Mini Project 1

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October 7, 2022

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Part I A

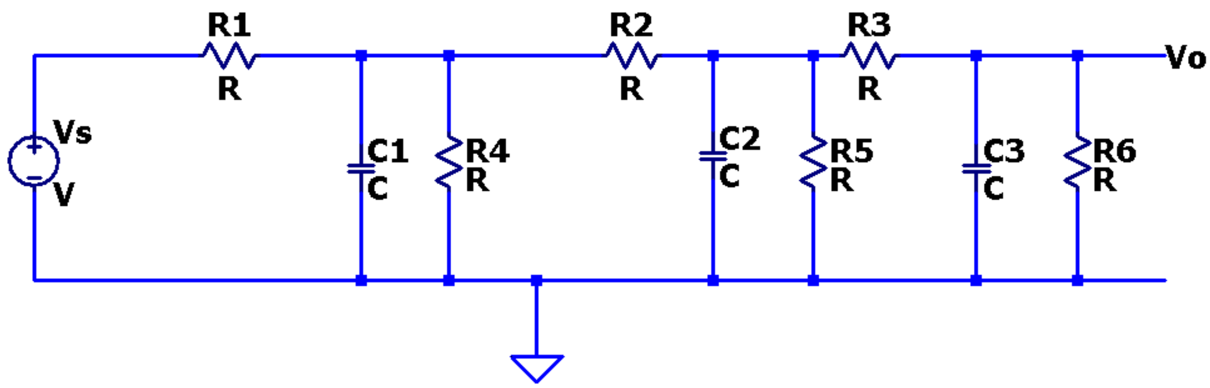


Fig 1.1: Circuit to be analyzed

The circuit that is being worked with is a low pass filter with the transfer function given as

$$\frac{V_o(s)}{V_s(s)} = 0.125 * \frac{5 * 10^5}{s + 5 * 10^5} * \frac{5 * 10^6}{s + 5 * 10^6} * \frac{5 * 10^7}{s + 5 * 10^7}$$

It is clear from the transfer function that the pass band gain is 0.125. The resistor values can be determined when the transfer function is securely in the pass band. At low frequencies, the capacitors can be represented with open circuits.

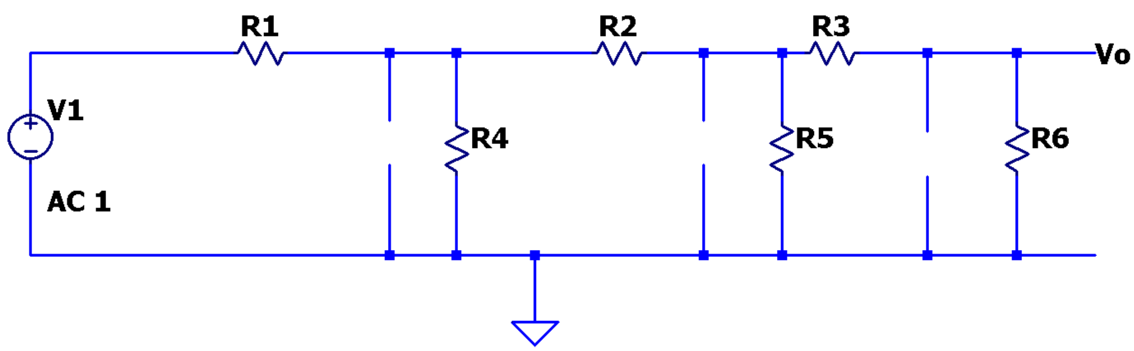


Fig 1.2: Circuit in pass band

By numerically solving a system of equations on an HP-Prime calculator, it was found that the transfer function of this circuit $\frac{V_o(s)}{V_s(s)}$ is equal to

$$\frac{R_4R_5R_6}{R_1R_2(R_3+R_5+R_6)+R_1R_3(R_4+R_5)+R_1R_4(R_5+R_6)+R_1R_5R_6+R_2R_3R_4+R_2R_4R_5+R_2R_4R_6+R_3R_4R_5+R_4R_5R_6}$$

By guessing and checking values, the transfer function could be made equal to 0.125 with the following resistances.

$R_1 = 500\Omega, R_2 = 500\Omega, R_3 = 500\Omega, R_4 = 1000\Omega, R_5 = 1000\Omega, R_6 = 500\Omega$

So that the open circuit short circuit approximation can be made, it will be assumed that $C_1 \geq 10 * C_2 \geq 10 * C_3$ .The open-circuit approximation involves shorting all the high frequency capacitors that contribute to the high pass cut-off, however there are none in this case. The remaining capacitances can be solved for by choosing the frequency to be around one of the poles. One can solve for the value of the capacitor that causes said pole by shorting the lower frequency capacitors, open circuiting the higher frequency capacitors, and shorting the source voltage. One can use the formula $\omega_{p_n} \approx \frac{1}{C_n * Req_n}$ where the pole at angular frequency ω_{pn} is caused by the capacitor C_n, and the capacitor sees an equivalent resistance Req_n.

At lower frequencies, the smaller capacitors C_2 and C_3 approach open circuits.

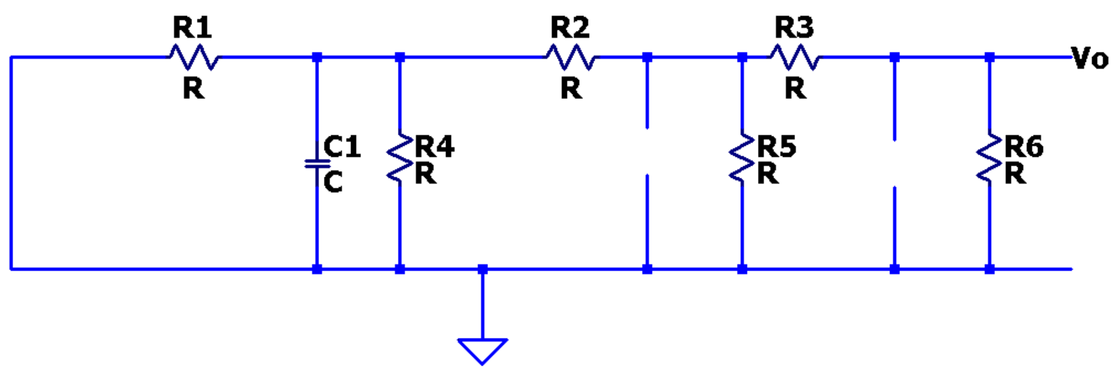


Fig 1.3: Open circuit short circuit test to determine C1

The equivalent resistance seen by capacitor C_1 is $R_1||R_4||(R_2 + R_5||(R_3 + R_6))$, and thus

$$5 * 10^5 \approx \frac{1}{C_1 * R_1||R_4||(R_2 + R_5||(R_3 + R_6))}$$

Solving for C_1, the result is $C_1 = 8 * 10^{-9}F = 8nF$

At high frequencies, the larger capacitors C_1 and C_2 approach short circuits and one can find the value of C_3.

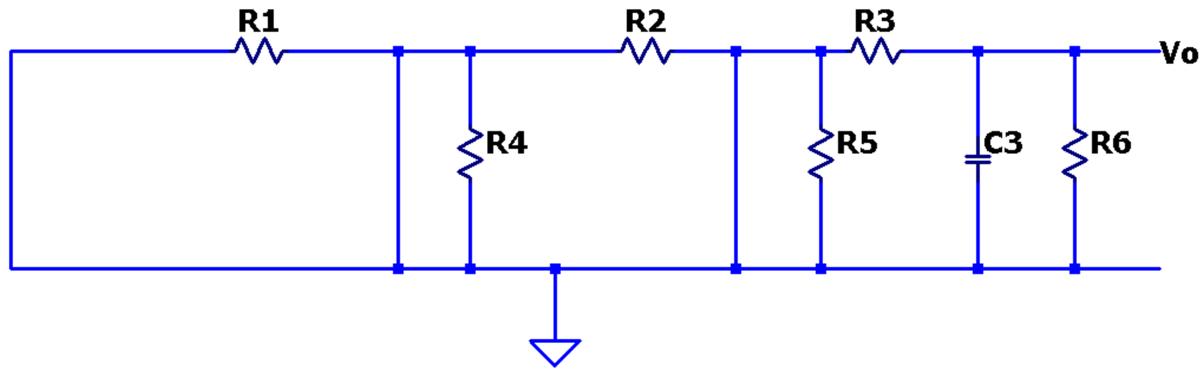


Fig 1.2: Open circuit short circuit test to determine C3

It can be found that, $R_{eq} = R_3||R_6$, $5 * 10^7 \approx \frac{1}{C_3 * R_3||R_6}$ so $C_3 = 8 * 10^{-11} = 80pF$

One can solve for C_2 by choosing a frequency around the middle pole $5 * 10^6$ where C_1 looks like a short circuit and C_3 looks like an open circuit.

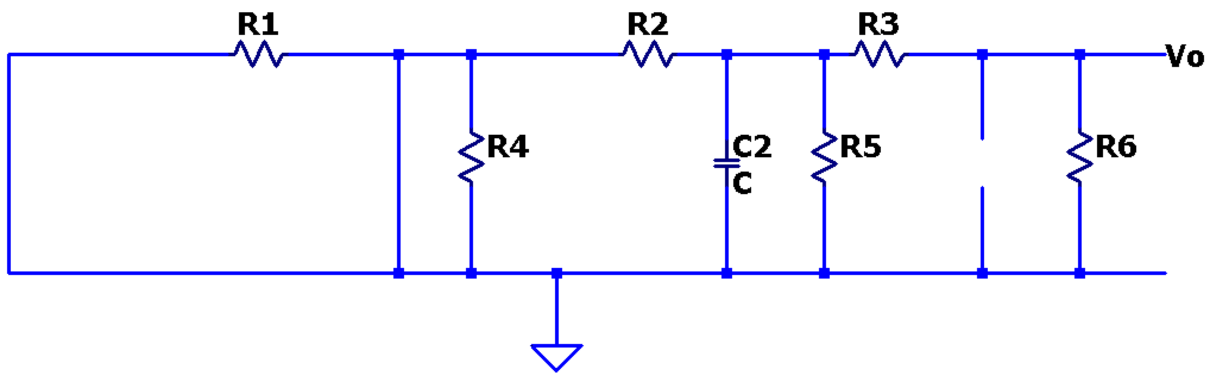


Fig 1.3: Open circuit test to determine C2

By solving for the equivalent resistance, it can be found that

$R_{eq} = R_5||(R_3 + R_6)||R_2$, $5 * 10^6 \approx \frac{1}{C_2 * R_5||(R_3+R_6)||R_2}$, so $C_2 = 8 * 10^{-10} = 800pF$

A bode plot containing both the approximated circuit and the transfer function will be included in part B.

Part I B

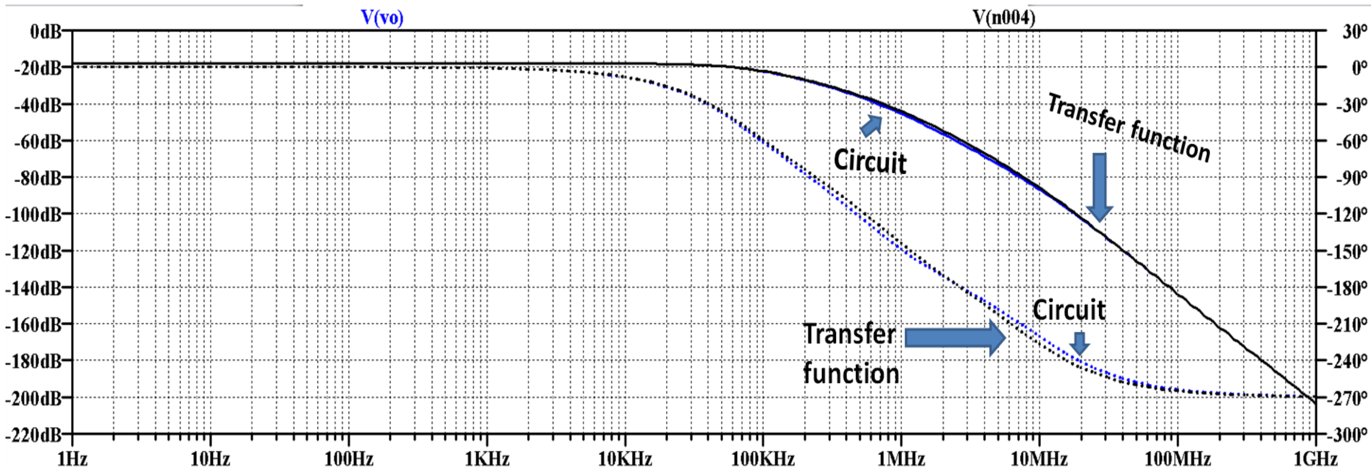


Fig 1.4: Bode plot of amplitude(solid line) and phase (dotted line)

The transfer function and the circuit approximation have excellent correspondence in the pass band, and the magnitudes are still close in the stop band. The phase plots correspond very well in the pass band, but they are a bit off in the stop band.

Problem II A

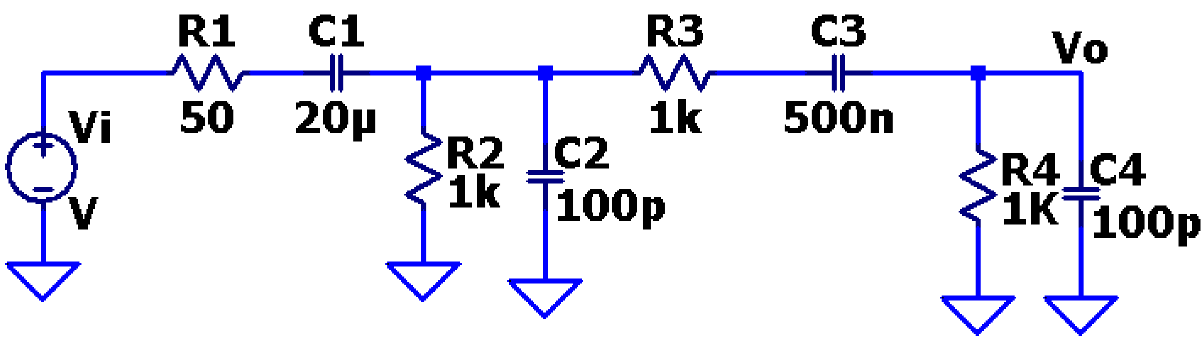


Fig 2.1: Band pass filter to be modelled, $C_3 = 500nF$

The poles can be graphically estimated from analysis of the bode plot generated in LTspice. The analysis is done for $C_3 = 1\mu F$.

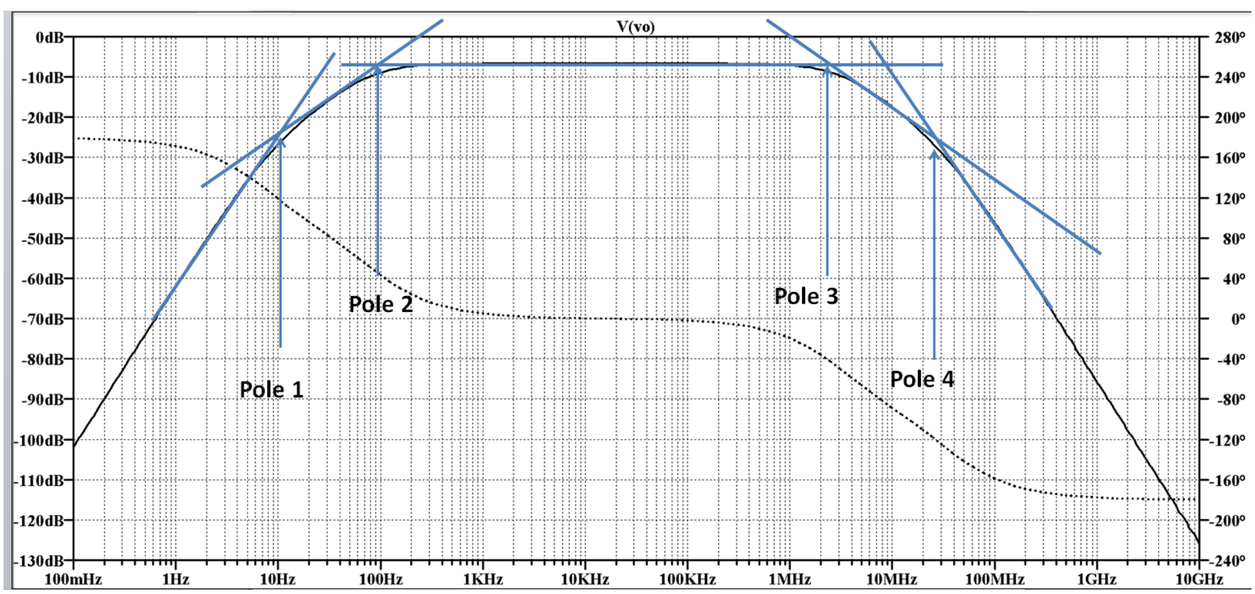


Fig 2.2: Approximating poles of the bode plot using tangent lines, $C_3 = 1\mu F$

From finding the intersection of the 40dB/decade line with the 20dB/decade line and the intersection between the 20dB/decade line with the 0dB/decade line, one can estimate the values of the low frequency poles. Similarly, one can estimate the values of the high frequency poles by looking for the intersection between the 0dB/decade line with the -20dB/decade line and the intersection between the -20dB/decade line and the -40dB/decade line.

From finding the locations of these intersections on the LTspice plot, the following pole frequencies were obtained.

$f_{p1} = 10.934Hz, f_{p2} = 94.970Hz, f_{p3} = 2.388MHz, f_{p4} = 25.366MHz$

The mid-band value of the bode plot is -6.649dB, so the 3dB frequencies can be found my moving the graph cursor to -9.649dB. The 3dB frequencies that were found from the LTspice simulations are displayed in the table below.

C_3	f_{3dB}
$500nF$	$160.153Hz$
$1\mu F$	82.939Hz
$2\mu F$	44.741Hz
$5\mu F$	22.699Hz
$10\mu F$	15.933Hz

Table 2.1: 3dB frequencies at different C3 values

Problem II B

3dB frequencies can be estimated from the circuit using the open circuit short circuit method. In order to analyze the low frequency corner of the bode plot, the high frequency (low capacitance) capacitors can be approximated as open circuits. After shorting the source and the other low frequency capacitor, the time constants can be calculated as $\tau_n = C_n Req_n$ where Req_n is the equivalent resistance seen by the nth capacitor. The time constants can then be used to calculate the low corner frequency.

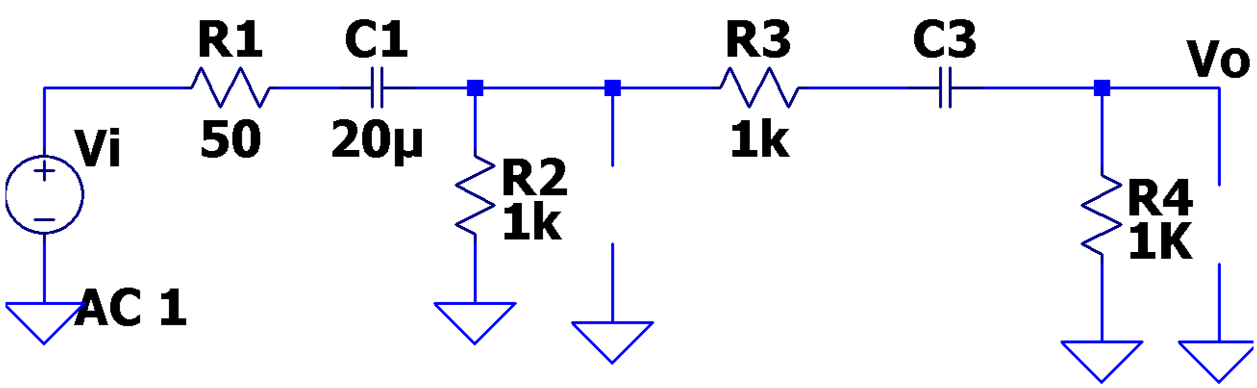


Fig 2.2: Low frequency circuit equivalent

To find the time constant τ_{C1} associated with C_1, C_3 and the source must be shorted.

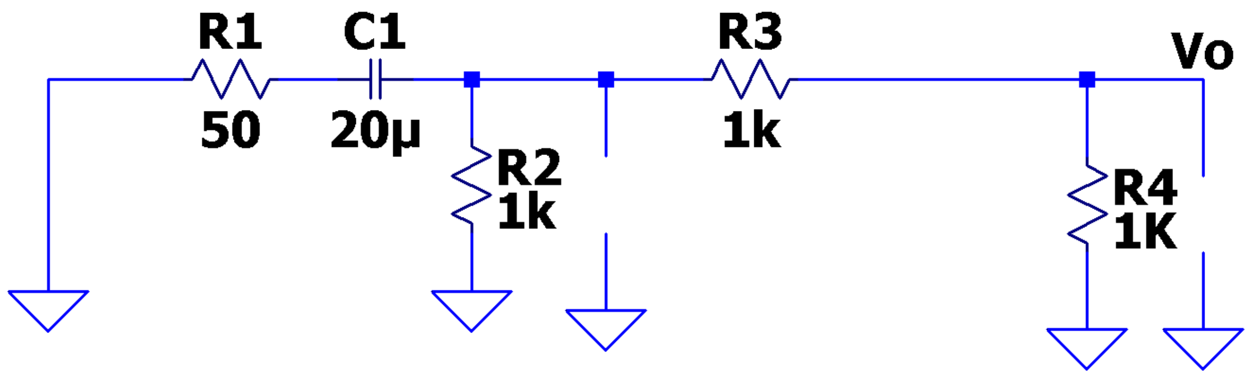


Fig 2.3: Open-short to determine $\tau C1$

$Req_{C1} = 50 + 1000||(1000 + 1000)\Omega = 716.667\Omega$

$\tau_{C1} = 716.667 * 20 * 10^{-6} = 1.433 * 10^{-2}s$

To find τ_{C3}, one must short the source and C_1.

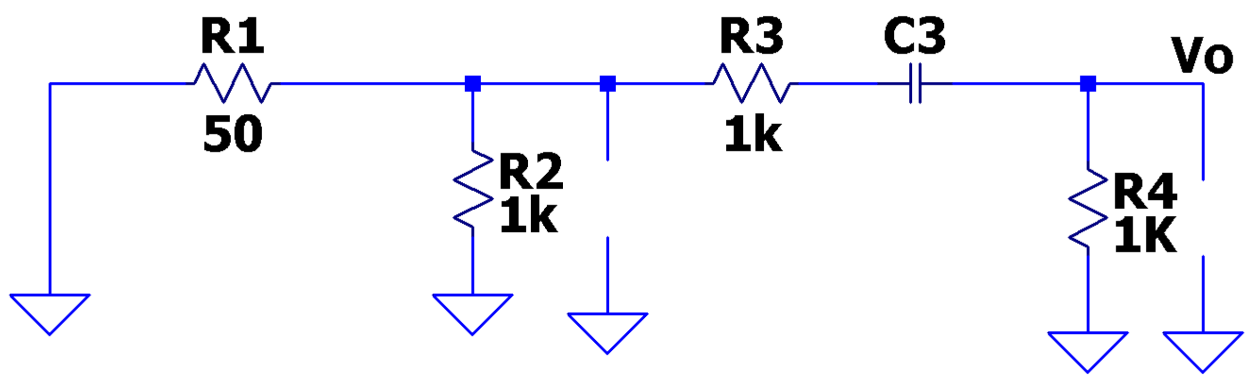


Fig 2.4: Open-short to determine $\tau C3$

$Req_{C3} = 1000 + 50||1000 + 1000\Omega = 2047.619\Omega$, $\tau_{c3} = C_3 * 2047.619$

The 3db angular frequency can be approximated as

$$\omega_{L3dB} \approx \frac{1}{\tau_{C1}} + \frac{1}{\tau_{C1}} = 69.7674 + \frac{1}{2047.619 * C_3} \frac{radians}{s}$$

To convert from radians to hertz, the formula is $f_{L3dB} = \frac{\omega_{L3dB}}{2\pi}$

Here, relative error will be calculated as the deviation of the open-short method's frequencies with respect to the graphical results.

$$error = \frac{|graphical - open,short|}{open,short} * 100\%$$

C_3 value	Graphical	Open-short estimate	Error
500nF	160.153Hz	166.557	3.999%
1µF	82.939Hz	88.831	7.104%
2µF	44.741	49.967	11.681%
5µF	22.699	26.649	17.402%
10µF	15.933	18.877	18.477%

Table 2.2: 3dB frequencies and their open-short approximations

It seems that the error is consistently under 20%. There is a clear trend of lower capacitances having more accurate lower 3dB frequency estimates. A potential explanation might be that the lower values of C_3 are further from the value of C_1 which results in the low frequency poles being spaced further apart which could improve the accuracy of the approximation.

Problem III A

The circuit to be analyzed is

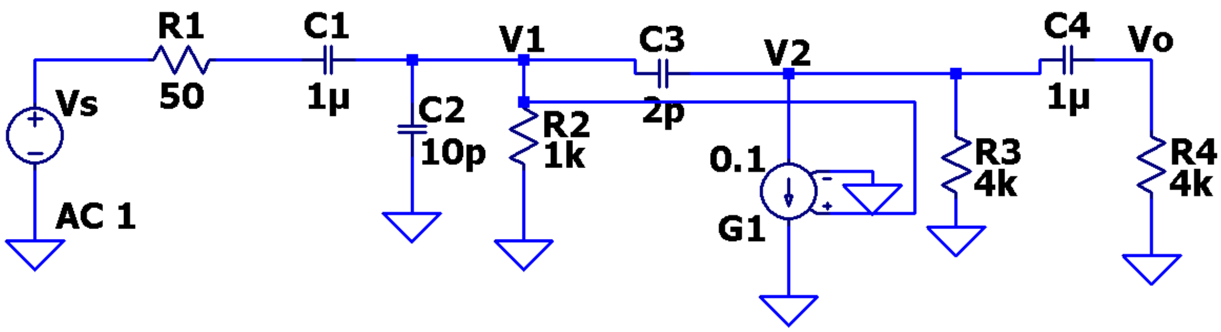


Fig 3.1: Band pass filter with controlled current source

The first step is to calculate the mid-band gain. In the mid-band, all the high frequency capacitors C_2 and C_3 become open circuits and all the low frequency capacitors C_1 and C_4 become short circuits. The simplified circuit will appear as below.

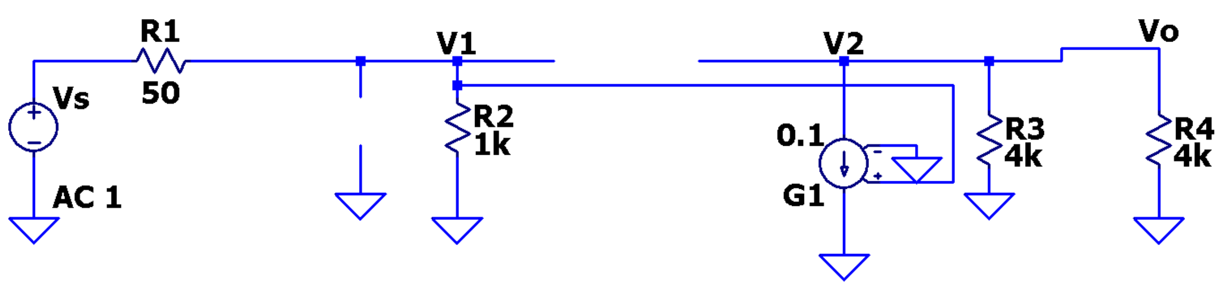


Fig 3.2: Mid band equivalent circuit

By setting up and solving a system of equations to solve for $\frac{V_O}{V_S}$ it was determined that the mid-band gain is -190.476.

At high frequencies, the circuit can be split using Miller's Theorem.

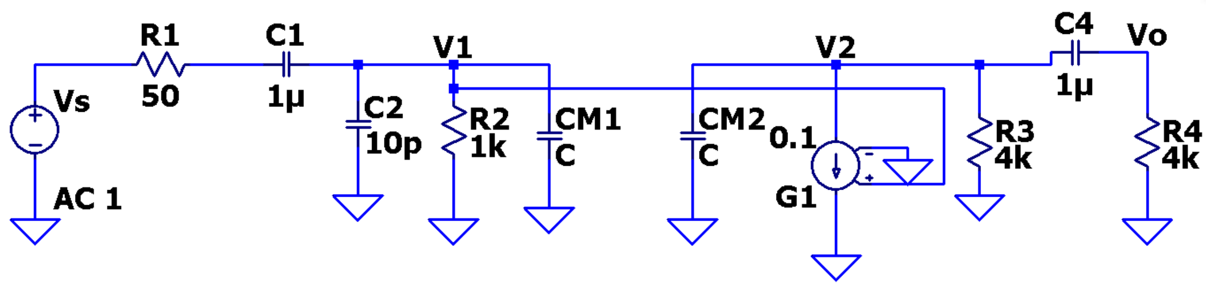


Fig 3.3 Circuit split with Miller's Theorem

At high frequencies, $C_4 = 1\mu F$ and $C_1 = 1\mu F$ can be approximated as short circuits which means that $V_2 = V_o$. By applying Miller's theorem to the capacitors, it can be found that $C_{M1} = C_3 * (1 - K)$ and $C_{M2} = C_3 * (1 - \frac{1}{K})$ where $K = \frac{V_2}{V_1} = \frac{V_o}{V_1}$.

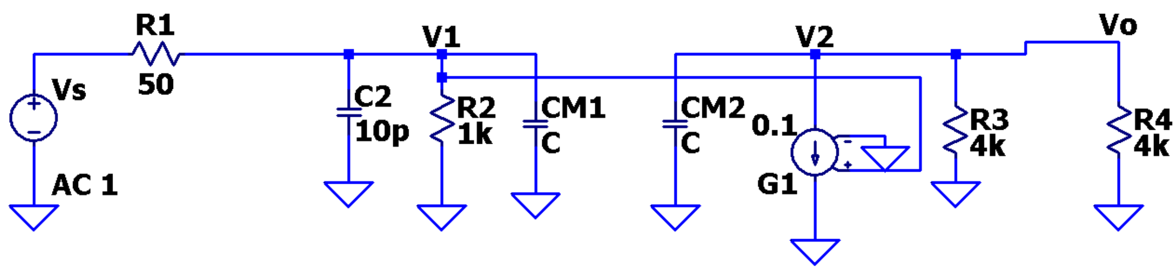


Fig 3.4 High frequency Miller's approximation of the circuit

If the current flowing through R_3 and R_4 is equal to the current flowing through the dependant source, it can be said that $V_o = -0.1 * V_1 * 4000||4000 = -200 * V_1$ and $K = -200$. This means that $C_{M1} = 402pF$ and $C_{M2} = 2.01pF$.

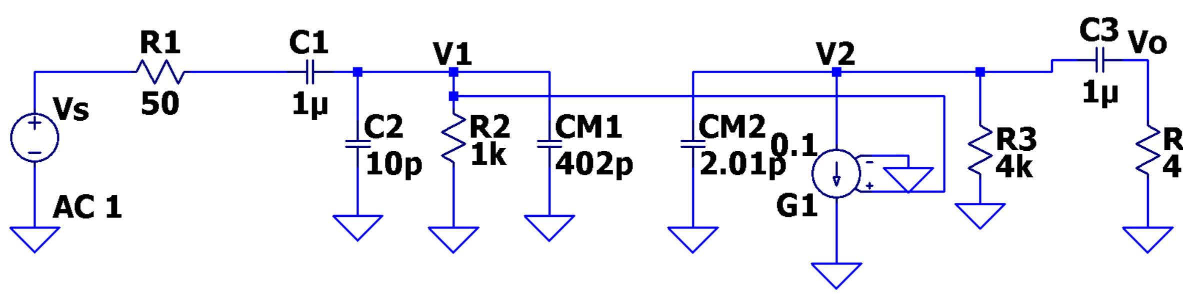


Fig 3.5 Circuit with Miller's capacitances added

High frequency poles can be calculated by opening the current source, shorting the voltage source, replacing two high frequency capacitors with open circuits and finding the equivalent resistance seen by the other. C_2 and C_{M1} will be combined into a new capacitor $C_{new} = 410pF$. For C_{new}, it can be found that $Req_2 = 50||1000 = 47.619\Omega$ and thus

$\tau_{C2} = 412 * 10^{-12} * 47.619 = 1.962 * 10^{-8}s$, and the associated pole's angular frequency is $\omega_{Cnew} = 5.097 * 10^{7}\frac{radians}{s}$. The pole frequency in Hz is $f_{Cnew} = 8.112 * 10^{6}$.

For C_{M2}, $Req_{CM2} = 4000||4000 = 2000\Omega$ and thus $\tau_{CM2} = 2000 * 2.01 * 10^{-12} = 4.02 * 10^{-9}$ and $\omega_{CM2} = 2.488 * 10^{8}\frac{radians}{s}$. Finally, the associated pole frequency is $f_{CM2} = 3.959 * 10^{7}Hz$

The high 3dB frequency can be estimated as $\omega_{H3dB} = \frac{1}{\tau_{Cnew}+\tau_{CM2}} = 4.23 * 10^{7}\frac{radians}{s}$ so $f_{H3dB} = 6.732 * 10^{6}Hz.$

In order to calculate the low frequency poles, the high frequency capacitors C2 and C3 can be approximated as open circuits.

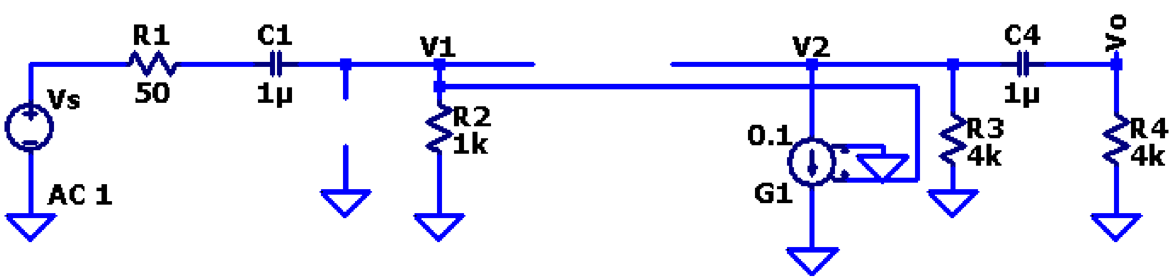


Fig 3.6: Low frequency equivalent circuit

Low frequency poles can be calculated by shorting the current source, opening the voltage source, replacing one low frequency capacitor with a short circuit, and finding the equivalent resistance seen by the other. For C_1, it can be found that $R_{eqC1} = 50 + 1000 = 1050\Omega$.

$\tau_{C1} = 1 * 10^{-6} * 1050 = 1.05 * 10^{-3}s$, $\omega_{PC1} = 952.381\frac{radians}{s}$, and $f_{PC1} = 151.576Hz$.

For C_4, it can be found that $R_{eqC4} = 4000 + 4000 = 8000\Omega$.

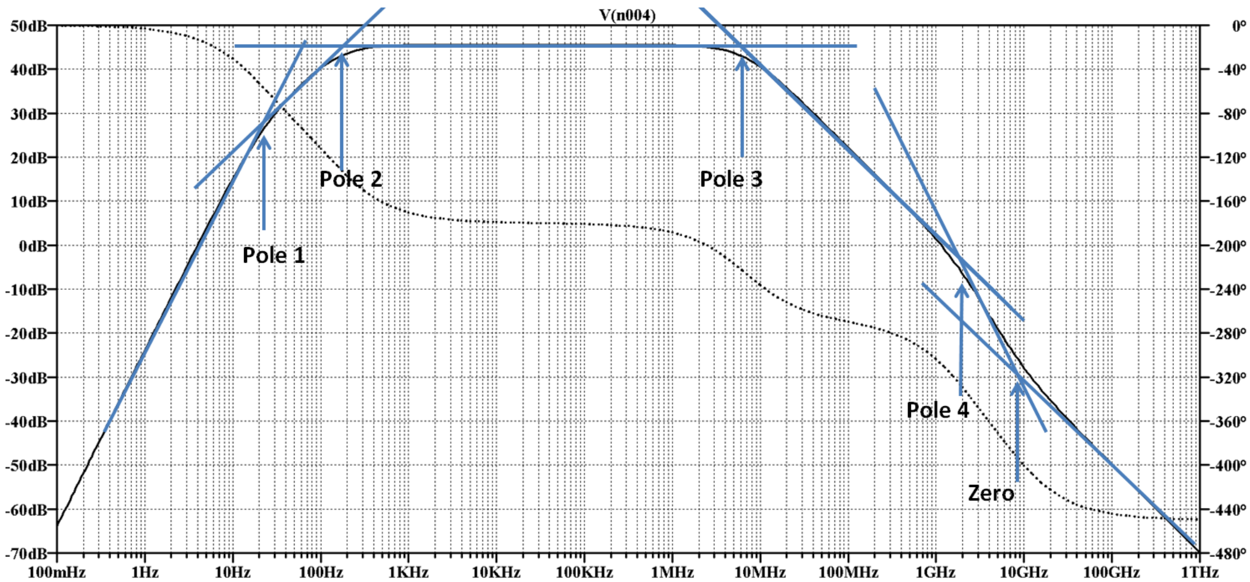
$\tau_{C4} = 8000 * 1 * 10^{-6} = 8 * 10^{-3}s$, $\omega_{PC4} = 125\frac{radians}{s}$, and $f_{PC1} = 19.894Hz$.

The low 3dB frequency can be estimated with the formula
$\omega_{L3dB} \approx \frac{1}{\tau_{C1}} + \frac{1}{\tau_{C4}} = 1077.381\frac{radians}{s}$ and $f_{L3dB} = 171.471Hz$

Listing the estimated pole frequencies in ascending order,

$f_{P1} = 19.894Hz$, $f_{P2} = 171.471Hz$, $f_{P3} = 8.112MHz$, and $f_{P4} = 39.591MHz$.

Problem III B



<u>Fig 3.7 Approximating poles on the bode plot</u>

By using a cursor on the graph it can be found that

Pole1=24.130Hz, Pole2=186.094Hz, Pole3=6.028MHz, Pole4=1.970GHz, Zero=8.198GHz. The results of this graphical analysis can be compared with the values obtained from the open circuit-short circuit method by finding the deviation of the graphically determined values with respect to the open-short approximated value deviation= $\frac{|graphical-open-short|}{open-short} * 100\%$. The location of the zero cannot be compared with any non-graphically estimated value.

	Graphical estimation	Open-short estimation	Relative deviation
Pole 1	24.130Hz	19.894Hz	21.293%
Pole 2	186.094Hz	171.471Hz	8.528%
Pole 3	6.028MHz	8.112MHz	25.690%
Pole 4	1.970GHz	39.591MHz	4875.878%
Zero	8.198GHz	N.A.	N.A.
High 3dB	6.769MHz	6.732MHz	0.550%
Low 3dB	154.981Hz	171.471Hz	9.617%

Table 3.1: Pole approximations

For poles 1, 2, and 3, the correspondence was impressive considering the haphazard nature of the graphical pole estimates. The 3dB frequencies also showed very good correspondence between the two methods. The graphical estimation of the 4th pole was wildly inaccurate; this may have to do with the closeness of the zero and the 4th pole on the bode plot. There does not appear to be any section of the high frequency stop band that actually has a slope of -40dB/decade, which makes it unfeasible to accurately locate the 4th pole on the graph. Since Miller's theorem will remove the zero, there may be a greater correspondence with a Bode Plot generated based on the circuit after applying Miller's theorem.

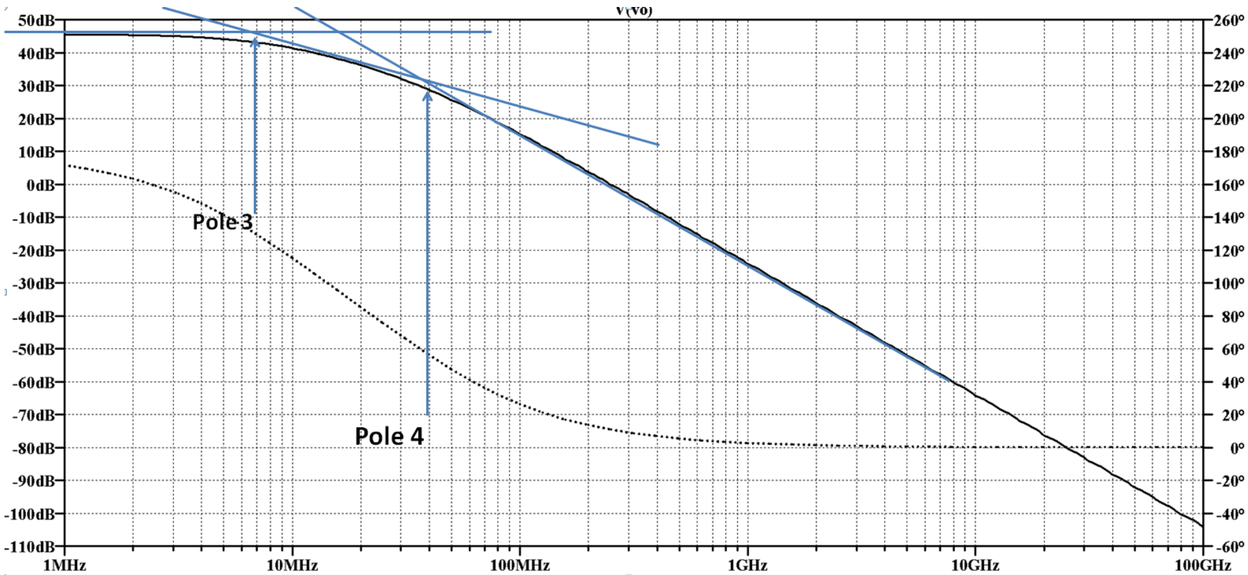


Fig: 3.8 Approximating the high frequency poles using the Miller circuit bode plot

	Graphical	Graphical (Miller's Circuit)	Open-Short Method	Error Between Open and Miller graph
Pole 3	6.028MHz	6.822MHz	8.112MHz	15.902%
Pole 4	1.970GHz	39.315MHz	39.591MHz	0.697%

Table 3.2: High frequency pole approximations

It seems that whether or not Miller's theorem is applied has a profound effect on the location of the 4th pole. With the previous result in mind, it is clear that, despite the uncertainty in the graphical estimate, applying Miller's theorem to the circuit has produced a highly inaccurate value for the 4th pole frequency. Since the frequencies of the first, second, and third poles are similar when calculated using either method, it is unclear which approximation is more accurate.

References

1) ELEC 301 Course notes
2) https://freebiesupply.com/logos/ubc-logo/ (Image credits)
3) http://jeastham.blogspot.com/2011/10/thick-lines-in-ltspice.html