AE3524 Assignment

Orbit Simulation for Formation Flying

Grgur Dujmović

Introduction

To start the assignment, the given values are determined. There are two satellites a simulation has to be created for. The following are the given/calculated specifications for each (calculated values are in **bold**):

Satellite 1:

- Size:
  - x – 300mm – 0.3m
  - y – 300mm – 0.3m
  - z – 300mm – 0.3m
- Mass – 30kg
- Orbit height – 500km
- Orbit circular and sun-synchronous, hence inclination ≈ 98^o and orbit eccentricity = 0
- **Velocity** ≈ 7.61268km/s
- **Orbital radius (**semi-major axis**)** ≈ 6878km
- **Orbital period** ≈ 1h34m37s

Satellite 2:

- Size:
  - x – 300mm – 0.3m
  - y – 300mm – 0.3m
  - z – 300mm – 0.3m
- Mass – 30kg
- **Velocity ≈** 7.61368km/s
- **Orbit height ≈** 498.19km
- Orbit circular and sun-synchronous, hence inclination ≈ 98^o and orbit eccentricity = 0
- **Orbital radius (**semi-major axis**)** ≈ 6876km
- **Orbital period** ≈ 1h34m35s

Task 1. Two satellite decaying orbit simulation

To perform this task, a python script was developed to simulate the decaying orbits of the satellites. To calculate the rate of change of orbital altitude, a simplified decay model is used:

$$\frac{dr}{dt} = \frac{\alpha_0(r)\ \times T(r)}{\pi}$$

r – distance of satellite to Earth centre

$\alpha_0(r)$ – sum of accelerations acting on the satellite as a function of r (in this model only atmospheric drag is considered)

T(r) – period of the satellite as a function of r.

To calculate α_0, the following equation is used:[1]

$$\alpha_0 = \frac{\rho(r)v^2c_d}{2} \times \frac{A}{m}$$

$\rho(r)$ – atmosphere density at r distance from origin

[1] Low, Samuel Y. W. (August 2018). "Assessment of Orbit Maintenance Strategies for Small Satellites". AIAA/USU Conference on Small Satellites.

v – orbital velocity

c_d – drag coefficient

The atmospheric density model was adapted from Braeunig[2].

Values that remain constant during the orbit are:

- **i -** the inclination of the orbit – since the only force except gravity acting on the satellites is drag, the shape of the orbital plane is constant, therefore so is the inclination of it
- **Ω** – the longitude of the ascending node – the orbital plane is constant, so is Ω
- **ω –** the argument of periapsis – the orientation of the orbital plane is constant and the orbit is circular, hence it does not have a periapsis

The orbit is circular, hence the **eccentricity** of it is 0, and as it decays it turns spiral, not having an eccentricity.

Values that are tracked during the orbit are:

- **a** – semi-major axis – this is expected to decrease as the orbit progresses and will be tracked using the altitude of the satellite
- **M** – mean anomaly – this is expected to increase linearly as the orbit progresses with a limit from -π to π radians
- **v** – velocity of the satellite – this is expected to increase as **a** decreases
- **D** – distance between satellites – the distance between satellites is expected to increase and then decrease because the satellite with the lower altitude will always be faster

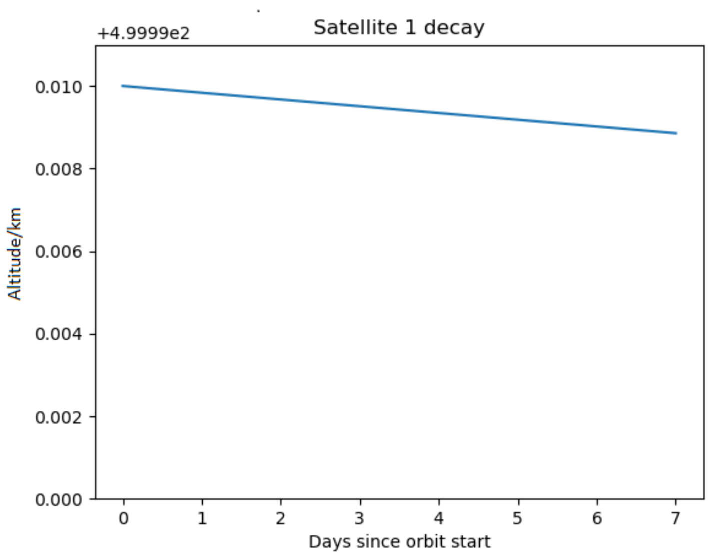

Figure 1

The decaying orbit was simulated for 1 week with a drag coefficient for a cube ($c_d = 0.8$ [3]). The altitude changes, shown in Figure 1 and 2, are minimal, with a change of 1.15m for satellite 1 and 1.20m for satellite 2. However, as the satellites decay, the atmosphere will become denser, resulting in higher drag hence more altitude loss. Estimated time for re-entry (the satellite dropping below 120km [4]) for satellite 1 is approximately 425 years and for satellite 2 it is 400 years.

To calculate the mean anomaly of the satellites, first the mean angular motion is calculated:

$$n = \frac{2\pi}{T}$$

where T is the orbital period. Then, for each time step, the change in mean anomaly is calculated:

[2] "Properties Of Standard Atmosphere". Braeunig.us, 2022, http://www.braeunig.us/space/atmos.htm.

[3] "Drag Coefficient". Engineeringtoolbox.Com, 2004, https://www.engineeringtoolbox.com/drag-coefficient-d_627.html.

[4] Reentry And Collision Avoidance. 2022, https://www.esa.int/Space_Safety/Space_Debris/Reentry_and_collision_avoidance.

$$M = M_0 + \pi + (n * ts)$$

where ts is the timestep of the simulation.

Figures 3 and 4 don't convey much information because a span of a week is a long time compared to the orbital period of around 1.5h but reducing the time frame on Figures 5 and 6 the orbital period can be read out. It is quite similar for both satellites and will only start to deviate later in the orbit.

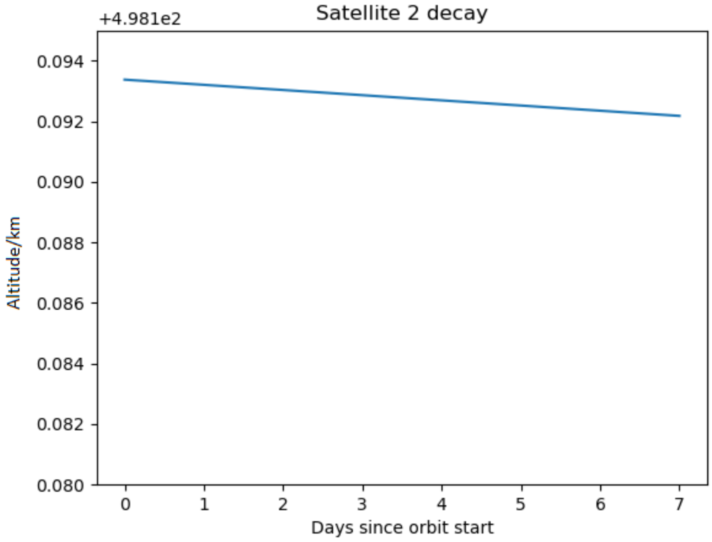

Figure 4

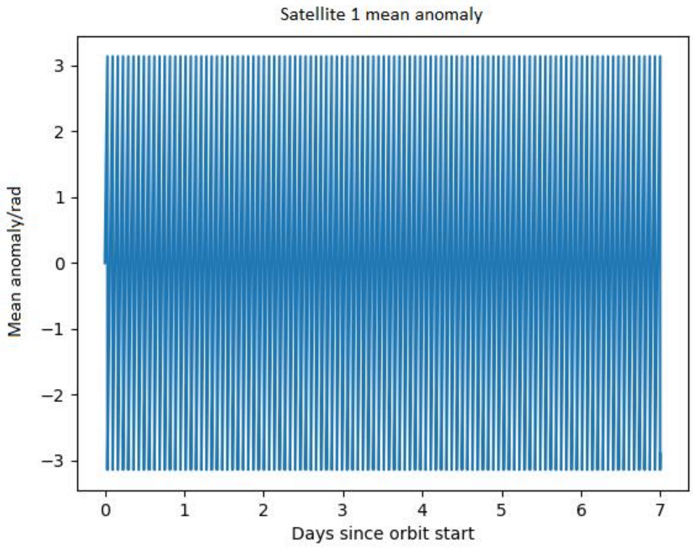

Figure 3

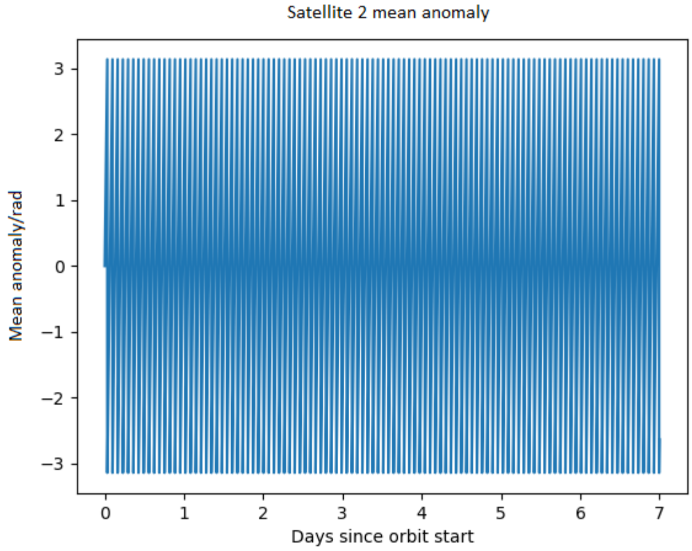

Figure 2

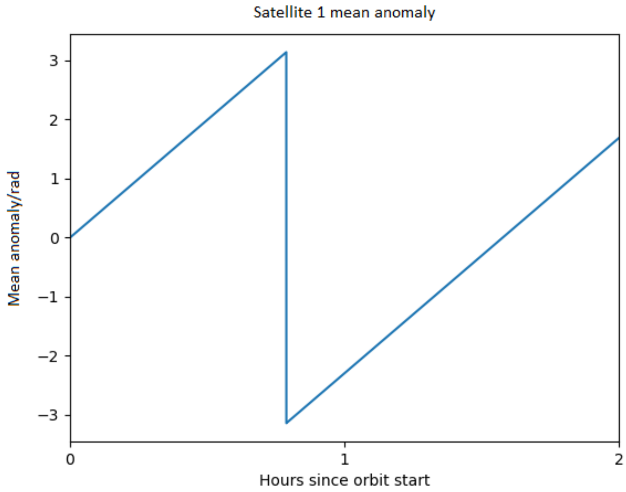

Figure 6

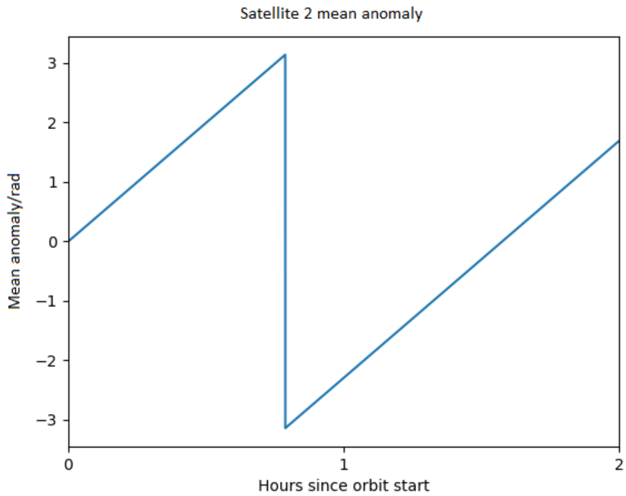

Figure 5

In addition to calculating mean anomaly and altitude changes, the change in distance between the satellites and velocities is also calculated. Velocity was calculated using the altitude of the satellite:

$$v = \sqrt{\frac{\mu}{a}}$$

where μ is the Earth's gravitational parameter, equal to $G \times M_{earth}$. The distance between satellites was calculated using a combination of the distance formula and the formula for the length of a circle chord:

$$l = 2 \times a \times \sin\left(\frac{\theta}{2}\right)$$

where θ is the difference in mean anomaly of the satellites to effectively calculate the angle between the satellites. Figure 7 shows how the formula for the distance was derived. To find the distance between the satellites, the distance formula is used:

$$distance = \sqrt{ad^2 + l^2}$$

where ad is the difference between the semi-major axes.

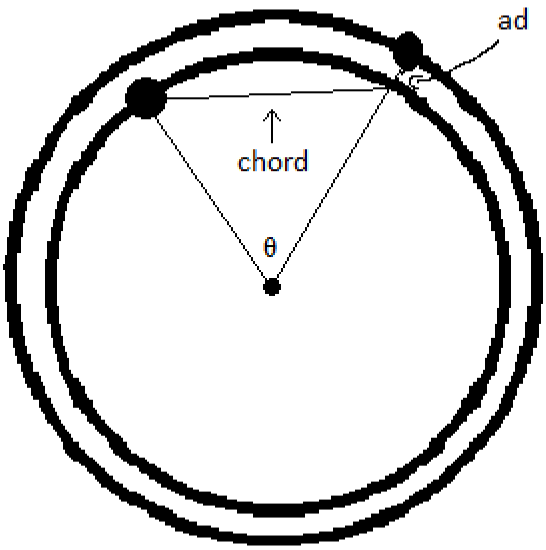

Figure 9

Figure 8 shows the change in distance over a period of a week, and figure 9 shows the change in distance over a year. Figure 9 displays how satellite 2, which is faster (check figure 10 and 11) oscillates the distance between it and satellite 1, effectively lapping it because of the minimal difference in velocity. The maximum distance is just under 1400km, which is in line with the diameter of Earth + altitudes of both satellites.

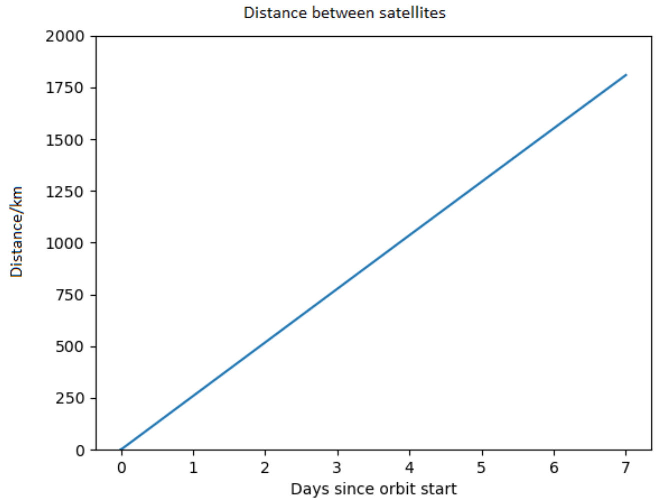

Figure 7

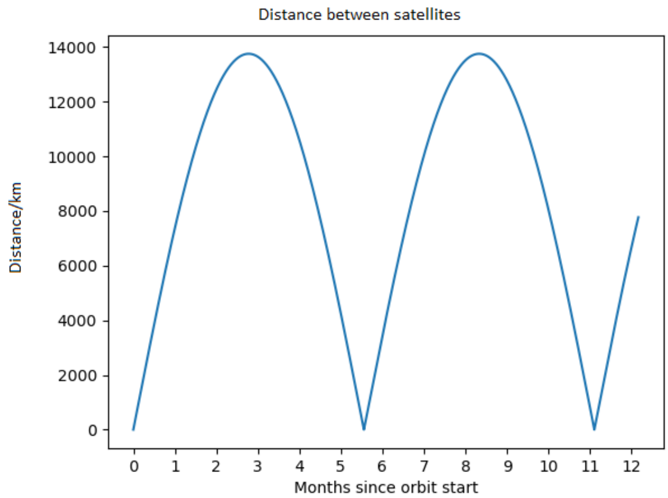

Figure 8

Lastly, the velocities of the satellites are shown in figures 10 and 11.

The initial velocity of satellite 1 is ≈ 7.6127km/s. It increased by 6.331×10^{-7}km/s. The initial velocity of satellite 2 is ≈ 7.6137. It increased by 6.627×10^{-7}km/s. It is evident that the increases of velocity per week are negligible, amounting to less than centimetres per second difference on velocities that are measured in kilometres per second. However, over a longer time period, the changes will be larger.

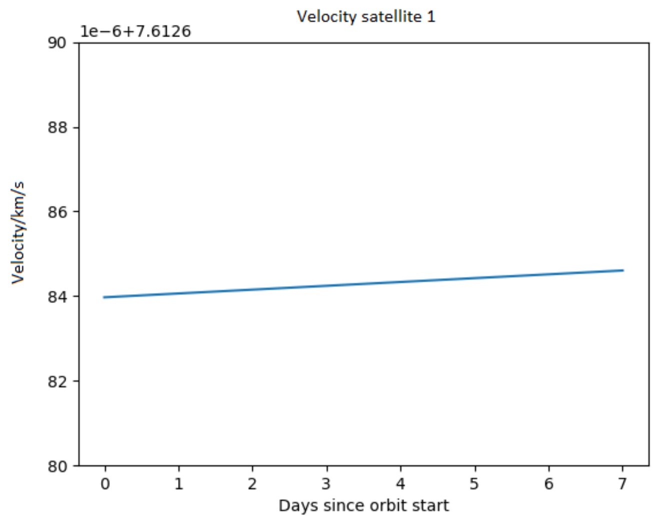

Figure 10

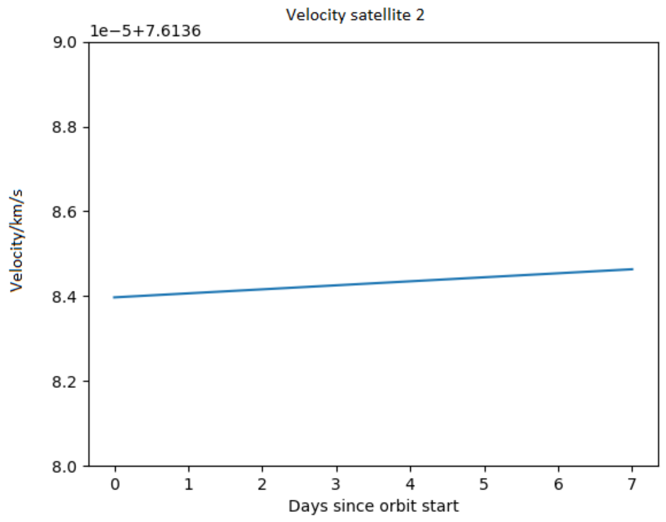

Figure 11

Task 2. Delta-v manoeuvres for along-track formation[5]

To achieve an along-track formation of the satellites, a coplanar rendezvous with a 100km offset is performed. Because the changes in altitude and velocity over 2 days for both satellites are within a meter and a meter per second respectively, it will not be taken into account.

Firstly, find the angular velocity of both of the satellites with:

$$\omega = \sqrt{\frac{\mu}{a^3}}$$

where ω is the angular velocity. The time of flight in the transfer orbit is also needed, to determine when satellite 2 should start the transfer:

$$T_{flight} = \pi \sqrt{\frac{a_{transfer}^3}{\mu}}$$

where $a_{transfer}$ is the semi-major axis of the transfer orbit. To get the timing right, satellite 2 will have to start the Hohmann transfer burn with a certain lead angle (α_{lead}):

$$\alpha_{lead} = \omega_{target} \times T_{flight}$$

This gives us the value to rendezvous with satellite 1, but satellite 2 has to be 100km behind it. To calculate how much α_{lead} has to be reduced, the circle chord formula will be reused:

$$l = 2 \times a \times \sin\left(\frac{\theta}{2}\right)$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{l}{2a}$$

$$\theta = 2\arcsin\left(\frac{l}{2a}\right)$$

Where θ is the angle difference between the satellites, l is the distance and a is semi-major axis. The angle α_{lead} has to be reduced by θ in order to create the needed separation.

During a Hohmann transfer, the satellite travels π radians, so calculating the final required phase angle is:

$$\phi_{final} = \pi - \alpha_{lead}$$

To determine how long satellite 2 will have to wait before starting the burn, we have to calculate the relation with the ϕ_{final} and the satellites angular velocities ω_{target} and $\omega_{original}$. Initial phase angle equals 0, hence it is omitted from the relation.

$$\phi_{final} = (\omega_{sat1} - \omega_{sat2}) \times t_{wait}$$

$$t_{wait} = \frac{\phi_{final}}{\omega_{sat1} - \omega_{sat2}}$$

For the calculations, the following values are required:

- $v_{original}$ – the velocity of the original orbit = 7.6137km/s

[5] Maneuvering In Space. USA Government, pp. 22-27, https://www.faa.gov/about/office_org/headquarters_offices/avs/offices/aam/cami/library/online_libraries/aerospace_medicine/tutorial/media/III.4.1.5_Maneuvering_in_Space.pdf.

- v_{target} – the velocity of the target orbit = 7.6127km/s
- $a_{original}$ – semi-major axis of the original orbit = 6876.19
- a_{target} – semi-major axis of the target orbit = 6878
- $a_{transfer}$ and $e_{transfer}$ – the semi-major axis and the eccentricity of the transfer orbit
  - a_t = 6877.095
  - e_t = 0.000132

For the transfer:

$$T_{flight} = \pi\sqrt{\frac{a_{transfer}^3}{\mu}} \approx 2838s = 47.2min$$

Then, the total delta-V is calculated by calculating delta-V needed for two manoeuvres.

$$\Delta V_1 = \sqrt{\mu\left(\frac{2}{a_{original}} - \frac{1}{a_t}\right)} - v_{original}$$

$$\Delta V_2 = v_{target} - \sqrt{\mu\left(\frac{2}{a_{target}} - \frac{1}{a_t}\right)}$$

$$\Delta V_{total} = |\Delta V_1| + |\Delta V_2|$$

$\Delta V_1 \approx 0.000500$km/s and $\Delta V_2 \approx 0.000500$km/s, making ΔVtotal ≈ 0.001 km/s. This is in line with the initial velocity difference given in the assignment. Next, the angular velocities are calculated:

$$\omega_{sat1} = \sqrt{\frac{\mu}{a_{target}^3}} \approx 0.00110682\frac{rad}{s}$$

$$\omega_{sat2} = \sqrt{\frac{\mu}{a_{original}^3}} \approx 0.00110725\frac{rad}{s}$$

With $\omega_{target} = \omega_{sat1}$, the lead angle is:

$$\alpha_{lead} = \omega_{target} \times T_{flight} \approx 3.1411552rad$$

The lead angle is almost equal to π. This occurs because during a Hohmann transfer, the spacecraft goes through a π angle, but as satellite 1 and 2 are not in the same orbit, the lead angle is a bit less than π so they rendezvous at the same point. But the satellites have to be at 100km separation, so for the final phase angle, θ is required:

$$\theta = 2\arcsin\left(\frac{l}{2a}\right) = 2\arcsin\left(\frac{100}{2\times\ 6878}\right) = 0.0145392rad$$

$$\phi_{final} = \pi - (\alpha_{lead} - \theta) = 0.0149767\text{rad}$$

Following with the wait time:

$$t_{wait} = \frac{\phi_{final}}{\omega_{sat1} - \omega_{sat2}} = 34830s = 580.5min = 9.675h$$

The condition was to establish a flying formation with a 100km distance withing 2 days. Considering the time it would take to complete the coplanar rendezvous with a 100km offset ($t_{wait} + T_{flight}$ = 34830s + 2838s ≈10.46h), the manoeuvre is within the limitations. The strategy to achieve the desired formation is: after a 34830 second wait time satellite 2 performs a $\Delta V_1 \approx 0.000500$km/s burn, and after 2838 seconds, it performs the second $\Delta V_2 \approx 0.000500$km/s burn, establishing the formation.

Task 3. Lengthening the satellite phase difference

After the satellites are in the formation orbit, it is up to satellite 1 to increase the distance by another 900km to achieve the 1000km distance formation. The principle is similar to task 2. The angle between satellite 1 and the required end point is:

$$\theta = 2\arcsin\left(\frac{l}{2a}\right) = 2\arcsin\left(\frac{900}{2\times\ 6878}\right) = 0.130946rad$$

Satellite 1 will slow down go into a lower orbit to gain speed and end up in a location 900km in front of its current location in orbit. The semimajor axis of the lower orbit must be determined:

Firstly, determine the angle the 900km distance point has to travel to reach the point of satellite 1:

$$\phi_{final} = 2\pi - \theta = 6.15224rad$$

This value is used to determine the required flight time of satellite 1 in the lower orbit, so it transfers back into the proper orbit at the 900km point. Knowing the angle it has to go through, to determine ω in the 500km orbit is:

$$\omega_{500} = \sqrt{\frac{\mu}{6878^3}} \approx 0.00110682\frac{rad}{s}$$

Hence:

$$t_{flight} = \frac{\phi_{final}}{\omega_{500}} = 2\pi\sqrt{\frac{a_{transfer}^3}{\mu}}$$

Satellite 1 has to reach the calculated angle in t_{flight}, so it is possible to calculate the semimajor axis of the transfer orbit. Performing basic algebra on the above relation, the resulting formula is:

$$a_{transfer} = \sqrt[3]{\mu(\frac{\phi_{final}}{2\pi\omega_{500}})^2} = 6782.08965km$$

Hence the delta-V is calculated. The first manoeuvre has a negative delta-V because it puts satellite 1 into the transfer orbit, while the second puts it back into the original orbit and has a positive delta-V. Only the first manoeuvre one has to be calculated because the magnitudes of both manoeuvres are equal.

$$\Delta V = \sqrt{\mu\left(\frac{2}{a_{500}} - \frac{1}{a_{transfer}}\right)} - \ v_{500}$$

$$\Delta V_{total} = 2\times\ |\Delta V| = 2\times\ |-0.05402| = 0.10804km/s$$

where a_{500} and v_{500} are the semimajor axis and velocity at a 500km altitude. Calculating transfer time:

$$t_{transfer} = t_{flight} = 2\pi\sqrt{\frac{a_{transfer}^3}{\mu}} = 5558.48s = 92.64min = 1.54h$$

The 2-day time limit is respected as the burn can start as soon as the satellites are in the formation as in Task 2, and the $t_{transfer}$ is approximately an hour and a half. The strategy to achieve the specified along-track formation with 1000km distance is for satellite 1 to start a $\Delta V \approx -0.05402$km/s burn, and after 5558.48 seconds, it starts a $\Delta V \approx 0.05402$km/s burn to get back into the original orbit.

Task 4. Maintenance delta-V

To calculate the delta-V manoeuvre over time needed to maintain the formation, it is required to calculate how much the 2% increase in drag would slow down the trailing satellite. At the start the persistent delta-V would be very small, but as the satellites decayed together, the atmosphere would be denser, and to maintain the formation within 1km, the delta-V would have to increase as the altitude decreased. Otherwise, the trailing satellite would decay faster and break the formation.

Limitations

For the purpose of this assignment, there were multiple things that were not taken into account, simplified.

1. The influence of solar radiance / solar and lunar gravity – the task specified to take "drag" into account and this was interpreted as purely air resistance. The specified effects were ignored as it would require significantly more computation, with determining the exact date of launch, and multiple orbital elements which were determined constant at the start of the report, as their values were not indicated in the assignment.
2. The atmospheric model – it was adapted from the mentioned source, but there are options of making a model more accurate. The creation of an accurate atmospheric model was not the focus of this assignment, therefore the used one was sufficient.
3. The decay of the orbit was ignored – for task 2 and 3, the decay of the orbit was determined to be negligible because it decays less than a meter for the 2-day time limitation of the manoeuvre.
4. The limitations of fuel and engines was not calculated – the calculations taking into account the engine range of 1N – 1kN and the mass of the satellite of 30kg was not mentioned, it was assumed to be enough to perform these quite simple and low intensity manoeuvres.
5. Impulse force was assumed for delta-V manoeuvres – the total delta-V required was 0.001m/s, which was determined to be a plausible amount for an impulse manoeuvre to be performed, and the time required to accelerate and decelerate was not considered.

Evaluation

The simulation and calculations in this task will be representative of the real world, as the assumptions made and factors ignored will not have a significant effect of the satellites in such a short timeframe of a couple of days. However, these calculations are not sufficient to use for an actual space mission, as they are usually planned for long timeframes, hence the slight inaccuracies in the calculations caused by omitting factors will have a significant effect on the outcome.

Bibliography

Low, Samuel Y. W. (August 2018). "Assessment of Orbit Maintenance Strategies for Small Satellites". AIAA/USU Conference on Small Satellites.

"Properties Of Standard Atmosphere". Braeunig.us, 2022, http://www.braeunig.us/space/atmos.htm.

"Drag Coefficient". Engineeringtoolbox.Com, 2004, https://www.engineeringtoolbox.com/drag-coefficient-d_627.html.

Reentry And Collision Avoidance. 2022, https://www.esa.int/Space_Safety/Space_Debris/Reentry_and_collision_avoidance.

Maneuvering In Space. USA Government, pp. 22-27, https://www.faa.gov/about/office_org/headquarters_offices/avs/offices/aam/cami/library/online_libraries/aerospace_medicine/tutorial/media/III.4.1.5_Maneuvering_in_Space.pdf.