

### Homework 3

1.

line	Cost	# of Times
1	$C_1$	$n - 6$
2	$C_2$	$n - 7$

$$T(n) = n(C_1 + C_2) - 6C_6 - 7C_2$$

Big-O:  $O(n)$

2.

line	Cost	# of Times
1	$C_1$	1
2	$C_2$	1
3	$C_3$	1

$$T(n) = C_1 + C_2 + C_3$$

Big-O:  $O(1)$

3.

line	Cost	# of Times
1	$C_1$	$n$
2	$C_2$	$n - 1$
3	$C_3$	$\frac{n(n+1)}{2} - 1$
4	$C_4$	$\frac{n(n+1)}{2} - n$
5	$C_5$	$n - 1$

$$T(n) = \frac{1}{2}n^2(C_3 + C_4) + n(C_1 + C_2 + \frac{1}{2}C_3 - \frac{1}{2}C_4 + C_5) - C_2 - C_3 - C_5$$

Big-O:  $O(n^2)$

4.

line	Cost	# of Times
1	$C_1$	$n + 1$
2	$C_2$	$\frac{(n+1)(n+2)}{2} - 1$
3	$C_3$	$\frac{n(n+1)(n+2)}{6}$
4	$C_4$	$\frac{n(n^2 + 6n + 5)}{2}$

$$T(n) = \frac{1}{6}n^3(C_3 + C_4) + n^2(\frac{1}{2}C_2 + \frac{1}{2}C_3 + C_4) + n(C_1 + \frac{1}{2}C_2 + \frac{1}{3}C_3 + \frac{5}{6}C_4) + C_1 + \frac{1}{2}C_2$$

Big-O:  $O(n^3)$

Work:

Lines 1 and 2 were calculated using the same methods from questions 1 – 3.

Lines three and four were calculated via the use of a table I had set up below:

i value	j for	k for	k body
1	2	2	1
2	3	2 + 3	1 + 2
3	4	2 + 3 + 4	1 + 2 + 3
...	...	...	...
n	n + 1	2 + 3 + ... + n + (n + 1)	1 + 2 + ... + n

Line 3 can be converted into the form  $\sum_{i=1}^n \frac{i(i+1)}{2}$ . This was then converted, using Wolfram alpha, to the form  $\frac{n(n+1)(n+2)}{6}$ .

Line 4 can be converted into the form  $\sum_{i=2}^{n+1} \frac{i(i+1)}{2} - 1$ . This was then converted, using Wolfram alpha, to the form  $\frac{n(n^2+6n+5)}{6}$ .

5. If  $f(n) = O(g(n))$ , then in  $\log(n) = O(n^\epsilon)$   $f(n)$  is equal to  $\log(n)$  and  $g(n)$  is equal to  $n^\epsilon$ .

$$\lim_{n \rightarrow \infty} \frac{\log(n)}{n^\epsilon} \rightarrow \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{n^\epsilon * \log(n)} \text{ (using l'Hospital's) } = \lim_{n \rightarrow \infty} \frac{1}{n * n^\epsilon * \log(n)} = \frac{1}{\infty * \infty * \infty} = 0$$

## Permutations:

begin lucky run

run n	time
1 250	0.0008
2 250	0.0012
3 250	0.0006
4 250	0.0007
5 250	0.0008
6 250	0.0006
7 250	0.0005
8 250	0.0005
9 250	0.0005
10 250	0.0007

avg for n = 250 for lucky is 0.0007 seconds

run n	time
1 500	0.0020
2 500	0.0030
3 500	0.0026
4 500	0.0022
5 500	0.0031
6 500	0.0024
7 500	0.0023
8 500	0.0022
9 500	0.0022
10 500	0.0021

avg for n = 500 for lucky is 0.0024 seconds

run	n	time
1	1000	0.0116
2	1000	0.0106
3	1000	0.0102
4	1000	0.0090
5	1000	0.0087
6	1000	0.0102
7	1000	0.0102
8	1000	0.0147
9	1000	0.0092
10	1000	0.0089

avg for n = 1000 for lucky is 0.0103 seconds

run	n	time
1	2000	0.0493
2	2000	0.0433
3	2000	0.0413
4	2000	0.0539
5	2000	0.0520
6	2000	0.0428
7	2000	0.0375
8	2000	0.0471
9	2000	0.0407
10	2000	0.0506

avg for n = 2000 for lucky is 0.0459 seconds

begin used run

run	n	time
1	2500	0.0003
2	2500	0.0003
3	2500	0.0003
4	2500	0.0002
5	2500	0.0003
6	2500	0.0003
7	2500	0.0003
8	2500	0.0003
9	2500	0.0003
10	2500	0.0003

avg for n = 2500 for used is 0.0003 seconds

run	n	time
1	5000	0.0006
2	5000	0.0007
3	5000	0.0006
4	5000	0.0010
5	5000	0.0010

6	5000	0.0006
7	5000	0.0008
8	5000	0.0007
9	5000	0.0006
10	5000	0.0006

avg for n = 5000 for used is 0.0007 seconds

run n	time
1 10000	0.0016
2 10000	0.0015
3 10000	0.0013
4 10000	0.0020
5 10000	0.0015
6 10000	0.0013
7 10000	0.0017
8 10000	0.0012
9 10000	0.0016
10 10000	0.0014

avg for n = 10000 for used is 0.0015 seconds

run n	time
1 20000	0.0045
2 20000	0.0025
3 20000	0.0027
4 20000	0.0031
5 20000	0.0029
6 20000	0.0027
7 20000	0.0034
8 20000	0.0030
9 20000	0.0027
10 20000	0.0030

avg for n = 20000 for used is 0.0031 seconds

run n	time
1 40000	0.0068
2 40000	0.0070
3 40000	0.0082
4 40000	0.0071
5 40000	0.0060
6 40000	0.0067
7 40000	0.0070
8 40000	0.0056
9 40000	0.0061
10 40000	0.0063

avg for n = 40000 for used is 0.0067 seconds

run n	time
1 80000	0.0159

2	80000	0.0202
3	80000	0.0129
4	80000	0.0150
5	80000	0.0132
6	80000	0.0183
7	80000	0.0184
8	80000	0.0147
9	80000	0.0137
10	80000	0.0165

avg for n = 80000 for used is 0.0159 seconds

begin knuth run

run	n	time
1	1000	0.0000
2	1000	0.0000
3	1000	0.0000
4	1000	0.0000
5	1000	0.0000
6	1000	0.0000
7	1000	0.0000
8	1000	0.0000
9	1000	0.0000
10	1000	0.0000

avg for n = 1000 for knuth is 0.0000 seconds

run	n	time
1	2000	0.0000
2	2000	0.0000
3	2000	0.0000
4	2000	0.0000
5	2000	0.0000
6	2000	0.0000
7	2000	0.0000
8	2000	0.0000
9	2000	0.0000
10	2000	0.0000

avg for n = 2000 for knuth is 0.0000 seconds

run	n	time
1	4000	0.0001
2	4000	0.0001
3	4000	0.0001
4	4000	0.0001
5	4000	0.0001
6	4000	0.0001
7	4000	0.0001

8 4000 0.0001

9 4000 0.0001

10 4000 0.0001

avg for n = 4000 for knuth is 0.0001 seconds

run n time

1 8000 0.0001

2 8000 0.0001

3 8000 0.0002

4 8000 0.0002

5 8000 0.0002

6 8000 0.0002

7 8000 0.0002

8 8000 0.0002

9 8000 0.0002

10 8000 0.0002

avg for n = 8000 for knuth is 0.0001 seconds

run n time

1 16000 0.0003

2 16000 0.0003

3 16000 0.0003

4 16000 0.0003

5 16000 0.0003

6 16000 0.0003

7 16000 0.0003

8 16000 0.0003

9 16000 0.0003

10 16000 0.0003

avg for n = 16000 for knuth is 0.0003 seconds

run n time

1 32000 0.0006

2 32000 0.0006

3 32000 0.0006

4 32000 0.0006

5 32000 0.0006

6 32000 0.0006

7 32000 0.0006

8 32000 0.0006

9 32000 0.0006

10 32000 0.0006

avg for n = 32000 for knuth is 0.0006 seconds

run n time

1 64000 0.0013

2 64000 0.0012

3 64000 0.0012

4 64000 0.0012

5 64000 0.0012

6 64000 0.0012

7 64000 0.0012

8 64000 0.0017

9 64000 0.0015

10 64000 0.0012

avg for n = 64000 for knuth is 0.0013 seconds