Note for g.s. fit

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1 Choice of fit method

1.1 Fit function

In the two-state fit method, we did joint fit with three parts: local matrix element $C(z=0,t)$, real and imag part of ratio $\frac{C(z,t)}{C(z=0,t)}$.

1. For local part
$$C_2(z=0,t) = ce^{-E_0 t}\left(1 + a_1 e^{-\Delta E t}\right)$$
2. For real/imag part of ratio
$$\frac{C_2(z,t)}{C_2(z=0,t)} = \frac{\phi_2\left(1 + b_{1(re/im)} e^{-\Delta E t}\right)}{1 + a_1 e^{-\Delta E t}}$$

In the 1-state fit method, we fit real/imag part of ratio directly,

$$\frac{C_2(z,t)}{C_2(z=0,t)} = \phi_{2(re/im)}$$

1-state fit v.s. two-state fit, which shoule we choose in the analysis of two point data? Two figures are helpful before we set about doing fits:

1. Effective mass plot of local matrix element.
$$\ln\left(\frac{C_2(z=0,t)}{C_2(z=0,t+1)}\right) = E_0 + \ln(1 + a_1 e^{-\Delta E t}) - \ln(1 + a_1 e^{-\Delta E t - \Delta E})$$
Therefore, if the effective mass plot is horizontal without decay behavior through t-axis, it is impossible to extract excited state contamination successfully.
2. Effective mass plot of non-local matrix element.

Only when both plots above show the decay behavior, two-state fit is suggested to be tried. Both the coefficient and the effective energy of excited states contamination should be fitted to non-zero results, so that we can claim the two-state fit is successful.

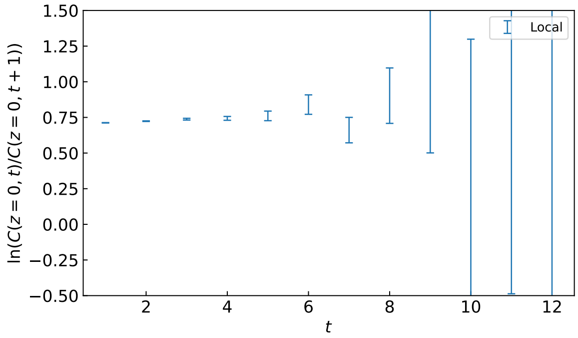
1.2 Example 1

Neither of two plots of effective mass in the Figure.1 shows the exponential decay behavior at small t region, so the 1-state fit method is suggested.

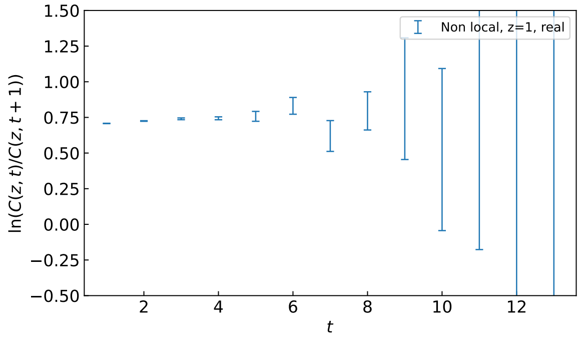
This is the output of two-state fit,

$$\begin{aligned}
&\text{Least Square Fit:}\\
&\text{chi2/dof [dof]} = 0.91[18] \quad Q = 0.57 \quad \log GBF = 96.901\\
&\text{Parameters:}\\
&g.s._{re} \quad 0.8964(25)\\
&a1 \quad -2.6(6.5)\\
&b1_{re} \quad -1.6(4.1)\\
&dE1 \quad 1.30(73)\\
&E0 \quad 0.7442(82)
\end{aligned} \tag{1}$$

From the output it can be found that the fit result of excited state's coefficient b_1 covers zero within the error, which means the two-state fit failed to extract the excited state contamination.



(a) Local matrix element



(b) Non-local matrix element

Figure 1: Effective mass

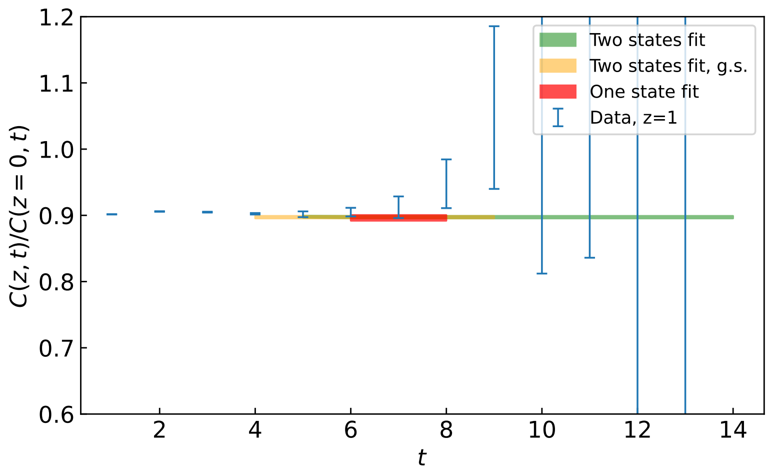
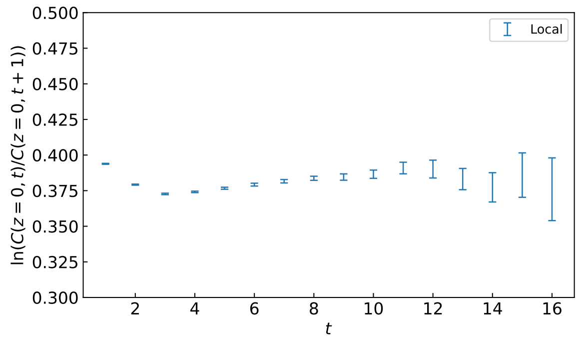
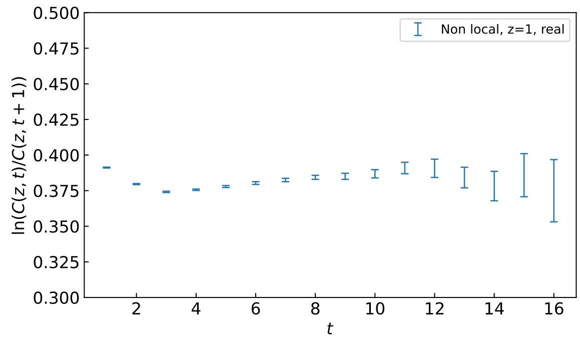


Figure 2: Fit result comparison



(a) Local matrix element



(b) Non-local matrix element

Figure 3: Effective mass

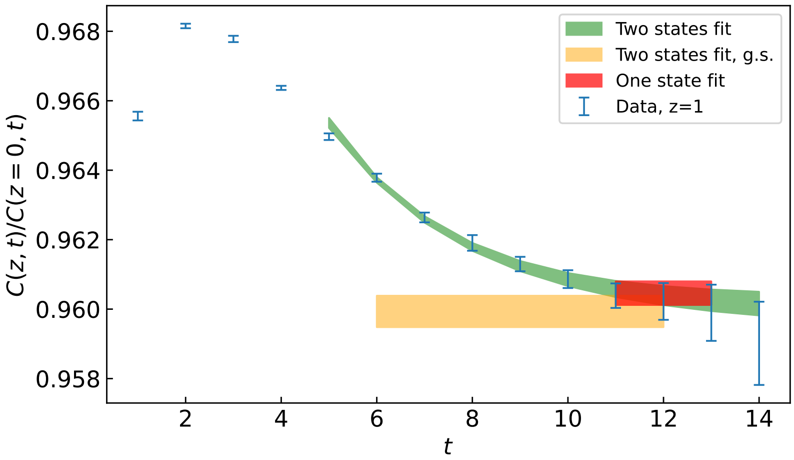


Figure 4: Fit result comparison

1.3 Example 2

Both of two plots of effective mass in the Figure.3 shows the exponential decay behavior at small t region, so the two-state fit is worthy to try.

This is the output of two-state fit,

$$\begin{aligned}
&\text{Least Square Fit:}\\
&\text{chi2/dof [dof]} \ = 0.2[21] \quad Q = 1 \quad \log GBF = 194.25\\
&\text{Parameters:}\\
&g.s._{re} \quad\ 0.95954(56)\\
&a1 \quad\ -0.231(41)\\
&b1_{re} \quad\ -0.206(40)\\
&dE1 \quad 0.294(53)\\
&E0 \quad 0.3899(20)
\end{aligned} \tag{2}$$

From the output it can be found that the fit result of excited state's energy ΔE and coefficient b_1 does not cover zero within the error, which means the two-state fit succeeded to extract the excited state contamination.

2 Factors affecting fit method

2.1 Source for generating data

In generating the two point function data $C_2(z,t)$, there're two sources, wall source and point source.

For wall source, the definition can be written

$$S_{wall}(x,t,\vec{P}_s) = \sum_{\vec{x}_0} e^{-i\vec{P}_s\vec{x}_0}\delta^3(\vec{x}-\vec{x}_0)$$

In the sum of space variable x_0, the excited states offset each other, causing it hard to determine the excited energy ΔE in local $C_2(z=0,t)$

$$C_2(z=0,t) = ce^{-E_0 t}(1 + a_1 e^{-\Delta E t})$$

For point source, the definition turns to

$$S_{point}(x,t,\vec{P}_s) = e^{-i\vec{P}_s\vec{x}_0}\delta^3(\vec{x}-\vec{x}_0)$$

there's not this effect, thus it is easier to determine the excited energy ΔE in local $C_2(z=0,t)$.

2.2 Hadron mass

If the excited energy is very close to ground state, the meson mass of $C_2(z=0,t)$, obviously, the two-state fit can not work well.

$$C_2(z=0,t) = ce^{-E_0 t}(1 + a_1 e^{-\Delta E t})$$

In another word, the first term in the right side hardly dominate when $\Delta E \sim 0$.

We have investigate three cases $\pi(130)$, $\eta_s(670)$, $J/\psi(3000)$ in the same lattice approach, the results are as follows in FIG. (xx).

Smaller excited state energy should be more suitable for two-state fit, coz the e.s. effects would not be invisible compared with the g.s.

2.3 Hadron momentum

With the same consideration, the hadron momentum also affect two-state fit. Because of the relative size for ground state energy and excited state energy.

Though big momentum will lead to bigger ratio $\frac{E_{g.s.}}{E_{e.s.}}$, or say smaller relative energy gap, but it also means much more significant suppression on $e^{-\Delta E t}$, so we should compare 1 with $e^{-\Delta E t}$ to say whether there is an obvious t-dependence to do two-state fit.

For pion two point function at the momentum $P^z = \{0, 6, 10\} \times \frac{2\pi}{L}$, the following figure shows the two-state fit results.