We have

$$\mathscr{R}_N^{(l)}(t) := \frac{\sqrt{\mathscr{V}_N^{(l)(t)}}}{\left|\mathscr{G}_N^{(l)}(t)\right|}, \quad 1 = 1, 2, 3, 4, 5 \qquad (4.8)$$

Let us analyze the infected compartment only for now:

For R_N(t) to blow up we need the denominator to be zero.

We know

$$\mathscr{G}_N^2(t) = -(\gamma_{IH} + \gamma_{IR} + \gamma_{ID})\boldsymbol{i}_N(t) + \beta_{SI}\boldsymbol{s}_N(t)\boldsymbol{i}_N(t),$$

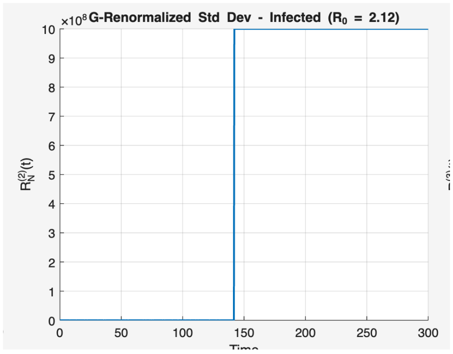
It is easy to see that this becomes 0 when

either i_N(t)=0

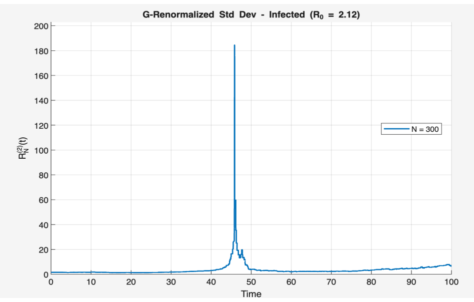
OR

s_N(t) = (p_IH+p_IR+p_ID) gamma/beta

Here is our plot for infected (for N=300) from our old code:



If we only look till T=100

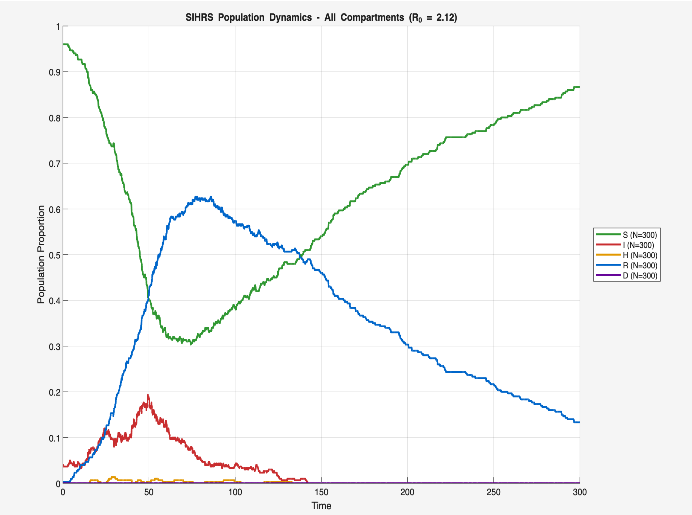


We know for our old code with params as

```
params.beta = 0.212;          % Infection rate (β > 0) - matching SIHRS.m
    params.pSI = 1.0;             % Infection probability (S to I)
    params.pII = 0.0;            % probability of I to I (stay infected)
    params.pIH = 0.04;            % probability of I to H
    params.pIR = 0.959;           % probability of I to R
    params.pID = 0.001;           % probability of I to D
    params.pHH = 0.01;            % probability of H to H (stay hospitalized)
    params.pHR = 0.9882;          % probability of H to R
    params.pHD = 0.0018;          % probability of H to D
    params.pRR = 0.02;            % probability of R to R (stay recovered)
    params.pRS = 0.98;            % probability of R to S
    params.gamma = 0.1;           % Infection transition rate (γ > 0)
    params.alpha = 0.1;           % Hospitalized transition rate (α > 0)
    params.lambda = 0.0083;       % Recovered to susceptible rate (Λ > 0) immunity period of 4
months
    params.T = 1000;              % Total simulation time
    params.dt = 0.01;             % Time step for interpolation
    params.N_values = [300, 1600,3000]; % Population sizes - matching SIHRS.m
    params.initial_s = 0.96;      % Initial susceptible fraction
    params.initial_i = 0.04;      % Initial infected fraction
    params.initial_h = 0;         % Initial hospitalized fraction
    params.initial_r = 0;         % Initial recovered fraction
    params.initial_d = 0;         % Initial dead fraction
    params.n_runs = 5;            % Number of stochastic runs
    params.colors = {'#0072BD','#77AC30', '#A2142F'}; % Colors matching SIHRS.m
```

(p_IH+p_IR+p_ID) gamma/beta is 0.471698113208.

Here is an IPC plot from the same parameters and we can clearly see that there is atleast one time around (t=50) where s_N(t) is ***around*** (p_IH+p_IR+p_ID) gamma/beta (=0.47169811320)



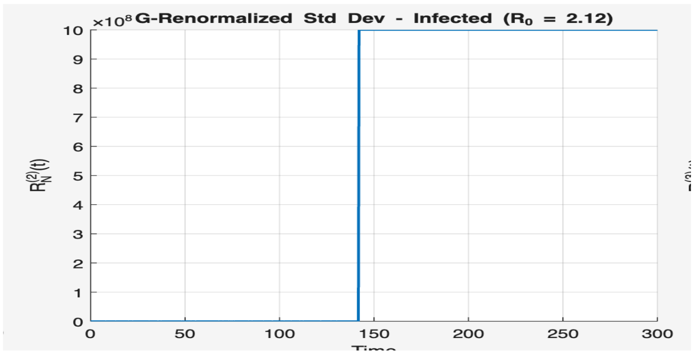
So according to our analysis

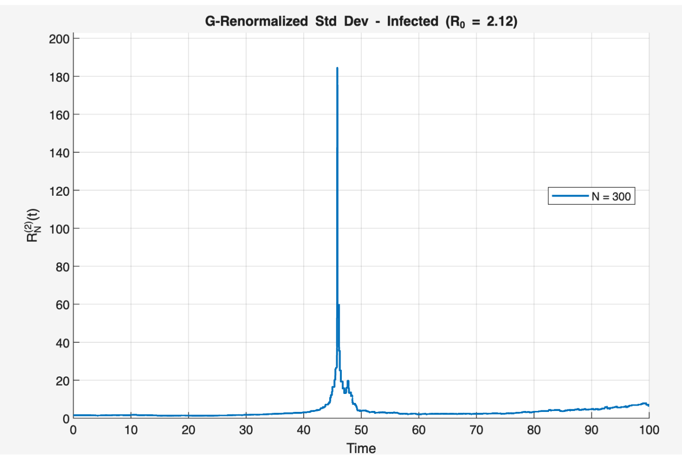
$$\mathscr{G}_N^2(t) = -(\gamma_{IH}+\gamma_{IR}+\gamma_{ID})\boldsymbol{i}_N(t)+\beta_{SI}\boldsymbol{s}_N(t)\boldsymbol{i}_N(t),$$

Should be 0 around t=50 and we should have a blow out around there too.

BUT

When we analyze the plot





We see that we don't see a blow out at around (t=50) instead we only see the blow out time when the disease dies out.

The problem is

```
For N = 300, s_N(t) = 0.471698113:
- Requires k = 141.5094 individuals
- But k must be integer!
- s_N(t) jumps: 142/300 = 0.4733... → 141/300 = 0.4700...
- s_critical = 0.471698 is "skipped over"
```

0.100346667

```
So what we do is

Engineer parameters so that s_critical= k/N for integer k, guaranteeing the
stochastic simulation can hit the critical value exactly.
```

Given: s_critical = γ(p_IH + p_IR + p_ID)/β
Want: s_critical = k/N for some integer k
Solution: Adjust γ = β × k/(N × (p_IH + p_IR + p_ID))

So we change gamma from 0.1 to 0.100346667

Original Parameters:
- β = 0.212, γ = 0.100000
- s_critical = 0.471698113 (impossible to hit exactly)

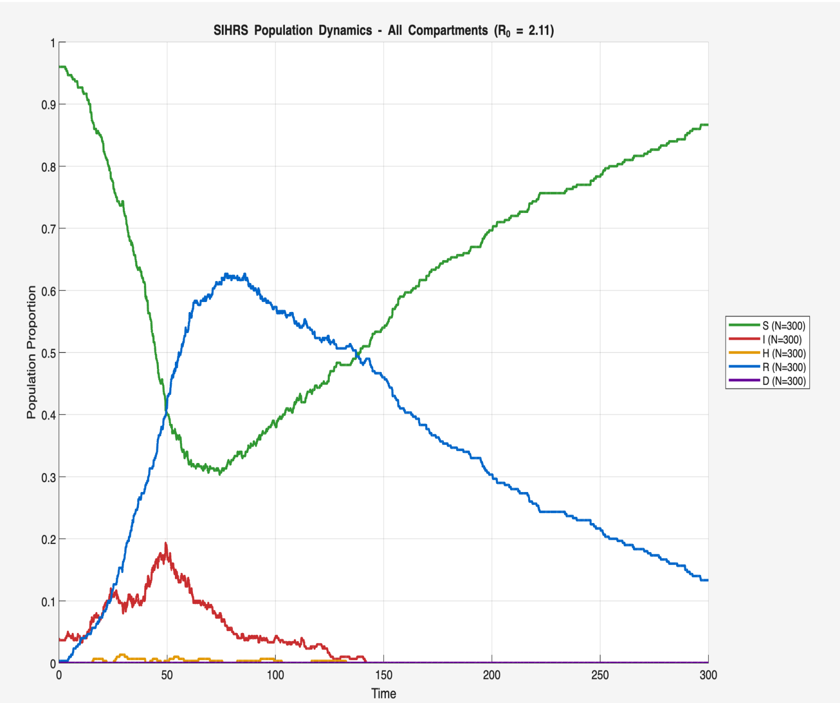
Engineered Parameters:
- β = 0.212 (unchanged)
- γ = 0.100346667 (adjusted)
- s_critical = 142/300 = 0.473333333 (exactly achievable!)

Impact on Model
- R_0 changes minimally: 2.120 → 2.113
- All other dynamics essentially unchanged
- Now guarantees true mathematical blow-up when S = 142/300

Now let us check the plots:

First let us check the IPC plots to see that we actually are guruanteed to get atleast one time where we get s_N(t) as 0.473333333 and occurs before the die out time.

And analyzing the IPC plot it does seem like it.



Finally here is our plot for the infected compartment

And as expected we got at-least 2 blowouts.