Mini Project 3

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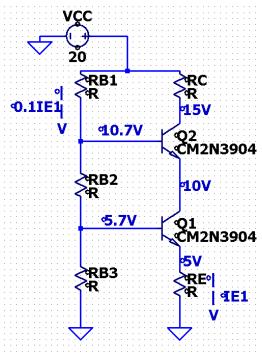
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Part 1)

Based on the LTSPICE netlist for the 2N3904, $\beta_1 = \beta_2 = \beta = 300 \frac{A}{A}$.



The DC model of our cascode amplifier has some voltages and currents given by the ¼ rule. At midband, we need $R_{out} = 2500 \pm 250\Omega$ and $R_L = 50K\Omega$. $R_{out} = R_C$ so we can pick a standard resistor value of $R_C = 2400\Omega$. Now, solving for R_E , R_{B1} , R_{B2} and R_{B3} , $I_{C2} = \frac{20-15}{2400} = 2.083mA$, $I_{B2} = \frac{I_{C2}}{\beta} = \frac{2.038*10^{-3}}{300} = 6.944\mu A$, $I_{E2} = I_{C1} = I_{B2} + I_{C2} = 2.09mA$, $I_{B1} = \frac{I_{C1}}{300} = 6.968\mu A$, $I_{E1} = I_{C1} + I_{B1} = 2.097mA$. $R_E = \frac{5}{I_{E1}} = 2384.1\Omega$, so we can use a standard value $R_E = 2400\Omega$. We will label the downwards currents in resistors R_{B1} , R_{B2} , and R_{B3} as I_1 , I_2 , and I_3 respectively, $I_1 = 0.1*I_{E1} = 0.2097mA$. $I_1 = \frac{20-10.7}{R_{B1}}$ so $R_{B1} = 44343.9\Omega$, the closest standard value is $R_{B1} = 43000\Omega$.

Figure 1.1: DC cascode model

By KCL, $I_2 = I_1 - I_{B2} = 0.20276 mA$ so $R_{B2} = \frac{10.7 - 5.7}{I_2} = 24657.2 \Omega$ which corresponds to a standard resistor value of $R_{B2} = 24000 \Omega$. $I_3 = I_2 - I_{B1} = 0.1957 mA$ so $R_{B3} = \frac{5.7}{I_3} = 29109.5 \Omega$ which corresponds to a standard resistor value of $R_{B3} = 30000 \Omega$.

Checking to see if the specs are met, at midband we need $R_{in} > 5000\Omega$. $R_{in} = R_{B3} \big| |R_{B2}| \big| r_{\pi 1}, r_{\pi 1} = \frac{\beta}{g_{m1}}, g_{m1} = \frac{I_{C1}}{V_T} = \frac{2.097*10^{-3}}{0.025} = 0.08389 \ so \ r_{\pi 1} = 3576.1\Omega \ and \ R_{in} = 30000 \big| |24000| \big| 3576.1 = 2819.8\Omega < 5000\Omega \ so the condition is not met. This can be fixed by adding a new resistor after the source right to the left of <math>C_{C1} R_{adj}$ Originally $R_{adj} = 2200\Omega$ was chosen, but this caused the simulated circuit to have an input resistance less than 5000Ω , and it was decided to use $R_{adj} = 3000\Omega$. Now $R_{in} = R_{B3} \big| |R_{B2}| \big| r_{\pi 1} + R_{adj} = 2819.8 + 3000 = 5819.8\Omega > 5000\Omega$. The overall circuit is now as shown in figure 2

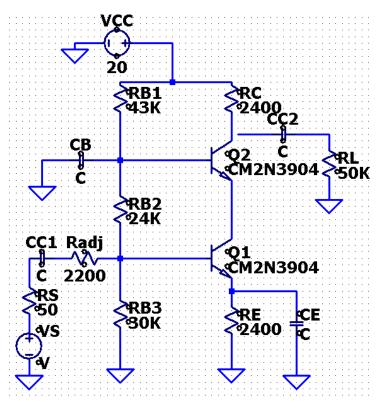


Figure 1.2: one node at top right is disconnected RADJ iswrong

We also need midband gain $|A_m| > 50$, $A_m = -g_{m2} * R_C ||R_L \frac{R_{B2}||R_{B3}||r_{\pi}}{R_{B2}||R_{B3}||r_{\pi} + R_S + R_{adj}}$. $g_{m2} = \frac{I_{C2}}{V_T} = \frac{2.083*10^{-3}}{0.025} = 0.08333 \text{ U}, r_{\pi 2} = \frac{\beta}{g_{m2}} = 3600 \Omega$. Plugging in our values, $A_m = -91.92787 \frac{V}{V}$ which meets the condition.

Based on what was discussed in class, the cost of purchasing capacitors can be minimized by calculating poles as

$$\begin{aligned} \tau_{CC1} &= (R_S + R_{adj} + R_{B2} \big| |R_{B3}| \big| \big(r_{\pi 1} + R_E (1 + \beta) \big)) * C_{C1}, & R_{eqCC1} &= 16142.8687 \Omega \\ \tau_{CC2} &= C_{C2} (R_C + R_L), & R_{eqCC2} &= 52400 \Omega \\ \tau_{CE} &= R_E || \left(\frac{\left(R_S + R_{adj} \right) \big| |R_{B2}| \big| R_{B3} + r_{\pi 1}}{1 + \beta} \right) * C_E, & R_{eqCE} &= 20.10416 \Omega \end{aligned}$$

Since C_E sees the smallest resistance by multiple orders of magnitude, it is almost entirely responsible for the cut-in frequency of 500Hz if the same values are chosen for C_{C1} , C_{C2} , C_{C3} . We can design it so that the highest pole meets the 3dB frequency condition, and then calculate all the poles from there.

 $\omega_{pCE} = 2\pi * 500 \approx \frac{1}{C_E 20.10416} C_E \approx \frac{1}{20.10416*500*2\pi} = 15.833035 \mu F$, we will use standard capacitor values of $22\mu F$ because estimating upwards will result in a 3dB frequency below our limit as opposed to above.

$$\begin{split} \omega_{pCC1} &= \frac{1}{22*10^{-6}*16142.9} = 2.8158, \omega_{pCC2} = \frac{1}{22*10^{-6}*52400} = 0.86745, \omega_{pCE} = \\ &\frac{1}{22*10^{-6}*18.138458} = 2505.9763, \ \omega_{z1} = \omega_{z2} = 0, \omega_{z3} = \frac{1}{R_E C_E} = \frac{1}{22*10^{-6}*2400} = 18.9393939 \\ \omega_{pCE} &= \frac{1}{22*10^{-6}*20.1041635} = 2260.95208 \end{split}$$

Now $\omega_{pCE} \gg \omega_{pCC1}$, ω_{pCC2} , ω_{z3} so ω_{3dBL} is basically just equal to ω_{CE} and we get a cut in of 2260.95208 $\frac{rad}{s}$ or 359.8417Hz.

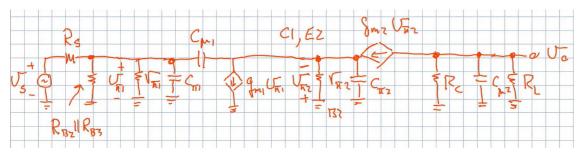


Figure 1.3: Small signal model of cascode amplifier

As demonstrated in the lectures

$$\omega_{HP1} = \frac{1}{(c_{\pi 1} + 2c_{\mu 1})(r_{\pi 1}||R_{B2}||R_{B3}||(R_S + R_{adj}))}, \ \omega_{HP2} = \frac{1 + \beta}{(c_{\pi 2} + 2c_{\mu 1})r_{\pi 2}}, \ \omega_{HP3} = \frac{1}{c_{\mu 2} * R_C||R_L}$$

$$c_{\pi} \text{ and } c_{\mu} \text{ can be estimated as } c_{\pi} = 2 * CJE + TF * g_{m} \text{ and } c_{\mu} = \frac{CJC}{\left(1 + \frac{V_C - V_B}{VJC}\right)^{MJC}} \text{ and they'll be}$$

$$\text{calculated as } c_{\pi 1} = c_{\pi 2} = 2 * 4.5 * 10^{-12} + 400 * 10^{-12} * 0.083333 = 42.5559 * 10^{-12}F$$

$$c_{\mu 1} = \frac{3.6 * 10^{-12}}{\left(1 + \frac{9.8877 - 5.6539}{0.75}\right)^{0.33}} = 1.92698 * 10^{-12}, c_{\mu 2} = \frac{3.6 * 10^{-12}}{\left(1 + \frac{15.087 - 10.553}{0.75}\right)^{0.33}}$$

So our poles can be calculated as

$$\omega_{HP1} = 14.705952*10^6, \omega_{HP2} = 1.8015798*10^9, \omega_{HP3} = 231.02286*10^8$$
 And the high 3dB frequency can be calculated as
$$\omega_{H3dB} = \frac{1}{\sqrt{\frac{1}{\omega_{HP1}^2} + \frac{1}{\omega_{HP2}^2} + \frac{1}{\omega_{HP3}^2}}} = 14.67576*10^6$$

or 2.3357199MHz

Part 1 A)
Calculated DC operating point

V_{B1}	V_{C1} and V_{E2}	V_{E1}	V_{B2}	V_{C2}
5.7V	10V	5V	10.7V	15V
I_{B1}	I_{C1} and I_{E2}	I_{E1}	I_{B2}	I_{C2}
6.968μΑ	2.0972mA	2.0972mA	6.944μΑ	2.083mA

Measured DC operating point

V_{B1}	V_{C1} and V_{E2}	V_{E1}	V_{B2}	V_{C2}
5.6539V	9.8877V	4.98787V	10.553V	15.087V
I_{B1}	I_{C1} and I_{E2}	I_{E1}	I_{B2}	I_{C2}
15.683μΑ	2.0626mA	2.0783mA	15.540μΑ	2.0471mA

Part 1 B)

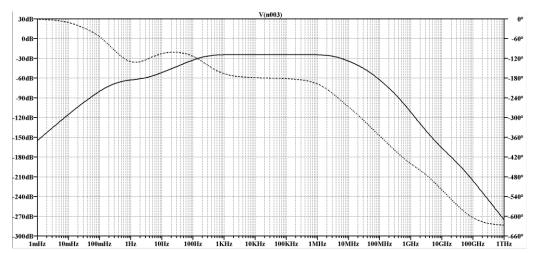


Figure 1.4: Bode plot of cascode amplifier, dotted line is phase, solid line is magnitude

	Measured	Calculated	
ω_{L3dB}	247.4462 <i>Hz</i>	359.8417Hz	
ω_{H3dB}	3.467906 <i>MHz</i>	2.3357199 <i>MHz</i>	

There seems to be a decent correspondence between the 3dB frequency of the low poles and the high poles with similar degrees of error. These errors probably come from the inherent inaccuracy that comes with the open-circuit short-circuit time constant approximation.

Part 1 C)

10KHz was the chosen value for a midband frequency

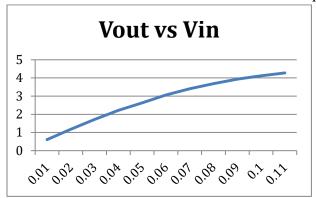


Figure 1.5: Output vs. input peak voltage of cascode amplifier at midband

At 10KHz, it seems that the output voltage starts becoming nonlinear and the gain begins decreasing at around 70mV, this is due to the transistors saturating.

Part 1 D)

At 10KHZ with an input voltage of 50mV, we get a peak current of 9.615 μ A for an input resistance of 5200.208 Ω . This is a decent bit below what was hypothesized, but it is within tolerance for our amplifier design, $R_{in} = 5200.208\Omega$.

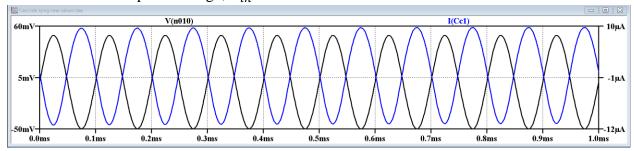


Figure 1.6: Input voltage vs. input current

With a voltage of 50mV on the output, a current of $20.805895\mu A$ was measured for an output resistance of 2403.1529Ω . This is extremely close to the value used in our calculations, so our approximations were accurate. $R_{out} = 2403.1529\Omega$

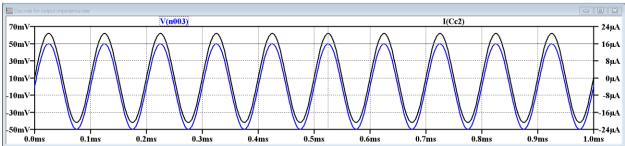


Figure 1.7: Output voltage vs. output current

With a 50mV input, the output is -2.6682757V for a midband gain of $A_m = -53.3655 \frac{V}{V}$. This is a good bit lower than the predicted midband gain, but it is within tolerance.

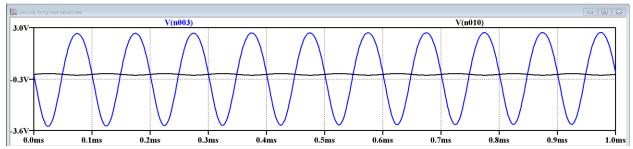


Figure 1.8: Input voltage vs. output voltage

Part 2 a)

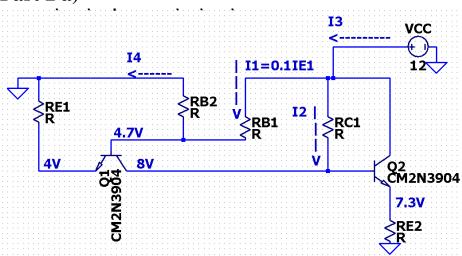


Figure 2.1: DC model of cascaded amplifiers circuit

The first conditions that must be satisfied are the conditions for the input impedance and output impedance at midband. R_{E1} goes to ground, $r_{\pi1}$ gets de-magnified, and R_{B2} gets shorted, so the input resistance can be estimated as $R_{in} = R_{E1} || \frac{r_{\pi1}}{1+\beta} = 50\Omega$. $r_{\pi1} = \frac{V_T}{I_{B1}} = \frac{V_T}{I_{E1}}$, $R_{E1} = \frac{V_{E1}}{I_{E1}}$, $R_{in} = \frac{V_{E1}}{I_{E1}} || \frac{V_T}{I_{E1}}|| \frac{V_$

 $R_{out} = 50\Omega = R_{E2} ||\frac{r_{\pi 2} + R_{C1}}{1 + \beta}|$, we need more equations to solve this system. We can say $r_{\pi 2} =$

$$\frac{V_T}{I_{B2}} = \frac{0.025}{I_{B2}}$$
 and $R_{E2} = \frac{7.3V}{I_{E2}} = \frac{7.3V}{301*I_{B2}}$. We get $50 = \frac{7.3}{301*I_{B2}} || \frac{\frac{0.025}{I_{B2}} + R_{C1}}{301}$

With two unknowns confined to standard resistor values and a range of allowable outputs, this problem is best solved by graphing R_{out} vs R_{E2} for different values of R_{C1} to find a pair of resistors that work.

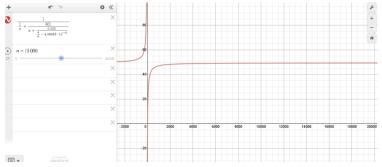


Figure 2.2: R_{out} vs. R_{E2} with R_{C1} as a parameter

We found a solution at $R_{E2}=R_{C1}=15K\Omega$. With these values, $r_{\pi 2}=\frac{V_T}{I_{B2}}=681.28656\Omega$

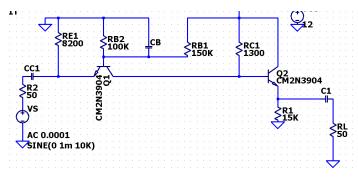


Figure 2.3: AC model of cascaded amplifiers

To calculate the capacitor values, based on the lecture notes on common base and common collector amplifiers, we can say that

$$\begin{split} \tau_{CC2} &= C_{C2}(\frac{R_{C1} + r_{\pi 2}}{1 + \beta} || R_{E2}) = 51.91698 * C_{C2} & R_{eqCC2} = 51.91698\Omega \\ \tau_{CC1} &= (\frac{1}{1 + \beta} r_{\pi 1} || R_{E1}) C_{C1} = 49.8661298 * C_{C1} & R_{eqCC1} = 49.8661298\Omega \\ \tau_{CB} &= C_{B} * (R_{B1} || R_{B2} || r_{\pi 1} + (1 + \beta) R_{E1})) & R_{eqCB} = 2.480265 M\Omega \end{split}$$

If we set $C_{C1} = C_{C2} = C_B$ then C_B does not produce a dominant pole and C_{C1} and C_{C2} are both dominant and must be chosen to set ω_{L3dB} .

We will choose $C_{C1} = C_{C2}$ and choose their values to approximate our desired corner frequency of 1000Hz or $2000\pi \frac{rad}{s}$ to be $\omega_{L3dB} = \sqrt{\frac{1}{(49.8661298*C_{C1})^2} + \frac{1}{(51.91698*C_{C1})^2}} = 2000\pi$ so

 $C_{C1} = 4.425414 \mu F$ so we can pick a standard values of $4.7 \mu F$ for C_{C1} and C_{C2} . We will chose $C_B = 4.7 \mu F$ and adjust it later to fine tune ω_{L3dB} in the simulation. With our choice of capacitors, we can estimate $\omega_{L3dB} = \sqrt{\frac{1}{(49.8661298*C_{C1})^2} + \frac{1}{(51.91698*C_{C1})^2}} = 5916.1$ or 941.57745Hz

Our initial guess at design specs is given in the table below

C_{C1}	C_{C2}	C_B	R_{E1}	R_{E2}	R_{B1}	R_{B2}	R_{C1}
$4.7\mu F$	4.7μF	$4.7\mu F$	8200Ω	15ΚΩ	150ΚΩ	100ΚΩ	15ΚΩ

Part 2 b)

We can pick 10KHz as a midband frequency for our measurements.

With an output source voltage of 1mV we get a current of $4.22\mu A$ for a resistance of 237 Ω . It seems that we need to adjust R_{E2} and R_{C1} . By trying out different values for R_{C1} it was found that $R_{C1} = 1300\Omega$ and $R_{E2} = 15000\Omega$ result in an output current of $17.651574\mu A$ and resistance $R_{Out} = 53.135\Omega$ which is within spec.

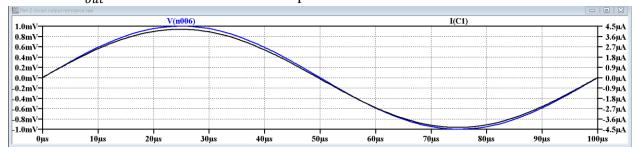


Figure 2.4: V_{out} vs I_{out} for $R_{C1} = 15000\Omega$

Measuring input resistance using a 1mV 10KHz source we get an input current of $19\mu A$ for a resistance of $R_{in} = 52.631579$ which is within spec.

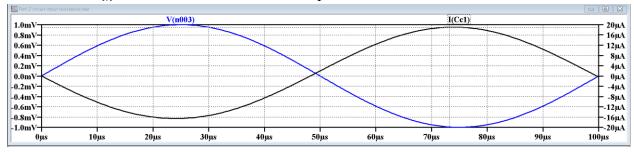


Figure 2.5: V_{in} vs. I_{in}

Finally, the midband gain was found to be $A_m = \frac{21.388763mV}{1mV} = 21.388763 \frac{V}{V}$

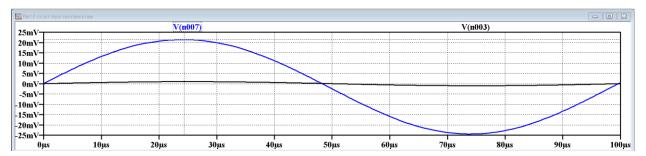


Figure 2.6: V_s vs. V_o

Part 2 c)

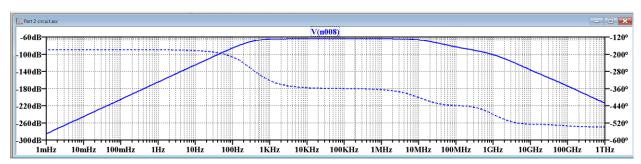


Figure 2.7: Bode plot of cascaded amplifiers

It was found that $\omega_{H3dB} = 11.206658 \text{MHz}$ and $\omega_{L3dB} = 534.06021 \text{Hz}$ which is off spec and is quite different from our predicted value of 941.57745 Hz. This error is likely from the error inherent in the short-circuit open-circuit time constant method. C_B is not a dominant pole so it can be changed significantly to produce relatively small effects on the 3dB frequencies.

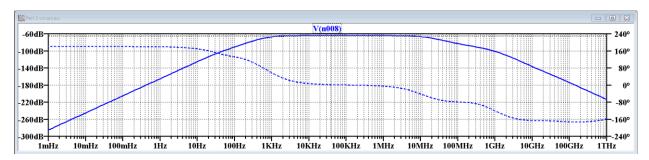


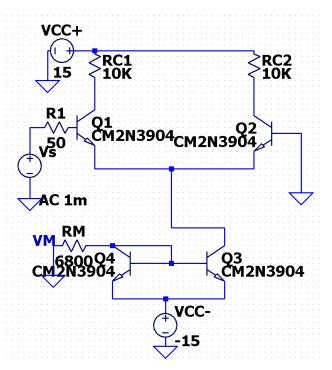
Figure 2.8: Bode plot of cascaded amplifiers with $C_B = 0.022 \mu F$

 C_B was decreased by multiple orders of magnitude and values were experimented with until settling on $0.022\mu F$ to get a new $\omega_{H3dB}=10.983609 \mathrm{MHz}$ and $\omega_{L3dB}=1016.2133 \mathrm{Hz}$ which is quite close to the desired value. We can recalculate our low corner frequency using the new

choice of
$$C_B$$
 as $f_{L3dB} = \frac{1}{2\pi} \sqrt{\frac{1}{(49.8661298*C_{C1})^2} + \frac{1}{(51.91698*C_{C1})^2} + \frac{1}{(2.480265*10^6*C_B)}} = 941.582Hz$, compared to our prediction of 941.57745Hz for $C_B = 4.7*10^{-6}$. While the real simulation can

be significantly affected by the changing value of C_B , it seems that the predicted 3dB frequency is negligibly influenced by it.

Part 3)



We must draw 1mA from the emitters of Q1 and Q2, so the current mirror must draw a total of 2mA. In class it was demonstrated

that
$$I_o = I_{E1} + I_{E2} = \frac{V_M - (\frac{V_E + V_{BE}}{R_M})}{(1 + \frac{2}{\beta})}$$
. Here $V = 0V$, $V_E = -15V$, and $V_{BE} \approx 0.7$ so the current mirror draws $\frac{-(\frac{-15 + 0.7}{R_M})}{(1 + \frac{2}{300})} = 2 * 10^{-3} A$

so R_M =7102.649 Ω and we can pick a standard value of R_M = 6800 Ω , plugging it back in we get I_0 = 2.0890144mA.

Figure 3.1: Differential amplifier with a current mirror



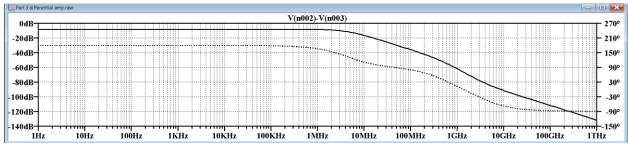


Figure 3.2: Differential amplifier bode plot

Based on plots in LTspice, it was found that $\omega_{H3dB} = 5.1703266MHz$ and the midband differential gain was found to be -386.57117 $\frac{V}{V}$

Part 3 b)

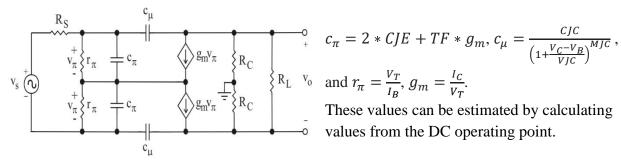


Figure 3.3: Small signal model of the differential amplifier

Assume
$$I_{E1} = I_{E2} = \frac{2.0890144mA}{2} = 1.0445072mA$$
, $I_{B1} = I_{B2} = \frac{I_{E1}}{1+\beta} = 3.4701236\mu A$, $I_{C1} = I_{C2} = \beta I_B = 1.041037mA$. $V_{C1} = V_{C2} = V_{CC+} - R_{C1}I_{C1} = 4.589629V$, and $V_{B1} = V_{B2} = 0$ because the AC source is shorted in DC.

We can now calculate
$$g_{m1} = g_{m2} = \frac{1.041037*10^{-3}}{0.025} = 0.04164148, r_{\pi 1} = r_{\pi 2} = \frac{0.025}{3.4701236*10^{-6}} = 7204.35436\Omega, c_{\pi 1} = c_{\pi 2} = 2*4.5*10^{-12} + 400*10^{-12}*0.04164148 = 25.6565932pF$$

$$c_{\mu 1} = c_{\mu 2} = \frac{3.6*10^{-12}}{\left(1 + \frac{4.589629V}{0.75}\right)^{0.33}} = 1.883623327pF.$$

It was shown in class that
$$\omega_{HP1}=\frac{1}{\left[\frac{c_{\pi}+^{c}\mu}{2}(1-k)\right](2r_{\pi})||R_{s}},$$
 $\omega_{HP2}=\frac{1}{\frac{c_{\mu}}{2}\left(1-\frac{1}{k}\right)R_{L}||(2R_{C})},$ since the bode plot was generated with an open load, $R_{L}=\infty$ and $\omega_{HP2}=\frac{1}{\frac{c_{\mu}}{2}\left(1-\frac{1}{k}\right)(2R_{C})}$. It was also shown in class that $k=-g_{m}R_{C}\frac{R_{L}}{R_{L}+2R_{C}},$ $\lim_{R_{L}\to\infty}k=-g_{m}R_{C}$ so our final formulas for the high frequency poles are $\omega_{HP1}=\frac{1}{\left[\frac{c_{\pi}+^{c}\mu}{2}(1+g_{m}R_{C})\right](2r_{\pi})||R_{s}}}$ and $\omega_{HP2}=\frac{1}{c_{\mu}R_{C}\left(1+\frac{1}{g_{m}R_{C}}\right)}$

Plugging in our numbers, we get $\omega_{HP1} = 4.9437672 * 10^7$ and $\omega_{HP2} = 5.29619849 * 10^7$ $\omega_{H3dB} = \frac{1}{\sqrt{\frac{1}{\omega_{HP1}^2} + \frac{1}{\omega_{HP2}^2}}} = 3.61394237 * 10^7$ or 5.7517679MHz which is relatively close to the 6.7701466MHz that was measured.

In class it was shown that $A_M = -g_m R_c \left[\frac{2r_\pi}{2r_\pi + R_s} \right] \left[\frac{R_L}{R_L + 2R_C} \right]$ and since there is no load $\lim_{R_L \to \infty} A_M = -g_m R_c \left[\frac{2r_\pi}{2r_\pi + R_s} \right] = -414.9748 \frac{V}{V}$ which is relatively close to our measured gain of -386.57117 $\frac{V}{V}$.

	Measured	Predicted
A_{M}	-386.57117	-414.9748
f_{H3dB}	6.7701466MHz	5.7517679MHz

Part 3 c)

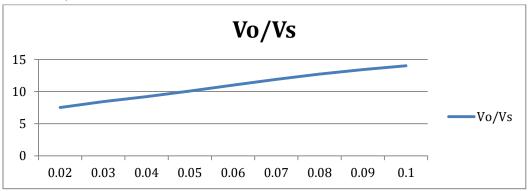


Figure 3.4: Output voltage vs. source voltage

Plotting output vs. input voltage, it seems like a slight nonlinearity is observed beginning around 0.08V or 80mV. This is probably due to the transistors saturating at 80mV.

Problem 4 a)

The differential output of the AM modulator was observed and graphed below

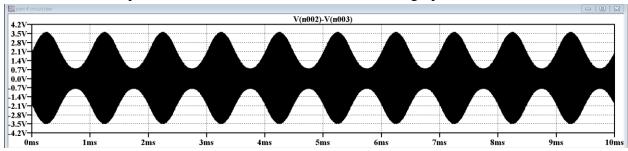


Figure 4.1: Modulated sine wave

It seems that there is a 100KHz carrier signal enveloped by the KHz signal at the input. The amplitude is modulated which seems to correspond well with the name "AM modulator".

Part 4b)

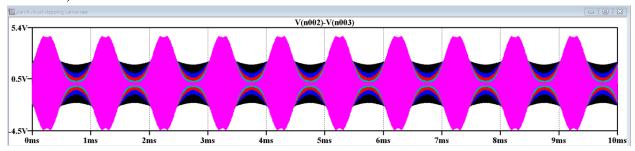


Figure 4.2: Carrier modulated by sine waves with different amplitudes, 100mV modulated signal is light grey

It seems that for the highest amplitude modulating input,100mV(light grey), the top of the waveform is cut off a little bit which indicates that we have saturated the transistors. The off cycle of the 100mV modulated signal also becomes thinner when compared with the other input voltage signals.

Part 4c)

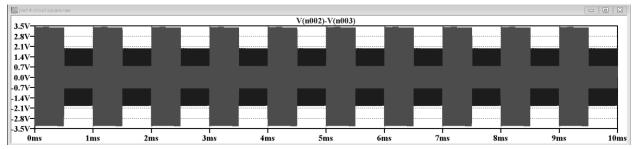


Figure 4.3: Carrier modulated by 100mV square wave(grey) on top of carrier modulated by 10mV(black)

It seems that a 100KHz sine wave gets its amplitude scaled by the square wave at 1KHz. Once the input signals get to 80mV, the peaks of the output voltages plateau at around 4V. As modulating input voltages become greater, the voltages on the off cycle of the modulating signal will become smaller.

Based on a source from electronicspost.com, it seems that the output of the am modulator can be calculated. With a carrier signal $V_c = A_c \sin(\omega_c t)$ and a modulating signal $V_s = A_s m(\omega_s t)$ the output can be modelled as $V_o = A_c \sin(\omega_c t) \left[1 + \frac{A_s}{A_c} m(\omega_s t)\right]$. This formula explains why the modulating signal will envelope the carrier signal because they are multiplied, and the +1 explains why it will never shrink it to zero amplitude. When the modulating signal is equal to zero, the output signal is not modulated to zero because of the +1 in the formula.

Cited sources:

- 1) ELEC 301 course notes
- 2) AM modulator model was found at https://electronicspost.com/derive-an-expression-for-single-tone-amplitude-modulated-wave-and-draw-its-frequency-spectrum/
- 3) LTspice netlist parameter definitions were found at http://www.ece.mcgill.ca/~grober4/SPICE/SPICE_Decks/1st_Edition_LTSPICE/chapter 4/Chapter%204%20BJTs%20web%20version.html
- 4) https://freebiesupply.com/logos/ubc-logo/