Mini Project 4

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Part A 1)

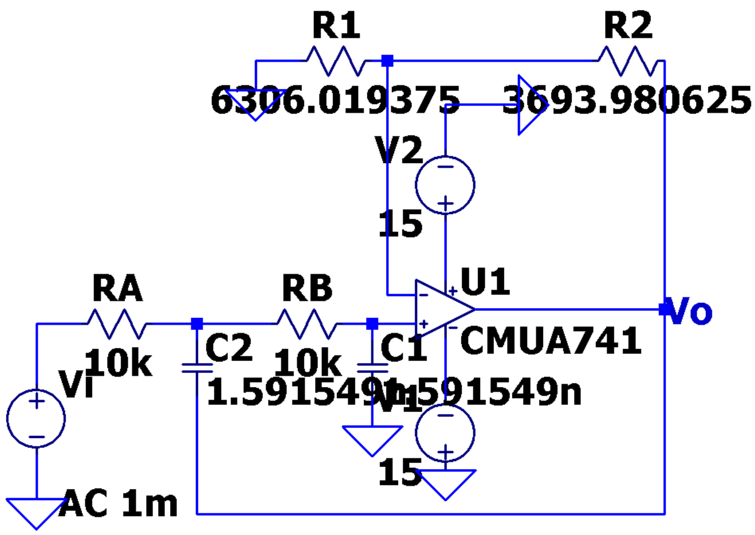

Figure A1: Active filter circuit

This active filter has a transfer function given by $H(s) = A_M \frac{\frac{1}{(RC)^2}}{s^2+s\frac{3-A_M}{RC}+\frac{1}{(RC)^2}}$ and $A_M = 1 + \frac{R_2}{R_1}$.

To be a 2nd order Butterworth filter, we need poles with angles of 135° and 225° and a magnitude equal to the pole frequency $\omega_p \approx \omega_{3dB} = 10000 * 2\pi$. The poles can be determined with the formula $s^2 + s\frac{3-A_M}{RC} + \frac{1}{(RC)^2} = s^2 + 2\zeta\omega_p s + \omega_p^2$ so $\omega_p = \frac{1}{RC}$ and $\zeta = \frac{3-A_M}{2}$. $R = 10K\Omega$. The Butterworth polynomial for a second order Butterworth filter with $\omega_p = 1$ is $s^2 + \sqrt{2}s + 1$ so $\zeta = \frac{1}{\sqrt{2}}$ and $\frac{3-A_M}{2} = \frac{1}{\sqrt{2}}$ so $A_M = 3 - \sqrt{2}$.

We have two simultaneous equations, $1 + \frac{R_2}{R_1} = 3 - \sqrt{2}$, $R_1 + R_2 = 10000$ with the solution $R_1 = 6306.019375\Omega$ and R_2=3693.980625 **Ω** which gets us $A_M = 1.5857864\frac{V}{V}$

We can solve for the capacitors required to get our desired pole,

$C = \frac{1}{R*\omega_p} = \frac{1}{10^4*10^4*2\pi} = 1.591549 * 10^{-9}F$.

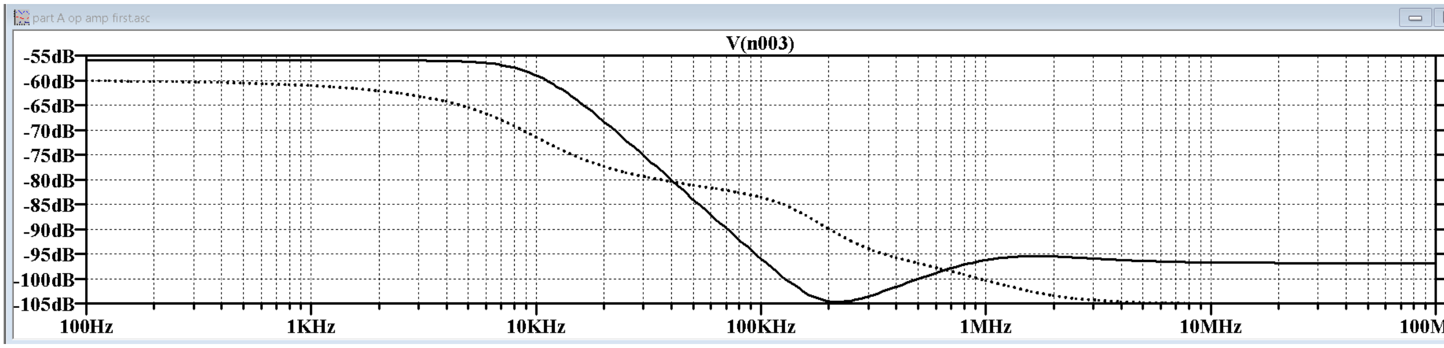

Figure A2: Bode plot of amplitude(solid) and phase(dotted)

Part A 2)

The amplifier will oscillate if σ=0 at the poles to ensure the response does not grow or decay in the time domain, the input is grounded to remove any forcing factor from the time domain differential equation, and a nonzero initial condition is placed on the output node, we choose Vout(t=0)=2mV. We can get σ =0 if the first order term is removed from the denominator, this happens if $A_M = 3$, so $R_2 = 6666.666\Omega$ and $R_1 = 3333.333\Omega$. The denominator becomes $s^2 + \frac{1}{(RC)^2}$ and the only poles are purely imaginary.

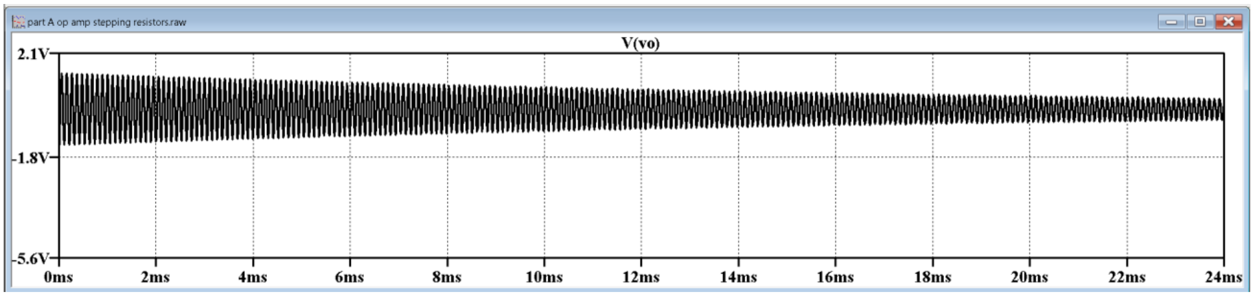

Figure A3: Oscillation with $R_2 = 6666.666\Omega$ and $R_1 = 3333.333\Omega$

The oscillation actually decays which indicates that σ is still negative, we will try new values $R_2 = 6766.666\Omega$ and $R_1 = 3233.333\Omega$

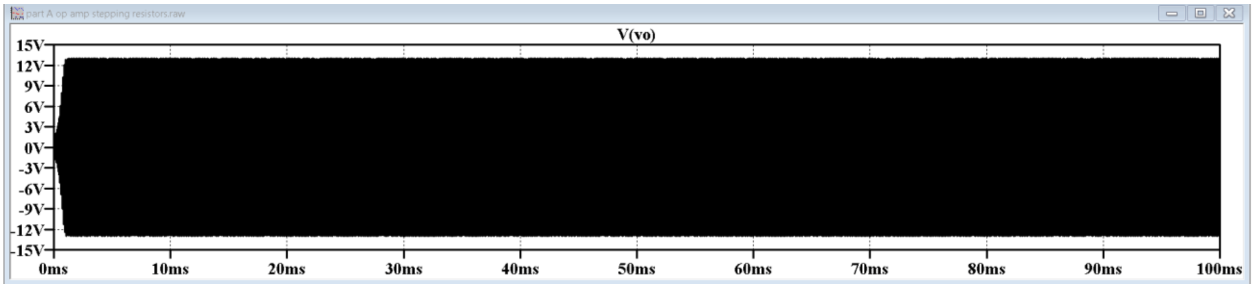

Figure A4: Oscillation with $R_2 = 6766.666\Omega$ and $R_1 = 3233.333\Omega$

We are able to achieve stable oscillation with $A_M = 1 + \frac{6766.666}{3233.333} = 3.09287\frac{V}{V}$ as opposed to the predicted $A_M = 3\frac{V}{V}$. By zooming in, we can determine the frequency of this oscillation as 8.2922519KHz.

We can draw a root locus corresponding to this circuit to describe what happened

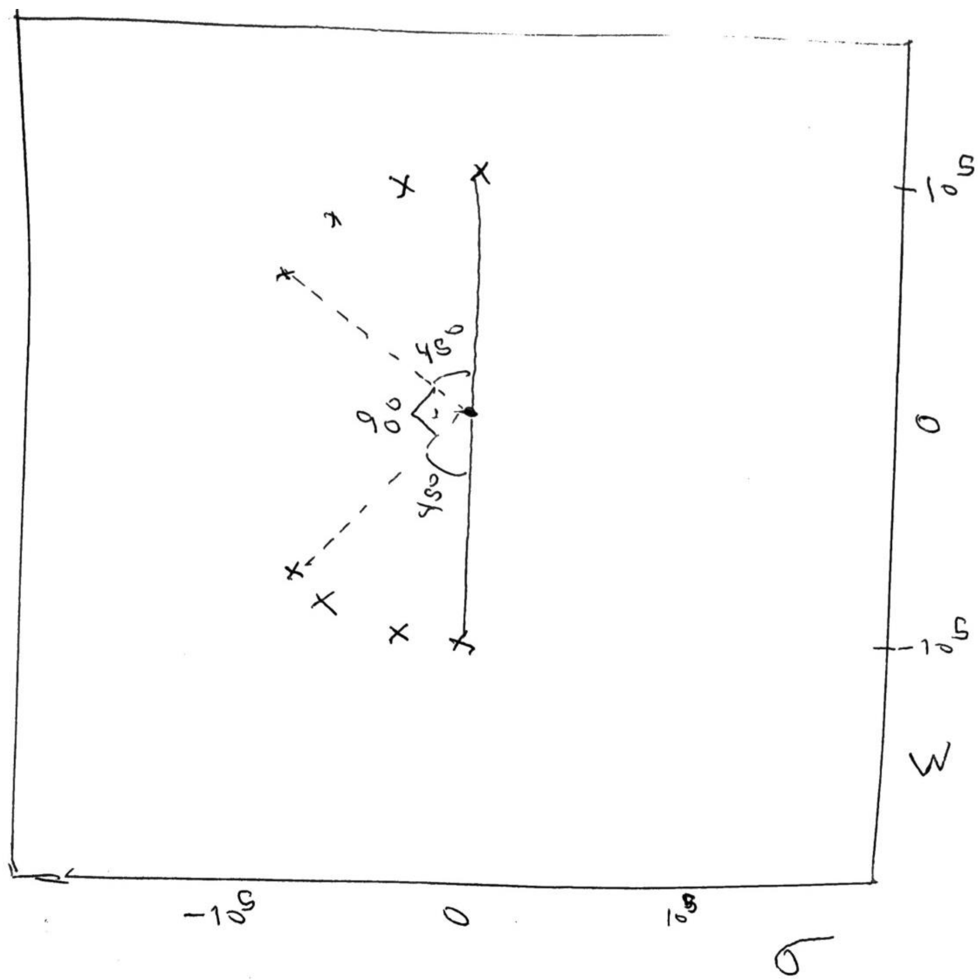

Figure A5: Root locus for Butterworth filter circuit (units in HZ)

The leftmost set of Xs correspond to the poles of the Butterworth filter. As A_M is changed, the pole frequency does not change and the poles migrate along the circumference of a circle with radius 10KHz. Once the Xs get on the jω axis, the circuit oscillates. Once the poles enter the positive σ region, the circuit becomes unstable.

Part B)

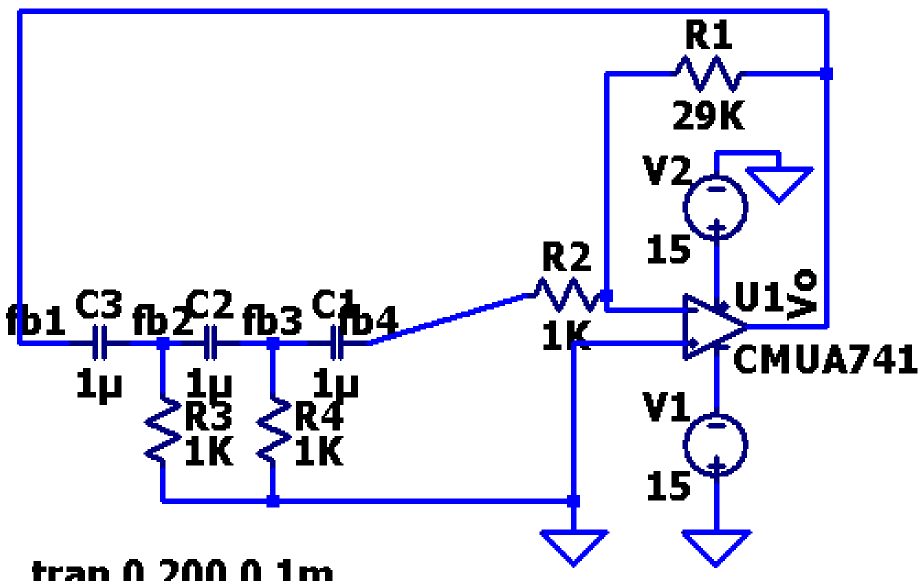

Figure B1: Phase shift oscillator model in LTspice

To trigger oscillation, the phase shift oscillator was set up with an initial condition of 1mV on the output node and the output was captured.

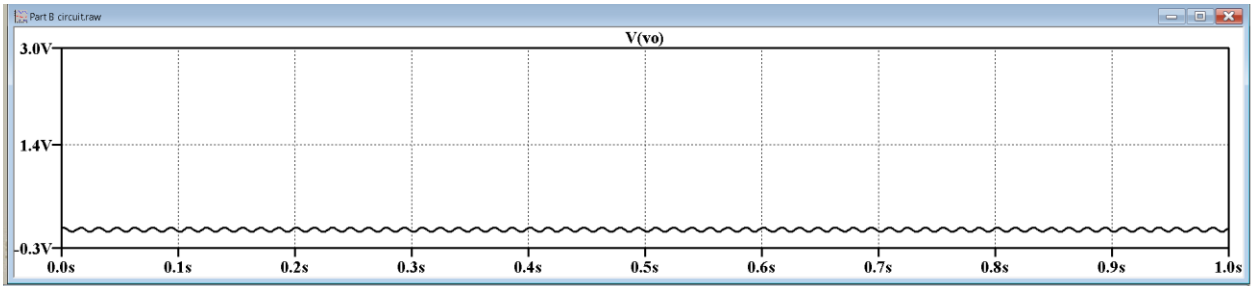

Figure B2: Phase shift oscillator waveform

The scale of the graph seemed much larger than the oscillation, so the initial condition was zoomed in on.

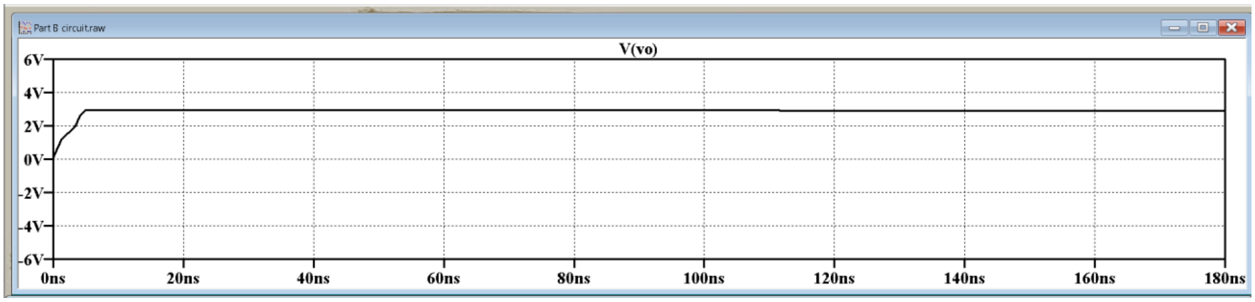

Figure B3: Initial condition on nanosecond scale

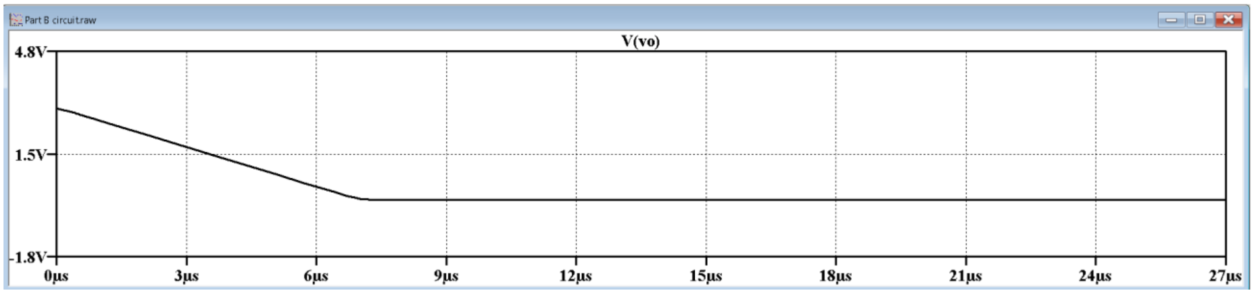

Figure B4: Initial condition on microsecond scale

We will try with a 1V initial condition next

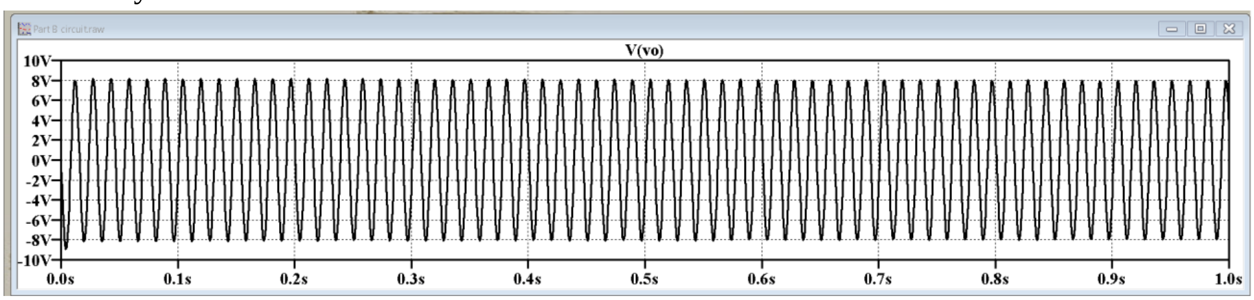

Figure B5: Phase shift oscillator response to 1V initial condition

There seems to be a relatively small initial voltage spike which quickly settles down to a steady state sine wave. This sine has parameters f=64.82593Hz, peak to peak voltage of 16.234735V, and a tiny DC offset of 4.8642mV which could just be cursor placement error from LTspice. There is essentially no DC offset. It seems that regardless of initial condition, the oscillator osscilates at around 64.8Hz, but it has a very strong relative response to small initial conditions, especially with the gigantic voltage spike at the beginning. For the coming tests, we will operate the oscillator with a 1V initial condition due to the less erratic response. However, I failed to simulate for long enough, this is the response observed when the timescale is extended with the 1V initial condition.

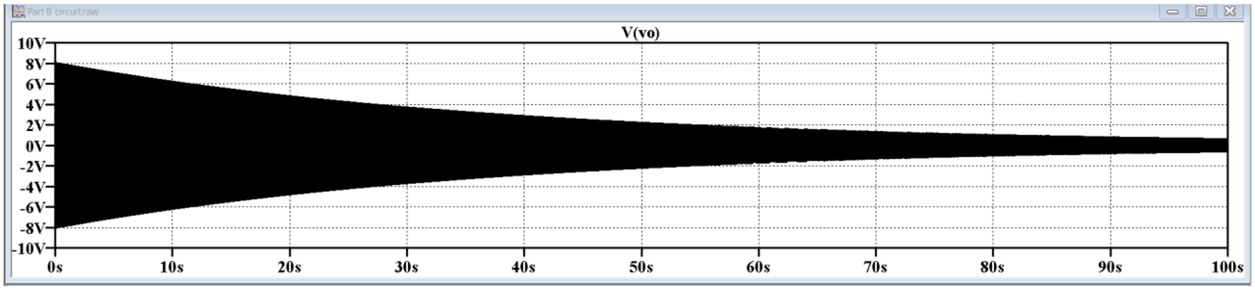

Figure B6: Phase shift oscillator at 1V initial condition on large timescale

It was found that with R_1=29K**Ω**, the signal decays, so R_1 was adjusted until it was found that no decay occurred with $R_1 = 29.1K\mathbf{\Omega}$

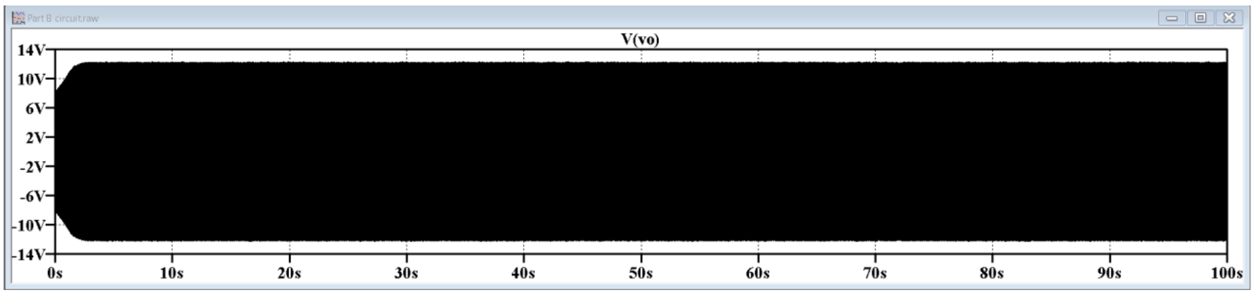

Figure B7: Phase shift oscillator at 1V initial condition on large timescale with R_1=29.1KΩ

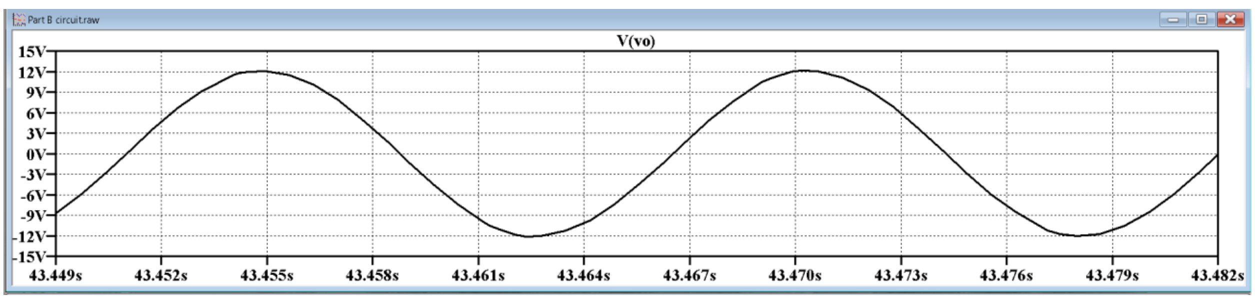

Figure B8: Phase shift oscillator wave on smaller time scale

The properties of the oscillation were measured as f=65.085956Hz, $V_{peak\ to\ peak}$=24.130424V And DC offset=-24.885mV which is close to zero. The oscillator was simulated with R and C doubled, then with R and C halved.

With R and C doubled, a frequency of 16.320529Hz was measured with $V_{peak\ to\ peak} = 25.873054$ and DC offset ≈ 0

With R and C halved, a frequency of 258.42044Hz was measured with $V_{peak\ to\ peak} = 23.5695\mathbf{V}$ and DC offset ≈ 0

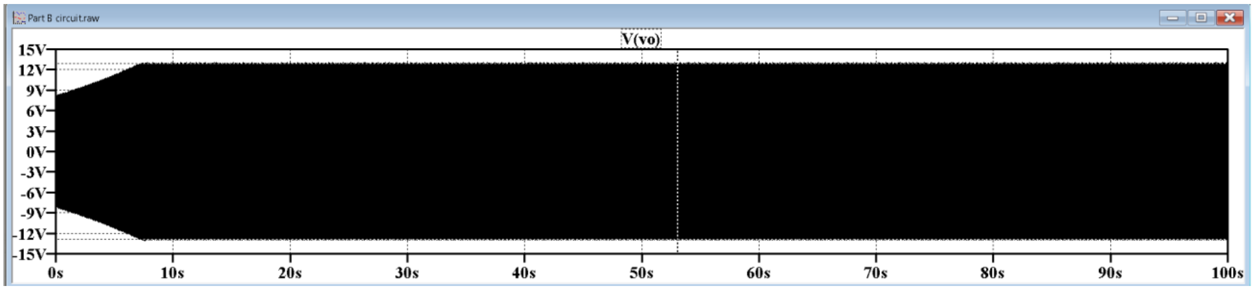

Figure B9: Phase shift oscillator wave with R and C doubled

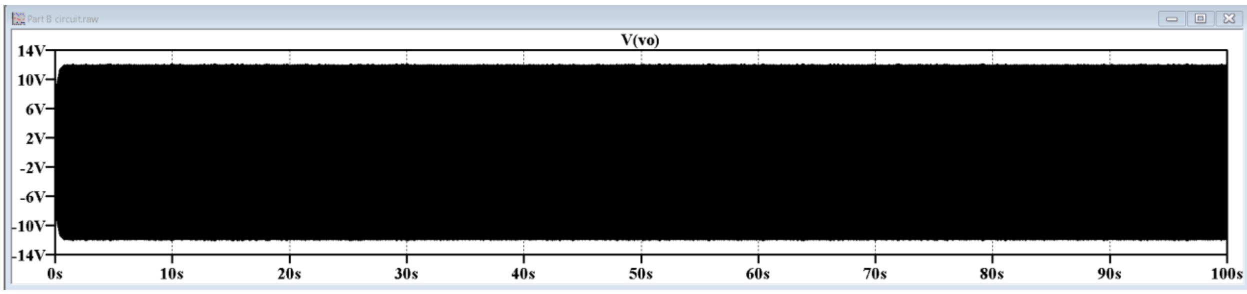

Figure B10: Phase shift oscillator wave with R and C halved

It seems that steady state is reached faster with smaller R and C; this is probably due to a higher frequency. It seems that when R and C are doubled, the frequency is halved and vice versa. This is likely due to the fact that the poles of the feedback stage have R and C in the denominator, so doubling or halving them at the same time quadruples or quarters the frequency. Also, we have negative feedback so, as given in the course notes, the frequency of the phase shift oscillator can be expressed with the formula $f = \frac{1}{2\pi\sqrt{6}RC}$. We can then calculate our predicted frequencies as $f = 64.9747$Hz for R=1000**Ω** and C=1µF, 16.2437Hz for R=2000**Ω** and C=2µF, and 259.8989Hz for R=500**Ω** and C=0.5µF, these values correspond extremely strongly to what was measured. The peak magnitude of the oscillation did not seem to depend on R and C. R_1 is 29.1R which results in the OP-OMP having a gain of 29.1R. The feedback network should result in attenuation by a factor of 29.1R and we must counter that by bringing the OP-AMP’s gain to the same level to make sure that there is no overall attenuation or magnification on the output and marginal stability is achieved. In theory, we should have to have an attenuation of $\frac{1}{29R}$ and an amplifier gain of 29R, but they are both 29.1R in our case which is not far off. We can learn more by probing the nodes at different stages of feedback, Vo,fb2,fb3, and fb4, as labelled in figure B1.

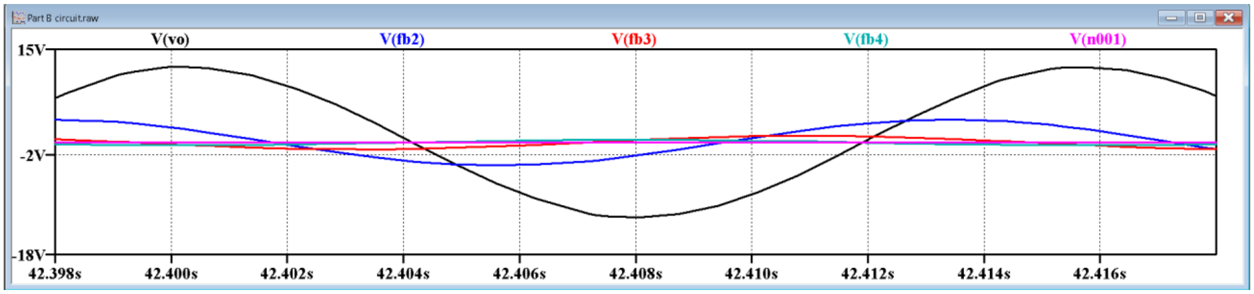

Figure B11: Feedback stages of phase shift oscillator

While the graphs cannot be differentiated in black and white, the feedback stages result in a phase shift of 60° each and they attenuate the signal. The final feedback stage, fb4 has a peak to peak to peak voltage of 828.4mV. Comparing the output voltage to the final feedback stage, we calculate $\frac{24.130424\text{V}}{828.4\text{mV}} = 29.12887$ as expected. The fb4 node is shifted 180° compared to the output and the OP-AMP then inverts it to cancel out that phase shift.

For curiosity, the oscillator was characterized with a zero initial condition.

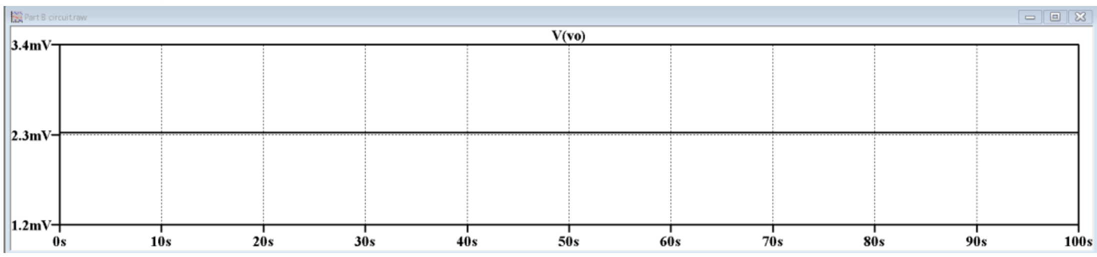

Figure B12: Phase shift oscillator with resistor at 29KΩ and initial condition of 0

It seems that no oscillation occurs at the start with a zero initial condition, what about when the resistor is adjusted to 29.1KΩ?

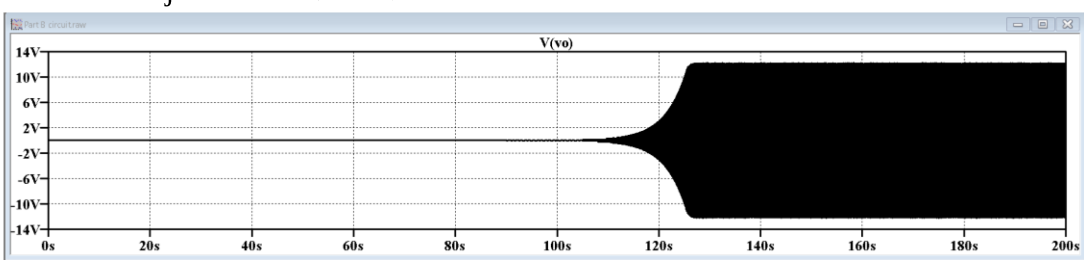

Figure B13: Waveform with resistor at 29.1K and zero initial condition

It seems that he oscillator takes a very long time to settle to an AC steady state output. The frequency was measured as 65.441176Hz with a peak to peak voltage of 24.08V. It seems that the frequency and amplitude of the oscillator's response is very similar with or without an initial condition on the output.

Part C 1)

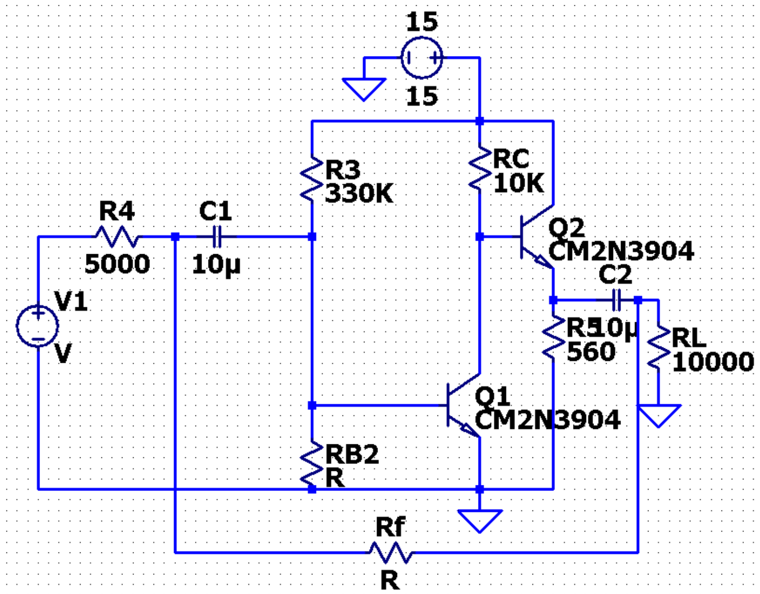

Figure C1: Feedback circuit diagram

It was found that the open loop gain at 1KHz was maximized with $R_{B2} = 20K\boldsymbol{\Omega}$ with a maximum value of $A_{M\ open\ loop} - 127.59063\frac{V}{V}$.

The DC bias will be measured with this resistor value in the DC equivalent circuit.

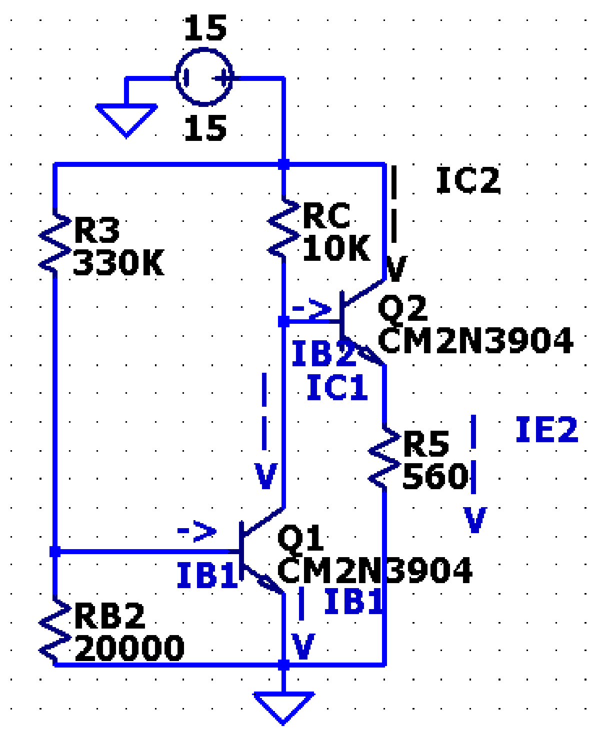

Figure C2: DC equivalent circuit

The operating point was measured as $I_{B1} = 10.769\mu A$, $I_{C1} = 1.294467mA$, $I_{E1} = 1.3052365mA$, $I_{B2} = 15.397461\mu A$, $I_{C2} = 2.1915634mA$, $I_{E2} = 2.2069768mA$.

$$V_{B1} = 654.06195mV, V_{C1} = 1.9013567V, V_{E1} = 0V, V_{B2} = 1.9013567V, V_{C2} = 15V, V_{E2} = 1.2358979V$$

$$r_{\pi1} = \frac{V_T}{I_{B1}} = 2321.4783\boldsymbol{\Omega}, r_{\pi2} = \frac{V_T}{I_{B2}} = 1623.644314\boldsymbol{\Omega}, h_{FE1} = \frac{I_{C1}}{I_{B1}} = 120.20308\frac{A}{A}, h_{FE2} = \frac{I_{C2}}{I_{B2}} = 142.332778, \frac{A}{A}\ g_{m1} = \frac{I_{C1}}{V_T} = 0.05177868℧, g_{m2} = \frac{I_{C2}}{V_T} = 0.0876625365℧.$$

Part C 2)

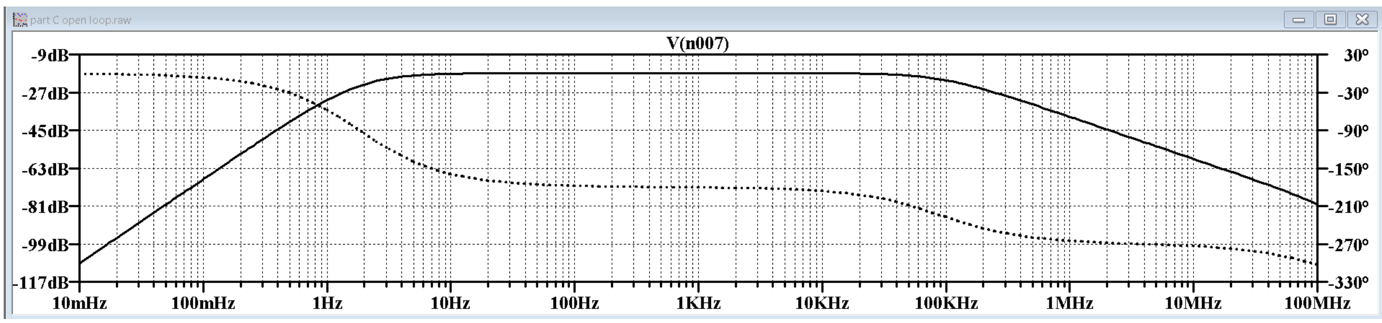

Figure C3: Open loop bode plot

It was found that $\omega_{L3dB\ open} = 2.9129039Hz$ and $\omega_{H3dB\ open} = 114.22407KHz$.

Comparing with the closed loop measurements with $R_f = 100K\Omega$

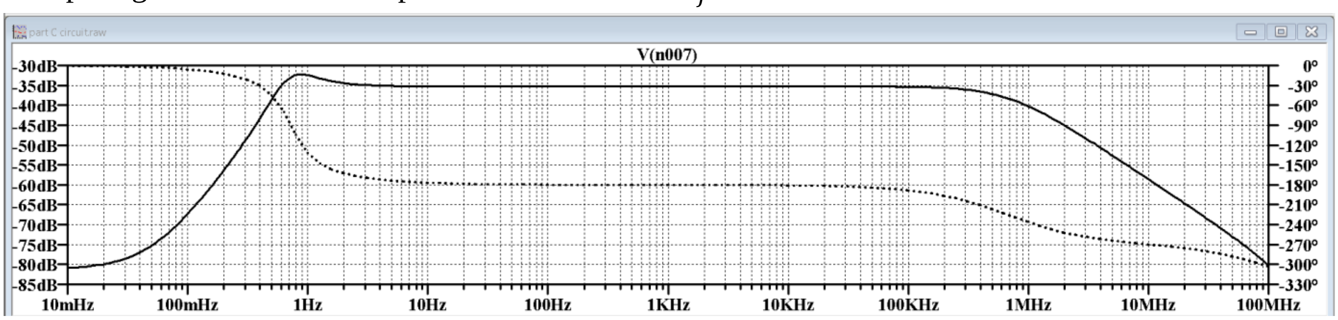

Figure C4: Closed loop bode plot

It was found that $\omega_{L3dB\ closed} = 0.51247575Hz$ and $\omega_{H3dB\ closed} = 680.54674KHz$

The open loop resistances can be measured by replacing Rin and Rout with a test source at 1mV 1KHz.

$R_{in\ open\ loop} = \frac{v_{in}}{i_{in}} = \frac{1mV}{383.83275nA} = 2605.301398\Omega$, $R_{out\ open\ loop} = \frac{1mV}{16.088495\mu A} = 62.156218\Omega$.

For $R_f = 100K\Omega$ we can measure the closed loop resistances with the same procedure

$R_{in\ closed\ loop} = \frac{V_{in}}{i_{in}} = \frac{1mV}{4.1255112\mu A} = 242.394203\Omega, R_{out\ closed\ loop} = \frac{1mV}{118.10419\mu A} = 8.4671\Omega.$

The closed loop midband gain must also be measured with a 1mV source at 1000Hz.

$$A_{M\ closed\ loop} = \frac{-17.207498mV}{1mV} = -17.207498\frac{V}{V}$$

Now, we must calculate the y parameters for the two port network$\left(\begin{matrix} I_{in} \\ I_{out} \end{matrix}\right)\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \left(\begin{matrix} V_{in} \\ V_{out} \end{matrix}\right)$. If we short the output $V_{out} = 0$ then we can say $y_{11} = \frac{I_{in}}{V_{in}} = \frac{1}{100K}$ and if we short the input port $V_{in} = 0$ then $y_{12} = \frac{I_{in}}{V_{out}} = -\frac{1}{100K\Omega}$ and $y_{22} = \frac{I_{out}}{Vout} = \frac{1}{100K\Omega}$ and we can approximate $y_{21} \approx 0$. The feedback β is the output voltage feeding back an input current so $\beta = \frac{I_{in}}{V_{out}}$ when $V_{in} = 0$ to contribute not current so $\beta = y_{12} = -\frac{1}{100K\Omega}$.
We can find $A'\ in\ units\ of\ \frac{V}{A}\ as\ \frac{V_{out}}{I_{in}}\ and\ I_{in} = \frac{V_{in}}{Rs}\ and\ A_{Mclosed\ loop} = \frac{V_{out}}{V_{in}}\ so\ A' = R_s A_M$

So our overall closed loop feedback gain in $\frac{V}{A}$ is $A'_{closed} = \frac{A'}{1+A'\beta}$ and plugging in our values we get -86499.004$\frac{V}{A}$. We can convert it back into $\frac{V}{V}$ by dividing by R_f so overall $A_{M\ closed} = -17.2898\frac{V}{V}$. The bandwidth will be extended by a unitless factor of $1 + A'\beta = 7.3795315$ so $\omega_{L3dB\ closed} = \frac{\omega_{L3db\ open}}{7.3795315} = 0.304727Hz$ and $\omega_{H3dB\ closed} = 7.3795315 * \omega_{H3dB\ open} = 842.920123KHz.$
The input and output impedances will both be reduced by the unitless factor $1 + A'\beta$, so using the measured closed loop values $R_{in\ closed} = \frac{R_{in\ open}}{7.3795315} = 353.04428\Omega$ and $R_{out\ closed} = \frac{R_{out\ open}}{7.3795315} = 8.422786\Omega.$

Comparing the predicted closed loop values vs the measured values

	$A_{M\ closed}$	$R_{in\ closed}$	$R_{out\ open}$	$\omega_{L3dB\ closed}$	$\omega_{H3dB\ closed}$
Predicted	-17.2898	353.04428	8.422786	$0.304727Hz$	$842.920123KHz$
Measured	-17.207498	242.394203	8.4671	$0.51247575Hz$	$680.54674KHz$

It seems that our analysis did a very good job of predicting the midband voltage gain, and output resistance after closing the feedback loop. However, our analysis overestimated the input resistance and high 3dB frequency, and underestimated the low 3dB frequency by a significant margin.

Part C 3)

It must be noted that the following bode plots are with respect to the input voltage that LTspice labels as -60dB, so they are all shifted down.

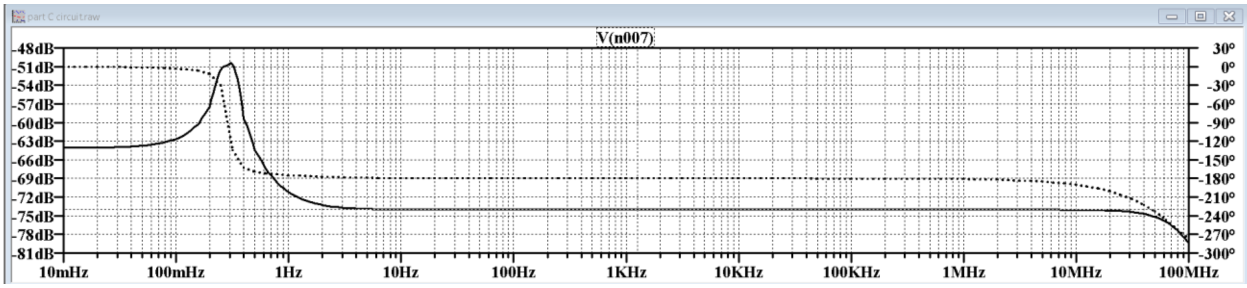

Figure C5: Bode plot with Rf=1K

For $R_f = 1K\Omega$ a midband gain of -14.018268dB was measured $A_M = -0.199107$, $R_s A_M = \frac{A'}{1+A'\beta}$ so $\beta = \frac{1}{A'} - \frac{1}{R_s A_M} = 1.002918 * 10^{-3}\mho$

Our predicted values are $\beta = -\frac{1}{R_f} = -10^{-3}\mho$, $A_M = \frac{A_{M\ open}}{1+A'\beta} = -0.199687\frac{V}{V}$

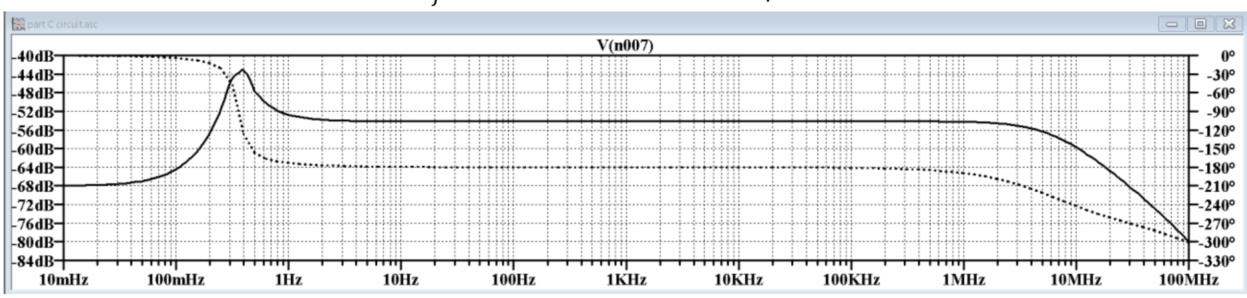

Figure C6: Bode plot with Rf=10K

For R_f=10KΩ a midband gain of 5.86177dB was measured $A_M = -1.96376\frac{V}{V}$

$$\beta = \frac{1}{A'} - \frac{1}{R_s A_M} = 1.002779 * 10^{-4}\mho$$

Our predicted values are $\beta = -\frac{1}{R_f} = -10^{-4}\mho$, $A_M = \frac{A_{M\ open}}{1+A'\beta} = -1.969134\frac{V}{V}$

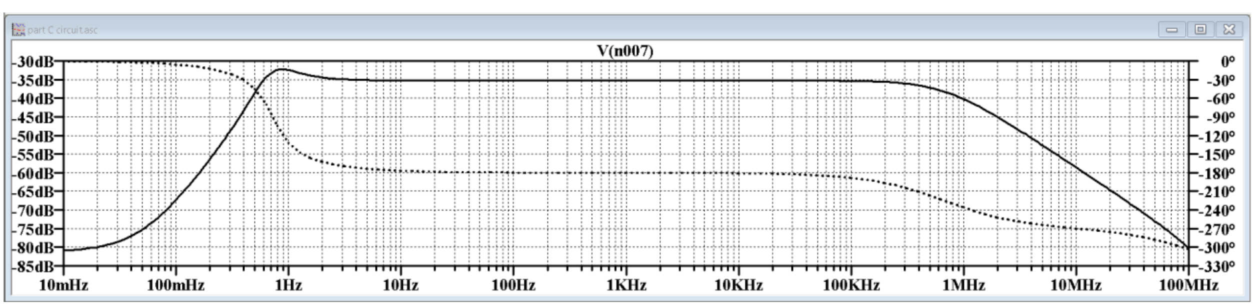

Figure C7: Bode plot with Rf=100K

For R_f=100KΩ a midband gain of 24.734749dB was measured so $A_M = -17.24795\frac{V}{V}$

$$\beta = \frac{1}{A'} - \frac{1}{R_s A_M} = 1.002807 * 10^{-5} \mho$$

Our predicted values are $\beta = -\frac{1}{R_f} = -10^{-5}\mho$, $A_M = \frac{A_{M\,open}}{1+A'\beta} = -17.2898\frac{V}{V}$

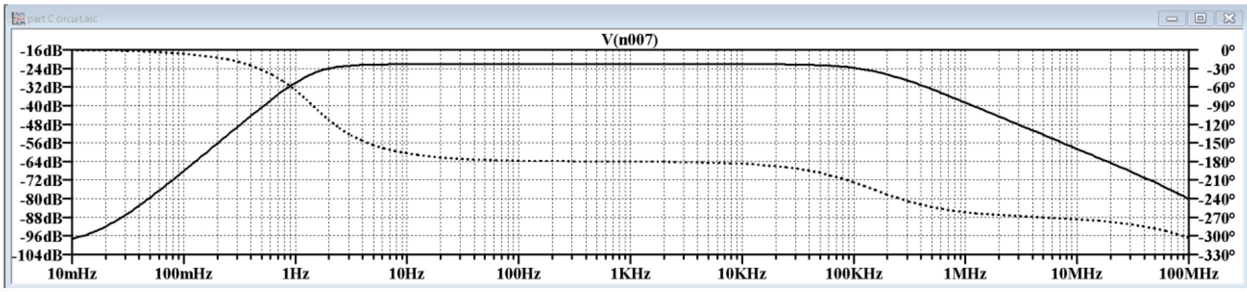

Figure C8: Bode plot with Rf=1M

For R_f=1MΩ a midband gain of 37.819447dB was measured $A_M = -77.7987\frac{V}{V}$

$$\beta = \frac{1}{A'} - \frac{1}{R_s A_M} = 1.0032238 * 10^{-6} \mho$$

Our predicted values are $\beta = -\frac{1}{R_f} = -10^{-6}\mho$, $A_M = \frac{A_{M\,open}}{1+A'\beta} = -77.89639$

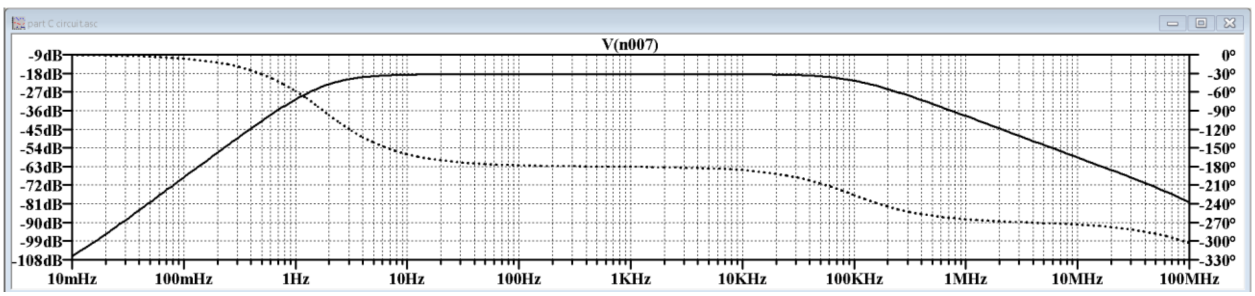

Figure C9: Bode plot with Rf=10M

For R_f=10M a midband gain of 41.575171dB was measured $A_M = -119.8832616\frac{V}{V}$

$$\beta = \frac{1}{A'} - \frac{1}{R_s A_M} = 1.0077639 * 10^{-7} \mho$$

Our predicted values are $\beta = -\frac{1}{R_f} = -10^{-7}\mho$, $A_M = \frac{A_{M\,open}}{1+A'\beta} = -119.93908\frac{V}{V}$

It seems that in all cases, the β and A_M values are extremely close to the predictions.

Part C 4)

With $R_f = 10K\Omega$, $R_{in} = \frac{1mV}{37.437185\mu A} = 26.7114\Omega$, $R_{out} = \frac{1mV}{907.80362\mu A} = 1.101559\Omega$
With $R_f = 100K\Omega$ $R_{in} = 242.394203\Omega$, $R_{out} = 8.4671\Omega$
With $R_f = 1M\Omega$, $R_{in} = \frac{1mV}{756.72392nA} = 1321.4849\Omega$, $R_{out} = \frac{1mV}{26.452618\mu A} 37.80344\Omega$
We can estimate β with the formuals $R_{in\ closed} = \frac{R_{in\ open}}{1+A'\beta}$ and $R_{out\ closed} = \frac{R_{out\ open}}{1+A'\beta}$ and average the two estimates

With $R_f = 10K\Omega$, $\beta = \frac{1}{2} * \left(\frac{1}{A'}\left(\frac{R_{in\ open}}{R_{in\ closed}} - 1\right) + \frac{1}{A'}\left(\frac{R_{out\ open}}{R_{out\ closed}} - 1\right)\right) = -1.191 * 10^{-4}\mho$
With $R_f = 100K\Omega$, $\beta = \frac{1}{2} * \left(\frac{1}{A'}\left(\frac{R_{in\ open}}{R_{in\ closed}} - 1\right) + \frac{1}{A'}\left(\frac{R_{out\ open}}{R_{out\ closed}} - 1\right)\right) = -1.26099 * 10^{-5}\mho$
With $R_f = 1M\Omega$, $\beta = \frac{1}{2} * \left(\frac{1}{A'}\left(1 - \frac{R_{in\ open}}{R_{in\ closed}}\right) + \frac{1}{A'}\left(1 - \frac{R_{out\ open}}{R_{out\ closed}}\right)\right) = -1.2663 * 10^{-6}\mho$
Comparing the amount of feedback (beta value) with the theoretical values

	$R_f = 10K\Omega$	$R_f = 100K\Omega$	$R_f = 1M\Omega$
Measured	$-1.191 * 10^{-4}\Omega$	$-1.26099 * 10^{-5}\Omega$	$-1.2663 * 10^{-6}\Omega$
Predicted	$-10^{-4}\Omega$	$-10^{-5}\Omega$	$-10^{-6}\Omega$

It seems that our prediction of the amount of feedback consistently underestimates the feedback determined from the measured input and output resistances.

Part C 5)

As demonstrated in the notes, the desensitivity factor is $1 + A'\beta$ where A' is the gain in ohms to cancel out the units of β and can be calculated for the closed loop case with $\frac{dA'_{closed}}{dA'_{open}} = \frac{dA_{M\ closed}*R_S}{dA_{M\ open}*R_S} = \frac{1}{(1+A'\beta)^2}$. When looking at the open loop circuit, $\beta = 0$ and so the desensitivity factor is just 1.
We can measure the open loop voltage gain with $R_C = 9.9K\Omega, 10K\Omega, 10.1K\Omega$

Open loop:
When $R_C = 9.9K\Omega$, $A_M = -126.45385\frac{V}{V}$, When $R_C = 10K\Omega$, $A_M = -127.59063\frac{V}{V}$, When $R_C = 10.1K\Omega, A_M = -127.8291\frac{V}{V}$
Closed loop:
When $R_C = 9.9K\Omega$, $A_M = -17.172354\frac{V}{V}$, When $R_C = 10K\Omega$, $A_M = -17.207498\frac{V}{V}$, When $R_C = 10.1K\Omega, A_M = -17.222461\frac{V}{V}$

We can calculate $1 + A'\beta = \frac{1}{\sqrt{\frac{dA_{M\ closed}}{dA_{M\ open}}}}$, and we can estimate $\frac{dA_{M\ closed}}{dA_{M\ open}}$ using the differences in measured gains as $-9.40847 * 10^{-3}$, we must take the absolute value in order to make the answer real, we can calculate our desensitivity factor as 10.309568. This is significantly different from the desensitivity factor calculated before of 7.37953, but not more than an order of magnitude off. This error is likely due to the relatively small changes in the gains resulting in it being very easy for measurement errors to skew the value.

Sources cited

1) ELEC 301 course notes
2) https://freebiesupply.com/logos/ubc-logo/