

## Part 1: Math Review

Limit Formula:  $\lim_{x \rightarrow 2} x^2 = 4$

Average speed formula:  $\frac{\text{distance travelled}}{\text{time taken}}$

Power rule for derivatives:  $\frac{d}{dx}(x^n) = nx^{n-1}$

### Exercise 1 (f, l, b, g)

f)  $x^2 + x \rightarrow x' + x^{-1}$

Step 1:  $f(x) = x^2 + x$  has two terms, so we differentiate each one:

I. Derivative of  $x^2$ :  $\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$

II. Derivative of  $x$ :  $\frac{d}{dx}(x^1) = 1x^{1-1} = 1$

III. Add together  $2x + 1$

Final answer:  $f'(x) = 2x + 1$

l)  $\frac{x^3 + 5}{x} \rightarrow f(x) = \frac{x^3}{x} + \frac{5}{x} \rightarrow x^2 + 5x^{-1}$

$x^2$ :  $\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$

$5x^{-1}$ :  $\frac{d}{dx}(x^2) = 5(-1)x^{-2} = -5x^{-2}$

Final answer:  $f'(x) = 2x - \frac{5}{x^2}$

b)  $\sqrt[3]{x} \rightarrow x^{\frac{1}{3}}$

$x^{\frac{1}{3}}$ :  $\frac{d}{dx}(x^{\frac{1}{3}}) \rightarrow \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} \rightarrow f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

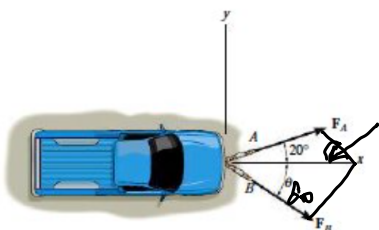
Final answer:  $f'(x) = \frac{1}{3x^{\frac{2}{3}}}$

g)  $\frac{5}{x^2\sqrt{x}} \rightarrow \sqrt{x} = x^{\frac{1}{2}} \rightarrow x^2\sqrt{x} = x^{2+1/2} = x^{5/2} \rightarrow f(x) = 5x^{-5/2}$

$f'(x) = 5 \cdot (-\frac{5}{2})x^{-5/2-1} \rightarrow f'(x) = -\frac{25}{2}x^{-7/2}$

Final answer:  $f'(x) = -\frac{25}{2x^{7/2}}$

2-14. The truck is to be towed using two ropes. Determine the magnitude of forces  $F_A$  and  $F_B$  acting on each rope in order to develop a resultant force of 950 N directed along the positive  $x$  axis. Set  $\theta = 50^\circ$ .

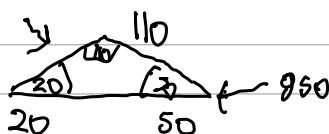


Prob. 2-14

Answer:

$F_A \approx 744.45 \text{ N}$

$F_B \approx 345.77 \text{ N}$



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{F_A}{\sin 50} = \frac{950}{\sin 110}$$

# Exercise 2

3. According to Wikipedia, the 2006 Renault R26) accelerates at the following rates:

- 0 to 100 km/h : 1.7 seconds
- 0 to 200 km/h: 3.8 seconds
- 0 to 300 km/h: 8.6 seconds

Compute the average accelerations of this car as it goes from 0 to 100 km/hr, from 100 km/hr to 200 km/hr and from 200 km/hr to 300 km/hr. Provide your answer in SI units.

In each case, how many g's does the driver experience? (  $1g = 9.8 \text{ m/s}^2$  )

4. An object is dropped from a certain height and hits the ground with speed 20 m/s.  
a) From what height was it dropped? b) How fast is it going when it is 5m from ground?

3. (1) We are given:

- 0 → 100 km/h in 1.7 s
- 0 → 200 km/h in 3.8 s
- 0 → 300 km/h in 8.6 s
- $1g = 9.8 \text{ m/s}^2$

(2) Convert speeds

- $100 \text{ km/h} = 100 \cdot \frac{1000}{3600} = 27.78 \text{ m/s}$
- $200 \text{ km/h} = 200 \cdot \frac{1000}{3600} = 55.56 \text{ m/s}$
- $300 \text{ km/h} = 300 \cdot \frac{1000}{3600} = 83.33 \text{ m/s}$

(3) interval 0 → 100 km/h (4) Answer

$$a = \frac{v-u}{t} = \frac{27.78-0}{1.7 \text{ s}} \quad \text{Avg acceleration:}$$

$$a_{0-100} = 16.34 \text{ m/s}^2 \approx 1.67g$$

$$a \approx 16.34 \text{ m/s}^2 \quad \text{In g's: } \frac{16.34}{9.8} \approx 1.67g$$

(7) Final Answer:

Interval	Speed Change	Time	Acceleration	g's
0 → 100 km/h	0 → 27.78	1.7 s	16.34	1.67g
100 → 200 km/h	27.78 → 55.56	2.1	13.26	1.35g
200 → 300 km/h	55.56 → 83.33	4.8	5.79	0.59g

(5) Interval 100 → 200 km/h

$$\Delta t = 3.8 - 1.7 = 2.1 \text{ s}$$

$$a = \frac{55.56 - 27.78}{2.1}$$

$$a \approx 13.26 \text{ m/s}^2$$

$$\text{In g's: } \frac{13.26}{9.8} \approx 1.35g$$

Answer: Avg acceleration  $a_{100-200}$   
 $13.26 \text{ m/s}^2 (\approx 1.35g)$

(6) Interval 200 → 300 km/h

$$\Delta t = 8.6 - 3.8 = 4.8 \text{ s}$$

$$a = \frac{83.33 - 55.56}{4.8} \approx 5.79 \text{ m/s}^2$$

$$\text{Answer: } 5.79 \text{ m/s}^2 (\approx 0.59g)$$

4. Using Kinetic equation:  $v^2 = u^2 + 2gh$

(a) From what height was it drop?

$$(20)^2 = 0 + 2(9.8)h$$

$$400 = 19.6h$$

$$h = \frac{400}{19.6}$$

$$h \approx 20.41 \text{ m}$$

$$\text{height} \approx 20.41 \text{ m}$$

(b) How fast is it going 5m from the ground?

$$20.41 - 5 = 15.41 \text{ m}$$

$$v^2 = 0 + 2(9.8)(15.41)$$

$$v^2 \approx 301.036$$

$$v \approx \sqrt{301.036} \approx 17.36 \text{ m/s}$$

Final Answer:

(a) 20.4 m

(b) 17.4 m/s