Agent-based modelling

A NOTE ON REPLICATOR DYNAMICS

(from Wiki : "…the replicator equation is a deterministic monotone non-linear and non-innovative game dynamic used in evolutionary game theory...Unlike the quasispecies equation, the replicator equation does not incorporate mutation and so is not able to innovate new types or pure strategies. ")

Examples are the prisoner's dilemma game, buyers/sellers in economics or pigs/piglets in animal behaviourism.		Agent X	
		P	Q
Agent Y	P	A_{11}, B_{11}	A_{12}, B_{12}
	Q	A_{21}, B_{21}	A_{22}, B_{22}

Each agent has two strategies to follow, P or Q. Since P and Q represent fractions of the population (number of agents X), we can say that number of agents X playing P is equal to (1 – Q) agents X. The same counts for agent Y. Payoff matrix A belongs to agent Y and payoff matrix B to agent X.

A number of agents X and Y is constant just the fractions P and Q change.

If we let the game run, agents need to somehow conclude what to play in each turn. An interesting algorithm that leads us to an equilibrium between the best play of each agent is replicator dynamics. It turns out that species converge to some behaviours via adaptation, not intelligence. Replicator dynamics is a basic case of learning in agents.

SIMPLIFIED EQUATION:

The idea is to increase the number of agents that perform better at the other strategy expense. So, for example, as the number of agents X playing P increases because of the better payoff, the number of agents X playing Q decreases.

If we write it down in an equation, we get:

$$X_P\,(t+1) = X_P(t)\,(1 + f_{XP})$$

$$\dot{X}_P - X_P\, f_{XP} = 0$$

f_{XP} isn't constant!

- $X_P(t+1)$ is the number of agents X playing P in the next generation (next loop pass/tick)
- $X_P(t)$ current number playing P
- f_P a function telling us if the generation is going to decrease or increase

Obviously, f_P must depend on the payoff matrix for this agent (matrix A if agent X and matrix B if agent Y) and the number of agents playing against that particular strategy. For example, if we want to know how agent X with strategy P performs, we have to consider how agent Y plays against agent's P strategy, in both cases. When we write it down as:

$$f_{XP} = Y_P * B_{11} + Y_Q * B_{21}$$

- Y_P is a fraction of agents Y playing P
- Y_P is a fraction of agents Y playing Q

Similarly, for other strategy Q for the same agent X we get:

$$X_Q\,(t+1) = X_Q(t)\,(1 + f_{XQ})$$

$$f_{XQ} = Y_P * B_{12} + Y_Q * B_{22}$$

And for agent Y we have:

$$Y_P\,(t+1) = Y_P(t)\,(1 + f_{YP})$$

$$f_{YP} = X_P * A_{11} + X_Q * A_{12}$$

$$Y_Q\,(t+1) = Y_Q(t)\,(1 + f_{YQ})$$

$$f_{YQ} = X_P * A_{21} + X_Q * A_{22}$$

In the 'replicator.nlogo' this is a commented 'go' function with a payoff matrix that isn't normalized (divided by the maximum number in the matrix).To keep the number of agents constant normalization of the total population is done on the end of the calculation of the number of agents playing each of the strategies.

In the beginning, we wipe out the whole generation (all four strategies) and build a new one based on these calculations, for both agents.

ORIGINAL EQUATION (involving average performance as well):

Throughout the literature, you will find a replicator equation in another form, where the difference between the number of agents playing a particular strategy is represented by the derivative. Another 'feature' is added in calculating the next generation for each of the strategies. That is a modification of the growth/decline factor. For example, in the first equation, we can consider agent X playing P to be dependent not only on how he performs against agent Y playing P and Q, but also on how agent X playing Q influences the result.

So, agents have an average performance factor in each generation:

$$f_{X\_average} = X_P * f_{XP} + X_Q * f_{XQ} = X_P * f_{XP} + (1 - X_P) * f_{XQ}$$

$$f_{Y\_average} = Y_P * f_{YP} + Y_Q * f_{YQ} = Y_P * f_{YP} + (1 - Y_P) * f_{YQ}$$

Therefore growth/decline factor for agent X playing P is modified:

$$X_P\ (t+1) = X_P(t)\ (1 + f_{XP} - f_{X\_average})$$

Be aware that payoff matrix should be normalized *. The rest of the equation are:

$$X_Q\ (t+1) = X_Q(t)\ \left(1 + f_{XQ} - f_{X_{average}}\right)$$

$$Y_P\ (t+1) = Y_P(t)\ \left(1 + f_{YP} - f_{Y\_average}\right)$$

$$Y_Q\ (t+1) = Y_Q(t)\ (1 + f_{YQ} - f_{Y\_average})$$

Each of the equation gives a fraction, so multiplication with the total number of agents X or Y is necessary. The total number of agents X or Y is constant.

* For the one who want to be thorough, in the bachelor thesis there are some notes on nonnegativity condition for payoff matrix, please check it out because I'm not sure.

Sources:

http://jmvidal.cse.sc.edu/papers/mas.pdf

https://uu.diva-portal.org/smash/get/diva2:824755/FULLTEXT01.pdf