Matthew Olsen
February 14, 2016
CSE 122: Lab 1

Lab 1: Proof by Induction

Equation 1:

Equation:

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, where\ n\ is\ a\ positive\ integer$$

Base Case: Let n = 1. Thereby,

$$1^2 = \frac{1(1+1)(2+1)}{6} = 1$$

Assumption: If n = k, then

$$1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Prove: If n = k + 1, then

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+1)}{6}$$

Proof:

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 =$$
$$\frac{k(k+1)(2K+1)}{6} + \frac{6k^2 + 12k + 6}{6} = by\ assumption$$
$$\frac{2k^3 + 3k^2 + k + 6k^2 + 12k + 6}{6} =$$
$$\frac{2k^3 + 9k^2 + 13k + 6}{6} =$$
$$\frac{(k^2 + 3k + 2)(2k+3)}{6} =$$
$$\frac{(k+1)(k+2)(2k+1)}{6}$$

Equation 2:

Equation:

$$1^3 + 2^2 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2, where\ n\ is\ any\ positive\ integer$$

Base Case: If n = 1, then

$$1^3 = \left[\frac{1(1+1)}{2}\right]^2 = 1$$

Assumption: If n = k, then

$$1^3 + 2^3 + \cdots + k^3 = \left[\frac{k(k+1)}{2}\right]^2$$

Prove: If n = k + 1, then

$$1^3 + 2^3 + \cdots + k^3 + (k+1)^3 = \left[\frac{(k+1)(k+2)}{2}\right]^2$$

Proof:

$$1^3 + 2^3 + \cdots + k^3 + (k+1)^3 =$$

$\left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3 =$ by assumption

$$\frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4} =$$

$$\frac{k^2(k+1)^3 + (4k+4)(k+1)^2}{4} =$$

$$\frac{(k^2+4k+4)(k+1)^2}{4} =$$

$$\frac{(k+2)^2(k+1)^2}{4} =$$

$$\left[\frac{(k+1)(k+2)}{2}\right]^2$$

Equation 3:

Equation:

$$1 * 1! + 2 * 2! + \cdots + n * n! = (n+1)! - 1, where\ n\ is\ a\ positive\ integer$$

Base Case: If n = 1, then

$$1 * 1! = (1+1)! - 1 = 1$$

Assumption: If n = k, then

$$1 * 1! + 2 * 2! + \cdots + k * k! = (k+1)! - 1$$

Prove: If n = k +1, then

$$1 * 1! + 2 * 2! + \cdots + k * k! + (k+1) * (k+1)! = (k+2)! - 1$$

Proof:

$1 * 1! + 2 * 2! + \cdots + k * k! + (k+1) * (k+1)! =$

$(k+1)! - 1 + (k+1) * (k+1)! =$ by assumption

$1(k+1)! + (k+1)(k+1)! - 1 =$

$(k+1+1)(k+1)! - 1 =$

$(k+2)! - 1$

Equation 4:

PART 1:

Equation:

$$2^n > n^2\ when\ n > 4$$

Base Case: If n > 5, then

$$2^5 > 5^2, or\ 32 > 25, which\ is\ true$$

Assumption: If n = k, then

$$2^k > k^2\ when\ k > 4$$

Prove: If n = k + 1, then

$$2^{k+1} > (k+1)^2\ when\ k+1 > 4$$

Proof:

$$(k+1)^2 = k^2 + 2k + 1 < 2^k + 2k + 1\ (by\ assumption) < 2^k + 2^k (see\ second\ proof) = 2^{k+1}$$

PART 2:

Equation:

$$2n + 1 < 2^n\ when\ n > 4$$

Base Case: If n = 5, then

$$2(5) + 1 < 5^2 \text{ } or \text{ } 11 < 25, which \text{ } is \text{ } true$$

Assumption: If n = k, then

$$2k + 1 < 2^k$$

Prove: If n = k + 1, then

$$2(k + 1) + 1 < 2^{k+1}$$

Proof:

$$2(k + 1) + 1 = (2k + 1) + 2 < 2^k + 2 \text{ } (by \text{ } assumption) <$$
$$2^k + 2^k \text{ } (because \text{ } base \text{ } case \text{ } 2^5 > 2) = 2^{k+1}$$

Equation 5:

Equation:

$$1^3 + 3^3 + \cdots + (2n - 1)^3 = n^2(2n^2 - 1), where \text{ } n \text{ } is \text{ } a \text{ } positive \text{ } integer$$

Base Case: If n=1, then

$$1^3 = 1^2(2(1)^2 - 1)$$

Assumption: If n=k, then

$$1^3 + 3^3 + \cdots + (2k - 1)^3 = k^2(2k^2 - 1)$$

Prove: If n=k+1, then

$$1^3 + 3^3 + \cdots + (2k - 1)^3 + (2k + 1)^3 = (k + 1)^2(2(k + 1)^2 - 1)$$

Proof:

$$1^3 + 3^3 + \cdots + (2k - 1)^3 + (2k + 1)^3 =$$
$$k^2(2k^2 - 1) + (2k + 1)^3 = \qquad \text{by assumption}$$
$$2k^4 - k^2 + 8k^3 + 12k^2 + 6k + 1 =$$
$$2k^4 + 8k^3 + 11k^2 + 6k + 1 =$$
$$(k^2 + 2k + 1)(2k^2 + 4k + 1) =$$
$$(k + 1)^2(2(K^2 + 2k + 1) - 1) =$$
$$(k + 1)^2(2(k + 1)^2 - 1)$$

Equation 6:

Equation:

$$\frac{1}{1*2} + \frac{1}{2*3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}, where \text{ } n \text{ } is \text{ } a \text{ } positive \text{ } integer$$

Base Case: If n=1, then

$$\frac{1}{1*2} = \frac{1}{1+1} = \frac{1}{2}$$

Assumption: If n=k, then

$$\frac{1}{1*2} + \frac{1}{2*3} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

Prove: If n=k+1, then

$$\frac{1}{1*2} + \frac{1}{2*3} + \cdots + \frac{1}{k(k+1)} + \frac{k}{k+1} = \frac{k+1}{k+2}$$

Proof:

$$\frac{1}{1*2}+\frac{1}{2*3}+\cdots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=$$
$$\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}= by\ assumption$$
$$\frac{k(k+2)+1}{(k+1)(k+2)}=$$
$$\frac{(k+1)^2}{(k+1)(k+2)}=$$
$$\frac{k+1}{k+2}$$

Equation 7:

Equation:

$$\sum_{j=0}^{n} ar^j = \frac{ar^{n+1}-a}{r-1}, where\ a, r, and\ n\ are\ a\ positive\ integer$$

Base Case: If n=1, a=2, r =3, then

$$\sum_{j=0}^{1} 2*3^j = \frac{2*3^{1+1}-2}{3-1}$$
$$2+6=8$$

Assumption: If n=k, then

$$\sum_{j=0}^{k} ar^j = \frac{ar^{k+1}-a}{r-1}$$

Prove: If n=k+1, then

$$\sum_{j=0}^{k+1} ar^j = \frac{ar^{k+2}-a}{r-1}$$

Proof:

$$\sum_{j=0}^{k+1} ar^j =$$
$$\sum_{j=0}^{k} ar^j + ar^{k+1} =$$
$$\frac{ar^{k+1}-a}{r-1}+\frac{(r-1)ar^{k+1}}{r-1}=$$
$$\frac{ar^{k+1}-a}{r-1}+\frac{ar^{k+2}-ar^{k+1}}{r-1}=$$
$$\frac{ar^{k+2}-a}{r-1}$$

Equation 8:

Equation:

$$\sum_{i=1}^{n+1} i * 2^i = n * 2^{n+2} + 2 \; when \; n \geq 0$$

Base Case: If n = 1, then

$$\sum_{i=1}^{1+1} i * 2^i = 1 * 2^{1+2} + 2$$

$$2 + 2 * 2^2 = 2^3 + 2 = 10$$

Assumption: If n = k, then

$$\sum_{i=1}^{k+1} i * 2^i = k * 2^{k+2} + 2 \; when \; k \geq 0$$

Prove: If n = k + 1, then

$$\sum_{i=1}^{k+2} i * 2^i = (k+1) * 2^{k+3} + 2$$

Proof:

$$\sum_{i=1}^{k+2} i * 2^i =$$

$$\sum_{i=1}^{n+1} i * 2^i + (k+2) *^{k+2} =$$

$k * 2^{k+2} + (k+2) * 2^{k+2} + 2 =$ by assumption

$(2k+2)2^{k+2} + 2 =$

$(k+1)(2)2^{k+2} + 2 =$

$(k+1)2^{k+3} + 2$