

by a compound nucleus in the capture of slow or moderately fast neutrons. For medium-weight nuclei, activation energies are of the order of 50 Mev, which is very large compared to the excitation energy of the compound nucleus due to the capture of any light nuclear projectile. Thus from the practical standpoint the fission process is primarily of interest as a mode of decay accessible to very heavy nuclei.

We shall now present the essence of the calculation of the activation energy made by Bohr and Wheeler.<sup>(12)</sup> This calculation is based upon the assumption that a heavy nucleus may be treated as a classical liquid droplet held together by a nuclear surface tension. According to this

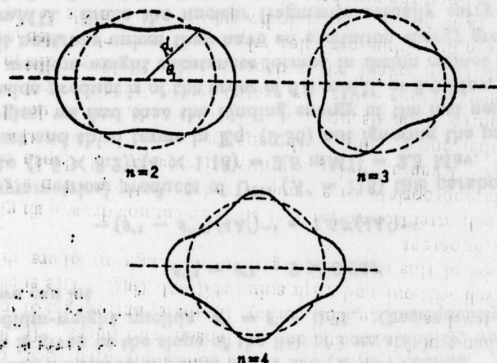


FIG. 9-7. Small deformations of a liquid drop of the type  $R' = R + \alpha_n P_n(\cos \theta)$  for  $n = 2, 3$ , and 4. The figure corresponding to  $n = 1$  is simply a circle displaced to the right a distance  $\alpha_1$ . (From Hill and Wheeler, Ref. 14.)

model, if a nucleus acquires the excitation energy  $W^*$ , it will undergo vibratory distortions. When the excitation energy exceeds  $W^*$ , the critical activation energy, then the nucleus in its vibratory motion may reach a critical deformation for fission before it loses part or all of its excitation energy by some other process, such as radiation or neutron emission. In this event it will spontaneously subdivide, releasing the very large fission energy. To calculate  $W^*$ , we must solve the equilibrium problem of a liquidlike droplet in various conditions of distortion, a problem which is one of the very difficult problems arising in classical physics. Nevertheless, by making certain simplifying assumptions and approximations, Bohr and Wheeler obtained a solution which provides a reasonable interpretation of the results for distortions arising in the cases which are of interest.

The only terms in the semiempirical nuclear energy formula which are changed appreciably by nuclear deformations are  $E_s$  and  $E_c$ . To treat

the energy changes connected with distortions, let us assume that, instead of a perfect sphere, the figure of the nucleus is described by a surface of revolution for which  $R' = R f(\theta)$ . Since one may express an arbitrary continuous function of a periodic variable as an expansion in the Legendre polynomials, let us write

$$f(\theta) = 1 + \alpha_1 P_1(\cos \theta) + \alpha_2 P_2(\cos \theta) + \dots + \alpha_n P_n(\cos \theta) + \dots \quad (9-38)$$

where (see Fig. 9-7)

$$P_1 = \cos \theta \quad (9-39)$$

$$P_2 = \frac{1}{2}(3 \cos^2 \theta - 1) \quad (9-40)$$

$$P_3 = \frac{1}{2}(5 \cos^3 \theta - 3 \cos \theta) \quad (9-41)$$

$$P_4 = \frac{1}{8}(35 \cos^4 \theta - 30 \cos^2 \theta + 3) \quad (9-42)$$

and the  $\alpha$ 's are a set of constants which characterize the distortion. For any given  $f(\theta)$  the deformation parameters (the  $\alpha$ 's) can be found by use of

$$\alpha_n = \frac{\int_0^\pi f(\theta) P_n(\cos \theta) \sin \theta d\theta}{\int_0^\pi P_n^2(\cos \theta) \sin \theta d\theta} \quad (9-43)$$

an equation which depends upon the well-known orthogonality properties of the Legendre functions. The surface area of any deformed droplet can then be calculated in terms of the deformation parameters by the formula

$$S = 2\pi R^2 \int_0^\pi f(\theta) \sin \theta \left[ f^2 + \left( \frac{df}{d\theta} \right)^2 \right]^{1/2} d\theta \quad (9-44)$$

The volume may be calculated using

$$V = \pi \int_{-1}^1 y^2 dx \quad (9-45)$$

where  $y = R' \sin \theta$  and  $x = R' \cos \theta$ . Since the incompressibility condition requires that the volume of the nucleus be constant, it imposes a restriction upon the distortion parameters, a condition which becomes important when these parameters are large. Most studies of the fission problem have been confined to the symmetrical case, for which the odd distortion parameters, that is,  $\alpha_1, \alpha_3, \dots$ , may be set to zero.

The total surface and coulomb energy can now in principle be calculated, the former rather simply since it is just the area multiplied by the surface tension. To obtain the coulomb energy, one must solve a rather complicated problem in electrostatics. Bohr and Wheeler in doing this