System modelling

Let’s start with a hydrophone’s voltage response which can be expressed as a function of the response from the desired wave hitting the hydrophone plus noise . where and are the responses for each hydrophone and is a time delay that depends on the angle from which the sound is arriving . =Our techniques can be illustrated using 3 inputs for 3 hydrophones, .

Since the response is the result of one wave hitting three hydrophones, and can be expressed as phase shifted versions of one base signal . The phase shifts depend on the angle which the sound is coming from , so. This system becomes easier to resolve out once we take the Fourier transform of the signals and convert the time delays into phase shifts so , . For our purposes, we will approximate out or ignore noise to varying degrees. In vector notation, we have . is simply a processed version of our hydrophone measurements so it is assumed to be known. is what our signal should ideally look like and it must be calculated from geometry TALK ABOUT r APPROXIMATOINS AND SHOW TO TO CALCULATE FROM GEOMETRY. All of our localization techniques hinge on finding a value such that is as close as possible to

NLS, nonlinear least squares

This is a pretty simple method where we just ignore noise(DO WE?), and try to minimize the difference between and . This can be accomplished by minimizing the square error of our estimate so

MUSIC method

Let’s start with a hydrophone’s voltage response which can be expressed as a function of the response from the desired wave hitting the hydrophone plus noise . where and are the responses for each hydrophone and is a time delay that depends on the angle from which the sound is arriving . The MUSIC technique can be illustrated using 3 inputs for 3 hydrophones, .

Since the response is the result of one wave hitting three hydrophones, and can be expressed as phase shifted versions of one base signal . The phase shifts depend on the angle which the sound is coming from , so. This system becomes easier to resolve out once we take the Fourier transform of the signals and convert the time delays into phase shifts so , . In vector notation we can say that . For the sake of ease, we will assume that the noise is uniform across all hydrophones . We need to find the autocorrelation matrix of our signal vector which is the expected value of our vector multiplied by its hermatian transpose . We will denote the autocorrelation matrices as and

We will start by taking the outer product of both sides with their haematin transposes.

When taking the outer product of a matrix of complex numbers, we have to take the conjugate of complex numbers.

is fully determined because it is based on a set of measurements, is also fully determined and is based on calculations from geometry. We want to find such that holds as true as possible, this can be done using what is called a subspace technique, analyzing the subspaces of these matrices through eigenvalue-eigenvector decomposition.

When we take the outer product of a 1XN vector with itself, we get back a matrix which can be fully defined by one nonzero eigenvalue with an eigenvector that is parallel to the vector, and N-1 zero eigenvalues with eigenvectors that are perpendicular to the vector . , so based on the assumption that noise does not vary between the different hydrophones, we can say that has no effect on the direction of the total vector and thus .

Let’s call the eigenvalues and eigenvectors of as and respectively. Recall that is a highly processed form of our hydrophone measurements and is fully known, so its eigenvalues and eigenvectors are can be found. Also recall that is a known vector that is calculated from geometry. and . If we want this to be true, then we must satisfy the condition that

. The way it is generally stated in literature is that and we want .