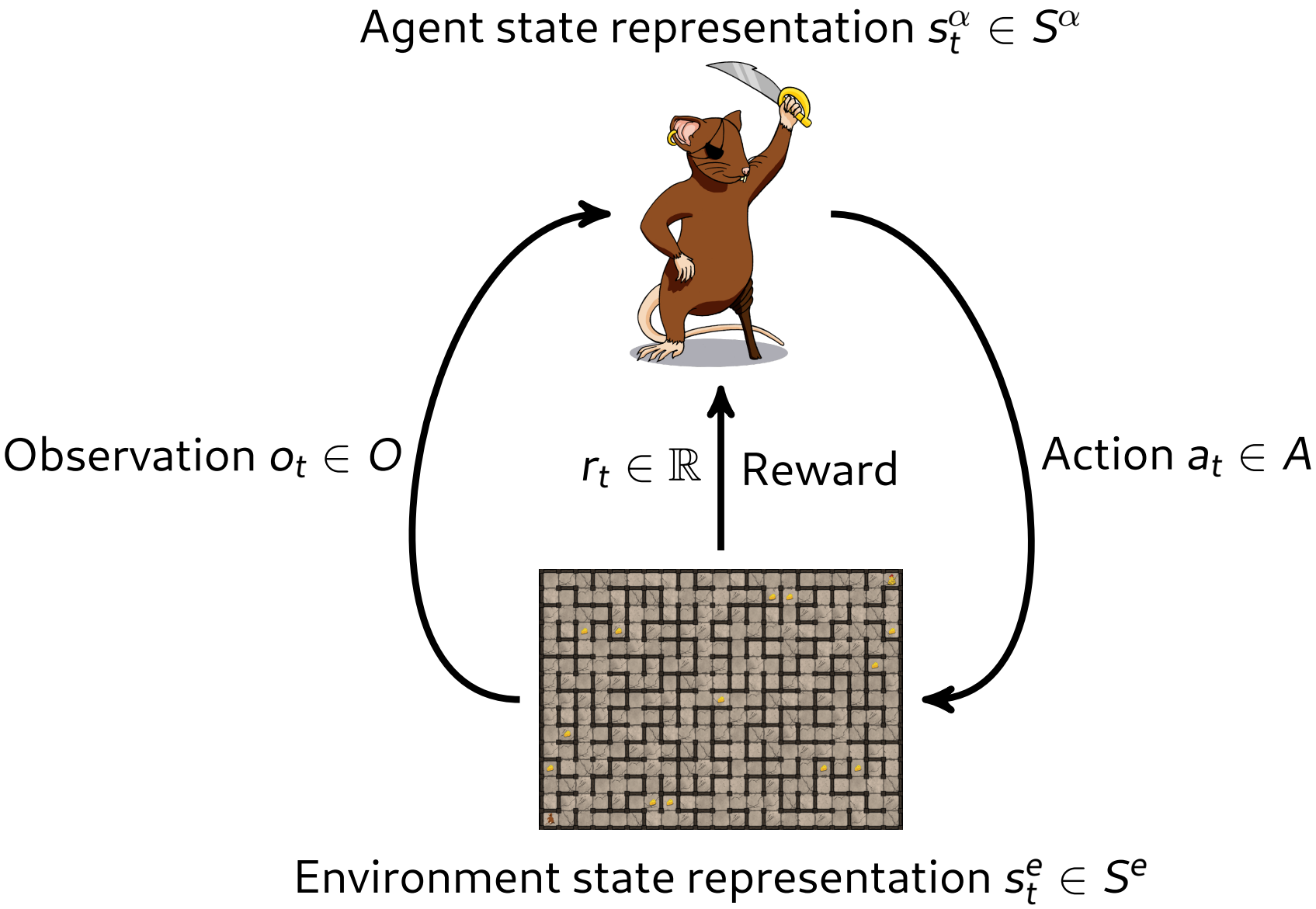

Definitions

- The agent α...
  1. analyzes previous actions, states, rewards and observations,
  2. computes action a_t,
  3. obtains reward r_t,
  4. obtains an observation o_{t+1},
  5. deduce a new state s^{α}_{t+1}.
- The environment...
  1. receives action a_t,
  2. produces reward r_t,
  3. deduce a new state s^{e}_{t+1},
  4. produces o_{t+1}.
- Observability:
  - Perfect: $s^{\alpha}_t = s^{e}_t = o_t$
  - Imperfect: no access to full environment state:
    - The agent indirectly observes the environment through o_t,
    - s^{α}_t is estimated by the agent and may differ from s^{e}_t.

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Example: PyRat

Definitions

- The agent is either the Rat or the Python,
- The opponent becomes part of the environment,
  - Note that the game can be with perfect observability if the opponent strategy is known,
- Seen this way, the game becomes sequential.

RL-based PyRat versus supervised approach

- Reward signal: number of picked up pieces of cheese,
- Delayed feedback: several moves required to reach a reward,
- Character's moves affect subsequent data it receives,
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Observability examples

- Perfect: $s_t^{rat} = s_t^o = o_t$
  - a_t: Last move of the rat,
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- Imperfect:
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Policy Function

Definition

The policy function of an agent α is:

$$\pi : \begin{cases} S^\alpha & \rightarrow & A \\ s_t^\alpha & \mapsto & a_t^\alpha \end{cases}$$

- π can be deterministic or stochastic.

Playout

The playout $(s_t^{\alpha,\pi})_{t\in\mathbb{N}}$ associated with a policy π and initial state s_0, is defined by considering agent α takes his/her actions using π.

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Fix $\gamma \in [0, 1[$, the value function v^π is defined as:

$$v^\pi : \begin{cases} S^\alpha & \to & \mathbb{R} \\ s_{t_0}^{\alpha,\pi} & \mapsto & \sum_{t=t_0}^{+\infty} \gamma^{t-t_0} r_t \end{cases}$$

- The value of a policy function is thus an expectation of cumulative future rewards, weakened by the geometrical coefficient γ to avoid divergence and prioritize short term reward,
- The best possible policy $\pi^*(s)$ is defined as:

$$\forall s \in S^\alpha, \forall \pi, V^{\pi^*}(s) \geq V^\pi(s).$$

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Q-learning

Definition

In Q-learning, we aim to find V^{π^*} as a solution to the recursive system of equations (Bellman equation):

$$\forall s \in S^{\alpha}, \forall a \in A, Q(s,a) = r_{s,a} + \gamma \max_{a'} Q(s(a), a'),$$

where $r_{s,a}$ is the reward agent α performs action a in state s and $s(a)$ is the state observed by agent α after performing action a.

Pros and cons

Pros:

- Can be learned even if the agent is not following any specific π,
- Self training is possible,

Cons:

- Scalability issues when S^{α} is large.

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Q-learning: value iteration

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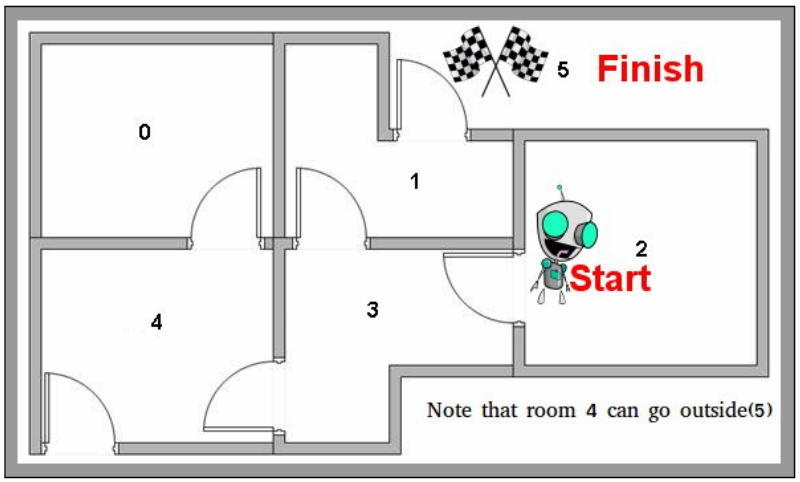
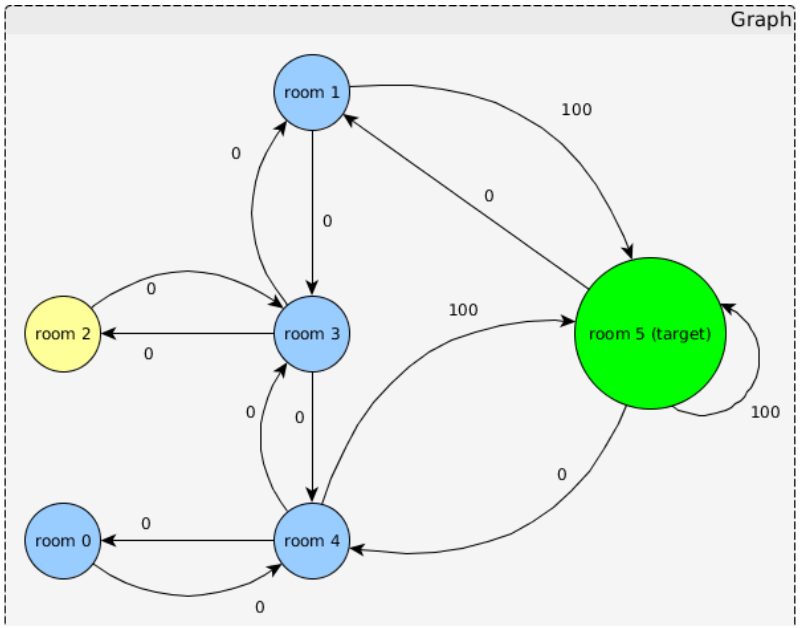
	UP	DOWN	RIGHT	LEFT
s1	+2	0	+1	+2.5
s2	0	+1	+4	+1
s3	0	+1	0	0
s4	0	+1	+2.5	+2.5

```
initialize Q[s,a] arbitrarily
observe initial state s
repeat
        select action a
        observe reward r and new state s'
        Q[s,a]=r + γ max(Q[s',a'])
        s=s'
until end
```

Example





https://leonardoaraujosantos.gitbook.io/artificial-inteligence/artificial_intelligence/reinforcement_learning/qlearning_simple

Example

Step 1: state 1, random action 5

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad R = \begin{array}{c} \\ \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} & & & \text{Action} & & \\ 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

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Update Q(1,5):

$Q(1,5)=R(1,5)+0.8.\max([Q(5,1),Q(5,4),Q(5,5)])=100+0.8(0)$

Step 2: ....

Approximated Q-learning

Definition

- Train a model to approximate Q,
  - Input is a state s and output is made of values of $Q(s, \cdot)$,
  - Representation learning can be used to compress S^{α}.

Problems

- Almost always needs a simulator for the game,
- Game duration can be bottleneck for training,
- Catastrophic forgetting and adversary specialization,
  - These effects can be alleviated by training using experience replay.

Experience replay

- Instead of using only the last decision to train, sample at random from the m previous decisions,
- Decisions taken before should remain considered now.

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Exploration/Exploitation

Dilemma

- Repeat with existing strategy (Exploitation)...
- ... or try a new strategy (Exploration)?

Example

- Always eating in restaurants that you know is exploitation,
- While that is a good heuristic, you have no way of knowing if you have the maximum reward possible,
- So exploring new restaurants from time to time may be needed to find the maximum reward.

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Lab Session 5

TP4 - PyRat with reinforcement learning

- Approximate Q-learning algorithm using experience replay and linear regression to beat the greedy algorithm,
- Approximation method (linear regression) and experience replay routine are given,
- Assemble all the primitives to perform Reinforcement Learning.

Challenge

You can continue working in the challenge after finishing TP4. You can now integrate reinforcement learning in your solution.