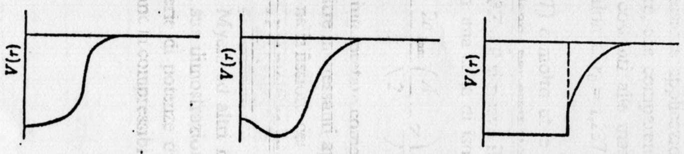
tial which may have the counterpart of the atomic role of the coulomb potential set up by the nucleus. Nevertheless, investigations were carried out based upon the assumption that each nucleon is effectively bound by an average central field created by the remaining nucleus. The shape of this central field determines the order of the quantum states and the corresponding degeneracy numbers. The earliest efforts based upon the one-particle model sought to find a well shape, which had a reasonable theoretical basis, which would give rise to the observed magic numbers. An excellent review of this early work is contained in a paper by Feenberg.[13]

In light nuclei a bell-shaped central field (see Fig. 11-5) serves nicely to produce the magic numbers 2, 8, and 20. However, in heavy nuclei the observed closed shells at 50 and 82 seem to require a "wine-bottle



The bell-shaped potential
FIG. 11-5.

The wine-bottle potential
FIG. 11-6.

The approximate nonlinear Meson potential
FIG. 11-7.

FIG. 11-5. The bell-shaped potential.
FIG. 11-6. The wine-bottle potential.
FIG. 11-7. The approximate nonlinear meson potential. (*From Malenka, Ref.* 17.)

potential" (see Fig. 11-6). Some justification for an effective potential with this shape has been given on the basis of the decreased central density of nucleons caused by the repulsion between protons. However, Swiatecki[14] has shown that this explanation is not adequate to produce a nucleus with a sufficiently hollow core to give rise to the magic numbers 50 and 82.

11-5. Spin-orbit Coupling Model. A promising proposal for generating the observed magic numbers has been made by Maria Mayer[15] and by Haxel, Jensen, and Suess,[16] who suggest that each nucleon possesses a strong spin-orbit energy of the type

$$W_{ls} = -\beta \frac{\mathbf{l}\cdot\mathbf{s}}{\hbar^2} \tag{11-40}$$

where β is a positive constant. Letting $\mathbf{i} = \mathbf{l} + \mathbf{s}$ denote the total angular momentum of an individual nucleon, it follows that $i = l \pm \frac{1}{2}$. Inserting these into the Landé formula, we find

$$\begin{aligned} \Delta W_{ls}(i = l - \tfrac{1}{2}) &= \tfrac{1}{2}\beta(l+1) \\ \Delta W_{ls}(i = l + \tfrac{1}{2}) &= -\tfrac{1}{2}\beta l \end{aligned} \tag{11-41}$$

These doublets are called inverted doublets, since the state of highest total angular momentum is the most stable, whereas in the well-known alkali-metal doublets the state of lowest total angular momentum is the most stable. By applying this spin-orbit splitting to the energy levels which go with the square well with rounded corners or the clipped oscillator potential and by using a reasonable value for β, it is possible to generate a level sequence which gives the magic numbers 2, 8, 20, 28, 50, 82, and 126 immediately.

Malenka[17] has carried out a quantitative calculation based upon the well (see Fig. 11-7)

$$\begin{aligned} V &= -V_0 & r < R \\ &= -\tfrac{3}{4}V_0 \exp\left[-\kappa(r-R)\right] & r > R \end{aligned} \tag{11-42}$$

with the well constants $V_0 = 34.1$ Mev, $R = 8.77 \times 10^{-13}$ cm, and $\kappa = 1.31 \times 10^{13}$ cm^{-1} and with the spin-orbit energy constant $\beta = 0.54$ Mev. This well, which has a possible theoretical basis in nonlinear meson theory, is quite similar to the square well with rounded corners so that the results obtained with it are consistent with those obtained from the scheme of Mayer and Haxel, Jensen, and Suess. In Fig. 11-8 we give the level sequence, degeneracy numbers, and magic numbers for Malenka's well without and with spin-orbit splitting. The success and reasonableness of this scheme for generating magic numbers should be immediately apparent.

We must call attention to the fact that we cannot draw a single set of energy levels appropriate to all nuclei and to both neutron states and proton states. Even assuming a universal nuclear well-depth constant V_0, the number and spacing of the energy levels vary with the radius, which, of course, depends upon A. Furthermore, the coulomb interaction between protons shifts the proton levels upward relative to the neutron levels to an extent which depends upon Z. In addition, interactions between individual particles, whose effects we shall discuss in the next section, raise or depress the individual-particle levels to an extent which depends upon the numbers of occupants of the levels. Nevertheless, it is possible to represent schematically the approximate energy levels of nuclei by a filling-order diagram which indicates the information which is most useful to the physicist. Figure 11-9 is such a diagram. The neutron orbitals shown are based upon the assignments of Klinkenberg,[20] which have been arrived at on the basis of the shell model with the aid of spin and magnetic-moment data. The proton orbitals have been positioned in relation to the neutron orbitals with consideration given to the effects of the coulomb interactions as well as the experimental spin and magnetic-moment data. Competing orbitals are grouped together. In nuclear physics this diagram may be used for all nuclei as